

European Research Workshop in International Trade (ERWIT)

held jointly with the

2nd EFIGE Scientific Workshop and Policy Conference

Rome, 16-18 June 2010

Income Distribution, Product Quality, and International Trade

Pablo Fajgelbaum, Gene Grossman and Elhanan Helpman

The views expressed in this paper are those of the author(s) and not those of the funding organization(s), which take no institutional policy positions.

Income Distribution, Product Quality, and International Trade*

Pablo Fajgelbaum
Princeton University

Gene M. Grossman
Princeton University

Elhanan Helpman
Harvard University

June 2009

Abstract

We develop a framework for studying trade in horizontally and vertically differentiated products. In our model, consumers have heterogeneous incomes and heterogeneous tastes. They purchase a homogeneous good as well as making a discrete choice of quality and brand of a differentiated product. The distribution of preferences in the population generates a nested logit demand structure. These demands are such that the fraction of consumers who buy a higher-quality product rises with income. We use the model to study the pattern of trade between countries that differ in size and income distributions but are otherwise identical. Trade—which is driven primarily by demand factors—derives from “home market effects” in the presence of transport costs. When these costs are sufficiently small, goods of a given quality are produced in a single country. The model provides a tractable framework for studying the welfare consequences of trade, transport costs, and trade policy for different income groups in an economy.

Keywords: monopolistic competition, vertical specialization, product quality, nested logit

JEL Classification: F12

*Grossman and Helpman thank the National Science Foundation and Fajgelbaum thanks the International Economics Section at Princeton University for financial support. We are grateful to Aykut Ahlatçioğlu and Gabriel Kreindler for research assistance and to Juan Carlos Hallak, Jon Vogel, and Yoram Weiss for helpful comments and suggestions.

1 Introduction

International trade flows reveal systematic patterns of vertical specialization. When rich and poor countries export goods in the same product category, the richer countries sell goods with higher unit values (Schott, 2004; Hummels and Klenow, 2005; Hallak and Schott, 2008). This suggests a positive association between per capita income and the quality of exports. Also, when a country exports goods in a product category to several destinations, the higher-quality goods are directed disproportionately to the higher-income markets (Hallak, 2006). Since wealthier households typically consume goods of higher quality (Bils and Klenow, 2001; Broda and Romalis, 2009), the pattern of vertical specialization has important implications for the distributional consequences of world trade.

In this paper, we propose a new analytic framework for studying trade in vertically-differentiated products. Our approach features non-homothetic preferences over goods of different quality, as is suggested by the observed consumption patterns. It allows trade patterns to depend on the distributions of income in trading partners and implies that the welfare consequences of trade vary across income groups in any country. It predicts that richer countries will be net exporters of higher-quality goods and net importers of lower-quality goods under reasonable assumptions about levels and distributions of national income. Our model implies that, in many circumstances, trade liberalization benefits especially the poorer households in wealthy countries and the richer households in poor countries.

We provide a demand-based explanation for the pattern of trade in goods of different quality. In this respect, our approach is reminiscent of Linder (1961), who hypothesized that firms in any country produce goods suited to the predominant tastes of their local consumers and sell them worldwide to others who share these tastes.¹ Our approach complements a flourishing literature that highlights various supply-side determinants of trade in vertically-differentiated goods. In Markusen (1986) or Bergstrand (1990), for example, the country with higher per-capita income exports the luxury good, because the luxury good happens to be capital intensive. Similarly, in Flam and Helpman (1987), Stokey (1991), Murphy and Shleifer (1997) and Matsuyama (2000), the pattern of trade follows from an assumption that richer countries have relative technological superiority in producing higher-quality goods.² More recently, Baldwin and Harrigan (2007) and Johnson (2008) have incorporated vertically-differentiated products into trade models with heterogeneous firms. They seek to explain the observation that more productive firms export higher-priced (and therefore,

¹Mitra and Trindade (2005) also offer a demand-based explanation for the pattern of trade in a model with non-homothetic preferences. However, their model implies that countries will export goods that are little demanded at home absent any supply-side differences between them. See also Foellmi et al. (2007), who model trade in horizontally-differentiated goods with discrete choice and non-homothetic preferences.

²See also Fieler (2008), who finds in her calibration exercise using a Ricardian framework à la Eaton and Kortum (2002) with two industries and many goods that the industry with a higher income elasticity of demand also has the greater spread in its productivity draws. As she shows, this gives the country with the higher technology level a comparative advantage in luxury goods.

presumably, higher quality) products with reference to the relatively greater incentive that such firms have to undertake quality-enhancing investments. Their approach would generate a supply-side explanation for the observed pattern of trade if richer countries are home to a disproportionate share of the high-productivity firms.³

The demand structure that we exploit has strong empirical roots. We assume that individuals consume varying quantities of a homogeneous good and a discrete choice of a product that is both horizontally and vertically differentiated. Consumers choose among different quality options of the good and from a set of distinctive products at each quality level that have idiosyncratic appeal. The assumed form of the utility function and the distribution of tastes are such that the system of aggregate demands exhibits a nested-logit structure. We draw on the theory of such demands that has been developed by McFadden (1978), Anderson et al. (1992) and Verboven (1996a), among others.⁴ Since the econometric methods for estimating such models are well developed, our analytic framework should be readily amenable to empirical research.

We posit a utility function that features complementarity between the quantity of the homogeneous good and the quality of the differentiated product. This property of the assumed preferences implies that the marginal value of quality is higher for those with greater income, who consume more of the homogeneous good. Yet consumers have idiosyncratic valuations of each of the differentiated products, which means that a wealthy consumer may fancy a particular low-quality variety while a poorer consumer favors one of the high-quality products. In the aggregate, the fraction of consumers that buys a high-quality product rises with income. This behavior generates heterogeneity in income elasticities of demand across different goods. Such heterogeneity has proven useful in explaining bilateral trade flows in work by Hunter and Markusen (1988), Bergstrand (1990), and Hunter (1991).

The non-homotheticities in demand forge a link between the shape of a country's income distribution and the pattern and intensity of its trade in vertically-differentiated products. We draw out some of the implications in our analysis, much as do Flam and Helpman (1987), Matsuyama (2000), and Mitra and Trindade (2005). Dalgin et al. (2008) and Choi et al. (2009) show that such links between income distribution and trade patterns are important in reality.⁵

In our model, patterns of aggregate demand translate into patterns of specialization and trade via "home-market effects." As Krugman (1980) argued, when transportation is costly, a large home market lends an advantage to local firms producing under increasing returns to scale. Therefore, countries tend to export the increasing-returns goods that are in great domestic demand.⁶ We derive

³See also Kugler and Verhoogen (2008) and Hallak and Sivadasan (2009), who provide empirical evidence on the relationship between firm size, firm productivity, and export unit values.

⁴The nested-logit demand structure has been applied to international trade by Goldberg (1995) and Verboven (1996b), and more recently by Khandelwal (2008) and Verhoogen (2008). The latter two include a vertical dimension of product differentiation in their discussion, but their focus is very different from ours.

⁵Choi et al. (2009) show that country pairs that share more similar income distributions also exhibit more similar distributions of import prices. Dalgin et al. (2008) find a positive correlation in a sample of developed countries between income dispersion and imports of luxury goods.

⁶Hanson and Xiang (2006) extend Krugman's argument to a setting with many industries that differ in transport costs and the extent of product differentiation. They provide empirical support for the proposition that larger

conditions under which a richer country, or one with a more dispersed distribution of income, has a larger home demand for high-quality goods and a smaller home demand for low-quality goods. Under such conditions, more firms enter to produce high-quality goods in the richer (or more unequal) country, while the opposite is true of firms producing low-quality products. Firms at a given quality charge the same ex-factory prices, so the number of producers predicts the direction of trade. Thus, our model can explain, for example, why Germany exports high-quality cars to Korea while importing low-quality cars from there.

Our framework provides a tractable and parsimonious tool for studying the distributional implications of changes in transport costs or trade policy. Since different income classes in a country consume different mixes of low- and high-quality products, the delocation of firms induced by changes in trading conditions affects the welfare of the various income classes differently. We find, for example, that trade liberalization in a rich country tends to favor the lower-income groups there, who benefit qua consumers from an expansion in the range of product offerings at the low-quality level and from a transfer of income from groups that consume greater shares of the high-quality good.

In Section 2, we develop our framework in the context of a closed economy. Each consumer buys one unit of some differentiated product and devotes all remaining income to the homogeneous good. Individuals have idiosyncratic evaluations of the various differentiated products, which also differ in quality. The distribution of taste parameters generates a nested-logit structure of aggregate demands. We combine these demands with a simple supply model that features a single factor of production, fixed costs plus constant unit costs that vary by quality level, and free entry into the differentiated-products sector. In the monopolistically-competitive equilibrium, each firm producing a differentiated product charges a fixed markup over its unit cost that depends on the quality level of its product and a parameter describing the distribution of idiosyncratic tastes. We show in Section 3 that the autarky equilibrium is unique and that it is characterized by positive numbers of producers of both low- and high-quality goods. We proceed to examine how changes in population size and in the level and spread of the income distribution affect the numbers of producers at each quality level and the welfare of different income groups.

Section 4 introduces international trade between countries that share similar supply characteristics but differ in their levels and distributions of income. We assume that differentiated products are costly to transport internationally, with per-unit shipping costs that may vary with the quality level. When shipping costs are sufficiently high, as we assume throughout that section, each country produces and trades both low- and high-quality goods. We examine how country sizes and income distributions combine to determine the pattern of trade. We also investigate the distributional implications of a decline in trading costs. When such costs decline sufficiently, the production of goods of a given quality must be concentrated in a single country, as we show in Section 5. For trading costs close enough to zero, each good is produced in the country that would have the larger

countries export more in industries with high transport costs and highly-differentiated products. See Davis and Weinstein (2003) for further empirical evidence of home-market effects in the pattern of trade.

home market in a hypothetical, integrated equilibrium. This implies, for example that if countries are of equal size and the income distribution in one first-order stochastically dominates that in the other, then the richer country produces and exports the higher-quality goods while the poorer country produces and exports the lower-quality goods.

In Section 6, we study commercial policy. Tariffs have no effect on ex-factory prices in our model. The welfare effects of a tariff derive from a composition effect and a redistribution effect. The former captures the change in the relative numbers of high- and low-quality products that results from protection. The latter reflects the transfer of tariff revenues from import purchasers to the average consumer. Section 7 offers concluding remarks and an appendix gathers the algebraic derivations and proofs.

2 The Model

We develop a model featuring income heterogeneity and non-homothetic preferences over goods of different quality. We describe the model in this section, characterize its autarky equilibrium in the next, and then move on to international trade in Section 4 below.

Each individual consumes a homogeneous good and his optimal choice from a finite set of differentiated products. Both types of goods are produced with labor alone. The homogeneous good requires one unit of effective labor per unit of output. This good is competitively priced and serves as numeraire. The differentiated products require a fixed input of labor and a constant variable input per unit of output. Monopolistic competition prevails in this industry. We assume that the labor supply is sufficiently large relative to aggregate demand for differentiated products to ensure a positive output of the numeraire good in any equilibrium. Then competition implies a wage rate for effective labor equal to one.

The economy is populated by a continuum of individuals who are endowed with different amounts of effective labor. This heterogeneity in endowments generates a distribution of income. We denote the income distribution by F_y , so that $F_y(y)$ is the fraction of the mass N of individuals with effective labor and wage income less than or equal to y . We assume throughout that every individual has sufficient income to purchase one unit of any brand of the differentiated product, including the most expensive, at the prevailing equilibrium prices.

Each consumer values only one unit of the differentiated product and thus faces a discrete consumption choice. Each buys the good that offers him the highest utility, considering the prices and characteristics of all available products. Varieties are distinguished by their quality level, as well as by other attributes that affect consumers' idiosyncratic valuations. We denote by Ω the finite set of available quality levels such that a good with index $i \in \Omega$ has quality q_i . For most of the paper, we will take $\Omega = \{H, L\}$ with $q_H > q_L$; i.e., consumers choose from among two quality levels, opting either for one of the high-quality products or one of the low-quality products. At each quality level, there is a large number of competing brands that consumers evaluate differently. So, each consumer chooses not only a quality level, but also a particular item from among the set

Q_i of brands with quality q_i .

2.1 Preferences and Demand

Consider an individual h who consumes z units of the homogenous good and a variety j of the

and

$$\rho_i(y) = \frac{\left(\sum_{j \in Q_i} e^{(y-p_{i,j})q_i/\theta_i}\right)^{\theta_i}}{\sum_{\omega \in \Omega} \left(\sum_{j \in Q_\omega} e^{(y-p_{\omega,j})q_\omega/\theta_\omega}\right)^{\theta_\omega}}$$

is the fraction of consumers that opt for a product of this quality. These fractions depend on the price vector, of course, and the latter fraction varies across the income distribution.

Readers familiar with the empirical literature on discrete-choice modeling will recognize the implied demand system as a *nested logit*, with choice over quality levels and brands of a given quality. In that literature, θ_i is known as the dissimilarity parameter; it measures the degree of heterogeneity in preferences over the brands in the set Q_i .⁸ The greater is θ_i , the smaller is the correlation between $\varepsilon_{i,j}$ and $\varepsilon_{i,j'}$ for j and j' in Q_i (see McFadden, 1978), and therefore the greater are the perceived differences among the various brands with quality q_i . If different brands of lower-quality products are better substitutes for one another than different brands of higher-quality products, then θ_i will be increasing in product quality. This ordering seems plausible inasmuch as higher-quality products typically embody richer sets of product characteristics, which expands the scope for horizontal differentiation. However, our analysis does not require any particular relationship between θ_i and q_i .⁹

Different income groups allocate their spending similarly among the various brands of a given quality. Variation in spending patterns across income groups arises solely from variation in the fraction of individuals who purchase a product with quality q_i , as reflected by the function $\rho_i(y)$. It follows that the market share of a good of quality q_i varies across income groups according to

$$\frac{1}{\rho_{i,j}(y)} \frac{d\rho_{i,j}(y)}{dy} = \frac{1}{\rho_i(y)} \frac{d\rho_i(y)}{dy} = q_i - q_a(y), \quad (2)$$

where

$$q_a(y) \equiv \sum_{i \in Q} q_i \rho_i(y)$$

is the average quality consumed by individuals with income y . Equation (2) implies that the fraction of individuals who purchase brand j of quality q_i rises with income if and only if $q_i > q_a(y)$; that is, if and only if q_i is higher than the average quality consumed by individuals in this income group. In particular, in the case of two qualities, $Q = \{q_H, q_L\}$, we have $q_H > q_a(y) > q_L$ for all y , so that the fraction of individuals who purchase high-quality products rises with income at all income levels. This is the key property of these non-homothetic preferences that will guide our analysis of

⁸Readers familiar with the trade literature will also recognize a similarity between the distribution of preference shocks here and the distribution of productivity shocks in the Ricardian model of Eaton and Kortum (2002). In their work, the productivity shocks are assumed to have a Type-II extreme value distribution in which θ parameterizes the dissimilarity of productivity levels across goods.

⁹With this specifications of preferences and the distribution of the idiosyncratic taste parameter, the ratio of fractions $\rho_{i,j}(y)/\rho_{i,j'}(y)$, for $j, j' \in Q_i$ does not depend on the set of varieties available in other quality classes, nor on the prices of these goods. This ratio does vary with θ_i , but not with the dissimilarity parameters that apply to other quality groups. On the other hand, the ratio of fractions $\rho_{i,j}(y)/\rho_{i',j'}(y)$, for $j \in Q_i$ and $j' \in Q_{i'}$ depends on the set of products available in both groups and on the dissimilarity parameters for each.

the trade flows.

From now on, we will focus on an economy with two qualities of the differentiated product. Then we can write the fraction of individuals with income y who purchase brand j of quality q_i as

$$\rho_{i,j}(y) = \frac{e^{-p_{i,j}q_i/\theta_i}}{\sum_{j' \in Q_i} e^{-p_{i,j'}q_i/\theta_i}} \frac{\left[\sum_{j' \in Q_i} e^{(y-p_{i,j'})q_i/\theta_i} \right]^{\theta_i}}{\sum_{\omega \in \{H,L\}} \left[\sum_{j' \in Q_\omega} e^{(y-p_{\omega,j'})q_\omega/\theta_\omega} \right]^{\theta_\omega}} \quad \text{for } i = H, L.$$

Using this expression, we calculate *per capita* demand for variety j of quality q_i ,

$$\begin{aligned} d_{i,j} &= \int_{y_{\min}}^{\infty} \rho_{i,j}(y) dF_y(y) \\ &= \frac{e^{-p_{i,j}q_i/\theta_i}}{\sum_{j' \in Q_i} e^{-p_{i,j'}q_i/\theta_i}} \int_{y_{\min}}^{\infty} \frac{\left[\sum_{j' \in Q_i} e^{(y-p_{i,j'})q_i/\theta_i} \right]^{\theta_i}}{\sum_{\omega \in \{H,L\}} \left[\sum_{j' \in Q_\omega} e^{(y-p_{\omega,j'})q_\omega/\theta_\omega} \right]^{\theta_\omega}} dF_y(y), \end{aligned} \quad (3)$$

where y_{\min} is the income of the poorest individual in this economy. Evidently, the aggregate demand $Nd_{i,j}$ for brand j of quality q_i depends on the price of this good and on aggregate indexes of the prices of competing products in each quality class.

2.2 Pricing and Profits

Firms can enter freely into the differentiated-products sector by employing a fixed input f_i of effective labor to produce a good of quality q_i . Each active firm produces a different brand, using c_i units of labor per unit of output for a good of quality q_i . The producers of differentiated products set prices to maximize profits taking the aggregate price indexes as given. Entry at each quality level proceeds until the next entrant would fail to cover its fixed cost. Let n_i be the number of firms that produce goods of quality q_i . The demand structure requires the number of brands in each quality class to be a finite integer, but we will take liberty in treating n_i as if it were a continuous variable to facilitate the exposition.

A firm that produces a brand j of the differentiated product with quality q_i earns profits of $\pi_{i,j} = Nd_{i,j}(p_{i,j} - c_i) - f_i$, where $d_{i,j}$ is given in (3). If the number of active producers of each quality level is large, the terms in the various sums in (3) vary only slightly with a firm's own price. We assume that the firm ignores this dependence, as is common in models of monopolistic competition. Then a firm producing any brand j with quality q_i maximizes profits by setting the price

$$p_i = c_i + \frac{\theta_i}{q_i} \quad \text{for } i = H, L. \quad (4)$$

Evidently, the markup over marginal cost differs for goods of different qualities. The markup reflects two properties of the class of goods. First, the higher is q_i , the greater is the marginal utility from consumption of the homogeneous good, due to the complementarity between z and q_i reflected in (1). A higher marginal utility from consumption of the homogeneous good makes consumers more

sensitive to price differences when choosing among the different brands in Q_i . Second, the greater is θ_i , the greater are the perceived differences among the various brands with quality q_i , as we have noted before. This greater degree of product differentiation tends to make demands less sensitive to price changes. These two forces may work in opposition as they affect price setting; the markup on high-quality goods will be greater than that on low-quality goods if and only if $\theta_H/q_H > \theta_L/q_L$.

With common prices, the firms that produce different varieties of a good of a given quality q_i achieve similar volumes of sales. Let d_i be the quantity demanded per capita of a typical brand in Q_i when all goods are priced according to (4). Then

$$d_i = \frac{1}{n_i} \mathbb{E} \left[\frac{n_i^{\theta_i} \phi_i(y)}{n_H^{\theta_H} \phi_H(y) + n_L^{\theta_L} \phi_L(y)} \right], \quad \text{for } i = H, L, \quad (5)$$

where

$$\phi_i(y) \equiv e^{(y-c_i)q_i - \theta_i}$$

captures the effect of income on demand and \mathbb{E} is the expectations operator with respect to the distribution of income, i.e., $\mathbb{E}[B(y)] \equiv \int_{y_{\min}}^{\infty} B(y) F_y(y) dy$. The markups of θ_i/q_i on sales of Nd_i yield a common profit π_i to all producers of brands with quality q_i , where

$$\pi_i(n_H, n_L) \equiv \frac{\theta_i}{q_i} \frac{N}{n_i} \mathbb{E} \left[\frac{n_i^{\theta_i} \phi_i(y)}{n_H^{\theta_H} \phi_H(y) + n_L^{\theta_L} \phi_L(y)} \right] - f_i, \quad \text{for } i = H, L. \quad (6)$$

These functions determine the profitability of entry at each quality level. In equilibrium, $n_i > 0$ implies $\pi_i(n_H, n_L) = 0$ while $\pi_i(n_H, n_L) \leq 0$ when $n_i = 0$. In the next section, we will use these free-entry conditions to characterize an equilibrium in a closed economy. Once the number of firms producing at a given quality level is known, sales of each brand of the differentiated products also are determined. Together, the firms selling goods with quality q_i capture aggregate sales of $n_i d_i N$, so that aggregate output of all differentiated products is $n_H d_H N + n_L d_L N = N$.¹⁰ This equality reflects the fact that each of the mass N of consumers buys one unit of some product.

The differentiated-products industry employs a total of $n_H(d_H c_H N + f_H) + n_L(d_L c_L N + f_L)$ units of effective labor. The difference between aggregate labor supply—which equals N times the mean value of y —and labor use in the differentiated-products industry gives the labor used in producing homogeneous goods. The market for homogeneous products clears by Walras' Law. Therefore, once we solve for the number of firms of each type in the differentiated-products industry, the remainder of the variables determined in the general equilibrium are readily found. Accordingly, we focus in what follows on the determinants of n_H and n_L .

¹⁰Note that $n_i d_i = \mathbb{E}[\rho_i(y)]$; i.e., the per capita demand for all goods with quality q_i matches the probability that a random consumer will choose to purchase a good with that quality.

3 Autarky Equilibrium

To characterize an equilibrium in a closed economy, we define x_i as the quantity that a firm producing a brand with quality q_i must sell in order to break even when it prices according to (4); i.e.,

$$x_i = \frac{f_i q_i}{\theta_i} \quad \text{for } i = H, L. \quad (7)$$

Notice that the break-even volume depends only on the size of the fixed cost and the size of the markup, as in Krugman (1980). So, (7) will pin down the output per brand for any quality of good that is available in equilibrium.

In an autarky equilibrium, if some positive number of firms produce goods with quality q_i the demand per brand must reach the break-even level. Otherwise, no firm producing this quality can profitably enter. In other words, if $n_i > 0$, $Nd_i = x_i$, whereas $Nd_i < x_i$ implies $n_i = 0$. In any case, the aggregate output of all differentiated products matches the population size N , or

$$n_H x_H + n_L x_L = N. \quad (8)$$

We will refer to this equation as the *aggregate demand* condition. It implies, of course, that at least one of n_H and n_L must be positive.

But notice from (5) that as n_i approaches zero with $n_{i'} > 0$ for $i' \neq i$, the demand for a typical brand with quality q_i grows infinitely large. This means that a producer of a brand with quality q_i will certainly be able to achieve the break-even volume when the number of its competitors offering a similar quality is sufficiently small. In equilibrium, brands of both qualities will be available to consumers.¹¹

Now that we know that n_H and n_L both must be positive, market-clearing for each brand requires $x_i = Nd_i$ or

$$x_i = N \mathbb{E} \left[\frac{n_i^{\theta_i-1} \phi_i(y)}{n_H^{\theta_H} \phi_H(y) + n_L^{\theta_L} \phi_L(y)} \right], \quad \text{for } i = H, L. \quad (9)$$

This pair of equations in turn implies¹²

$$\mathbb{E} \left[(n_L x_L)^{\theta_L} + \frac{(n_H x_H + n_L x_L) G_H(y)}{(n_H x_H)^{1-\theta_H} G_L(y) - (n_L x_L)^{1-\theta_L} G_H(y)} \right]^{-1} = 0, \quad (10)$$

¹¹In making this statement, we have ignored the integer constraint. The equilibrium “solution” for some n_i might be a fraction, in which case it might not be profitable for the first “whole” firm to enter in a quality segment. Moreover, we have assumed that many firms compete in order to justify our assumption that firms take price indexes as given. We will not divert attention to these details, but instead restrict ourselves to parameters for which our focus on an equilibrium with $n_H > 0$ and $n_L > 0$ is well justified.

¹²Note that the weighted sum of x_H and x_L from (9) implies (8), which means that only two of these three equations are independent. See the appendix for the derivation of equation (10).

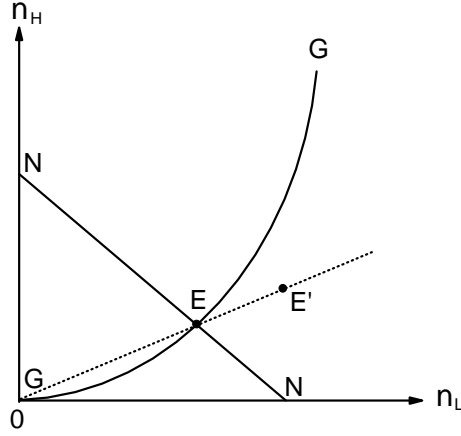


Figure 1: Autarky equilibrium

where

$$G_i(y) \equiv x_i^{-\theta_i} \phi_i(y) .$$

We will refer to (10) as the *composition of demand* condition; it gives a second relationship between n_H and n_L , once x_H and x_L are pinned down by (7).

Equations (8) and (10) have a unique solution, which indeed has $n_H > 0$ and $n_L > 0$. To see this, substitute (8) into (10), to derive

$$\mathbb{E} \left[(n_L x_L)^{\theta_L} + \frac{N G_H(y)}{(n_H x_H)^{1-\theta_H} G_L(y) - (n_L x_L)^{1-\theta_L} G_H(y)} \right]^{-1} = 0 . \quad (11)$$

Combinations of n_H and n_L that satisfy (11) are depicted by the curve GG in Figure 1. These are the numbers of firms in the two quality classes that are consistent with equal profitability across the two types of firms, given that the typical firm in Q_i sells x_i units and that aggregate sales are equal to N . Equation (11) is satisfied in the limit as $n_L = n_H \rightarrow 0$. Moreover, the curve GG is everywhere upward sloping. This reflects the fact that, for given aggregate demand, as n_L increases, the profitability of a brand in Q_L falls by more than one in Q_H , because the extra low-quality goods compete more directly with other low-quality goods than with the high-quality goods. To restore equal profitability in the two quality classes, the number n_H of producers of high-quality goods would have to rise in order to maintain the required composition of demand.

As the figure shows, if the GG curve begins at the origin and slopes upward, it surely must cross the NN line—which represents equation (8)—and can do so only once.

This establishes

Proposition 1 *There exists a unique autarky equilibrium. In the autarky equilibrium, $n_H > 0$ and $n_L > 0$.*

In the remainder of this section, we describe how the autarky equilibrium reflects the size of the economy and its income distribution.¹³ We also show how the model can be used to examine the welfare implications of changes in the economic environment for different income groups. These properties of the model will aid us in understanding the direction and distributional implications of trade in the sections that follow.

The size of the economy is captured by the parameter N . As N increases, the NN curve in Figure 1 shifts out by the same proportion as the increase in the consumer population. Equation (11) implies that if $\theta_H = \theta_L$, the GG curve shifts out radially by this same proportion, and the new equilibrium is at E' , where $\hat{n}_H = \hat{n}_L = \hat{N}$.¹⁴ That is, when the dissimilarity of low and high quality goods is the same, an expansion in the size of the economy leads to an equiproportionate increase in the number of goods of either quality level. If $\theta_H > \theta_L$ —that is, if low-quality goods substitute more closely for one another than high-quality goods, as seems most plausible—the GG curve does not shift quite so far radially as for the case of equal dissimilarity parameters. Then $\hat{n}_H > \hat{N} > \hat{n}_L$, and possibly $\hat{n}_L < 0$. Alternatively, if $\theta_H < \theta_L$, the GG curve shifts out radially by more than the proportional increase in population size, so that $\hat{n}_L > \hat{N} > \hat{n}_H$ and possibly $\hat{n}_H < 0$. In short, growth in market size causes an expansion in the relative number of the more horizontally-differentiated products.¹⁵

Now consider an upward shift in the income distribution in the sense of first-order stochastic dominance; i.e., at every income level y , the fraction of the population with income less than or equal to y declines. This has no effect on the NN curve, but it causes the GG curve to shift to the left.¹⁶ Intuitively, as the economy grows richer, demand shifts toward the high-quality goods, raising the relative profitability of these goods. This induces entry of firms that produce such goods and exit of producers of low-quality products. Therefore, when comparing two similarly-sized economies, the richer one (in the sense of first-order stochastic dominance) will have more varieties of the high-quality product and fewer varieties of the low-quality product.

Finally, consider an increase in income inequality, as represented by a mean-preserving spread of the distribution $F_y(\cdot)$. The effect on relative demand is in general ambiguous, as those at the top end of the distribution collectively buy more of the high-quality goods while those at the bottom end do just the opposite. However, if the initial equilibrium is such that a majority of every income class purchases low-quality products, then the former effect must dominate.¹⁷ Thus, a spread in income distribution in a poor economy (one in which $\rho_L(y) > \rho_H(y)$ for all y) induces a shift in the

¹³The algebra of the comparative statics that we describe here are derived more formally in the appendix.

¹⁴We use a circumflex to denote a proportional increase; i.e., $\hat{Z} = dZ/Z$.

¹⁵See Epifani and Gancia (2006) and Hanson and Xiang (2004) for similar results in a different context.

¹⁶The direction of shift in the GG curve depends upon the pattern of the changes in $G_L(y)/G_H(y)$ but

$$\frac{G_L(y)}{G_H(y)} = \frac{x_L^{-\theta_L} \phi_L(y)}{x_H^{-\theta_H} \phi_H(y)} = \frac{x_L^{-\theta_L}}{x_H^{-\theta_H}} e^{(\theta_H + c_H q_H) - (\theta_L + c_L q_L)} e^{y(q_L - q_H)}$$

decreases with y . This reflects the fact that the fraction of consumers who buy high quality is increasing in income.

¹⁷If $\rho_L(y) > \rho_H(y)$ for all y , then the term inside the expectation in (10) is a concave function of y for given n_H and n_L . Therefore, a mean-preserving spread in the distribution of y causes the GG curve to shift to the left in such circumstances. See the appendix for the details.

composition of firms toward producers of high-quality products.

We can readily examine the implications of these shifts for the welfare of different income groups. As McFadden (1978) has shown, the expected welfare among those with income y increases with

$$v(y) \equiv n_H^{\theta_H} \phi_H(y) + n_L^{\theta_L} \phi_L(y) . \quad (12)$$

As conditions change,

$$\hat{v}(y) = \rho_H(y) \theta_H \hat{n}_H + \rho_L(y) \theta_L \hat{n}_L .$$

In words, the change in average welfare at income y weights the changes in the number of products in each quality class by the probability that a consumer with income y purchases a good of that class times the degree of horizontal differentiation (dissimilarity) within the class.

From the aggregate demand condition (8), $\rho_H \hat{n}_H + \rho_L \hat{n}_L = \hat{N}$, where $\rho_i = n_i d_i = n_i x_i / N$ is the fraction of the overall population that purchases a good of quality i . Using this equation, we can write

$$\hat{v}(y) = \left[\theta_L \frac{\rho_L(y)}{\rho_L} + \theta_H \frac{\rho_H(y)}{\rho_H} \right] \hat{N} + \rho_H \rho_L \left[\theta_H \frac{\rho_H(y)}{\rho_H} - \theta_L \frac{\rho_L(y)}{\rho_L} \right] (\hat{n}_H - \hat{n}_L) . \quad (13)$$

The first term in the expression for $\hat{v}(y)$ is a pure *scale effect*. Holding constant the relative number of high- and low-quality products, an expansion of scale benefits individuals at all income levels, because it increases the number of varieties and therefore increases the likelihood that an individual will find one to his liking. The second term is a pure *composition effect*. For a given scale, an increase in the relative number of high-quality products benefits those who are more likely than average to consume such a product and harms those who are more likely than average to consume a low-quality product. An increase in the variety of high-quality products relative to the variety of low-quality products is more likely to benefit a given income group the more dissimilar are the brands of high-quality products and the more similar are the brands of low-quality products.

An increase in population size generates a scale effect that benefits all consumers and a composition effect that affects the various income groups differently. If $\theta_H = \theta_L$, $\hat{n}_H = \hat{n}_L$ and all income groups gain from population growth. If $\theta_H > \theta_L$, as seems most plausible, then $\hat{n}_H > \hat{n}_L$. The richest individuals in the economy are more likely to purchase the high-quality good than the average consumer and less likely to purchase the low-quality good, so $\rho_H(y_{\max}) / \rho_H > 1 > \rho_L(y_{\max}) / \rho_L$. These individuals must gain on average from population growth. The poorest consumers—who are more likely than average to purchase the low-quality good and less likely than average to consume the high-quality good—may gain or lose.

An upward shift in the income distribution (or a spread of the distribution in a poor economy) generates a shift in the composition of differentiated products toward high-quality goods, without changing the output-weighted number of products. With $\theta_H = \theta_L$, the associated composition effect benefits the members of the highest income group (on average) and harms the members of the lowest income group. The former are more likely than average to consume a high-quality good and so they gain from having a wider variety of choices. The latter are more likely than average

to consume a low-quality good and so they lose from having fewer choices. If $\theta_H > \theta_L$, the highest income group again must benefit, but the lowest income group may do so as well. Although the latter are more likely to consume a low-quality product, the contraction of variety does not hurt them so much because the goods are relatively similar; meanwhile, the expansion of variety of high-quality products is advantageous, because these goods are relatively dissimilar. If there are losers (on average) from a change in the composition of products that favors high-quality goods, it will be all those groups with income less than or equal to some critical value.

4 Trade with Incomplete Specialization

In this section, we introduce international trade. We assume that there are two countries, \mathcal{R} and \mathcal{P} , that differ only in size and in their distributions of efficiency labor. In other words, we do not allow for any comparative advantage in production. By doing so, we abstract from supply-side determinants of the pattern of trade in order to focus on those that derive from differences in income in the face of non-homothetic preferences. The country designations of \mathcal{R} and \mathcal{P} are meant to suggest “rich” and “poor,” although we do not insist on any particular relationship between their sizes or their income distributions except in some special cases.

We assume throughout that both countries have sufficient supplies of effective labor relative to the equilibrium labor demands by their producers of differentiated products so that some labor in each country is used to produce the homogeneous, numeraire good. This ensures that the wage of a unit of effective labor is equal to one in both countries.

We assume that the differentiated products are costly to trade.¹⁸ In particular, it takes τ_i units of effective labor to ship one unit of a variety with quality q_i from one country to the other.¹⁹ As τ_i grows large, national outputs converge on those of the autarky equilibria. In such a setting, as we now know, both countries produce goods of either quality. We will find that such incomplete specialization characterizes the trade equilibrium whenever the trading costs are sufficiently high. These are the circumstances that we consider now, whereas in the next section we will study equilibria in which each quality level is produced in only one country, as happens almost always when shipping costs are small.

Shipping requirements raise the cost of serving foreign consumers relative to domestic consumers. For a good with quality q_i , the marginal cost of a delivered export unit is $c_i + \tau_i$, whereas local consumers can be supplied at a cost of c_i . The arguments from Section 2.2 now imply that a firm producing a brand with quality q_i maximizes profits by charging foreign consumers the price $c_i + \tau_i + \theta_i/q_i$, whereas domestic consumers are charged the lower price $c_i + \theta_i/z_i$ (see (4)). In other words, mark-ups are θ_i/q_i for all sales, as firms fully pass on their shipping costs to their foreign

¹⁸Davis (1998) has shown that a home-market effect may not exist if differentiated products and homogeneous goods bear similar trading costs. But Amiti (1998) and Hanson and Xiang (2004) demonstrate that the home-market effect requires only that transport costs differ across sectors.

¹⁹Notice that the trading costs here are a certain amount per unit, not “iceberg costs” as is more typically (but unrealistically) assumed in models of monopolistic competition with trade. Iceberg costs could be readily incorporated into our framework as well.

customers.

Demands for domestic goods of quality q_i in country k reflect the prices of these goods, the prices of competing import goods, and the numbers of local and foreign imported brands at each quality level. Letting d_i^k represent the per capita demand by domestic consumers for a typical good of quality q_i produced in country k when all goods are priced optimally, (3) implies

$$d_i^k = \frac{1}{\tilde{n}_i^k} \mathbb{E}^k \left[\frac{(\tilde{n}_i^k)^{\theta_i} \phi_i(y)}{(\tilde{n}_H^k)^{\theta_H} \phi_H(y) + (\tilde{n}_L^k)^{\theta_L} \phi_L(y)} \right], \quad i = H, L \text{ and } k = \mathcal{R}, \mathcal{P} \quad (14)$$

where

$$\begin{aligned} \tilde{n}_i^k &= n_i^k + \lambda_i n_i^\ell, \\ \lambda_i &\equiv e^{-\tau_i q_i / \theta_i}, \end{aligned}$$

n_i^k is the number of brands of goods of quality q_i produced in country k , and \mathbb{E}^k is the expectation with respect to the income distribution in country k . Notice the similarity between (14) and (5). Now, domestic brands share the market with both domestic and foreign rivals but, inasmuch as imports of a given quality bear a higher price due to shipping costs, the foreign brands are less effective competitors. For domestic firms, demand per capita is the same as it would be in autarky with \tilde{n}_i^k local competitors producing quality q_i . The foreign firms are discounted in this measurement of “effective competitors” by an amount $\lambda_i \in (0, 1)$ that reflects the trading cost for goods of quality q_i , as well as the quality and dissimilarity of these products. Also, (3) implies that per capita demand for an imported variety of quality q_i in country k equals $\lambda_i d_i^k$; i.e., it is a fraction λ_i of the demand faced by a local brand.

A firm producing a variety with quality q_i in either country earns profits per sale of θ_i/q_i , considering the fixed mark-up it charges over delivered cost. In order to break even, such a firm, no matter where it is located, must make sales totalling $x_i = f_i q_i / \theta_i$ units, as per (7). In an equilibrium with producers of both qualities in both countries, we must have

$$x_i = N^k d_i^k + \lambda_i N^\ell d_i^\ell, \quad \ell \neq k, k = \mathcal{R}, \mathcal{P}, i = H, L,$$

where N^k is the population of consumers in country k . The right-hand side of this equation represents total sales by a firm located in country k , comprising domestic sales and exports sales, where the latter are a fraction λ_i of what a local producer in country ℓ makes of domestic sales. For these equations to hold for both $k = \mathcal{P}$ and $k = \mathcal{R}$ it must be that $N^{\mathcal{R}} d_i^{\mathcal{R}} = N^{\mathcal{P}} d_i^{\mathcal{P}}$ for $i = H, L$; that is, firms in each country must achieve the same volume of domestic sales. Since the size and the income distributions in the two countries may differ, the equality must be achieved by adjustments in the numbers of effective firms in each market. In particular, the equality between the required volume of total sales and the total demand faced by producers in market k implies

$x_i = N^k d_i^k (1 + \lambda_i)$ or

$$N^k \mathbb{E}^k \left[\frac{(\tilde{n}_i^k)^{\theta_i-1} \phi_i(y)}{(\tilde{n}_H^k)^{\theta_H} \phi_H(y) + (\tilde{n}_L^k)^{\theta_L} \phi_L(y)} \right] = \frac{1}{1 + \lambda_i} \frac{f_i q_i}{\theta_i}, \quad i = H, L, k = \mathcal{R}, \mathcal{P}. \quad (15)$$

The equations in (15) provide four independent relationships, two for country \mathcal{R} that jointly determine $\tilde{n}_L^{\mathcal{R}}$ and $\tilde{n}_H^{\mathcal{R}}$ and two for country \mathcal{P} that jointly determine $\tilde{n}_L^{\mathcal{P}}$ and $\tilde{n}_H^{\mathcal{P}}$. These equations have exactly the same form as those in (9) that describe the autarky equilibrium, except that x_i for the closed economy is replaced by $x_i/(1 + \lambda_i)$ for the open economy. In other words, trade effectively reduces sales per firm, as far as the solution for the effective number of firms in each country is concerned. The proof of Proposition 1 guarantees that (15) has a unique solution with a positive number of effective firms of each type in each country.

It is not enough, of course, that the number of effective firms in each country be positive for the solutions to (15) to represent a legitimate trade equilibrium. We require in addition that the actual number of varieties in each country be positive. Given the values of $\tilde{n}_L^{\mathcal{R}}$, $\tilde{n}_H^{\mathcal{R}}$, $\tilde{n}_L^{\mathcal{P}}$, and $\tilde{n}_H^{\mathcal{P}}$ that result from the solution of (15), we can solve for n_L^k and n_H^k using $\tilde{n}_i^k = n_i^k + \lambda_i n_i^\ell$. This gives

$$n_i^k = \frac{\tilde{n}_i^k - \lambda_i \tilde{n}_i^\ell}{1 - \lambda_i^2}, \quad \ell \neq k, k = \mathcal{R}, \mathcal{P}, i = H, L. \quad (16)$$

A positive solutions for n_i^k for all i and k requires

$$\frac{1}{\lambda_i} > \frac{\tilde{n}_i^k}{\tilde{n}_i^\ell} > \lambda_i, \quad \ell \neq k, k = \mathcal{R}, \mathcal{P}, i = H, L$$

which is always satisfied when λ_i is close to zero but rarely satisfied when λ_i is close to one.²⁰ This justifies our claim that a trade equilibrium with incomplete specialization always exists when transport costs are sufficiently high, but fails to exist (generically) when transport costs are low.

The trade equilibrium with incomplete specialization features intra-industry trade at each quality level. Some consumers in \mathcal{R} opt for preferred varieties of the high-quality good produced in \mathcal{P} , despite their higher price that includes a charge for shipping. Similarly, some consumers in \mathcal{R} choose to import a favorite foreign variety of the low-quality good. Consumers in \mathcal{P} will likewise import high- and low-quality goods produced in \mathcal{R} .

In fact, we know that the export sales by a typical producer of quality q_i are the same in both locations. Therefore, country \mathcal{R} exports more of goods of quality q_i to \mathcal{P} than it imports of that quality if and only if country \mathcal{R} has more firms producing goods of quality q_i than does country \mathcal{P} . But, from (16), $n_i^{\mathcal{R}} > n_i^{\mathcal{P}}$ if and only if $\tilde{n}_i^{\mathcal{R}} > \tilde{n}_i^{\mathcal{P}}$. Therefore, we can identify the equilibrium trade balance in each quality class by comparing the effective number of sellers of that quality in the two

²⁰When λ_i is close to one, the pair of inequalities can be satisfied only if $\tilde{n}_i^{\mathcal{R}} \approx \tilde{n}_i^{\mathcal{P}}$, which happens only under exceptional circumstances. For example, $N^{\mathcal{R}} = N^{\mathcal{P}}$ and $F_y^{\mathcal{R}} = F_y^{\mathcal{P}}$ implies $\tilde{n}_i^{\mathcal{R}} = \tilde{n}_i^{\mathcal{P}}$.

countries.²¹

The pair of equations that determine $\tilde{n}_i^{\mathcal{R}}$ and $\tilde{n}_i^{\mathcal{P}}$ are the same as those that determine the autarky numbers of producers of quality q_i in \mathcal{R} and \mathcal{P} , except that x_i in the latter is replaced by $x_i/(1 + \lambda_i)$ in the former. Therefore, we can use the comparative statics of the autarky equilibrium to identify the sectoral imbalances of the trade equilibrium with incomplete specialization. For example, suppose that the countries have the same distributions of income ($F_y^{\mathcal{R}}(y) = F_y^{\mathcal{P}}(y)$) but country \mathcal{R} is larger than country \mathcal{P} (i.e., $N^{\mathcal{R}} > N^{\mathcal{P}}$). We have seen that if $\theta_H = \theta_L$, the larger country has more autarky producers of both classes of goods—in proportion to its larger size. If instead high-quality goods are more differentiated than low-quality goods ($\theta_H > \theta_L$), then the larger country has in autarky an even greater relative abundance of producers of high-quality goods than its relative superiority in size, but it may support fewer (or more) producers of low-quality products. These comparisons carry over to the numbers of effective sellers in a trade equilibrium with incomplete specialization. That is, if $\theta_H = \theta_L$, then $\tilde{n}_i^{\mathcal{R}} > \tilde{n}_i^{\mathcal{P}}$ for $i = H$ and $i = L$; then the larger country is a net exporter of both high- and low-quality products. If $\theta_H > \theta_L$, $\tilde{n}_H^{\mathcal{R}} > \tilde{n}_H^{\mathcal{P}}$, but the comparison of effective numbers of producers of low-quality goods can run in either direction. In such circumstances, the larger country is a net exporter of high-quality goods but may be a net exporter or a net importer of low-quality products.

Now suppose that the two countries are identical in size ($N^{\mathcal{R}} = N^{\mathcal{P}}$) but the income distribution in the richer \mathcal{R} first-order stochastically dominates that in poorer \mathcal{P} . Then, in autarky, the rich country has more firms producing high-quality goods and the poor country has more firms producing low-quality goods. These comparisons carry over to the effective numbers of firms in the trade equilibrium with incomplete specialization, so that $\tilde{n}_H^{\mathcal{R}} > \tilde{n}_H^{\mathcal{P}}$ and $\tilde{n}_L^{\mathcal{P}} > \tilde{n}_L^{\mathcal{R}}$. It follows that the rich country \mathcal{R} is a net exporter of high-quality goods and a net-importer of low-quality goods.

Finally, suppose that a majority of consumers at every income level in both countries purchase low-quality goods. Let the countries be of similar size and with similar mean income, but suppose that the income distribution in \mathcal{R} is more spread than that in \mathcal{P} . As we have seen before, \mathcal{R} has more producers of high-quality goods and fewer producers of low-quality goods than does \mathcal{P} in autarky. With costly trade, \mathcal{R} becomes a net exporter of high-quality goods and a net importer of low-quality goods.

We summarize our findings about the pattern of trade in

Proposition 2 *If trade costs are sufficiently high, there exists a unique trade equilibrium in which each country produces both high- and low-quality differentiated products. In this equilibrium, (i) if $N^{\mathcal{R}} > N^{\mathcal{P}}$ and $F_y^{\mathcal{R}}(y) = F_y^{\mathcal{P}}(y)$ for all y , and if $\theta_H \geq \theta_L$, then \mathcal{R} exports on net the high-quality goods but may export or import on net the low-quality goods; (ii) if $N^{\mathcal{R}} = N^{\mathcal{P}}$ and $F_y^{\mathcal{R}}(y) < F_y^{\mathcal{P}}(y)$*

²¹Net exports from R to P of goods of quality q_i are given by

$$\begin{aligned} \lambda_i N^{\mathcal{P}} d_i^{\mathcal{P}} n_i^{\mathcal{R}} - \lambda_i N^{\mathcal{R}} d_i^{\mathcal{R}} n_i^{\mathcal{P}} &= \frac{\lambda_i}{1 + \lambda_i} x_i \left(n_i^{\mathcal{R}} - n_i^{\mathcal{P}} \right) \\ &= \frac{\lambda_i}{1 - \lambda_i^2} \frac{f_i q_i}{\theta_i} \left(n_i^{\mathcal{R}} - n_i^{\mathcal{P}} \right). \end{aligned}$$

for all y , then \mathcal{R} exports on net the high-quality goods and imports on net the low-quality goods; (iii) if $N^{\mathcal{R}} = N^{\mathcal{P}}$, $\rho_L(y) > \rho_H(y)$ for all income groups in \mathcal{R} and \mathcal{P} , and $F_y^{\mathcal{R}}(\cdot)$ is a mean-preserving spread of $F_y^{\mathcal{P}}(\cdot)$, then \mathcal{R} exports on net the high-quality goods and imports on net the low-quality goods.

Proposition 2 can be understood in terms of the “home-market effect” described by Krugman (1980). Take for example the case in which the countries are of similar size but the income distribution in \mathcal{R} first-order stochastically dominates that in \mathcal{P} . The greater income in \mathcal{R} compared to \mathcal{P} provides this country with a larger home market for high-quality goods. If the same numbers of producers of high-quality goods were to enter in both countries, those in \mathcal{R} would earn greater profits than those in \mathcal{P} , thanks to their ability to serve more consumers with sales that do not bear shipping costs. In order that producers of high-quality goods in both countries break even, there must be greater entry of such producers in the rich country, so that their finer division of the market offsets their local-market advantage. The same is true in the market for low-quality goods, where producers in \mathcal{P} enjoy an advantage due to their closer proximity to the larger market. Access to a large home market affords a competitive advantage that induces entry and ultimately dictates the pattern of trade.

We turn now to the effects of a reduction in trade costs, focusing particularly on the distributional consequences. For concreteness, consider first a decline in the cost of transporting high-quality goods.²² A fall in τ_H induces an increase in λ_H . To see the effects of this, we use Figure 2 to determine the effective numbers of sellers in country \mathcal{R} ; the numbers for country \mathcal{P} are determined similarly. In Figure 2, the $G^{\mathcal{R}}G^{\mathcal{R}}$ curve is analogous to the GG curve in Figure 1; it represents combinations of $\tilde{n}_H^{\mathcal{R}}$ and $\tilde{n}_L^{\mathcal{R}}$ that satisfy (11), when in the latter n_H and n_L are replaced by $\tilde{n}_H^{\mathcal{R}}$ and $\tilde{n}_L^{\mathcal{R}}$, respectively, and x_i is replaced by $x_i/(1 + \lambda_i)$. The line $N^{\mathcal{R}}N^{\mathcal{R}}$ depicts the equation²³

$$\frac{x_H}{1 + \lambda_H} \tilde{n}_H^{\mathcal{R}} + \frac{x_L}{1 + \lambda_L} \tilde{n}_L^{\mathcal{R}} = N^{\mathcal{R}},$$

which replaces the NN line of Figure 1.

Now let λ_H rise. This shifts the $N^{\mathcal{R}}N^{\mathcal{R}}$ line upward in proportion to the increase in $1 + \lambda_H$. Thus, point \tilde{E} represents an increase in the effective number of high-quality products relative to E in proportion to the increase $1 + \lambda_H$. But, as can be seen from (11), the $G^{\mathcal{R}}G^{\mathcal{R}}$ curve shifts upward more than proportionately to the percentage increase in $1 + \lambda_H$. As a result, the new equilibrium point E' has more effective brands of high-quality products and fewer effective brands of low-quality products than at E . Indeed, it is apparent that an increase in λ_H induces the same response as a reduction in the fixed cost of entry by producers of high-quality products, f_H . A similar analysis applied to a reduction in the cost of trading low-quality products would show a rise in the effective number of low-quality brands and a fall in the effective number of high-quality brands in country \mathcal{R} . Moreover, the analysis is qualitatively similar for country \mathcal{P} . Finally, it is evident that if $1 + \lambda_H$

²²The details of the algebra are provided in the appendix.

²³Note that $\kappa_H^{\mathcal{R}} N^{\mathcal{R}} d_H^{\mathcal{R}} + \kappa_L^{\mathcal{R}} N^{\mathcal{R}} d_L^{\mathcal{R}} = N^{\mathcal{R}}$, while $x_i = N^{\mathcal{R}} d_i^{\mathcal{R}} + \lambda_i N^{\mathcal{P}} d_i^{\mathcal{P}} = (1 + \lambda_i) N^{\mathcal{R}} d_i^{\mathcal{R}}$ for $i = H, L$. The equation for the $N^{\mathcal{R}}N^{\mathcal{R}}$ line follows from combining these two.

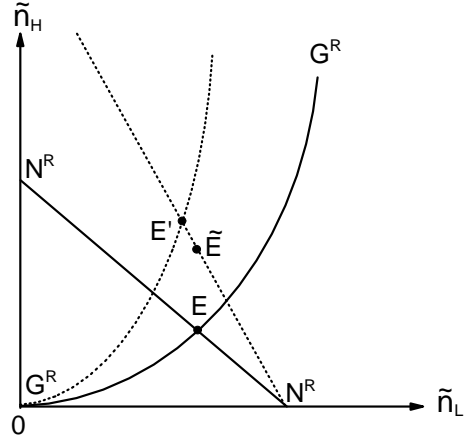


Figure 2: Trade equilibrium: Reduction in τ_H

and $1 + \lambda_L$ both rise equiproportionately, then the change in the effective number of high- and low-quality products in country k mimics that for a similar percentage increase in the population variable, N^k . From our analysis of the autarky equilibrium, we know that if $\theta_H = \theta_L$, population growth generates an equiproportionate increase in the effective numbers of low- and high-quality products in country k , whereas if $\theta_H > \theta_L$ the effective number of high-quality products expands more than in proportion to the increase in $1 + \lambda_H$ or $1 + \lambda_L$ while the effective number of low-quality products may rise or fall.

What are the welfare implications of these induced changes in the effective numbers of brands? In a world with costly trade, the average welfare of those with income y in country k increases with

$$v^k(y) = \left(\tilde{n}_H^k\right)^{\theta_H} \phi_H(y) + \left(\tilde{n}_L^k\right)^{\theta_L} \phi_L(y), \text{ for } k = \mathcal{R}, \mathcal{P}.$$

Welfare of individuals in country k depends on the effective numbers of brands available there, with foreign brands carrying less weight than domestic brands due to their higher prices. Differentiating the expression for $v^k(y)$ and rearranging terms, we can derive an expression for the change in average welfare of an income group analogous to (13), namely

$$\begin{aligned} \hat{v}^k(y) = & \left[\theta_L \frac{\rho_L^k(y)}{\rho_L^k} + \theta_H \frac{\rho_H^k(y)}{\rho_H^k} \right] \left[\rho_H^k \left(\widehat{1 + \lambda_H} \right) + \rho_L^k \left(\widehat{1 + \lambda_L} \right) \right] \\ & + \rho_H^k \rho_L^k \left[\theta_H \frac{\rho_H^k(y)}{\rho_H^k} - \theta_L \frac{\rho_L^k(y)}{\rho_L^k} \right] \left(\hat{n}_H^k - \hat{n}_L^k \right), \text{ for } k = \mathcal{R}, \mathcal{P}, \end{aligned} \quad (17)$$

where $\rho_i^k(y)$ is the fraction of consumers in country k with income y that buys a good with quality q_i , and ρ_i^k is the fraction of all consumers in country k that buys a good with quality q_i . The term in the first line of (17) is a pure *cost-savings effect*, analogous to the scale effect in (13). The term in the second line of (17) is a pure *composition effect*, analogous to the similarly-named term in (13).

The cost-savings effect benefits consumers at all levels of income; it reflects the fact that, for given relative numbers of effective brands of each quality, a fall in the cost of trade facilitates entry of new producers, which expands the range of available varieties and so the probability that a consumer will find one especially to his liking. The composition effect impacts different income classes differently. An expansion in the effective variety of high-quality goods relative to the effective variety of low-quality goods benefits those who are more likely to consume a high-quality product but harms those who are more likely to consume a low-quality product; and, of course, the likelihood of consuming a high-quality good rises with income.

Let us return to the effects of a reduction in trade costs. Consider first a decline in the cost of transporting high-quality goods. As we have seen, such a decline in τ_H expands the effective number of high-quality brands in each country, while contracting the effective number of low-quality brands. The cost-savings effect benefits all consumers. If $\theta_H \geq \theta_L$, the composition effect must benefit the average member of the highest income group in each economy, but it may harm the average member of the lowest income group. Therefore, $\theta_H \geq \theta_L$ suffices for a fall in τ_H to augment the average welfare of the most well-off consumers in each country. There may be gains from a decline in τ_H for all income groups in a given country, or else the gains may be limited to those income groups with y above some critical level.²⁴

Other reductions in trade costs can be analyzed similarly. For example, declines in τ_H and τ_L that increase $1 + \lambda_H$ and $1 + \lambda_L$ by the same proportions must benefit all income groups, if the two classes of goods are horizontally differentiated to the same extent ($\theta_H = \theta_L$); if, however, the high-quality goods are more dissimilar as a group than the low-quality goods, such an equiproportionate increase in $1 + \lambda_H$ and $1 + \lambda_L$ must benefit the highest income group in each country but may harm the poor.

Our analysis also sheds lights on the distribution of the gains from trade. The autarky equilibrium for either country is the solution to (15) with $\lambda_H = \lambda_L = 0$. The effects of trade can be found by integrating the increases in λ_H and λ_L from zero to their actual levels. This generates a cost-savings effect that benefits all consumers. It also generates a composition effect that may benefit some income groups at the expense of others. If the effective number of brands at both quality levels rises as a result of trade, then all consumers must gain. If the effective number of brands of some quality level declines, then income groups that buy this good with a probability that exceeds the economy-wide average may lose. Although trade may not benefit every income group, it always benefits some such groups.²⁵

We summarize our discussion of the distributional consequences of a reduction in trade costs in

Proposition 3 *In a trade equilibrium with incomplete specialization, a decline in the trade cost*

²⁴Even if $\theta_H < \theta_L$ there must be some income groups that gain from a reduction in τ_H . To see this, suppose the opposite were true. Then, the left-hand side of (15) increases for $i = L$ inasmuch as the numerator increases at every y (because π_L falls) and the denominator falls at every y (because average welfare has been assumed to fall). But the right-hand side of (15) is unchanged, which contradicts the requirement for equilibrium in the market for low-quality goods.

²⁵The proof of this statement follows along similar lines to that used in footnote 24.

τ_i raises the effective number of brands of quality q_i and reduces the effective number of brands of quality q_j , $j \neq i$, in both countries. Any reduction in trade costs must benefit the average member of some income group. If, as a result of a reduction in trade costs, the effective number of high-quality (low-quality) brands falls in some country, then the highest-income (lowest-income) groups in that country may lose.

We conclude this section with numerical examples that illustrate some of the points we have made. Figure 3 shows a case in which all income groups gain from a fall in the transport cost for high-quality goods. To generate this figure, we have assumed that income in each country has a displaced Gamma distribution, with $y_{\min}^R = y_{\min}^P = 1$, $y_{\text{median}}^R \approx 5.15$, and $y_{\text{median}}^P \approx 2.38$.²⁶ In the left-hand panel, the figure shows the effective numbers of high- and low-quality varieties in \mathcal{R} as λ_H is raised from zero to 0.8, while λ_L is held constant at 0.5.²⁷ The right-hand panel shows the average welfare among those with the lowest income in \mathcal{R} and among those with the median income there. In drawing the figure, we have normalized the (average) welfare level for each income group to equal 100 when $\lambda_H = 0$. The rich gain proportionately more than the poor from a fall in τ_H thanks to the composition effect that reflects the rise in the effective variety of high-quality goods and the fall in the effective variety of low-quality goods. Nonetheless, even the poorest consumers benefit from a fall in transport costs in this case. The figures for the poor country \mathcal{P} (not shown) are qualitatively similar.

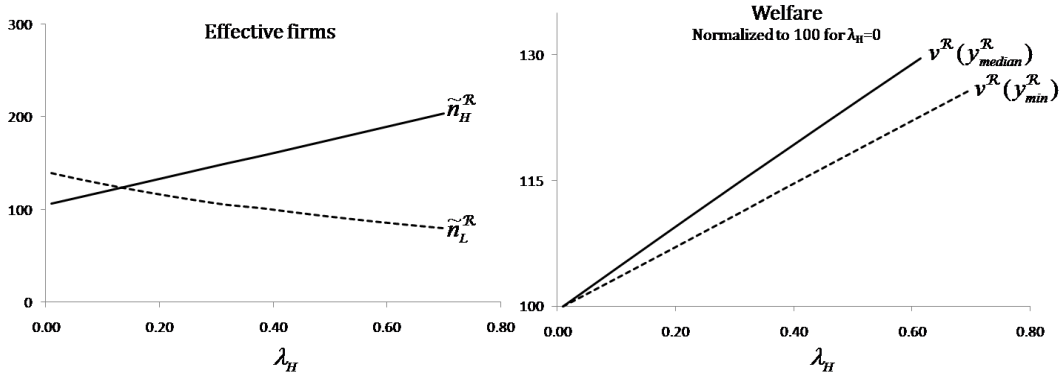


Figure 3: Increase in λ_H : All consumers gain

Figure 4 illustrates the possibility of distributional conflict. This figure uses the same distributions of income in the two countries as above, but it depicts a case in which the high-quality products are less differentiated (and the low-quality products more so) than before, so that the composition effect is more damaging to the poor.²⁸ Here, the median income group in country \mathcal{R}

²⁶The example takes $N^R = N^P = 1000$, $q_H = 1.05$, $q_L = 0.9$, $\theta_H = 0.7$, $\theta_L = 0.5$, $\lambda_L = 0.5$, $f_H = 5$, $f_L = 1.5$, $c_H = 0.3$, and $c_L = 0.05$. The distributions of income are such that $y - 1$ has a Gamma distribution in each country, with a shape parameter equal to 1 in each case. We take the scale parameter in \mathcal{R} to be 6 and that in \mathcal{P} to be 2, so that mean incomes are 7 and 3, respectively.

²⁷For $\lambda_H > 0.8$ in this example, \mathcal{P} specializes in the production of low-quality goods.

²⁸The parameters for Figure 4 are $N^R = N^P = 1000$, $q_H = 0.9$, $q_L = 0.75$, $\theta_H = \theta_L = 0.6$, $\lambda_L = 0.5$, $f_H = 20$,

gains from a reduction in the cost of trading high-quality goods, but the lowest income group in \mathcal{R} loses. Again, the figures for the poor country look qualitatively similar.

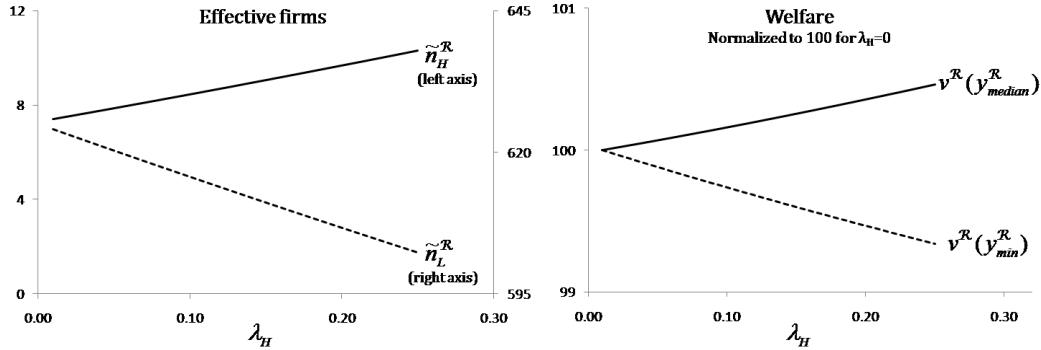


Figure 4: Increase in λ_H : Rich gain but poor lose

Finally, Figure 5 illustrates the patterns of specialization for different values of λ_H and λ_L . The figure is drawn for the same parameters as underlie Figure 3. In this case, the countries are similar in size but consumers in \mathcal{R} are richer than their counterparts in \mathcal{P} . When both trading costs are reasonably large, so that λ_H and λ_L are small, both countries are incompletely specialized, much as they are in autarky. A sufficient reduction in the cost of trading the high-quality goods, holding λ_L fixed at a reasonably low level, generates an equilibrium in which the poor country \mathcal{P} produces only low-quality goods, while the rich country \mathcal{R} produces both high- and low-quality goods. Similarly, a sufficient reduction in the cost of trading low-quality goods, holding λ_H at a reasonably low level, results in an equilibrium in which \mathcal{R} produces only high-quality goods while \mathcal{P} produces goods in both quality classes. If the cost of transporting both goods is sufficiently small, each class of goods is produced in a single location. We study this latter type of equilibrium in greater depth in the next section.

5 Trade with Specialization

In a trade equilibrium, high transport costs allow firms in each country to enter profitably in both quality segments of the market for differentiated products. Even if there are relatively many foreign producers of a given quality level, local firms can enter to sell to local customers thanks to the protection afforded by the high transport costs. As we have seen, when λ_H and λ_L are sufficiently close to zero, the trade equilibrium is characterized by incomplete specialization in both countries.

As transport costs fall, it becomes more difficult for firms in a smaller market to overcome the disadvantage of their lesser local demand. Eventually, as λ_i rises toward one, the number of producers of quality q_i in some country must fall to zero, as is implied by equation (16). For still

$f_L = 1.5$, $c_H = 0.3$, and $c_L = 0.05$.

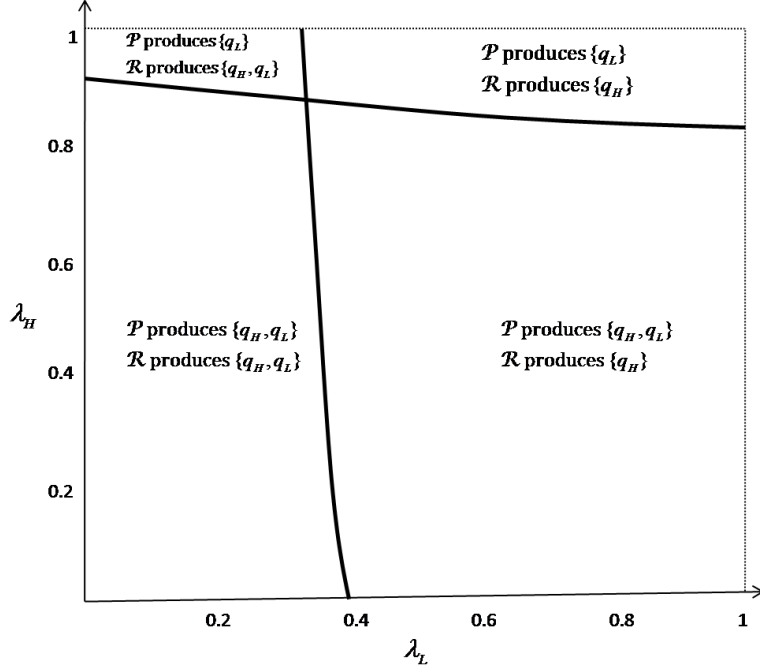


Figure 5: Patterns of specialization

smaller transport costs, all of the varieties with quality q_i are produced in a single country. In this section, we study trade equilibria with specialization of this sort. We are particularly interested in the limiting equilibrium, as transport costs approach zero. We will see that this equilibrium is unique and has a readily understood pattern of trade. Before beginning this analysis, however, it will prove useful to have a brief discussion of the integrated equilibrium, when transport costs for both quality levels are literally zero.

5.1 The Integrated Equilibrium

Suppose that $\tau_H = \tau_L = 0$, so that $\lambda_H = \lambda_L = 1$. With no supply-side sources of comparative advantage, there is nothing in our model to pin down the location of production. Factor-price equalization and zero transport costs means that the different goods can be produced in various combinations in the two countries, without consequence for any aggregate variables or anyone's welfare level. Although we cannot say anything about the pattern of trade, we can nonetheless characterize the integrated equilibrium in terms of the total numbers of brands of each quality that are produced and the average welfare of the different income groups.

In the absence of trade costs, the effective number of varieties with quality q_i is the same in both countries; i.e., $\tilde{n}_i^{\mathcal{R}} = \tilde{n}_i^{\mathcal{P}} = n_i^{\mathcal{R}} + n_i^{\mathcal{P}}$ for $i = H, L$. We can solve for these aggregate numbers of varieties using the autarky equilibrium conditions for an economy with population $N^{\mathcal{R}} + N^{\mathcal{P}}$ and an income distribution that is the composite of the separate distributions in the two countries. This gives \bar{n}_H and \bar{n}_L , the aggregate numbers of high- and low-quality products, respectively, that are produced in the integrated global economy. Armed with these variables, we can calculate per

capita demand in country k for a typical brand with quality q_i , which we denote by \bar{d}_i^k . That is,

$$\bar{d}_i^k = \frac{1}{\bar{n}_i} \mathbb{E}^k \left[\frac{(\bar{n}_i)^{\theta_i} \phi_i(y)}{(\bar{n}_H)^{\theta_H} \phi_H(y) + (\bar{n}_L)^{\theta_L} \phi_L(y)} \right]. \quad (18)$$

The impact of trade with zero transport costs on the welfare of an income group y in country k reflects a scale effect and a composition effect, as before. The scale effect—which arises because the integrated economy has a larger population than either separate economy—works to the benefit of all income groups in both countries. The composition effect benefits the high-income groups in country k if the relative number of high-quality brands in the integrated equilibrium exceeds the relative number of high-quality brands in the country's autarky equilibrium. Otherwise, the composition effect benefits the low-income groups in country k . The effect of an opening of trade on the relative numbers of brands of the different qualities levels reflects both the biased nature of growth when $\theta_H \neq \theta_L$, and the demand effects of a change in income distribution from one with the properties of the local economy to one with the properties of the global economy.

5.2 Trade Equilibrium with Small (but Positive) Trade Costs

Now we are ready to characterize the trade equilibrium when transport costs are positive but small. If a firm producing quality q_i in country k is to break even, it must attain total worldwide sales of $x_i = f_i q_i / \theta_i$. Each firm's sales comprise its home sales— $N^k d_i^k$ for a firm in country k —and its export sales, which are a fraction λ_i of the domestic sales of a foreign firm. For firms producing quality q_i to achieve the break-even volume of sales in both countries given the required relationship between the home sales of one and the export sales of the other requires that domestic sales be common to the two countries; i.e., $N^{\mathcal{R}} d_i^{\mathcal{R}} = N^{\mathcal{P}} d_i^{\mathcal{P}}$, as we have noted before.

But note that the per capita demand in country k for a typical variety with quality q_i approaches \bar{d}_i^k as transport costs go to zero. The per capita demands of the integrated equilibrium are given by (18) and are uniquely determined by parameters of the world economy. Only exceptionally will it happen that $N^{\mathcal{R}} \bar{d}_i^{\mathcal{R}} = N^{\mathcal{P}} \bar{d}_i^{\mathcal{P}}$ for $i = H$ or $i = L$. In other words, only exceptionally will it happen that firms in both countries producing a given quality can break even when transport costs are sufficiently small. Otherwise, goods of a particular quality are produced in a single country, while a potential entrant at that quality level in the other country finds insufficient demand (at its optimal price) to cover its fixed costs.

Which country produces each class of goods when trade costs are small? To answer this question, we look at national demands for products of a given quality in the integrated equilibrium. Suppose, for example that $N^{\mathcal{R}} \bar{d}_i^{\mathcal{R}} > N^{\mathcal{P}} \bar{d}_i^{\mathcal{P}}$ for products of quality q_i ; that is, the typical producer of a good with quality q_i makes greater sales in country \mathcal{R} than in country \mathcal{P} . With positive trade costs and optimal pricing, each firm's exports are a fraction of sales by a local producer in the destination market. It follows that when transport costs are sufficiently small, profits per firm for a producer of a brand with quality q_i in country \mathcal{R} must exceed those for a producer of that quality in country

\mathcal{P} .²⁹ More generally, all production of goods with quality q_i takes place in the country with the larger domestic market for goods of that quality in the integrated equilibrium. We summarize in

Proposition 4 *Suppose $N^k \bar{d}_i^k > N^\ell \bar{d}_i^\ell$ for $i \in \{H, K\}$; $k, \ell = \mathcal{R}, \mathcal{P}$; and $\ell \neq k$. Then, for τ_H and τ_L sufficiently close to zero, all goods of quality q_i are produced in country k .*

Let us apply Proposition 4 to some special cases that we have considered previously. Suppose, for example, that $F_y^{\mathcal{R}}(\cdot) = F_y^{\mathcal{P}}(\cdot)$ and $N^{\mathcal{R}} > N^{\mathcal{P}}$; i.e., the countries share the same income distribution but differ in size. Then, by (18), $\bar{d}_i^{\mathcal{R}} = \bar{d}_i^{\mathcal{P}}$ for $i = H, K$. Therefore, $N^{\mathcal{R}} \bar{d}_H^{\mathcal{R}} > N^{\mathcal{P}} \bar{d}_H^{\mathcal{P}}$ and $N^{\mathcal{R}} \bar{d}_L^{\mathcal{R}} > N^{\mathcal{P}} \bar{d}_L^{\mathcal{P}}$, so the larger country produces and exports all varieties of both the high-quality and low-quality differentiated products. Now suppose instead that $N^{\mathcal{R}} = N^{\mathcal{P}}$ while $F_y^{\mathcal{R}}(\cdot)$ first-order stochastically dominates $F_y^{\mathcal{P}}(\cdot)$. Then $\bar{d}_H^{\mathcal{R}} > \bar{d}_H^{\mathcal{P}}$ and $\bar{d}_L^{\mathcal{R}} < \bar{d}_L^{\mathcal{P}}$, so the richer country produces all of the high-quality goods while the poorer country produces all of the low-quality goods. We record these results in

Corollary 1 *Suppose that transport costs are small. (i) If $F_y^{\mathcal{R}}(y) = F_y^{\mathcal{P}}(y)$ for all y and $N^{\mathcal{R}} > N^{\mathcal{P}}$, then $n_H^{\mathcal{P}} = n_L^{\mathcal{P}} = 0$ and \mathcal{R} exports goods of quality q_H and q_L . (ii) If $N^{\mathcal{R}} = N^{\mathcal{P}}$ and $F_y^{\mathcal{R}}(y) < F_y^{\mathcal{P}}(y)$ for all y , then $n_H^{\mathcal{P}} = n_L^{\mathcal{R}} = 0$, \mathcal{R} exports goods of quality q_H , and \mathcal{P} exports goods of quality q_L .*

We can also readily examine the effects of a fall in trading costs in an equilibrium with specialization by quality level. Suppose, for example, that only \mathcal{R} produces high-quality goods while only \mathcal{P} produces low-quality goods, as when the countries are of similar size and the income distribution in \mathcal{R} first-order stochastically dominates that in \mathcal{P} . Since every consumer buys one unit of the differentiated product of some quality level or another,

$$n_L x_L + n_H x_H = N^{\mathcal{R}} + N^{\mathcal{P}}, \quad (19)$$

where n_i is the equilibrium number of varieties of quality q_i , all produced in \mathcal{R} for $i = H$ and all produced in \mathcal{P} for $i = L$, and x_i is the break-even quantity of sales per firm for producers of goods with quality q_i as before. This linear relationship implies that a change in trade costs that induces entry at one quality level in the country where that quality is produced also forces exit of producers of the other quality level, in the other country.

Next we can use (14) to calculate per capita domestic sales for each type of firm, recognizing that $\hat{n}_H^{\mathcal{R}} = n_H$, $\hat{n}_L^{\mathcal{R}} = \lambda_L n_L$, $\hat{n}_H^{\mathcal{P}} = \lambda_H n_H$, and $\hat{n}_L^{\mathcal{P}} = n_L$. Also, export sales per firm are

²⁹That is, for λ_i close to one,

$$\pi_i^{\mathcal{R}} \approx \frac{\theta_i}{q_i} \left(N^{\mathcal{R}} d_i^{\mathcal{R}} + \lambda_i N^{\mathcal{P}} d_i^{\mathcal{P}} \right) - f_i$$

and

$$\pi_i^{\mathcal{P}} \approx \frac{\theta_i}{q_i} \left(\lambda_i N^{\mathcal{R}} d_i^{\mathcal{R}} + N^{\mathcal{P}} d_i^{\mathcal{P}} \right) - f_i$$

so $N^{\mathcal{R}} \bar{d}_i^{\mathcal{R}} > N^{\mathcal{P}} \bar{d}_i^{\mathcal{P}}$ implies $\pi_i^{\mathcal{R}} > \pi_i^{\mathcal{P}}$, where π_i^k is the net profit of a typical producer of quality q_i in country k .

$N^{\mathcal{P}} (1 - n_L d_L^{\mathcal{P}}) / n_H$ for a typical producer of a high-quality good in \mathcal{R} and $N^{\mathcal{R}} (1 - n_H d_H^{\mathcal{R}}) / n_L$ for a typical producer of a low-quality good in \mathcal{P} . The fact that total sales by each type of firm must attain the break-even level gives us two more equations for n_H and n_L , one of which is redundant given (19).³⁰

Suppose now that the cost of transporting high-quality goods falls. This shifts demand toward the (imported) high-quality products in \mathcal{P} , without affecting demands in \mathcal{R} (at the initial numbers of brands). The shift in the composition of demand induces additional entry by producers of high-quality goods in \mathcal{R} , while some producers of low-quality goods in \mathcal{P} are forced to exit the market. Then, in both countries, the effective number of high-quality brands rises while the effective number of low-quality brands falls. From (17), we see that the wealthiest consumers in both countries must gain, while the poorest consumers in both countries may gain or lose. The consequences of a reduction in the cost of shipping low-quality goods are analogous—the variety of low-quality goods expands and that of high-quality goods contracts in each country, to the benefit of poor consumers and the possible detriment of those who are well-off.

6 Commercial Policy

We turn to the impact of commercial policy. With fixed markups over delivered costs, tariffs do not alter the terms of trade. All welfare effects of tariffs emanate from the induced entry and exit of firms producing at different quality levels and from the shifts in the distribution of income that result from the disposition of tariff revenue.

The novel effects of tariffs in our model are well illustrated in a simple setting in which transportation costs are positive but close to zero and each quality class is produced in only one country. We assume that \mathcal{R} produces all varieties of high-quality products while \mathcal{P} produces all varieties of low-quality products and examine a specific tariff of t per unit that is introduced in country \mathcal{R} .³¹ In this setting with non-homothetic demands, the manner of redistribution of government revenues influences the effects of tariffs on aggregate demand. For concreteness, we assume that tariffs are redistributed to consumers on an equal per-capita basis. Each consumer receives additional income of $r = t n_L d_L^{\mathcal{R}}$, where $d_L^{\mathcal{R}}$ is the per-capita demand for a typical (imported) low-quality product in country \mathcal{R} .

A tariff raises the relative price of low-quality goods in country \mathcal{R} . Considering both the price hike and the redistributed proceeds, (3) implies that the per capita demand for a typical low-quality product there becomes

$$d_L^{\mathcal{R}} = \frac{1}{n_L} \mathbb{E}^{\mathcal{R}} \left[\frac{(n_L)^{\theta_L} \phi_L(y) e^{(r-t)q_L}}{(n_L)^{\theta_L} \phi_L(y) e^{(r-t)q_L} + (n_H)^{\theta_H} \phi_H(y) e^{r q_H}} \right].$$

³⁰Details of these equations and their comparative statics can be found in the appendix.

³¹In this setting with specialization, a tariff on all imports in country \mathcal{R} is indistinguishable from a tariff on imports of low-quality goods.

The per capita demand for a typical high-quality product in \mathcal{R} is

$$d_H^{\mathcal{R}} = \frac{1}{n_H} \mathbb{E}^{\mathcal{R}} \left[\frac{(n_H)^{\theta_H} \phi_H(y) e^{rq_H}}{(n_L)^{\theta_L} \phi_L(y) e^{(r-t)q_L} + (n_H)^{\theta_H} \phi_H(y) e^{rq_H}} \right].$$

Note that $r = tn_L d_L^{\mathcal{R}} = t\rho_L^{\mathcal{R}} > 0$, while $r - t = -t(1 - \rho_L^{\mathcal{R}}) < 0$, so the tariff-cum-redistribution shifts demand in \mathcal{R} from low-quality goods to high-quality goods, for given numbers of each type of product. Consequently, the tariff induces entry of firms producing high-quality goods in \mathcal{R} and exit by firms producing low-quality goods in \mathcal{P} .

How does the tariff affect the welfare of different income groups in country \mathcal{R} ? The average welfare of individuals with income y in country \mathcal{R} can be written analogously to (12) as

$$v^{\mathcal{R}}(y) = n_H^{\theta_H} \phi_H(y) e^{rq_H} + n_L^{\theta_L} \phi_L(y) e^{(r-t)q_L}.$$

Differentiating the expression for $v^{\mathcal{R}}(y)$ at $t = 0$ and using (19) and the market-clearing condition, $n_i x_i = \rho_i^R N^R + \rho_i^P N^P$, we can express the impact of a small tariff on the average welfare of those with income y in country \mathcal{R} as

$$\hat{v}^{\mathcal{R}}(y)|_{t=0} = \bar{\rho}_H \bar{\rho}_L \left[\theta_H \frac{\rho_H^{\mathcal{R}}(y)}{\bar{\rho}_H} - \theta_L \frac{\rho_L^{\mathcal{R}}(y)}{\bar{\rho}_L} \right] (\hat{n}_H - \hat{n}_L) + \rho_H^{\mathcal{R}} \rho_L^{\mathcal{R}} \left[\frac{\rho_H^{\mathcal{R}}(y)}{\rho_H^{\mathcal{R}}} q_H - \frac{\rho_L^{\mathcal{R}}(y)}{\rho_L^{\mathcal{R}}} q_L \right] dt \quad (20)$$

where $\bar{\rho}_i = n_i x_i / (N^{\mathcal{R}} + N^{\mathcal{P}})$ is the fraction of consumers worldwide who buy a differentiated product with quality q_i . The right-hand side of (20) combines two terms, a *composition effect* and a *redistribution effect*. The composition effect should be familiar. It reflects the rise in the number of high-quality varieties and the fall in the number of low-quality varieties induced by the tariff. The richest income group in \mathcal{R} buys a greater fraction of high-quality goods and a smaller fraction of low-quality goods than the average consumer in the world economy; i.e., $\rho_H^{\mathcal{R}}(y_{\max}) > \bar{\rho}_H$ and $\rho_L^{\mathcal{R}}(y_{\max}) < \bar{\rho}_L$. Therefore, if $\theta_H \geq \theta_L$, the composition effect certainly benefits the richest income group in country \mathcal{R} . However, the poorest income group in that country may well lose from the change in the composition of differentiated products.

The redistribution effect reflects the transfers of income implied by the lump-sum redistribution of tariff revenues. The tariff transfers income from those who chose to purchase an imported, low-quality product to those who choose to purchase a domestic, high-quality product. The rich are more likely to buy a high-quality product than the poor, so they are most likely to benefit from these transfers. Indeed, for the richest income group, $\rho_H^{\mathcal{R}}(y_{\max}) > \rho_H^{\mathcal{R}}$ and $\rho_L^{\mathcal{R}}(y_{\max}) < \rho_L^{\mathcal{R}}$ and the members of this group must gain on average from the redistribution effect as well. But notice that the redistribution effect might also benefit the poor, since $q_H > q_L$. This is because the tariff transfers income from those in any income class who happen to prefer one of the low-quality varieties to those in that same class who happen to prefer one of the high-quality varieties. The latter group has a higher marginal utility of income due to the complementarity in preferences between quality

of the differentiated good and quantity of the numeraire good. If the quality difference between differentiated products is large, it may be that $[\rho_H^{\mathcal{R}}(y_{\min})/\rho_H^{\mathcal{R}}]q_H > [\rho_L^{\mathcal{R}}(y_{\min})/\rho_L^{\mathcal{R}}]q_L$, in which case the redistribution effects of a small tariff serves to benefit even the poorest income group in \mathcal{R} . Indeed, the combined composition and redistribution effect can be positive for those with income y_{\min} , in which case a tariff would raise the average welfare of every income group in \mathcal{R} , despite the absence of any terms-of-trade improvement.³²

7 Concluding Remarks

We have developed a tractable model of trade in vertically- and horizontally-differentiated products. The model features discrete quality choices by consumers who differ in income levels and nonhomothetic aggregate demands for goods of different qualities. The nonhomotheticity in demand reflects an assumed complementarity in individual preferences between the quality of the differentiated product and the quantity of a homogeneous good. Consumers have idiosyncratic components in their evaluations of the different varieties of a given quality of product. The distribution of taste parameters in the population generates a nested-logit system of product demands.

We have embedded such consumers in a simple, supply-side environment. Goods are produced from labor alone, with constant returns to scale in the homogeneous-good industry and fixed and constant-variable costs for the varieties of the differentiated products. The number of varieties at each quality level is determined by free entry in a monopolistically-competitive, general equilibrium. Transport costs impede trade between countries that differ in size and in their income distributions but are otherwise similar. In this setting, a large home market for goods of a given quality confers a competitive advantage to firms located there, which renders them as net exporters in the trade equilibrium.

We have used this framework to study the pattern of international trade. We find, for example, that the reflection of country size in the trade pattern depends on the extent of horizontal product differentiation at each quality level. If low- and high-quality goods are equally differentiated, then the larger country among two with similar distributions of income will be a net exporter of both types of differentiated products. But if varieties of high-quality goods are more dissimilar than varieties of low-quality goods, the small country may be a net exporter of the latter. When two countries have similar populations, the one that is richer (in the sense of first-order stochastic dominance) will be a net exporter of high-quality goods and a net-importer of low-quality goods. This prediction of the model is consistent with patterns observed in the data by Schott (2004), Hummels and Klenow (2005), and Hallak and Schott (2008).

Our framework lends itself to analysis of the incidence of changes in the trading environment. We can decompose the welfare impact on a particular income group of, for example, reductions in trading costs into a cost-savings effect and a composition effect. The former tends to benefit all consumers, whereas the latter—reflecting the induced change in the relative number of low-

³² A small tariff raises the average welfare of all income groups, for example, when the parameters take the values that were used to generate Figure 3.

and high-quality products—often benefits consumers at one end of the income distribution at the expense of those at the other.

We have also investigated the impacts of tariff policy. In our setting, tariffs have no effect on the terms of trade. They generate a composition effect while redistributing income across income groups. For example, a tariff in a country that imports low-quality goods transfers income from lower income groups—who disproportionately consume goods of lesser quality—to higher income groups, who do the opposite.

Our framework could readily be extended in several directions. First, it would not be difficult to incorporate several quality classes and additional countries. Then, the model could be used to address Hallak’s (2006) finding that when a country exports goods in a product category to several destinations, the higher-quality goods are directed disproportionately to the higher-income markets. We can show, for example, that a unique exporter of a high-quality good will sell more to a richer country than to a poorer country of similar size. Second, we could introduce direct foreign investment as an alternative means for firms to serve foreign markets. Then the model might shed light on the spread of Chinese and India multinational corporations to other developing countries (see Boston Consulting Group, 2006). Finally, the close affinity between our analytical framework and the empirical literature on discrete-choice demands makes the model ripe for empirical application. We hope to pursue these lines in future research.

References

- [1] Amiti, Mary (1998), “Inter-industry Trade in Manufactures: Does Country Size Matter?,” *Journal of International Economics* 44:2, 231-255.
- [2] Anderson, Simon P., De Palma, Andre and Thisse, Jacques-Francois (1992), *Discrete Choice Theory of Product Differentiation*, Cambridge MA: M.I.T. Press.
- [3] Baldwin, Richard E. and Harrigan, James (2007), “Zeros, Quality and Space: Trade Theory and Trade Evidence,” NBER Working Paper No. 13214.
- [4] Bergstrand, Jeffrey H. (1990), “The Heckscher-Ohlin-Samuelson Model, The Linder Hypothesis and the Determinants of Bilateral Intra-Industry Trade,” *Economic Journal* 100:403, 1216-1229.
- [5] Ben-Akiva, Moshe E. (1973), *Structure of Passenger Travel Demand Models*, Ph.D. Dissertation, Massachusetts Institute of Technology.
- [6] Bils, Mark and Klenow, Peter J. (2001), “Quantifying Quality Growth,” *American Economic Review* 91:4, 1006-1030.
- [7] Boston Consulting Group (2006), *The New Global Challengers: How 100 Top Companies from the Rapidly Developing Economies are Changing the World*, Boston: The Boston Consulting Group.
- [8] Broda, Christian and Romalis, John (2009), “The Welfare Implications of Rising Price Dispersion,” manuscript.
- [9] Choi, Yo Chul, Hummels, David and Xiang, Chong (2009), “Explaining Import Quality: The Role of the Income Distribution,” *Journal of International Economics* 77:2, 265-275.
- [10] Dalgin, Muhammed, Trindade, Vitor and Mitra, Devashish (2008), “Inequality, Nonhomothetic Preferences, and Trade: A Gravity Approach,” *Southern Economic Journal* 74:3, 747–774.
- [11] Davis, Donald R. (1998), “The Home Market, Trade, and Industrial Structure,” *American Economic Review*, 88:5, 1264-1276.
- [12] Davis, Donald R. and Weinstein, David E. (2003), “Market Access, Economic Geography and Comparative Advantage: An Empirical Test,” *Journal of International Economics* 59:1, 1-23.
- [13] Eaton, Jonathan and Kortum, Samuel (2002), “Technology, Geography and Trade,” *Econometrica* 70:5, 1741-1779.
- [14] Epifani, Paolo and Gancia, Gino (2006), “Increasing Returns, Imperfect Competition, and Factor Prices,” *Review of Economics and Statistics* 88:4, 583-598.

- [15] Fieler, Ana Cecilia (2008), “Non-Homotheticity and Bilateral Trade: Evidence and a Quantitative Explanation,” manuscript.
- [16] Flam Harry and Helpman, Elhanan (1987), “Vertical Product Differentiation and North-South Trade,” *American Economic Review* 77:5, 810-822.
- [17] Foellmi, Reto, Hepenstrick, Christian, and Zweimüller, Josef (2007), “Income Effects in the Theory of Monopolistic Competition and Trade,” manuscript.
- [18] Goldberg, Pınelopi K. (1995), “Product Differentiation and Oligopoly in International Markets: The Case of the U.S. Automobile Industry,” *Econometrica* 63:4, 891-951.
- [19] Hallak, Juan Carlos (2006), “Product Quality and the Direction of Trade,” *Journal of International Economics* 68:1, 238-265.
- [20] Hallak, Juan Carlos and Schott, Peter K. (2008), “Estimating Cross-Country Differences in Product Quality,” NBER Working Paper No. 13807.
- [21] Hallak, Juan Carlos and Sivadasan, Jagadeesh (2009), “Exporting Behavior under Quality Constraints,” manuscript.
- [22] Hanson, Gordon C. and Xiang, Chong (2004), “The Home-Market Effect and Bilateral Trade Patterns,” *American Economic Review* 94:4, 1108-1129.
- [23] Hummels, David and Klenow, Peter J. (2005), “The Variety and Quality of a Nation’s Exports,” *American Economic Review* 95:3, 704-723.
- [24] Hunter, Linda (1991), “The Contribution of Nonhomothetic Preferences to Trade,” *Journal of International Economics* 30:3-4, 345-358.
- [25] Hunter, Linda and Markusen, James R. (1988), “Per-capita Income as a Determinant of Trade,” in R. Feenstra (ed.), *Empirical Methods for International Trade*, Cambridge: MIT Press, 89-109.
- [26] Johnson, Robert C. (2008), “Trade and Prices with Heterogeneous Firms,” manuscript.
- [27] Khandelwal, Amit (2008), “The Long and Short (of) Quality Ladders,” manuscript.
- [28] Krugman, Paul R. (1980), “Scale Economies, Product Differentiation, and the Pattern of Trade,” *American Economic Review* 70:5, 950-959.
- [29] Kugler, Maurice and Verhoogen, Eric (2008), “The Quality-Complementarity Hypothesis: Theory and Evidence from Colombia,” NBER Working Paper No. 14418.
- [30] Linder, Staffan Burenstam (1961), *An Essay on Trade and Transformation*, Stockholm: Almqvist and Wiksell.

- [31] Markusen, James R. (1986), "Explaining the Volume of Trade: An Eclectic Approach," *American Economic Review* 76:5, 1002-1011.
- [32] Matsuyama, Kiminori (2000), "A Ricardian Model with a Continuum of Goods under Non-homothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade," *Journal of Political Economy* 108:6, 1093-1120.
- [33] McFadden, Daniel (1978), "Modelling the Choice of Residential Location," in A. Karlqvist, L. Lundqvist, F. Snickars and J. Weibull (eds.), *Spatial Interaction Theory and Planning Models*, Amsterdam: North Holland, 75-96.
- [34] Mitra, Devashish and Trindade, Vitor (2005), "Inequality and Trade," *Canadian Journal of Economics* 38:4, 1253-1271.
- [35] Murphy, Kevin M. and Shleifer, Andrei (1997), "Quality and Trade," *Journal of Development Economics* 53:1, 1-15.
- [36] Schott, Peter K. (2004), "Across-Product versus Within-Product Specialization in International Trade," *Quarterly Journal of Economics* 119:2, 647-678.
- [37] Stokey, Nancy L. (1991), "The Volume and Composition of Trade Between Rich and Poor Countries," *Review of Economic Studies* 58:1, 63-80.
- [38] Train, Kenneth E. (2003), *Discrete Choice Methods with Simulation*, New York: Cambridge University Press.
- [39] Verboven, Frank (1996a), "The Nested Logit Model and Representative Consumer Theory," *Economic Letters* 50:1, 57-63.
- [40] Verboven, Frank (1996b), "International Price Discrimination in the European Car Market," *RAND Journal of Economics* 27:2, 240-268.
- [41] Verhoogen, Eric (2008), "Trade, Quality Upgrading, and Wage Inequality in the Mexican Manufacturing Sector," *Quarterly Journal of Economics* 123:2, 489-530.

8 Appendix

8.1 Derivation of (10)

The pair of equations in (9) implies

$$\frac{1}{x_L} \mathbb{E} \left[\frac{n_L^{\theta_L-1} \phi_L(y)}{n_L^{\theta_L} \phi_L(y) + n_H^{\theta_H} \phi_H(y)} \right] = \frac{1}{x_H} \mathbb{E} \left[\frac{n_H^{\theta_H-1} \phi_H(y)}{n_L^{\theta_L} \phi_L(y) + n_H^{\theta_H} \phi_H(y)} \right]$$

or

$$\mathbb{E} \left[\frac{\frac{1}{x_L} n_L^{\theta_L-1} \phi_L(y)}{n_L^{\theta_L} \phi_L(y) + n_H^{\theta_H} \phi_H(y)} - \frac{\frac{1}{x_H} n_H^{\theta_H-1} \phi_H(y)}{n_L^{\theta_L} \phi_L(y) + n_H^{\theta_H} \phi_H(y)} \right] = 0.$$

The term in square brackets can be written as

$$\begin{aligned} & \left(\frac{n_L^{\theta_L} \phi_L(y) + n_H^{\theta_H} \phi_H(y)}{\frac{1}{x_L} n_L^{\theta_L-1} \phi_L(y) - \frac{1}{x_H} n_H^{\theta_H-1} \phi_H(y)} \right)^{-1} \\ &= \left(\frac{(n_L x_L)^{\theta_L} x_L^{-\theta_L} \phi_L(y) + (n_H x_H)^{\theta_H} x_H^{-\theta_H} \phi_H(y)}{(n_L x_L)^{\theta_L-1} x_L^{-\theta_L} \phi_L(y) - (n_H x_H)^{\theta_H-1} x_H^{-\theta_H} \phi_H(y)} \right)^{-1} \\ &= \left[(n_L x_L)^{\theta_L} + \frac{(n_H x_H + n_L x_L) x_H^{-\theta_H} \phi_H(y)}{(n_H x_H)^{1-\theta_H} x_L^{-\theta_L} \phi_L(y) - (n_L x_L)^{1-\theta_L} x_H^{-\theta_H} \phi_H(y)} \right]^{-1} (n_L x_L)^{\theta_L-1}. \end{aligned}$$

Since $n_L x_L$ must be finite, the expectation will be zero if and only if (10) is satisfied.

8.2 Comparative Statics of the Autarky Equilibrium

Define

$$\psi(y; n_H, n_L) \equiv \left[(n_L x_L)^{\theta_L} + \frac{N G_H(y)}{(n_H x_H)^{1-\theta_H} G_L(y) - (n_L x_L)^{1-\theta_L} G_H(y)} \right]^{-1}$$

so that (10) implies $\mathbb{E}[\psi(y; n_L, n_H)] = 0$. Since $G_H(y)/G_L(y)$ is increasing in y , $\partial\psi/\partial y > 0$. Note also that $\partial\psi/\partial n_H > 0$ and $\partial\psi/\partial n_L < 0$.

8.2.1 Population size

Divide the numerator and denominator of the expression in square brackets in the definition of ψ by N , to define $\tilde{\psi} \equiv \tilde{\psi}(y; \tilde{n}_H, \tilde{n}_L, N)$, where $\tilde{n}_i = n_i/N$. That is,

$$\tilde{\psi}(y; \tilde{n}_H, \tilde{n}_L, N) \equiv \left[(\tilde{n}_L x_L)^{\theta_L} N^{\theta_L} + \frac{G_H(y)}{(\tilde{n}_H x_H)^{1-\theta_H} N^{\theta_L-\theta_H} G_L(y) - (\tilde{n}_L x_L)^{1-\theta_L} G_H(y)} \right]^{-1}$$

If $\theta_H = \theta_L$, $\tilde{\psi}(\cdot)$ is homogeneous of degree zero in \tilde{n}_H , \tilde{n}_L , and N . With (8), this implies $\hat{n}_L = \hat{n}_H = \hat{N}$. If $\theta_H > \theta_L$, $\tilde{\psi}$ is increasing in both \tilde{n}_L and N . With (8), this implies $\hat{n}_L < \hat{N} < \hat{n}_H$.

8.2.2 First-order stochastic dominance

The fact that $\partial\psi/\partial y > 0$ implies, with (8), $\partial\psi/\partial n_H > 0$ and $\partial\psi/\partial n_L < 0$ that n_H rises and n_L falls as the income distribution shifts to the right.

8.2.3 Mean-preserving spread

A mean-preserving spread in $F_y(\cdot)$ will increase n_H and decrease n_L if $\psi(y; n_H, n_L)$ is concave in y . We calculate

$$\frac{\partial \psi}{\partial y} = \left[(n_L x_L)^{\theta_L} (n_H x_H)^{1-\theta_H} \frac{G_L(y)}{G_H(y)} - n_L x_L + N \right]^{-2} N (n_H x_H)^{1-\theta_H} \left(\frac{G_L(y)}{G_H(y)} \right)'$$

Since $[G_L(y)/G_H(y)]' = (q_L - q_H)[G_L(y)/G_H(y)]$ we have

$$\begin{aligned} \frac{\partial^2 \psi}{\partial y^2} &= -2 \left[(n_L x_L)^{\theta_L} (n_H x_H)^{1-\theta_H} \frac{G_L(y)}{G_H(y)} - n_L x_L + N \right]^{-3} (n_L x_L)^{\theta_L} (n_H x_H)^{2(1-\theta_H)} N (q_L - q_H)^2 \left(\frac{G_L(y)}{G_H(y)} \right)^2 \\ &\quad + \left[(n_L x_L)^{\theta_L} (n_H x_H)^{1-\theta_H} \frac{G_L(y)}{G_H(y)} - n_L x_L + N \right]^{-2} N (n_H x_H)^{1-\theta_H} (q_L - q_H)^2 \left(\frac{G_L(y)}{G_H(y)} \right) \end{aligned}$$

Therefore, $\partial^2 \psi / \partial y^2 < 0$ if and only if

$$n_H x_H = -n_L x_L + N < (n_L x_L)^{\theta_L} (n_H x_H)^{1-\theta_H} \frac{G_L(y)}{G_H(y)},$$

which implies that $\partial^2 \psi / \partial y^2 < 0$ if and only if

$$1 < \frac{(n_L x_L)^{\theta_L}}{(n_H x_H)^{\theta_H}} \frac{G_L(y)}{G_H(y)} = \frac{n_L^{\theta_L} \phi_L(y)}{n_H^{\theta_H} \phi_H(y)} \Leftrightarrow \rho_L(y) > \rho_H(y).$$

Therefore, if $\rho_L(y) > 1/2$ for all y , a mean-preserving spread will increase n_H and decrease n_L .

8.3 Comparative Statics of Trade Equilibrium with Incomplete Specialization

Let us define $\psi^k(y, \tilde{n}_H^k, \tilde{n}_L^k)$ analogously to $\psi(\cdot)$, namely

$$\psi^k(y, \tilde{n}_H^k, \tilde{n}_L^k) \equiv \left[\left(\tilde{n}_L^k \frac{x_L}{1+\lambda_L} \right)^{\theta_L} + \frac{N G_H(y)}{\left(\tilde{n}_H^k \frac{x_H}{1+\lambda_H} \right)^{1-\theta_H} G_L(y) - \left(\tilde{n}_L^k \frac{x_L}{1+\lambda_L} \right)^{1-\theta_L} G_H(y)} \right]^{-1}$$

where $G_i(y) \equiv [x_i / (1 + \lambda_i)]^{-\theta_i} \phi_i(y)$, so that $\mathbb{E}^k[\psi] = 0$ for $k = \mathcal{R}, \mathcal{P}$. Now we use

$$\frac{x_H}{1+\lambda_H} \tilde{n}_H^k + \frac{x_L}{1+\lambda_L} \tilde{n}_L^k = N$$

to solve for \tilde{n}_H^k and substitute into $G_i(y)$ and $\psi^k(\cdot)$ to derive

$$\begin{aligned} \psi^k(y) &= \left[\left(\tilde{n}_L^k \frac{x_L}{1+\lambda_L} \right)^{\theta_L} + \frac{N G_H(y)}{\left(N - \tilde{n}_L^k \frac{x_L}{1+\lambda_L} \right)^{1-\theta_H} \left(\frac{x_L}{1+\lambda_L} \right)^{-\theta_L} \phi_L(y) - \left(\tilde{n}_L^k \frac{x_L}{1+\lambda_L} \right)^{1-\theta_L} \left(\frac{x_H}{1+\lambda_H} \right)^{-\theta_H} \phi_H(y)} \right]^{-1} \\ &= \left[\left(\tilde{n}_L^k \right)^{\theta_L} + \frac{N G_H(y)}{\left(N - \tilde{n}_L^k \frac{x_L}{1+\lambda_L} \right)^{1-\theta_H} \phi_L(y) - \left(\tilde{n}_L^k \right)^{1-\theta_L} \frac{x_L}{1+\lambda_L} \left(\frac{x_H}{1+\lambda_H} \right)^{-\theta_H} \phi_H(y)} \right]^{-1} \left(\frac{x_L}{1+\lambda_L} \right)^{-\theta_L}. \end{aligned}$$

It follows that $d\psi^k/d\lambda_H < 0$ and therefore $d\tilde{n}_L^k/d\lambda_H < 0$.

Now consider an equiproportionate increase in $1 + \lambda_H$ and $1 + \lambda_L$. Define $\tilde{n}_i^k = n_i^k/N(1 + \lambda_i)$, so that

$$\psi^k(y) = N^{-\theta_L} \left[(\tilde{n}_L^k x_L)^{\theta_L} + \frac{x_H^{-\theta_H} \phi_H(y)}{\frac{(1+\lambda_L)^{\theta_L}}{(1+\lambda_H)^{\theta_H}} N^{\theta_L - \theta_H} (\tilde{n}_H^k x_H)^{1-\theta_H} x_L^{-\theta_L} \phi_L(y) - (\tilde{n}_L^k x_L)^{1-\theta_L} x_H^{-\theta_H} \phi_H(y)} \right]^{-1}.$$

It is clear from this equation that an equiproportionate increase in $1 + \lambda_H$ and $1 + \lambda_L$ has the same effect on \tilde{n}_i^k as an increase in N .

8.4 Equilibrium Numbers of Brands with Small Trade Costs

The aggregate demand condition is

$$n_L x_L + n_H x_H = N^R + N^P = N^W.$$

The zero-profit conditions imply

$$\begin{aligned} N^P \mathbb{E}^P \left[\frac{n_L^{\theta_L - 1} \phi_L(y)}{(\lambda_H n_H)^{\theta_H} \phi_H(y) + n_L^{\theta_L} \phi_L(y)} \right] + \lambda_L N^R \mathbb{E}^R \left[\frac{(\lambda_L n_L)^{\theta_L - 1} \phi_L(y)}{n_H^{\theta_H} \phi_H(y) + (\lambda_L n_L)^{\theta_L} \phi_L(y)} \right] &= x_L \\ \lambda_H N^P \mathbb{E}^P \left[\frac{(\lambda_H n_H)^{\theta_H - 1} \phi_H(y)}{(\lambda_H n_H)^{\theta_H} \phi_H(y) + n_L^{\theta_L} \phi_L(y)} \right] + N^R \mathbb{E}^R \left[\frac{n_H^{\theta_H - 1} \phi_H(y)}{n_H^{\theta_H} \phi_H(y) + (\lambda_L n_L)^{\theta_L} \phi_L(y)} \right] &= x_H. \end{aligned}$$

We multiply the first of these equations by x_H and the second by x_L , and subtract, to derive

$$N^P \mathbb{E}^P \left[\frac{\frac{1}{x_L} n_L^{\theta_L - 1} \phi_L(y) - \frac{1}{x_H} n_H^{\theta_H - 1} \lambda_H^{\theta_H} \phi_H(y)}{n_H^{\theta_H} \lambda_H^{\theta_H} \phi_H(y) + n_L^{\theta_L} \phi_L(y)} \right] + N^R \mathbb{E}^R \left[\frac{\frac{1}{x_L} n_L^{\theta_L - 1} \lambda_L^{\theta_L} \phi_L(y) - \frac{1}{x_H} n_H^{\theta_H - 1} \phi_H(y)}{n_H^{\theta_H} \phi_H(y) + n_L^{\theta_L} \lambda_L^{\theta_L} \phi_L(y)} \right] = 0.$$

Now we can follow similar steps to those used in the derivation of (10). Note that the first term in square brackets is the same as the term in square brackets in that derivation, except that we have $\lambda_H^{\theta_H} \phi_H(y)$ in place of $\phi_H(y)$. Therefore, we can replace N^W using the aggregate demand condition and rewrite the first term in square brackets as

$$\left[(n_L x_L)^{\theta_L} + \frac{N^W x_H^{-\theta_H} \lambda_H^{\theta_H} \phi_H(y)}{(n_H x_H)^{1-\theta_H} x_L^{-\theta_L} \phi_L(y) - (n_L x_L)^{1-\theta_L} x_H^{-\theta_H} \lambda_H^{\theta_H} \phi_H(y)} \right]^{-1} (n_L x_L)^{\theta_L - 1}.$$

Similarly, the second term in the square brackets can be written as

$$\left[(n_L x_L)^{\theta_L} + \frac{N^W x_H^{-\theta_H} \phi_H(y)}{(n_H x_H)^{1-\theta_H} x_L^{-\theta_L} \lambda_L^{\theta_L} \phi_L(y) - (n_L x_L)^{1-\theta_L} x_H^{-\theta_H} \phi_H(y)} \right]^{-1} (n_L x_L)^{\theta_L - 1}.$$

Therefore, the composition-of-demand condition can be written as

$$N^P \mathbb{E}^P [\psi^P] + N^R \mathbb{E}^R [\psi^R] = 0$$

where

$$\begin{aligned}\psi^{\mathcal{P}} &= \left[n_L + \frac{N^W \phi_H(y) \lambda_H^{\theta_H}}{x_H n_H^{1-\theta_H} \phi_L(y) - n_L^{1-\theta_L} x_L \phi_H(y) \lambda_H^{\theta_H}} \right]^{-1} \\ \psi^{\mathcal{R}} &= \left[n_L + \frac{N^W \phi_H(y)}{x_H n_H^{1-\theta_H} \phi_L(y) \lambda_L^{\theta_L} - n_L^{1-\theta_L} x_L \phi_H(y)} \right]^{-1}\end{aligned}$$

Note that $d\psi^k/dn_L < 0$, which implies that the equilibrium is unique. Moreover, $d\psi^{\mathcal{P}}/d\lambda_H < 0$ and $d\psi^{\mathcal{R}}/d\lambda_H = 0$. Therefore, $dn^L/d\lambda_H < 0$.