Informational Cycles in Search Markets*

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I show in a stationary environment that market participants' equilibrium beliefs can create fluctuations in the volume of trading. I study a sequential search model where buyers face an unknown distribution of offers. Each buyer learns about the distribution by observing whether a randomly chosen buyer traded yesterday. A cyclical equilibrium exists where the informational content of observing a trade fluctuates: a trade is good news about the distribution in every other period and bad news in the remaining periods. This leads to fluctuations in the volume of trading. The cyclical equilibrium can be more efficient than steady-state equilibria.

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1 Introduction

There are many search markets where searchers are uncertain about some market characteristics. For example, a job-seeker may be uncertain about the distribution of pay packages across relevant employers, how many other job-seekers are applying for jobs, and how impatient the others are. Searchers learn about these unknown characteristics from the offers they receive and from additional sources of information. A job-seeker learns from the pay packages of the interviewed employers, but also from the unemployment rate, from advertised wages, and by hearing if a friend has found a job.

In this paper, I show that in a stationary environment learning about unknown market characteristics from other searchers' actions can lead to cycles in the volume of trading. In reality, trade volume fluctuates on labour markets, but also on

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financial, real estate, and other search markets. I focus on one unknown market characteristic, the distribution of offers, and one additional source of information, other searchers' actions. In reality, an investor may not know the distribution of project qualities and learns by hearing if a competitor invested. Hearing that she invested is good news about the distribution if the investor believes that the competitor only invests into high-quality projects. Hearing that she invested can be, conversely, bad news if the investor believes that the competitor also invests into low-quality projects. I show that a cyclical equilibrium exists where a signal indicating that many other searchers traded is good news in every other period and bad news in the remaining periods. The fluctuations in the informational content of this "trade signal" translate into fluctuations in the volume of trading through the searchers' optimal behaviour. I thus show that information generated on a market can lead to endogenous cycles.

This informational explanation of fluctuations is appealing for three reasons. First, the driver of cycles is novel: a signal with an endogenous precision. The signal can generate cycles because its informational content can change over time. A searcher trades only if it is optimal to her. Depending on how a trade is interpreted in equilibrium, her decision to trade can either increase or decrease the probability that other searchers trade in the future. Second, endogenous cycles are delivered within an equilibrium of a parsimonious model. The environment is stationary (especially, lacks aggregate shocks), there are neither spillovers from other sectors nor adverse selection, and buyers are both rational and ex-ante homogeneous. Third, the type of information that I consider is realistic and present in many markets. In the model, a searcher observes if another searcher traded; in reality, a person hears if a friend found a job and if a competitor invested.

In my model a fixed amount of searchers enters the market in each period. I call them "buyers" for concreteness; in the motivating examples they are job-seekers and investors. Each buyer gets a randomly drawn offer in each period and decides whether to trade (i.e., to accept the offer) or to continue to search. Continuing is costly because buyers discount future payoffs. Buyers do not know if they face a good or a bad distribution of offers. The good distribution contains more high-value offers and fewer low-value offers than the bad distribution.[1] The

[1] The offer distribution can be thought of as being exogenously given, as is the distribution of project qualities. Alternatively, it can be the distribution of gains from trade with heterogeneous sellers. The model's results are the same as with an exogenous distribution if a gain is split according to Nash bargaining or if buyers make take-it-or-leave-it offers to sellers. The results are unchanged because my model has no adverse selection; in models with adverse selection, bargaining is complicated (see, e.g., Hörner and Vieille, 2009, and Fuchs et al., 2016).

distribution is fixed throughout the operation of the market. In the full model, a buyer learns about the unknown distribution from her own experience and from a private "trade signal". Her own experience is the offers that she gets. The trade signal reveals if yesterday a randomly drawn buyer traded ("a trade" is observed) or did not ("no trade" is observed).[2] This is the simplest way of modelling the idea that buyers learn from others' actions. The informational content of the trade signal is determined in equilibrium. A trade is good news about the unknown distribution if yesterday many buyers accepted high-value offers and bad news if many buyers accepted low-value offers.

I study stationary symmetric equilibria of the model, where stationarity means that the endogenous variables have the same value every $K \geq 1$ periods. If $K = 1$, the equilibrium is a steady state and if $K > 1$, a cyclical equilibrium. In any equilibrium, all buyers accept high-value offers. The only real decision is, thus, made by buyers who get low-value offers.

The three main results of the paper are as follows. First, I show that a cyclical equilibrium exists and characterise it. To argue that the sole driver of cycles is learning from others' actions, I show that in a benchmark where buyers learn only from their own experience, only steady-state equilibria exist. In the cyclical equilibrium the endogenous variables fluctuate between two values across time. The equilibrium strategy says that in every other period some buyers react to the private trade signal outcome (i.e., accept a low-value offer after observing bad news and continue after observing good news) and in the remaining periods no buyer reacts to the signal outcome.

Buyers' behaviour generates cycles in the volume of trading as follows. For the sake of the argument suppose that the bad distribution contains very few and the good distribution quite many high-value offers. Suppose now that yesterday very few buyers accepted low-value offers (while everyone accepts high-value offers). In this case, yesterday the volume of trading was quite low under both possible distributions. To create a cycle, the volume must be higher today (and again low tomorrow) under both distributions. The trade signal sustains such a cycle because not only does the absolute volume of trading change over time, but also the relative volume under the good distribution versus the bad distribution. I explain this dynamic in more detail.

Recall that yesterday most trades took place with high-value offers. Thus, the trade volume was much higher under the good distribution than the bad distribution because the good distribution contains many more high-value offers. As

[2]In the main model, no buyer observes the value of a past trade/no trade event, but the assumption is not crucial for the model's results. Please see Section 7 for details.

a result, buyers who today observe a trade from yesterday become more optimistic about the offer distribution. Yesterday's trade event is quite informative (i.e., moves buyers' beliefs a lot) because the volume of trading differed considerably under the two distributions. This means that buyers who today observe no trade from yesterday become very pessimistic about the distribution and accept low-value offers. The volume of trading is, thus, higher today than yesterday under each distribution. However, today the volume of trading is higher under the bad than under the good distribution because more buyers observe no-trade events and accept low-value offers under the bad distribution. Thus, a buyer who (tomorrow) observes a trade from today becomes more pessimistic about the distribution. But today's trade event is not very informative because today the volume of trading is similar under the two distributions. This is because today more buyers accept low-value offers under both distributions as compared to yesterday. The low informational content of a trade today means that, even after becoming more pessimistic about the distribution, very few buyers tomorrow (just like yesterday) become pessimistic enough to accept low-value offers.

The model generates a testable prediction about the volume of trading: fluctuations in the volume of trading are greater if the distribution of offers is bad rather than good. The reason is that the bad distribution contains more low-value offers. Their amount matters because information plays a role only for a buyer's decision about a low-value offer. The trade signal coordinates the actions of buyers more effectively within a period if the distribution is bad, which creates larger fluctuations. If we interpret the bad state as a bust and the good state as a boom, the model predicts larger fluctuations in busts than in booms.

The second main result is that the cyclical equilibrium can be more efficient than steady-state equilibria. The cyclical equilibrium coexists with one steady state for some parameter values and another steady state for others. The steady state where buyers do not react to the trade signal is less efficient than the cyclical equilibrium. In the cyclical equilibrium, the trade signal is informative, so reacting to its outcome moves the market closer to the efficient complete-information benchmark. The other steady-state equilibrium, where in all periods some buyers react to the signal outcome (i.e., accept a low-value offer after observing bad news), can also be less efficient than the cyclical equilibrium. This happens if the precision of the trade signal in the cyclical equilibrium is much higher than in this steady state, which compensates for the fact that buyers react less frequently to the signal in the cyclical equilibrium than in the steady state.

Since the cyclical equilibrium is more efficient than a steady state for some parameter values and less efficient for others, the model suggests that from efficiency viewpoint fluctuations on some real-life submarkets should be less worrisome than those on others. The cyclical equilibrium is more efficient than a steady state if buyers are more patient and if the value dispersion is larger. In reality, searchers with higher savings may be more patient. The value dispersion may be larger on a submarket for white-collar as opposed to blue-collar jobs, for riskier as opposed to safer investment projects, and for commercial as opposed to residential real estate. Thus, the model suggests that fluctuations should be more worrisome on submarkets for blue-collar jobs, for safer investment projects, for residential real estate, and where searchers are poorer.

As the third result, I show that a market that has a concrete starting date converges to the cyclical equilibrium for an open set of parameter values. The cyclical equilibrium is, thus, a natural limit that some markets reach rather than a curiosity that can be sustained only in the long run.

Literature. My paper integrates two branches of literature: on explanations to fluctuations within an equilibrium and on learning about an unknown state in search markets. The paper's main contribution is to propose a novel mechanism, information on others' trades, as a driver of fluctuations.

Many different drivers of fluctuations have been suggested earlier.[3] The real business cycle theory proposes exogenous shocks (see Frisch, 1933, and Slutzky, 1937, for the seminal contributions).[4] Other suggestions are spillovers (see, for example, Caplin and Leahy, 1993), boundedly rational agents (see, for example, De Bondt and Thaler, 1985, and Abreu and Brunnermeier, 2003), and ex ante heterogeneous agents (see, for example, Conlisk et al., 1984, Sobel, 1984, and Woodford, 1992). The suggestion most related in spirit to mine is informational: adverse selection.[5] In the adverse selection models, the volume of trading changes over time because the distribution of offers changes endogenously over time. In my model, only the buyers' equilibrium beliefs about the distribution of offers change over time, but the distribution of offers does not change.

Learning about an unknown state in search models has been studied earlier.[6] These models, if they are dynamic and have more than two time periods, focus on

[3]I focus on models where fluctuations occur within an equilibrium, as in my model. In many models with multiple equilibria, agents' different expectations about future payoffs sustain cycles. The most related among these are search models by Diamond and Fudenberg (1989) and Fershtman and Fishman (1992).

[4]The most related paper with exogenous shocks is Zeira (1994), where agents' learning about randomly changing demand generates cycles.

[5]See, for example, Janssen and Karamychev (2002), Janssen and Roy (2004), Daley and Green (2012), Kultti et al. (2015), Fuchs et al. (2016), and Maurin (2017).

[6]See, for example, Benabou and Gertner (1993), Dana (1994), Fishman (1996), Janssen et al. (2011), Lauermann (2012), Janssen and Shelegia (2015), Asriyan et al. (2017), Janssen et al. (2017), Kim (2017), Lauermann et al. (2017), Mauring (2017), and Kaya and Kim (2018).

steady-state equilibria. Asriyan et al. (2017) and Mauring (2017) study learning from a trade signal, and Kim (2017) and Kaya and Kim (2018), from delay.[7] In them, a trade or delay is always either bad or good news about the market conditions because these papers focus on steady states. Conversely, in my model's cyclical equilibrium a trade is bad news in some periods and good news in others.

In Section 2 I introduce the model and the equilibrium concept. Section 3 shows that only steady-state equilibria exist in two benchmark models. Section 4 shows that both steady-state and cyclical equilibria exist in the full model. I compare efficiency of the full model's equilibria in Section 5. Section 6 analyses a market with a concrete starting date. I discuss alternatives to the model's assumptions in Section 7.

2 Model

I first describe the setup of the model and then the equilibrium concept.

2.1 Setup

Time t is discrete and runs till $+\infty$. The market starts at $t_1 = -\infty$ (except in Section 6 where it starts at $t_1 = 1$). The market is characterised by state $\theta \in \{B, G\}$ that is fixed for all periods.

Buyers. In each period t a mass one of buyers enter the market. In the motivating examples they are job-seekers and investors. Each buyer has a unit demand and discounts the future at rate $\delta \in (0, 1)$. A buyer searches sequentially for a good offer. A buyer dies at the end of her second life period. I call the buyers who entered today "young" buyers and who entered yesterday and did not exit "old" buyers. The assumption that buyers are short-lived is not crucial for the model's results.[8] Buyers do not know the state θ and their prior belief is $\pi := P(\theta = G)$.

Offers. The state of the market, θ, determines the distribution of offers that buyers face. If $\theta = B$, I say that the state is bad and if $\theta = G$, that the state is good. In state θ, a fraction μ^θ of offers are high-value offers v_H and $1 - \mu^\theta$ are

[7] Learning from others' exits in strategic experimentation literature has been studied by Murto and Välimäki (2011), Cripps and Thomas (2017), and others. Observational learning literature studies learning from past players' actions in models where one player moves at a time (see Banerjee (1992) and Bikhchandani et al. (1992) for the seminal papers).

[8] In particular, suppose that buyers potentially live for ever, but die in each period with a constant probability δ. There is no further discounting. The main result, a cyclical equilibrium, exists in this version of the model. Please see Section 7 for details.

low-value offers v_L with $v_H > v_L > 0$ and $1 > \mu^G > \mu^B \geq 0$.[9] The utility that a buyer receives from an offer v is $u(v) = v$.

An alternative interpretation to this structure is the following. A searcher (i.e., a buyer) gets value $v \in \{v_L, v_H\}$ from matching with an agent from the other side of the market (i.e., an offer). The value of a match is observed upon meeting. These values are randomly and independently drawn across searchers and matches. The probability that a match value is high, v_H, is $\mu^\theta \in \{\mu^B, \mu^G\}$, which is common for all searchers and measures the fit between the two sides of the market.

Timing. First, new buyers enter and each buyer gets a random offer $v \in \{v_L, v_H\}$. A buyer sees the offer v and updates her beliefs about the state. Then she decides whether to accept the offer. The buyers who trade exit the market. At the end of the period old buyers die and young buyers who did not trade are carried over to the next period.

Information. Buyers update their beliefs about the state using Bayes' rule. In the main part of the paper, a buyer updates her beliefs based on her own experience and by observing something about others' actions. A buyer's own experience is the offer that she gets, $v \in \{v_L, v_H\}$. Offer v_L is bad news: a buyer's posterior belief after v_L is lower than her prior.

A buyer b learns at t from others' actions via a private "trade signal". A buyer b' is randomly drawn from amongst the buyers active at $t-1$ and buyer b observes at t whether b' traded at $t-1$, without observing the offer that b' got. If b observes that b' traded, I say that b observes a "trade" (outcome T_{t-1}) and if she observes that b' did not trade, I say that b observes "no trade" (outcome N_{t-1}). The assumption that no b observes the offer that b' saw is not crucial for the model's results.[10] Conditional on the state and date, the realisations of the signal are i.i.d. The signal's precision is determined in equilibrium: $P(T_t|\theta) =: \tau_t^\theta$ is the equilibrium probability that a randomly drawn buyer trades in state θ at date t. A trade at t is informative if the equilibrium probability of a trade at t differs across states.

[9] The interpretation of the offer distribution depends on the application. The offer distribution can be thought of as being exogenously given, as is the distribution of project qualities for investors. Alternatively, it can be the distribution of gains from trade with heterogeneous sellers. Such sellers are, for example, firms that want to hire and differ in productivity or, if buyers are house-hunters, owners of heterogeneous houses. A sale creates surplus v. Each seller is either infinitely lived and has infinite capacity or is replaced by an identical seller upon exit. The model's results are the same as with an exogenous offer distribution if surplus is split according to Nash bargaining or if buyers make take-it-or-leave-it offers to sellers.

[10] Specifically, let a fraction ε of buyers observe both a trade/no trade event and the value at which it took place, and the rest observe only a trade/no trade event. The main result, a cyclical equilibrium, exists if the fraction ε is small enough. Please see Section 7 for details.

Strategies. A young buyer's strategy specifies for each possible private history whether to accept the offer that she gets or to continue to search. A young buyer optimally accepts v_H: life does not get better in this model. An old buyer's strategy is whether to accept or reject the offer she gets and she optimally accepts both v_L and v_H. Thus, a relevant strategy only specifies whether a buyer who is born at t and gets an offer v_L accepts it or continues. Formally, a (relevant) strategy σ_t is a mapping from the space of a young buyer's private histories (conditional on getting an offer v_L) to the space of all probability distributions over her actions "accept" and "continue", $\sigma_t : v_L \times \{T_{t-1}, N_{t-1}\} \to \Omega(\{A, C\})$, where Ω is the set of all probability distributions over accepting v_L (A) and continuing (C).

2.2 Equilibrium

I study the model's symmetric stationary equilibria. A strategy profile σ_t^* is an equilibrium if for all (v, i), where $v \in \{v_L, v_H\}$ is the offer and $i \in \{T_{t-1}, N_{t-1}\}$ the signal outcome that the buyer observes, σ_t^* is

(a) optimal: σ_t^* is a best response of a buyer to all other buyers using σ_t^*;

(b) uses Bayes' updating: the posterior odds are $\frac{P(G|v,i)}{P(B|v,i)} = \frac{\pi}{1-\pi}\frac{P(v|G)}{P(v|B)}\frac{P(i|G)}{P(i|B)}$, where $P(x|\theta)$ is the equilibrium probability of event x in state θ;

(c) consistent: the buyers' beliefs are consistent with the strategy σ_t^*;

(d) stationary: for all endogenous variables x and periods t, $K \in \mathbb{N}$ exists such that $x_t = x_{t+K}$. If $K = 1$, the equilibrium is called a steady state.

Intuitively, in all equilibria a young buyer who gets an offer v_L accepts v_L if she is pessimistic enough about the state and continues if she is optimistic enough. A critical belief $\bar{\pi}$ plays a role throughout the analysis so I define it here:

$$\bar{\pi} := (\mu^G - \mu^B)^{-1}\left[\frac{(1-\delta)v_L}{\delta(v_H - v_L)} - \mu^B\right]. \tag{1}$$

If a young buyer's posterior belief equals the critical belief, she is just indifferent between accepting the low-value offer and continuing. The critical belief decreases in the potential benefit of continuing, $v_H - v_L$ and $\mu^G - \mu^B$, and in the discount factor, δ. For the rest of the paper, I make the following assumption:

Assumption 1. $0 < \bar{\pi} < 1$.

The assumption ensures that, if the state is known, a young buyer's optimal behaviour depends on the true state (please see Section 3.1 for details). Only then is the buyer interested in learning about the state if the state is unknown.

3 Benchmarks

I show that only steady-state equilibria exist in two benchmark cases. In the first benchmark, buyers know the state. In the second, buyers do not know the state and learn about it only from their own experience. In both benchmarks, any optimal strategy for a single buyer is an equilibrium because there is no interaction between buyers.

3.1 Buyers know the state

Only steady-state equilibria exist if buyers know the state. For this subsection only, I relax Assumption 1 in order to explain its significance.

Proposition 1. *Suppose that buyers know the state θ. A strategy whereby a young buyer who gets an offer v_L*

(i) *accepts v_L is the unique equilibrium if $\theta = B$ and $\bar{\pi} > 0$.*

(ii) *continues is the unique equilibrium if $\theta = B$ and $\bar{\pi} < 0$.*

(iii) *accepts v_L is the unique equilibrium if $\theta = G$ and $\bar{\pi} > 1$.*

(iv) *continues is the unique equilibrium if $\theta = G$ and $\bar{\pi} < 1$.*

Proof. In the Appendix. □

The result is intuitive. In either state, a young buyer optimally continues after drawing a low-value offer if the critical belief $\bar{\pi}$ (defined in (1)) is low enough: if the potential benefit of continuing, $v_H - v_L$ and $\mu^G - \mu^B$, and the discount factor, δ, are large. The condition is easier to satisfy if the state is good because there are more high-value offers in the market than if the state is bad.

Assumption 1 ensures that a young buyer who gets an offer v_L accepts v_L and continues in the unique equilibrium of the bad and good state respectively. If the condition did not hold, all equilibria in the rest of the paper would be trivial.

3.2 Buyers do not know the state and learn only from their own experience

I show that only steady-state equilibria exist if buyers do not know the state and learn only from their own experience.

Proposition 2. *Suppose that buyers do not know the state θ and learn only from their own experience. A strategy whereby a young buyer who gets an offer v_L*

(i) accepts v_L *(steady state 0) is the unique equilibrium if* $\frac{\pi}{1-\pi} < \frac{\bar{\pi}}{1-\bar{\pi}}\frac{1-\mu^B}{1-\mu^G}$.

(ii) continues is the unique equilibrium if $\frac{\pi}{1-\pi} > \frac{\bar{\pi}}{1-\bar{\pi}}\frac{1-\mu^B}{1-\mu^G}$.

Proof. In the Appendix. □

The result is intuitive. A young buyer optimally continues after receiving a low-value offer if she is optimistic enough and accepts the offer if she is pessimistic enough. Her posterior belief is lower than the prior because a low-value offer is bad news: there are more low-value offers in the market if the state is bad rather than good. Thus, to sustain the equilibrium where the buyer continues, it is not sufficient that her prior odds exceed the critical odds $\frac{\bar{\pi}}{1-\bar{\pi}}$, she must be more optimistic ex ante: $\frac{\pi}{1-\pi} > \frac{\bar{\pi}}{1-\bar{\pi}}\frac{1-\mu^B}{1-\mu^G}$ where $\mu^B < \mu^G$. Figure 1a (on page 12) illustrates the regions of the parameter space that support the two steady states for $\mu^G = \frac{1}{2}$ and $\mu^B = 0$.[11]

4 Buyers do not know the state and learn from their own experience and the others' trades

In this section I derive the steady-state equilibria and a cyclical equilibrium of the full model where buyers learn from their own experience and from the trade signal. The signal reveals whether one randomly drawn buyer traded or did not trade yesterday. The trade signal introduces interaction between buyers' optimal policies: a buyer trades only if it is optimal for her to do so and all buyers' trading decisions together determine the content of the trade signal, thus, the optimal decision of each buyer tomorrow.

4.1 Steady-state equilibria

I show here that three different steady-state equilibria in pure strategies exist.[12]

Proposition 3. *A strategy whereby a young buyer who gets an offer* v_L

(i) accepts v_L *(steady state 0) is an equilibrium if* $\frac{\pi}{1-\pi} < \frac{\bar{\pi}}{1-\bar{\pi}}\frac{1-\mu^B}{1-\mu^G}$.

[11] I express the results in terms of odds ratios as this is more concise than using the beliefs and carries the same information. I depict the results in terms of beliefs, however, as this is more standard.

[12] I focus on pure-strategy equilibria here because degenerate sets of parameter values support mixed-strategy equilibria if the market starts at $t_1 = 1$.

(ii) *accepts* v_L *after observing no trade and continues after observing a trade (steady state 1) is an equilibrium if* $\frac{\bar{\pi}}{1-\bar{\pi}}\frac{1-\mu^B}{1-\mu^G}\frac{\tau^B}{\tau^G} < \frac{\pi}{1-\pi} < \frac{\bar{\pi}}{1-\bar{\pi}}\frac{1-\mu^B}{1-\mu^G}\frac{1-\tau^B}{1-\tau^G}$, *where the probability of trading in state* $\theta = B, G$ *is* $\tau^\theta = \frac{\sqrt{5-4\mu^\theta}-1}{2(1-\mu^\theta)}$.

(iii) *continues (steady state 2) is an equilibrium if* $\frac{\pi}{1-\pi} > \frac{\bar{\pi}}{1-\bar{\pi}}\frac{1-\mu^B}{1-\mu^G}\frac{1-\tau^B}{1-\tau^G}$, *where the probability of trading in state* $\theta = B, G$ *is* $\tau^\theta = (2-\mu^\theta)^{-1}$.

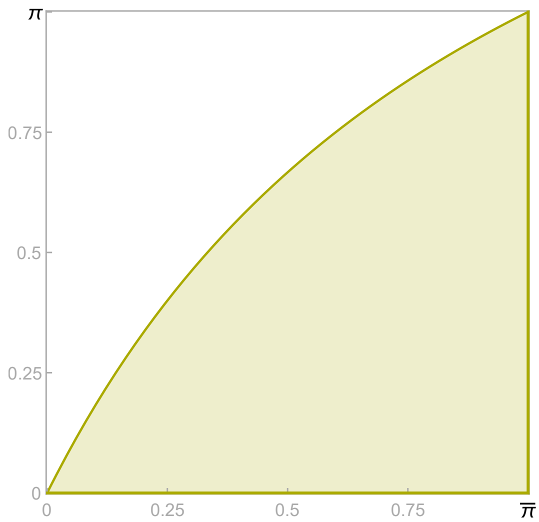
In the steady-state equilibria, a trade is (weakly) good news: $\tau^G \geq \tau^B$.
No steady-state equilibria where a trade is strictly bad news exist.

Proof. In the Appendix. □

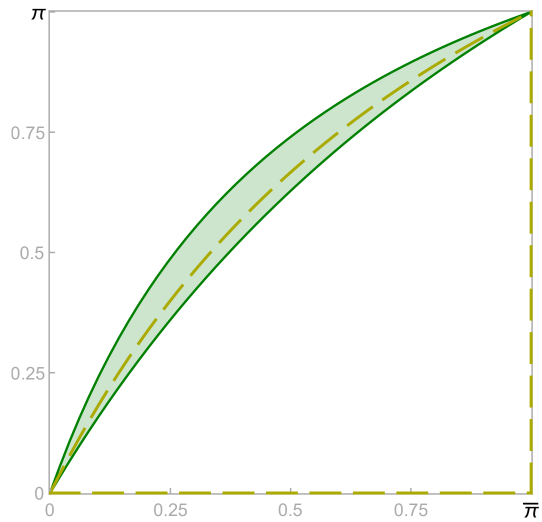
The steady states are named so that a young buyer is more likely to continue in a steady state with a higher number. In all steady states either a trade provides no news (because all buyers trade in their entry period in both states) or is good news about the state. A trade is (weakly) good news in all steady states because a trade is (weakly) more likely in the good state. A trade is more likely because there are more offers that are traded with a high probability in the good state: high-value offers trade with probability one, whereas low-value offers trade with a (weakly) lower probability.

The regions of the parameter space that support the steady states are ordered intuitively. The more optimistic buyers are ex ante, the more often they continue in equilibrium. Figures 1a, 1b, and 1c (on page 12) illustrate the regions of the parameter space that support the steady states for $\mu^G = \frac{1}{2}$ and $\mu^B = 0$.

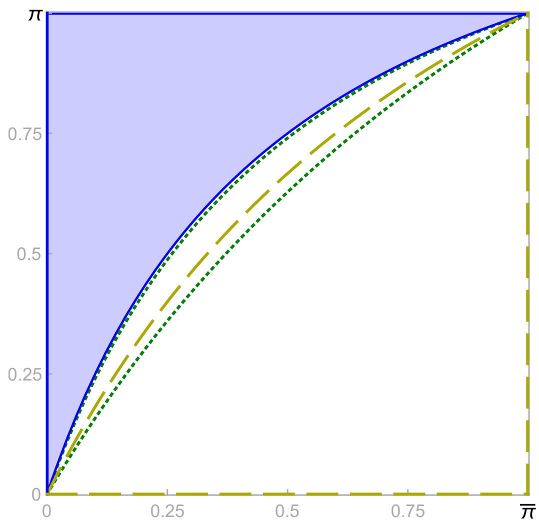
The posteriors of young buyers are more dispersed and the trade signal thus more informative in steady state 2 than in steady state 1. To understand the relation between signal informativeness and efficiency in this model, assume for this paragraph that the two steady states coexist for some parameter values. Then the higher signal informativeness in steady state 2 would translate into steady state 2 being more efficient than steady state 1 if buyers took the same actions after each signal outcome in both steady states. But they do not. Specifically, in steady state 2 a young buyer who gets an offer v_L never reacts to the signal outcome and, thus, in the good state she always takes the efficient action (to continue) and in the bad state never takes the efficient action (to accept v_L). The buyer, conversely, reacts to the signal outcome in steady state 1: she continues after a trade and accepts v_L after no trade. By reacting to the signal, she chooses the efficient action more frequently on average because the trade signal is informative: a trade is more likely in the good state (and no trade in the bad state). I argue in Section 5 that this force makes the cyclical equilibrium more efficient than steady state 2.



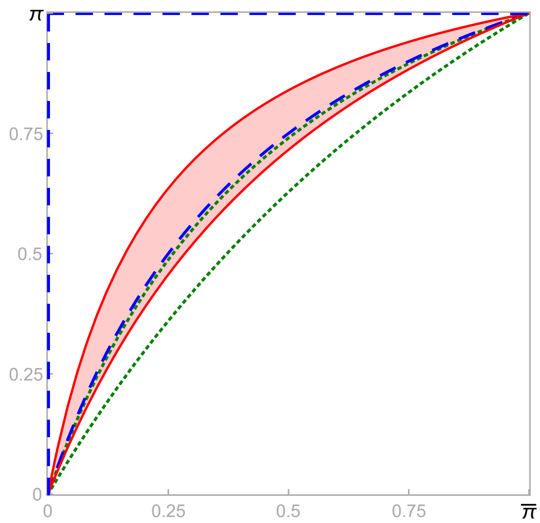
(a) *Steady state 0 (shaded region).*



(b) *Steady state 1 (shaded region; steady state 0 between dashed lines).*



(c) *Steady state 2 (shaded region; steady state 0 between dashed and steady state 1 between dotted lines).*



(d) *Cyclical equilibrium (shaded region; steady state 1 between dotted and steady state 2 between dashed lines).*

Figure 1: *A market that starts at* $t_1 = -\infty$ ($\mu^G = \frac{1}{2}$ *and* $\mu^B = 0$). *A young buyer who gets an offer* v_L: *in steady state 0, accepts* v_L; *in steady state 1, accepts* v_L *after observing no trade and continues after observing a trade; in steady state 2, continues; in the cyclical equilibrium, in an odd period continues, and in an even period accepts* v_L *after observing no trade and continues after observing a trade.*

4.2 A cyclical equilibrium

I show that a cyclical equilibrium, where only information on trades sustains the cycles, is supported by an open set of parameter values. In this equilibrium, the volume and probability of trading fluctuate between two values over time. A trade is good news in one period, and bad news in the next period. I call a "trade at t" the event that a buyer trades at t despite this event being observed only at $t+1$. For concreteness, I call periods $2t$ even and periods $2t+1$ odd.

Proposition 4. *Consider a strategy whereby a young buyer who gets an offer v_L*

(i) in an odd period, continues, and

(ii) in an even period, accepts v_L after observing no trade and continues after observing a trade.

The necessary and sufficient conditions for the strategy profile to be an equilibrium are that

1. *a trade in an odd period is good news ($\tau^G_{odd} > \tau^B_{odd}$) and a trade in an even period is bad news ($\tau^G_{even} < \tau^B_{even}$), where the probabilities of trading are $\tau^\theta_{odd} = \frac{\sqrt{(4-3\mu^\theta)\mu^\theta}-\mu^\theta}{2(1-\mu^\theta)}$ and $\tau^\theta_{even} = 1-(2-\mu^\theta)^{-1}(1-\mu^\theta)\tau^\theta_{odd}$ for $\theta = B, G$.*

2. *no buyer wants to deviate, i.e., that*

$$\frac{\bar{\pi}}{1-\bar{\pi}}\frac{1-\mu^B}{1-\mu^G}\frac{\tau^B_{even}}{\tau^G_{even}} < \frac{\pi}{1-\pi} < \frac{\bar{\pi}}{1-\bar{\pi}}\frac{1-\mu^B}{1-\mu^G}\frac{1-\tau^B_{odd}}{1-\tau^G_{odd}}.$$

One sufficient condition for the equilibrium to exist is that $\mu^G < \bar{\mu} := \frac{2}{7}(3-\sqrt{2})$. Another sufficient condition is that $\mu^B = 0$.

In the cyclical equilibrium, a trade in an odd period is good news and in an even period is bad news. The probability and the volume of trading fluctuate more in the bad state than in the good state.

Proof. In the Appendix. □

The first sufficient condition guarantees that the trade probabilities are ordered as necessary for the cyclical equilibrium: τ^θ_{odd} increases in μ^θ for all μ^θ and τ^θ_{even} decreases in μ^θ for all $\mu^\theta < \bar{\mu}$. But both sufficient conditions are much stronger than the necessary and sufficient conditions. Broadly, the cyclical equilibrium also exists for an open set of parameter values if μ^B is close to zero and μ^G is not too close to one.[13]

[13] See Figure 5 in Appendix A for a depiction of τ^θ_{odd} and τ^θ_{even}.

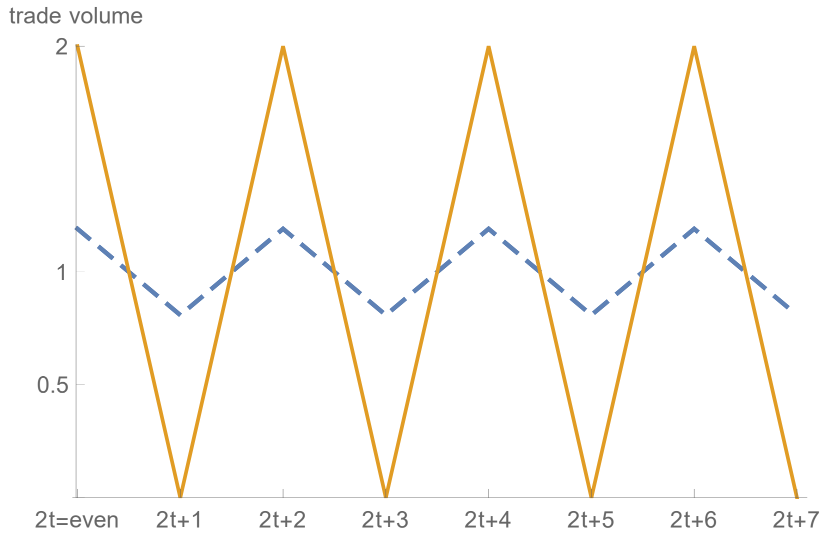


Figure 2: *The volume of trading in the cyclical equilibrium in the good state (blue dashed) and bad state (orange) for* $\mu^G = \frac{1}{2}$ *and* $\mu^B = 0$.

I explain how the equilibrium strategy sustains cycles in the volume of trading (the argument for the probability of trading is analogous). Young buyers' optimal actions differ in odd and even periods, which leads to different amounts of young buyers trading not only across periods, but also across states. This is the main driver of cycles in the volume of trading. Old buyers' actions are not crucial for the cycles because their optimal behaviour is the same across periods and states, and their amounts are relatively similar across states in any period.

To understand the effect of young buyers' actions, suppose that trade volume is low in period $2t-1$ because young buyers are only willing to trade at value v_H. This makes a trade good news for young buyers at $2t$: those who observe a trade are less willing and those who observe no trade are more willing to trade at v_L. If the pessimistic ones trade, then trade volume is high at $2t$ and a trade can be bad news for buyers at $2t+1$. This is because buyers at $2t+1$ correctly infer that many of the trades at $2t$ took place at v_L, which are abundant if the state is bad. A trade that is bad news due to this, however, is not as bad news as no trade was for buyers at $2t$. This is because the signal becomes less precise if some (rather than no) young buyers accept low-value offers. Thus, at $2t+1$ both young buyers who observe a trade and who observe no trade are optimistic enough to not trade at v_L. But then young buyers only trade at value v_H at $2t+1$, just like at $2t-1$. Recall that only steady-state equilibria exist if buyers learn only from their own experience: cycles are sustained by information on trades. Figure 2 illustrates the volume of trading for $\mu^G = \frac{1}{2}$ and $\mu^B = 0$.

Figure 1d (on page 12) illustrates the region of the parameter space that supports the cyclical equilibrium for $\mu^G = \frac{1}{2}$ and $\mu^B = 0$.[14] The figure illustrates that, in general, the regions that support steady state 1 and steady state 2 can overlap with the region that supports the cyclical equilibrium. This is intuitive: in the cyclical equilibrium buyers use the strategy of steady state 1 in even periods and the strategy of steady state 2 in odd periods. For general μ^G and μ^B, steady state 0 and the cyclical equilibrium coexist for no parameter values. I compare the efficiency of the equilibria that are supported by the same parameter values in Section 5 for general μ^G and μ^B.

The model provides two testable predictions. First, we should observe fluctuations in (sub)markets that are characterised by intermediate rather than extreme values of the critical belief $\bar{\pi}$ (see (1)). If buyers across (sub)markets have similar discount factors, we should observe fluctuations in (sub)markets where the range of offers in the market, $v_H - v_L$, or the wedge between the good and bad distributions, $\mu^G - \mu^B$, are intermediate rather than extreme. Second, the fluctuations in the volume of trading are larger if the underlying state is bad rather than good. If we interpret the bad state as a bust and the good state as a boom, the model predicts larger fluctuations in busts than in booms.[15]

5 Efficiency

I compare the efficiency of the different equilibria in the regions of the parameter space where multiple equilibria exist. Efficiency is measured by an entering buyer's expected value from participating in the market. I show that the cyclical equilibrium can be more efficient than the coexistent steady state.

Proposition 5. *In the region of the parameter space where the cyclical equilibrium coexists with*

(i) the steady state where a young buyer continues after v_L (steady state 2), the cyclical equilibrium is more efficient.

(ii) the steady state where a young buyer accepts v_L only after no trade (steady state 1), a sufficient condition for the cyclical equilibrium to be more efficient is that $\mu^B < \hat{\mu}$, where $\hat{\mu} := \frac{2-\sqrt{3}}{3}$.

Proof. In the Appendix. □

[14]Please note, first, that the size of the shaded region depends on the fractions of high-value offers in both states, and, second, that there is no obvious way to say if the region is "small" or "large" because its size depends on which values of π and $\bar{\pi}$ are "reasonable".

[15]I provide empirical evidence that supports this prediction in an Online Appendix that is available on `https://sites.google.com/site/eevamauring/research`.

The reason why the cyclical equilibrium is more efficient than steady state 2 is analogous to the argument provided at the end of Section 4.1: young buyers react to the informative trade signal more often in the cyclical equilibrium than in steady state 2. A young buyer who gets an offer v_L reacts to the trade signal in every other period in the cyclical equilibrium and in no period in steady state 2. The trade signal is informative in the cyclical equilibrium, so reacting to it guides the market towards the efficient complete-information benchmark.

The cyclical equilibrium can also be more efficient than steady state 1 despite the fact that buyers react to the trade signal less frequently in the cyclical equilibrium than in steady state 1.[16] The cyclical equilibrium can be more efficient because the precision of the signal that buyers react to is higher in the cyclical equilibrium. That is, the bad- and good-state trade probabilities differ more in an odd period of the cyclical equilibrium than in steady state 1.

I explain the intuition for small μ^B and large μ^G. If μ^G is large, the good-state trade probability is driven by μ^G and is, thus, similar in the two equilibria. However, the bad-state trade probability is much lower in an odd period of the cyclical equilibrium than in steady state 1. In steady state 1, the probability is relatively high because all old and some young buyers accept low-value offers. But in an odd period of the cyclical equilibrium, only old buyers accept low-value offers so the probability of trading is much lower. If this discrepancy is large enough, the cyclical equilibrium is more efficient than steady state 1.

For a fixed prior belief, the cyclical equilibrium is more efficient than the co-existent steady state if the critical belief $\bar{\pi}$ is lower. The model suggests, thus, that fluctuations are less worrisome in (sub-)markets where $v_H - v_L$ or δ is higher (see (1)). Fluctuations are less worrisome if the support of offered wages is larger (for example, white-collar as opposed to blue-collar jobs), if the spread of investment projects' expected values is larger (for example, riskier as opposed to safer projects), if the support of real estate values net of prices is larger (for example, commercial as opposed to residential real estate), and if searchers are more patient (for example, have higher as opposed to lower savings).

6 A market that starts at $t_1 = 1$

Consider the full model that starts at $t_1 = 1$ instead of $t_1 = -\infty$. I show that the cyclical equilibrium as described in Proposition 4 is reached from an open set

[16] It is easy to construct an example of parameter values such that the cyclical equilibrium is less efficient than steady state 1. For example, if $\bar{\pi} = 0.5$, $\mu^B = 0.5$ and $\mu^G = 0.75$, then the two equilibria coexists if $\pi \in (0.65, 0.76)$ and steady state 1 is more efficient if $\pi \in (0.65, 0.7)$.

of parameter values. The cyclical equilibrium is, thus, a natural limit that some markets reach.

Proposition 6. *Suppose that the market starts at* $t_1 = 1$*.*

(i) The steady state where a young buyer accepts v_L *(steady state 0) is reached at* $t = 1$ *if* $\frac{\pi}{1-\pi} < \frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G}$*.*

(ii) The steady state where a young buyer continues after v_L *(steady state 2) is reached at* $t = 2$ *if* $\frac{\pi}{1-\pi} > \frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G} \frac{1-\tau_1^B}{1-\tau_1^G}$*, where* $\tau_1^\theta = \mu^\theta$ *for* $\theta = B, G$*.*

(iii) The cyclical equilibrium is reached as $t \to \infty$ *for an open set of parameter values. If either* $\mu^B = 0$*, or* $\mu^B < \mu^G < \varepsilon$ *for* $\varepsilon > 0$ *small, the necessary and sufficient conditions for the market to converge to the cyclical equilibrium are* $\frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G} \frac{\tau_{even}^B}{\tau_{even}^G} < \frac{\pi}{1-\pi} < \frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G} \frac{1-\tau_1^B}{1-\tau_1^G}$*, where* $\tau_1^\theta = \mu^\theta$*,* $\tau_{odd}^\theta = \frac{\sqrt{(4-3\mu^\theta)\mu^\theta} - \mu^\theta}{2(1-\mu^\theta)}$*, and* $\tau_{even}^\theta = 1 - (2-\mu^\theta)^{-1}(1-\mu^\theta)\tau_{odd}^\theta$ *for* $\theta = B, G$*.*

Proof. In the Appendix. □

The above necessary and sufficient conditions that guarantee convergence to the cyclical equilibrium are insufficient if the parameter values are such that the informativeness of an even-period trade (captured by τ_{2t}^B/τ_{2t}^G) is not monotone increasing in time. If the informativeness is not monotone, I cannot find a closed-form constraint that ensures that a young buyer who gets an offer v_L and sees the trade of any even period optimally continues.

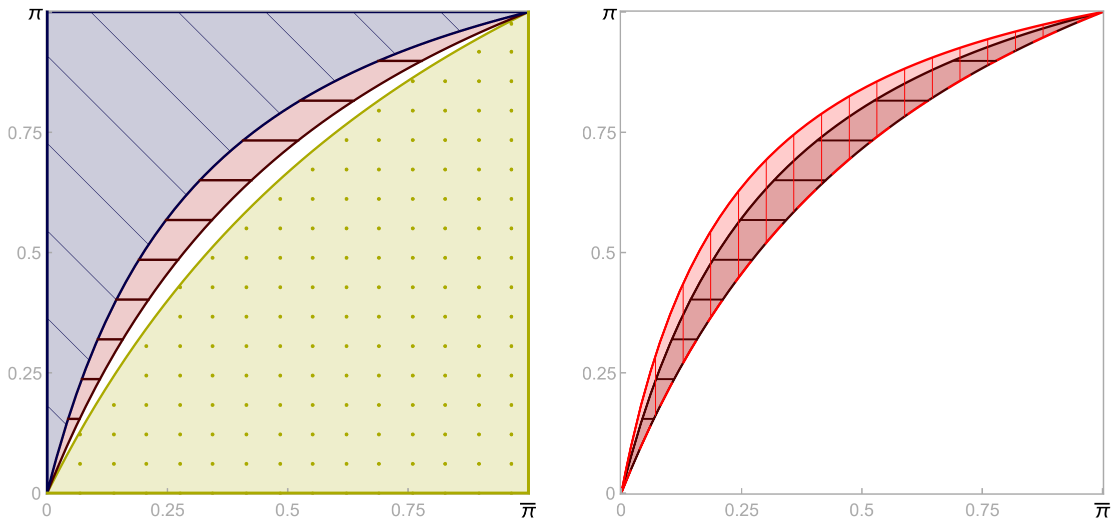
The result of Proposition 6 is illustrated in Figure 3a for $\mu^G = \frac{1}{2}$ and $\mu^B = 0$. The ordering of the three regions is intuitive: if buyers are more optimistic ex ante, they are more willing to continue in equilibrium.[17]

Figure 3b illustrates that in general a smaller region of the parameter space sustains a cycle in a market that starts at $t_1 = 1$ as compared to a market that starts at $t_1 = -\infty$. One reason is that at $t = 1$ no trade signal is observed if $t_1 = 1$: the young buyers' incentives to continue at $t = 1$ are different on a market without a past (i.e., if $t_1 = 1$) as opposed to a market with an infinite past (i.e., if $t_1 = -\infty$).[18]

Figure 4 illustrates that the market's convergence to the cyclical equilibrium's long-run values can be fast. I depict the probabilities of trading in the bad and

[17] For some parameter values in the white region in Figure 3a, longer cyclical equilibria are reached in the limit. In these equilibria, a trade is good news in one period and bad news in the next consecutive $K - 1 > 1$ periods.

[18] Another reason, which is only relevant for some parameter values, is that the informativeness of an even-period trade signal is not monotone in time.



(a) *Limit equilibria: cyclical equilibrium (dark red horizontal lines), steady state 0 (yellow dots) and 2 (blue diagonal lines).*

(b) *The cyclical equilibrium in a market that starts at* $t_1 = 1$ *(dark red horizontal lines) versus* $t_1 = -\infty$ *(red vertical lines).*

Figure 3: *A market that starts at* $t_1 = 1$ $(\mu^G = \frac{1}{2}$ *and* $\mu^B = 0)$.

good states as a function of time and their long-run equilibrium values for $\mu^G = \frac{1}{2}$ and $\mu^B = 0$. In the bad state, the probability converges to its long-run value immediately. In the good state, at $t = 6$ the probability is about 0.1% higher than the long-run value τ^G_{even} and at $t = 7$, about 0.1% lower than τ^G_{odd}.

7 Concluding discussion

I discuss several modifications to the model's assumptions. In Appendix B I show formally that neither the assumption of short-lived buyers nor the assumption that the value at which a trade/no trade takes place is never observed are crucial for the existence of cycles. I also discuss the possibility of endogenous information acquisition, a noisy trade signal, and Markov state.

Long-lived buyers. The cyclical equilibrium exists for an open set of parameter values also if buyers are long-lived. In Proposition 7 in Appendix B, I present the necessary and sufficient conditions for a cyclical equilibrium to exist if buyers potentially live for ever, but die exogenously with probability $\delta \in (0, 1)$ in each period. The strategies of buyers are more complicated in this version of the model because a buyer who has been in the market for more than a period has seen multiple trade and/or no trade events. The equilibrium that I focus on is sustained by a strategy that is close to the one that buyers use in the main model's

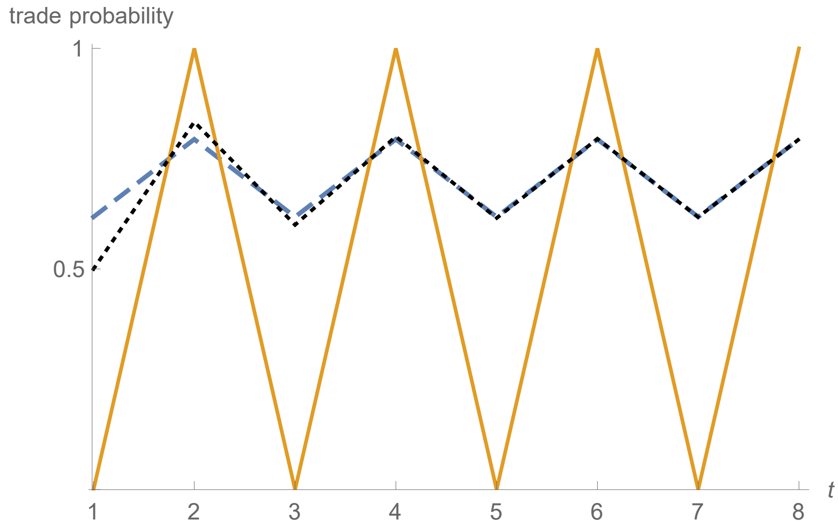


Figure 4: *The convergence of trading probabilities to the cyclical equilibrium for* $\mu^G = \frac{1}{2}$ *and* $\mu^B = 0$*: the probability of trading in the good state if* $t_1 = 1$ *(black dotted) and if* $t_1 = -\infty$ *(blue dashed); and in the bad state (orange solid).*

cyclical equilibrium (please see Proposition 7 for details).[19]

Partially observed value of trade. The cyclical equilibrium exists for an open set of parameter values if some buyers observe the trade signal as in the main model and others observe not only whether another buyer traded, but also the value v at which this trade/no trade took place. I prove in Proposition 8 in Appendix B that if the fraction of these buyers is small enough, a cyclical equilibrium is sustained by a strategy that is similar to the one that buyers use in the main model's cyclical equilibrium.[20] Thus, the existence of cycles does not depend crucially on the assumption that the values at which trades take place are unobservable.

Noisy trade signal. Suppose that some buyers observe the trade signal and others observe a completely uninformative private signal. Then the young buyers who observe the uninformative signal optimally decide about accepting v_L based on their prior only, whereas the rest behave roughly as in the main model of this paper. The presence of uninformed young buyers changes the precision and informational content of the trade signal, but should not change the paper's results unless their mass becomes very large.

Alternatively, suppose that the trade signal is noisy: a buyer observes the trade event of a buyer who was active yesterday with probability p and the trade event of a buyer who was active earlier with probably $1 - p$. This decreases the informational content of a trade signal. The cyclical equilibrium, however, should

[19]For convenience, I let $\mu^G = \frac{1}{2}$ and $\mu^B = 0$ in the Proposition, but these values are not crucial for the result.

[20]See footnote 19.

still exist for p large enough.

Endogenous information acquisition. Suppose that buyers are able to choose how many trade signal realisations to see. If each realisation is costless, a buyer chooses to see as many signal realisations as possible in all equilibria where the trade signal is informative. Each realisation tells the buyer which state is more likely so she is able to make a better decision about whether to continue or stop.

If each realisation costs, a buyer optimally sees a finite number of signal realisations. The buyer never optimally sees an infinite number of signal realisations because an additional observation is very uninformative, thus, not valuable, if the buyer is relatively sure about the state. My model can be seen as the limiting case where obtaining one signal realisation is cheap whereas further observations are prohibitively expensive.

Markov state. Consider a model where the state of the market may change in each period. Suppose that the state is persistent: in each period, the state is more likely to remain the same than to change. The model without a starting date (i.e., if $t_1 = -\infty$) cannot be solved anymore because the changing state and the information structure make the environment nonstationary. In particular, the signal introduces dependencies across periods $t+1$, t, and $t-1$, which means that the entire history of states matters for a buyer's optimal decision.

In a model with a starting date (i.e., if $t_1 = 1$), I can show for $t = 1, ..., 6$ that for certain parameter values the expected trade probabilities fluctuate as required for a cyclical equilibrium: a trade in periods $t = 1, 3, 5$ is good news and a trade in periods $t = 2, 4, 6$ is bad news. I do not know if in the limit the market converges to a cycle in expectations.

A Appendix A

Appendix A contains the proofs of the Propositions in the main part of the paper.

A.1 Benchmarks

Proof of Proposition 1. Here buyers know the state. For each state, I derive the conditions under which a young buyer prefers continuing to accepting a low-value offer and vice versa.

A young buyer's continuation value in state θ is

$$V^\theta = \delta[v_L + \mu^\theta(v_H - v_L)]. \tag{2}$$

The buyer accepts any offer when old. She gets v_L for sure and the extra value $v_H - v_L$ only if she gets offer v_H. The probability of getting an offer v is equal to the fraction of these offers because of random matching. She prefers to continue after v_L if $v_L < V^\theta$. Rearranging gives that she prefers to continue in the bad state if $\bar{\pi} < 0$ and in the good state if $\bar{\pi} < 1$, where $\bar{\pi}$ is defined in (1). □

Proof of Proposition 2. Here, a buyer does not know the state and learns about it only from her own experience, i.e., from the offers that she gets. I derive the conditions under which a young buyer prefers continuing to accepting a low-value offer.

Consider any period t. For a young buyer with posterior belief π', the expected value of continuing and accepting either offer when old is

$$V(\pi') = \delta \left\{ v_L + [\pi' \mu^G + (1 - \pi')\mu^B](v_H - v_L) \right\}. \tag{3}$$

She gets utility v_L for sure and the extra value $v_H - v_L$ only if she gets an offer v_H. She gets an offer v_H with probability μ^G if the state is good and μ^B if the state is bad. The young buyer optimally continues after offer v_L if $V(\pi') > v_L$, or if $\pi' > \bar{\pi}$, where the cutoff belief $\bar{\pi}$ is the same as in the known-state benchmark, defined in (1).

Let $\pi(v_L)$ denote a young buyer's posterior belief that the state is good after getting an offer v_L. Her posterior odds are

$$\frac{\pi(v_L)}{1 - \pi(v_L)} = \omega \frac{P(v_L|G)}{P(v_L|B)} = \omega \frac{1 - \mu^G}{1 - \mu^B},$$

where $\omega := \frac{\pi}{1-\pi}$ denotes the prior odds. I focus on the posterior odds throughout because the odds contain the same information as the posterior belief but are easier to interpret. The posterior odds are lower than the prior odds because getting a low-value offer makes the buyer more pessimistic about the state.

A young buyer optimally continues after getting an offer v_L if $V(\pi' = \pi(v_L)) > v_L$ or, equivalently, if $\pi(v_L) > \bar{\pi}$, which is the condition in the Proposition if rearranged. The argument holds both if the economy starts at $t_1 = 1$ and at $t_1 = -\infty$ because a buyer's optimal decision only depends on the distribution of offers and that is given. □

A.2 Buyers do not know the state and learn from their own experience and the others' trades

Proof of Proposition 3.

(i) Steady state 0: A young buyer who gets an offer v_L accepts v_L.

If buyers use this strategy, a randomly drawn buyer trades with probability one in both states. This makes the trade signal uninformative. But if the signal is uninformative, then the equilibrium exists for the same parameter values as in a market where buyers learn only from their own experience (see Proposition 2).

(ii) Steady state 1: A young buyer who gets an offer v_L accepts v_L after observing no trade and continues after observing a trade.

I first derive the equilibrium objects (amounts of old buyers and trading probabilities) assuming that the strategy profile constitutes an equilibrium and then derive the conditions under which no buyer has an incentive to deviate. Let O_t^θ denote the amount of old buyers and τ_t^θ the probability of a randomly drawn buyer trading at date t in state θ. The amounts of buyers are measured at the start of a period, after entry.

Consider any period t and state θ. Given the above strategy, the probability of a trade at t in state θ is

$$\tau_t^\theta = 1 - \frac{(1-\mu^\theta)\tau_{t-1}^\theta}{1+O_t^\theta}. \tag{4}$$

The only buyers who do not trade are young buyers who got an offer v_L and observed a trade (T_{t-1}). The probability of getting an offer v_L is equal to the fraction of these offers (because of random matching) and the probability of observing a trade is the probability that a randomly drawn buyer traded at $t-1$. The total amount of buyers is the sum of the amounts of young and old buyers.

The young buyers who become old, i.e., buyers who are carried over to $t+1$ are young buyers who get low-value offers and see a trade. So the amount of old buyers at $t+1$ in state θ is

$$O_{t+1}^\theta = (1-\mu^\theta)\tau_{t-1}^\theta. \tag{5}$$

Imposing the steady state condition that $x_t = x_{t+1}$ for all endogenous variables x and solving equations (4) and (5) for $\theta = B, G$ gives that τ^θ solves $(\tau^\theta)^2(1-\mu^\theta) = 1-\tau^\theta$. A trade is good news, as required, because $\tau^G > \tau^B$. The trade probability is explicitly

$$\tau^\theta = \frac{\sqrt{5-4\mu^\theta}-1}{2(1-\mu^\theta)}.$$

The proposed strategy is optimal if no young buyer wants to deviate. At t, the posterior odds of a young buyer who gets an offer v_L and observes no trade

N_{t-1} are

$$\frac{\pi(v_L, N_{t-1})}{1-\pi(v_L, N_{t-1})} = \omega \frac{P(v_L|G)}{P(v_L|B)} \frac{P(N_{t-1}|G)}{P(N_{t-1}|B)} = \omega \frac{1-\mu^G}{1-\mu^B} \frac{1-\tau^G_{t-1}}{1-\tau^B_{t-1}}. \tag{6}$$

The posterior odds of a young buyer who gets an offer v_L and observes a trade T_{t-1} are

$$\frac{\pi(v_L, T_{t-1})}{1-\pi(v_L, T_{t-1})} = \omega \frac{P(v_L|G)}{P(v_L|B)} \frac{P(T_{t-1}|G)}{P(T_{t-1}|B)} = \omega \frac{1-\mu^G}{1-\mu^B} \frac{\tau^G_{t-1}}{\tau^B_{t-1}}. \tag{7}$$

For a young buyer with belief π', the value of continuing and accepting either offer when old is given by (3). Thus, the cutoff belief is still $\bar{\pi}$ as defined in (1) and the proposed equilibrium strategy is optimal if $\pi(v_L, N) < \bar{\pi} < \pi(v_L, T)$, which, if rearranged, give the exact conditions in the Proposition. A trade being good news ($\tau^G > \tau^B$) guarantees that the conditions are satisfied for an open set of parameter values.

(iii) Steady state 2: A young buyer who gets an offer v_L continues.

I go through the same two steps as in (ii): I derive first the equilibrium objects given the strategy profile and then the conditions under which no buyer deviates. According to the above strategy, all young buyers who get an offer v_L continue at t so the probability of a trade at t in state $\theta = B, G$ is

$$\tau^\theta_t = 1 - \frac{1-\mu^\theta}{1+O^\theta_t}, \tag{8}$$

and the amount of old buyers at $t+1$ is

$$O^\theta_{t+1} = 1 - \mu^\theta. \tag{9}$$

Imposing the steady state condition gives $\tau^\theta = (2-\mu^\theta)^{-1}$, so $\tau^G > \tau^B$.

The strategy is optimal if the most pessimistic young buyer (one who gets an offer v_L and observes no trade) does not want to deviate and accept v_L. Her posterior odds are given by (6), where the trade probabilities are $\tau^\theta = (2-\mu^\theta)^{-1}$ for $\theta = B, G$. Thus, steady state 2 exists if $\bar{\pi} < \pi(v_L, N)$, which, once rearranged, gives the exact condition in the Proposition.

Finally, I prove by contradiction that a trade cannot be bad news in a pure-strategy steady-state equilibrium. Since in the above steady states a trade is (weakly) good news, I have to consider only one candidate steady state: where young buyers who get an offer v_L continue after observing no trade and accept

v_L after observing a trade. The derivation is analogous to that of steady state 1 (where a trade event is replaced by a no trade event) so I am brief.

Given the proposed strategy, the buyers who become old are young buyers who get a low-value offer and see no trade so the amount of old buyers is $O^\theta = (1-\mu^\theta)(1-\tau^\theta)$ in state θ. The probability of a trade in state θ is $\tau^\theta = 1 - \frac{(1-\mu^\theta)(1-\tau^\theta)}{1+O^\theta}$. Combining the equations gives $\tau^G = 1$ and $\tau^B = 1$. But a trade is bad news only if $\tau^G < \tau^B$, a contradiction. □

Proof of Proposition 4. In the cyclical equilibrium, in an odd period $2t-1$ (for any integer t) buyers behave as in steady state 2: no young buyer accepts v_L. I assume that a trade at $2t-1$, T_{2t-1}, (which is observed at $2t$) is good news. In an even period $2t$ buyers behave as in steady state 1: the young buyers who observe bad news about the state (i.e., no trade N_{2t-1}), accept v_L. I assume that a trade at $2t$ is bad news, but comparatively "less bad news" than no trade at $2t-1$ (so that all young buyers at $2t+1$ optimally reject v_L, whereas the pessimistic young buyers at $2t$ accept v_L). Let O_t^θ denote the amount of old buyers and τ_t^θ the probability of a randomly drawn buyer trading in period t in state θ.

I first derive the equilibrium objects O_{2t-1}^θ, O_{2t}^θ, τ_{2t-1}^θ and τ_{2t}^θ given the proposed strategy and then show under which conditions they satisfy the assumptions made above. I then derive the conditions under which no buyer has an incentive to deviate from the strategy. Finally, I show that the probability and volume of trading fluctuate more in the bad than in the good state.

Consider an odd period $2t-1$. Since at $2t-1$ buyers behave as in steady state 2, the equations for the probability of a trade at $2t-1$ and the amount of old buyers at $2t$ are given by equations (8) and (9) respectively (where $2t-1$ replaces t).

Consider an even period $2t$. Since at $2t$ buyers behave as in steady state 1, the equations for the probability of a trade at $2t$ and the amount of old buyers at $2t+1$ are given by equations (4) and (5) respectively (where $2t$ replaces t).

To complete this step, I impose the condition that the cycle is two periods long, i.e., that for $t' = 2t-1, 2t$ and all endogenous variables x, $x_{t'+2} = x_{t'}$. I denote the equilibrium values of the endogenous variables with subscripts "odd" and "even". The probabilities of trading solve $(\tau_{odd}^\theta)^2(1-\mu^\theta) = \mu^\theta(1-\tau_{odd}^\theta)$ and $\tau_{even}^\theta = 1-(2-\mu^\theta)^{-1}(1-\mu^\theta)\tau_{odd}^\theta$ for $\theta = B, G$, which give, explicitly

$$\tau_{odd}^\theta = \frac{\sqrt{\mu^\theta(4-3\mu^\theta)} - \mu^\theta}{2(1-\mu^\theta)}, \tag{10}$$

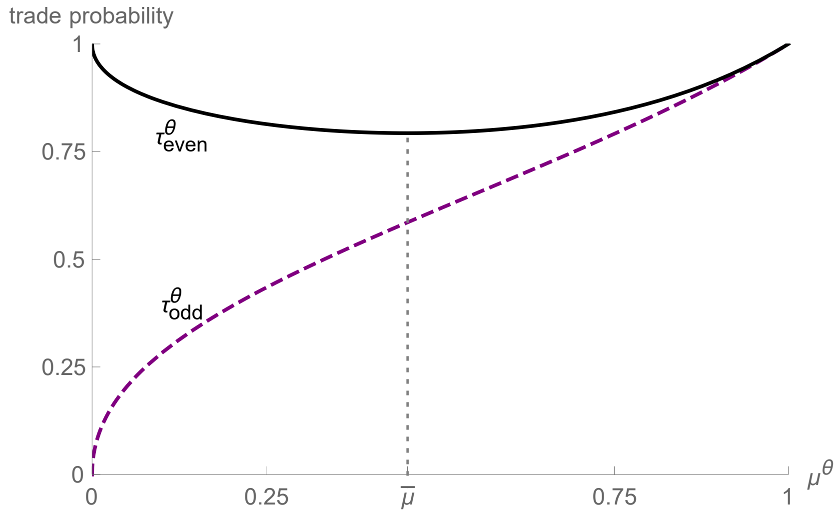


Figure 5: *The proposed equilibrium probabilities of trading in odd periods (purple dashed) and in even periods (black).*

and

$$\tau^{\theta}_{even} = 1 - \frac{\sqrt{\mu^{\theta}(4 - 3\mu^{\theta})} - \mu^{\theta}}{2(2 - \mu^{\theta})}. \tag{11}$$

The probability of trading in an odd period, τ^{θ}_{odd}, strictly increases in μ^{θ} while the probability of trading in an even period, τ^{θ}_{even}, is convex in μ^{θ}. It is easy to check that $\tau^{\theta}_{even} > \tau^{\theta}_{odd}$ for all μ^{θ}. Figure 5 depicts equations (10) and (11).

Next I derive the conditions under which the informational content of trades satisfy the assumptions made earlier. I need that

(a) a trade in an odd period is good news, $\tau^{G}_{odd} > \tau^{B}_{odd}$. This holds because τ^{θ}_{odd} strictly increases in μ^{θ}.

(b) a trade in an even period is bad news, $\tau^{G}_{even} < \tau^{B}_{even}$. This holds for sure if either $\mu^{B} = 0$ or if $\mu^{G} \leq \bar{\mu}$, where $\bar{\mu} := \frac{2}{7}(3 - \sqrt{2})$ minimises τ^{θ}_{even}, but not necessarily for other values of μ^{B} and μ^{G}. Broadly, the condition is also satisfied if μ^{B} is close to zero and μ^{G} is not too close to one (see Figure 5).

Next I determine the parameter values for which the proposed strategy is optimal for young buyers. For a young buyer with belief π', the value of continuing and accepting either offer when old is given by equation (3). So the cutoff belief is still $\bar{\pi}$ (defined in (1)). At period $t' = 2t - 1, 2t$, a young buyer's posterior odds after getting an offer v_L and observing signal outcome $N_{t'-1}$ are given by (6) and after signal outcome $T_{t'-1}$ by (7) where $t = t'$. The proposed strategy constitutes an equilibrium if a young buyer who gets an offer v_L in an odd period optimally continues regardless of the signal outcome, i.e., if $\bar{\pi} < \pi(v_L, T_{even})$,

$\pi(v_L, N_{even})$, and in an even period optimally continues after a trade, but accepts v_L after no trade, i.e., if $\pi(v_L, N_{odd}) < \bar{\pi} < \pi(v_L, T_{odd})$. If the conditions on the informational content of trades are satisfied, the strategy is optimal if $\pi(v_L, N_{odd}) < \bar{\pi} < \pi(v_L, T_{even})$, which, once rearranged, are the conditions in the Proposition. The posteriors satisfy the conditions for a range of parameter values as long as the conditions on the informational content of trades hold.

Finally, I show that the probability and volume of trading fluctuate more in the bad than in the good state. The trade probability fluctuates more in the bad than in the good state because, together, $\tau^G_{odd} > \tau^B_{odd}$, $\tau^G_{even} < \tau^B_{even}$, and $\tau^\theta_{even} > \tau^\theta_{odd}$ for all μ^θ imply that $\tau^B_{even} > \tau^G_{even} > \tau^G_{odd} > \tau^B_{odd}$.

In an odd period, the volume of trading is

$$Vol^\theta_{odd} = \tau^\theta_{odd}(1 + O^\theta_{odd}) = \frac{\sqrt{\mu^\theta(4 - 3\mu^\theta)} + \mu^\theta}{2}.$$

In an even period, the volume is

$$Vol^\theta_{even} = \tau^\theta_{even}(1 + O^\theta_{even}) = \frac{4 - \mu^\theta - \sqrt{\mu^\theta(4 - 3\mu^\theta)}}{2},$$

with $Vol^\theta_{even} - Vol^\theta_{odd} > 0$ for all μ^θ. The fluctuations in trading volume are larger in the bad state if $Vol^\theta_{even} - Vol^\theta_{odd}$ decreases in μ^θ. Differentiation yields

$$\frac{\partial}{\partial \mu^\theta} Vol^\theta_{even} - Vol^\theta_{odd} \propto -[\mu^\theta(4 - 3\mu^\theta)]^{1/2} + 3\mu^\theta - 2 < -\mu^\theta + 3\mu^\theta - 2 < 0. \quad \square$$

A.3 Efficiency

Proof of Proposition 5. In steady state 1, a young buyer who gets an offer v_L accepts the offer only if she also observes no trade. A buyer's expected utility from participating in the market is

$$W_{ss1} = \sum_{\theta = B,G} P(\theta)\{\mu^\theta v_H + (1 - \mu^\theta)[(1 - \tau^\theta)v_L + \tau^\theta V^\theta]\},$$

where V^θ is the discounted value of accepting any offer tomorrow (defined in (2)) and τ^θ solves $(\tau^\theta)^2(1 - \mu^\theta) = 1 - \tau^\theta$. If the buyer gets an offer v_H when young (which happens with probability μ^θ in state θ), she accepts the offer. If she instead gets an offer v_L, she accepts the offer only if she observes no trade and continues otherwise. If she continues, she accepts any offer when old.

In steady state 2, no young buyer who gets an offer v_L accepts the offer. A buyer's expected value from participating in the market is

$$W_{ss2} = \sum_{\theta=B,G} P(\theta)[\mu^\theta v_H + (1-\mu^\theta)V^\theta].$$

In the cyclical equilibrium, a buyer's expected utility depends on her entry period. For a buyer born in an odd period (when no young buyer who gets an offer v_L accepts v_L), the expected utility from participating in the market, $W_c(odd)$, is exactly the same as in steady state 2: $W_c(odd) = W_{ss2}$. For a buyer born in an even period (when a young buyer who gets an offer v_L accepts the offer only if she also observes no trade), the expected utility from participating in the market is as in steady state 1, except for the different trade probabilities:

$$W_c(even) = \sum_{\theta=B,G} P(\theta)\{\mu^\theta v_H + (1-\mu^\theta)[(1-\tau^\theta_{odd})v_L + \tau^\theta_{odd}V^\theta]\},$$

where τ^θ_{odd} solves $(\tau^\theta_{odd})^2(1-\mu^\theta) = \mu^\theta(1-\tau^\theta_{odd})$.

Comparing efficiency in steady state 2 and in the cyclical equilibrium is the same as comparing W_{ss2} to $W_c(even)$. The cyclical equilibrium is more efficient if

$$\frac{\pi}{1-\pi} < \frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G} \frac{1-\tau^B_{odd}}{1-\tau^G_{odd}}, \tag{12}$$

where I have used the fact that $\frac{\bar{\pi}}{1-\bar{\pi}} = \frac{v_L - V^B}{V^G - v_L}$. The condition holds because it must be satisfied for the cyclical equilibrium to exist (see Proposition 4).

Comparing efficiency in steady state 1 and in the cyclical equilibrium is the same as comparing W_{ss1} to $\frac{W_c(odd)+W_c(even)}{2}$. The cyclical equilibrium is more efficient if

$$\frac{\pi}{1-\pi}\frac{1-\mu^G}{1-\mu^B}\frac{1-\tau^G_{odd}}{2} + \frac{\bar{\pi}}{1-\bar{\pi}}\frac{1+\tau^B_{odd}}{2} < \frac{\pi}{1-\pi}\frac{1-\mu^G}{1-\mu^B}(1-\tau^G) + \frac{\bar{\pi}}{1-\bar{\pi}}\tau^B. \tag{13}$$

An equilibrium is the more efficient the less frequently young buyers make mistakes: accept v_L in the good state and continue after v_L in the bad state. Condition (13) exactly compares the relative cost of the two types of mistakes in the cyclical equilibrium (LHS) and in steady state 1 (RHS). Equation (13) can be rewritten as

$$\frac{\pi}{1-\pi}(1-\mu^G)\left(2\tau^G - 1 - \tau^G_{odd}\right) < \frac{\bar{\pi}}{1-\bar{\pi}}(1-\mu^B)\left(2\tau^B - 1 - \tau^B_{odd}\right).$$

Finding an interior root to $2\tau^\theta - 1 - \tau^\theta_{odd} = 0$ is equivalent to finding the root of $1 - 12\mu^\theta + 9(\mu^\theta)^2 = 0$. The unique interior root is $\hat{\mu} := \frac{2-\sqrt{3}}{3}$. The expression $2\tau^\theta - 1 - \tau^\theta_{odd}$ is positive for small μ^θ and negative for large μ^θ. Thus, a sufficient condition for the cyclical equilibrium to be more efficient than the steady state 1 is $\mu^B < \hat{\mu} < \mu^G$.

If $\mu^G < \hat{\mu}$, condition (13) can be rewritten as

$$\frac{\pi}{1-\pi} < \frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G} \frac{2\tau^B - 1 - \tau^B_{odd}}{2\tau^G - 1 - \tau^G_{odd}}.$$

I can show that this conditions holds if the two equilibria coexist because the RHS of this inequality is larger than the RHS of (12) if $\mu^G < \hat{\mu}$. It suffices to show that $\frac{2\tau^\theta - 1 - \tau^\theta_{odd}}{1-\tau^\theta_{odd}}$, decreases in μ^θ, or that $\frac{1-\tau^\theta_{odd}}{1-\tau^\theta}$ decreases in μ^θ. Thus, I want to show that for all $\mu^B < \mu^G < \hat{\mu}$,

$$\tau^G - \tau^B < \tau^G_{odd} - \tau^B_{odd} + \tau^G \tau^B_{odd} - \tau^B \tau^G_{odd}. \tag{14}$$

But we saw above that $\mu^B < \mu^G < \hat{\mu}$ is the same as saying that $2\tau^G - 1 - \tau^G_{odd} < 2\tau^B - 1 - \tau^B_{odd}$, or, equivalently, $2(\tau^G - \tau^B) < \tau^G_{odd} - \tau^B_{odd}$. But then condition (14) holds for sure if

$$\tau^G_{odd} - \tau^B_{odd} < 2(\tau^G_{odd} - \tau^B_{odd} + \tau^G \tau^B_{odd} - \tau^B \tau^G_{odd}),$$

is satisfied. The last condition simplifies to $\tau^B_{odd}(1-\tau^G) < \tau^G_{odd}(1-\tau^B)$, which holds for sure because $\tau^B_{odd} < \tau^G_{odd}$ and $\tau^B < \tau^G$. Thus, together with the previous paragraph, a sufficient condition for the cyclical equilibrium to be more efficient than steady state 1 is that $\mu^B < \hat{\mu}$.

If $\mu^B > \hat{\mu}$, condition (13) can be rewritten as

$$\frac{\pi}{1-\pi} > \frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G} \frac{2\tau^B - 1 - \tau^B_{odd}}{2\tau^G - 1 - \tau^G_{odd}}.$$

Parameter values exists for which this condition is not satisfied: steady state 1 is more efficient than the cyclical equilibrium for some parameter values. Everything else constant, the condition is easier to satisfy if $\bar{\pi}$ is smaller. □

A.4 A market that starts at $t_1 = 1$

Proof of Proposition 6. The proof for part (i) is separate and for parts (ii) and (iii) is joint. Let $\bar{\omega} := \frac{\bar{\pi}}{1-\bar{\pi}}$ denote the critical odds, where $\bar{\pi}$ is defined in (1).

(i) $\omega < \bar{\omega}\frac{1-\mu^B}{1-\mu^G}$: all buyers trade in their entry period. Steady state 0 is reached at $t = 1$.

Assume that at $t = 1$, a young buyer who gets an offer v_L optimally accepts v_L

(i.e., that $\omega < \bar{\omega}\frac{1-\mu^B}{1-\mu^G}$). But then there are no old buyers at $t = 2$, exactly as at $t = 1$. Hence, all buyers trade at $t = 2$ and in all the following periods. In other words, steady state 0 is reached at $t = 1$.

(ii) $\omega > \bar{\omega}\frac{1-\mu^B}{1-\mu^G}\frac{1-\tau_1^B}{1-\tau_1^G}$, where $\tau_1^G = \mu^G$ and $\tau_1^B = \mu^B$: at all t, young buyers who get an offer v_L continue. Steady state 2 is reached at $t = 2$.

(iii) $\bar{\omega}\frac{1-\mu^B}{1-\mu^G}\frac{\tau_{even}^B}{\tau_{even}^G} < \omega < \bar{\omega}\frac{1-\mu^B}{1-\mu^G}\frac{1-\tau_1^B}{1-\tau_1^G}$, where $\tau_{even}^\theta = 1-(2-\mu^\theta)^{-1}(1-\mu^\theta)\tau_{odd}^\theta$, τ_{odd}^θ solves $(\tau_{odd}^\theta)^2(1-\mu^\theta) = \mu^\theta(1-\tau_{odd}^\theta)$, and $\tau_1^\theta = \mu^\theta$ for $\theta = B, G$: if $\mu^B = 0$ or $\mu^B < \mu^G$ and $\mu^G \to 0$, convergence to cycles where a young buyer who gets an offer v_L in an odd period continues and in an even period accepts v_L if she observes no trade. The cyclical equilibrium is reached in the limit as $t \to \infty$.

I start the market off at $t_1 = 1$ and show that it converges to the two equilibria in the specified times.

At $t = 1$, I assume that a young buyer who gets an offer v_L optimally continues (i.e., that $\omega > \bar{\omega}\frac{1-\mu^B}{1-\mu^G}$). Then $\tau_1^\theta = \mu^\theta$ so a trade at $t = 1$ is good news.

Consider $t = 2$. The amount of old buyers is $O_2^\theta = 1-\mu^\theta$. At $t = 2$ a young buyer who gets an offer v_L and sees good news (a trade), is more optimistic about the state than a young buyer who get an offer v_L at $t = 1$. Since at $t = 1$ the young buyer optimally continued, the more optimistic young buyer at $t = 2$ optimally continues, too. The same argument holds for all subsequent periods: a young buyer who gets an offer v_L and sees good news optimally continues. At $t = 2$, a young buyer who gets an offer v_L and sees bad news (no trade) optimally either continues or accepts v_L. I consider both cases in turn.

(a) Assume that a young buyer who gets an offer v_L and sees bad news at $t = 2$ optimally continues. I show that the necessary and sufficient condition for this is $\omega > \bar{\omega}\left(\frac{1-\mu^B}{1-\mu^G}\right)^2$.

If at $t = 2$ the pessimistic young buyers continue, only old buyers accept v_L at $t = 2$ and the trade probability in state θ is given by (8) where $t = 2$. The solution is $\tau_2^\theta = (2-\mu^\theta)^{-1}$. Thus, a trade at $t = 2$ is good news, but not as good news as at $t = 1$. The pessimistic young buyers do not want to deviate at $t = 2$ if $\pi(v_L, N_1) > \bar{\pi}$. The posterior odds are given by (6) where $t = 2$, explicitly, $\frac{\pi(v_L,N_1)}{1-\pi(v_L,N_1)} = \omega\left(\frac{1-\mu^G}{1-\mu^B}\right)^2$. The inequality $\pi(v_L, N_1) > \bar{\pi}$ is thus equivalent to $\omega > \bar{\omega}\left(\frac{1-\mu^B}{1-\mu^G}\right)^2$. Since all young buyers who get an offer v_L continue, the amount of old buyers at $t = 3$ is $O_3^\theta = 1-\mu^\theta$.

Consider $t = 3$ and recall that a trade at $t = 2$ is good news as $\tau_2^G > \tau_2^B$. At $t = 3$, the pessimistic young buyers' posterior odds are given by (6) where $t = 3$.

The odds are explicitly $\frac{\pi(v_L,N_2)}{1-\pi(v_L,N_2)} = \omega\frac{1-\mu^G}{1-\mu^B}\frac{2-\mu^B}{2-\mu^G}$, which are higher than for the pessimistic young buyers at $t = 2$. Thus, at $t = 3$ the pessimistic young buyers continue, the trade probabilities are exactly like at $t = 2$, and steady state 2 is reached at $t = 2$. The condition that ensures convergence to steady state 2 is $\omega > \bar{\omega}\left(\frac{1-\mu^B}{1-\mu^G}\right)^2$.

(b) Assume now that a young buyer who gets an offer v_L and sees bad news at $t = 2$, no trade, accepts v_L. We know from Part (a) that the necessary and sufficient condition for this to be optimal is that

$$\omega < \bar{\omega}\left(\frac{1-\mu^B}{1-\mu^G}\right)^2. \tag{15}$$

I show that if this condition holds, the market converges to the cyclical equilibrium for an open set of parameter values. In particular, I derive the necessary and sufficient conditions for the market to converge to the cyclical equilibrium separately for μ^G small (and $0 \leq \mu^B < \mu^G$) and for $\mu^B = 0$ (and any $\mu^G > \mu^B$).

I show that if μ^G is small and $\mu^B \in (0, \mu^G)$, the relative sizes of trade probabilities are as required for all t and as $t \to \infty$, converge to the trading probabilities of the cyclical equilibrium in Proposition 4. If that is the case, a necessary and sufficient condition for the market to converge to the cyclical equilibrium is that no buyer wants to deviate, i.e., that the most optimistic (pessimistic) of the buyers who at any t is supposed to accept v_L (continue) optimally does so.

First, I show for all $t \geq 1$ that a trade that takes place in an odd period $2t+1$ is good news for any μ^G and μ^B because $\tau_1^G > \tau_1^B$. In particular, I show that $\tau_1^G > \tau_1^B$ implies that $\frac{\tau_{2t+1}^G}{\tau_{2t+1}^B} > 1$ for any $t \geq 1$. Equations (5) and (8) together imply that in an odd period $2t+1$, the trade probability can be written as

$$\tau_{2t+1}^\theta = 1 - (1-\mu^\theta)[1 + (1-\mu^\theta)\tau_{2t-1}^\theta]^{-1}. \tag{16}$$

Thus, in any odd period

$$\frac{\partial \tau_{2t+1}^\theta}{\partial \mu^\theta} = (1-\tau_{2t+1}^\theta)(1-\mu^\theta)^{-1} + (1-\tau_{2t+1}^\theta)^2(1-\mu^\theta)^{-1}\frac{\partial \tau_{2t-1}^\theta}{\partial \mu^\theta}.$$

Altogether, $\frac{\partial \tau_1^\theta}{\partial \mu^\theta} > 0$ implies that $\frac{\partial}{\partial \mu^\theta}\tau_{2t+1}^\theta > 0$ for all odd $2t+1 \geq 3$.

Second, I show that τ_{2t+1}^θ and τ_{2t}^θ as sequences in t converge respectively to τ_{odd}^θ and τ_{even}^θ, the trade probabilities of the cyclical equilibrium in a market without a starting date. From (16) it follows that $\tau_{2t+1}^\theta \geq \tau_{2t-1}^\theta$ if $\mu^\theta(1-\tau_{2t-1}^\theta) \geq (1-\mu^\theta)(\tau_{2t-1}^\theta)^2$, which holds with strict inequality for $\tau_{2t-1}^\theta = \tau_1^\theta$ and with equality for $\tau_{2t-1}^\theta = \tau_{odd}^\theta$. So we know that $\tau_3^\theta > \tau_1^\theta$. But this implies that $\tau_{2t+1}^\theta > \tau_{2t-1}^\theta$ for all

t because τ^θ_{2t+1} increases in τ^θ_{2t-1}: $\frac{\partial \tau^\theta_{2t+1}}{\partial \tau^\theta_{2t-1}} = (1-\mu^\theta)^2[1+(1-\mu^\theta)\tau^\theta_{2t-1}]^{-2} > 0$. Also, $\tau^\theta_{2t+1}/\tau^\theta_{2t-1}$ decreases in τ^θ_{2t-1}:

$$\frac{\partial}{\partial \tau^\theta_{2t-1}} \frac{\tau^\theta_{2t+1}}{\tau^\theta_{2t-1}} \propto -2\tau^\theta_{2t-1}\mu^\theta(1-\mu^\theta) - \mu^\theta - (\tau^\theta_{2t-1})^2(1-\mu^\theta)^2 < 0.$$

Thus, the sequence τ^θ_{2t+1} converges in t to τ^θ_{odd}. Note that from (4) and (9) it follows that for any even period $2t$,

$$\tau^\theta_{2t} = 1 - (1-\mu^\theta)(2-\mu^\theta)^{-1}\tau^\theta_{2t-1},$$

and we know that in the cyclical equilibrium in a market without a starting date, $\tau^\theta_{even} = 1-(1-\mu^\theta)(2-\mu^\theta)^{-1}\tau^\theta_{odd}$. Thus, τ^θ_{2t} converges to τ^θ_{even}.

Now I show that $\frac{\tau^G_{2t}}{\tau^B_{2t}} < 1$ for all t if μ^G is small and $\mu^B \in [0, \mu^G)$. I will show that τ^θ_{2t} decreases in μ^θ in the limit. The derivative is

$$\frac{\partial \tau^\theta_{2t}}{\partial \mu^\theta} \propto \tau^\theta_{2t-1} - (1-\mu^\theta)(2-\mu^\theta)\frac{\partial \tau^\theta_{2t-1}}{\partial \mu^\theta}.$$

As $\mu^\theta \to 0$, $\tau^\theta_{2t-1} \to 0$ and

$$\lim_{\mu^\theta \to 0} \frac{\partial \tau^\theta_{2t-1}}{\partial \mu^\theta} = 1 + \lim_{\mu^\theta \to 0} \frac{\partial \tau^\theta_{2t-3}}{\partial \mu^\theta} \geq 1.$$

But then $\lim_{\mu^\theta \to 0} \frac{\partial \tau^\theta_{2t}}{\partial \mu^\theta} \propto -2 \lim_{\mu^\theta \to 0} \frac{\partial \tau^\theta_{2t-1}}{\partial \mu^\theta} < 0$, as required.

Finally, I derive the conditions under which the most optimistic (pessimistic) of the buyers who along the path is supposed to accept v_L (continue) optimally does so. Thus, I need that a young buyer who gets an offer v_L and sees no trade of any odd period wants to accept v_L. I show that the most optimistic of these buyers is the one who sees no trade of the first period. It is easy to show that

$$\frac{\partial}{\partial \mu^\theta} \frac{1-\tau^\theta_{2t+1}}{1-\tau^\theta_{2t-1}} = -(1-\tau^\theta_{2t-1})^{-1}\frac{\partial \tau^\theta_{2t+1}}{\partial \mu^\theta} + (1-\tau^\theta_{2t+1})(1-\tau^\theta_{2t-1})^{-2}\frac{\partial \tau^\theta_{2t-1}}{\partial \mu^\theta},$$

which in the limit is $\lim_{\mu^\theta \to 0} \frac{1-\tau^\theta_{2t+1}}{1-\tau^\theta_{2t-1}} = -1$, where I have used $\lim_{\mu^\theta \to 0} \tau^\theta_{2t-1} = 0$ and the above limit results. If $\frac{1-\tau^\theta_{2t+1}}{1-\tau^\theta_{2t-1}}$ decreases in μ^θ, then, equivalently, $\frac{1-\tau^G_{2t-1}}{1-\tau^B_{2t-1}} > \frac{1-\tau^G_{2t+1}}{1-\tau^B_{2t+1}}$. Thus, all young buyers who get an offer v_L and see no trade of an odd period want to accept v_L if the condition $\frac{\pi(v_L,N_1)}{1-\pi(v_L,N_1)} \leq \bar{\omega}$, or, (15), holds.

I also need that a young buyer who gets an offer v_L and sees trade of an even period wants to continue. It is easy to show that

$$\frac{\partial}{\partial\mu^\theta}\frac{\tau^\theta_{2t+2}}{\tau^\theta_{2t}} \propto \tau^\theta_{2t}\left[\tau^\theta_{2t+1}-(1-\mu^\theta)(2-\mu^\theta)\frac{\partial\tau^\theta_{2t+1}}{\partial\mu^\theta}\right]-\tau^\theta_{2t+2}\left[\tau^\theta_{2t-1}-(1-\mu^\theta)(2-\mu^\theta)\frac{\partial\tau^\theta_{2t-1}}{\partial\mu^\theta}\right],$$

which in the limit is

$$\lim_{\mu^\theta\to 0}\frac{\partial}{\partial\mu^\theta}\frac{\tau^\theta_{2t+2}}{\tau^\theta_{2t}} \propto -2.$$

If $\frac{\tau^\theta_{2t+2}}{\tau^\theta_{2t}}$ decreases in μ^θ, then, equivalently, $\frac{\tau^G_{2t}}{\tau^B_{2t}} > \frac{\tau^G_{2t+2}}{\tau^B_{2t+2}}$. Thus, all young buyers who get an offer v_L and see a trade of an even period want to continue if the limit condition $\frac{\pi(v_L,T_{even})}{1-\pi(v_L,T_{even})} = \omega\frac{1-\mu^G}{1-\mu^B}\frac{\tau^G_{even}}{\tau^B_{even}} \geq \bar{\omega}$ holds, or, equivalently, if

$$\omega \geq \bar{\omega}\frac{1-\mu^B}{1-\mu^G}\frac{\tau^B_{even}}{\tau^G_{even}}. \tag{17}$$

I show that conditions (15) and (17) are necessary and sufficient for any $\mu^G < 1$ if $\mu^B = 0$. If $\mu^B = 0$, $\tau^B_{2t-1} = 0$ and $\tau^B_{2t} = 1$ for all $t \geq 1$. Thus, for $\frac{1-\tau^G_{2t-1}}{1-\tau^B_{2t-1}} > \frac{1-\tau^G_{2t+1}}{1-\tau^B_{2t+1}}$ and $\frac{\tau^G_{2t}}{\tau^B_{2t}} > \frac{\tau^G_{2t+2}}{\tau^B_{2t+2}}$ to hold, it is sufficient that $\tau^G_{2t+1} > \tau^G_{2t-1}$. But we know that this condition holds along the path. Hence, for any $\mu^G < 1$, the necessary and sufficient conditions for the market to converge to the cyclical equilibrium are (15) and (17) if $\mu^B = 0$. □

B Appendix B

Appendix B contains details on the robustness checks presented in the Concluding discussion. For brevity, I focus on the case where $\mu^G = \frac{1}{2}$ and $\mu^B = 0$, but the assumption is necessary for neither of the below results.

B.1 Long-lived buyers

Consider a model where a mass one of buyers enters in each period and each buyer can live for ever, but survives till the next period with a fixed probability $\delta \in (0,1)$. The survival probability replaces the discount factor and ensures the existence of a steady-state equilibrium. A buyer observes in each period of life whether a randomly drawn buyer traded in the previous period or did not trade. That is, the buyer can tell trades apart from exits due to the exogenous destruction rate. In this version of the model, some buyers learn the state if the state is good so the equilibrium strategy must specify their behaviour. A cyclical equilibrium is sustained by an open set of parameter values.

Proposition 7. *Let $\mu^G = \frac{1}{2}$, $\mu^B = 0$, buyers be infinitely-lived and in each period survive to the next period with probability $\delta \in (0,1)$. Sufficient conditions for a strategy whereby a buyer who gets an offer v_L*

(i) and knows that the state is good, continues,

(ii) and does not know the state,

- *in odd periods, continues and*
- *in even periods, continues after observing a trade, and accepts v_L after observing no trade,*

to be an equilibrium are that $\frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G} \frac{\tau^B_{even}}{\tau^G_{even}} < \pi < \frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G} \frac{1-\tau^B_{odd}}{1-\tau^G_{odd}}$, where the probabilities of trading are $\tau^B_{odd} = 0$, $\tau^B_{even} = 1$, $\tau^G_{odd} = 1/2$, and

$$\tau^G_{even} = \frac{1}{2\delta^3}\left(16 + 6\delta - 2\delta^2 + \delta^3 - \sqrt{256 + 192\delta - 28\delta^2 - 40\delta^3 + 4\delta^5 + \delta^6}\right).$$

In equilibrium, a trade in an odd period is good news and a trade in an even period is bad news. The probabilities of trading are ordered $\tau^B_{odd} < \tau^G_{odd} < \tau^G_{even} < \tau^B_{even}$. The probability fluctuates more in the bad than in the good state.

Proof. I construct an equilibrium where

(i) buyers with posterior belief $\pi' = 1$ only accept v_H,

(ii) in any odd period $2t+1$, buyers with a posterior belief $\pi' < 1$ continue after getting an offer v_L (regardless of whether they see N_{2t} or T_{2t}), and

(iii) in any even period $2t$, buyers with a posterior belief $\pi' < 1$ continue after getting an offer v_L and observing T_{2t-1}, and accept v_L after getting an offer v_L and observing N_{2t-1}.

I first derive the probability of trading and amounts of buyers given the above strategy and then the conditions under which no buyer has an incentive to deviate. Let the mass of uninformed buyers (i.e., buyers who are uncertain of the state) be denoted by M^θ_t and the total mass of buyers by N^θ_t at t and in state θ as measured at the start of t, after entry.

The probability of trading at an odd period $2t-1$ is $\tau^\theta_{2t-1} = \mu^\theta$, because all buyers accept a high-value offer and no buyer accepts a low-value offer. A trade at $2t-1$ is good news because $\tau^G_{2t-1} > \tau^B_{2t-1}$. The probability of trading at $2t$ is

$$\tau^\theta_{2t} = \mu^\theta + (1-\mu^\theta)\frac{M^\theta_{2t}}{N^\theta_{2t}}(1-\tau^\theta_{2t-1}),$$

because all buyers accept an offer v_H and a buyer accepts v_L if she is uncertain of the state and observes no trade. The probabilities at $2t$ are explicitly $\tau^G_{2t} = \frac{1}{2}\left(1 + \frac{M^G_{2t}}{N^G_{2t}}\frac{1}{2}\right)$ and $\tau^B_{2t} = \frac{M^B_{2t}}{N^B_{2t}} = 1$, where the last equality follows from the fact that no buyer can know that the state is good if the state is in fact bad. A trade at $2t$ is bad news because $\tau^G_{2t} < 1 = \tau^B_{2t}$. Note that if the cycles are two periods long, then $\tau^G_{2t} = \tau^G_{2t-2}$ and observing no trade at $2t-1$ (N_{2t-2}) or a trade at $2t$ (T_{2t-1}), reveal the good state (so the equilibrium strategy has to specify what buyers who know that the state is good do).

Now I derive M^G_{2t} and N^G_{2t} to get a closed-form solution for τ^G_{2t}. What are the flows into these masses of buyers? First, consider period $2t-1$. How many buyers who start at $2t-1$ as uninformed, M^G_{2t-1}, reach $2t$ as uninformed? An uninformed buyer does not learn the state at $2t-1$ if she gets an offer v_L and observes a trade (T_{2t-2}). These buyers continue and they reach $2t$ with probability δ. In addition to them, the buyers who enter at $2t$ start off as uninformed. Thus,

$$M^G_{2t} = M^G_{2t-1}(1-\mu^G)\tau^G_{2t-2}\delta + 1.$$

Consider an even period $2t$. How many buyers who start at $2t$ as uninformed, M^G_{2t}, reach period $2t+1$ as uninformed? An uninformed buyer does not learn the state at $2t$ if she gets an offer v_L and observes no trade (N_{2t-1}). But all of these buyers accept v_L according to the proposed strategy so no uninformed buyers are carried over to $2t+1$ from $2t$. All buyers who enter at $2t+1$ start off as uninformed. Thus, $M^G_{2t+1} = 1$.

How many buyers who start at $2t-1$ as informed, $N^G_{2t-1} - M^G_{2t-1}$, reach period $2t$ as informed? All informed buyers remain informed, but some of them exit: only those reach period $2t$ who get an offer v_L and survive. Buyers who start $2t-1$ off as uninformed, in the amount M^G_{2t-1}, reach period $2t$ as informed if they become informed, don't exit at $2t-1$, and survive. They become informed if they get an offer v_H or observe N_{2t-2}. They continue if they get an offer v_L, regardless of the signal outcome. Thus, the amount of informed buyers at $2t$ is

$$N^G_{2t} - M^G_{2t} = (N^G_{2t-1} - M^G_{2t-1})(1-\mu^G)\delta + M^G_{2t-1}(1-\mu^G)(1-\tau^G_{2t-2})\delta.$$

Finally, how many buyers who start at $2t$ as informed, $N^G_{2t} - M^G_{2t}$, reach period $2t+1$ as informed? All informed buyers remain informed, but only those reach $2t+1$ who at $2t$ see v_L and survive. Buyers who start $2t$ off as uninformed, M^G_{2t}, reach period $2t+1$ as informed if they get an offer v_L, observe T_{2t-1}, and survive.

Thus, the amount of informed buyers at $2t+1$ is

$$N^G_{2t+1} - M^G_{2t+1} = (N^G_{2t} - M^G_{2t})(1-\mu^G)\delta + M^G_{2t}(1-\mu^G)\tau^G_{2t-1}\delta.$$

Combining these equations and imposing that $x_{t'+2} = x_{t'}$ for $t' = 2t, 2t-1$ and all endogenous variables x, gives a solution

$$\tau^G_{2t} = \frac{1}{2\delta^3}(16 + 6\delta - 2\delta^2 + \delta^3 - \sqrt{256 + 192\delta - 28\delta^2 - 40\delta^3 + 4\delta^5 + \delta^6}),$$

which decreases in δ and is in the interval $[\frac{21-\sqrt{385}}{2} \approx 0.69, \frac{3}{4}]$ for all $\delta \in (0,1)$.

Finally, I derive the conditions under which the proposed strategy is optimal. For a buyer with belief π', the value of continuing for one more period and then accepting either offer is given by equation (3) so the critical belief is again $\bar{\pi}$.

Let the beliefs of a buyer who has seen h of T_{2t-1}, i of N_{2t-1}, j of T_{2t}, and k of N_{2t}, be $\pi(h,i,j,k)$. Since odd and even periods alternate, it must be that $h+i \in \{j+k-1, j+k, j+k+1\}$. A sufficient condition for the proposed strategy to be optimal is that a buyer who is supposed to continue according to the strategy wants to continue for at least one period and that a buyer who is supposed to accept v_L according to the strategy prefers accepting v_L to continuing for one more period. Then the strategy is optimal if the following three sets of conditions hold:

(i) buyers who know the state to be good prefer to continue after v_L: $\pi(h,i,j,k) > \bar{\pi}$ for all $h,k \geq 1$,

(ii) buyers who do not know the state and have not seen N_{2t-1} continue after v_L: $\pi(0,0,j,0) > \bar{\pi}$ for all j (i.e., for $j = 0,1$), and

(iii) buyers who do not know the state and have seen at least one N_{2t-1} accept v_L: $\pi(0,i,j,0) < \bar{\pi}$ for all $i \geq 1$ and all j.

The set of conditions in (i) is satisfied as $\pi(h,i,j,k) = 1$ for all $h,k \geq 1$. Of the conditions in set (ii), the stricter is for the more pessimistic buyer, i.e., for $j=1$ since T_{2t} is bad news. The stricter condition, $\pi(0,0,1,0) > \bar{\pi}$, can be written as

$$\frac{\pi(0,0,1,0)}{1-\pi(0,0,1,0)} = \omega\frac{1-\mu^G}{1-\mu^B}\frac{\tau^G_{2t}}{\tau^B_{2t}} = \frac{\omega\tau^G_{2t}}{2} > \frac{\bar{\pi}}{1-\bar{\pi}}.$$

Of the conditions in set (iii), the strictest is for the most optimistic buyer, i.e., for $i=1$ and $j=0$ because both N_{2t-1} and T_{2t} are bad news. The strictest condition,

$\pi(0,1,0,0) < \bar{\pi}$, can be written as

$$\frac{\pi(0,1,0,0)}{1-\pi(0,1,0,0)} = \omega \frac{1-\mu^G}{1-\mu^B}\frac{1-\tau^G_{2t-1}}{1-\tau^B_{2t-1}} = \frac{\omega}{4} < \frac{\bar{\pi}}{1-\bar{\pi}}.$$

The conditions can be satisfied simultaneously because $\frac{1}{4} < \frac{\tau^G_{2t}}{2}$. I rearrange the two inequalities to get the exact conditions in the Proposition. □

B.2 Partially observed value of trade

Consider the main model with short-lived buyers, except that a fraction $1-\varepsilon$ of the buyers observe a trade signal as before (that is, without observing the value at which the trade/no trade took place) and a fraction ε observe not only whether another buyer traded, but also the value v at which this trade/no trade took place. For small enough ε, a cyclical equilibrium is sustained by a strategy that is similar to the one described in Proposition 4.

The modification in the strategy comes about because a trade (no trade) at value v_L tells a buyer not only about the event of a trade (no trade) at v_L, but also that another buyer got an offer v_L. It is as bad news as observing another low-value offer. If a trade at v_L in an even period is bad news, a young buyer who in an odd period gets an offer v_L and observes a trade at v_L can become pessimistic enough about the state that she accepts v_L in equilibrium.

Proposition 8. *Let $\mu^G = \frac{1}{2}$ and $\mu^B = 0$. A strategy whereby a young buyer who gets an offer v_L*

(i) in an odd period, accepts v_L after observing a trade at value v_L and continues otherwise, and

(ii) in an even period, accepts v_L after observing no trade or no trade at value v_L and continues otherwise,

is an equilibrium for an open set of parameter values if $\varepsilon < \bar{\varepsilon}$ for some $\bar{\varepsilon} > 0$.

In the cyclical equilibrium, a trade in an odd period is good news and in an even period is bad news. A trade at value v_L in an odd period is good news and in an even period is bad news.

Proof. Let λ^θ_t denote the conditional probability and TL_t the event that a randomly drawn buyer who gets an offer v_L trades at t in state θ. Let NL_t denote the event that the buyer does not trade.

I first derive the probabilities of trading and amounts of buyers given the proposed strategy and then the conditions under which no buyer has an incentive to deviate. Finally, I derive conditions under which a trade (at value v_L) in an odd period is good news and in an even period is bad news.

Consider an odd period $2t-1$ and state θ. The only young buyers who at $2t-1$ get an offer v_L also accept v_L (and do not become old) are those who observe a trade at value v_L (TL_{2t-2}). The amount of old buyers at $2t$ is thus

$$O^{\theta}_{2t} = (1-\mu^{\theta})\left[1-\varepsilon(1-\mu^{\theta})\lambda^{\theta}_{2t-2}\right],$$

where the second $(1-\mu^{\theta})$ accounts for the fact that the buyer whose trade information is observed at $2t-1$ got an offer v_L at $2t-2$. The unconditional trading probability at $2t-1$ is

$$\tau^{\theta}_{2t-1} = [O^{\theta}_{2t-1} + \mu^{\theta} + (1-\mu^{\theta})\varepsilon(1-\mu^{\theta})\lambda^{\theta}_{2t-2}](1+O^{\theta}_{2t-1})^{-1}.$$

because all old buyer trade, young buyers trade at value v_H always and at value v_L if they observe a trade at value v_L. The probability of a trade at value v_L is

$$\lambda^{\theta}_{2t-1} = [O^{\theta}_{2t-1} + \varepsilon(1-\mu^{\theta})\lambda^{\theta}_{2t-2}](1+O^{\theta}_{2t-1})^{-1},$$

because, conditional on getting an offer v_L, a buyer accepts v_L if she is old or if she is young and observes a trade at v_L.

In an even period $2t$, young buyers continue if they get an offer v_L and see a trade (T_{2t-1}), a trade at v_L (TL_{2t-1}), or a trade/no trade at value v_H. The amount of old buyers at $2t+1$ is thus

$$O^{\theta}_{2t+1} = (1-\mu^{\theta})\left[(1-\varepsilon)\tau^{\theta}_{2t-1} + \varepsilon(1-\mu^{\theta})\lambda^{\theta}_{2t-1} + \varepsilon\mu^{\theta}\right].$$

The unconditional trading probability at $2t$ is

$$\tau^{\theta}_{2t} = \left\{O^{\theta}_{2t} + \mu^{\theta} + (1-\mu^{\theta})\left[(1-\varepsilon)(1-\tau^{\theta}_{2t-1}) + \varepsilon(1-\mu^{\theta})(1-\lambda^{\theta}_{2t-1})\right]\right\}(1+O^{\theta}_{2t})^{-1},$$

because all old buyer trade, young buyers trade value v_H always and value v_L if they observe no trade or no trade at v_L. The probability of a trade at v_L is

$$\lambda^{\theta}_{2t} = [O^{\theta}_{2t} + (1-\varepsilon)(1-\tau^{\theta}_{2t-1}) + \varepsilon(1-\mu^{\theta})(1-\lambda^{\theta}_{2t-1})](1+O^{\theta}_{2t})^{-1},$$

because, conditional on getting an offer v_L, a buyer accepts v_L if she is old or if she is young and observes either no trade or no trade at value v_L. Note that $\tau^{B}_{t'} = \lambda^{B}_{t'}$ and $\tau^{G}_{t'} > \lambda^{G}_{t'}$ for $t' = 2t-1, 2t$ because $\mu^{B} = 0$ and $\mu^{G} > 0$.

Imposing the condition that $x_{t'+2} = x_{t'}$ for all endogenous x and $t' = 2t-1, 2t$

and solving the system of equations for the bad state gives trade probabilities $\lambda^B_{2t-1} = \tau^B_{2t-1} = \sqrt{\varepsilon\tau^B_{2t}}$, $\lambda^B_{2t} = \tau^B_{2t}$, and τ^B_{2t} solves

$$\sqrt{\varepsilon\tau^B_{2t}} = (2 - \varepsilon\tau^B_{2t})(1 - \tau^B_{2t}).$$

The probability τ^B_{2t} decreases in ε. Letting $y := 3 - \sqrt{5 + 2\varepsilon\lambda^G_{2t}}$, the system of equations for the good state collapses to, $\lambda^G_{2t-1} = 1 - y$, $\tau^G_{2t-1} = \frac{1}{2} + \frac{\lambda^G_{2t-1}}{2}$, $\tau^G_{2t} = \frac{1}{2} + \frac{\lambda^G_{2t}}{2}$, and λ^G_{2t} solves $4(2 - \varepsilon\lambda^G_{2t}) = y[14 - \lambda^G_{2t}(6 - \varepsilon\lambda^G_{2t} + \varepsilon)]$.

The equilibrium exists for parameter values such that no buyer wants to deviate. For a young buyer with belief π', the value of continuing and accepting either offer when old is given by equation (3) so the critical belief is again $\bar{\pi}$ (defined in (1)). A young buyer who gets an offer v_L and sees a trade (no trade) updates according to equation (7) (equation (6)) respectively. A young buyer who at $t' = 2t - 1, 2t$ gets an offer v_L and observes that a buyer b' accepted and offer v_L updates her beliefs as

$$\frac{\pi(v_L, TL_{t'})}{1 - \pi(v_L, TL_{t'})} = \omega \frac{1 - \mu^G}{1 - \mu^B} \frac{\lambda^G_{t'}}{\lambda^B_{t'}} \frac{1 - \mu^G}{1 - \mu^B} = \frac{\omega}{4} \frac{\lambda^G_{t'}}{\lambda^B_{t'}},$$

where the second fraction $\frac{1-\mu^G}{1-\mu^B}$ accounts for the fact that b' must have received an offer v_L. A young buyer who at $t' = 2t - 1, 2t$ gets an offer v_L and observes no trade at offer v_L updates her beliefs as

$$\frac{\pi(v_L, NL_{t'})}{1 - \pi(v_L, TL_{t'})} = \omega \frac{1 - \mu^G}{1 - \mu^B} \frac{1 - \lambda^G_{t'}}{1 - \lambda^B_{t'}} \frac{1 - \mu^G}{1 - \mu^B} = \frac{\omega}{4} \frac{1 - \lambda^G_{t'}}{1 - \lambda^B_{t'}}.$$

No buyer wants to deviate from the proposed equilibrium strategy if

$$\min\{\pi(v_L, T_{2t-1}), \pi(v_L, TL_{2t-1}), \pi(v_L, N_{2t}), \pi(v_L, T_{2t}), \pi(v_L, NL_{2t})\} \geq \bar{\pi}, \quad (18)$$

and

$$\max\{\pi(v_L, N_{2t-1}), \pi(v_L, NL_{2t-1}), \pi(v_L, TL_{2t})\} \leq \bar{\pi}. \quad (19)$$

I show how to considerably reduce the set of constraints (18) and (19). Note that $\pi(v_L, T_{2t-1}) > \pi(v_L, TL_{2t-1})$ because $\tau^G_{2t-1} > \lambda^G_{2t-1}$ and $\tau^B_{2t-1} = \lambda^B_{2t-1}$. Since no trades take place only at value v_L, the information contained in $N_{t'}$ is exactly the same as that contained in $NL_{t'}$ for any t'. If a trade in an even period $2t$ is bad news, then $\pi(v_L, N_{2t}) > \pi(v_L, T_{2t})$. The set of constraints thus reduces to $\min\{\pi(v_L, TL_{2t-1}), \pi(v_L, T_{2t}))\} \geq \bar{\pi}$, and $\max\{\pi(v_L, N_{2t-1}), \pi(v_L, TL_{2t})\} \leq \bar{\pi}$.

I show that the equilibrium exists for ε small enough. The limits of the trading probabilities as $\varepsilon \to 0$ are $\tau^B_{2t-1} = \lambda^B_{2t-1} \to 0$, and $\tau^B_{2t} = \lambda^B_{2t} \to 1$ in state B and $\tau^G_{2t-1} \to \frac{\sqrt{5}-1}{2}$, $\lambda^G_{2t-1} \to \sqrt{5}-2$, $\tau^G_{2t} \to \frac{7-\sqrt{5}}{6}$ and $\lambda^G_{2t} \to \frac{17-7\sqrt{5}}{3(3-\sqrt{5})}$ in state G. It is easy to check that, in the limit, the inequalities on the posteriors that are required for the proposed strategy to constitute an equilibrium can be satisfied for an open set of parameter values.

Now I show that for small ε, the trading probabilities are true probabilities so that a solution exists for ε close to zero. In state B, the condition $(2 - \varepsilon\tau^B_{2t})(1 - \tau^B_{2t}) = \sqrt{\varepsilon\tau^B_{2t}}$ holds. As τ^B_{2t} decreases in ε, the only way this equality can hold is if $\varepsilon\tau^B_{2t}$ increases in ε. But $\tau^B_{2t-1} = \sqrt{\varepsilon\tau^B_{2t}}$ is thus positive for $\varepsilon > 0$. As τ^B_{2t} decreases in ε, both τ^B_{2t-1} and τ^B_{2t} are less than one for $\varepsilon > 0$.

Similarly, I can show that as ε increases, $\varepsilon\lambda^G_{2t}$ must increase (and thus y decrease) so that λ^G_{2t-1} and τ^G_{2t-1} both increase in ε. Close to $\varepsilon = 0$, both are thus positive (and far from zero). Both λ^G_{2t} and τ^G_{2t} are below one for any positive ε, τ^G_{2t-1}, and λ^G_{2t-1}. Because everything is continuous, the proposed strategy is an equilibrium for an open set of parameter values for all $\varepsilon < \bar{\varepsilon}$ for some $\bar{\varepsilon} > 0$. □

Intuitively, the informational content of a trade at an unknown value can fluctuate, but it is ex ante plausible that a trade at value v_L is bad news in all periods. The reason why a trade at v_L is good news in an odd period is because of the composition effect: there are many more old buyers if the state is good and they accept low-value offers.

If ε is large, an equilibrium in these strategies does not exist because the probability that a low-value offer trades is higher in the bad than in the good state in all periods. If $\varepsilon = 1$, for example, in the good state half of the young buyers observe a trade at v_H, learn that the state is good and reject v_L. Conversely, in the bad state, all buyers observe trade information about low-value offers, which leads to more than a half of the young buyers accepting v_L. Thus, a trade at v_L is bad news in all periods.

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