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Rollover Risk and Market Freezes

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Abstract

The sub-prime crisis of 2007-2009 has been characterized by a sudden freeze in the market for short-term, secured borrowing. We present a model that can explain a sudden collapse in the amount that can be borrowed against assets with little credit risk. The borrowing in this model takes the form of asset-backed commercial paper that has to be rolled over several times before the underlying assets mature and their true value is revealed. In the event of default, the creditors (holders of commercial paper) can seize the collateral. We assume that there is a small cost of liquidating the assets. The debt capacity of the assets (the maximum amount that can be borrowed using the assets as collateral) depends on the information state of the economy. At each date, in general there is either "good news" (the information state improves), "bad news" (the information state gets worse), or "no news" (the information state remains the same). When rollover risk is high, because debt must be rolled over frequently, we show that the debt capacity is lower than the fundamental value of the asset and in extreme cases may be close to zero. This is true even if the fundamental value of the assets is high in all states. Thus, a small change in information, as measured by a change in the fundamental value, can lead to a "market freeze."

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1 Introduction

1.1 Motivation

One of the many striking features of the sub-prime crisis of 2007-2009 has been the sudden freeze in the market for the rollover of short-term debt. While the rationing of firms in the unsecured borrowing market is not uncommon and has a long-standing theoretical underpinning (see, for example, the seminal work of Stiglitz and Weiss, 1981), what is puzzling is the almost complete inability of financial institutions to issue (or roll over) short-term debt backed by assets with relatively low credit risk. From a theoretical standpoint, this is puzzling because the ability to pledge assets and provide collateral has been considered one of the most important tools available to firms in order to get around credit rationing (Bester, 1985). From an institutional perspective, the inability to borrow overnight against high-quality assets has been a striking market failure that effectively led to the demise of a substantial part of investment banking in the United States. More broadly, it led to the collapse in the United States, the United Kingdom, and other countries of banks and financial institutions that had relied on the rollover of short-term wholesale debt in the asset-backed commercial paper (ABCP) and overnight secured repo markets.

The first such collapse occurred in the summer of 2007. On July 31, 2007, two Bear Stearns hedge funds based in the Cayman Islands and invested in sub-prime assets filed for bankruptcy. Bear Stearns also blocked investors in a third fund from withdrawing money. In the week to follow, more news of problems with sub-prime assets hit the markets. Finally, on August 7, 2007, BNP Paribas halted withdrawals from three investment funds and suspended calculation of the net asset values because it could not "fairly" value their holdings:[1]

> "[T]he complete evaporation of liquidity in certain market segments of the US securitization market has made it impossible to value certain assets fairly regardless of their quality or credit rating... Asset-backed securities, mortgage loans, especially sub-prime loans don't have any buyers... Traders are reluctant to bid on securities backed by risky mortgages because they are difficult to sell on... The situation is such that it is no longer possible to value fairly the underlying US ABS assets in the three above-mentioned funds."

This announcement appeared to cause investors in ABCP, primarily money market funds, to shy away from further financing of ABCP structures. These investors could no longer be guaranteed that there was minimal risk in ABCP debt. In particular, many ABCP vehicles had recourse to sponsor banks that set up these vehicles as off-balance-sheet structures but provided them with liquidity and credit enhancements. If ABCP debt could not be rolled

[1] Source: "BNP Paribas Freezes Funds as Loan Losses Roil Markets" (Bloomberg.com, August 9, 2007).

over, the sponsor banks would have to effectively take assets back onto their balance-sheets. But, given the assets had little liquidity, the banks' ability to raise additional finance – typically rollover debt in the form of financial CP – would be limited too. Money market funds thus faced the risk that the assets underlying ABCP would be liquidated at a loss. This liquidation and rollover risk produced a "freeze" in the ABCP market, raised concerns about counter-party risk amongst banks, and caused LIBOR to shoot upwards. The sub-prime crisis truly took hold as the European Central Bank pumped 95 billion Euros in overnight lending into the market that same day in response to the sudden demand for cash from banks.

The failure of Bear Stearns in mid-March 2008 was the next example of a market freeze.[2] As an intrinsic part of its business, Bear Stearns relied day-to-day on its ability to obtain short-term finance through secured borrowing. At this time, they were reported to be financing $85 billion of assets on the overnight market (Cohan, 2009). Beginning late Monday, March 10, 2008 and increasingly through that week, rumors spread about liquidity problems at Bear Stearns and eroded investor confidence in the firm. Even though Bear Stearns continued to have high quality collateral to provide as security for borrowing,[3] counterparties became less willing to enter into collateralized funding arrangements with the firm. This resulted in a crisis of confidence late in the week, where counterparties to Bear Stearns were unwilling to make even secured funding available to the firm on customary terms. This unwillingness to fund on a secured basis placed enormous stress on the liquidity of Bear Stearns. On Tuesday, March 11, the holding company liquidity pool declined from $18.1 billion to $11.5 billion. On Thursday, March 13, Bear Stearns' liquidity pool fell sharply and continued to fall on Friday. In the end, the market rumors about Bear Stearns' liquidity problems became self-fulfilling and led to the near failure of the firm. Bear Stearns was adequately capitalized at all times during the period from March 10 to March 17, up to and including the time of its agreement to be acquired by J.P. Morgan Chase. Even at the time of its sale, Bear Stearns' capital and its broker dealers' capital exceeded supervisory standards. In particular, the capital ratio of Bear Stearns was well in excess of the 10% level used by the Federal Reserve Board in its well-capitalized standard.

In his analysis of the failure of Bear Stearns, the Federal Reserve Chairman Ben Bernanke observed:[4]

> "[U]ntil recently, short-term repos had always been regarded as virtually risk-

[2] The discussion that follows is based on the Security and Exchange Commission's Chairman Christopher Cox's Letter to the Basel Committee in Support of New Guidance on Liquidity Management, available at: http://www.sec.gov/news/press/2008/2008-48.htm

[3] This high quality collateral mainly consisted of highly rated mortgage-backed assets which had low but not inconsequential credit risk by this time in the sub-prime crisis.

[4] The quotes are based on Ben Bernanke's remarks to the Risk Transfer Mechanisms and Financial Stability Workshop at the Bank for International Settlements, May 29, 2008.

> free instruments and thus largely immune to the type of rollover or withdrawal risks associated with short-term unsecured obligations. In March, rapidly unfolding events demonstrated that even repo markets could be severely disrupted when investors believe they might need to sell the underlying collateral in illiquid markets. Such forced asset sales can set up a particularly adverse dynamic, in which further substantial price declines fan investor concerns about counterparty credit risk, which then feed back in the form of intensifying funding pressures. . . In light of the recent experience, and following the recommendations of the President's Working Group on Financial Markets (2008), the Federal Reserve and other supervisors are reviewing their policies and guidance regarding liquidity risk management to determine what improvements can be made. In particular, future liquidity planning will have to take into account the possibility of a sudden loss of substantial amounts of secured financing."

1.2 Model and results

Our paper is an attempt to provide a theoretical model of such market freezes. We consider the debt capacity of a finitely-lived asset under the following three conditions:

(i) the debt is short-term in nature and, hence, needs to be frequently rolled over;

(ii) in the event of default by the borrower, the underlying assets are sold to buyers who also use short term finance and a (small) liquidation cost is incurred;

(iii) information arrives slowly relative to the frequency of refinancing the debt.

These are essentially the features alluded to in the preceding discussion of the conditions surrounding the freeze in the market for ABCP and the collapse of Bear Stearns.

The use of short term financing gives rise to what we call *rollover risk*, that is, the possibility that the borrower may not be able to obtain a new source of finance to pay off the current lender. In these circumstances, a lender who is unwilling or unable to extend credit until the assets mature has to take into account the risk that the borrower will not be able to find another lender, that is, to roll over his debt. Does such rollover risk diminish the debt capacity of an asset? The answer to this question depends crucially on the rate at which information is released relative to the rate at which debt is being rolled over.

When debt is short-term in nature, it is natural to assume that uncertainty about the credit risk of the underlying asset will not be fully resolved by the date of the next rollover. In fact, debt may have to be rolled over several times before information about the asset is completely revealed. In the period of the 2007-2009 crisis, there were numerous examples of the slow release of information about the quality of assets held by banks or used as collateral

for asset-backed securities. On the one hand, the difficulty of valuing complex securities or assets for which no markets existed led to repeated changes in the losses announced by financial institutions. On the other, these same institutions were loath to reveal all their holdings of troubled assets. For both these reasons, information about the true extent of the financial distress dribbled out over many months.

Although the preceding discussion makes it clear that the release of information is endogenous to the market, from the point of view of lenders it can be treated as an exogenous signal. We model the release of information in terms of the arrival of "news." There is a finite number of possible information states. At each date, one of three things can happen: either there is "good news" (the information state improves), there is "bad news" (the information state gets worse), or there is "no news." If the period between roll over dates is sufficiently short, it is most likely that "no news" will have been released by the time the debt has to be refinanced.

Our main objective in this paper is to characterize the debt capacity[5] of an asset used as collateral for short-term borrowing. The standard result in efficient markets is that the debt capacity of an asset is equal to its NPV or "fundamental" value. We show in the sequel that this result almost never holds. It may be approximately true in very favorable states of the economy, but this is rare. Using the methods developed in this paper, we show that, under mild assumptions, the debt capacity of an asset is *always* smaller than its fundamental value. In addition, our main result shows that, in the worst case, the debt capacity equals the *minimum possible value of the asset* when the number of rollovers is large enough. We call this phenomenon a "market freeze." This remarkable and perhaps counter-intuitive result holds for *arbitrarily small credit risks*, capturing the scenario that Bear Stearns experienced during its failure in March 2008.

The intuition for the market freeze result can be explained as follows. When information arrives slowly relative to the rollover frequency, it is likely that no new information will have arrived by the next time the debt has to be refinanced. The upper bound on the amount of money that can be repaid is the debt capacity at the next rollover date. Since there is a small liquidation cost, issuing debt with face value greater than the debt capacity is unattractive, so the best the borrower can do is to issue debt with a face value equal to the debt capacity assuming no new information. But this locks the borrower into a situation in which he is forced to act as if his condition remains the same forever. In fact, his situation is somewhat worse, because there is always the possibility of bad news arriving, which will force him to default and realize the liquidation cost. These two facts, the need to set the face value low and the possibility of default if bad news arrives, together guarantee that the debt capacity is always less than the fundamental value. In fact, the difference between the fundamental

[5] The *rollover debt capacity* is the maximum amount of debt that can be obtained when the debt has to be rolled over each period.

value and the debt capacity can be very large, because the fundamental value reflects the substantial probability of good news arriving whereas the debt capacity ignores this upside risk. Thus, a small change in the fundamental value of the assets may be accompanied by a drastic fall in the debt capacity, resulting in a market freeze.

Among the key assumptions of our model, we take as given the short-term nature of debt and liquidation costs. That investment banks are (or used to be!) funded with rollover debt and that debt capacity can be higher with short-term debt under some circumstances for many underlying assets, are interesting facts in their own right. Indeed, there exist agency-based explanations in the literature (for example, Diamond, 1989, 1991, 2004, Calomiris and Kahn, 1991, and Diamond and Rajan, 2001a, 2001b) for the existence of short-term debt as optimal financing in such settings. In contrast to this literature, Brunnermeier and Oehmke (2009) consider a model where a financial institution is raising debt from multiple creditors and argue that there may be excessive short-term debt in equilibrium as short-term debt issuance dilutes long-term debt values and creates among various creditors a "maturity rat race." Our model presents another counter-example to the claim that short-term debt maximizes debt capacity: debt capacity through short-term borrowing may in fact be arbitrarily small, suggesting that institutions ought to arrange for this possibility by funding themselves also through sufficiently long-term financing. Providing a micro (for example, an agency-theoretic) foundation for debt maturity in a model where the information about the underlying assets' fundamental value can change as in our model is a fruitful goal for future research, but one that is beyond the scope of this paper.

Our model also assumes that when a lender needs to seize assets, these can only be sold to another buyer who must also finance the assets with short-term debt. Further, the maximum liquidation price the lender can obtain is proportional to, but smaller than, the new debt capacity of assets.[6] From the standpoint of our paper, these two features – short-term borrowing and liquidation costs – imply that market freezes are most likely when the borrowing and/or lending horizon is short-term and the underlying assets are "crowded" in the sense that most financial institutions are on one side of the market for these assets. The first feature generates rollover risk and the second feature generates discounts in asset liquidations.

[6] There is a large body of literature in finance and economics justifying or empirically verifying liquidation costs of assets, and in particular, fire sale discounts in asset liquidation prices during periods of industry- or economy-wide shocks. On the theoretical front, Williamson (1988) and Shleifer and Vishny (1992) link this to the notion of specificity of assets, that is, the non-transferability of assets across industries. On the empirical front, Pulvino (1998), Krugman (1998), Aguiar and Gopinath (2005), Coval and Stafford (2006), Acharya, Bharath and Srinivasan (2007), and Acharya, Shin and Yorulmazer (2007) have provided evidence of fire sales in real and financial markets in a variety of settings. And, on an applied front, a large literature on financial crises (starting with Allen and Gale 1994, 1998) has employed fire sales as a key modeling device.

The rest of the paper proceeds as follows. Section 2 illustrates our basic idea with a two-period, two-state numerical example. Section 3 derives the general result on market freeze for the two-state model when the number of rollovers becomes unbounded. Section 4 generalizes the result to an arbitrary number of states (the proof is in the appendix). Section 5 explains why our results have implications for repo haircuts observed in markets during stress times. Section 6 discusses policy implications regarding ex-post and ex-ante avoidance of market freezes. Section 7 discusses the related literature. Section 8 ends.

2 Numerical example

To illustrate the method of calculating debt capacity in the presence of roll over risk, we present a simple numerical example. The parameter values are chosen for convenience rather than realism.

Suppose that an asset purchased at time 0 will mature at time 1. If the information state at time 1 is H the value of the asset is $V_1^H = 100$; if the state at time 1 is L the value of the asset is $V_1^L = 40$. The purchase of the asset is financed by issuing short-term debt with maturity 0.5, which means that debt issued at time 0 must be rolled over at time 0.5.

The information state evolves between time t and time $t' = t + 0.5$ according to the transition matrix

$$\begin{bmatrix} 0.95 & 0.05 \\ 0.20 & 0.80 \end{bmatrix}.$$

In other words, conditional on being in state H at t, there is a 5% chance of moving to state L at time t', whereas conditional on being in state L at time t, there is a 20% chance of switching to state H at time t'. The fundamental value of the asset in state s at time t is denoted by V_t^s and defined to be the expected terminal value of the asset conditional on the current state and time. Using the terminal values given above and the transition probabilities, we can calculate the fundamental value of the asset in different states at different points of time. These are:

$$V_0^H = 94.75,\ V_0^L = 61;$$
$$V_{0.5}^H = 97, V_{0.5}^L = 52;$$
$$V_1^H = 100,\ V_1^L = 40.$$

At time 1, the fundamental values are 100 in state H and 40 in state L by assumption. At time 0.5, the fundamental value in state L is $52 > 40$ because there is a 20% chance of switching to state H and a value of 100. Similarly, in state H at time 0.5 the fundamental value is $97 < 100$ because there is a 5% chance of switching to state L and a value of 40. The further we get from the terminal date, the closer the fundamental values in the two states get.

We want to calculate the debt capacity at date 0, which is denoted by B_0, and defined to be the maximum amount that can be borrowed using the asset as collateral. For this purpose we assume universal risk neutrality (or risk neutral probabilities) and a zero risk free interest rate. We will denote the maximum debt capacity as B and possible face values of the debt as D, with subscripts denoting the time point and superscripts the state.

We make the important assumption that, if the debt cannot be paid by rolling over the debt at date 0.5 or with asset's cash flows at date 1, then there is a costly liquidation by creditors. In particular, the liquidation value of the asset is 60% of its maximum debt capacity. In other words, a buyer of the asset being liquidated pays 60% of the maximum debt financing that can be raised against the asset.

Consider now the debt capacities at the rollover date 0.5. We need to choose the face value D to maximize the expected value of the debt. It is never optimal to choose $D > 100$ because this leads to default in both states, with associated liquidation costs, but without any increase in the payoff. For values of $D < 100$, the expected value of the debt is increasing in D holding constant the probability of default. Then it is clear that the relevant face values of debt (D) to consider are 40 and 100. For any other value of D we could increase D without changing the probability of default.

If we set the face value $D = 40$, the debt can be paid off at date 1 in both states and the expected value of the debt is 40. If we set the face value $D = 100$, there will be default in state L but not in state H. Then the expected value of the debt depends on the state of the world at date 0.5 because the transition probabilities depend on the state.

$$\begin{aligned} \text{state H} &: \quad 0.95(100) + 0.05(0.6)(40) = 96.20, \\ \text{state L} &: \quad 0.20(100) + 0.80(0.6)(40) = 39.20. \end{aligned}$$

For example, if the state is H at date 0.5, then with probability 0.95 the state is H at date 1 and the debt pays off 100 and with probability 0.05 the state is L at date 1, the asset must be liquidated and the creditors only realize $0.6 \times 40 = 24$.

Comparing the expected values of the debt with the two different face values, we can see that the optimal face value will depend on the state. In state H, the expected value of the debt when $D = 100$ is $96.20 > 40$, so it is optimal to set $D_{0.5}^H = 100$. In state L, on the other hand, the expected value of the debt with face value $D = 100$ is only $39.20 < 40$, so it is optimal to set the face value $D_{0.5}^L = 40$. The debt capacities in the two states are

$$\begin{aligned} B_{0.5}^H &= 96.20, \\ B_{0.5}^L &= 40. \end{aligned}$$

Next, consider the debt capacities at date 0. Now, the relevant face values to consider are 40 and 96.20 (since these are the maximum amounts that can be repaid in each state without incurring default and the associated liquidation costs).

If $D = 40$, the debt capacity is 40 too since the debt capacity at date 0.5 is greater than or equal to 40 in both states and, hence, the debt can always be rolled over. In contrast, if $D = 96.20$, the debt cannot be rolled over in state L at date 0.5 and the liquidation cost is incurred. Thus, the expected value of the debt depends on the state at date 0:

$$\begin{aligned}\text{state H} &: \quad 0.95(96.20) + 0.05(0.6)(40) = 92.57,\\ \text{state L} &: \quad 0.20(96.20) + 0.80(0.6)(40) = 38.06.\end{aligned}$$

Comparing the expected value corresponding to different face values of the debt, we see that the optimal face value is $D_0^H = 96.20$ in state H and $D_0^L = 40$ in state L, so that the debt capacities are

$$\begin{aligned}B_0^H &= 92.57,\\ B_0^L &= 40.\end{aligned}$$

Comparing the debt capacity in state L to the fundamental value, we see that the debt capacity $B_0^L = 40$ is equal to the most pessimistic cash flow at date 1, whereas the fundamental value is $V_0^L = 61$. The fundamental value is higher because it captures the probability of switching to the high state H. Because of the liquidation costs of default, it is never optimal to set the face value of the debt higher than 40 and hence it is never possible to capture any of the upside potential.

In the numerical example above, the liquidation cost was chosen to be sufficiently high to get the debt capacity in low state to always be equal to the most pessimistic cash flow. We show formally in Section 3 that if we hold constant the probability distribution of cash flows at maturity and increase the number of rollover dates, then as long as there is some liquidation cost in default (however tiny), the debt capacity in the low state is equal to the most pessimistic cash flow at all rollover dates whenever the frequency of rollovers is sufficiently large. That is, through a backward-induction argument such as the one in the example, we can show that no matter how high the fundamental value gets, the borrower is always forced to act as if the value of the asset is 40 in the low state at the next rollover date.

The two-state example only allows for no news or bad news in state H and for no news or good news in state L. In general, there will typically be good news, bad news or no news. The model presented in Section 4 extends the setup and the results to an arbitrary number of information states.

3 Model

In this section we introduce the essential ideas in terms of a two-state example. For concreteness, consider the case of a bank that sets up a SIV to hold a collection of assets. The

sponsoring bank's objective is to maximize the value of the assets transferred to the SIV. The SIV is financed by issuing ABCP. The maximum amount the bank can extract as payment for the assets will be equal to the maximum value of the commercial paper that can be sold by the SIV, using the assets as collateral. Thus, in equilibrium, the value of the assets to the sponsoring bank will be equal to their debt capacity.

3.1 Time

Time is represented by the unit interval, $[0, 1]$, and the SIV is set up at the initial date $t = 0$. The assets in the SIV have a finite life (e.g., mortgages) which we normalize to one unit. To keep the analysis simple, we assume that the assets have a terminal value at $t = 1$, but generate no income at dates $0 \leq t < 1$. We also assume that the risk-free interest rate is 0 and that all market participants are risk neutral.

Commercial paper is assumed to have a fixed maturity, denoted by $0 < \tau < 1$, and the debt must be rolled over N times, where

$$\tau = \frac{1}{N+1}.$$

The unit interval is divided into intervals of length τ by a series of dates denoted by t_n and defined by

$$t_n = n\tau, \ \ n = 0, 1, ..., N+1,$$

where t_0 is the date the SIV is set up, t_n is the date of the n-th rollover (for $n = 1, ..., N$), and t_{N+1} is the final date at which the assets mature and their terminal value is realized.

For the time being we treat the maturity of the commercial paper τ and the number of rollovers N as fixed. Later, we will be interested to see what happens when the maturity of commercial paper τ becomes very small and the number of rollovers N becomes correspondingly large.

3.2 Information

There are two states, denoted by s_1 and s_2, where $s_1 = H$ is the "high" state and $s_2 = L$ is the "low" state. Transitions between states occur at the dates t_n and are governed by a stationary transition probability matrix

$$A(\tau) = \begin{bmatrix} a_{11}(\tau) & a_{12}(\tau) \\ a_{21}(\tau) & a_{22}(\tau) \end{bmatrix} = \begin{bmatrix} 1 - p\tau & p\tau \\ q\tau & 1 - q\tau \end{bmatrix},$$

where $a_{ij}(\tau)$ is the probability of a transition from state s_i at date t_n to state s_j when the period length is $\tau > 0$. If the economy is in the high state at date t_n, it remains in the same state at date t_{n+1} with probability $1 - p\tau$ and switches to the low state state at date

t_{n+1} with probability $p\tau$. So, in terms of the story we told in the introduction, $p\tau$ is the probability of "bad news" and $1 - p\tau$ is the probability of "no news." Similarly, in the low state, the probability of "good news" is $q\tau$ and the probability of "no news" is $1 - q\tau$. The shorter the period length, the more likely it is that no news arrives before the next rollover date.

The terminal value of the asset is determined by the information state at the terminal date $t_{N+1} = 1$. If the state is high, the value is V^H and if the state is low, the value is V^L, where $0 < V^L < V^H$. For simplicity, we assume that the asset has a yield of zero and the risk-free interest rate is 0. We also assume that all market participants are risk neutral.

3.3 Liquidation

If the SIV is forced to default and liquidate the assets, we assume that the assets can be sold for a fraction $\lambda \in [0, 1]$ of the *maximum amount of finance that could be raised by the SIV as a going concern*. It is important to note that the recovery rate λ is applied to the maximum debt capacity rather than to the fundamental value of the assets. If the buyer of the assets were a wealthy investor who could buy and hold the assets until maturity, the fundamental value would be the relevant benchmark and the investor might well be willing to pay some fraction of the fundamental value, only demanding a discount to make sure that he does not mistakenly overpay for the assets. What we are assuming here is that the buyer of the assets is another financial institution that must also issue short term debt in order to finance the purchase. Hence, the buyer is constrained by the same forces that determined the debt capacity in the first place. While the debt capacity provides an upper bound on the purchase price, there is no reason to think that the assets will reach this value. In fact, we assume $\lambda < 1$ in what follows.

3.4 Debt capacity

The debt capacity of the assets can be determined by backward induction. Suppose that the economy is in the low state at date t_N, which is the last of the rollover dates. Let D be the face value of the debt issued by the SIV. If $D > V^H$, the SIV will default in both states at date t_{N+1} and the creditors will receive λV^H in the high state and λV^L in the low state. Clearly, the market value of the debt at date t_N would be greater if the face value were $D = V^H$, so it cannot be optimal to choose $D > V^H$. Now suppose that the SIV issues debt with face value D, where $V^L < D < V^H$. This will lead to default in the low state at date t_{N+1} and the creditors will receive D in the high state and λV^L in the low state. Clearly, this is dominated by choosing a higher value of D. Thus, either $D = V^H$ or $D \leq V^L$. An exactly similar argument shows that it cannot be optimal to choose $D < V^L$, so we are left with only two possibilities, either $D = V^H$ or $D = V^L$. In the first case, the market value of

the debt is $(1 - q\tau)\lambda V^L + q\tau V^H$ and in the second case it is V^L. Clearly, for τ sufficiently small,

$$(1 - q\tau)\lambda V^L + q\tau V^H < V^L.$$

In fact, we can calculate that the critical value of τ is

$$\tau^* = \frac{(1 - \lambda) V^L}{q(V^H - V^L)}.$$

For any $\tau < \tau^*$ it is strictly optimal to put $D = V^L$. Then we have found the debt capacity in the low state at date t_N, which we denote by $B_N^L = V^L$.

Now consider the high state at date t_N. It is easy to see, as before, that the only candidates for the face value of the debt are $D = V^H$ and $D = V^L$. If the SIV issues debt with face value V^H, there will be default in the low state. The creditors will receive λV^L in the low state and V^H in the high state and the market value of the debt at date t_N will be $(1 - p\tau) V^H + p\tau\lambda V^L$. If the SIV issues debt with face value V^L, there will be no default, the creditors will receive V^L in both states at date t_{N+1} and the market value of the debt at date t_N will be V^L. If the period length τ is sufficiently short, we can see that

$$(1 - p\tau) V^H + p\tau\lambda V^L > V_L.$$

In fact, the critical value of τ is

$$\tau^{**} = \frac{V^H - V^L}{p(V^H - \lambda V^L)}.$$

So if $\tau < \tau^{**}$, it is optimal to put $D = V^H$. Then we have found the debt capacity in the high state at date t_N, which we denote by $B_N^H = (1 - p\tau) V^H + p\tau\lambda V^L$.

Now suppose that we have calculated the debt capacities B_k^H and B_k^L for $k = n+1, ..., N$. We show by induction that

Proposition 1 *For τ sufficiently small, the debt capacities of the asset in states H and L are given respectively as:*

$$B_k^H = (1 - p\tau)^{N-k} V^H + \left[1 - (1 - p\tau)^{N-k}\right] \lambda V^L, \text{ for } k = n+1, ..., N, \tag{1}$$

and

$$B_k^L = V^L, \text{ for } k = n+1, ..., N. \tag{2}$$

Consider what happens in the low state at date t_n. If the face value of the debt issued by the SIV is D at date t_n, then the SIV will default in the low state if $V^L < D < B_{n+1}^H$ and the SIV will default in both states if $D > B_{n+1}^H$. By our usual argument, the only candidates for

the optimal face value are $D = V^L$ and $D = B^H_{n+1}$. If the face value is $D = V^L$, the creditors will receive V^L in both states at date t_{n+1} and the market value of the debt at date t_n will be V^L. On the other hand, if the face value of the debt is $D = B^H_{n+1}$, the creditors receive B^H_{n+1} in the high state and λV^L in the low state, so the market value of the debt at date t_n is

$$(1 - q\tau)\,\lambda V^L + q\tau B^H_{n+1} \leq (1 - q\tau)\,\lambda V^L + q\tau V^H,$$

since $B^H_{n+1} \leq V^H$. But $\tau < \tau^*$ implies that $(1 - q\tau)\,\lambda V^L + q\tau V^H < V^L$, so the debt capacity is $B^L_n = V^L$.

Now consider the high state. Again, our two candidates for the face value of the debt are $D = B^H_{n+1}$ and $D = V_L$, which yield market values at date t_n of $(1 - p\tau)\,B^H_{n+1} + p\tau\lambda V^L$ and V_L, respectively. From our induction hypothesis,

$$\begin{aligned}(1 - p\tau)\,B^H_{n+1} + p\tau V^L &= (1 - p\tau)\left\{(1 - p\tau)^{N-n-1} V^H + \left[1 - (1 - p\tau)^{N-n-1}\right]\lambda V^L\right\} + p\tau\lambda V^L \\ &= (1 - p\tau)^{N-n} V^H + \left[1 - (1 - p\tau)^{N-n}\right]\lambda V^L.\end{aligned}$$

For τ sufficiently small, this expression is clearly greater than V^L. In fact, the critical value of τ is

$$\tau^n = \frac{1}{p} - \frac{1}{p}\left[\frac{(1-\lambda)\,V^L}{(V^H - \lambda V^L)}\right]^{\frac{1}{N-n}}.$$

For λ close to one, the expression on the right is decreasing in n, so a sufficient condition is $\tau < \tau^0$. Assuming this condition is satisfied, we can see that the face value of the debt will be set equal to $D = B^H_{n+1}$ at date t_n and, by induction, for all dates $t_0, ..., t_N$. We have proved that the debt capacities are given by the formulae in (1) and (2) for all $n = 0, ..., N$.

The qualitative properties of the debt capacities characterized in Proposition 1 are the same as in the numerical example in Section 2. In the low state, the debt capacity B^L_n is constant and equal to the lowest possible terminal value, V^L. The fundamental value of the asset in the low state V^L_n is greater than the debt capacity at every date t_n except at the terminal date, when they are both equal to V^L. In the high state, the debt capacity B^H_n is always less than the fundamental value V^H_n, except at the terminal date when both are equal to V^H. We call this behavior of debt capacity of the asset as a "market freeze" since a switch in the information state from high state to the low state can produce a sudden, sharp drop in debt capacity that is much larger than the drop in fundamental value associated with the switch.

These properties are illustrated in Figure 1. In contrast to our numerical example of Section 2, liquidation cost here is tiny ($\lambda = 0.99$); other parameters are chosen to be $p = 5\%$, $q = 90\%$ and $N = 200$. In other words, the liquidation cost is tiny, but the number of rollovers is large, approximately four times per week for an asset of one-year duration.

— Insert figures 1 and 2 here —

Figure 2 shows the critical role played by the number of rollovers in ensuring that debt capacity in the low state is simply the lowest possible terminal value. With same parameters as in Figure 1, it varies the number of rollovers N in the set $\{10, 20, 50, 100, 200\}$. As the number of rollovers increases, debt capacity is farther and farther away from fundamental value and more so with greater time left to maturity. Also for the first four value of N (other than 200), debt capacity is greater than the lowest possible terminal value of 40, highlighting the important role played by maturity mismatch in generating the market freeze result.

3.5 Intuitive explanation of the market freeze

The intuition for the market freeze result can be explained in terms of the tradeoff between the costs of default and the face value of the debt. Suppose we are in the low information state at date t_n. If the period length τ is sufficiently short, it is very likely that the information state at the next rollover date t_{n+1} will be the low state. Choosing a face value of the debt greater than B^L_{n+1} will increase the payoff to the creditors if good news arrives at the next date (the state switches to H), but it will also lead to default if there is no news (the state remains L). Since there is a liquidation cost, issuing debt with face value greater than the debt capacity is always unattractive if the probability of good news is sufficiently small. Then, the best the borrower can do is to issue debt with a face value equal to the debt capacity assuming no new information. But this implies that the debt capacity in the low state is V^L at every date. In other words, no matter how high the fundamental value is in state L, the borrower is forced to act as if the asset is only worth V^L in order to avoid default.

3.6 Factors driving market freezes

Before we move on to the general case, it may be useful to review the various components of the model to identify the key drivers of the market freeze phenomenon in our setup.

- **Credit risk** While some credit risk is necessary for the market freeze result, an arbitrarily small amount of credit risk is sufficient.

- **Liquidation risk** The market freeze result holds as long as there is a positive liquidation cost $1 - \lambda > 0$. If there is zero liquidation cost then, regardless of credit risk, rollover risk and information structure, the debt capacity of the asset is equal to the fundamental value of the asset (set $D = V^H_{n+1}$ at each date t_n).

- **Rollover frequency** Regardless of the credit risk and the liquidation cost, the debt capacity of buy-to-hold debt is equal to the fundamental value of the asset. Hence, rollover risk is critical to obtaining the market freeze result.

- **Information structure** There are two aspects of the information structure that are crucial. First, information is a Poisson process in which the probability of "news" is proportional to the period length. Secondly, because we are assuming the period is short (the rollover frequency is high), information arrives slowly relative to the rollover frequency.

We conclude that the critical features of the model are the necessity of frequent rollovers and the Poisson arrival of information.

4 Debt capacity in the general case

To extend the analysis of the two state case, we allow for a finite number of information states or signals, denoted by $\mathcal{S} = \{s_1, ..., s_I\}$. At each date t_n, the market observes a public signal $S_n \in \mathcal{S}$. The signals $\{S_n\}$ form a stationary finite Markov chain with transition probabilities given by

$$\Pr\left[S_{n+1} = s_j | S_n = s_i\right] \equiv a_{ij}(\tau) = \begin{cases} p_{ij}\tau & \text{if } i \neq j \\ 1 - \sum_{k\neq i} p_{ik}\tau & \text{if } i = j. \end{cases}$$

The constants $\{p_{ij}\}$ satisfy

$$p_{ij} > 0 \text{ and } \sum_{j\neq i} p_{ij} \leq 1, \text{ for } i \neq j \text{ and } i, j = 1, ..., I.$$

We say that there is "no news" at date t_{n+1} if $S_{n+1} = S_n$ so we regard the current state as the status quo and say that news arrives only if a new information state is observed. Of course, "no news" is also informative so beliefs will be updated even if the information state remains the same. What is important to note is that, when the period length τ is short, the probability of "news" becomes small $p_{ij}\tau$ and the probability of "no news" $1 - \sum_{k\neq i} p_{ik}\tau$ becomes correspondingly large. In that case, the informational content of "no news" is also small.

The terminal value of the assets is a function of the information state at date $t = 1$. We denote by v_i the value of the assets if the terminal state is $S_{N+1} = s_i$ and assume that the values $\{v_1, ..., v_I\}$ satisfy

$$0 < v_1 < \ldots < v_I.$$

Let V_n^i denote the fundamental value of the asset at date t_n in state i. Then clearly the values $\{V_n^i\}$ are defined by putting $V_{N+1}^i = v_i$ for $i = 1, ..., I$ and

$$V_n^i = \sum_{j=1}^{I} a_{ij}(\tau) V_{n+1}^j, \text{ for } n = 0, ..., N \text{ and } i = 1, ..., I.$$

Higher information states are are assumed to be "better" in the sense that

$$V_{in} < V_{i+1,n}, \text{ for all } i = 1, ..., I-1 \text{ and } n = 0, ..., N+1. \tag{3}$$

A sufficient condition for (3) is that $\{a_{i+1,j}(\tau)\}$ strictly dominates $\{a_{i,j}(\tau)\}$ in the sense of first-order stochastic dominance. That is, for all $i = 1, ..., I-1$,

$$\sum_{j=1}^{k} a_{ij}(\tau) > \sum_{j=1}^{k} a_{i+1,j}(\tau), \text{ for all } i, k = 1, ..., I-1. \tag{4}$$

Let B_n^i denote the equilibrium debt capacity of the assets in state s_i at date t_n. By convention, we set $B_{N+1}^i = v_i$ for all i.

Proposition 2 *Suppose that (4) is satisfied. Then there exists $\tau^* > 0$ such that for all $0 < \tau < \tau^*$, for any $n = 0, ..., N$ and $i = 1, ..., I$,*

$$B_n^i = \sum_{j=1}^{i-1} a_{ij}(\tau) \lambda B_{n+1}^j + \sum_{j=i}^{I} a_{ij}(\tau) B_{n+1}^i.$$

Proof. See the appendix. ■

The properties established in Proposition 2 generalize those of Proposition 1. First, in the lowest state, s_1, the debt capacity is constant and equal to v_1, which is strictly less than the fundamental value except at the terminal date. For any state s_i, with $i > 1$, the debt capacity is increasing over time and converges to v_i from below. Furthermore, the debt capacity is strictly less than the fundamental value except at the terminal date. From the formula for the debt capacity, it is clear that the debt capacity is increasing in i for any date $t_n < 1$.

5 Application: Repo haircuts

Our results can alternatively be stated in terms of the so-called "haircut" of an asset when it is pledged for secured borrowing or used in a repo transaction. We can measure the haircut in our model as $1 - \frac{B}{V}$, that is, one minus the ratio of the debt capacity of an asset to the asset's expected or fundamental value (or in other words, the debt capacity for buy-to-hold debt). Our model shows that while the haircut is affected by the fundamental risk of the asset, its primary determinant is the maturity of the debt relative to that of the asset or in other words the "maturity mismatch" in the funding structure of the asset. Naturally, the loss incurred in the event of liquidation of the assets by the lender is a critical determinant too, but our results show that when rollover frequency of debt is very high, even a small liquidation cost may suffice to generate large haircuts.

Shin (2008) documents based on data from Bloomberg, that the typical haircuts on treasuries, corporate bonds, AAA asset-backed securities, AAA residential mortgage-backed securities and AAA jumbo prime mortgages are respectively, less than 0.5%, 5%, 3%, 2% and 5%, whereas, in March 2008, these haircuts respectively rose to between 0.25% and 3%, 10%, 15%, 20% and 30%. Brunnermeier and Pedersen (2008) also discuss the widening of haircuts in stress times. Interestingly, while some of the collateralized debt obligations (CDO) have had no secured borrowing capacity at all during the crisis of 2007-2009, equities – which are in principle riskier assets – had only around a 20% haircut (see Box 1.5 from Chapter 1, Page 42 of IMF (2008)).

Our model can potentially explain this apparently puzzling evidence. The model shows that the haircut in borrowing against an asset should depend upon the maturity of the debt in borrowing, and equally importantly, on what is the rollover risk involved, that is, how other potential owners of the asset fund its purchase. It is reasonable to conclude that a large number of mortgage-backed and other asset-backed securities were funded by almost all potential buyers before and during this crisis by short-term rollover debt, whereas equities in contrast are held by relatively long-term investors. This difference in funding structure may be sufficient to generate lower haircuts for equities even if equities be riskier assets overall than asset-backed securities.

Our results also apply to other institutional settings. The most natural candidate, as we have discussed, is the practice of taking assets off-balance-sheet, putting enough capital and liquidity/credit enhancements to make them "bankruptcy-remote" and AAA-rated, and then borrowing short-term against these assets. Such structures, characterized by a maturity mismatch between assets and liabilities, were prevalent in many forms ("structured investment vehicles" or SIVs, "conduits," and others) in the period leading up to the sub-prime crisis (Crouhy, Jarrow and Turnbull, 2007). Yet another candidate is the commercial paper market accessed primarily by financial institutions, but also by highly-rated industrial corporations, where rollover at short maturities is a standard feature. Consider the balance-sheet of a financial institution (such as that of Northern Rock or of the investment banks Bear Stearns and Lehman Brothers) where the funding model inherently resembles that of a SIV, that is, long-term risky assets such as mortgages are funded by short-term, asset-backed commercial paper. These markets experienced severe stress during the sub-prime crisis and froze (large haircut) for many days at a stretch once the expectations about the quality of mortgage assets became pessimistic, even though, prior to this period, they appeared to be the cheapest form of financing available (near-zero haircut).

While an arbitrarily small liquidation cost is sufficient for the market freeze result, the magnitude of λ is an important determinant of debt capacity and haircut for a given number of rollovers. A small value of λ is most likely where the likely acquirers of assets are all on one side of the market. That is, when potential acquirers are likely to be hit by a common

shock or the underlying trade is "crowded", then its liquidation will lead to fire sales. An alternative interpretation is that a market freeze is likely in assets that are complex and where owners of assets have expertise or management skills which are not transferable when lenders try to seize and liquidate the asset (Williamson, 1988, Shleifer and Vishny, 1992). A fitting example here might be mortgage-backed securities since prepayment and default risk of households on home loans are borne by the financial sector as a whole, but the risk gets repackaged also within the financial sector through these securities. Hence, a common shock to the financial sector would render this asset class relatively illiquid, as witnessed briefly during the Long Term Capital Management crisis of 1998 and more extensively during the sub-prime crisis of 2007-2009.

Overall, the sub-prime crisis of 2007-2009 illustrates our key assumptions: Assets were funded with short-term rollover debt; once the fundamentals deteriorated and crisis broke out, further short-term financing was difficult to obtain for most institutions; and there have been substantial discounts in the sale of assets in SIVs and conduits.[7]

6 Policy implications

It is tempting to ask the question: Can a regulator do something to unfreeze the market? Note that our model is partial equilibrium. It shows how the nature of information arrival affects rollover debt capacity. It does not endogenize either the short-term nature of debt or the liquidation cost in case of default. Hence, any policy implications we draw below must also be viewed as partial equilibrium ones, narrowly focused at alleviating the market freeze without attention to any other related efficiency issues or unintended consequences.

We discuss two possible policy interventions, one that affects liquidation values (λ) and other that affects the maturity mismatch in funding of risky assets (N, the number of rollovers).

6.1 Improving the liquidation value of assets

Recall that in our model, debt capacity shrinks with rollovers since in case of default, asset must be sold to another player also constrained to borrow using rollover debt. If the maturity

[7]See, for instance, "SIV restructuring: A ray of light for shadow banking," *Financial Times, June 18 2008*; and "Creditors find little comfort in auction of SIV Portfolio assets," *Financial Times, July 18 2008*, which both report that net asset values due to asset fire sales have fallen below 50% of paid-in capital. As the first article reports: "[W]hen defaults on US subprime mortgages rose last summer, ABCP investors stopped buying [short-term ABCP] notes – creating a funding crisis at SIVs. . . . This situation prompted deep concern about the risk of a looming firesale of assets. The prospect was deemed so alarming that the US Treasury attempted to organize a so-called "super-SIV" last autumn, which was supposed to purchase SIV assets."

of borrowing of alternative buyers of assets lengthens, then the haircut in borrowing would fall and so would liquidation costs. During a systemic crisis, however, there are few private agents with long-term horizons because the ultimate providers of finance – the households – themselves become short-term focused. A regulator such as the Central Bank or the Treasury can, in principle, directly attempt to improve the liquidation value (λ) of affected assets by lending against the asset as a collateral based on its full buy-and-hold value.

This argument could provide a rationale for the wide variety of lending facilities created by the Federal Reserve during the sub-prime crisis to lend to a large number of borrowers, against a wide variety of collateral, at minimal (if any) haircut.[8] In practice, such facilities had a temporary effect on most markets they intervened in, but they appear to have failed to resolve the market freezes completely. While the reason behind this failure remains an important puzzle, one possible explanation could be the following. Our model suggests that for haircuts to vanish or become minimal, at *each* rollover date until maturity of the asset, the lender should anticipate *guaranteed* regulatory lending against the assets to potential buyers. The Federal Reserve facilities were, however, newly introduced and their horizon of provision was announced as short-term, with discretionary extension in future. The residual uncertainty left by such discretion may have been a factor that prevented a complete thawing of asset-backed debt markets in spite of substantial short-term interventions.

6.2 Requiring higher "capital" in asset-backed finance

An alternative policy implication of our model is that while short-term rollover debt entails only a small financing cost in good times, its availability can dry up suddenly when fundamentals deteriorate. The excessive reliance on rollover finance creates a severe maturity mismatch for borrowers such as SIV's and conduits and exposes them to low likelihood but high magnitude funding risk. It may be more prudent for such borrowers to account for such funding risk and complement rollover debt in their capital structure with forms of capital

[8]For example, in addition to the traditional tools the Fed uses to implement monetary policy (e.g., Open Market Operations, Discount Window, and Securities Lending program), five new programs were implemented during August 2007 to March 2008: 1) Term Discount Window Program (announced 8/17/2007) - extended the length of discount window loans available to institutions eligible for primary credit from overnight to a maximum of 90 days; 2) Term Auction Facility (TAF) (announced 12/12/2007) - provides funds to primary credit eligible institutions through an auction for a term of 28 days; 3) Single-Tranche OMO (Open Market Operations) Program (announced 3/7/2008) - allows primary dealers to secure funds for a term of 28 days. These operations are intended to augment the single day repurchase agreements (repos) that are typically conducted; 4) Term Securities Lending Facility (TSLF) (announced 3/11/2008) - allows primary dealers to pledge a broader range of collateral than is accepted with the Securities Lending program, and also to borrow for a longer term — 28 days versus overnight; and, 5) Primary Dealer Credit Facility (PDCF) (announced 3/16/2008) - is an overnight loan facility that provides funds directly to primary dealers in exchange for a range of eligible collateral.

such as long-term debt and equity capital that face lower rollover risk.

Of course, long-term finance may be difficult to raise once a crisis erupts due to information reasons or if the crisis is systematic in nature. Hence, the reduced reliance on rollover debt must be a part of prudential capital structure choice in good times rather than being undertaken during bad times. While financial institutions should have privately recognized the risks of rollover finance, it cannot be ruled out that such risks were not yet fully understood. Going forward, a prudential regulator could play the role of a supervisory watchdog, looking out for excessive reliance on rollover finance (not just in regulated but also in the shadow banking sector), and encouraging in such cases a greater reliance on long-term capital, through moral suasion or rule-based policies. Since long-term capital may be infeasible beyond a point for opaque, off-balance-sheet structures such as SIV's and conduits, such regulatory push will most likely reduce their incidence in the first place.

7 Related literature

At a general level, our result on market freezes can be considered a generalization of the Shleifer and Vishny (1992) result that when potential buyers of assets of a defaulted firm are themselves financially constrained, there is a reduction in the ex-ante debt capacity of the industry as a whole. We expand on their insight by considering short-term debt financing of long-term assets with rollovers to be met by new short-term financing or liquidations to other buyers also financed through short-term debt. Our "market freeze" result can be considered as a particularly perverse dynamic arising through the Shleifer and Vishny (1992) channel at each rollover date, that through backward induction, can in the worst case drive short-term debt capacity of an asset to its most pessimistic cash flow.

More specifically, our paper is related to the literature on haircuts, freezes and runs in financial markets. Rosenthal and Wang (1993) use a model where owners occasionally need to sell their assets for exogenous liquidity reasons through auctions with private information. Because of the auction format, sellers may not be able to extract the full value of the asset and this liquidation cost gets built into the market price of the asset, making the market price systematically lower than the fundamental value. In our model, the source of the haircut is not the private information of potential

He and Xiong (2009) consider a model of dynamic bank runs in which bank creditors have supplied debt maturing at differing maturities and each creditor faces the risk at the time of rolling over that fundamentals may deteriorate before remaining debt matures causing a fire sale of assets. In their model, volatility of fundamentals plays a key role in driving the runs even when the average value of fundamentals has not been affected. Our model of freeze or "run" of short-term debt is complementary to theirs, and somewhat different in the sense that *both* average value and uncertainty about fundamentals are held constant in our model,

but it is the nature of revelation of uncertainty over time – whether good news arrives early or bad news arrives early – that determines whether there is rollover risk in short-term debt or not.

Huang and Ratnovski (2008) model the behavior of short-term wholesale financiers who prefer to rely on noisy public signals such as market prices and credit ratings, rather than producing costly information about the institutions they lend to. Hence, wholesale financiers run on other institutions based on imprecise public signals, triggering potentially inefficient runs. While their model is about runs in the wholesale market as is ours, their main focus is to challenge the peer-monitoring role of wholesale financiers, whereas our main focus is the role of rollover and liquidation risk in generating such runs.

An alternative modelling device to generate market freezes is to employ the notion of Knightian uncertainty (see Knight, 1921) and agents' overcautious behavior towards such uncertainty. Gilboa and Schmeidler (1989) build a model where agents become extremely cautious and consider the worst-case among the possible outcomes, that is, agents are uncertainty averse and use maxmin strategies when faced with Knightian uncertainty. Dow and Werlang (1992) apply the framework of Gilboa and Schmeidler (1989) to the optimal portfolio choice problem and show that there is an interval of prices within which uncertainty-averse agents neither buy nor sell the asset. Routledge and Zin (2004) and Easley and O'Hara (2005, 2008) use Knightian uncertainty and agents that use maxmin strategies to generate widening bid-ask spreads and freeze in financial markets. Caballero and Krishnamurthy (2007) also use the framework of Gilboa and Schmeidler (1989) to develop a model of flight to quality during financial crises: During periods of increased Knightian uncertainty, agents refrain from making risky investments and hoard liquidity, leading to flight to quality and freeze in markets for risky assets.

As opposed to these models, in our model agents maximize their expected utility and the main source of the market freeze is rollover and liquidation risk which become relevant when the rate of arrival of good news is slower than the rate at which debt is being rolled over. While both types of models can generate market freezes, we believe our model, by emphasizing the rollover and liquidation risk, better captures important features of the recent crisis where the market for rollover debt completely froze while equities - typically considered to be more risky - continued to trade with haircuts of only 20%.

8 Conclusion

In this paper, we have attempted to provide a simple information-theoretic model for freezes in the market for secured borrowing against finitely lived assets. The key ingredients of our model were rollover risk, liquidation risk and slow arrival of good news relative to the frequency of debt rollovers. In particular, our model could be interpreted as a micro-foundation

for the funding risk arising in capital structures of financial institutions or special purpose vehicles that have extreme maturity mismatch between assets and liabilities.

In future work, it would be interesting to embed an agency-theoretic role for short-term debt, which we assumed as given, and see how the desirability of such rollover finance is affected when information problems can lead to complete freeze in its availability. While we took the pattern of release of information about the underlying asset as either ordained by nature or determined by investors' expectations, it seems worthwhile to reflect on its deeper foundations, and thereby assess whether a strategic disclosure of information by agents in charge of the asset can alleviate (or aggravate) the problem of freezes to some extent.

Appendix: Proofs

We can solve for the equilibrium debt capacities by backward induction. Let D denote the face value of the debt issued in state s_i at date t_n. This debt will pay off D in state s_j at date t_{n+1} if $D_n^i \leq B_{n+1}^j$ and λB_{n+1}^j otherwise. In other words, the market value of the debt is given by the formula

$$\sum_{B_{n+1}^j < D} a_{ij}(\tau)\,\lambda B_{n+1}^j + \sum_{B_{n+1}^j \geq D} a_{ij}(\tau)\,D$$

and the debt capacity is given by

$$B_n^i = \max_D \left\{ \sum_{B_{n+1}^j < D} a_{ij}(\tau)\,\lambda B_{n+1}^j + \sum_{B_{n+1}^j \geq D} a_{ij}(\tau)\,D \right\}.$$

Proposition 3 $B_n^i \leq B_n^{i+1}$, *for* $i = 1, ..., I-1$ *and* $n = 0, ..., N+1$.

Proof. The claim is clearly true by definition when $n = N+1$, so suppose it is true for some arbitrary number $n+1$. Then, for any D and $i = 1, ..., I-1$, it is clear that

$$\sum_{B_{n+1}^j < D} a_{ij}(\tau)\,\lambda B_{n+1}^j + \sum_{B_{n+1}^j \geq D} a_{ij}(\tau)\,D \leq \sum_{B_{n+1}^j < D} a_{i+1,j}(\tau)\,\lambda B_{n+1}^j + \sum_{B_{n+1}^j \geq D} a_{i+1,j}(\tau)\,D,$$

because $\{a_{ij}(\tau)\}$ is first-order stochastically dominated by $\{a_{i+1,j}(\tau)\}$. It follows immediately that $B_n^i \leq B_n^{i+1}$ for $i = 1, ..., I$. The claim follows by induction. ■

Let D_n^i denote the optimal face value of the debt in state i at date t_n. It is clear that market value of the debt is maximized by setting the face value $D = B_{n+1}^j$, for some value of $j = 1, ..., n$. Thus, we can write the equilibrium condition as

$$B_n^i = \max_{k=1,...,I} \left\{ \sum_{j=1}^{k-1} a_{ij}(\tau)\,\lambda B_{n+1}^j + \sum_{j=k}^{I} a_{ij}(\tau)\,B_{n+1}^k \right\},$$

for $i = 1, ..., I$ and $n = 0, ..., N$.

Proposition 4 *For all $\tau > 0$ sufficiently small, $D_n^i = B_{n+1}^i$ for all $i = 1, ..., I$ and $n = 0, ..., N$.*

Proof. The proof is by backward induction. Consider first the case $n = N$, so $B_{n+1}^i = v_i$ for $i = 1, ..., I$. Then, for any $k > i$,

$$\begin{aligned}
& \sum_{j=1}^{i-1} a_{ij}(\tau)\lambda v_j + \sum_{j=i}^{I} a_{ij}(\tau) v_i - \sum_{j=1}^{k-1} a_{ij}(\tau)\lambda v_j - \sum_{j=k}^{I} a_{ij}(\tau) v_k \\
= \ & \sum_{j=i}^{k-1} a_{ij}(\tau)(v_i - \lambda v_j) + \sum_{j=k}^{I} a_{ij}(\tau)(v_i - v_k) \\
= \ & \left(1 - \sum_{j \neq i} p_{ij}\tau\right)(v_i - \lambda v_i) + \sum_{j=i+1}^{k-1} p_{ij}\tau(v_i - \lambda v_j) + \sum_{j=k}^{I} p_{ij}\tau(v_i - v_k) \\
\geq \ & \left(1 - \sum_{j \neq i} p_{ij}\tau\right)(1-\lambda) v_i + \tau(v_1 - v_I),
\end{aligned}$$

since $(v_i - \lambda v_j) \geq (v_i - v_j) \geq (v_1 - v_I)$ for $j = i+1, ..., I$ and $\sum_{j=i+1}^{I} p_{ij} \leq 1$. Then it is clear that for $\tau > 0$ sufficiently small, the last expression above is positive.

Similarly, for any $k < i$,

$$\begin{aligned}
& \sum_{j=1}^{i-1} a_{ij}(\tau)\lambda v_j + \sum_{j=i}^{I} a_{ij}(\tau) v_i - \sum_{j=1}^{k-1} a_{ij}(\tau)\lambda v_j - \sum_{j=k}^{I} a_{ij}(\tau) v_k \\
= \ & \sum_{j=k}^{i-1} a_{ij}(\tau)(\lambda v_j - v_k) + \sum_{j=i}^{I} a_{ij}(\tau)(v_i - v_k) \\
= \ & \sum_{j=k}^{i-1} p_{ij}\tau(\lambda v_j - v_k) + \left(1 - \sum_{j=k}^{i-1} p_{ij}\tau\right)(v_i - v_k) \\
= \ & \sum_{j=k}^{i-1} p_{ij}\tau(\lambda v_j - v_i) + (v_i - v_k) \\
\geq \ & \tau(\lambda v_1 - v_I) + (v_i - v_k),
\end{aligned}$$

since $(\lambda v_j - v_i) \geq (\lambda v_1 - v_I)$ for $j = k, ..., i-1$ and $\sum_{j=k}^{i-1} p_{ij} \leq 1$. Then it is clear that, for $\tau > 0$ sufficiently small, the last expression is non-negative.

This completes the proof that $D_N^i = v_i$.

Now suppose that we have shown that $D_n^i = B_{n+1}^i$ for some n and $i = 1, ..., I$. Then the formula for B_n^i can be rewritten as follows:

$$\begin{aligned}
B_n^i &= \sum_{j=1}^{i-1} a_{ij}(\tau)\,\lambda B_{n+1}^j + \sum_{j=i}^{I} a_{ij}(\tau)\, B_{n+1}^i \\
&= \sum_{j=1}^{i-1} a_{ij}(\tau)\,\lambda B_{n+1}^j + \left(1 - \sum_{j=1}^{i-1} a_{ij}(\tau)\right) B_{n+1}^i \\
&= B_{n+1}^i - \sum_{j=1}^{i-1} a_{ij}(\tau)\left(B_{n+1}^i - \lambda B_{n+1}^j\right).
\end{aligned}$$

>From this formula, it is clear that

$$\begin{aligned}
B_n^{i+1} - B_n^i &= B_{n+1}^{i+1} - B_{n+1}^i - \sum_{j=1}^{i} a_{i+1,j}(\tau)\left(B_{n+1}^{i+1} - \lambda B_{n+1}^j\right) + \sum_{j=1}^{i-1} a_{ij}(\tau)\left(B_{n+1}^i - \lambda B_{n+1}^j\right) \\
&\geq B_{n+1}^{i+1} - B_{n+1}^i - \sum_{j=1}^{i} a_{i+1,j}(\tau)\left(B_{n+1}^{i+1} - \lambda B_{n+1}^j\right) \\
&\geq B_{n+1}^{i+1} - B_{n+1}^i - \tau\left(v_I - v_1\right)
\end{aligned}$$

since $B_{n+1}^{i+1} - \lambda B_{n+1}^j \leq v_I - \lambda v_1$ for $j = 1, ..., i$ and $\sum_{j=1}^{i} a_{i+1,j}(\tau) \leq \tau$. Similarly,

$$\begin{aligned}
B_n^{i+1} - B_n^i &= B_{n+1}^{i+1} - B_{n+1}^i - \sum_{j=1}^{i} a_{i+1,j}(\tau)\left(B_{n+1}^{i+1} - \lambda B_{n+1}^j\right) + \sum_{j=1}^{i-1} a_{ij}(\tau)\left(B_{n+1}^i - \lambda B_{n+1}^j\right) \\
&\leq B_{n+1}^{i+1} - B_{n+1}^i + \sum_{j=1}^{i-1} a_{ij}(\tau)\left(B_{n+1}^i - \lambda B_{n+1}^j\right) \\
&\leq B_{n+1}^{i+1} - B_{n+1}^i + \tau\left(v_I - \lambda v_1\right)
\end{aligned}$$

since $B_{n+1}^i - \lambda B_{n+1}^j \leq v_I - \lambda v_1$ and $\sum_{j=1}^{i} a_{i+1,j}(\tau) \leq \tau$. Repeated substitution in these inequalities shows that

$$v_{i+1} - v_i - (N + 1 - n)\,\tau\left(v_I - \lambda v_1\right) \leq B_n^{i+1} - B_n^i \leq v_{i+1} - v_i + (N + 1 - n)\,\tau\left(v_I - \lambda v_1\right).$$

We will use these inequalities in the proof of the induction step.

Consider what happens at date t_n. The argument is similar to the one used for $n = N$. For any $k > i$,

$$\begin{aligned}
& \sum_{j=1}^{i-1} a_{ij}(\tau)\lambda B_{n+1}^{j} + \sum_{j=i}^{I} a_{ij}(\tau) B_{n+1}^{i} - \sum_{j=1}^{k-1} a_{ij}(\tau)\lambda B_{n+1}^{j} - \sum_{j=k}^{I} a_{ij}(\tau) B_{n+1}^{k} \\
= & \sum_{j=i}^{k-1} a_{ij}(\tau)\left(B_{n+1}^{i} - \lambda B_{n+1}^{j}\right) + \sum_{j=k}^{I} a_{ij}(\tau)\left(B_{n+1}^{i} - B_{n+1}^{k}\right) \\
= & \left(1 - \sum_{j\neq i} p_{ij}\tau\right)\left(B_{n+1}^{i} - \lambda B_{n+1}^{i}\right) + \sum_{j=i+1}^{k-1} p_{ij}\tau\left(B_{n+1}^{i} - \lambda B_{n+1}^{j}\right) + \sum_{j=k}^{I} p_{ij}\tau(B_{n+1}^{i} - B_{n+1}^{k}) \\
\geq & \left(1 - \sum_{j\neq i} p_{ij}\tau\right)(1-\lambda) v_1 + \tau\left(v_1 - v_I\right),
\end{aligned}$$

since $B_{n+1}^{i} \geq v_1$, $B_{n+1}^{i} - \lambda B_{n+1}^{j} \geq B_{n+1}^{i} - B_{n+1}^{j} \geq (v_1 - v_I)$ for $j = i+1, ..., I$ and $\sum_{j=i+1}^{I} p_{ij} \leq 1$. Then it is clear that, for τ sufficiently small, the last expression above is positive.

Similarly, for $k < i$,

$$\begin{aligned}
& \sum_{j=1}^{i-1} a_{ij}(\tau)\lambda B_{n+1}^{j} + \sum_{j=i}^{I} a_{ij}(\tau) B_{n+1}^{i} - \sum_{j=1}^{k-1} a_{ij}(\tau)\lambda B_{n+1}^{j} - \sum_{j=k}^{I} a_{ij}(\tau) B_{n+1}^{k} \\
= & \sum_{j=k}^{i-1} a_{ij}(\tau)\left(\lambda B_{n+1}^{j} - B_{n+1}^{k}\right) + \sum_{j=i}^{I} a_{ij}(\tau)\left(B_{n+1}^{i} - B_{n+1}^{k}\right) \\
\geq & \sum_{j=k}^{i-1} p_{ij}\tau\left(\lambda B_{n+1}^{j} - B_{n+1}^{k}\right) + \left(1 - \sum_{j\neq i} p_{ij}\tau\right)\left(B_{n+1}^{i} - B_{n+1}^{k}\right) \\
\geq & \tau\left(\lambda v_1 - v_I\right) + \left(1 - \sum_{j\neq i} p_{ij}\tau\right)\left(v_i - v_k - (N-n)\tau\left(v_I - v_1\right)\right),
\end{aligned}$$

since $\lambda B_{n+1}^{j} - B_{n+1}^{k} \geq \lambda v_1 - v_I$, $\sum_{j=k}^{i-1} p_{ij} \leq 1$ and $B_{n+1}^{i} - B_{n+1}^{k} \geq v_i - v_k - (N-n)\tau(v_I - v_1)$. Then, it is clear that, for τ sufficiently small, the last expression is positive. This proves that $D_n^i = B_{n+1}^i$ for $i = 1, ..., I$ and by induction, we have shown that this result is true for all $n = 0, ..., N$. ■

References

Acharya, Viral, Sreedhar Bharath and Anand Srinivasan (2007). "Does Industry-Wide Distress Affect Defaulted Firms? — Evidence from Creditor Recoveries," *Journal of Financial Economics* **85**(3), 787-821.

Acharya, Viral V., Hyun-Song Shin and Tanju Yorulmazer (2007). "Fire-sale FDI," Working Paper, London Business School.

Aguiar, Mark, and Gita Gopinath, 2005, "Fire-Sale FDI and Liquidity Crises," *The Review of Economics and Statistics* **87**(3): 439-542.

Allen, Franklin and Douglas Gale (1994). "Liquidity Preference, Market Participation and Asset Price Volatility," *American Economic Review* **84**, 933–955.

Allen, Franklin and Douglas Gale (1998). "Optimal Financial Crises," *Journal of Finance* **53**, 1245–1284.

Coval, Joshua and Erik Stafford (2006) "Asset Fire Sales in Equity Markets,' *Journal of Financial Economics*, forthcoming.

Bester, H. (1985). "Screening vs. Rationing in Credit Markets with Imperfect Information," *American Economic Review* **75**(4), 850–855.

Brunnermeier, Markus and Martin Oehmke (2009). "The Maturity Rat Race," Working Paper, Princeton University.

Brunnermeier, Markus and Lasse H. Pedersen (2005). "Market Liquidity and Funding Liquidity," *Review of Financial Studies*, forthcoming.

Caballero, Ricardo J. and Arvind Krishnamurthy (2007). "Collective Risk Management in a Flight to Quality Episode," *Journal of Finance*, forthcoming.

Calomiris, Charles and Charles Kahn (1991). "The Role of Demandable Debt in Structuring Optimal Banking Arrangements," *American Economic Review* **81**, 497–513.

Cohan, William D. (2009). *House of Cards: A Tale of Hubris and Wretched Excess on Wall Street.* New York: Knopf Doubleday.

Crouhy, Michel G., Robert A. Jarrow and Stuart M. Turnbull (2007). "The Subprime Credit Crisis of 2007," Working Paper, Cornell University.

Diamond, Douglas (1989). "Reputation Acquisition in Debt Markets," *Journal of Political Economy* **97** (4), 828-862.

Diamond, Douglas (1991). "Debt Maturity Structure and Liquidity Risk," *Quarterly Journal of Economics* **106** (3),709-737.

Diamond, Douglas (2004). "Presidential Address: Committing to Commit: Short Term Debt When Enforcement is Costly, " *Journal of Finance* **59** (4), 1447-1479.

Douglas W. Diamond and Raghuram G. Rajan (2001a). "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking," *Journal of Political Economy* **109**(2), 287-327.

Diamond, Douglas and Raghuram G. Rajan (2001b). "Banks, Short Term Debt and Financial Crises: Theory, Policy Implications and Applications," *Carnegie Rochester Conference on Public Policy* **54**, 37–71.

Dow, James and Sergio Werlang (1992). "Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio," *Econometrica* **60**, 197-204.

Easley, David and Maureen O'Hara (2005). "Regulation and Return: The Role of Ambiguity," Working Paper, Cornell University.

Easley, David and Maureen O'Hara (2008). "Liquidity and Valuation in an Uncertain World," Working Paper, Cornell University.

Feller, William (1968). *An Introduction to Probability Theory and Its Applications, Volume I,* Third Edition. New York: John Wiley and Sons.

Gilboa, Itzhak and David Schmeidler (1989). "Maxmin Expected Utility with Non-unique Priors," *Journal of Mathematical Economics* **18**, 141-153.

He, Zhiguo and Wei Xiong (2009). "Dynamic Bank Runs," Working Paper, Princeton University.

Huang, Rocco and Lev Ratnovski (2008). "The Dark Side of Wholesale Bank Funding," Working Paper, Federal Reserve Bank of Philadelphia.

International Monetary Fund (IMF) (2008). "Assessing Risks to Global Financial Stability," mimeo.

Knight, Frank (1921) *Risk, Uncertainty and Profit* (Houghton Mifflin, Boston).

Krugman, Paul (1998). "Fire-sale FDI," available at
http://web.mit.edu/krugman/www/FIRESALE.htm.

Pulvino, Todd C. (1998). "Do Asset Fire Sales Exist: An Empirical Investigation of Commercial Aircraft Sale Transactions," *Journal of Finance* **53**, 939–978.

Rosenthal, Robert W. and Ruqu Wang (1993). "An Explanation of Inefficiency in Markets and a Justification for Buy-and-Hold Strategies," *Canadian Journal of Economics* **26**(3), 609–624.

Routledge, Bryan and Stanley Zin (2004). "Model Uncertainty and Liquidity," NBER Working Paper # 8683.

Shin, Hyun Song (2008). "Reflections on Modern Bank Runs: A Case Study of Northern Rock," Working Paper, Princeton University.

Shleifer, Andrei and Robert Vishny (1992). "Liquidation values and debt capacity: A market equilibrium approach," *Journal of Finance* **47**, 1343–1366.

Stiglitz, Joseph and Weiss, A. (1981). "Credit Rationing in Markets with Imperfect Information," *American Economic Review* **71**, 393–410.

Williamson, Oliver E. (1988). "Corporate Finance and Corporate Governance," *Journal of Finance* **43**, 567–592.