CATs and DOGs

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Abstract

The role of intermediaries in international trade is a topic of renewed interest. Recent …rm level evidence has shown that it is not only specialized wholesalers and retailers that engage in the trading of other …rms' goods, but manufacturing …rms are also exporting products that they do not produce themselves. Bernard et. al. (2012) call this "Carry-Along Trade" (CAT) and show that it is a widespread phenomenon among Belgian manufacturing exports. In this paper, we study why manufacturing …rms may decide to have their products carried-along instead of exporting their products themselves. We show that if the "Delivery of Own Goods" (DOG) is an alternative option, the pro…tability of CAT is related to the pro…tability of collusion and the degree of transportation costs synergies. Consequently, CAT dominates DOG if the synergies from transporting and selling products jointly exceed the disadvantages of de-specialization. By focussing on demand linkages and transportation cost synergies, we are able to derive a number of testable predictions regarding …rm/market characteristics that tend to reinforce the appearance of CAT.

Keywords: Carry Along Trade, Multi-Product Firms.

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1 Introduction

The traditional view of how goods get exported has been that …rms produce and export their own products. The availability of better models and better data regarding exporting …rm behavior has led to a great deal of new work studying precisely how …rms export. Much of this work has focused on the use of intermediaries to assist …rms in their export activities. In recent work by Bernard, Blanchard,Van Beveren and Vandenbussche (2012) they identify and document using Belgian data, a di¤erent form of intermediation called carry-along trade (CAT). CAT occurs when Belgian …rms export products produced by other Belgian …rms in addition to their own products. The di¤erence between CAT and typical intermediation in export markets is that CAT …rms produce and export their own goods in addition to exporting goods produced by other …rms. Bernard, Blanchard,Van Beveren and Vandenbussche (2012) show that CAT is quantitatively important,

"We …nd that Carry-Along Trade, i.e. exports of goods where the …rm exports more than it produces, is widespread and important, occurring at more than 90 percent of exporters, appearing in more than 95 percent of exported products and accounting for more than 30 percent of export value."

We develop a model to explain why CAT occurs. In our model, …rms di¤er in their inherent ability to produce and transport. In addition, they are able to invest in transportation technology. Firms have to choose whether to engage in CAT or DOG (deliver own goods.) We show that the CAT versus DOG decision comes down to three factors, demand linkages, transportation cost synergies, and relative price-cost margins of the CAT good and the produced good. We show that if demand linkages are strong enough, transportation synergies large enough, and the relative price cost margin of the CAT good is low enough, then CAT is pro…table. Our paper is related to a growing literature on intermediaries.

Rauch and Watson (2004) use a network approach and focus on the problem of matching buyers and sellers in the export market. They develop a model to determine which firms decide to become intermediaries and whether the amount of intermediation in the market is sufficient. Blum, Claro and Horstmann. (2010) document facts regarding intermediation. They find that small exporters tend to match with large importers. Import intermediaries tend to become large by matching with large sellers and buying few products. Large intermediaries are the source of imports for both small countries and low value products. Ahn, Khandelwal and Wei (2011) argue that intermediaries are an important aspect of exporting. They develop and test a model in which firms chose their mode of export. They can export directly or through an intermediary. They show that less productive firms tend to use intermediaries more and that markets that have higher export costs have more exports handled by intermediaries.

Intermediation in international trade is the focus of an interesting paper by Antràs and Costinot (2011). They consider two ways that intermediation can become "globalized." They first look at what they call W-integration in which there is one centralized market where all of the world's trades take place. In the second type of integration, M-integration, intermediaries can locate in foreign markets if they chose, but there is no direct integration of markets. This is analogous to the difference between integrated and segmented markets. They show that the results of these different types of intermediation vary and in fact, M-intermediation can lower welfare in the smaller country. Akerman (2012) builds a model of intermediation based on firm heterogeneity. In his model the most productive firms export without using an intermediary and the least productive firms do not export. There exists a range of firms with intermediate productivity that use intermediaries to export. He derives a number of predictions that are then tested using Swedish export data.

This paper is also related to recent work on multi-product firms. Bernard,

Jensen, Redding, and Schott (2007) show that "...engaging in international trade is an exceedingly rare activity: of the 5.5 million …rms operating in the United States in 2000, just 4 percent were exporters. Among these exporting …rms, the top 10 percent accounted for 96 percent of total U.S. exports....In 2000, 42.2 percent of exporting …rms exported a single product abroad ...these exporters represented a small share of aggregate exports, just 0.4 percent...Firms exporting …ve or more products accounted for 25.9 percent of …rms but 98 percent of export value." This illustrates the importance of multi-product …rms in exporting activities. Bernard, Blanchard,Van Beveren and Vandenbussche (2012) illustrate in Figure 1 of their paper that "Except for the category of single-product exporters, …rms in every other category report greater numbers of products exported than products produced. Multi-product exporters are also multi-product domestic producers but the number of exported products increases much more rapidly than the number of produced products."

Our model of CAT combines elements of intermediation and multi-product …rm behavior to produce a unique exporting environment. We begin in section 2 by developing the model, section 3 solves for equilibrium, we characterize when CAT occurs in section 4 and section 5 concludes.

2 The Model

There are two domestic …rms (…rm 1 and …rm 2) who export to the foreign country. We assume they face identical, linear demands, hence the inverse demand functions are given by

$$p_1 = a - bq_1 - b\theta q_2 \tag{1}$$

$$p_2 = a - bq_2 - b\theta q_1 \tag{2}$$

where $\theta \in [-1, 1]$ represents the degree of product differentiation between the two products that the firms produce. If the two goods are perfect substitutes (i.e. homogeneous goods) then $\theta = 1$. If the goods are perfectly differentiated then $\theta = 0$ indicating that the demand for the two products is completely independent. Finally if $\theta < 0$ this indicates that the products are complements.

Marginal costs c_i ($i = 1, 2$) are constant and product specific. To simplify the exposition, we assume that firm 1 always transports it's own good, but also can possibly transport firm 2's goods. We will call Firm 1 the CAT firm. Firm 2 decides whether to transport it's own goods or have Firm 1 "carry along" its exports. Both firms can invest in transportation technology which will result in lower transportation costs. Unit transport costs for Firm 1 are given by

$$
\begin{aligned}
t_{11} &= t_1 - 2\tau (k_1)^{\frac{1}{2}} && (3)\\
t_{12} &= \xi \left(t_2 - 2\tau (k_1)^{\frac{1}{2}}\right) + (1-\xi)\, t_2 = t_2 - 2\tau\xi (k_1)^{\frac{1}{2}} && (4)
\end{aligned}
$$

where t_{11} is Firm 1's cost of exporting its' own goods and t_{12} is Firm 1's cost of exporting Firm 2's goods. k_1 is Firm 1's investment in transportation technology and the parameter τ indicates how effective this investment is in reducing transportation cost. Investment in transportation technology by the Firm 1 is tailored to its own product, but there may be spillovers to Firm 2's product. These spillovers are captured by parameter ξ, $\xi \in [0, 1]$. If ξ is zero then the products of Firm 1 and 2 are sufficiently different that investments in transportation technology by Firm 1 only reduce the cost of transporting its' own product and a value of ξ equal to one indicates that from the point of view of transportation, the two products are identical.

Unit transportation costs for firm 2, t_2 are

$$t_2 = t_2 - 2\tau (k_2)^{\frac{1}{2}} \qquad (5)$$

where k_2 is Firm 2's investment in transportation technology. Notice that transportation costs depend on two factors: (i) an exogenous factor that is product speci…c (t_i), and (ii) investments in cost reduction by the two …rms. Note that if $\xi = 0$, then $t_{12} = t_2 \geq t_2 = t_2 - 2\tau (k_2)^{\frac{1}{2}}$. Hence, if $\xi = 0$, the CAT …rm has no transportation cost advantage relative to …rm 2. Next, we solve for the equilibrium in the DOG case.

3 Equilibrium

3.1 Solving for DOG Equilibrium

Solving for the DOG equilibrium is relatively straightforward. Essentially it is a standard duopoly Nash equilibrium with endogenous investment. From …rm 1^0s point of view with DOG it means that (3) and (4) are replaced by

$$t_1 = t_1 - 2\tau (k_1)^{\frac{1}{2}} \tag{6}$$

Pro…t for the two …rms is

$$\begin{aligned} \pi_1 &= (p_1 - c_1 - t_1)\, q_1 - k_1 \\ \pi_2 &= (p_2 - c_2 - t_2)\, q_2 - k_2 \end{aligned}$$

Taking …rst order condition with respect to output and simplifying we get the following equation for Firm 1

$$2bq_1 = a - c_1 - t_1 - b\theta q_2 \tag{7}$$

The …rst order condition with respect to transportation investment yields

$$k_1^t = \tau^2 (q_1)^2$$

which means that transportation costs are

$$t_1 = t_1 - 2\tau^2 (q_1) \tag{8}$$

Solving (7) and (8) we get the aggregated best response function for Firm 1

$$2\left(b-\tau^2\right) q_1 = a - c_1 - t_1 - b\theta q_2 \tag{9}$$

By symmetry Firm 2's best response function is

$$2\left(b-\tau^2\right) q_2 = a - c_2 - t_2 - b\theta q_1 \tag{10}$$

Next, we solve for the two outputs. To guarantee that a solution exists we need to assume that

$$b > 2\tau^2 \tag{11}$$

Solving the two best response functions (9) and (10) we get

$$q_1 = \frac{2\left(b-\tau^2\right)(a-c_1-t_1) - b\theta (a-c_2-t_2)}{4\left(b-\tau^2\right)^2 - b^2\theta^2} \tag{12}$$

$$q_2 = \frac{2\left(b-\tau^2\right)(a-c_2-t_2) - b\theta (a-c_1-t_1)}{4\left(b-\tau^2\right)^2 - b^2\theta^2} \tag{13}$$

Substituting the above into the pro…t functions we get

$$\pi_1 = \left(b-\tau^2\right)\left(\frac{2\left(b-\tau^2\right)(a-c_1-t_1) - b\theta (a-c_2-t_2)}{4\left(b-\tau^2\right)^2 - b^2\theta^2}\right)^2 \tag{14}$$

$$\pi_2 = \left(b-\tau^2\right)\left(\frac{2\left(b-\tau^2\right)(a-c_2-t_2) - b\theta (a-c_1-t_1)}{4\left(b-\tau^2\right)^2 - b^2\theta^2}\right)^2 \tag{15}$$

Now we solve for the CAT equilibrium.

3.2 Solving for CAT Equilibrium

In the CAT equilibrium we assume that …rm 1 acts like a multi-product monopolist. In this case, we assume that …rm 1 can purchase good 2 from …rm 2

at firm 2's cost which is c_2. We first compute firm 1's profits assuming firm 2 agrees to use carry along trade (CAT).

$$\pi_1 = (p_1 - c_1 - t_{11})\, q_1 + (p_2 - c_2 - t_{12})\, q_2 - k_1 \tag{16}$$

Maximizing profits under CAT gives rise to the following first order conditions for output

$$2b\,(q_1 + \theta q_2) = a - c_1 - t_{11} \tag{17}$$

$$2b\,(\theta q_1 + q_2) = a - c_2 - t_{12} \tag{18}$$

We next compute the optimal investment in technology to reduce transportation costs

$$\frac{d\pi_1}{dk_1} = \tau\,(k_1)^{\frac{1}{2}}\, q_1 + \tau\xi\,(k_1)^{\frac{1}{2}}\, q_2 - 1 = 0 \tag{19}$$

which implies,

$$(k_1)^{\frac{1}{2}} = \tau\,(q_1 + \xi q_2) \tag{20}$$

and this can be simplified to

$$1 = \frac{1}{(q_1 + \xi q_2)} q_1 + \xi \frac{1}{(q_1 + \xi q_2)} q_2$$

Next substitute the expression in (3), (4) and (20) into (17) and (18) to obtain

$$\begin{aligned} 2bq_1 + 2b\theta q_2 &= a - c_1 - t_1 + 2\tau^2\,(q_1 + \xi q_2) \\ 2b\theta q_1 + 2bq_2 &= a - c_2 - t_2 + 2\xi\tau^2\,(q_1 + \xi q_2) \end{aligned}$$

which simplifies to

$$2\left(b - \tau^2\right) q_1 + 2\left(b\theta - \tau^2\xi\right) q_2 = a - c_1 - t_1 \tag{21}$$

$$2\left(b\theta - \xi\tau^2\right) q_1 + 2\left(b - \xi^2\tau^2\right) q_2 = a - c_2 - t_2 \tag{22}$$

Stability requires that

$$\left(b-\tau^2\right)\left(b-\xi^2\tau^2\right) > \left(b\theta-\xi\tau^2\right)^2 \tag{23}$$

Solving (21) and (22) we have

$$q_1 = \frac{\left(b-\xi^2\tau^2\right)(a-c_1-t_1)-\left(b\theta-\xi\tau^2\right)(a-c_2-t_2)}{2\left(\left(b-\tau^2\right)\left(b-\xi^2\tau^2\right)-\left(b\theta-\tau^2\xi\right)^2\right)} \tag{24}$$

$$q_2 = \frac{\left(b-\tau^2\right)(a-c_2-t_2)-\left(b\theta-\xi\tau^2\right)(a-c_1-t_1)}{2\left(\left(b-\tau^2\right)\left(b-\xi^2\tau^2\right)-\left(b\theta-\tau^2\xi\right)^2\right)} \tag{25}$$

Note that for their to be positive carry along trade i.e. $q_2 > 0$ it requires that

$$\frac{(a-c_2-t_2)}{(a-c_1-t_1)} > \frac{b}{b-\tau^2}\theta - \frac{\tau^2}{b-\tau^2}\xi \tag{26}$$

How do we interpret the expression $\frac{(a \quad c_2 \quad \bar{t}_2)}{(a \quad c_1 \quad t_1)}$? Note that the price-cost margin can be expressed as $p_1 - c_1 - t_{11} = b(q_1 + \theta q_2)$. However, this does not include the costs of investment in transportation cost reduction. The problem that we have is that investment in transportation is used for transporting two di¤erent goods so we need to assign the costs of investment k_1 to the two products. Formally, suppose we let $k_1 = \kappa_1 k_1 + \kappa_2 k_1$. Then we can allocate the costs of investment to the two products based on marginal pro…tability. If we let

$$\begin{aligned} \kappa_1 &\equiv \tau(k_1)^{\frac{1}{2}} q_1 \\ \kappa_2 &\equiv \tau\xi(k_1)^{\frac{1}{2}} q_2 \end{aligned}$$

then we can use equation (19) to assign cost shares of investment to the two products.

$$\frac{d\pi_1}{dk_1} = \underbrace{\tau(k_1)^{\frac{1}{2}} q_1}_{\text{Share of costs assigned to product 1: } \kappa_1} + \underbrace{\tau\xi(k_1)^{\frac{1}{2}} q_2}_{\text{Share of costs assigned to product 2: } \kappa_2} - 1$$

Hence, investment costs assigned to product 1 are

$$\kappa_1 k_1 = \tau \left(k_1\right)^{\frac{1}{2}} q_1 = \tau^2 \left(q_1 + \xi q_2\right) q_1$$

and for product 2 we have

$$\kappa_2 k_1 = \tau \xi \left(k_1\right)^{\frac{1}{2}} q_2 = \tau^2 \left(q_1 + \xi q_2\right) \xi q_2$$

We can write full cost price margins as $p_1 - c_1 - t_{11} - \frac{\kappa_1 k_1}{q_1} = b\left(q_1 + \theta q_2\right) - \tau^2 \left(q_1 + \xi q_2\right)$ and $p_2 - c_2 - t_{12} - \frac{\kappa_2 k_1}{q_2} = b\left(\theta q_1 + q_2\right) - \tau^2 \left(q_1 + \xi q_2\right) \xi$. Using (21) and (22) we get

$$p_1 - c_1 - t_{11} - \frac{\kappa_1 k_1}{q_1} = \frac{1}{2}\left(a - c_1 - t_1\right) \tag{27}$$

$$p_2 - c_2 - t_{12} - \frac{\kappa_2 k_1}{q_2} = \frac{1}{2}\left(a - c_2 - t_2\right) \tag{28}$$

Equations (27) and (28) mean that we can think of $\frac{(a \quad c_2 \quad \bar{t}_2)}{(a \quad c_1 \quad t_1)}$ as relative full price-cost margins. Therefore, we can interpret the condition (26) for positive CAT output as

$$\underbrace{\frac{\left(a - c_2 - t_2\right)}{\left(a - c_1 - t_1\right)}}_{\text{Relative price-cost margin of CAT product}} > \underbrace{\frac{b}{b - \tau^2}\theta}_{\text{Cannibalization e¤ect}} - \underbrace{\frac{\tau^2}{b - \tau^2}\xi}_{\text{Transportation costs synergies}} \tag{29}$$

specifying that the the relative price-cost margin for good 2 (the CAT product) has to be larger than the cannibalization e¤ect net of transportation synergies.

The cannibalization e¤ect depends on θ.For example, if, $\theta = 0$ the products are unrelated and the cannibalization e¤ect is zero. A larger θ means that the two products are closer substitutes and hence, to maximize the pro…t of selling

both of them quantity will have to be lowered more (cannibalized). If the products are complements then the cannibalization e¤ect is negative. Transportation cost synergies depend on ξ, the is larger ξ the more transportation cost synergies there are and this counteracts the cannibalization e¤ect. To summarize, CAT is pro…table if the CAT product has a relatively high price-cost margin, or if the cannibalization e¤ect is small, or if transportation costs synergies are large.

It therefore, follows from (29) that if $\theta \leq 0$, CAT is always pro…table. Also, if $\theta = \xi$, then CAT is pro…table if $\frac{(a \quad c_2 \quad \bar{t}_2)}{(a \quad c_1 \quad \bar{t}_1)} > \theta$. Finally, if $\frac{(a \quad c_2 \quad \bar{t}_2)}{(a \quad c_1 \quad \bar{t}_1)} = 1$, then CAT is pro…table if $\frac{b}{\tau^2} > \frac{(1 \quad \xi)}{(1 \quad \theta)}$. Next, we need to compute CAT pro…ts.

Using our earlier results on full price-cost margins

$$\begin{aligned}
\pi_1 &= \left(p_1 - c_1 - t_{11} - \frac{\kappa_1 k_1}{q_1}\right) q_1 + \left(p_2 - c_2 - t_{12} - \frac{\kappa_2 k_1}{q_2}\right) q_2 \\
&= \frac{1}{2}\left(a - c_1 - t_1\right) q_1 + \frac{1}{2}\left(a - c_2 - t_2\right) q_2
\end{aligned}$$

Substituting (24) and (25) into the above and simplifying we have

$$\begin{aligned}
\pi^{CAT} &= \frac{\left(b - \xi^2\tau^2\right)\left(a - c_1 - t_1\right)^2}{4\left(\left(b - \tau^2\right)\left(b - \xi^2\tau^2\right) - \left(b\theta - \tau^2\xi\right)^2\right)} - \frac{2\left(b\theta - \xi\tau^2\right)\left(a - c_1 - t_1\right)\left(a - c_2 - t_2\right)}{4\left(\left(b - \tau^2\right)\left(b - \xi^2\tau^2\right) - \left(b\theta - \tau^2\xi\right)^2\right)} \\
&\quad + \frac{\left(b - \tau^2\right)\left(a - c_2 - t_2\right)^2}{4\left(\left(b - \tau^2\right)\left(b - \xi^2\tau^2\right) - \left(b\theta - \tau^2\xi\right)^2\right)}
\end{aligned} \tag{30}$$

Next, we turn to determining if CAT or DOG will be chosen in equilibrium.

3.3 Parameter Restrictions

We simplify notation by de…ning

$$\varphi \equiv \frac{(a - c_2 - t_2)}{(a - c_1 - t_1)}$$

$$\beta \equiv \frac{b}{\tau^2}$$

where φ is the relative price-cost margin of the CAT product- good 2 and β is the inverse market size relative to the marginal e¤ectiveness of transportation investment.

Substituting the above two expressions into (12) and (13) we obtain the following expressions for outputs in the DOG case.

$$\frac{\tau^2}{(a-c_1-t_1)}q_1 = \frac{2(\beta-1)-\beta\theta\varphi}{(2(\beta-1)-\beta\theta)(2(\beta-1)+\beta\theta)} \tag{31}$$

$$\frac{\tau^2}{(a-c_1-t_1)}q_2 = \frac{2(\beta-1)\varphi-\beta\theta}{(2(\beta-1)-\beta\theta)(2(\beta-1)+\beta\theta)} \tag{32}$$

We do the same for the CAT case using equations (24) and (25) and obtain the following.

$$\frac{\tau^2}{(a-c_1-t_1)}q_1 = \frac{\left(\beta-\xi^2\right)-(\beta\theta-\xi)\varphi}{2\left((\beta-1)\left(\beta-\xi^2\right)-(\beta\theta-\xi)^2\right)} \tag{33}$$

$$\frac{\tau^2}{(a-c_1-t_1)}q_2 = \frac{(\beta-1)\varphi-(\beta\theta-\xi)}{2\left((\beta-1)\left(\beta-\xi^2\right)-(\beta\theta-\xi)^2\right)} \tag{34}$$

We assume that $\beta > 2$.[1] We show in the Appendix that positive DOG output implies that the following two parameter restrictions hold[2].

$$\theta \in (-1, 1) \tag{35}$$

$$\varphi \in \begin{cases} fe\left(\frac{\beta\theta}{2(\beta\ \ 1)}, \frac{2(\beta\ \ 1)}{\beta\theta}\right) & \text{if} \quad 0 < \theta < 1 \\ (0, \infty) & \text{if} \quad -1 < \theta \leq 0 \end{cases} \tag{36}$$

Likewise, to guarantee positive outputs in the CAT case we assume that the following hold.

$$\theta \in \left(\frac{\xi-\sqrt{\left(\beta-\xi^2\right)(\beta-1)}}{\beta}, \frac{\xi+\sqrt{\left(\beta-\xi^2\right)(\beta-1)}}{\beta}\right) \tag{37}$$

$$\varphi \in \begin{cases} \left(\frac{(\beta\theta\ \ \xi)}{(\beta\ \ 1)}, \frac{(\beta\ \ \xi^2)}{(\beta\theta\ \ \xi)}\right) & \text{if} \quad \frac{\xi}{\beta} < \theta < \frac{\xi+\sqrt{(\beta\ \ \xi^2)(\beta\ \ 1)}}{\beta} \\ (0, \infty) & \text{if} \quad \frac{\xi\ \ \sqrt{(\beta\ \ \xi^2)(\beta\ \ 1)}}{\beta} < \theta \leq \frac{\xi}{\beta} \end{cases} \tag{38}$$

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[1] Without this assumption we cannot be sure that demand complementarities are possible in the CAT case.

[2] Note that stability requires that $\frac{}{2(\ \ -1)} < 1 < \frac{2(\ \ -1)}{}$.

4 When is Cat Chosen?

To see whether CAT or DOG is chosen we simply compare aggregate pro…ts under the two regimes. If total pro…ts under CAT are larger than total pro…ts under DOG then there exists a price p_2. that …rm 1 pays …rm 2 for its' goods that yield higher pro…ts for both …rms under CAT. There would actually be a range of p_2 that would work but the determination of p_2 (which determines how the extra pro…ts from CAT are shared between the two …rms) is a bargaining problem that we will ignore. Pro…ts under DOG will be the sum of individual …rm pro…ts.

$$\frac{\tau^2}{(a-c_1-t_1)^2}\sum \pi^{DOG} = \frac{(\beta-1)\left((2(\beta-1)-\beta\theta\varphi)^2+(2(\beta-1)\varphi-\beta\theta)^2\right)}{\left(4(\beta-1)^2-\beta^2\theta^2\right)^2} \quad (39)$$

$$= \frac{(\beta-1)\left(\left(4(\beta-1)^2+\theta^2\beta^2\right)\left(\varphi^2+1\right)-8\theta\beta(\beta-1)\varphi\right)}{(2(\beta-1)-\beta\theta)^2(2(\beta-1)+\beta\theta)^2} \quad (40)$$

For CAT we compute the pro…ts of …rm 1 under the CAT equilibrium which is

$$\frac{\tau^2}{(a-c_1-t_1)^2}\pi^{CAT} = \frac{\left(\beta-\xi^2\right)-2(\beta\theta-\xi)\varphi+(\beta-1)\varphi^2}{4\left((\beta-1)\left(\beta-\xi^2\right)-(\beta\theta-\xi)^2\right)}$$

$$= \frac{(\beta-1)\varphi^2-2(\beta\theta-\xi)\varphi+\left(\beta-\xi^2\right)}{4\beta\left((\beta-1)\left(1-\theta^2\right)-(\xi-\theta)^2\right)} \quad (41)$$

Next, we de…ne the relative pro…tability of CAT to be

$$\equiv \frac{\tau^2}{(a-c_1-t_1)^2}\left(\pi_1^{CAT}-\sum \pi^{DOG}\right)$$

which substituting equations (40) and (41) becomes

$$(\beta,\varphi,\xi,\theta) = \frac{(\beta-1)\varphi^2-2(\beta\theta-\xi)\varphi+\left(\beta-\xi^2\right)}{4\left((\beta-1)\left(\beta-\xi^2\right)-(\beta\theta-\xi)^2\right)} \quad (42)$$

$$-\frac{(\beta-1)\left(\left(4(\beta-1)^2+\theta^2\beta^2\right)\left(\varphi^2+1\right)-8\theta\beta(\beta-1)\varphi\right)}{(2(\beta-1)-\beta\theta)^2(2(\beta-1)+\beta\theta)^2}$$

If $(\beta, \varphi, \xi, \theta) > 0$ then CAT is pro…table. We next turn to determining under what circumstances CAT is pro…table.

4.1 Two Special Cases

We begin by looking at the case in which there are no demand linkages or transportation cost synergies. This means that

$$\theta = \xi = 0 \tag{43}$$

Substituting (43) into (42) we obtain,

$$(\beta, \varphi, 0, 0) = -\frac{1}{4\beta}\frac{\varphi^2}{\beta - 1} < 0$$

which means that CAT is not pro…table. Thus, we have our …rst result.

Proposition 1 Without demand linkages or transportation cost synergies, CAT is not pro…table

In this case, the markets for the products are not linked and hence there are no gains from internalizing the cannibalization e¤ect. There are however, disadvantages in transportation costs. These disadvantages are due to the fact that the CAT …rm's investment in its transportation costs technology has no e¤ect on the transportation costs of the potential CAT product. On the other hand, …rm 2's investment is tailored to its product's transportation technology and therefore, …rm 2 will have an advantage in transporting its own product.

Now consider the case in which transportation cost synergies are the highest possible, $\xi = 1$. If $\xi = 1$ and $\theta \to 1$[3] then

$$\lim_{\theta \,!\, 1^-} (\beta, \varphi, 1, \theta) = \text{signum}\left(\frac{1}{\beta}(\varphi - 1)^2\right)\infty = \infty$$

[3] Note that $\theta^{\max}(\xi = 1) = \frac{1+\sqrt{(\ \ -1)(\ \ -1)}}{} = 1$

Thus, when cost synergies are maximal and demand linkages are maximal (for the case of substitutes) then CAT is always pro…table. What if the two goods are complements? First, given that ξ = 1 the smallest θ can be is $-\frac{\beta \quad 2}{\beta}$, i.e. $\theta^{\min}(\xi = 1) = \frac{1 \quad \sqrt{(\beta \quad 1)(\beta \quad 1)}}{\beta} = -\frac{\beta \quad 2}{\beta}$. Then, it follows that

$$\lim_{\theta\ !\ \frac{\beta-2}{\beta}+} \qquad (\beta, \varphi, 1, \theta) = \text{signum}\left(\frac{1}{\beta}(\varphi + 1)^2\right)\infty = \infty$$

This gives us our second result

Proposition 2 If demand linkages (both for the case of substitutes and complements) and transportation costs synergies are maximal, CAT is always pro…table

So, we know that with no demand linkages or transportation cost synergies, CAT is not pro…table and with maximal linkages and synergies, CAT is always pro…table. The remaining task is to sketch out the degree to which demand linkages and transportation cost synergies are required to make CAT pro…table.

4.2 When is CAT Profitable?

Next, we show that transportation cost synergies are not necessary for CAT to be pro…table. Let ξ = 0, then suppose that the two products are complements. If $\theta \to \theta^{\min} = \frac{\xi \quad \sqrt{(\beta \quad \xi^2)(\beta \quad 1)}}{\beta} = -\sqrt{\frac{(\beta \quad 1)}{\beta}}$ then

$$\begin{aligned}
&\lim_{\theta\ !\ \frac{(\beta-1)}{\beta}+} \left(\begin{array}{c} \frac{(\beta \quad 1)\varphi^2 \quad 2\beta\theta\varphi + \beta}{4\left((\beta \quad 1)\beta \quad (\beta\theta)^2\right)} \\ -\frac{(\beta \quad 1)\left((2(\beta \quad 1) \quad \beta\theta\varphi)^2 + (2(\beta \quad 1)\varphi \quad \beta\theta)^2\right)}{\left(4(\beta \quad 1)^2 \quad \beta^2\theta^2\right)^2} \end{array} \right) \\
= \quad &\text{signum}\left(\frac{\beta + (\beta - 1)\varphi^2 + 2\beta\varphi\sqrt{\frac{1}{\beta}(\beta - 1)}}{\beta^2\sqrt{\frac{1}{\beta}(\beta - 1)}}\right)\infty = \infty
\end{aligned}$$

Hence: $\left(\beta, \varphi, 0, \theta^{\min}\right) > 0$, if the two goods are complementary enough CAT is pro…table. Next, for the case in which the two goods are substitutes we

have $\theta \rightarrow \theta^{\max} = \frac{\beta \;\; 1}{\beta}\varphi$ so

$$\begin{aligned}
&\lim_{\theta \; ! \; \frac{\beta-1}{\beta}\varphi^-} \left(\begin{array}{c} \frac{(\beta \;\; 1)\varphi^2 \;\; 2\beta\theta\varphi+\beta}{4\left((\beta \;\; 1)\beta \;\; (\beta\theta)^2\right)} \\ -\frac{(\beta \;\; 1)\left((2(\beta \;\; 1) \;\; \beta\theta\varphi)^2+(2(\beta \;\; 1)\varphi \;\; \beta\theta)^2\right)}{\left(4(\beta \;\; 1)^2 \;\; \beta^2\theta^2\right)^2} \end{array} \right) \\
= \;\; & \frac{1}{4}\varphi^2 \frac{4-3\varphi^2}{(\beta-1)\left(\varphi^2-4\right)^2} > 0
\end{aligned}$$

Hence: $\quad (\beta, \varphi, 0, \theta^{\max}) > 0$. These results are summarized in the following proposition

Proposition 3 CAT is pro…table if demand linkages are large enough, transportation cost synergies are not necessary.

It is also the case that demand linkages are not necessary. Suppose there are no demand linkages, $\theta = 0$ but that transportation cost synergies are maximal, $\xi = 1$. Then we have

$$(\beta, \varphi, 1, 0) = \frac{\varphi^2 + 2\varphi(\beta-1) + 1}{4\beta\left(\beta^2 - 3\beta + 2\right)} > 0$$

This is summarized below

Proposition 4 CAT is pro…table if transportation cost synergies are large enough, demand linkages are not necessary.

So, we know that CAT is pro…table if either transportation cost synergies or demand linkages are large enough. Next, we turn to a more complete characterization of when CAT is pro…table.

4.3 CAT versus DOG

From previous propositions we know that the three key variables that determine whether CAT or DOG is chosen are φ, ξ, and θ. We can illustrate the tradeo¤s between these variables by considering the $\quad = 0$ locus in (ξ, θ) space (see

Figure 1). By de…nition of the parameters the diagram is restricted to the parameter space $\xi \in [0,1]$ and $\theta \in [-1,1]$ and the …gure is drawn for a given value of φ. We begin by …rst characterizing the dominant mode of entry for the case of $\varphi = 1$, and then we analyze how changes in φ a¤ect the entry mode.

The indi¤erence condition between CAT and DOG for $\varphi = 1$ is given by

$$(\beta, 1, \xi, \theta) = \frac{(\beta - 1) - 2(\beta\theta - \xi) + \left(\beta - \xi^2\right)}{4\left((\beta - 1)\left(\beta - \xi^2\right) - (\beta\theta - \xi)^2\right)} - \frac{2(\beta - 1)}{(2(\beta - 1) + \theta\beta)^2} = 0. \tag{44}$$

In Figure 1 we provide a complete characterization of the $(\beta, 1, \xi, \theta) = 0$ locus. First note that this locus is well behaved and twice di¤erentiable for $\xi \in [0,1]$ and $\theta \in \left(\frac{\xi}{\beta} - \frac{1}{\beta}\sqrt{\left(\beta - \xi^2\right)(\beta - 1)}, \frac{\xi}{\beta} + \frac{\beta\ \ 1}{\beta}\right)$. Hence, the $(\beta, 1, \xi, \theta) = 0$ locus provides a lower bound for the pro…tability of carry-along trade in (ξ, θ) space. Since $_{\xi} > 0$, CAT is pro…table above the locus, while the area below the locus indicates pairs of (ξ, θ) where DOG is more pro…table than CAT.

Looking, at Figure 1 one can see that CAT requires either su¢ ciently large demand linkages, or su¢ ciently large transportation cost synergies, or both. In fact, we can show analytically that relative CAT pro…ts are increasing in transportation costs synergies ξ. First, we …nd that

$$\frac{\partial}{\partial \xi} \quad (\beta, \varphi, \xi, \theta) = -\frac{\beta\left((\theta - \xi)\varphi - (1 - \theta\xi)\right)\left((\beta - 1)\varphi - (\beta\theta - \xi)\right)}{2\left((\beta - 1)\left(\beta - \xi^2\right) - (\beta\theta - \xi)^2\right)^2}$$

It follows that $((\beta - 1)\varphi - (\beta\theta - \xi)) > 0$ because $\varphi \in \left(\frac{(\beta\theta\ \ \xi)}{(\beta\ \ 1)}, \frac{(\beta\ \ \xi^2)}{(\beta\theta\ \ \xi)}\right)$. Next, observe that $((\theta - \xi)\varphi - (1 - \theta\xi)) < 0$ if $\theta \leq \xi$. It follows then, that for the case of $\theta \leq \xi \Rightarrow \frac{\partial}{\partial \xi} \quad (\beta, \varphi, \xi, \theta) > 0$.

If $\theta > \xi$, then

$$(\theta\xi - 1 + (\theta - \xi)\varphi) = (\theta - \xi)\left(\varphi - \frac{\left(\beta - \xi^2\right)}{(\beta\theta - \xi)}\right) - \frac{\xi\left((\beta - 1)\left(1 - \theta^2\right) - (\xi - \theta)^2\right)}{(\beta\theta - \xi)} < 0$$

This follows because $\theta - \xi > 0$ if $\theta > \xi$ and $\left(\varphi - \frac{(\beta \quad \xi^2)}{(\beta\theta \quad \xi)}\right) < 0$ because $\varphi < \frac{(\beta \quad \xi^2)}{(\beta\theta \quad \xi)}$.Looking at the last term, $\xi\left((\beta-1)\left(1-\theta^2\right)-(\xi-\theta)^2\right) > 0$ from the stability condition and $(\beta\theta - \xi) > 0$ if $\theta > \xi$. This proves that $\frac{\partial}{\partial \xi} \quad (\beta, \varphi, \xi, \theta) >$ 0, which means that CAT pro…ts are increasing in transportation costs synergies, ξ. This result is quite intuitive because it means that the more that investments in improving transportation technology involve spillovers to the CAT good the more likely we are to see CAT in equilibrium. This result also means that demand complementarities are neither a necessary nor a su¢ cient condition for CAT. They are not necessary because CAT can be pro…table without any demand linkages (if transportation cost synergies are large enough), or (in the absence of any transportation cost synergies) if products are close substitutes.

Next, let us analyze how the relative price-cost margin of the CAT product, φ, a¤ects the pro…tability of CAT. The derivative of with respect to φ, evaluated at = 0, is

$$\frac{\partial}{\partial \varphi} \quad (\beta, \varphi, \xi, \theta) = \frac{2}{\varphi}\left(\begin{array}{c} \frac{(\beta \quad 1)\varphi^2 \quad (\beta\theta \quad \xi)\varphi}{4\left((\beta \quad 1)(\beta \quad \xi^2) \quad (\beta\theta \quad \xi)^2\right)} \\ -\frac{(\beta \quad 1)(2(\beta \quad 1)\varphi(2(\beta \quad 1)\varphi \quad \beta\theta) \quad \beta\theta\varphi(2(\beta \quad 1) \quad \beta\theta\varphi))}{\left(4(\beta \quad 1)^2 \quad \beta^2\theta^2\right)^2} \end{array}\right)$$

we show in the Appendix that this is unambiguously negative. This implies that an decrease in φ will shift the = 0 locus downwards (see Figure 2), making CAT more pro…table. Hence, if the relative price-cost margins of the potential CAT product falls, CAT becomes more pro…table.

4.4 Prices

We can illustrate the impact of carry-along trade on prices in Figure 3. To do this we compute the change in price that occurs when going from DOG to CAT and plot it on Figure 1. So, Figure 3 is the same as Figure 1 with the addition of two p = 0 loci - $p_1(\beta, 1, \xi, \theta)$ for the DOG product and $p_2(\beta, 1, \xi, \theta)$ for

the CAT product.

$$p_1(\beta,1,\xi,\theta) = -\frac{\left(\beta-\xi^2\right)-(\beta\theta-\xi)+\theta\left((\beta-1)-(\beta\theta-\xi)\right)}{2\left((\beta-1)\left(\beta-\xi^2\right)-(\beta\theta-\xi)^2\right)}+\frac{1+\theta}{2(\beta-1)+\beta\theta}$$

$$p_2(\beta,1,\xi,\theta) = -\frac{(\beta-1)-(\beta\theta-\xi)+\theta\left(\left(\beta-\xi^2\right)-(\beta\theta-\xi)\right)}{2\left((\beta-1)\left(\beta-\xi^2\right)-(\beta\theta-\xi)^2\right)}+\frac{1+\theta}{2(\beta-1)+\beta\theta}$$

We begin by discussing some special cases before characterizing the two loci more generally.

We begin with $\xi = \theta = 0$. In this case, $p_1(\beta,\varphi,0,0) = 0$ and $p_2(\beta,\varphi,0,0) = \frac{1}{2\beta(\beta-1)} > 0$. Hence, in the absence of both demand linkages and transportation cost synergies, the price of the CAT product clearly rises while the price of the product produced by firm 1 is unaffected. This is intuitive since in this case neither the output decision nor the investment decision of firm 1 is affected by carry-along trade so it makes sense that the price of good 1 is unchanged. Since firm 1 has higher transportation costs for product 2, it follows that the price of the CAT product, good 2, rises. Given how CAT affects the two prices it is not surprising that in this case CAT is never profitable.

Next, let us discuss the case where $\theta = 0$ but $\xi = 1$. In this case, $p_1(\beta,1,1,0) = p_2(\beta,1,1,0) = -\frac{1}{2(\beta-2)(\beta-1)} < 0$. Thus, if transportation costs synergies are perfect, but there are no demand linkages, both product prices are clearly lower with CAT. This is also very intuitive because these synergies change the investment behavior of the potential CAT firm. If firm 1 sells not only its own product but also the CAT product it has a larger marginal benefit from investing in lower transportation costs. Thus, it will invest more in reducing transportation costs, thereby lowering the marginal sales costs and hence, the prices of both products.

Next, we look at how prices are affected if demand linkages are large. First, let $\theta \to \theta^{\min} \equiv \frac{1}{\beta}\left(\xi-\sqrt{\left(\beta-\xi^2\right)(\beta-1)}\right)$. In the limit, both prices fall: $\lim_{\theta\to\theta^{\min}} p_1(\beta,1,\xi,\theta) = \lim_{\theta\to\theta^{\min}} p_2(\beta,1,\xi,\theta) = -\infty < 0$. This result holds independent of transportation costs synergies. Hence, if the two prod-

ucts are complements, prices tend to be lower. This is different if products are substitutes. To see what happens when products are substitutes let $\theta \to \theta^{\max} \equiv \frac{\xi}{\beta} + \frac{\beta - 1}{\beta}$. In this case, $p_1(\beta, 1, \xi, \theta^{\max}) = \frac{1}{2}\frac{(\beta - 2)(\beta - 1 + \xi)}{\beta(\beta - 1)(3(\beta - 1) + \xi)} > 0$ and $p_2(\beta, 1, \xi, \theta^{\max}) = \frac{(\beta + (1 - \xi)) - \frac{\xi}{(\beta - 1)}(\beta - (1 - \xi))}{2\beta(3(\beta - 1) + \xi)} > 0$ (note that $\frac{\beta + (1 - \xi)}{\beta - (1 - \xi)} > 1 > \frac{\xi}{\beta - 1}$). If products are substitutes, prices are higher with CAT than if sold by separate firms. Hence, CAT leads to lower prices when the products are complements and higher prices if they are substitutes. The reason for this lies in the cannibalization effect. Carry-along trade essentially makes the CAT firm a multi-product firm that internalizes demand linkages. This internalization pushes prices in different directions depending on whether outputs of the two products are strategic substitutes or complements. If they are strategic substitutes ($\theta > 0$), a higher output of one product reduces the optimal output of the other product. The CAT firm internalizes this negative externality and sells less of both products at higher prices. If the two outputs are strategic complements, a higher output of one product generates a positive externality on the other product. If the CAT firm internalizes this positive externality it sells more of both products, and prices of both products are lower.

This result shows that carry-along trade can have very different effects on prices depending on whether the products carried-along are substitutes or complements. While we don't model welfare explicitly, this result suggests an interesting perspective on the welfare effects of CAT. If the CAT products are complements, since CAT lowers prices consumes face it raises consumer rents relative to DOG. For the case of substitutes, prices increase and consumer rents are lower.

5 Conclusion

We have developed a model to explain why CAT occurs. We show that the CAT versus DOG decision comes down to demand linkages and transportation cost synergies. In addition, it turns out that the relative price-cost margins of the CAT good and the exporter's good a¤ects the CAT-DOG decision. If demand linkages are strong enough and transportation synergies large enough, then CAT is pro…table. We have ignored certain aspects of the problem for the sake of simplicity, but these issues are of interest. In particular, we ignore how the bene…ts of CAT are divided between the CAT …rm and the actual producer of the good that is carried along. In addition, we do not consider the case of competition among CAT …rms.

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6 Appendix

Simplify notation:

$$\varphi \equiv \frac{(a - c_2 - t_2)}{(a - c_1 - t_1)}$$

$$\beta \equiv \frac{b}{\tau^2}$$

Explain parameters:

$\beta \equiv \frac{b}{\tau^2} > 1$: Inverse market size relative to marginal e¤ectiveness of transportation investment

$\varphi \equiv \frac{(a \quad c_2 \quad \bar{t}_2)}{(a \quad c_1 \quad \bar{t}_1)}$: Relative price-cost margins of CAT product 2

$\xi \in [0, 1]$: Degree of spillover (synergies) from transportation cost investment

$\theta \in [-1, 1]$: Degree of product di¤erentiation

Output DOG:

$$\frac{\tau^2}{(a-c_1-\bar{t}_1)}q_1 = \frac{2(\beta-1)-\beta\theta\varphi}{4(\beta-1)^2-\beta^2\theta^2} = \frac{2(\beta-1)-\beta\theta\varphi}{(2(\beta-1)-\beta\theta)(2(\beta-1)+\beta\theta)}$$
$$\frac{\tau^2}{(a-c_1-\bar{t}_1)}q_2 = \frac{2(\beta-1)\varphi-\beta\theta}{4(\beta-1)^2-\beta^2\theta^2} = \frac{2(\beta-1)\varphi-\beta\theta}{(2(\beta-1)-\beta\theta)(2(\beta-1)+\beta\theta)}$$

Stability:

If $\theta > 0$: $2(\beta-1)-\beta\theta > 0$: $\theta < 2\frac{\beta-1}{\beta}$: This is binding, if $1 < \beta \leq 2$

If $\theta < 0$: $2(\beta-1)+\beta\theta > 0$: $\theta > -2\frac{\beta-1}{\beta}$: This is also binding, if $1 < \beta \leq 2$

Output 1: $2(\beta-1)-\beta\theta\varphi > 0$: $\varphi < 2\frac{(\beta-1)}{\beta\theta}$

Output 2: $2(\beta-1)\varphi-\beta\theta > 0$: $\varphi > \frac{\beta\theta}{2(\beta-1)}$

Since $\varphi \in (0,\infty)$, this is not binding if $\theta \leq 0$. Otherwise: $\varphi \in \left(\frac{\beta\theta}{2(\beta-1)}, \frac{2(\beta-1)}{\beta\theta}\right)$

Note (by stability): $\frac{\beta\theta}{2(\beta-1)} < 1 < \frac{2(\beta-1)}{\beta\theta}$

Output CAT:

$$\frac{\tau^2}{(a-c_1-\bar{t}_1)}q_1 = \frac{\left(\beta-\xi^2\right)-(\beta\theta-\xi)\varphi}{2\left((\beta-1)(\beta-\xi^2)-(\beta\theta-\xi)^2\right)}$$
$$\frac{\tau^2}{(a-c_1-\bar{t}_1)}q_2 = \frac{(\beta-1)\varphi-(\beta\theta-\xi)}{2\left((\beta-1)(\beta-\xi^2)-(\beta\theta-\xi)^2\right)}$$

Restrictions:

Stability: $(\beta-1)\left(\beta-\xi^2\right) > (\beta\theta-\xi)^2$: $\theta \in \left(\frac{\xi-\sqrt{(\beta-\xi^2)(\beta-1)}}{\beta}, \frac{\xi+\sqrt{(\beta-\xi^2)(\beta-1)}}{\beta}\right)$

[Alternatively: $\left(1-\theta^2\right)(\beta-1) > (\xi-\theta)^2$]

Note that $\frac{\xi-\sqrt{(\beta-\xi^2)(\beta-1)}}{\beta} > -1$: $(\xi+\beta) > \sqrt{\left(\beta-\xi^2\right)(\beta-1)}$: $(\xi+1)^2 > 0$

And note that $\frac{\xi+\sqrt{(\beta-\xi^2)(\beta-1)}}{\beta} < 1$: $\sqrt{\left(\beta-\xi^2\right)(\beta-1)} < (\beta-\xi)$: $(\xi-1)^2 >$
0

Note that $\lim_{\beta\to 1}\left(\frac{\xi-\sqrt{(\beta-\xi^2)(\beta-1)}}{\beta}\right) = -1$ and $\lim_{\beta\to 1}\left(\frac{\xi+\sqrt{(\beta-\xi^2)(\beta-1)}}{\beta}\right) =$
1.

Note: If $\xi > \sqrt{\left(\beta-\xi^2\right)(\beta-1)}$, i.e. if $\xi > \sqrt{\beta-1}$, then restrictions exclude complementarities! This can only be if $\beta < 2$.

Hence, we may want to assume that $\beta \equiv \frac{b}{\tau^2} > 2$

CAT output 1: $\left(\beta-\xi^2\right) > (\beta\theta-\xi)\varphi$

CAT output 2: $(\beta-1)\varphi > (\beta\theta-\xi)$

Two cases:

$(\beta\theta-\xi) < 0$: Both conditions hold trivially.

$(\beta\theta - \xi) > 0$: $\frac{(\beta - \xi^2)}{(\beta\theta - \xi)} > \varphi$ and $\varphi > \frac{(\beta\theta - \xi)}{(\beta - 1)}$. Note that stability requires $\frac{(\beta - \xi^2)}{(\beta\theta - \xi)} > \frac{(\beta\theta - \xi)}{(\beta - 1)}$

$\frac{(\beta - \xi^2)}{(\beta\theta - \xi)} > 1$: $\beta(1-\theta) > 0 > \xi(\xi - 1)$ always holds

$1 > \frac{(\beta\theta - \xi)}{(\beta - 1)}$: $\beta > \frac{1-\xi}{1-\theta}$. Holds if $\xi > \theta$, but does not hold for $\theta^{\max} \equiv \frac{\xi + \sqrt{(\beta - \xi^2)(\beta - 1)}}{\beta}$

Note that restriction on output 2 is our profitability condition from above: $\varphi > \frac{\beta}{\beta - 1}\theta - \frac{1}{\beta - 1}\xi$.

Note that $\lim_{\beta \to \infty}\left(\frac{\beta}{\beta - 1}\theta - \frac{1}{\beta - 1}\xi\right) = \theta$. Hence, if the destination market is really small [or, an alternative interpretation of $\beta \to \infty$, but with a trivial interpretation: if investment in transportation costs is ineffective ($\tau \to 0$)], transportation costs synergies become irrelevant.

Summarize restrictions:

$\beta > 2$

DOG:

$\theta \in (-1, 1)$

$$\varphi \in \begin{cases} \left(\frac{\beta\theta}{2(\beta - 1)}, \frac{2(\beta - 1)}{\beta\theta}\right) & \text{if} \quad 0 < \theta < 1 \\ (0, \infty) & \text{if} \quad -1 < \theta \leq 0 \end{cases}$$

CAT:

$$\theta \in \left(\frac{\xi - \sqrt{(\beta - \xi^2)(\beta - 1)}}{\beta}, \frac{\xi + \sqrt{(\beta - \xi^2)(\beta - 1)}}{\beta}\right)$$

$$\varphi \in \begin{cases} \left(\frac{(\beta\theta - \xi)}{(\beta - 1)}, \frac{(\beta - \xi^2)}{(\beta\theta - \xi)}\right) & \text{if} \quad \frac{\xi}{\beta} < \theta < \frac{\xi + \sqrt{(\beta - \xi^2)(\beta - 1)}}{\beta} \\ (0, \infty) & \text{if} \quad \frac{\xi - \sqrt{(\beta - \xi^2)(\beta - 1)}}{\beta} < \theta \leq \frac{\xi}{\beta} \end{cases}$$

6.0.1 Effect of φ

$$(\beta,\varphi,\xi,\theta) = \frac{(\beta \quad 1)\varphi^2 \quad 2(\beta\theta \quad \xi)\varphi + \left(\beta \quad \xi^2\right)}{4\left((\beta \quad 1)(\beta \quad \xi^2) \quad (\beta\theta \quad \xi)^2\right)} - \frac{(\beta \quad 1)\left((2(\beta \quad 1) \quad \beta\theta\varphi)^2 + (2(\beta \quad 1)\varphi \quad \beta\theta)^2\right)}{\left(4(\beta \quad 1)^2 \quad \beta^2\theta^2\right)^2}$$

$$\frac{\partial}{\partial\varphi} \quad (\beta,\varphi,\xi,\theta) = \frac{2}{\varphi}\left(\frac{(\beta \quad 1)\varphi^2 \quad (\beta\theta \quad \xi)\varphi}{4\left((\beta \quad 1)(\beta \quad \xi^2) \quad (\beta\theta \quad \xi)^2\right)} - \frac{(\beta \quad 1)(2(\beta \quad 1)\varphi(2(\beta \quad 1)\varphi \quad \beta\theta) \quad \beta\theta\varphi(2(\beta \quad 1) \quad \beta\theta\varphi))}{\left(4(\beta \quad 1)^2 \quad \beta^2\theta^2\right)^2}\right)$$

$\frac{\partial}{\partial\varphi} \quad (\beta,\varphi,\xi,\theta) < 0$ implies that $\frac{(\beta \quad 1)\varphi^2 \quad (\beta\theta \quad \xi)\varphi}{4\left((\beta \quad 1)(\beta \quad \xi^2) \quad (\beta\theta \quad \xi)^2\right)} < \frac{(\beta \quad 1)(2(\beta \quad 1)\varphi(2(\beta \quad 1)\varphi \quad \beta\theta) \quad \beta\theta\varphi(2(\beta \quad 1) \quad \beta\theta\varphi))}{\left(4(\beta \quad 1)^2 \quad \beta^2\theta^2\right)^2}$ I

1 $\frac{(\beta \quad 1) \quad (\beta\theta \quad \xi)}{4\left((\beta \quad 1)(\beta \quad \xi^2) \quad (\beta\theta \quad \xi)^2\right)} < \frac{(\beta \quad 1)(2(\beta \quad 1)(2(\beta \quad 1) \quad \beta\theta) \quad \beta\theta(2(\beta \quad 1) \quad \beta\theta))}{\left(4(\beta \quad 1)^2 \quad \beta^2\theta^2\right)^2}$

Evaluate inequality at $\quad (\beta,\varphi,\xi,\theta) = 0$: $\frac{(\beta \quad 1)\varphi^2 \quad 2(\beta\theta \quad \xi)\varphi + \left(\beta \quad \xi^2\right)}{4\left((\beta \quad 1)(\beta \quad \xi^2) \quad (\beta\theta \quad \xi)^2\right)} - \frac{(\beta \quad 1)\left((2(\beta \quad 1) \quad \beta\theta\varphi)^2 + (2(\beta \quad 1)\varphi \quad \beta\theta)^2\right)}{\left(4(\beta \quad 1)^2 \quad \beta^2\theta^2\right)^2}$

1 $\frac{(\beta \quad 1) \quad 2(\beta\theta \quad \xi) + \left(\beta \quad \xi^2\right)}{4\left((\beta \quad 1)(\beta \quad \xi^2) \quad (\beta\theta \quad \xi)^2\right)} - \frac{(\beta \quad 1)\left((2(\beta \quad 1) \quad \beta\theta)^2 + (2(\beta \quad 1) \quad \beta\theta)^2\right)}{\left(4(\beta \quad 1)^2 \quad \beta^2\theta^2\right)^2}$

Sub $\left(4(\beta-1)^2 - \beta^2\theta^2\right)^2 = 4\left((\beta-1)\left(\beta-\xi^2\right) - (\beta\theta-\xi)^2\right)\frac{(\beta \quad 1)\left((2(\beta \quad 1) \quad \beta\theta\varphi)^2 + (2(\beta \quad 1)\varphi \quad \beta\theta)^2\right)}{((\beta \quad 1)\varphi^2 \quad 2(\beta\theta \quad \xi)\varphi + (\beta \quad \xi^2))}$

$$\frac{(\beta \quad 1)\varphi^2 \quad (\beta\theta \quad \xi)\varphi}{4\left((\beta \quad 1)(\beta \quad \xi^2) \quad (\beta\theta \quad \xi)^2\right)} < \frac{(\beta \quad 1)(2(\beta \quad 1)\varphi(2(\beta \quad 1)\varphi \quad \beta\theta) \quad \beta\theta\varphi(2(\beta \quad 1) \quad \beta\theta\varphi))}{4\left((\beta \quad 1)(\beta \quad \xi^2) \quad (\beta\theta \quad \xi)^2\right)\frac{(\beta-1)\left((2(\beta-1)-\beta\theta\varphi)^2+(2(\beta-1)\varphi-\beta\theta)^2\right)}{\left((\beta-1)\varphi^2-2(\beta\theta-\xi)\varphi+\left(\beta-\xi^2\right)\right)}}$$

$\left((\beta-1)\varphi^2 - (\beta\theta-\xi)\varphi\right)\left((2(\beta-1)-\beta\theta\varphi)^2 + (2(\beta-1)\varphi-\beta\theta)^2\right) < (2(\beta-1)\varphi(2(\beta-1)\varphi-\beta\theta) - \beta\theta$

$((\beta-1)\varphi - (\beta\theta-\xi))\left((2(\beta-1)-\beta\theta\varphi)^2 + (2(\beta-1)\varphi-\beta\theta)^2\right) < (2(\beta-1)(2(\beta-1)\varphi-\beta\theta) - \beta\theta(2(\beta$

$((\beta-1)\varphi - (\beta\theta-\xi))\left((2(\beta-1)-\beta\theta\varphi)^2 + (2(\beta-1)\varphi-\beta\theta)^2\right) < (2(\beta-1)(2(\beta-1)\varphi-\beta\theta) - \beta\theta(2(\beta$

$((2(\beta-1))(2(\beta-1)-\beta\theta\varphi) - \beta\theta(2(\beta-1)\varphi-\beta\theta))((\beta-1)\varphi-(\beta\theta-\xi)) <$

$(2(\beta-1)(2(\beta-1)\varphi-\beta\theta) - \beta\theta(2(\beta-1)-\beta\theta\varphi))\left(\left(\beta-\xi^2\right)-(\beta\theta-\xi)\varphi\right)$

$$\frac{((2(\beta \quad 1))(2(\beta \quad 1) \quad \beta\theta\varphi) \quad \beta\theta(2(\beta \quad 1)\varphi \quad \beta\theta))}{(2(\beta \quad 1)(2(\beta \quad 1)\varphi \quad \beta\theta) \quad \beta\theta(2(\beta \quad 1) \quad \beta\theta\varphi))} < \frac{\left(\left(\beta \quad \xi^2\right) \quad (\beta\theta \quad \xi)\varphi\right)}{((\beta \quad 1)\varphi \quad (\beta\theta \quad \xi))}$$

If $\varphi = 1$

$1 < \frac{\left(\beta \quad \xi^2\right) \quad (\beta\theta \quad \xi)}{(\beta \quad 1) \quad (\beta\theta \quad \xi)}$

This clearly holds because $\xi^2 < 1$.

This proves that $\frac{\partial}{\partial\varphi} \quad (\beta,1,\xi,\theta)\Big|_{\Delta\Pi=0} < 0$

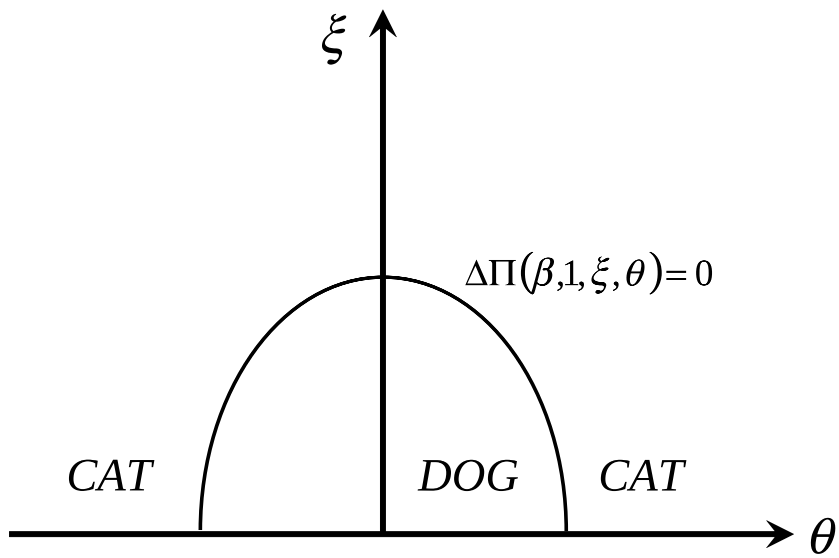

Figure 1: CAT versus DOG

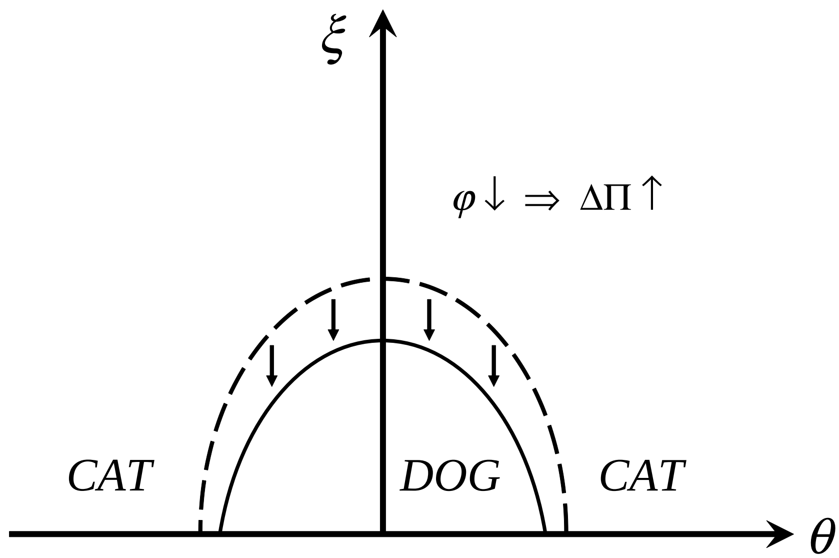

Figure 2: E¤ect of lower φ

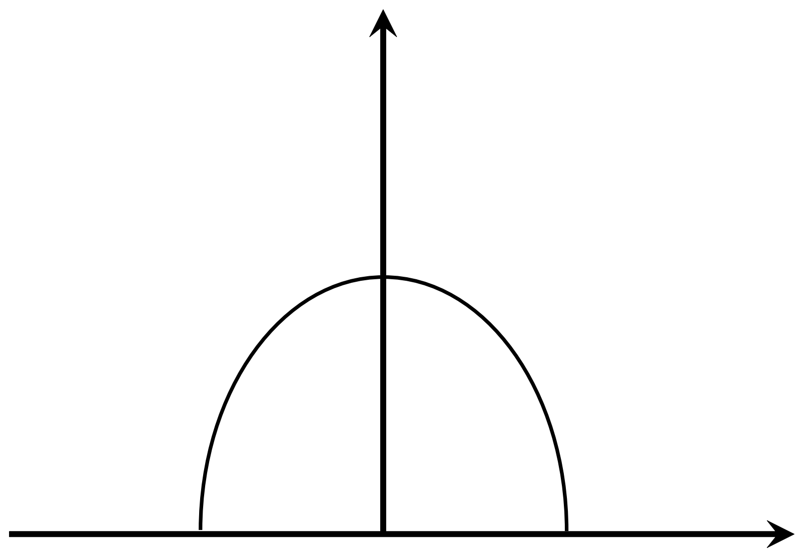

Figure 3: Prices