Marketmaking Middlemen*

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Very Preliminary

Abstract

This paper develops a model in which market structure is determined endogenously by the choice of intermediation mode. We consider two representative business modes of intermediation that are widely used in real-life markets: one is a *market-making mode* by which an intermediary offers a platform for buyers and sellers to trade by their own; the other is a *middleman mode* by which an intermediary holds inventories which he stocks from sellers for the purpose of reselling to buyers. In our model, buyers and sellers have an option of searching in an outside market as well as using the service offered by a monopolistic intermediary. We derive the conditions under which the mixture of the two intermediation modes is selected over an exclusive use of either of the modes.

Keywords: Middlemen, Marketmakers, Platform, Search

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1 Introduction

This paper considers a framework in which market structure is determined by the intermediation service offered to customers. We consider two representative business modes of intermediation that are widely used in real-life markets. In one mode, an intermediary acts as a *middleman* (or a *merchant*), who is specialized in buying and selling by his own account and typically operates by holding inventory (e.g. supermarkets, and traditional brick and mortar retailers). In the other mode, an intermediary acts as a *marketmaker*, who offers a marketplace or a platform where the participating buyers and sellers trade with each other (e.g. eBay).

In most real-life markets, however, intermediaries are not one of those extremes but operate both as a middleman and a marketmaker at the same time. This is what we call a marketmaking middleman.[1] For example, the well-known electronic intermediary Amazon, one of the largest marketmaking middlemen nowadays, started off as a pure middleman, buying and reselling products in its own name since its founding in 1994. In the early 2000s, Amazon moved toward a marketmaker, by allowing other suppliers to participate as independent sellers. In 2014, those independent sellers accounted for 50% of gross merchandise volume of Amazon. Similar operation patterns are used by both Rakuten and JD.com, which are Amazon's counterparts in Japan and China. Other examples of marketmaking middlemen include real-estate agents, dealer/brokers in financial markets, and department stores.[2]

We present a model in which the intermediated-market structure is determined endogenously as a result of the strategic choice of a monopolistic intermediary. In our model, there are two markets open to agents, one is an intermediated market operated by the intermediary, and the other is a decentralized market where buyers and sellers search randomly. The intermediated market combines two business modes: acting as a middleman, the intermediary is prepared to serve many buyers at a time by holding inventories; acting as a marketmaker, the intermediary offers a platform. The intermediary can choose how to allocate the attending buyers among these two business modes.

[1] In the finance literature, different terminologies are used to classify the type of intermediaries: brokers refer to intermediaries who do not trade for their own account, but act merely as conduits for customer orders, akin to our marketmakers; dealers refer to intermediaries who do trade for their own account, akin to our middlemen/merchants. The marketmakers (or specialists) in financial markets correspond broadly to our market-making middlemen, since they quote prices to buy or sell the asset as well as take market positions.

[2] In particular, real-estate agents match buyers and sellers, and also stock properties themselves for sale; dealer/brokers in financial markets engage in trading securities on behalf of clients as well as for their own accounts; some department stores also rent shelf spaces to manufacturers. Further examples can be found in Hagiu and Wright (2015).

In either way of intermediated trade, we formulate the intermediated market as a directed search market in order to feature the intermediary's technology of spreading price and capacity information efficiently.[3] In addition, this approach enables us to highlight the middleman's advantage in the high selling capacity that mitigates search frictions and provides customers with proximity. Thus, the middleman mode outperforms the marketmaker mode in allocation efficiency. The decentralized market represents an individuals' outside option that determines the lower bound of their market utility.

With this set up, we consider two situations, *single-market search* versus *multiple-market search*. With the single market search, agents have to choose which market to search in advance, either the decentralized market or the intermediated market. This implies that the intermediary needs to subsidize either buyers or sellers with their expected value in the decentralized market, but once they participate, the intermediated market operates without fear of competitive pressure outside. In this situation, the intermediary can extract the individual surplus monopolistically for each realized transaction. This can be done either by setting a monopolistic intermediation fee as a marketmaker, or by charging a monopolistic price for the inventory as a middleman – in either way, the per-transaction profits are the same. Given the middleman mode is more efficient in realizing transactions, the intermediary uses the middleman-mode exclusively when agents search a single market.

When agents are allowed to search multiple markets, attracting buyers and sellers becomes less costly compared to the single-market search case — the intermediary does not need to subsidy either of the sides to induce participation. However, this time, the prices/fees charged in the intermediated market must be acceptable relative to the available option in the decentralized market. Thus, with the multiple-market search, the outside option creates competitive pressure to the overall intermediated market. On the one hand, a higher capacity of the middleman leads to a larger number of successful transactions in the intermediated market. This happens due to the demand stimulating effect: a higher capacity induces more buyers to buy from the middleman, and fewer buyers to search in the platform, which increases the intermediary's profit. On the other hand, the demand effect makes sellers less likely to be successful in the platform, so that more sellers are available when a buyer attempts to search in the decentralized market. Accordingly, the buyers expect a higher value from the decentralized market, which causes a competitive pressure on

[3] Using the search function in the Amazon website, for example, one can receive instantly all relevant information such as prices, stocks of individual sellers and Amazon's inventories.

the price that the intermediary can charge. Hence, the intermediary trade-offs a larger quantity against lower price/fees to operate as a larger-scaled middleman. This trade-off pins down an optimal structure of intermediation modes.

In real-life economy, the single-market search may correspond to the traditional search technology for supermarkets or brick and mortar retailers. Over the course of shopping trip under such a circumstance, consumers usually have to search, buy and even transport the purchased products during a fixed amount of time. Given the time constraint, they visit a limited number of shops – typically one supermarket –, and appreciate the proximity provided by its inventory. In contrast, the multi-market search may be related to the advanced search technologies available in the digital economy. Such a technology allows the online-customers to search with ease and convenience, and to compare various options. A similar can happen in the market for durable goods such as housing or expensive items where customers are usually exposed to the market for a sufficiently long time period to ponder multiple available options.

The comparison of the above two cases delivers an interesting policy implication of intermediated markets. In our economy, the optimal business mode from the social planner's viewpoint is the middleman mode because it minimizes market frictions and can implement an efficient trade. In the presence of outside competition, however, using a platform can be profitable for the intermediary and it reduces its inventory from the efficient level. Hence, competition for intermediation can deteriorate efficiency and welfare.

This paper contributes to two strands of literature. One is the literature of middlemen developed by Rubinstein and Wolinsky (1987), Duffie, Garleanu, and Pedersen (2006), Lagos and Rocheteau (2009), Shevichenko (2004), Johri and Leach (2002), Masters (2007), and Wong and Wright (2014).[4] Using a directed search approach, Watanabe (2010, 2013) provide a model of an intermediated market operated by middlemen with high inventory holdings. The middlemen's high capacity enables them to serve many buyers at a time, thus to lower the likelihood of stock-out, which generates a retail premium of inventories. This mechanism is adopted by the middleman in our model as well. Hence, if intermediation fees are not available, then our model is a simplified version of Watanabe where we added an outside market. It is worth mentioning that in Watanabe

[4]Rubinstein and Wolinsky (1987) show that an intermediated market can be active under frictions, when it is operated by middlemen who have an advantage in the meeting rate over the original suppliers. Given some exogenous meeting process, two main reasons have been considered for the middlemens advantage in the rate of successful trades: a middleman may be able to guarantee the quality of goods (Biglaiser, 1993, Li, 1998), or to satisfy buyers demand for a varieties of goods (Shevichenko, 2004). While these are clearly sound reasons for the success of middlemen, the buyers search is modeled as an undirected random matching process, thereby the middlemens capacity cannot influence buyers search decisions in these models.

(2010, 2013), the middleman's inventory is modeled as a discrete unit, i.e., a positive integer, so that the middlemen face a non-degenerate distribution of their selling units as other sellers do. In contrast, we model the inventory as measured by mass, assuming more flexible inventory technologies, so that the middleman faces a degenerate distribution of sales. This simplification allows us to characterize the middleman's profit maximizing level of inventory – in Watanabe (2010) the inventory level of middlemen is determined by aggregate demand-supply balancing. Recently, Holzner and Watanabe (2015) study a labor market equilibrium using a directed search approach to model a job-brokering service (i.e., a market-making service), offered by Public Employment Agencies, but the choice of intermediation mode is not the scope of their paper.

The other related strand is the two-sided market literature, e.g. Rochet and Tirole (2003, 2006), Caillaud and Jullien (2003), Rysman (2004), Armstrong (2006), Hagiu (2006) and Weyl (2010).[5] The critical feature of platform is the presence of the cross-group externality, i.e. the participants' expected gains from a platform depend positively on the number of participants on the other side of it. When such a cross-group externality exists, the marketmaker can use "divide-and-conquer" strategies, namely, subsidizing one group of participants in order to attract another group and extract the ensuing externality benefit (see, Caillaud and Jullien, 2003). This strategy is adopted by the intermediary in our model as well. Broadly speaking, if there is no middleman mode, then our model is a directed search version of Caillaud and Jullien (2003) in combination with a decentralized market.

Finally, without considering search frictions, Hagiu and Wright (2015) study the profitability of intermediation modes as is determined by marketing activities. In their model, it is assumed that the owner of a product has private information on how effective their marketing activity will be. They show that the optimal design of intermediation mode is determined, among others, by the cross-product spillovers of marketing activity, and the degree of owners' informational advantage. For each product, an intermediary only takes the preferred extreme mode instead of a hybrid one, and their theory explains which products the intermediary should offer in which mode. In contrast, with considering search frictions, we show that even for a homogeneous product, a hybrid intermediation mode can occur in equilibrium. Our theory explains how the intermediated market is structured depending on the search/competitive environment. Rust and Hall (2003)

[5]Closely related papers based on a similar spirit can be found in Baye and Morgan (2001), Rust and Hall (2003), Parker and Van Alstyne (2005), Nocke et al. (2007), Galeotti and Moraga-Gonzalez (2009), Loertscher and Niedermayer (2012) and Edelman and Wright (2015). For earlier contributions of this strand of literature, see, e.g., Stahl (1988), Gehrig (1993), Yavas (1994, 1996), Spulber (1996) and Fingleton (1997).

consider two types of intermediaries, one is "middlemen" whose market requires costly search and a monopolistic "market maker" who offers a frictionless market. They show that agents segment into different markets depending on heterogeneous production costs and consumption values, thus these two types of intermediaries can coexist in equilibrium. Their model is very different from ours in many respects. For instance, selling capability and inventory does not play any role in their formulation of sequential search, but it is the key ingredient in our model. Further, we show that a monopolistic intermediary can pursue different types of intermediation modes even faced with homogeneous agents. We believe a similar property would survive in an extended environment with entry or oligopolistic intermediaries.

The rest of the paper is organized as follows. Section 2 presents the basic setup. Section 3 studies the choice of intermediation mode for single-market technology and it provides a benchmark of our economy. Section 4 extends the analysis to allow for multiple-market technology and gives the key finding of the paper. Section 5 discusses the welfare and other modeling issues. Section 6 concludes. Some proofs are in the Appendix.

2 Setup

We consider a large economy with two types of agents: a mass B of buyers and a mass S of sellers. Agents of each type are homogeneous. Each buyer has unit demand of a homogeneous good, and each seller has a production technology of that good. The consumption value for the buyer is normalized to 1. The marginal production cost is constant and without loss of generality, we normalize it to zero. Sellers are able to produce as much as they want but their selling/trading capability is limited so that each seller can sell only one unit of the good.

There are two retail markets, a centralized market and a decentralized market (see Figure 1). The decentralized market (hereafter D market) is featured by random matching and bilateral bargaining. We assume that the flow of contacts between sellers and buyers in the D market is given by a matching technology $M = M(B^D, S^D)$ where B^D and S^D denote the amount of buyers and sellers that actually participate in the D market. The function M is continuous, nonnegative, with $M(0, S^D) = M(B^D, 0) = 0$ for all B^D, $S^D \geq 0$. Without loss of generality, we further assume that for B^D, $S^D > 0$ a buyer finds a seller with probability $\lambda^b = \frac{M(B^D,S^D)}{B^D}$ and a seller finds a buyer with probability $\lambda^s = \frac{M(B^D,S^D)}{S^D}$, satisfying $B^D\lambda^b = S^D\lambda^s$ and $\lambda^b, \lambda^s \in (0,1)$ is a constant. This linear matching technology is extended to general non-linear matching functions in Section

6. Matched partners follow an efficient bargaining process, which yields a linear sharing of the total surplus, with $\beta \in (0, 1)$ as the share for the buyer, and $1 - \beta$ for the seller.

The centralized market (hereafter C market) is operated by a monopolistic intermediary. The intermediary can perform two different intermediation activities. As a middleman, he purchases a good from sellers in a wholesale market, and holds it as an inventory to resell to buyers. The wholesale market is operated by sellers, who have no limit in producing the good. We assume the wholesale market to be competitive so that the inventory demand of the middelman is always satisfied at the competitive wholesale-price equal to marginal production cost (normalized to zero). We will describe later the case with positive production costs. The middleman can stock inventories in advance so that he is prepared to serve buyers the retail markets.

As a marketmaker, he does not buy and sell but instead provides a platform where buyers and sellers can interact with each other for trade upon paying fees.

We assume that the C market is subject to directed search frictions. In a directed search environment, the prices and capacities of all the individual suppliers are publicly observable. The intermediary has technologies to spread this information. Still, given that individual buyers cannot coordinate their search activities, the limited selling capacity of individual sellers creates a possibility that some units remain unsold and some demands are not satisfied. This is the standard notion of directed search frictions, see e.g., Peters (1991, 2001), Moen (1997), Acemoglu and Shimer (1999), Burdett, Shi and Wright (2001), Albrecht, Gautier, and Vroman (2006), and Guerrieri, Shimer and Wright (2010), and in this sense, the platform in our economy is frictional. As will be detailed below, however, there is no such a friction in the middleman's trade since its inventory management technologies allow for the selling capability to be comparable to the population of potential buyers in magnitude.

The timing of events is as follows.

1. Two retail markets, a C market and a D market, open. In the C market, the intermediary announces a set of fees $F \equiv \left\{f^b, f^s, g^b, g^s\right\}$ for the platform, where $f^b,\ f^s \in [0, 1]$ is a transaction fee charged to a buyer or a seller, respectively, and $g^b,\ g^s \in [-1, 1]$ is a registration fee charged to a buyer or a seller, respectively.

2. Observing the announced fees F, buyers and sellers simultaneously decide whether to participate in the C market. We consider two different search technologies of agents: the single-market search, with which agents can attend only one market, and the multi-market

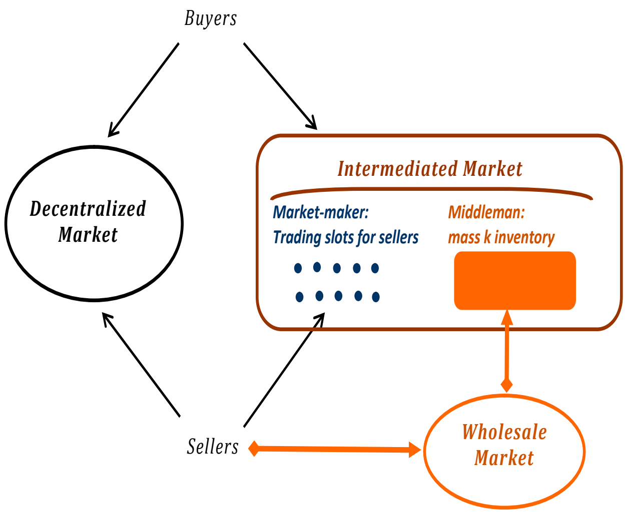

Figure 1: Overview

search with which agents can attend both markets.

3. In the C market, the participating buyers, sellers and middleman are engaged in a directed search game. In the D market, agents search randomly and follow the efficient bargaining sharing rule for the trade surplus.

We work backward and first characterize a directed search equilibrium in the C market. Suppose that a mass B buyers and a mass S sellers have decided to participate in the C market, and that the intermediary has stocked $K \in [0, B] \subset \mathrm{R}_+$ inventories. Then, the C market has the following stages. In the first stage, all the suppliers, i.e., the participating sellers with the unit selling-capacity and the middleman with the K capacity, simultaneously post a price which they are willing to sell at. Observing the prices and the capacities, all buyers simultaneously decide which supplier to visit in the second stage. Each buyer can visit one supplier. Assuming buyers cannot coordinate their actions over which supplier to visit, we follow Peters (1991, 2001) and investigate a symmetric equilibrium where all buyers use an identical visiting strategy for any configuration of the announced prices. Each individual seller has a queue x^s of buyers with an equilibrium price p^s and the middleman has a queue x^m of buyers with an equilibrium price p^m. These quantities should satisfy two requirements. The first requirement is the accounting identity,

$$Sx^s + x^m = B, \tag{1}$$

which states that the number of buyers visiting individual sellers Sx^s and the middleman x^m should sum up to the total population of participating buyers B. The second requirement is the buyers' optimal search:

$$x^m = \begin{cases} B & \text{if } V^m(B) \geq V^s(0) \\ (0, B) & \text{if } V^m(x^m) = V^s(x^s) \\ 0 & \text{if } V^m(0) \leq V^s(\frac{B}{S}), \end{cases} \tag{2}$$

where $V^i(x^i)$ is the equilibrium value of buyers to visit a seller if $i = s$ and the middleman if $i = m$ (yet to be specified below). Conditions (1) and (2) define the counterpart for $x^s \in [0, \frac{B}{S}]$.

As for the intermediation mode in the C market, we shall adopt the following terminology.

Definition 1 *Suppose B buyers and S sellers participate in the C market. Then, we say that the intermediary acts as:*

- *a pure middleman if* $x^m = B$ $(\Leftrightarrow x^s = 0)$*;*
- *a market-making middleman if* $x^m \in (0, B)$ $(\Leftrightarrow x^s \in (0, \frac{B}{S}))$*.*
- *a pure market-maker if* $x^m = 0$ $(\Leftrightarrow x^s = \frac{B}{S})$*;*

3 Single-market search

We start analysis for the single-market search technologies where agents can participate in only one market.

Given the participation of B buyers and S sellers, if agents have single-market search technologies, then the intermediary will act as a pure middleman by selecting $f^* = f^{b*} + f^{s*} > 1$ and $K^* = B$. This leads to the highest possible profits with $p^m = 1$ in the C market,

$$\Pi(p^m, f^*, K^*) = B.$$

Under these price and fees, all the buyers buy from the middleman, $x^m = B$, and the platform is inactive, $x^s = 0$. This is indeed an equilibrium since $V^m(B) = 0 > V^s(0)$ under $p^m = 1$ and $f^* > 1$, and for any $p^s \in [0, 1]$, sellers make zero profit in the C market.

In what follows, we illustrate that creating an active platform is not profitable. Suppose that intermediation fees are $f = f^b + f^s \leq 1$. Then, the platform generates a non-negative trade surplus $1 - f \geq 0$. An active platform will be frictional and is described as follows.

Given the symmetric strategy of buyers' search, the number of buyers visiting one individual seller is a random variable, denoted by N, which has a Poisson distribution, $\text{Prob}[N = n] = \frac{e^{-x}x^n}{n!}$, if he has an expected queue of buyers $x \geq 0$. Hence, the limiting selling capacity (i.e., an ability to sell only a single unit) implies that a seller with an expected queue $x^s \geq 0$ of buyers has a probability $1 - e^{-x^s}$ of successfully selling the good, where $e^{-x^s} = \text{Prob}[N = 0]$. The expected value of a seller with a price p^s and an expected queue x^s in the platform, denoted by $W = W(x^s)$, is

$$W(x^s) = (1 - e^{-x^s})(p^s - f^s), \tag{3}$$

where the seller succeeds with probability $1 - e^{x^s}$ in which case he receives p^s minus the transaction fee f^s. If not successful, then he receives zero payoff. The expected value of a buyer who visits a

seller with a price p^s and an expected queue x^s in the platform is

$$V^s(x^s) = \frac{1 - e^{-x^s}}{x^s}(1 - p^s - f^b), \tag{4}$$

where the buyer is served with probability $\frac{1-e^{-x^s}}{x^s}$ and obtains $1 - p^s$ minus the transaction fee f^b. If not served, then his payoff is zero.

The middleman sector is described as follows. If a middleman's price $p^m \in [0, 1]$ attracts $x^m \geq 0$ buyers, then since the middleman has a mass of inventory, the expected profit from the middleman sector is given by $\min\{K, x^m\}p^m$. The expected value of buyers to visit the middleman is

$$V^m(x^m) = \min\{\frac{K}{x^m}, 1\}(1 - p^m). \tag{5}$$

There are two cases for active platform. First, suppose that the intermediary shuts down the middleman sector, by holding no inventory at all $K = 0$, so that all the buyers search in the platform, i.e., $x^m = 0$ and $x^s = \frac{B}{S}$. Then, the intermediary makes an expected revenue $S(1 - e^{-x^s})f$ in the platform but nothing from the middleman sector. The profits are

$$\Pi(p^m, f, 0) = S(1 - e^{-\frac{B}{S}})f < B = \Pi(1, f^*, K^*)$$

for all $f \leq 1$, since $1 - e^{-\frac{B}{S}} < \frac{B}{S}$. Hence, the exclusive use of the platform is not profitable. Next, suppose that the intermediary make both sectors active with some $p^m \in [0, 1]$ and $K \in (0, B]$, i.e., $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$ that satisfy the add-up requirement (1) and the indifferent condition (2). Then, the resulting profit satisfies

$$\begin{aligned} \Pi(p^m, f, K) &= S(1 - e^{-x^s})f + \min\{K, x^m\}p^m \\ &< Sx^s f + x^m p^m \\ &\leq (Sx^s + x^m)\max\{f, p^m\} \\ &\leq B = \Pi(1, f^*, K^*), \end{aligned}$$

for all $f \leq 1$ and $p^m \leq 1$, where the last inequality from the add-up condition (1), $Sx^s + x^m = B$. Hence, this is not profitable either. All in all, any active platform is not profitable.

The intuition behind the occurrence of a pure middleman is as follows. Given the frictions in the intermediated platform, a larger middleman sector creates more transactions. To guarantee that all participating buyers are in the middleman sector, the intermediary can shut down the platform by setting high enough fees $f > 1$, and exploit them by charging $p^m = 1$ to the inventory.

In a nutshell, the middleman's inventory is the efficient way to allocate the good and, if agents search within a single market, then the intermediary is guaranteed with the full surplus of it.

We now consider how the intermediary can induce agents to participate in the C market. As each retail-market is faced with two-sided participation, an issue arises for the belief-dependent multiplicity of equilibria – the participation decision of buyers (sellers) depends on their belief on the participation of sellers (buyers). For the selection of beliefs, we follow the literature of two sided-market and assume that agents hold pessimistic beliefs on the participation decision of agents on the other side of the market (Caillaud and Jullien, 2003). Accordingly, agents believe that the intermediary would never hold inventory unless the C market attracts some buyers. For instance, a pessimistic belief of sellers means that sellers believe the number of buyers participating in the C market is zero whenever

$$\lambda^b \beta > -g^b,$$

where $\lambda^b \beta$ is the expected payoff of buyers in the D market and it is believed that the C market is empty so that all buyers receive in the C market is $-g^b$ (it is a participation subsidy when $g^b < 0$).

To induce participation under those beliefs, the best the intermediary can do is to use a divide-and-conquer strategy, denoted by h. To divide buyers and conquer sellers, referred to as $h = D_b C_s$, it is required that

$$D_b \quad : \quad -g^b \geq \lambda^b \beta, \tag{6}$$

$$C_s \quad : \quad W - g^s \geq 0. \tag{7}$$

The divide-condition D_b tells us that the intermediary should subsidy the participating buyers so that they are guaranteed at least what they would get in the D market, even if the C market is empty. This makes sure that buyers will join the C market. The conquer-condition C_s guarantees the participation of sellers, by giving them a nonnegative payoff – the participation fee $g^s \geq 0$ should be no greater than the expected value of sellers in the C market, $W \geq 0$. Observing that the intermediary offers buyers with a sufficiently generous subsidy, sellers understand that all buyers are in the C market, the D market is empty, and so the expected payoff from the D market is zero. Here, the expected value of sellers in the C market W is defined under the sellers' rational expectation that the intermediary will select the inventory level optimally given the full participation of buyers.

Similarly, a strategy to divide sellers and conquer buyers, referred to as $h = D_s C_b$, requires

that

$$D_s \quad : \quad -g^s \geq \lambda^s(1-\beta), \tag{8}$$

$$C_b \quad : \quad V - g^b \geq 0. \tag{9}$$

where $V = \max\{V^m(x^m), V^s(x^s)\} \geq 0$ is the expected value of buyers in the C market.

Given the participation decision of agents described above, the intermediary's problem of determining the intermediation fees $F = \{f^b, f^s, g^b, g^s\}$ for $h = \{D_bC_s, D_sC_b\}$ is described as

$$\Pi = \max_{F,h}\{Bg^b + Sg^s + \Pi(p^m, f, K)\},$$

subject to (6) and (7) if $h = D_bC_s$, or (8) and (9) if $h = D_sC_b$. Here, Bg^b and Sg^s are participation fees from buyers and sellers, respectively, and $\Pi(\cdot)$ is the expected profit in the C market described above. Under either of the divide-and-conquer strategies, the choice of participation fees g^i, $i = b, s$, does not influence anyone's behaviors in the C market. The choice of transaction fees affects the expected value of agents and hence the participation fees and intermediary's profits. However, it does not alter the intuition provided above and a pure middleman remains optimal.

Proposition 1 (Pure middleman) *Given single-market search technologies, the intermediary will act as a pure middleman with $f^* > 1$ and $K^* = B$. All the buyers buy from the middleman, $x^m = B$, and the platform is inactive, $x^s = 0$, satisfying $V = 0 = W$, for any $p^s \in [0, 1]$. The intermediary makes profits,*

$$\Pi = -\min\{B\lambda^b\beta, S\lambda^s(1-\beta)\} + B,$$

guaranteeing the participation of agents by:

- $h = D_bC_s$ *if* $\beta < \frac{1}{2}$, *with the binding* D_b *and* C_s *conditions (6) and (7) at* $p^m = 1$;
- $h = D_sC_b$ *if* $\beta > \frac{1}{2}$, *with the binding* D_s *and* C_b *conditions (8) and (9) for any* $p^m \in [0, 1]$.

Proof. In Appendix. ■

The allocation characterized in Proposition 1 serves as a benchmark of our economy. It is indeed Pareto efficient: The middleman's inventory is the efficient way to allocate the good. If agents search within a single market, then the intermediary is guaranteed with the full surplus of pursuing this intermediation mode. To make sure the participation of agents in the C market,

the intermediary can use a divide and conquer strategy to overcome pessimistic beliefs of agents by subsidizing either side of the market. Which side should be subsidized depends on the outside option in the D market. If $\beta < \frac{1}{2}$ then buyers are less costly to convince, and the buyers' trade surplus can be exploited fully in the middleman sector by charging $p^m = 1$. If $\beta > \frac{1}{2}$ then sellers are less costly to subsidize, and when to exploit the participating buyers does not matter – the intermediary makes the maximum revenue B for any $p^m \in [0, 1]$.

4 Multi-market search

In this section, we extend our analysis to multiple-market search technologies where agents can search both the C market and the D market.

Since agents can search both of the markets, a timing issue arises on which market should open first. Below, we present the analysis of the setup that the C market opens prior to the D market. Apart from the fact that this appears to be the most natural setup in our economy, we are motivated by the first mover advantage of the intermediary: its expected profit is higher if the C market opens before, rather than after, the D market. Hence, our setup will arise endogenously if the intermediary is allowed to select the timing of the market sequence to enjoy the first mover advantage. [6]

Directed search equilibrium in the C market: We work backward and first describe a directed search equilibrium in the C market assuming that there are B buyers and S sellers. As before any directed search equilibrium in the C market has to satisfy the add-up restriction (1) and the symmetric search strategy of buyers (2). Given the multiple-market search of agents, agents expect a non-negative value for the D market when deciding whether or not to accept an offer in the C market. Hence, whenever the platform is active $x^s > 0$, the set of transaction fees f^i, $i = b, s$, must satisfy the constraints:

$$1 - p^s - f^b \geq \lambda^b e^{-x^s} \beta \tag{10}$$

$$p^s - f^s \geq \lambda^s \xi(x^m, K)(1 - \beta). \tag{11}$$

[6] As a real-life correspondence of such a situation, online retailers have in principle unlimited opening hours a day, whereas such a flexible business practice is physically infeasible offline. In addition, many online retailers are enthusiastic in making their websites fast and easy, providing a wide range of information on merchandize and offering personalized service such as special offer emails tailored to a customer's interest, etc. These efforts would enhance customer experiences and create loyalty, which may increase their chance to become a first-mover. In a recent study without intermediation, Armstrong and Zhou (2014) show that a seller often makes it harder or more expensive to buy its product later than at the first opportunity.

The constraint of buyers (10) states that the offered price/fee in the C market is acceptable only if the offered payoff $1 - p^s - f^b$ is no less than the expected value in the D market: the outside payoff is not zero this time, but is β if he is matched in the D market with a seller who has failed to trade in the C market. This happens with probability $\lambda^b e^{-x^s}$. Hence, the larger the platform size x^s, the higher the chance that a seller is successful in the C market, and the lower the chance that a buyer can trade successfully in the D market and the lower his expected outside value. Similarly, the constraint of sellers (11) states that the payoff in the C market $p^s - f^s$ should be no less than the expected value in the D market where $\xi(x^m, K)$ represents the probability that a buyer has failed to trade in the C market and is given by

$$\begin{aligned} \xi(x^m, K) &\equiv 1 - \left(\frac{x^m}{B}\eta^m(x^m) + \frac{Sx^s}{B}\eta^s(x^s)\right) \\ &= 1 - \frac{1}{B}\left(\min\{K, x^m\} + S(1 - e^{-(\frac{B-x^m}{S})})\right), \end{aligned}$$

where $\eta^m(x^m) \equiv \min\{\frac{K}{x^m}, 1\}$ is the probability of the buyer to be served in the middleman sector and $\eta^s(x^s) \equiv \frac{1-e^{-x^s}}{x^s}$ in the platform. In this expression, the terms in the parethesis represent the expected chance of a buyer to trade in the C market. A buyer has been in the middleman sector with probability $\frac{x^m}{B}$ and served with probability $\eta^m(x^m)$, or in the platform with probability $\frac{Sx^s}{B}$ and served with probability $\eta^s(x^s)$.

We now determine the equilibrium price p^s in an active platform, $x^s > 0$. Suppose a seller deviates to a price $p \neq p^s$ that attracts an expected queue x of buyers. Note that given the limited selling-capacity, this deviation has measure zero and does not affect the expected utility in the C market, $V = \max\{V^s(x^s), V^m(x^m)\}$. Given the symmetry of buyers' search strategy and $x^s > 0$, since buyers must be indifferent between visiting any sellers (including the deviating seller), the equilibrium market-utility should satisfy

$$V = \eta^s(x)(1 - p - f^b) + (1 - \eta^s(x))\lambda^b e^{-x^s}\beta, \tag{12}$$

where $\eta^s(x) \equiv \frac{1-e^{-x}}{x}$ is the probability that the buyer is served by this seller. If not served, which occurs with probability $1 - \eta^s(x)$, his expected payoff in the D market is $\lambda^b e^{-x^s}\beta$. Given V, (12) determines the relationship between x and p, which we denote by $x = x(p \mid V)$.

Given the directed search of buyers described above, the seller's problem of choosing an optimal price, denoted by $p^s(V)$, is described as

$$p^s(V) = \text{argmax}_p \left\{(1 - e^{-x(p|V)})(p - f^s) + e^{-x(p|V)}\lambda^s\xi(x^m, K)(1 - \beta)\right\},$$

taking the market utility V as given. Substituting out p using (12), the objective function denoted by $W(x)$ can be written as

$$W(x) = 1 - f - \lambda^b e^{-x^s}\beta - e^{-x}(v(x^m, K) - f) - x(V - \lambda^b e^{-x^s}\beta),$$

where $x = x(p \mid V)$ satisfies (12) and

$$v(x^m, K) \equiv 1 - \lambda^b e^{-x^s}\beta - \lambda^s \xi(x^m, K)(1-\beta)$$

is the total trading surplus net of the outside option in the D market. The first order condition is:

$$\frac{\partial W(x)}{\partial x} = e^{-x}(v - f) - (V - \lambda^b e^{-x^s}\beta) = 0.$$

The second order condition is satisfied under the constraints (10) and (11). Arranging the first order condition using (12) and evaluating it at $x^s = x(p^s \mid V)$, we obtain the equilibrium price $p^s = p^s(V)$ which can be written as

$$p^s - f^s = (1 - \beta_C)(v - f) + \lambda^s \xi(x^m, K)(1-\beta), \tag{13}$$

where $\beta_C \equiv \frac{x^s e^{-x^s}}{1-e^{-x^s}}$. Given the equilibrium price p^s described above in an active platform $x^s > 0$, the equilibrium value of a buyer visiting an independent seller should satisfy $V = V^s(x^s)$, where

$$V^s(x^s) = \eta^s(x^s)(1 - p^s - f^b) + (1 - \eta^s(x^s))\lambda^b e^{-x^s}\beta. \tag{14}$$

The equilibrium value of active sellers in the platform is given by

$$W(x^s) = x^s \eta^s(x^s)(p^s - f^s) + (1 - x^s\eta^s(x^s))\lambda^s \xi(x^m, K)(1-\beta). \tag{15}$$

Note again that for the platform to be active the price and fees must satisfy the constraints (10) and (11). Combining these constraints, we obtain:

$$f \le v(x^m, K), \tag{16}$$

which states that whenever the platform is active $x^s > 0$ the total transaction fee f should not be greater than the total trade surplus net of the expected outside values, $v(x^m, K)$.

Similarly, whenever the middleman sector is non-empty $x^m > 0$, the middleman's price p^m must be acceptable to buyers:

$$1 - p^m \ge \lambda^b e^{-x^s}\beta. \tag{17}$$

The determination of the equilibrium price p^m depends on the intermediation mode selected (see below). The equilibrium value of a buyer who selects the middleman sector with a price p^m and an expected queue $x^m > 0$ is given by

$$V^m(x^m) = \eta^m(x^m)(1 - p^m) + (1 - \eta^m(x^m))\lambda^b e^{-x^s}\beta. \tag{18}$$

Using Definition 1, we can describe a directed search equilibrium depending on the intermediation mode, given that B buyers and S sellers are in the C market for given values of intermediation fees F and inventory stocks $K \in [0, B]$. A directed search equilibrium in the C market with multi-market search is characterized by

- $x^m = B$ and $x^s = 0$, with the equilibrium values in (14) and (18) satisfying $V = V^m(B) \geq V^s(0)$, and the equilibrium price p^m satisfying (17), when the intermediary is a pure middleman;
- $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$, with the equilibrium values in (14) and (18) satisfying $V = V^m(x^m) = V^s(x^s)$, and the transaction fees f^i, $i = b, s$, and the equilibrium price p^m satisfying (10), (11) and (17), when the intermediary is a market-making middleman;
- $x^m = 0$ and $x^s = \frac{B}{S}$, with the equilibrium values in (14) and (18) satisfying $V = V^s(B/S) \geq V^m(0)$, and the transaction fees f^i, $i = b, s$ satisfying (10) and (11), when the intermediary is a pure market-maker.

Finally, we determine the equilibrium price p^m. It has to maximize the intermediary's profit in the C market:

$$\Pi(p^m, f, K) = S(1 - e^{-x^s})f + \min\{K, x^m\}p^m, \tag{19}$$

subject to the constraint (17). If the intermediary is a pure middleman, then $x^m = B$ and $x^s = 0$ and so (17) should be binding, $p^m = 1 - \lambda^b\beta$. If the intermediary is a market-making middleman, then $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$, satisfying $V^m(x^m) = V^s(x^s)$. Applying p^s in (13) to (14), this indifference condition generates an expression of price $p^m = p^m(x^m)$:

$$p^m(x^m) = 1 - \lambda^b e^{-x^s}\beta - \frac{x^m e^{-x^s}}{\min\{K, x^m\}}\left(v(x^m, K) - f\right), \tag{20}$$

where $v(x^m, K) \equiv 1 - \lambda^b e^{-x^s}\beta - \lambda^s\xi(x^m, K)(1 - \beta)$ is the total surplus net of outside values as defined above. Observe that (16) and (20) imply (17) is redundant, and using (1) and (20), we

can transform the problem into the choice of $x^m \geq 0$ to maximize the expected profits in (19) subject to (16).

Lemma 1 *Suppose that buyers' indifference condition $V^m(x^m) = V^s(x^s)$ holds for the C market. Then, for any $f \geq 0$ satisfying (16) and for any $B, S > 0$ and $\beta, \lambda^s, \lambda^b \in (0,1)$, for $K > \underline{K}$, some $\underline{K} < B$, the profit maximizing queues of buyers satisfy $x^m \leq K$ and $x^s > 0$.*

Proof. In Appendix. ■

The lemma shows that having more buyers than the inventory is not profitable since it requires a lower price to attract those extra buyers, $x^m - K > 0$, at least for not too small K's, but it does not increase the amount of transactions. Hence, for $K > \underline{K}$ the middleman implements $\eta^m(x^m) = \min\{\frac{K}{x^m}, 1\} = 1$. The first order condition of $x^m \in [0, K]$ is

$$-e^{-x^s} f + p^m(x^m) + x^m \frac{\partial p^m(x^m)}{\partial x^m} - \mu_k + \mu_v \frac{\partial v(x^m, K)}{\partial x^m} = 0, \tag{21}$$

where

$$\frac{\partial p^m(x^m)}{\partial x^m} = -\frac{e^{-x^s}}{S} \left[v(x^m, K) - f + \lambda^b (1 - e^{-x^s}) \right] < 0.$$

The profit maximizing buyers' queue x^m trades-off a larger quantity in the middleman sector against an expected loss associated with shrinking the platform size x^s, which consists of a lower transaction fee and a lower inventory price p^m. The latter occurs because a smaller x^s implies a lower chance for sellers to trade successfully in the C market, which increases the populations of sellers in the D market who still hold the good. Since this increases the option value of buyers for the D market, the inventory price the middleman can charge will be lower. Of course, on top of this tradeoff, whenever relevant, we have to take into account its effect on the binding constraint with the Lagrange multiplier $\mu_v \geq 0$ for $v(x^m, K) \geq f$ and μ_k for $x^m \in [0, K]$ (with $x^m = K$ if $\mu_k > 0$ and $x^m = 0$ if $\mu_k < 0$). Finally, whenever buyers are indifferent between the middleman sector and the platform, the intermediary's profit maximization generates an active platform $x^s > 0$. This result is extended to show the following theorem.

Theorem 1 (Un-profitability of pure middleman) *With multiple-market search, suppose that B buyers and S sellers are in the C market. For any $B, S > 0$ and $\beta, \lambda^s, \lambda^b \in (0,1)$, define a subset:*

$$\mathfrak{K}_f \equiv \left\{ (\underline{K}, B] \mid v(x^m, K) \geq f \;\; and \max_{x^m \in [0,B]} \Pi(p^m, f, K) > \Pi(1 - \lambda^b \beta, f^*, K^*) \right\},$$

where $p^m = p^m(x^m) \geq 0$ is given by (20), and $f^* > 1$ and $K^* = B$ (as in Proposition 1) lead to a pure middleman with $p^m = 1 - \lambda^b \beta$ satisfying (17) and profits $\Pi(1 - \lambda^b \beta, f^*, K^*) = B(1 - \lambda^b \beta)$. Then, $\mathfrak{K}_f$ is non-empty.

Proof. In Appendix. ■

With multiple-market search, a pure middleman has to keep the inventory price low enough $p^m = 1 - \lambda^b \beta$ to be acceptable, relative to the outside option value, as in (17). On the other hand, an active platform creates a chance that sellers can sell in the C market, which makes it possible that a buyer is matched with someone not available in the D market. Since this lowers the outside value of buyers, the intermediary can set a higher inventory price with an active platform and make a higher profit. While this argument is conditioned on $K \in \mathfrak{K}_f$, the intermediary will choose such a level whenever (K, f) are endogenous.

Intermediation mode and fees: Concerning the divide and conquer strategy, with multiple-market search, any negative registration fee ensures that agents are in the C market, since they need not to give up the D market to do so. So, attracting one side of the market becomes less costly.

The $D_s C_b$ condition with multiple-market search is

$$\begin{aligned} D_s &: -g^s \geq 0, \\ C_b &: V - g^b \geq \lambda^b e^{-x^s} \beta. \end{aligned}$$

The divide seller condition D_s tells that now a non-positive fee is sufficient to convince one side to participate. The conquer buyer condition C_b allows for exploiting the full net surplus of buyers in the C market. Similarly, the $D_b C_s$ condition becomes

$$\begin{aligned} D_b &: -g^b \geq 0, \\ C_s &: W - g^s \geq \lambda^s \xi(x^m, K)(1 - \beta). \end{aligned}$$

The intermediary's problem of determining the intermediation fees $F = \{f^b, f^s, g^b, g^s\}$ for $h = \{D_b C_s, D_s C_b\}$ and the optimal choice of $K \in [0, B]$ are described as:

$$\Pi = \max_{F,h,K} \{Bg^b + Sg^s + \max_{x^m \in [0,B]} \Pi(p^m, f, K)\},$$

where $\Pi(p^m, f, K)$ is given in (19); For a pure middleman mode that implements $x^s = 0$, it is subject to the price given by $p^m = 1 - \lambda^b \beta$ satisfying (17); For the other modes that implements $x^s > 0$, it is subject to (16); For a market-making middleman, it is subject to p^m satisfying (20). The binding divide-and-conquer condition yields

$$g^b = 0,\ g^s = (1 - e^{-x^s} - x^s e^{-x^s})(v(x^m, K) - f),$$

if $h = D_b C_s$, or

$$g^s = 0,\ g^b = e^{-x^s}(v(x^m, K) - f).$$

if $h = DsCb$.

There are several observations on an optimal solution with multi-market search, denoted by $f^{**}, g^{s**}, g^{b**}, K^{**}$. First, the pure middleman mode has no advantage in terms of the participation fees for either h, and so Theorem 1 implies that it will never be selected with an endogenous capacity $K^{**} \in \mathfrak{K}_f$. Second, since the equilibrium queue always satisfies $\eta^m(x^m) = 1$ (see Lemma 1), hence the intermediary's profit is maximized for all $K^{**} \in \mathfrak{K}_f$. Finally, the intermediary trades-off a higher transaction fee against the resulting lower equilibrium value of buyers and sellers, which reduces their registration fees. However, the overall effect is positive since the divide and conquer strategies allows for exploiting only one side, either from buyers or from sellers, but not from the both. Hence, the constraint should be $f^{**} = v(x^m, K)$.

Proposition 2 (Market-making middleman/Pure market-maker) *With multiple-market search, the profit maximizing solution is $g^{i**} = 0$, $i = b, s$ and $f^{**} = v(x^m, K^{**})$, and $K^{**} \in \mathfrak{K}_{f^{**}} \neq \varnothing$, with $x^m \in [0, B)$ satisfying (21) and $x^s \in (0, \frac{B}{S}]$. If parameter values are such that*

$$1 - \lambda^b e^{-\frac{B}{S}} \beta - e^{-\frac{B}{S}} v(0, K^{**})(1 - \beta) + \lambda^b (1 - e^{-\frac{B}{S}})(1 - e^{-\frac{B}{S}} - \beta) < 0, \tag{22}$$

then a pure market-maker mode that implements $x^s = \frac{b}{S}$, $x^m = 0$ is selected. Otherwise, a market-making middleman emerges.

Proof. In Appendix ■

Setting $S = 1 - B$, we visualize inequality (22) in the space of (β, B, λ^b), as shown in Figure 2. The shadowed region is the parameter set that yields a pure market-maker $x^s = \frac{B}{S}$. This occurs when the mass of buyers B is small and the buyer's outside option is high (λ^b and β are large).

Indeed, the same logic applied to the comparative statics. If the buyers' outside market value $e^{-x^s}\lambda^b\beta$ increases with the buyer's bargaining power β, then so does the profitability of using the marketmaker mode. As a result, the the platform size x^s should increase.

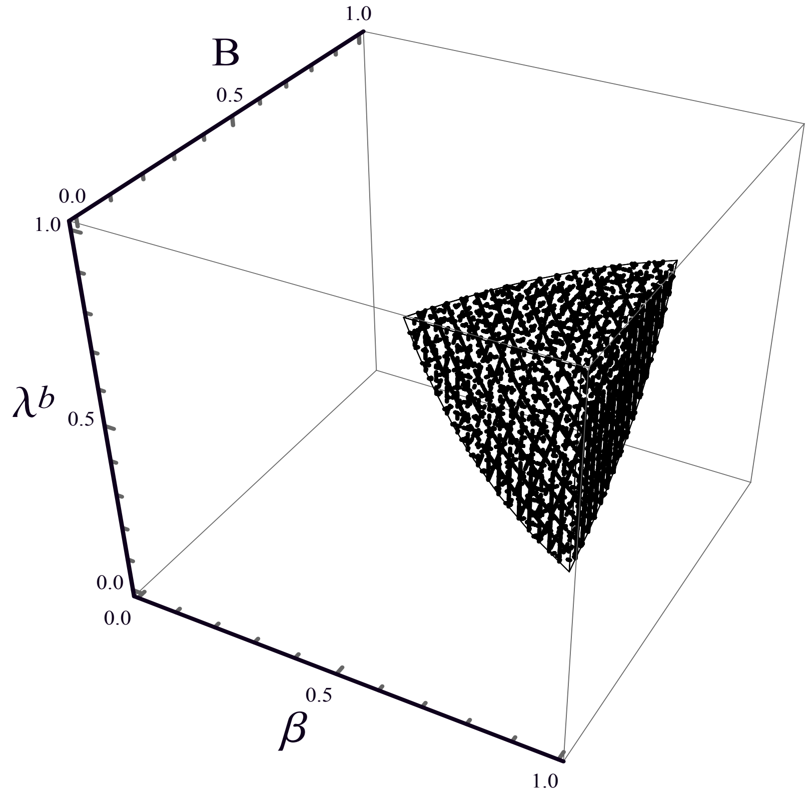

Figure 2: Illustration of the parameter space (shadowed region) where pure marketmaker mode is optimal

Corollary 1 (Comparative statics) *In a marker-making middleman mode, an increase in buyer's bargaining power β in the decentralized market leads to a smaller middleman sector x^m and a larger market-making sector x^s.*

Proof. In Appendix ■

Comparison of Proposition 1 and 2 yields the following welfare implication of search technology to intermediation mode.

Corollary 2 (Efficiency/Welfare) *Efficiency/welfare is higher with single-market search than with multiple-market search of agents.*

In the above analysis with zero cost and zero wholesale price, the profit maximizing capacity has a range of values $K^{**} \in \mathfrak{K}_{f^{**}}$. One simple way to pin down a unique value, without changing substantially the analysis, is to introduce a non-zero marginal production cost, denoted $c \geq 0$. With a marginal production cost c, the suppliers' payoff upon successful trade is $p^i - c$ (rather than p^i), $i = m, s$, and the the competitive wholesale price of inventory is c (rather than zero) and so the intermediary's inventory cost becomes Kc. This determines a unique value $K^{**} = x^m < B$ that satisfies the first order condition similar to (21) with μ_k replaced by c. With this extension our previous intuition can be stated in terms of the capacity choice. On the one hand, a higher capacity of the middleman leads to a larger number of successful transactions in the intermediated market. This happens due to the demand stimulating effect: a higher capacity induces more buyers to buy from the middleman, and fewer buyers to search in the platform, which increases the intermediary's profit. On the other hand, the demand effect makes sellers less likely to be successful in the platform, so that more sellers are available when a buyer attempts to search in the decentralized market. Accordingly, the buyers expect a higher value from the decentralized market, which causes a competitive pressure on the price that the intermediary can charge. Hence, the intermediary trade-offs a larger quantity against lower price/fees to operate as a larger-scaled middleman. This trade-off pins down an optimal structure of intermediation modes, in terms of capacity $K^{**} < B$.

5 Discussion

General matching function in D market: The matching function in the D market can be geenralized. Instead of a linear matching function, we consider the flow of contacts between sellers and buyers in the D market is given by a matching technology represented by

$$M = M(B^D, S^D),$$

where B^D and S^D denote the amount of buyers and sellers thatparticipate in the D market. As is standard, we assume that the function M is continuous and nonnegative, and satisfies $M(0, S^D) = M(B^D, 0) = 0$ for all $B^D, S^D > 0$.

Define the matching probabilities $\lambda^b(B^D, S^D) = \frac{M(B^D, S^D)}{B^D}$ and $\lambda^s(B^D, S^D) = \frac{M(B^D, S^D)}{S^D}$ for buyers and sellers, respectively. With single-market search, we will have the same result with these matching probabilities, $\lambda^b = \lambda^b(B, S)$, $\lambda^s = \lambda^s(B, S)$.

With multiple-market search, under a fairly reasonable condition that $\lambda^b(B^D, S^D)$ is decreasing in B^D and increasing in S^D, we can show that our main result is valid. The matching in the D market depends on how many buyers and sellers are not matched in the C market. In particular, $\min\{K, x^m\} + S(1 - e^{-x^s})$ buyers and $S(1 - e^{-x^s})$ sellers are matched in the C market, and thus

$$\begin{aligned} B^D &= B - \min\{K, x^m\} - S(1 - e^{-x^s}), \\ S^D &= S - S(1 - e^{-x^s}). \end{aligned}$$

Following the same derivation as before, we have the intermediary's profits

$$\Pi(K) = \left\{ \begin{array}{c} S(1 - e^{-x^s})(1 - \lambda^b(B^D, S^D)\beta - \lambda^s(B^D, S^D)(1 - \beta)) \\ +K(1 - \lambda^b(B^D, S^D)\beta) \end{array} \right\} (1 - c),$$

where the first term in the bracket is the platform profit, and the second term is the middleman profit. The first order derivative yields

$$\frac{\partial \Pi(K)}{\partial K} = \left\{ \begin{array}{c} -e^{-x^s}\left(1 - \lambda^b\left(B^D, S^D\right)\beta - \lambda^s\left(B^D, S^D\right)(1 - \beta)\right) \\ +S\left(1 - e^{-x^s}\right)\left(1 - \frac{\partial\lambda^b\left(B^D, S^D\right)}{\partial K}\beta - \frac{\partial\lambda^s\left(B^D, S^D\right)}{\partial K}(1 - \beta)\right) \\ +\left(1 - \lambda^b\left(B^D, S^D\right)\beta\right) - K\frac{\partial\lambda^b\left(B^D, S^D\right)}{\partial K}\beta \end{array} \right\} (1 - c).$$

Evaluating this derivative at $K = B$, we get

$$\frac{\partial \Pi(K)}{\partial K}|_{K=B} = -B\frac{\partial\lambda^b\left(B^D, S^D\right)}{\partial K}\beta(1 - c).$$

To make a pure middleman mode non-optimal ($\frac{\partial\Pi(K)}{\partial K}|_{K=B} < 0$), we need $\frac{\partial\lambda^b(B^D, S^D)}{\partial K} > 0$, which can be decomposed into two parts,

$$\frac{\partial\lambda^b\left(B^D, S^D\right)}{\partial K} = \frac{\partial\lambda^b\left(B^D, S^D\right)}{\partial B^D}\frac{\partial B^D}{\partial K} + \frac{\partial\lambda^b\left(B^D, S^D\right)}{\partial S^D}\frac{\partial S^D}{\partial K}.$$

Since $\frac{\partial B^D}{\partial K} < 0$ and $\frac{\partial S^D}{\partial K} > 0$, we need $\frac{\partial\lambda^b\left(B^D, S^D\right)}{\partial B^D} \leq 0$, $\frac{\partial\lambda^b\left(B^D, S^D\right)}{\partial S^D} \geq 0$, $\frac{\partial\lambda^b\left(B^D, S^D\right)}{\partial B^D} \times \frac{\partial\lambda^b(B^D, S^D)}{\partial S^D} \neq 0$. With this setup of general matching function, our main conclusion derived under the linear matching technologies can still go though.

6 Conclusion

This paper developed a model in which market structure is determined endogenously by the choice of intermediation mode. We considered two representative business modes of intermediation that are widely used in real-life markets, a *market-making mode* and a *middleman mode*. We showed

that the mixture of the two modes can emerge. With single-market search, the intermediary can set the intermediation fees and the inventory price without fear of outside competition, so that it will act as a pure middleman in order to create as many transactions as possible. With multiple-market search, the price/fees have to be made acceptable relative to the outside option of agents. We find that the intermediary's creation of an active marketplace, or platform, is the best response to such a competitive pressure, even though the total amount of transactions is not as high as in the pure middleman mode.

For future research, it is interesting to study oligopolistic intermediaries. In particular, we could examine whether the oligopolistic competition, rather than outside option, can shape the emergence of market-making middlemen. Further, some robust patterns of entry behaviors, which are empirically relevant, could be explored among incumbent and entering intermediaries. Some more relevant implications might be obtained with endogenous market power in the wholesales market, which is currently treated as competitive. We believe that the model can be extended to analyze the market-making behaviors of intermediaries in financial markets.

Appendix

Proof of Proposition 1

The proof follows a similar procedure to the one described above. What needs to be added is to show that even though a lower level of $f \leq 1$ increases the expected value of agents in the C market and the participation fees, it does not affect the optional solution, $f^* > 1$ and $K^* = B$, that leads to the emergence of a pure middleman.

Consider first $h = D_b C_s$. Then, by (6) and (7), $g^b = -\lambda^b \beta$ and $g^s = W$. For $f^* > 1$, no buyers go to the platform $x^s = 0$ and all are in the middleman sector $x^m = B$, yielding $g^s = W = 0$. By selecting $K^* = B$ and $p^m = 1$, the intermediary makes profits,

$$\Pi = -B\lambda^b \beta + \Pi(p^m, f^*, K^*) = (-\lambda^b \beta + 1)B.$$

Suppose $f = f^b + f^s \leq 1$ and $K = 0$. Then, $x^s = \frac{B}{S}$ and $x^m = 0$, and $g^s = W(B/S) \geq 0$ as given in (3), if there is a non-negative surplus in the platform for buyers, $f^b + p^s \leq 1$, and for sellers, $f^s \leq p^s$. The resulting profit satisfies

$$\begin{aligned} Bg^b + Sg^s + \Pi(p^m, f, 0) &= -B\lambda^b\beta + S(1 - e^{-\frac{B}{S}})(p^s - f^s) + S(1 - e^{-\frac{B}{S}})f \\ &= -B\lambda^b\beta + S(1 - e^{-\frac{B}{S}})(f^b + p^s) \\ &< -B\lambda^b\beta + B = \Pi \end{aligned}$$

for all $f^b + p^s \leq 1$. Hence, this is not profitable. Suppose $f = f^b + f^s \leq 1$ and $K \in (0, B]$, and both sectors have a non-negative surplus to buyers, i.e., $p^m \leq 1$ and $f^b + p^s \leq 1$. This leads to $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$ that satisfy the add-up requirement (1) and the indifferent condition (2). Then, $g^s = W(x^s) \geq 0$ as given in (3), and the resulting profit is

$$\begin{aligned} & Bg^b + Sg^s + \Pi(p^m, f, K) \\ &\quad = -B\lambda^b\beta + S(1 - e^{-x^s})(p^s - f^s) + S(1 - e^{-x^s})f + \min\{K, x^m\}p^m \\ &\quad < -B\lambda^b\beta + Sx^s(f^b + p^s) + x^m p^m \\ &\quad \leq -B\lambda^b\beta + (Sx^s + x^m)\max\{f^b + p^s, p^m\} \\ &\quad \leq -B\lambda^b\beta + B = \Pi \end{aligned}$$

for all $f^b + p^s \leq 1$ and $p^m \leq 1$. Hence, this is not profitable either. All in all, any deviation is not profitable for $h = D_b C_s$.

Consider next $h = D_s C_b$. Then, by (8) and (9), $g^s = -\lambda^s(1 - \beta)$ and $g^b = V$. When $f^* > 1$, no one go to the platform $x^s = 0$ and all are in the middleman sector $x^m = B$ as long as $p^m \leq 1$. This yields $g^b = V = V^m(B) \geq 0$ as given in (5) and $\Pi(p^m, f, B) = Bp^m$ with $K^* = B$. The profits are

$$\Pi = -S\lambda^s(1 - \beta) + B(1 - p^m) + \Pi(p^m, f^*, K^*) = -S\lambda^s(1 - \beta) + B.$$

Suppose $f = f^b + f^s \leq 1$ and $K = 0$. Then, $x^s = \frac{B}{S}$ and $x^m = 0$, and $g^b = V = V^s(B/S) \geq 0$ as given in (4), if there is a non-negative surplus in the platform for buyers, $f^b + p^s \leq 1$, and for sellers, $f^s \leq p^s$. This leads to

$$\begin{aligned} Sg^s + Bg^b + \Pi(p^m, f, 0) &= -S\lambda^s(1 - \beta) + S(1 - e^{-\frac{B}{S}})(1 - p^s - f^b) + S(1 - e^{-\frac{B}{S}})f \\ &= -S\lambda^s(1 - \beta) + S(1 - e^{-\frac{B}{S}})(1 - p^s + f^s) \\ &< -S\lambda^s(1 - \beta) + B = \Pi \end{aligned}$$

for all $f^s \leq p^s$. Hence, this is not profitable. Suppose $f = f^b + f^s \leq 1$ and $K \in (0, B]$, and both sectors have a non-negative surplus to buyers, i.e., $p^m \leq 1$ and $f^b + p^s \leq 1$. This leads to $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$ that satisfy the add-up constraint (1), $Sx^s + x^m = B$, and the indifferent condition (2), $V^s(x^s) = V^m(x^m)$. Then, $g^b = V = V^s(x^s)$, and the resulting profit is

$$\begin{aligned} & Sg^s + Bg^b + \Pi(p^m, f, K) \\ &\quad = -S\lambda^s(1 - \beta) + B\frac{1 - e^{-x^s}}{x^s}(1 - p^s - f^b) + S(1 - e^{-x^s})f + \min\{K, x^m\}p^m. \end{aligned}$$

There are two cases. Suppose $K \geq x^m$. Then, the indifferent condition implies that

$$p^m = 1 - \frac{1 - e^{-x^s}}{x^s}(1 - p^s - f^b).$$

Applying this expression to the profits, we get

$$\begin{aligned}
&Sg^s + Bg^b + \Pi(p^m, f, K) \\
&= -S\lambda^s(1-\beta) + B\frac{1 - e^{-x^s}}{x^s}(1 - p^s - f^b) + S(1 - e^{-x^s})f + x^m \left(1 - \frac{1 - e^{-x^s}}{x^s}(1 - p^s - f^b)\right) \\
&= -S\lambda^s(1-\beta) + (B - x^m)\frac{1 - e^{-x^s}}{x^s}(1 - p^s - f^b) + S(1 - e^{-x^s})f + x^m \\
&= -S\lambda^s(1-\beta) + S(1 - e^{-x^s})(1 - p^s + f^s) + x^m \\
&< -S\lambda^s(1-\beta) + B
\end{aligned}$$

for all $f^s \leq p^s$. Suppose $K < x^m$. Then, the indifferent condition implies that

$$p^m = 1 - \frac{x^m}{K}\frac{1 - e^{-x^s}}{x^s}(1 - p^s - f^b).$$

Applying this expression to the profits, we get

$$\begin{aligned}
&Sg^s + Bg^b + \Pi(p^m, f, K) \\
&= -S\lambda^s(1-\beta) + B\frac{1 - e^{-x^s}}{x^s}(1 - p^s - f^b) + S(1 - e^{-x^s})f + K \left(1 - \frac{x^m}{K}\frac{1 - e^{-x^s}}{x^s}(1 - p^s - f^b)\right) \\
&= -S\lambda^s(1-\beta) + (B - x^m)\frac{1 - e^{-x^s}}{x^s}(1 - p^s - f^b) + S(1 - e^{-x^s})f + K \\
&= -S\lambda^s(1-\beta) + S(1 - e^{-x^s})(1 - p^s + f^s) + K \\
&< -S\lambda^s(1-\beta) + B
\end{aligned}$$

for all $f^s \leq p^s$. Hence, any deviation is not profitable for $h = D_sC_b$.

Finally, since the intermediary makes the maximum revenue B for either h, which side should be subsidized is determined by the required costs: noting $B\lambda^b = S\lambda^s$ under the pessimistic belief, we have $B\lambda^b\beta \gtreqless S\lambda^s(1-\beta) \iff \beta \gtreqless \frac{1}{2}$. This completes the proof of Proposition 1. ■

Proof of Lemma 1

Substituting $p^m = p^m(x^m)$ in (20) into the profit function in (19), we have:

$$\begin{aligned}
\max_{x^m \in [0,B]} \Pi^m &= S(1 - e^{-x^s})f + \min\{K, x^m\}(1 - \lambda^b e^{-x^s}\beta) - x^m e^{-x^s}(v(x^m, K) - f) \\
s.t.\ \ f &\leq v(x^m, K),
\end{aligned}$$

where $v(x^m, K) \equiv 1 - \lambda^b e^{-x^s}\beta - \lambda^s\xi(x^m, K)(1-\beta)$, $\xi(x^m, K) = 1 - \frac{\min\{K, x^m\}}{B} - \frac{S}{B}(1 - e^{-x^s})$ and $x^s = \frac{B - x^m}{S}$.

Below, we first show that any $x^m > K$ cannot be a solution. Suppose $x^m \geq K$. Then, the Lagrangian is:

$$\mathcal{L} = S(1 - e^{-x^s})f + K(1 - \lambda^b e^{-x^s}\beta) - x^m e^{-x^s}(v(x^m, K) - f) + \mu_k(x^m - K) + \mu_v(v(x^m, K) - f),$$

where $\mu_v \geq 0$ is a Lagrange multiplier for $v(x^m, f) \geq f$ and $\mu_k \geq 0$ for $x^m \geq K$. The maximum in $x^m \in [K, B]$ is characterized by the Kuhn Tucker condition,

$$\frac{\partial \mathcal{L}}{\partial x^m} = -\frac{e^{-x^s}}{S}\left[K\lambda^b\beta + Sv(x^m, K) + x^m(v(x^m, K) - f) - \lambda^b e^{-x^s}x^m\right] + \mu_k - \mu_v\frac{\lambda^b e^{-x^s}}{S} = 0. \tag{23}$$

Define

$$\Phi(x^m) \equiv K\lambda^b\beta + Sv(x^m, K) - \lambda^b e^{-x^s}x^m.$$

Observe that

$$\frac{\partial \Phi(x^m)}{\partial x^m} = -\lambda^b e^{-x^s}(2 + \frac{x^m}{S}) < 0,$$

and

$$\Phi(B) = K\lambda^b\beta + S\left[1 - \lambda^s(1 - \frac{K}{B})(1-\beta) - \lambda^b\beta\right] - \lambda^b B = K\lambda^b + S(1 - \lambda^b\beta) - B\lambda^b(2-\beta) > 0$$

$\iff$

$$K > \frac{1}{\lambda^b}\left[-S(1-\lambda^b\beta) + B\lambda^b(2-\beta)\right] = B\left[\frac{\lambda^b\beta - 1}{\lambda^s} + 2 - \beta\right] \equiv \underline{K} < B,$$

where we use $\lambda^s S = \lambda^b B$. Hence, for $K > \underline{K}$, it holds that $\Phi(x^m) > 0$ for all $x^m \in [K, B]$. Since $v(x^m, K) \geq f$, this implies that for $K > \underline{K}$, the Kuhn Tucker condition (23) holds if and only if $\mu_k > 0$. Hence, any $x^m > K$ cannot be a solution for $K > \underline{K}$. If $K < B$ then a solution $x^m \leq K$ implies we must have $x^s = \frac{B - x^m}{S} > 0$.

What remains to show is $x^s > 0$ if $K = B$. The Kuhn Tucker condition for $x^m \in [0, K]$ is

$$\frac{\partial \mathcal{L}}{\partial x^m} = 1 - e^{-x^s}\left[\lambda^b\beta + v(x^m, K) + \frac{x^m}{S}\left(v(x^m, K) - f - \lambda^b(1 - e^{-x^s})\right)\right] - \mu_k + \mu_v\frac{\lambda^s(1 - e^{-x^s} - \beta)}{S} = 0, \quad (24)$$

where the lagrange multiplier μ_k is for $x^m \in [0, K]$ with $x^m = K$ if $\mu_k > 0$ and $x^m = 0$ if $\mu_k < 0$. Evaluating this equation for $\mu_k > 0$, which implies $x^m = K = B$ and $x^s = 0$, we have

$$\frac{\partial \mathcal{L}}{\partial x^m}\ |_{x^m = K = B} = -\frac{B}{S}\left(1 - \lambda^b\beta - f\right) - \mu_k - \mu_v\frac{\lambda^s\beta}{S} < 0,$$

since $v(B, B) = 1 - \lambda^b\beta \geq f$. Hence, $x^m = K = B$ cannot be a solution, implying that we must have $x^s > 0$ if $K = B$. This completes the proof of Lemma 1. ■

Proof for Theorem 1

The proof of Lemma 1 shows that an inactive platform $x^s = 0$ cannot be a solution of the intermediary's problem for all $f \leq v(x^m, K)$, including a special case $f = v(x^m, K)$ where $p^m(B) = 1 - \lambda^b\beta$. Now remember that $f^* > 1$ and $K^* = B$ (as in Proposition 1) lead to a pure middleman with $p^m = 1 - \lambda^b\beta$ satisfying (17). Hence, for$p^m = p^m(x^m) \geq 0$ given by (20), we must have

$$\begin{aligned}
\Pi(1 - \lambda^b\beta, f^*, K^*) = B(1 - \lambda^b\beta) &= \Pi(p^m(B), v(B, B), B) \\
&< \max_{x^m \in [0,B]} \Pi(p^m(x^m), v(x^m, B), B) \\
&\leq \max_{K \in (\underline{K}, B]} \max_{x^m \in [0,B]} \Pi(p^m(x^m), v(x^m, K), K).
\end{aligned}$$

Hence, a subset

$$\mathfrak{K}_f \equiv \left\{(\underline{K}, B] \mid v(x^m, K) \geq f \text{ and } \max_{x^m \in [0,B]} \Pi(p^m, f, K) > \Pi(1 - \lambda^b\beta, f^*, K^*)\right\}$$

is non-empty. This completes the proof of Theorem 1. ■

Proof for Proposition 2

Consider $h = D_b C_s$ where it holds that $g^b = 0$ and $g^s = (1 - e^{-x^s} - x^s e^{-x^s})(v(x^m, K) - f)$. Applying these fees we obtain

$$\Pi = S(1 - e^{-x^s} - x^s e^{-x^s})v(x^m, K) + S x^s e^{-x^s} f + x^m p^m.$$

Using the envelope condition in relation to the Kuhn Tucker condition (24), one can show that Π is strictly increasing in f, implying that the solution must be with the binding constraint $f^{**} = v(x^m, K)$. Applying $\mu_k < 0$ to the first order condition (21), implying $x^m = 0$, we reach the inequality (22) in the Proposition. A similar procedure applies to examine the case $h = D_s C_b$. The choice of $K \in \mathfrak{K}_f$ is immediate, since given $x^m \leq K$, it does not influence the equilibrium allocation x^m, x^s. This completes the proof of Proposition 2. ■

Proof for Corollary 1

When $x^m = 0$ and $x^s = \frac{B}{S}$, a change in β has no influence on the equilibrium allocation. Remember that $x^m > 0$ is determined by the condition (24). In this case, define $\Psi(x^m, \beta) = \frac{\partial \mathcal{L}}{\partial x^m} = 0$. For $f^{**} = v(x^m, K^{**})$ and $K^{**} \in \mathfrak{K}_{f^{**}}$, it holds that

$$\frac{\partial \Psi(x^m, \beta)}{\partial x^m} = -\frac{e^{-x^s}}{S} v(x^m, K^{**}) - \frac{3\lambda^b}{S} e^{-x^s}(1 - e^{-x^s}) - \frac{x^m}{S^2} \lambda^b e^{-x^s} \beta < 0$$

and

$$\frac{\partial \Psi(x^m, \beta)}{\partial \beta} = -e^{-x^s} \lambda^s \xi(x^m, K^{**}) - \lambda^b(1 - e^{-x^s}) - \frac{S(1 - e^{-x^s}) + x^m}{S} \lambda^b e^{-x^s} - \lambda^b(1 - e^{-x^s})^2 < 0.$$

Hence, $\frac{\partial x^m}{\partial \beta} = -\frac{\frac{\partial \Psi(x^m, \beta)}{\partial \beta}}{\frac{\partial \Psi(x^m, \beta)}{\partial x^m}} < 0$. This completes the proof of Corollary 1. ■

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