

Explaining the Spread of Temporary Jobs and its Impact on Labor Turnover

Pierre Cahuc¹

Ecole Polytechnique, CREST, IZA, CEPR

Olivier Charlot²

Université Cergy-Pontoise, THEMA

Franck Malherbet³

Université de Rouen, Ecole Polytechnique, CREST, IZA

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¹Contact: pierre.cahuc@polytechnique.edu.

²Contact: ocharlot@u-cergy.fr

³Contact: franck.malherbet@univ-rouen.fr

Abstract

This paper provides a simple model which explains the choice between permanent and temporary jobs. This model, which incorporates important features of actual employment protection legislations neglected by the economic literature so far, reproduces the main stylized facts about entries into permanent and temporary jobs observed in Continental European countries. We show that the stringency of legal constraints on the termination of permanent jobs has a strong positive impact on the turnover of temporary jobs. We also find that job protection has very small effects on total employment but induces large substitution of temporary jobs for permanent jobs which significantly reduces aggregate production.

Key words: Temporary jobs, Employment protection legislation.

JEL classification: J63, J64, J68.

1 Introduction¹

It is recurrently argued that the dramatic spread of temporary jobs in Continental European countries is the consequence of the combination of stringent legal constraints on the termination of permanent jobs and of weak constraints on the creation of temporary jobs. It is also argued that this combination creates labor market segmentation and traps workers in a recurring sequence of frequent unemployment spells.² However, strikingly, very little is known about the creation of temporary and permanent jobs in as much as very few contributions have analyzed the choice between these two types of job. There are also very few explanations of the duration of temporary jobs.

The aim of our paper is to contribute to fill this gap. The originality of our approach is to account for important features of employment protection legislations which have been neglected by the literature so far. In most countries, it is costly to dismiss temporary workers before the date of termination of the contract stipulated when the job starts. More precisely, in the ‘French type’ regulation, that covers Belgium, France, Greece, Italy and Germany, temporary contracts cannot be terminated before their expiration date,³ whereas in the ‘Spanish type’ regulation, which covers Spain and Portugal, the rule for dismissals before the expiration date of temporary contracts is the same as for permanent contracts. Hence, for a given employment spell, it is generally at least as costly to terminate a temporary contract before its expiration date as to terminate a regular contract. In the previous literature, it is generally assumed that it is costly to terminate permanent contracts while temporary contracts can be terminated at no cost at any time. This assumption, made for the sake of technical simplicity, is at odds with many actual regulations. It implies that employers prefer temporary jobs, which can be destroyed at no cost, to permanent jobs, which are costly to destroy, thus making it difficult to explain the choice between permanent and temporary jobs. We show that the choice between

¹We thank Samuel Bentolila, Tito Boeri, Werner Eichhorst, Øivind Nilsen, Oskar Nordström Skans, Pedro Portugal, Kostas Tatsiramos, Bruno Van der Linden and Frank Walsh for providing us with information about employment protection legislation and temporary jobs in different OECD countries. We thank Fabio Berton, Juan Dolado, Bruno Decreuse, Renato Faccini, Pietro Garibaldi, Juan Jimeno, Etienne Lehmann, Jean-Baptiste Michau, Claudio Michelacci, Fabien Postel-Vinay, Francesco Pappadà, Barbara Petrongolo, Jean-Luc Prigent, Jose Silva, Eric Smith and Hélène Turon for useful comments.

²See Boeri (2011) for a synthesis.

³There are obviously exceptions to this general rule, for instance for missbehavior of one of the parties. The legislations are described in appendix A. For a given employment spell, it appears that it is generally at least as costly to terminate a temporary contract before its date of termination as to terminate a regular contract.

permanent and temporary jobs can be explained easily when it is assumed that temporary jobs cannot be terminated before their expiration date or when the rule for dismissals is the same for temporary and permanent contracts. Moreover, this assumption allows us to reproduce important stylized facts about the turnover of temporary and permanent jobs and to shed new light on the consequence of job protection.

We consider a job search and matching model where firms hire workers to exploit production opportunities of different expected durations. Some production opportunities are expected to end (i.e. to become unproductive) quickly, others are expected to last longer. This assumption accounts for the heterogeneity of expected durations of jobs which is an important feature of modern economies. For instance, firms can get orders for their products for several days, several months or several years and it is not certain that these orders will be renewed. In the model, jobs can be either permanent or temporary. Permanent employees are protected by dismissal costs. Temporary jobs can be destroyed at zero cost at their expiration date, which is chosen at the instant when workers are hired. But employers either have to keep and pay their employees until the date of termination of temporary jobs in the ‘French type regulation’ or can dismiss them at the same cost as for permanent contracts in the ‘Spanish type’ regulation. These assumptions about employment legislation, which aim at accounting for the main features of Continental European labor regulations, do not induce Pareto optimal allocations. However, permanent workers protected by firing costs may support such regulations.⁴

When firing costs are sufficiently small, we find that all production opportunities are exploited with permanent jobs. When firing costs are relatively large, permanent jobs are chosen to exploit production opportunities expected to go on for a long time, while temporary jobs are used for production opportunities with short expected durations. In this framework, higher firing costs increase the share of entries into temporary jobs.

We show that this model matches the main stylized facts concerning entries into permanent and temporary jobs in Continental European countries. Moreover, calibration exercises show that the durations of temporary jobs are much shorter than that of production opportunities. Therefore, higher firing costs, by increasing the share of temporary jobs, induce a strong excess of labor turnover on production opportunities with relatively short durations. This excess of labor turnover is detrimental to temporary workers whose expected job duration becomes shorter

⁴See Saint-Paul, (1996), (2002).

when the employment protection of permanent jobs becomes more stringent. In this context, increases in the protection of permanent jobs have very small negative effects on aggregate employment. However, this small aggregate impact is the consequence of two large counteracting effects: a strong decrease in the number of permanent jobs and a strong increase in the number of temporary jobs. This large reallocation of jobs, in line with empirical evidence,⁵ decreases aggregate production, because the production (net of labor turnover costs) of temporary jobs is much smaller than that of permanent jobs. All in all, our model shows that protection of permanent jobs has very small effects on aggregate employment, but induces employment composition effects that drastically reduce aggregate production. Changes in aggregate production are about 20 times larger than changes in aggregate employment.

Our paper is related to at least three strands of the literature.

First, we introduce heterogeneity of idiosyncratic productivity shock arrival rates in the job search model. This yields a framework useful to explain the distribution of job durations and the coexistence of creations of temporary and permanent jobs. This approach sheds light on the impact of temporary contracts from a different perspective from that which considers temporary contracts as a way of screening workers before they are promoted into permanent jobs.⁶ Actually, in all countries, permanent contracts comprise probationary periods, with no firing cost and very short notice, which are used to screen workers into permanent jobs. The maximum mandatory duration of probationary periods is around several months, depending on countries, industries and skills.⁷ To the extent that temporary jobs cannot be terminated before their expiration date, it can be profitable to screen workers by means of temporary contracts only if the duration of the probationary period is too short, at least shorter than that of temporary contracts.⁸ Accordingly, the view which considers that temporary contracts are used to screen workers can be useful to explain the spread of temporary jobs with long spell, longer than that of probationary periods. But this approach cannot explain the huge amount of creation of temporary contracts of very short spell, much shorter than that of probationary periods.⁹ For instance, in France, the average duration of temporary jobs is about one month

⁵See among others Autor (2003), Kahn (2010), Centeno and Novo (2011), Cappellari et al. (2011).

⁶See Bucher (2010), Faccini (2008), Kahn (2010), Nagypal (2002), Portugal and Varejão (2009).

⁷See: http://www.ilo.org/dyn/eplex/termmain.home?p_lang=uk.

⁸In general, the probationary period of temporary jobs is much shorter than that of permanent jobs. Furthermore, when a temporary job is transformed into a permanent job, the duration of the temporary job has to be deduced from the duration of the probationary period of the permanent job.

⁹To the extent that workers can be dismissed at zero cost during probationary periods, at first sight it

and a half, while the probationary periods last at least two months and can go to eight months.¹⁰

Second, we complement the literature on the impact of employment protection legislation by explaining the choice between permanent and temporary jobs.¹¹ Most of this literature does not explain this choice.¹² Usually, in this literature, temporary jobs, which can be destroyed at zero cost, are preferred to permanent jobs, which are costly to destroy, and it is either assumed that all new jobs are temporary, or that the regulation forces firms to create permanent jobs. As far as we know, four papers explain the choice between temporary and permanent jobs in a dynamic setting.¹³ Berton and Garibaldi (2006) propose a matching model with directed search and exogenous wages in which firms are willing to open permanent jobs inasmuch as their job filling rate is faster than that of temporary jobs. The model features a sorting of firms and workers into permanent and temporary jobs. This model, which provides an endogenous explanation for the coexistence of permanent and temporary contracts, predicts that temporary workers have shorter unemployment durations than permanent workers, which appears to be true in empirical analysis. Caggese and Cunat (2008) consider the optimal dynamic employment policy of a firm that faces capital market imperfections and can hire two types of labor: one that is totally flexible (fixed-term contracts) and one that is subject to firing costs (permanent contracts). They assume that both are perfect substitutes but permanent employment is relatively more productive. This implies that a firm without financing constraints would hire permanent workers up to the point where expected firing costs are equal to the productivity gain with respect to temporary workers. Cao, Shao and Silos (2010) provide a matching model where firms find it optimal to offer high-quality matches a permanent contract because temporary workers search on the job while permanent workers do not. Finally,

is more profitable to exploit job opportunities expected not to last long with permanent contracts that are terminated at no cost during the probationary periods, rather than with temporary contracts that cannot be terminated before their date of termination even if the job becomes non profitable. However this type of behavior is unlawful. An employer who systematically hires workers with permanent contracts and dismisses them during the probationary period instead of using temporary contract takes the risk of being sentenced. Our paper does not account for probationary periods which are left for future research. We merely assume that permanent workers are protected by firing costs from the start of their contract.

¹⁰In France, the legal maximum duration of the probationary period for permanent contract goes from 2 months for blue collar workers to 4 months for white collar workers. The probationary period can be renewed once if this is stipulated in the labor contract.

¹¹See among others, Lazear (1990), Bentolila and Saint-Paul (1992), Saint-Paul (1996), Ljungqvist (2002), l'Haridon and Malherbet (2009).

¹²See, among others: Blanchard and Landier (2002), Cahuc and Postel-Vinay (2002), Boeri and Garibaldi (2007), Sala et al. (2011), Costain et al. (2010), Bentolila et al. (2010), Saint-Paul (1996).

¹³Kahn (2010) provides a static two period model where temporary jobs are used to screen workers.

Alonso-Borrego, Galdon-Sanchez and Fernandez-Villaverde (2011) assume that permanent and temporary jobs have different firing costs and hiring costs. In these papers, the duration of temporary jobs is exogenous and it is assumed that firms can dismiss workers before the date of termination of temporary contracts. We use an alternative approach, consistent with actual employment protection legislations of Continental European countries, where the duration of temporary jobs is chosen by employers and workers and where workers cannot be dismissed before the date of termination of temporary contracts or where the rule for dismissals is the same for temporary and permanent contracts .

Third, some papers explain why short-term contracts and long-term contracts may coexist in the absence of employment protection legislation. This issue is particularly relevant to understanding the emergence of temporary contracts in labor markets where there is little difference between the termination costs of temporary and permanent contracts, as in some Anglo-Saxon countries. Smith (2007) has provided a stock-flow matching model where it can be optimal to hire low profitable workers on a temporary basis in order to try to hire more profitable workers when the stock of job seekers has been sufficiently renewed. This model offers an underlying rationale for why some employment is limited in duration. It also explains the duration of temporary contracts. In our approach, which is complementary, the utilization of temporary contracts does not hinge on a stock-flow matching model but on the heterogeneity of expected production opportunity durations in an environment where there is a legal menu of contracts. Moreover, contrary to Smith, we assume a labor market with free entry. Macho-Stadler, Pérez-Castrillo and Porteiro (2011) provide an alternative explanation where long-term contracts allow the better provision of incentives because firms can credibly transfer payments from early to late periods in the life of the workers, and this transfer alleviates the incentive compatibility constraint. In this setup, short-term contracts can emerge in equilibrium because they allow the market to ensure a better matching between agents' abilities and firms' needs.

Our paper is organized as follows. Some stylized facts are provided in section 2. The search and matching model is presented in section 3. Section 4 presents calibration exercises that enable us to evaluate the impact of the regulation of job protection on labor turnover, employment and aggregate production. Section 5 introduces some extensions: it shows that 'French type' and 'Spanish type' regulations yield the same outcome. Then, it introduces mandatory limits on the duration of temporary contracts and the possibility to renew temporary contracts. Section

	France	Spain
Number of jobs (stock)	15.9	12.9
Annual entries into temporary jobs	26.7	14.4
Annual entries into permanent jobs	3.2	1.6
Number of entries/Number of jobs	1.88	1.24

Table 1: Number of jobs and number of entries (in millions) into employment according to the type of contract. Private non agricultural sector. Period 2000q1 2010q2 for France and 2005q1 2010q2 for Spain. Source: ACROSS and Spanish State Employment Office.

6 concludes.

2 Stylized facts

This section presents three important properties of entries into employment in France and in Spain.¹⁴ First, most entries are into temporary jobs.¹⁵ In both countries, about 90 percent of entries are into temporary jobs over the period 2000-2010. Second, the average spell of temporary jobs is very short. For instance, in France, the average duration of temporary jobs is about 1.5 months. As a consequence of these two properties, the number of entries into employment is very large in both countries, as shown by table 1. In France, the ratio of annual entries into employment over the stock of jobs is equal to 1.88. In Spain, the ratio is about 1.24. This ratio is smaller than in France. This might be due to the fact that not all entries into employment are reported in Spain.

The third property, illustrated by figures 1 and 2, is that the main part of fluctuations in employment inflows is due to inflows into temporary jobs. In France changes in total employment inflow are mainly driven by temporary jobs, as shown by figure 1. The average gap between the number of entries and its trend is seven times larger for temporary jobs than for permanent jobs. In particular, at the beginning of the recession that started in 2008, there is a strong drop of entries into temporary jobs, much larger than the drop of entries into permanent jobs. Figure 2 shows that employment inflows follow a similar pattern in Spain where the average gap between the number of entries and its trend is eleven times larger for temporary

¹⁴The choice of France and Spain is motivated by the availability of data (ACOSS and DARES for France, Spanish State Employment Office for Spain). As far as we are aware, other continental European countries have only limited information on entries into employment by type of labor contracts.

¹⁵Temporary jobs include all fixed-term jobs, including jobs of temporary work agencies.

jobs than for permanent jobs. The collapse of employment inflow in 2008 comes from the drop of entries into temporary jobs. Over the period accounted for in figure 2, short run fluctuations in employment inflow are mostly driven by temporary jobs.

3 The model

For the sake of clarity, we begin by presenting a simple benchmark model where production opportunities become unproductive at constant Poisson rates. This setup is then extended to include productivity shocks as in the framework of Mortensen and Pissarides (1994) before analyzing the labor market equilibrium.

3.1 The benchmark setup

There is a continuum of infinitely-lived risk-neutral workers and firms, with a common discount rate $r > 0$. Workers are identical and their measure is normalized to 1. Firms are competitive and create jobs to produce a numéraire output, using labor as sole input. All jobs produce the same quantity of output per unit of time, denoted by $y > 0$, but jobs differ by the rate at which they become unproductive, denoted by $\lambda > 0$. The type λ of the job is assumed to be randomly selected in $[\lambda_{\min}, +\infty)$, $\lambda_{\min} > 0$, according to a sampling distribution with cumulative distribution function G and density g . The distribution of λ has positive density over all its support and no mass point. Jobs and workers are brought together pairwise through a sequential, random and time consuming search process. Unemployed workers sample job offers sequentially at a Poisson rate $\alpha > 0$. This rate will be made endogenous later in the paper.

There are two types of contracts: temporary and permanent. Permanent contracts are the ‘regular’ type of contracts. Permanent contracts stipulate a fixed wage that can be renegotiated by mutual agreement only: renegotiations thus occur only if one party can credibly threaten the other to leave the match for good if the latter refuses to renegotiate. Permanent contracts do not stipulate any pre-determined duration. Permanent jobs can be terminated at any time at cost F , paid by the employer. F is a red-tape cost, not a transfer from the firm to the worker (such as severance pay). There is a (small) cost to write a contract, either temporary or permanent, which is denoted by $c > 0$.

Temporary contracts stipulate a wage and a fixed duration. Temporary contracts are neither renegotiable nor renewable. The employer must pay to the worker the wage stipulated in the

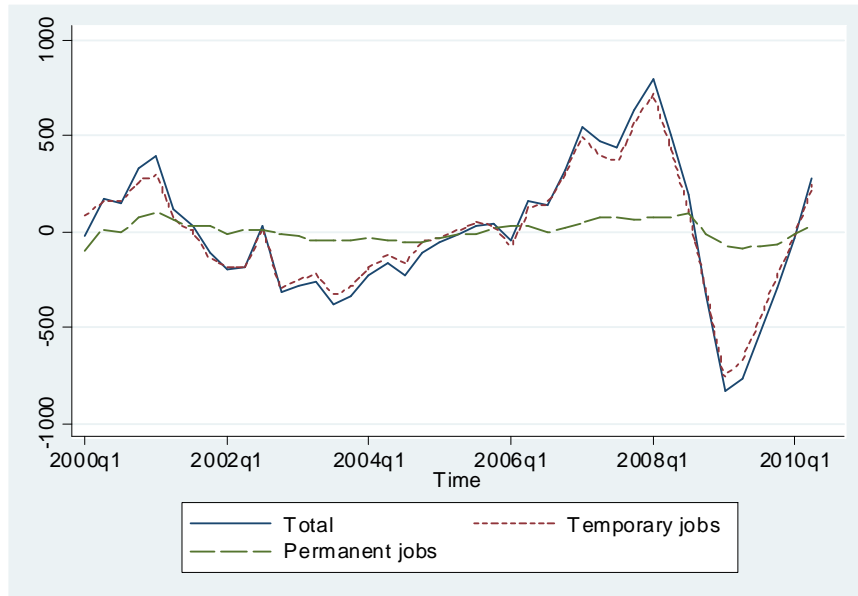


Figure 1: Number of entries into employment per quarter (in thousands) in France in the private non agricultural sector. Deviations with respect to trends (Hodrick and Prescott filter). Source: ACOSS and DARES.

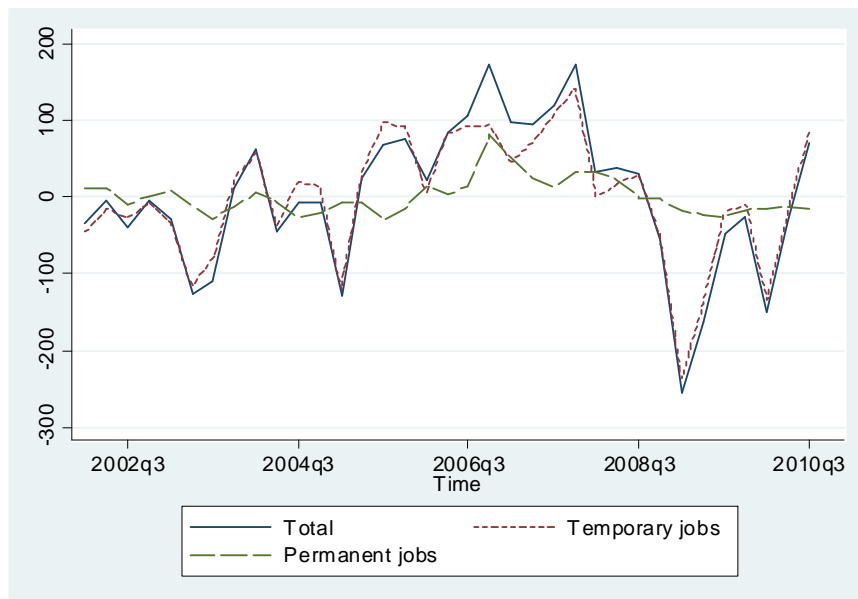


Figure 2: Number of entries into employment per quarter (in thousands) in Spain in the private non agricultural sector. Deviation with respect to trends (Hodrick and Prescott filter). Source: Spanish State Employment Office.

contract until the date of termination even if the job becomes unproductive before this date.¹⁶ At their date of termination, temporary jobs can be either destroyed at zero cost or transformed into permanent jobs. Then, new permanent contracts can be bargained over.¹⁷

When they meet, workers and employers bargain over a contract that maximizes the surplus of the starting job, which can be either temporary or permanent. A temporary contract is chosen if it yields a higher surplus than a permanent contract. If a temporary contract is selected, the wage profile and the duration of the contract are chosen once for all in the starting contract because it is not allowed to renegotiate the contract.

3.1.1 The surplus of permanent and temporary jobs

Let us denote by U the value of unemployment to the worker. It is assumed that a vacant job has zero value to the employer.¹⁸ The surplus of starting permanent jobs with shock arrival rate λ can be written as

$$S_p(\lambda) = (y - rU) \int_0^\infty e^{-(r+\lambda)\tau} d\tau - F \int_0^\infty \lambda e^{-(r+\lambda)\tau} d\tau - c.$$

In this equation, the first term, $(y - rU) \int_0^\infty e^{-(r+\lambda)\tau} d\tau$, stands for the present value of the expected instantaneous surpluses, equal to the difference between y , the production, and rU , the reservation utility. The term in the integral is equal to the discount factor $e^{-r\tau}$ times the survival function $e^{-\lambda\tau}$. The second term, $-F \int_0^\infty \lambda e^{-(r+\lambda)\tau} d\tau$, stands for the present value of the expected firing costs. The last term, c , is the cost to write the contract. The surplus of a starting permanent contract can be rewritten as

$$S_p(\lambda) = \frac{y - rU - \lambda F}{r + \lambda} - c. \quad (1)$$

Similarly, the surplus of starting temporary jobs with shock arrival rate λ and duration Δ can be written as (see appendix B)

$$S_t(\lambda, \Delta) = \int_0^\Delta (ye^{-\lambda\tau} - rU) e^{-r\tau} d\tau + \max[S_p(\lambda), 0] e^{-(r+\lambda)\Delta} - c. \quad (2)$$

¹⁶We consider the ‘French type’ regulation as a benchmark because it is easier to analyze. We show that the Spanish type regulation, where the rule for dismissals is the same for temporary and permanent contracts, yields exactly the same outcome in section 5.1.

¹⁷In many countries, there is a mandatory limit on the duration of temporary contracts and temporary contracts can be renewed several times. These two features of labor contracts will be analyzed in section 5.

¹⁸This condition holds true at market equilibrium as shown below.

The first term, $\int_0^\Delta (ye^{-\lambda\tau} - rU) e^{-r\tau} d\tau$, stands for the present value of the expected instantaneous surpluses over the duration of the job. In this expression, the level of production y is multiplied by the survival function because the production can drop to zero at rate λ . The term rU is not multiplied by the survival function because the employer has to keep and pay the employee until the date of termination of the contract. The second term, $\max[S_p(\lambda), 0] e^{-(r+\lambda)\Delta}$, is an option value associated with the possibility to transform the temporary job into a permanent job at the date of termination of the temporary contract. This option value decreases with the duration of the contract because time is discounted at rate r and because the probability that the job is productive at the date of termination of the contract decreases with the spell of the contract. The last term is the cost of writing the contract.

3.1.2 Optimal duration of temporary jobs

The optimal duration of temporary jobs maximizes the surplus of starting temporary jobs. Therefore, the optimal duration of a temporary job with shock arrival rate λ is defined by the first order condition¹⁹

$$ye^{-\lambda\Delta} - rU - (r + \lambda) e^{-\lambda\Delta} \max[S_p(\lambda), 0] = 0. \quad (3)$$

In this expression, the term $ye^{-\lambda\Delta}$ stands for the marginal gain of an increase in the duration of the job. This gain decreases with the duration of the job because the survival probability of production opportunities decreases with the job spell. It goes to zero when the duration goes to infinite. The marginal cost is equal to the sum of the two other terms. The first term, rU , is the flow of value that the employee can get if the job is terminated. The second term is the option value associated with the possibility to transform the temporary job into a permanent job. The marginal cost decreases with the duration of the job and has a strictly positive lower bound, equal to rU .

The first order condition yields, together with equation (1), the optimal duration as a

¹⁹The second order condition is always fulfilled. When $S_p(\lambda) \leq 0$, the second order condition is $-\lambda ye^{-\lambda} < 0$. When $S_p(\lambda) > 0$, the derivative of the first order condition with respect to Δ is

$$-\lambda ye^{-\lambda} + e^{-\lambda} (r + \lambda) \lambda S_p(\lambda),$$

which is equal to (using the first order condition): $-\lambda rU < 0$

function of λ , denoted by

$$\Delta(\lambda) = \begin{cases} \frac{1}{\lambda} \ln \left(\frac{rU + \lambda F + (r + \lambda)c}{rU} \right) & \text{if } \lambda \leq \lambda_p \\ \frac{1}{\lambda} \ln \left(\frac{y}{rU} \right) & \text{if } \lambda \geq \lambda_p \end{cases} \quad (4)$$

where λ_p is defined by the condition

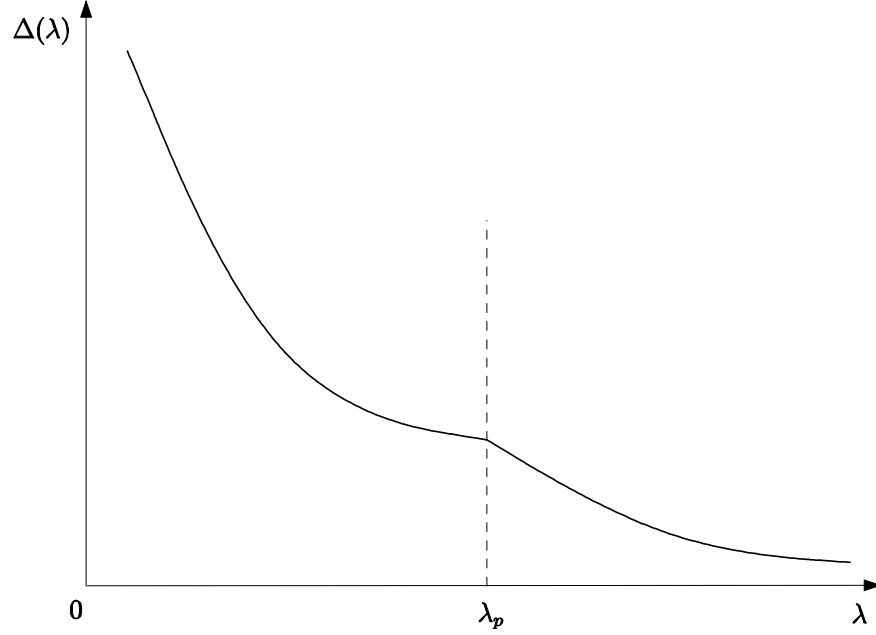


Figure 3: The relation between the shock arrival rate λ and the optimal duration of temporary jobs $\Delta(\lambda)$.

3.1.3 Choice between temporary and permanent contracts

When a job is created, firms and workers choose the type of contract that provides the highest surplus. Figure 4 displays the surplus of permanent jobs and the surplus of temporary jobs for all possible values of the shock arrival rate λ .²²

The surplus of permanent jobs is positive when λ is below the threshold value λ_p . When the shock arrival rate is above λ_p , permanent jobs cannot be created. However, it can be worth creating temporary jobs if the surplus of starting temporary jobs, $S_t(\lambda) = \max_{\Delta} S_t(\lambda, \Delta)$, is positive for some values of $\lambda \geq \lambda_p$, which is equivalent to (as shown in appendix C):

$$y \left[\frac{1 - e^{-(r+\lambda_p)\Delta(\lambda_p)}}{r + \lambda_p} - e^{-\lambda_p\Delta(\lambda_p)} \frac{1 - e^{-r\Delta(\lambda_p)}}{r} \right] > c. \quad (6)$$

Condition (6) can be fulfilled if c is small. λ_p also has to be small, which corresponds to situations where F is large. In other words, this condition means that it can be worth creating temporary jobs when permanent jobs are not profitable if firing costs are high and if the cost to write labor contracts is small. If condition (6) is not fulfilled, there are no temporary

²²See appendix C.

jobs. Since we want to study equilibria with temporary jobs, let us assume for now that this condition holds.²³ It follows that the surplus of temporary jobs is positive on the non empty interval $[\lambda_p, \lambda_t]$ (see appendix C), where λ_t stands for the threshold value of the shock arrival rate above which the surplus of temporary jobs is negative.

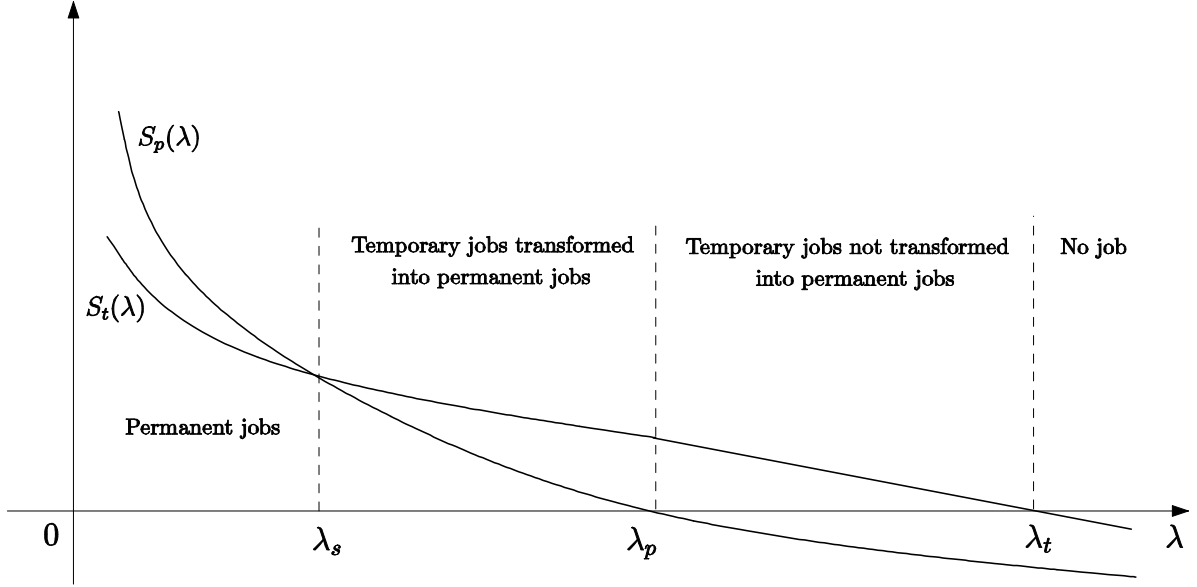


Figure 4: The relation between the shock arrival rate and the type of job creation when there is no mandatory limit on the duration of temporary contracts.

When the shock arrival rate is smaller than λ_p , the surplus of temporary and permanent jobs is positive. Firms and workers choose the type of contract that yields the highest surplus. In that case, if firing costs are not too large, i.e. if $F < U$ (see appendix C), there exists a threshold value of the shock arrival rate, denoted by λ_s , such that it is preferable to create permanent jobs when the shock arrival rate is smaller than λ_s . Otherwise, when firing costs are above U , it is always preferable to create temporary jobs whatever the shock arrival rate. Figure 4 summarizes the relation between the shock arrival rate and the type of job creation in the case where $F < U$.

It is worth stressing that there exists a trade-off between permanent jobs and temporary jobs because there are costs to write contracts and contracts are incomplete. If it is not costly to write (or to renegotiate) contracts, it is always preferable to hire workers on temporary jobs,

²³(6) is always satisfied when $c = 0$.

possibly for very short periods of time, and then to transform temporary jobs into permanent jobs rather than directly hiring workers on permanent jobs.²⁴ We also exclude the possibility to write a single contract that stipulates a contingent transformation of temporary contract into permanent contract at the instant when the worker is hired. It is likely that such contracts are not observed in the real world because they are too costly to verify.

Finally, it is worth noting that our model implies that temporary jobs pay lower wages than permanent jobs even when their productivity is the same. There are two reasons for this property, consistent with empirical evidence.²⁵ First, the duration of temporary jobs is shorter than that of permanent jobs. This induces a lower average surplus for temporary jobs as shown by figure 4. Second, the impossibility to terminate temporary contracts before their date of termination implies that there are situations where employers pay positive wages to unproductive temporary workers. This reduces their entry wage which is not renegotiated.

3.2 Productivity shocks

Let us now provide an extension of the benchmark model where it is assumed that shocks do not strike down productivity to zero once for all, but imply a new value of the productivity drawn in a stationary distribution as in the model of Mortensen and Pissarides (1994). Let us assume now that the production of an employee is a random variable with distribution $H(y)$ which has upper support y_u and no mass point. The productivity of each employee changes at Poisson rate λ . When productivity changes, there is a drawing from the fixed distribution $H(y)$. For the sake of simplicity, it is assumed that the productivity of new matches is equal to the upper support of the distribution, as in Mortensen and Pissarides (1994). In what follows, we show that the model with productivity shocks can be solved in a similar way as the benchmark model.

²⁴Formally, it can be checked that $S_p(\lambda) < S_t(\lambda)$ when $c = 0$. In the simulations, c takes very small values relative to y .

²⁵Empirical evidence shows that temporary workers get lower wages than permanent workers controlling for a large cluster of observable characteristics. For instance, Booth et al. (2002) find that temporary workers in Britain earn less than permanent workers (men 8.9 percent and women 6 percent). Hagen (2002) finds an even larger gap, about 23 percent in Germany, controlling for selection on unobservable characteristics.

3.2.1 The surplus of permanent and temporary jobs

The surplus of a continuing permanent job with shock arrival rate λ and productivity y , denoted by $S_c(y, \lambda)$ satisfies the Bellman equation

$$rS_c(y, \lambda) = y - r(U - F) + \lambda \left(\int_{-\infty}^{y_u} \max[S_c(x, \lambda), 0] dH(x) - S_c(y, \lambda) \right). \quad (7)$$

Continuing permanent jobs are destroyed when their surplus becomes negative. Since $S_c(y, \lambda)$ increases with y , jobs are destroyed if their productivity drops below the reservation value, denoted by $R(\lambda)$, such that $S_c(R, \lambda) = 0$. This reservation productivity satisfies

$$R(\lambda) = r(U - F) - \lambda \int_{R(\lambda)}^{y_u} \frac{y - R(\lambda)}{r + \lambda} dH(y). \quad (8)$$

This equation implies that $R(\lambda)$ is a decreasing function of λ .

The surplus of starting permanent jobs with shock arrival rate λ and productivity y , denoted by $S_p(y, \lambda)$, is equal to $S_c(y, \lambda) - F - c$. The creation of permanent jobs can proceed from entries of unemployed workers into employment. In that case, permanent jobs can be created only if $S_p(y_u, \lambda) \geq 0$, or in other words, if the shock arrival rate is below the threshold value λ_p such that $S_p(y_u, \lambda_p) = 0$. The creation of permanent jobs can also proceed from the transformation of temporary jobs. In that case, the starting productivity of permanent jobs is not necessarily equal to y_u because temporary jobs are hit by productivity shocks. The reservation productivity above which temporary jobs are transformed into permanent jobs, denoted by $T(\lambda)$, such that $S_p(T, \lambda) = 0$, is

$$T(\lambda) = R(\lambda) + (r + \lambda)(F + c). \quad (9)$$

The surplus of starting temporary jobs with shock arrival rate λ and duration Δ can be written as (see appendix D)

$$\begin{aligned} S_t(\lambda, \Delta) = & \int_0^\Delta \left(e^{-\lambda\tau} y_u + (1 - e^{-\lambda\tau}) \int_{-\infty}^{y_u} y dH(y) - rU \right) e^{-r\tau} d\tau + \\ & e^{-(r+\lambda)\Delta} \max[S_p(y_u, \lambda), 0] + (1 - e^{-\lambda\Delta}) e^{-r\Delta} \int_{-\infty}^{y_u} \max[S_p(y, \lambda), 0] dH(y) - c. \end{aligned} \quad (10)$$

The integral of the first row stands for the present value of the instantaneous surpluses obtained over the duration of the temporary contract. The terms of the second row correspond to the present value of the gains expected at the date of termination of the temporary contract minus the cost to write the contract.

3.2.2 Optimal duration of temporary contracts

Once the value of starting jobs is known, it is possible to determine the optimal duration of temporary jobs and the choice between temporary and permanent contracts.

The optimal duration of temporary jobs is the value of Δ , denoted by $\Delta(\lambda)$, which maximizes $S_t(\lambda, \Delta)$. We get (see appendix E)

$$\Delta(\lambda) = \begin{cases} \frac{1}{\lambda} \ln \left(\frac{y_u - \bar{y} - (r + \lambda)[S_p(y_u, \lambda) - \chi]}{rU - \bar{y} + r\chi} \right) & \text{if } \lambda \leq \lambda_p \\ \frac{1}{\lambda} \ln \left(\frac{y_u - \bar{y}}{rU - \bar{y}} \right) & \text{if } \lambda \geq \lambda_p \end{cases} \quad (11)$$

where $\bar{y} = \int_{-\infty}^{y_u} y dH(y)$, $\chi = \int_{T(\lambda)}^{y_u} S_p(y, \lambda) dH(y)$, and λ_p is defined by the condition $S_p(y_u, \lambda_p) = 0$.

This expression of the optimal duration of temporary contracts looks like that obtained in the benchmark model (see equation (4)). The optimal duration decreases with the shock arrival rate λ and increases with the productivity of starting jobs.

3.2.3 Choice between temporary and permanent contracts

The choice between the creation of temporary and permanent jobs is determined by the comparison of the values of the surplus of starting jobs. As in the benchmark model, there are values of the parameters such that temporary jobs are preferred to permanent jobs if the shock arrival rate is above a threshold denoted by λ_s , which satisfies $S_p(y_u, \lambda_s) = S_t(\lambda_s)$ (see appendix F). Below this threshold, permanent jobs are created. There also exists an upper finite value of the shock arrival rate, λ_t , such that $S_t(\lambda_t) \equiv \max_{\Delta} S_t(\lambda_t, \Delta) = 0$, above which no job is created. Temporary jobs with shock arrival rate λ belonging to the interval (λ_s, λ_t) are transformed into permanent jobs only if their productivity is above the reservation value $T(\lambda)$. Otherwise, they are destroyed.

3.3 Labor market equilibrium

Let us now describe the process of job creation, the matching between workers and jobs and the bargaining between workers and employers in order to determine the labor market equilibrium.

Firms must invest $\kappa > 0$ to find a production opportunity. κ is a sunk cost. As described above, all production opportunities start with the same level of productivity y_u . Then, they are hit by shocks at Poisson rates λ that differ across jobs. Firms draw production opportunities in

the distribution $G(\lambda)$ just after the sunk cost κ has been paid. When a production opportunity is found, a job vacancy can be created. The value of a type- λ vacant job (i.e. with shock arrival rate λ) is denoted by $V(\lambda)$. Free entry implies that the expected value of vacant jobs is equal to the investment cost

$$\kappa = \int \max[V(\lambda), 0] dG(\lambda). \quad (12)$$

Unemployed workers and job vacancies are brought together through a constant returns to scale matching technology which implies that vacant jobs are filled at rate $q(\theta)$, $q'(\theta) < 0$, where $\theta = v/u$ denotes the labor market tightness, equal to the ratio of vacancies, v , over unemployment u . For the sake of simplicity, it is assumed that the instantaneous cost of vacancies equals zero and that firms must re-invest to find new job opportunities when matches are broken. Moreover, bargaining allows workers to get the share $\beta \in (0, 1)$ of the job surplus. Therefore, the value of type- λ vacant jobs satisfies

$$rV(\lambda) = q(\theta) [(1 - \beta)S(\lambda) - V(\lambda)] \quad (13)$$

where $S(\lambda)$ denotes the surplus of type- λ starting filled jobs. Firms create type- λ vacancies only if their expected value is positive. Since it has been shown above that all (temporary and permanent) job surpluses $S(\lambda)$ decrease with λ and become negative when λ goes to infinite, this implies that type- λ vacant jobs are created only if $\lambda < \lambda_{\text{sup}}$ where λ_{sup} equals either λ_t (see figure 4) if the equilibrium comprises temporary and permanent jobs or λ_p if there are permanent jobs only, which occurs when firing costs are sufficiently small.

The matching technology implies that unemployed workers sample job offers at rate $\alpha = \theta q(\theta)$. Thus, denoting by z the instantaneous income of unemployed workers, the value of unemployment satisfies

$$rU = z + \theta q(\theta) \beta \int_{\lambda_{\min}}^{\lambda_{\text{sup}}} \frac{S(\lambda)}{G(\lambda_{\text{sup}})} dG(\lambda).$$

Combining the three previous equations, we get

$$rU = z + \frac{\beta \theta [r + q(\theta)]}{(1 - \beta) G(\lambda_{\text{sup}})} \kappa. \quad (14)$$

This equation shows that increases in labor market tightness, which increase the arrival rate of job offers, improve the expected gains of unemployed workers.

There are two possible types of labor market equilibrium. One where there are only permanent jobs and another where there are permanent and temporary jobs.²⁶

²⁶ An equilibrium with temporary jobs only can exist in our framework if there is a sufficiently low upper

3.3.1 Equilibrium with permanent jobs only

When firing costs are sufficiently small, all jobs are permanent because the surplus of permanent jobs is always larger than that of temporary jobs. It is possible to find a system of two equations that defines the equilibrium value of (θ, λ_p) . From equations (12) and (13), the free entry condition can be written as

$$\kappa = \frac{q(\theta)(1 - \beta)}{r + q(\theta)} \int_{\lambda_{\min}}^{\lambda_p} S_p(y_u, \lambda) dG(\lambda), \quad (15)$$

where $S_p(y_u, \lambda)$ is defined by equation (1) and U , which shows up in the expression of $S_p(y_u, \lambda)$, by equation (14). We get another relation between θ and λ_p using the condition that defines the threshold value of shock arrival rates above which no jobs are created:

$$S_p(y_u, \lambda_p) = 0. \quad (16)$$

Equations (15) and (16) define a unique equilibrium value of (θ, λ_p) provided that the conditions of existence are satisfied, which is assumed.²⁷

3.3.2 Equilibrium with permanent and temporary jobs

When firing costs are sufficiently large, starting jobs can be either temporary, with surplus $S_t(\lambda)$, or permanent, with surplus $S_p(y_u, \lambda)$. The free entry condition becomes

$$\kappa = \frac{q(\theta)(1 - \beta)}{r + q(\theta)} \left[\int_{\lambda_{\min}}^{\lambda_s} S_p(y_u, \lambda) dG(\lambda) + \int_{\lambda_s}^{\lambda_t} S_t(\lambda) dG(\lambda) \right]. \quad (17)$$

This equation defines a relationship between θ and the thresholds. In turn, the conditions

$$S_t(\lambda_t) = 0 \quad (18)$$

$$S_p(y_u, \lambda_s) = S_t(\lambda_s) \quad (19)$$

$$S_p(y_u, \lambda_p) = 0, \quad (20)$$

define the thresholds as a function of θ , once the relation between rU and θ has been taken into account in the expressions of the surpluses S_t and S_p . Then,²⁸ equations (18), (19), and

bound on the expected durations of job opportunities. We rule out this possibility for the sake of realism. We also rule out the trivial equilibrium without entries into employment.

²⁷See appendix G.1.

²⁸See appendix G.2.

(20) together with (17) define a unique equilibrium value of the 4-uple $(\lambda_s, \lambda_p, \lambda_t, \theta)$ provided that it exists, which is supposed.

3.4 Unemployment

Once the equilibrium value of the labor market tightness and of the thresholds λ_s, λ_p and λ_t are known, it is possible to define unemployment, and the mass of temporary and permanent jobs at equilibrium (for the sake of simplicity, we only focus on steady state).

Let us begin to define the steady state unemployment rate in the equilibrium where there are permanent jobs only. The mass of permanent jobs with shock arrival rate λ is denoted by $\ell(\lambda)$. By definition, the unemployment rate is

$$u = 1 - \int_{\lambda_{\min}}^{\lambda_p} \ell(\lambda) d\lambda \quad (21)$$

In steady state, the equality between entries and exits in type- λ jobs is

$$u\alpha_p g(\lambda) = \ell(\lambda)/\phi(\lambda). \quad (22)$$

where $\phi(\lambda) = 1/\lambda H[R(\lambda)]$ is the expected duration of type- λ jobs and $\alpha_p = \alpha/G(\lambda_p) = \theta q(\theta)/G(\lambda_p)$.

Equations (21) and (22) imply

$$u = \frac{1}{1 + \alpha_p \int_{\lambda_{\min}}^{\lambda_p} \phi(\lambda) dG(\lambda)} \quad (23)$$

This equation shows that the unemployment rate decreases with $\theta q(\theta)$, the arrival rate of job offers, and with the duration of jobs.

Let us now analyze the equilibrium with temporary and permanent jobs. $s_t(\lambda)$ denotes the mass of type- λ temporary jobs which are transformed into permanent jobs. $s_n(\lambda)$ denotes the mass of type- λ temporary jobs which are not transformed into permanent jobs and u denotes the unemployment rate. We can write

$$u = 1 - \int_{\lambda_{\min}}^{\lambda_p} \ell(\lambda) d\lambda - \int_{\lambda_s}^{\lambda_p} s_t(\lambda) d\lambda - \int_{\lambda_p}^{\lambda_t} s_n(\lambda) d\lambda \quad (24)$$

There are permanent jobs over the interval $[\lambda_{\min}, \lambda_p]$. The equality between entries into and exits out of permanent jobs with expected duration $\phi(\lambda)$ can be written as

$$\begin{cases} \frac{s_t(\lambda)[1-H(T(\lambda))(1-e^{-\lambda\Delta(\lambda)})]}{\Delta(\lambda)} = \frac{\ell(\lambda)}{\phi(\lambda)} & \text{if } \lambda \in [\lambda_s, \lambda_p] \\ u\alpha_t g(\lambda) = \frac{\ell(\lambda)}{\phi(\lambda)} & \text{if } \lambda \in [\lambda_{\min}, \lambda_s] \end{cases} \quad (25)$$

where $\alpha_t = \alpha/G(\lambda_t) = \theta q(\theta)/G(\lambda_t)$. The first row of equation (25) accounts for the transformations of temporary jobs into permanent jobs. The second row accounts for the entries of unemployed workers into permanent jobs. The equality between entries into and exits out of temporary jobs with expected duration $\Delta(\lambda)$ can be written as

$$u\alpha_t g(\lambda) = \frac{s_t(\lambda)}{\Delta(\lambda)} \text{ if } \lambda \in [\lambda_s, \lambda_p] \quad (26)$$

$$u\alpha_t g(\lambda) = \frac{s_n(\lambda)}{\Delta(\lambda)} \text{ if } \lambda \in [\lambda_p, \lambda_t] \quad (27)$$

Equations (24) to (27) imply:

$$u = \frac{1}{1 + \alpha_t \left[\int_{\lambda_s}^{\lambda_t} \Delta(\lambda) dG(\lambda) + \int_{\lambda_{\min}}^{\lambda_s} \phi(\lambda) dG(\lambda) + \int_{\lambda_s}^{\lambda_p} \phi(\lambda) [1 - H(T(\lambda)) (1 - e^{-\lambda \Delta(\lambda)})] dG(\lambda) \right]}$$

This equation shows that the unemployment rate decreases with the arrival rate of job offers and with the duration of jobs.

4 Simulation exercises

In this section, we calibrate the model to explore its property. In particular, we show that the model is able to reproduce the main properties of entries into employment observed in countries like France and Spain where there is a stringent employment protection legislation and a large share of temporary jobs. The model is first calibrated to match the labor market of the US economy where firing costs are close to zero. Then, firing costs are increased to evaluate their impact on entries into permanent and temporary jobs.

4.1 The benchmark economy without firing costs

The parameters and targets used in the calibration refer to the US economy which represents the benchmark economy without firing costs. Admittedly, this assumption is an approximation, to the extent that we neglect the exceptions to the employment at will doctrine which induce firms to use some temporary contracts (see e.g. Autor, 2003). However, employment protection legislation remains very weak in the US relative to most other OECD countries, and especially to Continental European countries (Venn, 2009).

The values of the parameters are in the range of those chosen in the literature (see e.g. Mortensen and Pissarides, 1999, Shimer, 2005, and Mortensen and Nagypal, 2007). We define

the time period to be one quarter, and consequently set the discount rate r to 1.23%, which corresponds to a 5 percent annual discount rate. The value of the bargaining power parameter β is set to 0.5 and the income of unemployed workers (the value of leisure), z , is equal to 0.3. As in Mortensen and Pissarides (1994), the distribution of idiosyncratic shocks is assumed to be uniform in the range $[y_{\min}, 1]$. We follow the literature and assume a Cobb-Douglas matching technology of the form $H(v, u) = hu^\eta v^{1-\eta}$, where h is a mismatch parameter and η is the elasticity of the matching function with respect to unemployment. We assume η to be equal to 0.5, which is in the range of the estimates obtained by Petrongolo and Pissarides (2001). The sampling distribution of type- δ jobs, $\delta = 1/\lambda$, is a truncated log normal distribution. The range of expected durations of production opportunities is comprised between one day (1/65 quarter, 65 being the number of days worked per quarter, given that there are 13 weeks per quarter and 5 days of work per week) and 45 years (180 quarters).

Then, assuming that the bottom equilibrium value of $\delta_p = 1/\lambda_p$, is equal to that of the truncated distribution of expected durations (one day) and as in Shimer (2005) that the average v-u ratio is equal to one, we are left with 6 unknown parameter values: the parameter of the cdf of the productivity distribution, y_{\min} , the two parameters of the cdf of the sampling distribution of durations of production opportunities, c , the cost of writing contracts, h , the mismatch parameter and κ , the investment cost. We determine the values of these parameters with six equations assuming that $F = 0$. First, equations (15) and (16) pin down the values of θ and δ_p . Second, two equations define the median and the mean value of the expected durations of production opportunity. The median and the mean durations, equal to 4 years (16 quarters) and 6.67 years (26.678 quarters) respectively, are obtained from the CPS, Displaced Workers, Employee Tenure, and Occupational Mobility Supplement, for the private sector in 2008. Third, one equation targets an average quarterly job finding rate of 1.35 (see eg: Shimer, 2005 or Nagypal and Mortensen, 2007). Finally, equation (23) is used to match the unemployment rate, equal to 6 percent.

Accordingly, the minimum match product is $y_{\min} = 0.017$, the values of the shape and of the log-scale parameters of the sampling distribution are equal to $\sigma = 0.9360$ and $\mu = 1.9032$ respectively. The cost of writing contracts is very small, $c = 0.0017$, which is roughly equal to 0.1756 percent of the average quarterly production of a job. The mismatch parameter is set to $h = 1.35$ and the cost to find production opportunities is $\kappa = 0.4389$. Baseline and calibrated

parameters are summarized in Table 2.

Table 2: Benchmark parameters values

Baseline parameters			Calibrated parameters		
Parameter	Notation	Value	Parameter	Notation	Value
Bargaining power	β	0.5	Cost of a contract	c	0.0017
Matching elasticity	η	0.5	Mismatch parameter	h	1.35
Discount rate	r	1.23%	$\log N$ - shape parameter	σ	0.9360
Value of leisure	z	0.3	$\log N$ - scale parameter	μ	1.9032
Maximum match product	y_{\max}	1	Minimum match product	y_{\min}	0.017
			Investment cost	κ	0.4389

4.2 The economy with firing costs and temporary contracts

Let us now look at the consequence of firing costs.²⁹ This exercise allows us to illustrate the mechanism of the model and to see whether it can potentially reproduce the three stylized facts presented above in section 2.³⁰

The first fact is that the share of entries into temporary jobs strongly increases with job protection. Figure 5 shows that the model predicts that firing costs do have a strong impact on the share of entries into temporary jobs. Firms begin to use temporary contracts when firing costs reach about one percent of the average quarterly production of an employee. Then, when firing costs increase, the share of entries into temporary jobs rises steadily. It amounts to 90 percent of all entries into employment when firing costs equal about 20 percent of the average quarterly production of a job, which is a reasonable order of magnitude given the available estimates.³¹ All in all, the model allows us to explain the large share of entries into temporary jobs observed in Continental European countries.

²⁹We focus on steady states only.

³⁰Obviously, this exercise is illustrative. It is not meant to reproduce the labor market of a specific country, but more generally to illustrate the consequences of the introduction of firing costs in a labor market with frictions and flexible wages. Dealing with a specific country, with strong job protection, would require accounting for the influence of minimum wage and/or collective bargaining that play an important role in countries with strong job protection. This issue, which is beyond the scope of this paper, is left for future research.

³¹Kramarz and Michaud (2010) estimate that the termination of the contract of a marginal permanent job amounts to 16 percent of the annual wage for an individual layoff and to 50 percent of the annual wage for a collective layoff in France. Since about three over four layoffs are individual layoffs, the average cost is 25 percent of the annual wage, which corresponds to 2/3 of the quarterly production if the share of wages in production is 2/3. This implies that red-tape costs amount to about 1/3 of the total layoff costs if layoff costs equal 20 percent of the average quarterly production of a job.

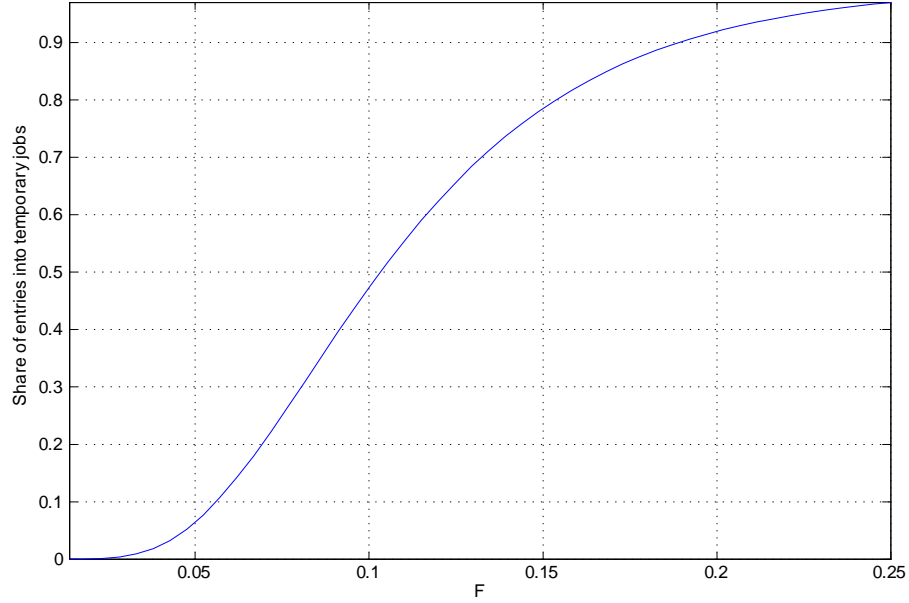


Figure 5: The relation between firing costs (in shares of quarterly production of an employee) and the share of entries into temporary jobs in total employment inflows.

The predictions of the model are also in line with the second stylized fact according to which the average duration of temporary jobs is very low, about 1.5 months in France. Indeed, the model predicts that the average duration of temporary jobs is 0.45 quarter when 90 percent of entries are into temporary jobs.

Figure 6 shows that the model fits the third stylized fact, according to which changes in entries into temporary jobs account for the main share of changes in the total number of entries into employment. This figure represents the relation between changes in the cost of finding production opportunities (parameter κ) and changes in the number of entries into temporary and permanent jobs. Increases in the cost of finding job opportunities induce much larger drops in entries into temporary jobs than into permanent jobs. The drop in entries into temporary jobs is about nine times larger than the drop of entries into permanent jobs. This order of magnitude is in line with the facts observed in France and Spain over the period 2000-2010, as stressed in section 2.

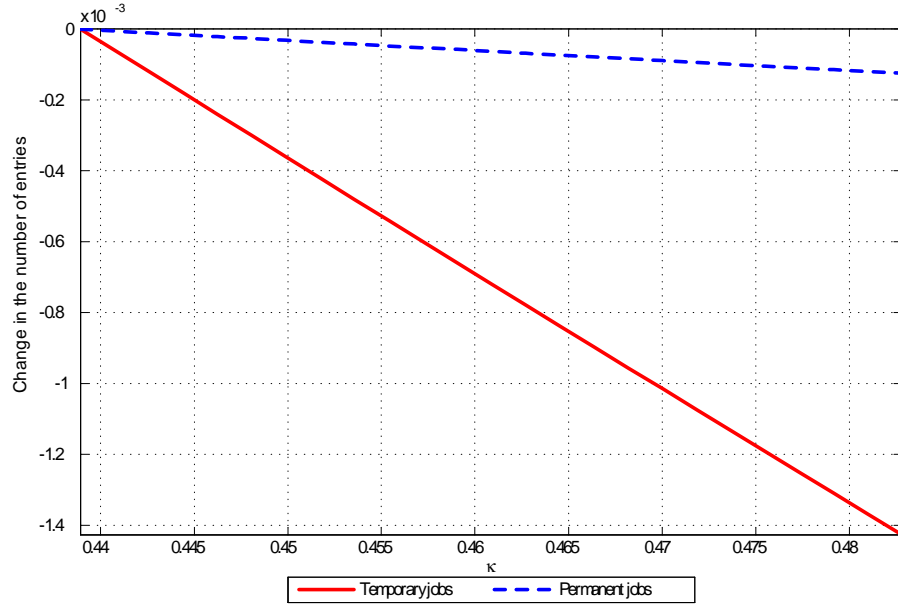


Figure 6: Changes in the number of entries into temporary and permanent employment induced by changes in κ , the cost of job creation.

4.3 Job protection and the excess of job turnover

Our model is particularly useful to evaluate the impact of job protection on job turnover, employment and production.

4.3.1 Job turnover

Our model predicts, in line with empirical evidence, that the average duration of temporary jobs is short, about 1.5 months, when the share of temporary jobs in entries corresponds to that observed in France or Spain. It is interesting to compare this duration with that of jobs that would be used to exploit the same production opportunities (on the same range of type- λ jobs) in the absence of job protection, where all jobs are permanent according to our model. In the absence of job protection, the average duration of these jobs is much longer, equal to 35 months. This result is illustrated on figure 7, which shows that the duration of temporary jobs is much shorter, than that of permanent jobs that would be in place to exploit the same production opportunities in the absence of firing costs. For instance, when the expected duration of permanent jobs in the absence of firing costs equals one quarter, the duration of temporary jobs is about three weeks. The difference is indeed quite large, and it increases with the expected

spell of production opportunities. The reason is that firms want to avoid situations where they have to pay unproductive workers. This implies that firing costs can induce, through their impact on the creation of temporary jobs, an important excess of job turnover which can induce large production losses. Obviously, this result hinges on the assumption that the shock arrival rate follows a Poisson process with constant instantaneous probability. Empirical estimates usually find non monotonous separation rates that begin to increase with tenure and then decrease toward a level that is lower than that observed at the beginning of the employment spell (see e.g. Booth et al., 1999). The relative high level of separation rates at the beginning of employment spells suggests that the shock arrival rate is higher at the beginning of job spells. This feature should induce employers to shorten the duration of temporary contracts with respect to a situation where the shock arrival rate is constant. Accordingly, it is likely that the assumption of constant shock arrival rate leads to underestimation of the discrepancy between the duration of temporary jobs and that of production opportunities. Figure 7 also shows that the durations of temporary jobs and permanent jobs react in opposite directions when firing costs increase: when there are higher firing costs, the average expected duration of new temporary jobs is less than the average expected duration of jobs that would have been created to exploit the same production opportunities in the absence of job protection. In other words, firing costs have opposite effects on the duration of jobs created to exploit production opportunities with short duration and on the duration of jobs created to exploit production opportunities with long expected duration. As shown by Figure 8, which represents the density of jobs durations, higher firing costs increase the dispersion of job durations. When firing costs are higher, there are more jobs with long durations. But there are also more jobs with short durations, because there are more temporary jobs.

It turns out that these two counteracting effects have a total positive impact on the average job duration in our model. The average job duration can be computed in two different ways. We can compute either the average duration of the stock of existing jobs or the average expected duration of new jobs created at any date. As shown by Figure 9, increases in firing costs raise the average duration of the stock of jobs (left hand side panel) and of new jobs (right hand side panel). The effects are nevertheless small: increasing dismissal costs from the level observed in the US (equal to zero in the calibration) to that observed in a Continental European country like France or Spain (equal to about 20 percent of the average quarterly production of jobs) ,

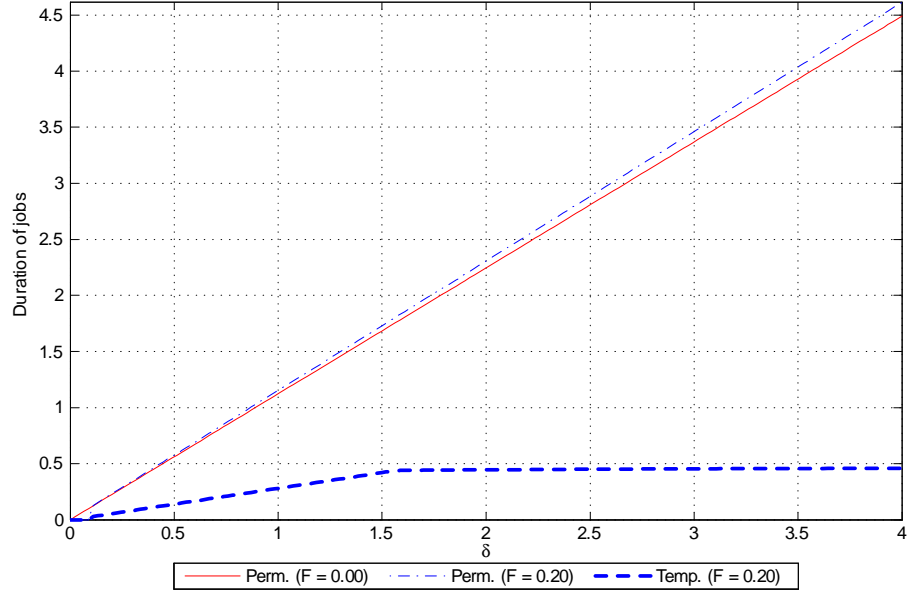


Figure 7: The relation between the expected duration of production opportunities (δ) and the duration of temporary jobs for different values of firing costs (in share of quarterly production of an employee).

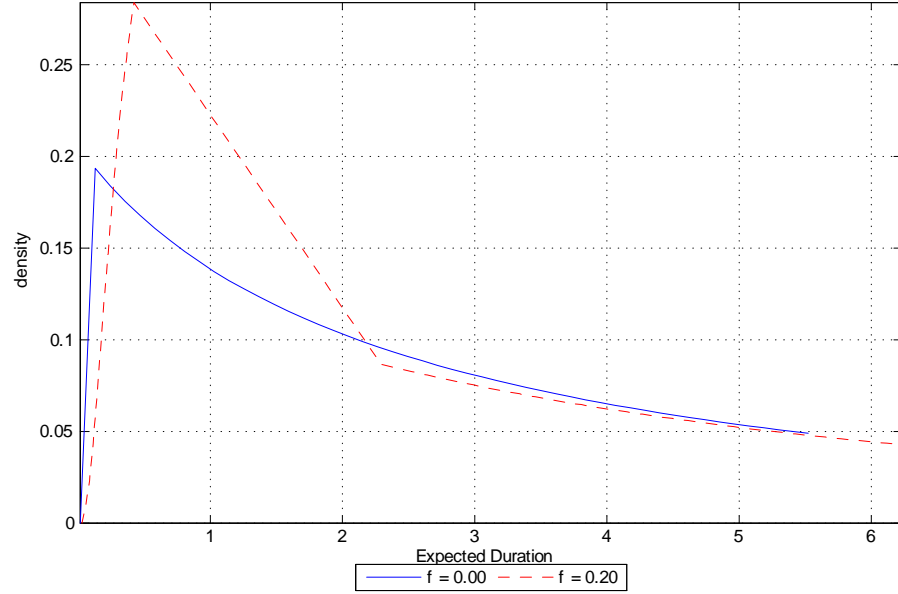


Figure 8: The density of expected job durations (in quarters) of new jobs for different values of firing costs (in share of quarterly production of an employee).

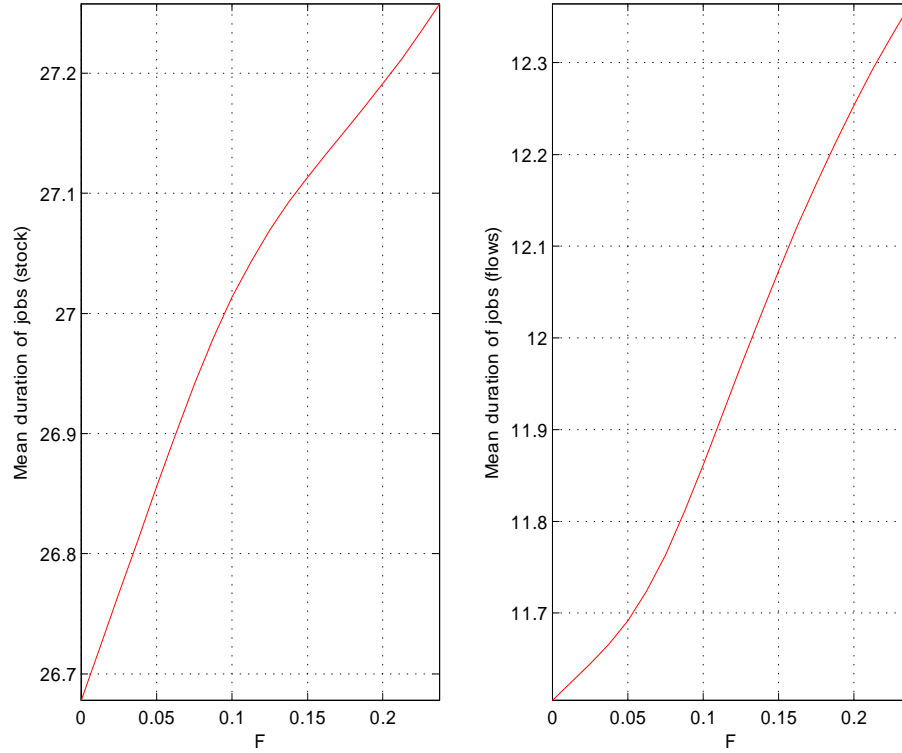


Figure 9: Mean job duration (in quarters) and firing costs (in share of quarterly production of an employee). Left hand side panel: Mean duration of the stock of jobs. Right hand side panel: Mean expected duration of new jobs.

raises the average duration of the stock of jobs by 1.5 percent and the average expected duration of the new jobs by 2 percent. This small impact is the result of the two counteracting effects of firing costs on job durations.

4.3.2 Employment and production

According to our simulation exercises, aggregate production is 2 percent lower in the economy with firing costs equal to 20 percent of the average quarterly production of jobs than in the economy without job protection. Employment is 0.06 percent lower. This shows that changes in production are much larger than changes in employment. Table 3, which displays the impact of an increase in firing costs from 20 percent to 21 percent of the quarterly average production of jobs, sheds more light on this issue. The three bottom rows show that job protection induces a strong decrease in the number of permanent jobs which is almost compensated by the increase in the number of temporary jobs, so that the impact of job protection on total employment is

very small, equal to 0.005 percent. The variation in total employment is very small compared to that of permanent jobs, meaning that job protection entails a strong reallocation of jobs and negligible effects on total employment. This reallocation has important consequences on production (defined as the market production net of hiring costs). Rows 2 and 3 of table 3 show that job protection decreases the production of permanent jobs and raises the production of temporary jobs. As for employment, these two counteracting effects entail a small negative effect on total production, equal to 0.1 percent. However, the relative drop in production is twenty times larger than the relative drop in employment. This large difference is the consequence of the increase in the share of unstable jobs which raises labor turnover costs. In other words, the detrimental effects of job protection are mainly due to its impact on the reallocation between permanent and temporary jobs.

Variation in aggregate production	ΔY	-0.0832
Variation in temp. jobs production	ΔY_s	0.1491
Variation in perm. jobs production	ΔY_p	-0.2323
Variation in the number of jobs	$\Delta(1 - u)$	-0.0046
Variation in the number of temp. jobs	Δs	0.2103
Variation in the number of perm. jobs	Δp	-0.2149

Table 3: Decomposition of the impact of an increase in F from 0.20 to 0.21 on production and employment. At $F=0.20$, employment is equal to 93.95 and production to 88.16.

5 Extensions

Our benchmark model can easily be extended to account for the main characteristics governing the regulation of employment contracts in Europe. In this spirit, we first show that the ‘Spanish type’ regulation, which allows to terminate temporary contracts before their expiration date at the same cost as for permanent contracts yields the same outcome as the ‘French type’ regulation. Then, we introduce a mandatory limit on the duration of temporary contracts. Finally, we allow for the renewal of temporary contracts. For the sake of simplicity, these extensions are introduced in the benchmark setup where jobs are hit by shocks that strike down productivity to zero once for all.

5.1 ‘Spanish type’ versus ‘French type’ regulations

In Spain and Portugal, the red-tape destruction cost of temporary jobs before their expiration date is the same as that of permanent jobs. Therefore, in the ‘Spanish type’ regulation, when the productivity of a temporary job drops to zero, it is possible either to pay the layoff cost F or to continue to pay the worker until the expiration date of the contract. In this situation, it is shown in appendix H that the initial surplus of a temporary job with arrival shock λ and duration Δ can be written as

$$S_t(\lambda, \Delta) = \begin{cases} y \frac{1 - e^{-(r+\lambda)\Delta}}{r+\lambda} + \max[S_p(\lambda), 0] e^{-(r+\lambda)\Delta} - U[1 - e^{-r\Delta}] - c & \text{if } \Delta \leq \tilde{\Delta} \\ [S_p(\lambda) + c] (1 - e^{-(r+\lambda)\Delta}) + e^{-(r+\lambda)\Delta} A - c & \text{if } \Delta \geq \tilde{\Delta} \end{cases} \quad (28)$$

where $\tilde{\Delta} = -\frac{1}{r} \log(1 - \frac{F}{U})$ and

$$A = \max[S_p(\lambda), 0] + \lambda(U - F) \frac{1 - e^{(r+\lambda)\tilde{\Delta}}}{r + \lambda} + U(e^{\lambda\tilde{\Delta}} - 1)$$

is a term that does not depend on Δ . Equation (28) shows that the expression of the surplus is identical to that obtained in the ‘French type’ regulation (defined by equation (2)) when the length of temporary jobs is shorter than $\tilde{\Delta}$ because it is always preferable to pay the worker until the expiration date of the contract in that case. When the contract length is longer than $\tilde{\Delta}$, it can become less expensive to pay the dismissal cost rather than waiting the expiration date of the contract.

Let us now compute the optimal length of temporary jobs. When $\Delta \geq \tilde{\Delta}$,

$$\frac{\partial S_t(\lambda, \Delta)}{\partial \Delta} = (r + \lambda) e^{-(r+\lambda)\Delta} [S_p(\lambda) + c - A].$$

Let us remark that the expression of the surplus (28) implies that

$$S_t(\lambda, \Delta) - S_p(\lambda) = e^{-(r+\lambda)\Delta} [A - S_p(\lambda) - c].$$

The two last equations imply that for all values of $\Delta > \tilde{\Delta}$, $\frac{\partial S_t(\lambda, \Delta)}{\partial \Delta} < 0$ when $S_t(\lambda, \Delta) > S_p(\lambda)$, i.e. the surplus of temporary jobs always decreases with Δ for values of λ for which temporary jobs are preferred to permanent jobs, which is the only relevant situation for the analysis of the optimal length of temporary jobs. This implies that the optimal value of Δ cannot be larger than $\tilde{\Delta}$.

So, let us turn to the only relevant case where $\Delta \leq \tilde{\Delta}$. The expression of the surplus is the same as in the ‘French type’ regulation. Thus, the optimal length is equal to the minimum value

of $\tilde{\Delta}$ and $\Delta(\lambda)$, defined by equation (4), which is the same as in the benchmark model, where temporary jobs can be destroyed at their expiration date only. It appears that the inequality $\Delta(\lambda) \leq \tilde{\Delta}$ is always satisfied.³² Therefore, the solution is $\Delta(\lambda)$ defined by equation (4).

As a consequence, the optimal length of temporary contracts is necessarily sufficiently short to ensure that it is always preferable to keep workers until the expiration date of the contract rather than paying the layoff costs. All in all, this result means that the possibility to destroy temporary jobs at the same red-tape cost as permanent jobs yields the same outcome as the situation where firms have to pay workers until the expiration date of the contract. Hence, ‘French type’ and ‘Spanish type’ regulations yield the same outcome. The reason is that the optimal length of temporary contracts is short enough, provided that temporary contracts are preferred to permanent contracts, to ensure that it is always preferable to pay workers until the expiration date of the temporary contract rather than paying layoff costs.

5.2 The mandatory limit on the duration of temporary contracts

Let us now consider the case where the duration of temporary contracts is upward bounded by the mandatory limit $\bar{\Delta}$. The duration of temporary contracts with shock arrival rate λ is equal to

$$\Delta^*(\lambda) = \min [\bar{\Delta}, \Delta(\lambda)] . \quad (29)$$

Since $\Delta(\lambda)$, defined in equation (4), is decreasing, there exists a threshold value of the shock arrival rate, denoted by $\bar{\lambda}$, such that the duration of temporary jobs is equal to the upper bound $\bar{\Delta}$ when the shock arrival rate is below $\bar{\lambda}$. Figure 10 displays the values of the surplus of temporary and permanent jobs in the case where temporary and permanent jobs are created, and where the upper limit on the duration of temporary jobs is binding for some jobs.

First, it turns out that the mandatory limit on the duration of temporary jobs, denoted by $\bar{\Delta}$, reduces the surplus of temporary jobs for which this constraint is binding, over the interval $[\lambda_s, \bar{\lambda}]$, i.e. $S_t(\lambda, \bar{\Delta}) \leq S_t(\lambda)$. Therefore, the condition (6) of existence of creation of temporary jobs is necessary, but not sufficient any more. To allow for the creation of temporary jobs, it

³²Let us show that $\Delta(\lambda) \leq \tilde{\Delta}$ when $S_t(\lambda) = S_t(\lambda, \Delta(\lambda)) > S_p(\lambda)$. Assume that $\Delta(\lambda) > \tilde{\Delta}$ provided that $S_t(\lambda) > S_p(\lambda)$. This implies that there exist values of $\Delta > \tilde{\Delta}$ such that the value of the surplus with the ‘French type’ regulation, defined equation (2), is strictly larger than that defined by the second row of equation (28), with the ‘Spanish type’ regulation. This is impossible because ‘French type’ regulation cannot yield higher surplus than the ‘Spanish type’ regulation to the extent that the only difference between the two regulations is that the ‘Spanish type’ gives the opportunity to terminate temporary jobs at lower cost.

has to be assumed that $\bar{\Delta}$, the maximum duration of temporary jobs, is strictly larger than $\Delta(\lambda_t)$. If this assumption were not satisfied, it would never be worth creating temporary jobs. When conditions (6) and $\Delta(\lambda_t) < \bar{\Delta}$ are satisfied, temporary jobs are created over the non empty interval $[\lambda_p, \lambda_t]$ (see appendix I.1). Let us assume for now that these two conditions are fulfilled.

When the shock arrival rate is smaller than λ_p , the surplus of temporary and permanent jobs is positive. Like in the previous case, where there is no limit on the duration of temporary jobs, there is a threshold value of the shock arrival rate below which it is preferable to create permanent jobs (see appendix I.2). Figure 10 summarizes the relation between the shock arrival rate and the type of job creation when the mandatory limit on the duration of temporary contract is binding for some jobs, i.e. when $\bar{\lambda}$, such that $\Delta(\bar{\lambda}) = \bar{\Delta}$, is larger than the threshold value λ_s below which permanent jobs are created.³³

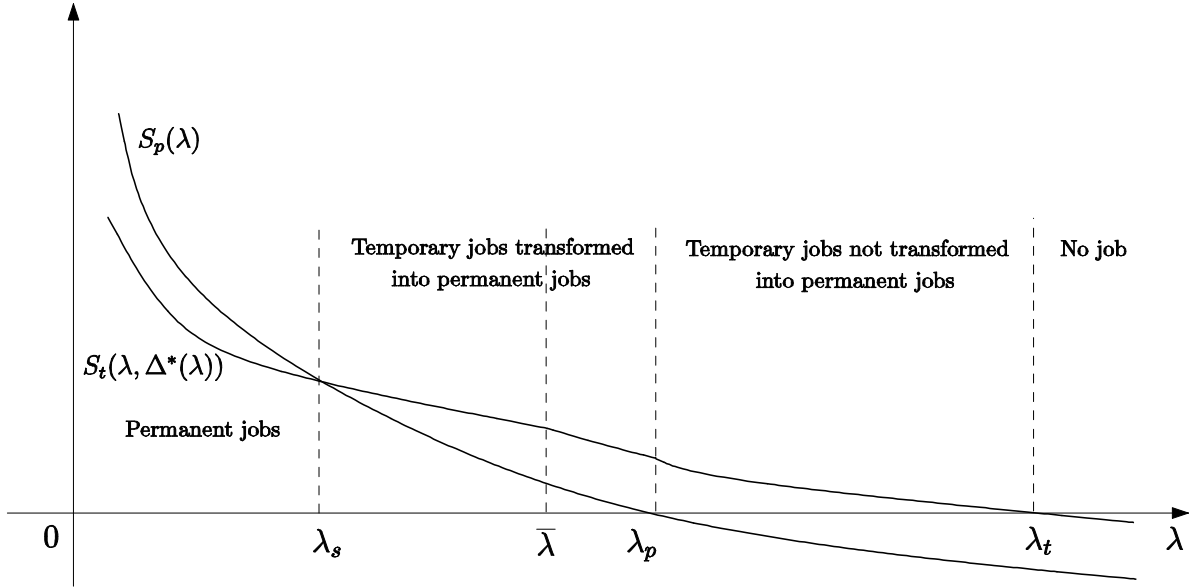


Figure 10: The relation between the shock arrival rate and the type of job creation when there is a mandatory limit on the duration of temporary contracts.

In partial equilibrium, drops in the mandatory maximal duration of temporary jobs do not change the total number of entries into employment but increase the share of entries into

³³In what follows, it is also assumed that $\bar{\lambda} < \lambda_p$, but the relation between the arrival rate of shocks and the type of job creation remains the same when $\bar{\lambda} \geq \lambda_p$ (assuming that $\bar{\lambda} < \lambda_t$).

permanent jobs because λ_t and λ_p do not change, but λ_s increases. The labor market equilibrium in the presence of a mandatory limit on the duration of temporary contract can be computed as in the case where there is no limit, except that $\Delta^*(\lambda)$ is substituted for $\Delta(\lambda)$ and $S_t(\lambda, \Delta^*(\lambda))$ is substituted for $S_t(\lambda)$. Obviously, in labor market equilibrium, the mandatory maximal duration of temporary jobs reduces expected profits and then decreases job creation.

5.3 Renewal of temporary contracts

Let us now assume that temporary contracts can be renewed. For the sake of simplicity, we only allow for an unique renewal. Temporary contracts are now indexed by the subscripts 1 and 2. Let Δ_1 be the duration of the first temporary contract and Δ_2 the duration of the second temporary contract. The surpluses S_{t_1} and S_{t_2} of, respectively, starting and renewed temporary jobs can be written as

$$S_{t_1}(\lambda, \Delta_1) = \int_0^{\Delta_1} (ye^{-\lambda\tau} - rU) e^{-r\tau} d\tau + \max[S_p(\lambda), S_{t_2}(\lambda, \Delta_2), 0] e^{-(r+\lambda)\Delta_1} - c \quad (30)$$

$$S_{t_2}(\lambda, \Delta_2) = \int_0^{\Delta_2} (ye^{-\lambda\tau} - rU) e^{-r\tau} d\tau + \max[S_p(\lambda), 0] e^{-(r+\lambda)\Delta_2} - c \quad (31)$$

Equation (31) is similar in all dimensions to (2). Likewise, the interpretation of (30) is almost similar to that of the benchmark model except for the term $\max[S_p(\lambda), S_{t_2}(\lambda, \Delta_2), 0] e^{-(r+\lambda)\Delta_1}$, which accounts for the possibility to transform, renew or terminate a temporary job at the end of the first contract, Δ_1 .

The model can easily be solved backward. Given the similarity between S_{t_2} and S_t , it turns out that the optimal duration $\Delta_2(\lambda)$ of the second temporary contract is similar to that of $\Delta(\lambda)$ in the benchmark case, and has the same comparative static properties.³⁴

Then, the optimal duration of the first temporary contract Δ_1 maximizes (30). The first

³⁴Indeed we get the following first order condition (see appendix J):

$$ye^{-\lambda\Delta_2} - rU - (r + \lambda) e^{-\lambda\Delta_2} \max[S_p(\lambda), 0] = 0$$

The second order condition is always met, as in the benchmark case. This leads to:

$$\Delta_2(\lambda) = \begin{cases} \frac{1}{\lambda} \ln \left(\frac{rU + \lambda F + (r+\lambda)c}{rU} \right) & \text{if } \lambda \leq \lambda_p \\ \frac{1}{\lambda} \ln \left(\frac{y}{rU} \right) & \text{if } \lambda \geq \lambda_p \end{cases}$$

where the threshold λ_p such that $S_p(\lambda_p) = 0$ is not altered by the possibility to renew temporary contracts.

order condition is given by

$$ye^{-\lambda\Delta_1} - rU - (r + \lambda) e^{-\lambda\Delta_1} \max[S_p(\lambda), S_{t_2}(\lambda, \Delta_2), 0] = 0$$

Let λ_{t_1} and λ_{s_1} be respectively the threshold values above which the surplus of a temporary job is negative and above which the surplus of a temporary job is greater than that of a permanent job. The optimal duration $\Delta_1(\lambda)$ can be written³⁵

$$\Delta_1(\lambda) = \begin{cases} \frac{1}{\lambda} \ln \left(\frac{rU + \lambda F + (r + \lambda)c}{rU} \right) & \text{if } \lambda \leq \lambda_{s_2} \\ \frac{1}{\lambda} \ln \left(\frac{(r + \lambda)[U(1 - e^{-r\Delta_2}) + c] + e^{-(r + \lambda)\Delta_2}[rU + \lambda F + (r + \lambda)c]}{rU} \right) & \text{if } \lambda_{s_2} \leq \lambda \leq \lambda_p \\ \frac{1}{\lambda} \ln \left(\frac{ye^{-(r + \lambda)\Delta_2} + (r + \lambda)U(1 - e^{-r\Delta_2}) + (r + \lambda)c}{rU} \right) & \text{if } \lambda_p \leq \lambda \leq \lambda_{t_2} \\ \frac{1}{\lambda} \ln \left(\frac{y}{rU} \right) & \text{if } \lambda \geq \lambda_{t_2} \end{cases} \quad (32)$$

where the thresholds λ_{s_2} and λ_{t_2} , such that, respectively, $S_p(\lambda_{s_2}) = S_{t_2}(\lambda_{s_2})$, and $S_{t_2}(\lambda_{t_2}) = 0$, are similar to λ_s and λ_t obtained in the benchmark case, and share the same comparative static properties.



Figure 11: The relation between the shock arrival rate and the type of job creation when temporary contracts can be renewed.

³⁵See appendix J for more details.

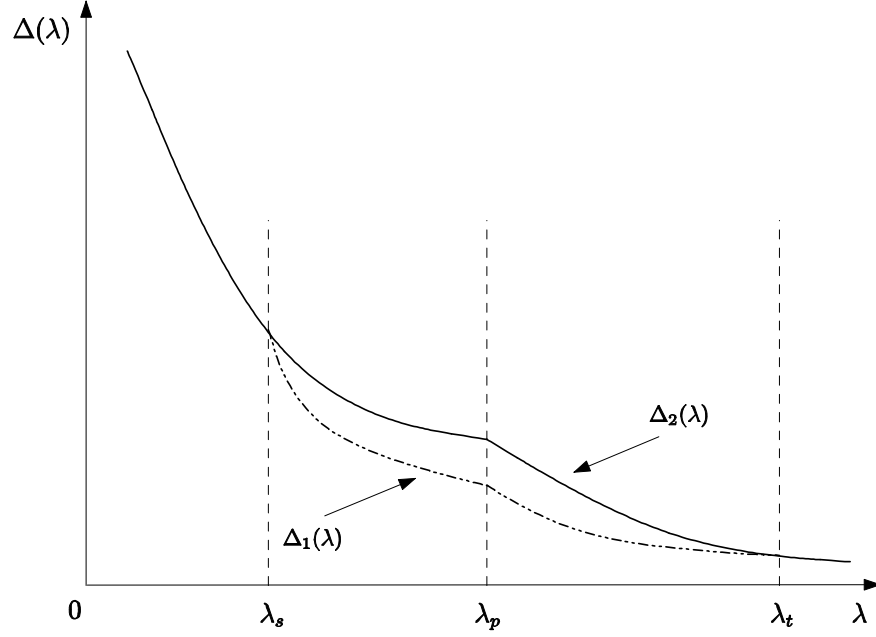


Figure 12: The relation between the shock arrival rate λ and the optimal duration of temporary jobs $\Delta(\lambda)$ when temporary contracts can be renewed.

At first sight, expression (32) may appear awkward compared to (4). It is however possible to show (see appendix J.3 and J.4), that the top and bottom rows of $\Delta_1(\lambda)$ are in fact irrelevant: it is better to offer permanent contracts for $\lambda \leq \lambda_{s_2}$ and temporary contracts are no longer profitable above λ_{t_2} . Indeed, as depicted in figure 11, we get that $S_{t_1}(\lambda) \geq S_{t_2}(\lambda)$ and that³⁶ $\lambda_{t_1} = \lambda_{t_2} = \lambda_t$ and $\lambda_{s_1} = \lambda_{s_2} = \lambda_s$. This has three main consequences. First, the condition for the existence of temporary jobs is almost the same as in the benchmark model.³⁷ Second, all starting temporary contracts are renewed in the absence of productivity shocks. Namely, for $\lambda \in [\lambda_s, \lambda_p]$, temporary jobs are renewed at the expiration of the first temporary contract provided that productivity shocks did not occur, and may be converted into a permanent contract at the end of the second contract, while for $\lambda \in [\lambda_p, \lambda_t]$, temporary contracts end at the expiration date of the second contract. Third, in partial equilibrium, the possibility to renew temporary jobs does not affect the number or the share of temporary jobs in entries into employment. Renewal only increases the total length of temporary contracts, from $\Delta(\lambda)$

³⁶See appendix J.3 and J.4 for details.

³⁷More precisely, we should have $F > U$ as in the benchmark, and the analogue of condition (6), $S_{t_1}(\lambda_p) \geq S_p(\lambda_p) = 0$ should be fulfilled.

in the benchmark case to $\Delta_1(\lambda) + \Delta_2(\lambda)$ when renewal is permitted, and thus increases the share of temporary contracts in the stock of employment. Notice that $\Delta_1(\lambda) \leq \Delta_2(\lambda) = \Delta(\lambda)$, meaning that the possibility to renew temporary contracts leads firms to shorten the duration of starting temporary jobs: when a temporary job is created, it is always preferable to do so for a shorter duration, rather than increase the risk of maintaining a job unproductively for a longer duration, and then renew the contract at the date of termination Δ_1 if it has not been hit by a shock.³⁸ This is illustrated on figure 12.

6 Conclusion

By taking into account the fact that temporary contracts cannot be destroyed at zero cost before their date of termination or that there are identical dismissal rules for temporary and permanent contracts, we have been able to explain not only the choice between temporary and permanent contracts but also the duration of temporary contracts in a search and matching model of the labor market. This model reproduces some important stylized facts about temporary jobs observed in Continental European countries. This framework shows that job protection of permanent jobs has a negligible impact on total employment but entails a strong substitution of temporary jobs for permanent jobs which decreases total production. All in all, this model is useful to explain and to understand the consequences of the huge creation of temporary jobs observed in Continental European countries characterized by stringent job protection legislations.

³⁸Alternatively, this can be inferred from first order conditions on Δ_1 and Δ_2 , where $S_{t_1}(\lambda) \geq S_{t_2}(\lambda)$ implies that the marginal cost of raising the duration of a temporary contract is larger for starting than for renewed temporary jobs, leading to $\Delta_1(\lambda) \leq \Delta_2(\lambda)$. See appendix for more details.

References

- [1] Alonso-Borrego, C., Galdon-Sanchez, J., and J. Fernandez-Villaverde, 2011, Evaluating Labor Market Reforms: A General Equilibrium Approach, working paper, Universidad Carlos III de Madrid.
- [2] Autor, D., 2001, Why do temporary help firms provide free general skills training? *Quarterly Journal of Economics*, 116, 1409-1448.
- [3] Autor, D., 2003, Outsourcing at Will: The Contribution of Unjust Dismissal Doctrine to the Growth of Employment Outsourcing, *Journal of Labor Economics*, 21, 1-41.
- [4] Bentolila, S., Cahuc, P., Dolado, J. and T. Le Barbanchon, 2010, Two-Tier Labor Markets in a Deep Recession: France vs. Spain, forthcoming in *Economic Journal*.
- [5] Bentolila, S., J. J. Dolado, and J. F. Jimeno, 2008, Two-tier Employment Protection Reforms: The Spanish Experience, CESifo DICE Report 4/2008.
- [6] Bentolila, S. and G. Saint-Paul, 1992, The Macroeconomic Impact of Flexible Labor Contracts, with an Application to Spain", *European Economic Review*, 36, 1013-1047.
- [7] Berton, F. and Garibaldi, P., 2006, Workers and Firm Sorting into Temporary Jobs, University of Turin, LABORatorio R. Revelli, forthcoming in *Economic Journal*.
- [8] Blanchard, O. J. and A. Landier, 2002, The Perverse Effects of Partial Labor Market Reform: Fixed Duration Contracts in France, *Economic Journal*, 112, 214-244.
- [9] Boeri, T. 2011, Institutional Reforms in European Labour Markets, *Handbook of Labor Economics*, Elsevier, vol. 4B, chap 13, 1174-1235.
- [10] Boeri T. and P. Garibaldi , 2007, Two Tier Reforms of Employment Protection Legislation. A Honeymoon Effect?, *Economic Journal*, 117, F357-F385.
- [11] Booth, A., Francesconi, M., Garcia-Serrano, C., 1999, Job Tenure and Job Mobility in Britain, *Industrial and Labor Relations Review*, 53, 43-47.
- [12] Bucher, A., 2010, Hiring Practices, Employment Protection and Temporary Jobs, TEPP working paper 2010-13.
- [13] Cahuc, P. and F. Postel-Vinay, 2002, Temporary Jobs, Employment Protection and Labor Market Performance, *Labour Economics*, 9, 63-91.
- [14] Caggese, A. and Cunat, V., 2008, Financing Constraints and Fixed-Term Employment Contracts, *Economic Journal*, 118, 2013-2046.

- [15] Cao, S., Shao, E., and P. Silos, 2010, Fixed-Term and Permanent Employment Contracts: Theory and Evidence, Atlanta Fed Working Papers 2010-13.
- [16] Cappellari, L., Dell’Arlinga, C. and M. Leonardi, 2011, Temporary employment, job flows and productivity: A tale of two reforms, CESifo Working paper 3520,
- [17] Centeno, M. and A. Novo, 2011, Excess worker turnover and fixed-term contracts: Causal evidence in a two-tier system, IZA Discussion Paper 6239.
- [18] Costain, J., Jimeno, J. and C. Thomas, 2010, Employment Fluctuations in a Dual Labor Market, Banco d’Espana, Working paper 1013.
- [19] Faccini R., 2008, Reassessing Labor Market Reforms: Temporary Contracts as a Screening Device, EUI Working Papers 2008-27.
- [20] Kahn, L., 2010, Employment protection reforms, employment and the incidence of temporary jobs in Europe: 1996–2001, *Labour Economics*, 17, 1-15.
- [21] Kramarz, F. and M. L. Michaud, 2010, The Shape of Hiring and Separation Costs, *Labour Economics* 17, 27-37.
- [22] Lazear, E., 1990, Job Security Provisions and Employment, *Quarterly Journal of Economics*, 105, 699-726.
- [23] L’Haridon, O. and F. Malherbet, 2009, Employment Protection Reform in Search Economies, *European Economic Review*, 53, 255-273.
- [24] Ljungqvist, L. 2002, How Do Layoff Costs Affect Employment?”, *Economic Journal*, 112, 829-853.
- [25] Macho-Stadler, I., Pérez-Castrillo, D. and N. Porteiro, 2011, Optimal Coexistence of Long-term and Short-term contracts in Labor Markets, Working paper 11.08, Department of Economics, Universidad Pablo Olavide.
- [26] Malcomson, J. M. 1999, Individual Employment Contracts, in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, ch. 35, 2291-2372.
- [27] Mortensen, D. T. and C. A. Pissarides, 1994, Job Creation and Job Destruction in the Theory of Unemployment, *Review of Economic Studies*, 61, 397-415.
- [28] Mortensen, D.T., and E. Nagypal, 2007, More on Unemployment and Vacancy Fluctuations, *Review of Economic Dynamics*, 10, 327-347.

- [29] Mortensen, D T. and C. Pissarides, 1999. New developments in models of search in the labor market, *Handbook of Labor Economics*, in: O. Ashenfelter & D. Card (ed.), *Handbook of Labor Economics*, edition 1, volume 3, chapter 39, 2567-2627, Elsevier.
- [30] Nagypal, E., 2002, Learning Capital and the Lack Thereof: Why Low-Skilled Workers are More Likely to Become Unemployed, mimeo.
- [31] Petrongolo, B. and C. Pissarides, 2001, Looking into the black box: A survey of the matching function, *Journal of Economic Literature* 39: 390-431.
- [32] Pissarides, C. A. 2000, *Equilibrium Unemployment Theory*, 2nd ed., The MIT Press, Cambridge, MA.
- [33] Portugal, P. and J. Varejão, 2009, Why Do Firms Use Fixed-Term Contracts?. IZA discussion paper 4380.
- [34] Saint-Paul, G. 1996, *Dual Labor Markets*, The MIT Press, Cambridge, MA.
- [35] Saint-Paul, G., 2002, The Political Economy of Employment Protection, *Journal of Political Economy*, 110, 672-704.
- [36] Sala, H., J. I. Silva, and M. E. Toledo, 2011, Flexibility at the Margin and Labor Market Volatility in OECD Countries, *Scandinavian Journal of Economics*, forthcoming.
- [37] Shimer, R., 2005, The Cyclical Behavior of Equilibrium Unemployment and Vacancies, *American Economic Review*, 95, 25-49.
- [38] Smith, E., 2007, Limited Duration Employment, *Review of Economic Dynamics*, 10, 444-471.
- [39] Venn, D., 2009, Legislation, Collective Bargaining and Enforcement: Updating the OECD Employment Protection Indicators. OECD Social, Employment and Migration Working paper 89.
- [40] Wasmer, E., 1999, Competition for Jobs in a Growing Economy and the Emergence of Dualism in Employment, *Economic Journal*, 109, 349-371.

APPENDIX

A Termination of temporary contracts

This appendix describes the legal rules for the termination of temporary contracts for 7 OECD countries. Other rules concerning the condition of creation, the maximal duration and the renewal of temporary jobs are described in detail in the ILO Employment protection legislation database - EPLex³⁹ and in the OECD indicator of job protection⁴⁰.

Belgium: In principle regular dismissals (as in the case of an open ended contract) are not possible for temporary jobs. The contract has to expire. The party which breaks the contract before the date of expiration without serious cause has to provide a severance payment whose amount is equal to the minimum of the payment due until the date of expiration of the contract, and twice the payment due during the advance notice if the contract had been permanent.

France: Regular dismissals (as in the case of an open ended contract) are not possible for temporary jobs. The contract has to expire. Employers can only dismiss fixed-term workers if there is a credible “valid reason” which makes the continuation of employment unacceptable, e.g. fraudulent behavior of the employee. The employee can quit if he finds an open ended contract.

Germany: Regular dismissals (as in the case of an open ended contract) are not possible for temporary jobs. The contract has to expire. Employers can only dismiss fixed-term workers if there is a credible “valid reason” which makes the continuation of employment unacceptable, e.g. fraudulent behavior of the employee.

Greece: Employers can only dismiss fixed-term workers if there is a credible “valid reason” which makes the continuation of employment unacceptable, e.g. fraudulent behavior of the employee. If the contract expires and the worker is still employed under the same conditions doing similar or the same work then the worker is considered as being under an open ended contract with the corresponding rules applying.

Italy: Regular dismissals (as in the case of an open ended contract) are not possible. The contract has to expire. Employers can only dismiss fixed-term workers if there is a credible "valid reason" which makes the continuation of employment unacceptable, e.g. fraudulent behavior of the employee. The employee can quit if he finds an open ended contract.

Portugal: the rule for individual dismissal is the same for fixed-term and open-ended contracts.

³⁹See <http://www.ilo.org/dyn/terminate/>

⁴⁰See www.oecd.org/employment/protection

Individual dismissals can be carried out solely for disciplinary reasons, which implies a fairly long disciplinary process. Among OECD countries Portugal is the one with the most stringent legislation for individual dismissals. So, in practice employers avoid this route, either waiting for the end of the fixed term contract (typically a one year contract, renewable for up to three years) or paying the corresponding severance pay (a minimum of three months); or, in the case of open-ended contracts they negotiate a separation very often paying the ruled severance pay (one month for each year of tenure).

Spain: If the employer wishes to terminate the contract in advance, he would follow exactly the same procedures as a permanent contract and therefore would pay 20 days for an economic dismissal, but workers can go to court and the employer would normally pay at least the penalty rate of 45 days. So, usually, employers wait for expiration, unless the worker has committed a really serious offence (fraud, etc.).

B Surplus of temporary jobs in the benchmark model

Temporary jobs can be in one of the following two states: (i) “productive” with productivity $y > 0$; (ii) “unproductive” with zero productivity. All jobs start with productivity y . They are hit by shocks which arrive at idiosyncratic Poisson rate λ . When there is a shock, productivity irreversibly goes to zero and the job stays idle until the contract expires, whereupon the job is destroyed. Conversely, if the productivity of the job remains constant throughout the duration of the contract, the job can be converted into a permanent contract.

Let us denote by Δ the duration of the temporary contract, which is decided when the job starts. Let us denote by τ the spell of the job from its date of creation. Once the cost c to sign the contract has been paid, the present discounted value of the surplus of a temporary job with shock arrival rate λ , contract duration Δ , spell τ and productivity $x = y, 0$, is denoted by $S_t(\lambda, \Delta, x, \tau)$. The Bellman equations satisfy:

$$rS_t(\lambda, \Delta, y, \tau) = y - rU + \lambda[S_t(\lambda, \Delta, 0, \tau) - S_t(\lambda, \Delta, y, \tau)] + \dot{S}_t(\lambda, \Delta, y, \tau) \quad (\text{B1})$$

$$rS_t(\lambda, \Delta, 0, \tau) = -rU + \dot{S}_t(\lambda, \Delta, 0, \tau) \quad (\text{B2})$$

where $\dot{S}_t(\lambda, \Delta, x, \tau) = \partial S_t(\lambda, \Delta, x, \tau) / \partial \tau$.

At the date of termination of the temporary contract, there are two possible outcomes. On one hand, if the job has not been hit by a productivity shock, it can be converted into a permanent job.

The formal condition reads, when $\tau = \Delta$, as

$$S_t(\lambda, \Delta, y, \Delta) = \max[S_p(\lambda), 0] \quad (\text{B3})$$

where $S_p(\lambda)$ denotes the value of a permanent job. On the other hand, if a shock occurred, the job is destroyed as soon as the contract reaches its term. The formal condition reads, when $\tau = \Delta$, as

$$S_t(\lambda, \Delta, 0, \Delta) = 0. \quad (\text{B4})$$

Let us find the solution to the system of equations (B1), (B2) with terminal conditions (B3), (B4).

A general solution to the first-order linear differential equation (B2) (with constant coefficient and constant term) is given by:

$$S_t(\lambda, \Delta, 0, \tau) = Ae^{r\tau} + B \quad (\text{B5})$$

where A and B are constants to be determined. Differentiation with respect to τ yields $\dot{S}_t(\lambda, \Delta, 0, \tau) = rAe^{r\tau}$. Plugging this expression together with (B5) into (B2), one gets: $rAe^{r\tau} + rB = -rU + rAe^{r\tau} \leftrightarrow B = -U$. Making use of the terminal condition $S_t(\lambda, \Delta, 0, \Delta) = 0$, we get $Ae^{r\Delta} + B = Ae^{r\Delta} - U = 0$, and it follows that: $A = Ue^{-r\Delta}$. Finally, using (B5) we get:

$$S_t(\lambda, \Delta, 0, \tau) = -\left(1 - e^{-r(\Delta-\tau)}\right)U \quad (\text{B6})$$

Let now rewrite (B1) as:

$$\dot{S}_t(\lambda, \Delta, y, \tau) = (r + \lambda)S_t(\lambda, \Delta, y, \tau) - (y - rU) - \lambda S_t(\lambda, \Delta, 0, \tau)$$

which is a first-order linear differential equation (with constant coefficient and variable term) of the form

$$\dot{S}_t(\lambda, \Delta, y, \tau) = CS_t(\lambda, \Delta, y, \tau) + D(\tau)$$

where $C = (r + \lambda)$ and $D = -(y - rU) - \lambda S_t(\lambda, \Delta, 0, \tau)$. A general solution to this equation is given by:

$$\begin{aligned} S_t(\lambda, \Delta, y, \tau) &= e^{C(\tau-\Delta)} \left[S_t(\lambda, \Delta, y, \Delta) + \int_{\Delta}^{\tau} e^{-C(\zeta-\Delta)} D(\zeta) d\zeta \right] \\ &= e^{(r+\lambda)(\tau-\Delta)} \left[S_t(\lambda, \Delta, y, \Delta) - \underbrace{\int_{\Delta}^{\tau} e^{-(r+\lambda)(\zeta-\Delta)} [(y - rU) + \lambda S_t(\lambda, \Delta, 0, \zeta)] d\zeta}_{\Gamma} \right] \end{aligned} \quad (\text{B7})$$

Using (B6), it is possible to rewrite Γ as:

$$\Gamma = [y - (r + \lambda)U] \left(\frac{1 - e^{-(r+\lambda)(\tau-\Delta)}}{r + \lambda} \right) + U \left(1 - e^{-\lambda(\tau-\Delta)} \right)$$

Multiplying both sides of this expression by $e^{(r+\lambda)(\tau-\Delta)}$, we get:

$$\Gamma e^{(r+\lambda)(\tau-\Delta)} = [y - (r + \lambda)U] \left(\frac{e^{(r+\lambda)(\tau-\Delta)} - 1}{r + \lambda} \right) + U \left(e^{(r+\lambda)(\tau-\Delta)} - e^{r(\tau-\Delta)} \right)$$

Using this expression together with (B7) yields:

$$S_t(\lambda, \Delta, y, \tau) = y \frac{1 - e^{(r+\lambda)(\tau-\Delta)}}{r + \lambda} - U \left[1 - e^{r(\tau-\Delta)} \right] + e^{(r+\lambda)(\tau-\Delta)} S_t(\lambda, \Delta, y, \Delta)$$

Finally, using the fact that $S_t(\lambda, \Delta, y, \Delta) = \max[S_p(\lambda), 0]$, then setting $\tau = 0$ and rearranging, the starting value of a temporary job writes as:

$$S_t(\lambda, \Delta, y, 0) = y \frac{1 - e^{-(r+\lambda)\Delta}}{r + \lambda} - U (1 - e^{-r\Delta}) + \max[S_p(\lambda), 0] e^{-(r+\lambda)\Delta} \quad (\text{B8})$$

Dropping the last two arguments of $S_t(\lambda, \Delta, y, 0)$ in order to alleviate the notations, we denote as $S_t(\lambda, \Delta) = S_t(\lambda, \Delta, y, 0) - c$, the starting value of the surplus of a job, including the cost of writing contracts. This last expression is similar to equation (2) displayed in the main text.

C The properties of functions $S_p(\lambda)$ and $S_t(\lambda)$

This section proves the properties of the surplus $S_p(\lambda)$ and $S_t(\lambda) = \max_{\Delta} S_t(\lambda, \Delta)$ displayed on figure 4. We begin by analyzing the properties of $S_p(\lambda)$, then we continue with the properties of $S_t(\lambda)$ and finally we study the intercept of these two functions.

- First,

$$S_p(\lambda) = \frac{y - rU - \lambda F}{r + \lambda} - c \quad (\text{C9})$$

is continuous, decreasing and $S_p(0) = \frac{y}{r} - U - c > 0$, $\lim_{\lambda \rightarrow +\infty} S_p(\lambda) = -c - F < 0$.

- Second, let us show that $S_t(\lambda)$ is continuous and decreasing with $\lim_{\lambda \rightarrow 0} S_t(\lambda) = \frac{y}{r} - U - c > 0$, $\lim_{\lambda \rightarrow \infty} S_t(\lambda) = -c$. When $\lambda \geq \lambda_p$, we get, from equation (2):

$$S'_t(\lambda) = y \frac{e^{-(r+\lambda)\Delta(\lambda)} [(r + \lambda)\Delta(\lambda) + 1] - 1}{(r + \lambda)^2},$$

which is negative because $e^{-x} < 1/(x + 1)$ when $x > 0$. Equations (2) and (3) allow us to write

$$S_t(\lambda) = y \frac{1 - e^{-(r+\lambda)\Delta(\lambda)}}{r + \lambda} - U \left[1 - e^{-r\Delta(\lambda)} \right] - c.$$

From the definition of $\Delta(\lambda)$ we know that $\lim_{\lambda \rightarrow \infty} \Delta(\lambda) = 0$ and $e^{-\lambda \Delta(\lambda)} = rU/y$, so that $\lim_{\lambda \rightarrow \infty} \frac{1 - e^{-(r+\lambda)\Delta(\lambda)}}{r+\lambda} = 0$ and $\lim_{\lambda \rightarrow \infty} 1 - e^{-r\Delta(\lambda)} = 0$. Therefore, $\lim_{\lambda \rightarrow \infty} S_t(\lambda) = -c$. When $\lambda < \lambda_p$, we get, from equation (2):

$$S'_t(\lambda) = y \frac{e^{-(r+\lambda)\Delta(\lambda)} [(r+\lambda)\Delta(\lambda) + 1] - 1}{(r+\lambda)^2} - e^{-(r+\lambda)\Delta(\lambda)} [\Delta(\lambda)S_p(\lambda) - S'_p(\lambda)]$$

which is negative, since it has just been shown that the first term $y \frac{e^{-(r+\lambda)\Delta(\lambda)} [(r+\lambda)\Delta(\lambda) + 1] - 1}{(r+\lambda)^2}$ is negative. Moreover, $S_p(\lambda) > 0$ when $\lambda < \lambda_p$, and $S'_p(\lambda) < 0$. From the definition of the surplus we get

$$\lim_{\lambda \rightarrow 0} S_t(\lambda) = \lim_{\lambda \rightarrow 0} S_p(\lambda) = \frac{y}{r} - U - c.$$

- Let us now look at the intercept of $S_t(\lambda)$ and $S_p(\lambda)$. From the definition of the surplus we get

$$\lim_{\lambda \rightarrow 0} S'_t(\lambda) = -\frac{y}{r^2}, \quad \lim_{\lambda \rightarrow 0} S'_p(\lambda) = -\frac{y + rF - rU}{r^2},$$

which implies that

$$\lim_{\lambda \rightarrow 0} S'_t(\lambda) < \lim_{\lambda \rightarrow 0} S'_p(\lambda) \iff F < U.$$

Thus, in the neighborhood of $\lambda = 0$, there exist values of $\lambda > 0$ such that $S_p(\lambda) > S_t(\lambda)$ if and only if $F < U$. Let us assume for now that this condition is satisfied. Therefore, $S_p(\lambda)$ and $S_t(\lambda)$ have at least one intercept for positive values of $S_t(\lambda)$ if $S_p(\lambda_p) = 0 < S_t(\lambda_p)$. Using equations (2) and (3), we can write

$$S_t(\lambda) = \max[S_p(\lambda), 0] - c + rU \left(e^{\lambda \Delta(\lambda)} \frac{1 - e^{-(r+\lambda)\Delta(\lambda)}}{r+\lambda} - \frac{1 - e^{-r\Delta(\lambda)}}{r} \right). \quad (\text{C10})$$

This equation implies that $S_t(\lambda_p) > 0$ (where λ_p is defined by $S_p(\lambda_p) = 0$) is equivalent to condition (6). This condition ensures the existence of creation of temporary jobs. If it is satisfied, there exists at least a threshold value of λ , denoted by λ_s , such that $S_p(\lambda) > S_t(\lambda)$ if $\lambda < \lambda_s$. Moreover, when condition (6) is satisfied, there exists a threshold value $\lambda_t > \lambda_p$ such that $S_t(\lambda) < 0$ if $\lambda > \lambda_t$ because $S_t(\lambda)$ is decreasing and is strictly negative, equal to $-c$, when λ goes to infinite.

D Surplus of temporary jobs in the model with productivity shocks

Let us denote by $S_t(\lambda, \Delta, y, \tau)$ the value at date τ of temporary jobs with shock arrival rate λ , duration Δ and productivity y . When there is a shock, there is a draw from the constant distribution H of

productivities. $S_t(\lambda, \Delta, y, \tau)$ satisfies the Bellman equation

$$rS_t(\lambda, \Delta, y, \tau) = y - rU + \lambda \left[\int S_t(\lambda, \delta, x, \tau) dH(x) - S_t(\lambda, \Delta, y, \tau) \right] + \dot{S}_t(\lambda, \Delta, y, \tau) \quad (\text{D11})$$

with $\dot{S}_t(\lambda, \Delta, y, \tau) = \partial S_t(\lambda, \Delta, y, \tau) / \partial \tau$. At date $\tau = \Delta$:

$$S_t(\lambda, \Delta, y, \Delta) = \max [S_p(y, \lambda), 0] \quad (\text{D12})$$

We proceed in two steps. Let us (i) determine $\int S_t(\lambda, \Delta, y, \tau) dH(y)$, and then (ii) solve for $S_t(\lambda, \Delta, y, \tau)$ in (D11).

D.1 Part (i)

Integrating (D11) over productivity, we get

$$r \int S_t(\lambda, \Delta, y, \tau) dH(y) = \int (y - rU) dH(y) + \int \dot{S}_t(\lambda, \Delta, y, \tau) dH(y) \quad (\text{D13})$$

This equation takes the form $\dot{x}(\tau) = rx(\tau) + b$ with $b = -\int (y - rU) dH(y)$, and with terminal condition $x(\Delta) = \int \max [S_p(x, \lambda), 0] dH(x)$. Its general solution is :

$$x(\tau) = Ae^{r\tau} + B$$

with

$$B = \frac{1}{r} \int (y - rU) dH(y)$$

and

$$A = \left(\int \max [S_p(\lambda, y), 0] dH(y) - \frac{1}{r} \int (y - rU) dH(y) \right) e^{-r\Delta}$$

Thus

$$\int S_t(\lambda, \Delta, y, \tau) dH(y) = \left(\frac{1 - e^{-r(\Delta - \tau)}}{r} \right) \int (y - rU) dH(y) + e^{-r(\Delta - \tau)} \int \max [S_p(\lambda, y), 0] dH(y)$$

D.2 Part (ii)

Equation (D11):

$$\dot{S}_t(\lambda, \Delta, y, \tau) = (r + \lambda)S_t(\lambda, \Delta, y, \tau) - (y - rU) - \lambda \int S_t(\lambda, \Delta, x, \tau) dH(x)$$

is a first order linear differential equation with constant coefficient $C = (r + \lambda)$ and variable term, $D(\tau) = -(y - rU) - \lambda \int S_t(\lambda, \Delta, x, \tau) dH(x)$. Its general solution is:

$$\begin{aligned} S_t(\lambda, \Delta, y, \tau) &= e^{C(\tau-\Delta)} \left[S_t(\lambda, \Delta, y, \tau) + \int_{\Delta}^{\tau} e^{-C(\zeta-\Delta)} D(\zeta) d\zeta \right] \\ &= e^{(r+\lambda)(\tau-\Delta)} \left[S_t(\lambda, \Delta, y, \tau) + \underbrace{\int_{\Delta}^{\tau} e^{-(r+\lambda)(\zeta-\Delta)} D(\zeta) d\zeta}_{\Gamma} \right] \end{aligned}$$

The term Γ can be rewritten:

$$\begin{aligned} \Gamma &= - \left(\frac{1 - e^{-(r+\lambda)(\tau-\Delta)}}{(r + \lambda)} \right) (y - rU) - \frac{\lambda}{r} \int (x - rU) dH(x) \left(\frac{1 - e^{-(r+\lambda)(\tau-\Delta)}}{(r + \lambda)} - \frac{1 - e^{-\lambda(\tau-\Delta)}}{\lambda} \right) \\ &\quad - \int \max[S_p(x, \lambda), 0] dH(x) (1 - e^{-\lambda(\tau-\Delta)}) \end{aligned}$$

Thus :

$$\begin{aligned} S_t(\lambda, \Delta, y, \tau) &= e^{(r+\lambda)(\tau-\Delta)} \left[S_t(\lambda, \Delta, y, \Delta) + \int_{\Delta}^{\tau} e^{-(r+\lambda)(\zeta-\Delta)} D(\zeta) d\zeta \right] \\ &= e^{(r+\lambda)(\tau-\Delta)} S_t(\lambda, \Delta, y, \Delta) \\ &\quad - (y - rU) e^{(r+\lambda)(\tau-\Delta)} \left(\frac{1 - e^{-(r+\lambda)(\tau-\Delta)}}{r + \lambda} \right) \\ &\quad - \frac{\lambda}{r} e^{(r+\lambda)(\tau-\Delta)} \int (x - rU) dH(x) \left(\frac{1 - e^{-(r+\lambda)(\tau-\Delta)}}{(r + \lambda)} - \frac{1 - e^{-\lambda(\tau-\Delta)}}{\lambda} \right) \\ &\quad - e^{(r+\lambda)(\tau-\Delta)} \int \max[S_p(x, \lambda), 0] dH(x) (1 - e^{-\lambda(\tau-\Delta)}) \end{aligned}$$

Since $S_t(\lambda, \Delta, y, \Delta) = \max[S_p(\lambda, x), 0]$, we get

$$\begin{aligned} S_t(\lambda, \Delta, y, 0) &= (y - rU) \left(\frac{1 - e^{-(r+\lambda)\Delta}}{r + \lambda} \right) + e^{-(r+\lambda)\Delta} \max[S_p(y, \lambda), 0] \\ &\quad - \frac{\lambda}{r} \int (x - rU) dH(x) \left(\frac{e^{-(r+\lambda)\Delta} - 1}{(r + \lambda)} - \frac{e^{-(r+\lambda)\Delta} - e^{-r\Delta}}{\lambda} \right) \\ &\quad - \int \max[S_p(x, \lambda), 0] dH(x) (e^{-(r+\lambda)\Delta} - e^{-r\Delta}) \end{aligned}$$

Rearranging a little bit, and taking account of the fact that jobs always start at y_u yields:

$$\begin{aligned} S_t(\lambda, \Delta) &= y_u \left(\frac{1 - e^{-(r+\lambda)\Delta}}{r + \lambda} \right) + \int y dH(y) \left(\frac{\lambda(1 - e^{-r\Delta}) + r(e^{-(r+\lambda)\Delta} - e^{-r\Delta})}{r(r + \lambda)} \right) \\ &\quad + e^{-(r+\lambda)\Delta} \max[S_p(y_u, \lambda), 0] + e^{-r\Delta} (1 - e^{-\lambda\Delta}) \int \max[S_p(y, \lambda), 0] dH(y) \\ &\quad - U(1 - e^{-r\Delta}) - c \end{aligned}$$

which is formally equivalent to the expression given equation (10).

E Optimal duration of temporary jobs in the model with productivity shocks

The optimal duration of temporary jobs maximizes the surplus of starting temporary jobs. We first consider temporary jobs which are not transformed into permanent jobs because the shock arrival rate is above the threshold value λ_p . Then, the case of temporary jobs that can be transformed into permanent jobs is studied in a second step.

E.1 Case 1: $\lambda \geq \lambda_p$

If $\lambda \geq \lambda_p$, the surplus of a temporary job with shock arrival rate λ and duration Δ is

$$S_t(\lambda, \Delta) = \int_0^\Delta \left(e^{-\lambda\tau} y_u + \left(1 - e^{-\lambda\tau}\right) \int_{-\infty}^{y_u} y dH(y) - rU \right) e^{-r\tau} d\tau - c$$

The first order condition, $\partial S_t(\lambda, \Delta)/\partial \Delta = 0$, can be written as

$$e^{-\lambda\Delta} y_u + \left(1 - e^{-\lambda\Delta}\right) \int_{-\infty}^{y_u} y dH(y) - rU = 0.$$

The second order condition:

$$-\lambda e^{-\lambda\Delta} \int_{-\infty}^{y_u} (y_u - y) dH(y) < 0$$

is always satisfied. Then, the optimal duration is

$$\Delta(\lambda) = \frac{1}{\lambda} \ln \frac{y_u - \int_{-\infty}^{y_u} y dH(y)}{rU - \int_{-\infty}^{y_u} y dH(y)},$$

which corresponds to the expression given by equation (11).

E.2 Case 2. $\lambda \leq \lambda_p$

When $\lambda \leq \lambda_p$, the surplus of a permanent job with shock arrival rate λ and duration Δ is

$$\begin{aligned} S_t(\lambda, \Delta) = & \int_0^\Delta \left(e^{-\lambda\tau} y_u + \left(1 - e^{-\lambda\tau}\right) \int_{-\infty}^{y_u} y dH(y) - rU \right) e^{-r\tau} d\tau + \\ & e^{-(r+\lambda)\Delta} S_p(y_u, \lambda) + \left(1 - e^{-\lambda\Delta}\right) e^{-r\Delta} \int_{T(\lambda)}^{y_u} S_p(y, \lambda) dH(y) - c. \end{aligned}$$

where $T(\lambda)$ is defined by equation (9).

The first order condition, $\partial S_t(\lambda, \Delta)/\partial \Delta = 0$, can be written as

$$e^{-\lambda \Delta} y_u + \left(1 - e^{-\lambda \Delta}\right) \int_{-\infty}^{y_u} y dH(y) - rU - r \int_{T(\lambda)}^{y_u} S_p(y, \lambda) dH(y) \\ + (r + \lambda) e^{-\lambda \Delta} \left(S_p(y_u, \lambda) - \int_{T(\lambda)}^{y_u} S_p(y, \lambda) dH(y) \right) = 0$$

The second order condition:

$$-\lambda e^{-\lambda \Delta} \left[\int_{-\infty}^{y_u} (y_u - y) dH(y) - (r + \lambda) \left(S_p(y_u, \lambda) - \int_{T(\lambda)}^{y_u} S_p(y, \lambda) dH(y) \right) \right] < 0$$

is always satisfied. Thus, the optimal duration is

$$\Delta(\lambda) = \frac{1}{\lambda} \ln \frac{y_u - \int_{-\infty}^{y_u} y dH(y) - (r + \lambda) \left[S_p(y_u, \lambda) - \int_{T(\lambda)}^{y_u} S_p(y, \lambda) dH(y) \right]}{rU - \int_{y_{\min}}^{y_{\max}} y dH(y) + r \int_{T(\lambda)}^{y_u} S_p(y, \lambda) dH(y)}$$

which corresponds to the expression given equation (11).

F The properties of functions $S_p(y_u, \lambda)$ and $S_t(\lambda)$ in the model with productivity shocks

F.1 Properties of $S_p(y_u, \lambda)$

The surplus of a permanent job with shock arrival rate λ

$$S_p(y_u, \lambda) = \frac{y_u - rU - \lambda F}{r + \lambda} - c + \frac{\lambda}{r + \lambda} \int_{R(\lambda)}^{y_u} \frac{x - R(\lambda)}{r + \lambda} dH(x)$$

From this expression, it is straightforward to prove, assuming that $y_u - rU > rF$, that S_p is continuous in λ and decreases from

$$\lim_{\lambda \rightarrow 0} S_p(y_u, \lambda) = \frac{y_u}{r} - U - c$$

to

$$\lim_{\lambda \rightarrow +\infty} S_p(y_u, \lambda) = -F - c$$

F.2 Properties of $S_t(\lambda)$

The surplus of a temporary job with shock arrival rate λ and optimal duration $\Delta(\lambda) = \max_{\Delta} S_t(\lambda, \Delta)$ is, using equation (10)

$$S_t(\lambda) = \int_0^{\Delta(\lambda)} \left(e^{-\lambda \tau} y_u + \left(1 - e^{-\lambda \tau}\right) \int_{-\infty}^{y_u} y dH(y) - rU \right) e^{-r\tau} d\tau + \\ e^{-(r+\lambda)\Delta(\lambda)} \max[S_p(y_u, \lambda), 0] + \left(1 - e^{-\lambda \Delta(\lambda)}\right) e^{-r\Delta(\lambda)} \int_{-\infty}^{y_u} \max[S_p(y, \lambda), 0] dH(y) - c. \quad (\text{F14})$$

From this expression, it is easily checked that $S_t(\lambda)$ is continuous and decreasing from $\lim_{\lambda \rightarrow 0} S_t(\lambda) = \frac{y_u}{r} - U - c$ to $\lim_{\lambda \rightarrow +\infty} S_t(\lambda) = -c$.

It turns out that $\lim_{\lambda \rightarrow +\infty} S_t(\lambda) = -c$ because equation (11) implies that $\lim_{\lambda \rightarrow +\infty} \Delta(\lambda) = 0$ and $\max[S_p(y_u, \lambda), 0] = 0$ when $\lambda \rightarrow +\infty$.

Moreover, $\lim_{\lambda \rightarrow 0} S_t(\lambda) = \frac{y_u}{r} - U - c$ because equation (11), which implies $\lim_{\lambda \rightarrow 0} \Delta(\lambda) = +\infty$, yields, using expression (F14)

$$\lim_{\lambda \rightarrow 0} S_t(\lambda) = \frac{y_u}{r} - U - c.$$

F.3 Intercept of S_t and S_p in the model with productivity shocks:

Using the expression of $S_t(\lambda)$, keeping in mind that $\lim_{\lambda \rightarrow 0} \Delta(\lambda) = +\infty$, the result is that the slope of this surplus in the neighborhood of $\lambda = 0$ writes:

$$\lim_{\lambda \rightarrow 0} S'_t(\lambda) = \frac{-y_u + \int_{-\infty}^{y_u} y dH(y)}{r^2} \quad (\text{F15})$$

Similarly, using the expression of $S_p(y_u, \lambda)$, keeping in mind that $\lim_{\lambda \rightarrow 0} R(\lambda) = rU - rF$, the result is that

$$\lim_{\lambda \rightarrow 0} S'_p(y_u, \lambda) = -\frac{y_u - r(U - F)}{r^2} + \frac{\int_{r(U-F)}^{y_u} [y - r(U - F)] dH(y)}{r^2} \quad (\text{F16})$$

Using (F15) and (F16) we get :

$$\lim_{\lambda \rightarrow 0} S'_p(y_u, \lambda) > \lim_{\lambda \rightarrow 0} S'_t(\lambda) \Leftrightarrow \int_{-\infty}^{r(U-F)} [y - r(U - F)] dH(y) < 0,$$

which holds if and only if $U > F$.

Since $\lim_{\lambda \rightarrow 0} S_t(\lambda) = \lim_{\lambda \rightarrow 0} S_p(y_u, \lambda)$, the fact that $\lim_{\lambda \rightarrow 0} S'_p(y_u, \lambda) > \lim_{\lambda \rightarrow 0} S'_t(\lambda)$ if and only if $U > F$ implies that there exists a value of $\lambda > 0$ such that $S_p(y_u, \lambda) > S_t(\lambda)$ in the neighborhood of $\lambda = 0$, if and only if $U > F$. Let us assume that this is the case. Then S_p and S_t have at least one positive intercept for positive values of $S_t(\lambda)$ if $S_p(y_u, \lambda_p) = 0 < S_t(\lambda_p)$. This yields a condition similar to (6) in the benchmark. We checked that this intercept is unique in the calibration exercises.

G Labor market equilibrium

G.1 Equilibrium with permanent jobs only

The free entry condition and the condition that defines the threshold value of shock arrival rates above which no jobs are created are respectively:

$$\kappa = \frac{q(\theta)(1 - \beta)}{r + q(\theta)} \int_{\lambda_{\min}}^{\lambda_p} S_p(y_u, \lambda) dG(\lambda) \quad (\text{G17})$$

$$S_p(y_u, \lambda_p) = 0. \quad (\text{G18})$$

with

$$\begin{aligned}(r + \lambda)S_p(y_u, \lambda) &= y_u - rU + \lambda \left[\int_{-\infty}^{y_u} \max[S_c(x, \lambda), 0] dH(y) \right] - rc \\ rU &= z + \frac{\beta\theta [r + q(\theta)]}{(1 - \beta)G(\lambda_p)} \kappa\end{aligned}$$

so that the threshold value of shock arrival rates λ_p above which no jobs are created $S_p(y_u, \lambda_p) = 0$, can be restated as $\lambda_p \equiv \lambda_p(\theta)$. Then, the free entry condition, which defines the equilibrium value of θ , can be written as follows

$$\Gamma(\theta) \equiv \kappa - \frac{q(\theta)(1 - \beta)}{r + q(\theta)} \int_{\lambda_{\min}}^{\lambda_p(\theta)} S_p(y_u, \lambda) dG(\lambda) = 0 \quad (\text{G19})$$

Differentiating (G19) with respect to θ yields

$$\Gamma'(\theta) = (1 - \beta) \frac{q'(\theta)r}{[r + q(\theta)]^2} \int_{\lambda_{\min}}^{\lambda_p(\theta)} S_p(y_u, \lambda) dG(\lambda) + \frac{q(\theta)(1 - \beta)}{r + q(\theta)} \int_{\lambda_{\min}}^{\lambda_p(\theta)} \frac{dS_p}{d\theta} dG(\lambda)$$

where $\frac{dS_p}{d\theta} = \frac{dS_p}{dr} \frac{dr}{d\theta} \leq 0$, so that $\Gamma'(\theta) \leq 0$. This implies that (G19) defines a unique value of θ provided that the conditions of existence of θ are satisfied.

G.2 Equilibrium with permanent and temporary jobs

The free entry condition and the condition that defines the threshold value of shock arrival rates above which no jobs are created are respectively:

$$\kappa = \frac{q(\theta)(1 - \beta)}{r + q(\theta)} \left[\int_{\lambda_{\min}}^{\lambda_s} S_p(y_u, \lambda) dG(\lambda) + \int_{\lambda_s}^{\lambda_t} S_t(\lambda) dG(\lambda) \right] \quad (\text{G20})$$

$$S_t(\lambda_t) = 0. \quad (\text{G21})$$

Let us note that, as previously in the regime without temporary jobs, we can write $\lambda_s \equiv \lambda_s(\theta)$, $\lambda_p \equiv \lambda_p(\theta)$, $\lambda_t \equiv \lambda_t(\theta)$, so that the free entry condition can be written

$$\kappa = \frac{q(\theta)(1 - \beta)}{r + q(\theta)} \left[\int_{\lambda_{\min}}^{\lambda_s(\theta)} S_p(y_u, \lambda) dG(\lambda) + \int_{\lambda_s(\theta)}^{\lambda_p(\theta)} S_t(\lambda) dG(\lambda) + \int_{\lambda_p(\theta)}^{\lambda_t(\theta)} S_t(\lambda) dG(\lambda) \right] \quad (\text{G22})$$

Again, the free entry condition, which defines the equilibrium value of θ , can be written as

$$\Gamma(\theta) \equiv \kappa - \frac{q(\theta)(1 - \beta)}{r + q(\theta)} \left[\int_{\lambda_{\min}}^{\lambda_s(\theta)} S_p(y_u, \lambda) dG(\lambda) + \int_{\lambda_s(\theta)}^{\lambda_p(\theta)} S_t(\lambda) dG(\lambda) + \int_{\lambda_p(\theta)}^{\lambda_t(\theta)} S_t(\lambda) dG(\lambda) \right] = 0$$

Differentiating Γ with respect to θ , and keeping in mind that $S_p(y_u, \lambda_s(\theta)) = S_t(\lambda_s(\theta))$ and that $S_t(\lambda_t(\theta)) = 0$, yields

$$\begin{aligned}\Gamma'(\theta) &= (1 - \beta) \frac{q'(\theta)r}{[r + q(\theta)]^2} \left[\int_{\lambda_{\min}}^{\lambda_s(\theta)} S_p(y_u, \lambda) dG(\lambda) + \int_{\lambda_s(\theta)}^{\lambda_t(\theta)} S_t(\lambda) dG(\lambda) \right] \\ &\quad + \frac{q(\theta)(1 - \beta)}{r + q(\theta)} \int_{\lambda_{\min}}^{\lambda_s(\theta)} \frac{dS_p}{d\theta} dG(\lambda) + \int_{\lambda_s(\theta)}^{\lambda_t(\theta)} \frac{dS_t}{d\theta} dG(\lambda)\end{aligned}$$

where $\frac{dS_p}{d\theta} = \frac{dS_p}{drU} \frac{drU}{d\theta} \leq 0$ and $\frac{dS_t}{d\theta} = \frac{dS_t}{drU} \frac{drU}{d\theta} \leq 0$, so that $\Gamma'(\theta) \leq 0$. Again, the unicity of the equilibrium value of θ follows.

H ‘Spanish type’ versus ‘French type’ regulations

In ‘Spanish type’ regulation, the red-tape destruction cost of temporary jobs before their expiration date is the same as that of permanent jobs. Therefore, when the productivity of a temporary job drops to zero, it is possible either to pay the layoff cost F or to continue to pay the worker until the expiration date of the contract. In this situation, using the same notations and method as in appendix B, the law of motion of the surplus of temporary jobs can be written

$$(r + \lambda) S_t(\lambda, \Delta, y, \tau) = y - rU + \lambda \max [S_t(\lambda, \Delta, 0, \tau), -F] + \dot{S}_t(\lambda, \Delta, y, \tau) \quad (\text{H23})$$

$$rS_t(\lambda, \Delta, 0, \tau) = -rU + \dot{S}_t(\lambda, \Delta, 0, \tau) \quad (\text{H24})$$

where $\dot{S}_t(\lambda, \Delta, x, \tau) = \partial S_t(\lambda, \Delta, x, \tau) / \partial \tau$.

At the date of termination of the temporary contract, there are two possible outcomes: i) if the job has not been hit by a productivity shock, it can be converted into a permanent job. The formal condition reads, when $\tau = \Delta$, as

$$S_t(\lambda, \Delta, y, \Delta) = \max [S_p(\lambda), 0] \quad (\text{H25})$$

where $S_p(\lambda)$ denotes the value of a permanent job. ii) If a shock occurred, the job is destroyed. The formal condition reads, when $\tau = \Delta$, as

$$S_t(\lambda, \Delta, 0, \Delta) = 0. \quad (\text{H26})$$

Let us find the solution to the system of equations (H23), (H24) with terminal conditions (H25), (H26). A general solution to the first-order linear differential equation (H24) is given by:

$$S_t(\lambda, \Delta, 0, \tau) = Ae^{r\tau} + B \quad (\text{H27})$$

where A and B are constants to be determined. Differentiation with respect to τ yields $\dot{S}_t(\lambda, \Delta, 0, \tau) = rAe^{r\tau}$. Plugging this expression together with (H27) into (H24), one gets: $B = -U$. Making use of the terminal condition $S_t(\lambda, \Delta, 0, \Delta) = 0$, it follows that $A = Ue^{-r\Delta}$. Finally, using (H27) we get:

$$S_t(\lambda, \Delta, 0, \tau) = - \left(1 - e^{-r(\Delta-\tau)} \right) U \quad (\text{H28})$$

Let now rewrite (H23) as:

$$\dot{S}_t(\lambda, \Delta, y, \tau) = (r + \lambda)S_t(\lambda, \Delta, y, \tau) - (y - rU) - \lambda \max[S_t(\lambda, \Delta, 0, \tau), -F]$$

A general solution to this equation is given by:

$$S_t(\lambda, \Delta, y, \tau) = e^{(r+\lambda)(\tau-\Delta)} \left[S_t(\lambda, \Delta, y, \Delta) - \underbrace{\int_{\Delta}^{\tau} e^{-(r+\lambda)(\zeta-\Delta)} [(y - rU) + \lambda \max[S_t(\lambda, \Delta, 0, \tau), -F]] d\zeta}_{\Gamma} \right]$$

Using (H28), it is possible to rewrite Γ as:

$$\Gamma = \int_{\Delta}^{\tau} e^{-(r+\lambda)(\zeta-\Delta)} \left[(y - rU) - \lambda \max \left[\left(1 - e^{-r(\Delta-\zeta)}\right) U, F \right] \right] d\zeta$$

Let us define $\bar{\tau} \geq 0$ such that $S_t(\lambda, \Delta, 0, \bar{\tau}) = -F$, we get:

$$1 - e^{-r(\Delta-\bar{\tau})} = \frac{F}{U} \Leftrightarrow \bar{\tau} = \Delta + \frac{1}{r} \ln \left(1 - \frac{F}{U} \right)$$

Remarking that:

$$\max[S_t(\lambda, \Delta, 0, \tau), -F] = \begin{cases} -F & \text{if } \tau \leq \bar{\tau} \\ -(1 - e^{-r(\Delta-\tau)}) U & \text{if } \tau > \bar{\tau} \end{cases}$$

Γ can be written as:

$$\Gamma = (y - rU) \frac{1 - e^{-(r+\lambda)(\tau-\Delta)}}{r + \lambda} - \lambda U \left[\frac{1 - e^{-(r+\lambda)(\bar{\tau}-\Delta)}}{r + \lambda} - \frac{1 - e^{-\lambda(\bar{\tau}-\Delta)}}{\lambda} \right] - \lambda F \frac{e^{-(r+\lambda)(\bar{\tau}-\Delta)} - e^{-(r+\lambda)(\tau-\Delta)}}{r + \lambda}$$

Thus

$$\begin{aligned} S_t(\lambda, \Delta, y, \tau) &= e^{(r+\lambda)(\tau-\Delta)} \max[S_p(\lambda), 0] - (y - rU) \frac{e^{(r+\lambda)(\tau-\Delta)} - 1}{r + \lambda} \\ &\quad + \lambda U \left[\frac{e^{(r+\lambda)(\tau-\Delta)} - e^{-(r+\lambda)(\bar{\tau}-\tau)}}{r + \lambda} - e^{(r+\lambda)(\tau-\Delta)} \frac{1 - e^{-\lambda(\bar{\tau}-\Delta)}}{\lambda} \right] + \lambda F \frac{e^{-(r+\lambda)(\bar{\tau}-\tau)} - 1}{r + \lambda} \end{aligned}$$

Evaluated in $\tau = 0$:

$$\begin{aligned} S_t(\lambda, \Delta, y, 0) &= y \frac{1 - e^{-(r+\lambda)\Delta}}{r + \lambda} + \max[S_p(\lambda), 0] e^{-(r+\lambda)\Delta} - \lambda F \frac{1 - e^{-(r+\lambda)(\Delta-\tilde{\Delta})}}{r + \lambda} \\ &\quad + U \left[\lambda \frac{1 - e^{(r+\lambda)\tilde{\Delta}}}{r + \lambda} e^{-(r+\lambda)\Delta} - e^{-(r+\lambda)\Delta} \left[1 - e^{\lambda\tilde{\Delta}} \right] - r \frac{1 - e^{-(r+\lambda)\Delta}}{r + \lambda} \right] \end{aligned}$$

with $\tilde{\Delta} = -\frac{1}{r} \log \left(1 - \frac{F}{U} \right)$. This equation defines the value of the surplus when $\Delta \geq \tilde{\Delta}$, which corresponds to the expression of the second row of the right hand side of equation (28), where we denote as $S_t(\lambda, \Delta) = S_t(\lambda, \Delta, y, 0) - c$. The first row of the right hand side of equation (28) defines the value of the surplus when $\Delta \leq \tilde{\Delta}$, which is taken from equation (2).

I Mandatory limit

I.1 The threshold value λ_t when there is a mandatory limit on the duration of temporary jobs

Let us define the conditions under which temporary jobs can be created when there is a mandatory limit on the duration of temporary jobs. Temporary jobs can be created if

$$\begin{cases} S_t(\lambda) \geq 0 & \text{when } \lambda \geq \bar{\lambda} \\ S_t(\lambda, \Delta) \geq 0 & \text{when } \lambda \leq \bar{\lambda} \end{cases}$$

where $\bar{\lambda}$ satisfies $\Delta(\bar{\lambda}) = \Delta$. It has been shown in appendix C that there exists a threshold value of λ , denoted by λ_t , such that $S_t(\lambda_t) = 0$, above which no jobs are created.

We know that $S_t(\lambda, \Delta) \leq S_t(\lambda)$, and that $\bar{\lambda}$ is defined by $\Delta(\bar{\lambda}) = \Delta$, which is equivalent to $S_t(\bar{\lambda}, \Delta) = S_t(\bar{\lambda})$. Thus, if $\Delta \leq \Delta(\lambda_t)$, which is equivalent to $\bar{\lambda} \geq \lambda_t$, no temporary job can be created. Then, let us henceforth assume that $\bar{\lambda} < \lambda_t$. Since $S_t(\lambda)$ and $S_t(\lambda, \Delta)$ are decreasing with respect to λ , and $S_t(\bar{\lambda}) = S_t(\bar{\lambda}, \Delta)$, it is worth creating temporary jobs $\lambda \leq \lambda_t$ (see figure 10).

I.2 The threshold value λ_s when there is a mandatory limit on the duration of temporary jobs

There exist values of Δ such that $S(\lambda, \Delta) > S_p(\lambda)$ if λ is sufficiently close to λ_p when condition (6) is fulfilled, as depicted in figure 10. From equations (1) and (2), $S_t(\lambda, \Delta) - S_p(\lambda)$ can be written as

$$S_t(\lambda, \Delta) - S_p(\lambda) = \left(\frac{rU + \lambda F + (r + \lambda)c}{r + \lambda} \right) (1 - e^{-(r+\lambda)\Delta}) - U(1 - e^{-r\Delta}) - c.$$

Which implies that $\lim_{\lambda \rightarrow 0} [S_t(\lambda, \Delta) - S_p(\lambda)] = -ce^{-r\Delta}$. Therefore, there exists at least a threshold value of the shock arrival rate, denoted by λ_s , which statisfies

$$S_t(\lambda_s, \Delta) = S_p(\lambda_s),$$

such that permanent jobs are created if the shock arrival rate is below λ_s .

J Renewal

Notice first that the expression of S_p , the surplus of a temporary job, is not altered by the possibility to renew temporary contracts. Hence, the threshold λ_p , such that $S_p(\lambda_p) = \frac{y - rU - \lambda_p F}{r + \lambda_p} - c = 0$ is the same as in the benchmark case.

The model can then be solved as follows: we first consider renewed temporary contracts and then starting temporary contracts. Third, we show that the equality between the thresholds for the two

types of temporary contracts, λ_{t_1} and λ_{t_2} , is between λ_{s_1} and λ_{s_2} . Fourth, we derive the condition for the existence of both types of contracts. Finally, we conclude by comparing surpluses S_{t_1} and S_{t_2} and durations $\Delta_1(\lambda)$ and $\Delta_2(\lambda)$.

J.1 Renewed temporary contract

From (31), the surplus of a renewed temporary contract writes:

$$S_{t_2}(\lambda, \Delta_2) = \int_0^{\Delta_2} (ye^{-\lambda\tau} - rU) e^{-r\tau} d\tau + \max[S_p(\lambda), 0] e^{-(r+\lambda)\Delta_2} - c$$

The optimal duration of a renewed temporary contract, Δ_2 , then maximizes S_{t_2} . The FOC writes:

$$ye^{-\lambda\Delta_2} - rU - (r + \lambda) e^{-\lambda\Delta_2} \max[S_p(\lambda), 0] = 0 \quad (\text{J29})$$

while the SOC is always met, as in the benchmark case. Therefore, the optimal duration of the contract is:

$$\Delta_2(\lambda) = \begin{cases} \frac{1}{\lambda} \ln \left(\frac{rU + \lambda F + (r+\lambda)c}{rU} \right) & \text{if } \lambda \leq \lambda_p \\ \frac{1}{\lambda} \ln \left(\frac{y}{rU} \right) & \text{if } \lambda \geq \lambda_p \end{cases}$$

where $\lambda_p = \frac{y - rU - rc}{F + c}$, as in the benchmark case. Hence, the surplus S_{t_2} decomposes as:

$$S_{t_2}(\lambda) = \begin{cases} \int_0^{\Delta_2(\lambda)} (ye^{-\lambda\tau} - rU) e^{-r\tau} d\tau + e^{-(r+\lambda)\Delta_2(\lambda)} S_p(\lambda) - c & \text{if } \lambda \leq \lambda_p \\ \int_0^{\Delta_2(\lambda)} (ye^{-\lambda\tau} - rU) e^{-r\tau} d\tau - c & \text{if } \lambda \geq \lambda_p \end{cases}$$

This expression is similar to that of $S_t(\lambda)$ in the benchmark, and $S_{t_2}(\lambda)$ admits the same properties as $S_t(\lambda)$. Therefore, the thresholds λ_{s_2} and λ_{t_2} such that $S_p(\lambda_{s_2}) = S_{t_2}(\lambda_{s_2})$ and $S_{t_2}(\lambda_{t_2}) = 0$ have expressions similar to λ_s and λ_t in the benchmark case, and have identical comparative static properties.

J.2 Initial temporary contract

From (30), we have

$$S_{t_1}(\lambda, \Delta_1) = \int_0^{\Delta_1} (ye^{-\lambda\tau} - rU) e^{-r\tau} d\tau + \max[S_p(\lambda), S_{t_2}(\lambda, \Delta_2), 0] e^{-(r+\lambda)\Delta_1} - c$$

The optimal duration Δ_1 maximizes the surplus of the first temporary contract. The first order condition writes:

$$ye^{-\lambda\Delta_1} - rU - (r + \lambda) e^{-\lambda\Delta_1} \max[S_p(\lambda), S_{t_2}(\lambda, \Delta_2), 0] = 0 \quad (\text{J30})$$

while the second order condition is always fulfilled. This leads to:

$$\Delta_1(\lambda) = \begin{cases} \frac{1}{\lambda} \ln \left(\frac{rU + \lambda F + (r + \lambda)c}{rU} \right) & \text{if } \lambda \leq \lambda_{s_2} \\ \frac{1}{\lambda} \ln \left(\frac{y - (r + \lambda) \left[\int_0^{\Delta_2} (ye^{-\lambda\tau} - rU) e^{-r\tau} d\tau + e^{-(r + \lambda)\Delta_2} S_p(\lambda) - c \right]}{rU} \right) & \text{if } \lambda_{s_2} \leq \lambda \leq \lambda_p \\ \frac{1}{\lambda} \ln \left(\frac{y - (r + \lambda) \left[\int_0^{\Delta_2} (ye^{-\lambda\tau} - rU) e^{-r\tau} d\tau - c \right]}{rU} \right) & \text{if } \lambda_p \leq \lambda \leq \lambda_{t_2} \\ \frac{1}{\lambda} \ln \left(\frac{y}{rU} \right) & \text{if } \lambda \geq \lambda_{t_2} \end{cases}$$

Notice that $\Delta_1(\lambda) = \Delta_2(\lambda)$ for $\lambda \leq \lambda_{s_2}$ and for $\lambda \geq \lambda_{t_2}$. Hence, $\Delta_1(\lambda)$ continuously decreases from $\lim_{\lambda \rightarrow 0} \Delta_1(\lambda) = \lim_{\lambda \rightarrow 0} \Delta_2(\lambda) = +\infty$ to $\lim_{\lambda \rightarrow \infty} \Delta_1(\lambda) = \lim_{\lambda \rightarrow \infty} \Delta_2(\lambda) = 0$. Hence, it follows that S_{t_1} is continuously decreasing from $\lim_{\lambda \rightarrow 0} S_{t_1}(\lambda) = \frac{y}{r} - U - c > 0$ to $\lim_{\lambda \rightarrow \infty} S_{t_2}(\lambda) = -c$.

J.3 Equality of λ_{s_1} and λ_{s_2}

By definition $S_p(\lambda_{s_1}) = S_{t_1}(\lambda_{s_1})$ and $S_p(\lambda_{s_2}) = S_{t_2}(\lambda_{s_2})$ which translates into:

$$S_{t_1}(\lambda_{s_1}) = \int^{\Delta_1(\lambda_{s_1})}$$

J.4 Equality of λ_{t_1} and λ_{t_2}

Notice that as $\lambda_{t_1} \geq \lambda_p$ and $\lambda_{t_2} \geq \lambda_p$, we have $\Delta_1(\lambda) = \frac{1}{\lambda} \ln\left(\frac{y}{rU}\right) = \Delta_2(\lambda)$ and

$$S_{t_1}(\lambda_{t_1}) = \int_0^{\Delta_1(\lambda_{t_1})} \left(ye^{-\lambda\tau} - rU \right) e^{-r\tau} d\tau + \max[S_{t_2}(\lambda_{t_1}), 0] e^{-(r+\lambda)\Delta_1(\lambda_{t_1})} - c = 0$$

while

$$S_{t_2}(\lambda_{t_2}) = \int_0^{\Delta_2(\lambda_{t_2})} \left(ye^{-\lambda\tau} - rU \right) e^{-r\tau} d\tau - c = 0$$

Then, two cases must be considered : (i) if $\lambda_{t_1} \geq \lambda_{t_2}$ or equivalently, $S_{t_1}(\lambda_{t_1}) = 0 \geq S_{t_2}(\lambda_{t_1})$, then

$$0 = \int_0^{\Delta_1(\lambda_{t_1})} \left(ye^{-\lambda\tau} - rU \right) e^{-r\tau} d\tau - c \geq \int_0^{\Delta_2(\lambda_{t_1})} \left(ye^{-\lambda\tau} - rU \right) e^{-r\tau} d\tau - c$$

hence, $S_{t_1}(\lambda_{t_1}) = S_{t_2}(\lambda_{t_1}) = 0$ and $\lambda_{t_1} = \lambda_{t_2}$.

(ii) If $\lambda_{t_1} \leq \lambda_{t_2}$ or equivalently, $S_{t_1}(\lambda_{t_1}) = 0 \leq S_{t_2}(\lambda_{t_1})$, then

$$0 = \int_0^{\Delta_1(\lambda_{t_1})} \left(ye^{-\lambda\tau} - rU \right) e^{-r\tau} d\tau - c + e^{-(r+\lambda)\Delta_1(\lambda_{t_1})} S_{t_2}(\lambda_{t_1}) \leq \int_0^{\Delta_2(\lambda_{t_1})} \left(ye^{-\lambda\tau} - rU \right) e^{-r\tau} d\tau - c$$

As $\Delta_1(\lambda_{t_1}) = \Delta_2(\lambda_{t_1})$, this is possible if and only if $S_{t_1}(\lambda_{t_1}) = S_{t_2}(\lambda_{t_1})$. Hence, $\lambda_{t_1} = \lambda_{t_2}$.

J.5 Comparison of S_{t_1} and S_p

Let us now study the intercept of S_{t_1} and S_p : the logic of the proof is the same as in the benchmark case. S_{t_1} is decreasing from $\lim_{\lambda \rightarrow 0} S_{t_1}(\lambda) = \frac{y}{r} - U - c$ to $\lim_{\lambda \rightarrow \infty} S_{t_1}(\lambda) = -c$ while S_p is decreasing from $\lim_{\lambda \rightarrow 0} S_p(\lambda) = \lim_{\lambda \rightarrow 0} S_{t_1}(\lambda)$ to $\lim_{\lambda \rightarrow \infty} S_p(\lambda) = -c - F$.

From the definition of the two surpluses, we have: $\lim_{\lambda \rightarrow 0} S'_{t_1}(\lambda) = \frac{-y}{r^2}$ while $\lim_{\lambda \rightarrow 0} S'_p(\lambda) = -\frac{y-rU+rF}{r^2}$. Thus in the neighborhood of $\lambda = 0$ there exist values of $\lambda > 0$ such that $S_p(\lambda) > S_t(\lambda)$ if and only if $F < U$. Let us assume that this is the case. Then, S_p and S_t have at least one positive intercept for positive values of $S_t(\lambda)$ if $S_p(\lambda_p) = 0 < S_{t_1}(\lambda_p)$. This yields a condition similar to (6) in the benchmark.

J.6 Comparison of S_{t_1} and S_{t_2}

First, notice that $\max[S_p(\lambda), S_{t_2}(\lambda, \Delta_2), 0] \geq \max[S_p(\lambda), 0]$ implying $\Delta_1(\lambda) \leq \Delta_2(\lambda)$ from the first order conditions (J30) and (J29).

Second, using the FOC (J30), and substituting in S_{t_1} , the surplus of a starting temporary contract writes :

$$S_{t_1}(\lambda) = \frac{y - U(r + \lambda(1 - e^{-r\Delta_1(\lambda)}))}{r + \lambda} - c$$

Similarly, using the FOC (J29), and substituting, we get :

$$S_{t_2}(\lambda) = \frac{y - U(r + \lambda(1 - e^{-r\Delta_2(\lambda)}))}{r + \lambda} - c$$

From these two expressions, it is straightforward to establish that $\Delta_1(\lambda) \leq \Delta_2(\lambda)$ implies $S_{t_1}(\lambda) \geq S_{t_2}(\lambda)$ for all $\lambda \geq 0$. In particular, for $\lambda \leq \lambda_s$ and for $\lambda \geq \lambda_t$, we have $\Delta_1(\lambda) = \Delta_2(\lambda)$, so that $S_{t_1}(\lambda) = S_{t_2}(\lambda)$. This leads to the figures in the text.