The Puzzling Inventory Growth Risk Premium*

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January 19, 2010

Abstract

In the cross-section of U.S. publicly traded firms there is a large inventory growth risk premium. We document that the spread in expected returns between firms with high versus low inventory growth rates is as high as 7% per annum, after controlling for differences across the firm's capital investment rate. We investigate the ability of two leading macroeconomic models of inventory behavior to simultaneously match the cross-sectional properties of asset prices and real quantities in the data. Calibrated to match quantities as close to the data as possible, we find that none of the models considered here can quantitatively explain the observed large inventory growth risk premium. Consistent with applications of standard Q-theory of investment in asset pricing, adding convex adjustment costs in inventory investment slightly improves the ability of these models to match the asset pricing facts. However, the adjustment costs deteriorate the fit of the models along the quantity dimension. We conclude that the strong negative link between inventory growth and firms' risk documented here is a quantitative puzzle for existing macroeconomic models of inventory behavior.

JEL Classification: E22, E23, E44, G12

Keywords: Stockout Costs, Stock Return Predictability, Cross-Sectional Asset Pricing, Q-theory, Accruals Anomaly

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*We thank Robert Goldstein, Selale Tuzel and Lu Zhang for helpful discussions. We thank Daniel Metzger for providing research assistance at the early stage of this project. Xiaoji Lin thanks the research support of Financial Market Group (FMG) and Sticcerd Research Grant. All errors are our own.

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1 Introduction

Inventories are an important source of business cycle variation: according to the evidence in Ramey and West (1999), the decline in inventory investment during recessions explains about 50% of the decline in GDP in the U.S. post-war economy. At the same time, the asset pricing literature documents substantial variation in risk premia over the business cycle both in the time-series and in the cross-section. Both facts are naturally linked if inventory investment, like physical capital investment, responds to changes in risk premia, a measure of firms' cost of capital. Thus understanding the economic mechanism behind these two facts seems important to understand both the business cycle itself and the variation in risk premia over the business cycle, which are central questions in macroeconomics and finance.

This paper provides an empirical and theoretical analysis of the link between inventory investment, physical capital investment and risk premia in the cross-section of U.S. publicly traded firms. The focus on the cross-section allows us to examine a large dataset and thus to provide a robust characterization of the properties of asset prices and real quantities in the data. On the asset price side, there is a large inventory growth risk premium. A spread portfolio of stocks that goes long on firms with low inventory growth rates and short on firms with high inventory growth rate generates a significant value-weighted spread as large as 7% per annum, after controlling for the firm's physical capital investment rate. So, similarly to physical capital investment, firms inventory investment responds significantly to changes in risk premia.[1] On the real quantity side, we document that the firm level physical capital and inventory investment are both volatile and procyclical, that inventory investment is more volatile than capital investment, and that the inventory-to-sales ratio is smooth and persistent. We provide a quantitative assessment of the ability of two leading alternative macroeconomic models of inventory behavior to simultaneously match the empirical facts. Our central finding is that some variations of the models considered here can match the

[1] An incomplete list of papers including Cochrane 1991, Titman, Wei and Xie 2004, Cooper, Gulen and Schill 2007, Polk and Sapienza 2008, Xing 2009, Bazdresch, Belo and Lin 2009, etc, document that physical capital investment and expected returns are negatively related.

real quantity firm level facts reasonably well, but all models have difficulty in matching the magnitude of the inventory growth risk premium with reasonable parameter values. We conclude that the strong link between inventory growth and firms' risk documented here is a quantitative puzzle for existing macroeconomic models of inventory behavior.

We specify a general production-based model with heterogenous firms that conveniently nests two leading macroeconomic models of inventory behavior as special cases. The two macroeconomic models differ in the motive by which firms hold inventories in equilibrium and each model corresponds to a particular parameterization of the firm's production technology, which facilitates the comparison across the different models. The first model is the stockout avoidance model which posits that firms hold inventories in equilibrium to avoid the risk implied by the delay between the commitment of factor inputs and the realization of the shocks affecting firms' sales and costs. For example, stockout costs arise when the firm is unable to satisfy a customer's order because the order is larger than the amount of goods the firms has available for sale. Examples of this approach include Kahn (1987, 1992), Bils and Kahn (1998) and Ramey and West (1999). The second model is the inventory as an input factor which posits that, in order to generate sales, firms are required to have some inventories. For example, cars in the showroom are necessary to generate sales (Bils and Kahn, 2000). Examples of this approach include Kydland and Prescott (1982), Christiano (1988), Jones and Tuzel (2009) and Gomes, Kogan and Yogo (2009). A key distinction between the two types of models is that inventory holding is endogenously determined in the stockout motive model, but is exogenously assumed in the inventory as an input model (see Khan and Thomas, 2007a, for a discussion of this feature). Our analysis excludes the production smoothing model of inventory behavior, another leading macroeconomic model of inventory behavior, because of its shortcomings explaining the dynamics of inventory investment (Ramey and West, 1999).

We also examine extensions of the two macroeconomic models of inventory behavior by adding adjustment costs in both capital and inventory investment, similar to applications of

the standard q-theory of investment to asset pricing (e.g. Cochrane, 1991). Adjustment costs have been shown to be important to match several asset pricing facts in macro asset pricing models with non-trivial production sectors both in the cross-section (e.g. Zhang, 2005 and Liu, Whited and Zhang, 2009) and at the aggregate level (e.g. Cochrane, 1991 and Jermann, 1998). For inventories, we investigate the role of both convex and non-convex adjustment costs. Khan and Thomas (2007b) show that an (S,s) rule for inventories generated by non-convex adjustment (delivery) costs helps macroeconomic models matching several business cycle facts related to inventory investment.

In the general production-based model, firms make capital investment and inventory investment decisions to maximize the value of the firm for shareholders. Optimal capital and inventory investment determines the firm's dividends and market value thus establishing an endogenous link between the firm's capital and inventory investment rates, and the firm's level of risk and expected stock return. Through simulations, we then examine the cross-sectional moments related to both quantities and asset prices to investigate which specifications better fits the data with reasonable parameter values.

The simulation results for each model can be summarized as follows. On the asset price side, all models considered here are qualitatively consistent with the data, but they cannot quantitatively match the observed magnitude of the inventory growth risk premium. Without convex inventory adjustment costs, a spread portfolio of stocks that goes long on firms with low inventory growth rates and short on firms with high inventory growth rates generates a positive spread of at most 0.92% per annum for the stockout motive model and 1.3% per annum for the inventory as an input factor model, after controlling for differences in firms' physical capital investment rates. This spread is much smaller than the maximum spread of 7% per annum observed in the data. Consistent with applications of the q-theory model to asset pricing, adding convex inventory adjustment costs slightly improves the fit of both models along the asset pricing dimension: the spread in returns across the inventory growth portfolios increases to 2.4% per annum for the stockout motive model and to 1.65%

per annum for the inventory as an input factor, but this spread is still too low compared with the data. This slightly improved fit, however, comes at the cost of deteriorating the fit along the real quantity dimension. With inventory convex adjustment costs, the inventory growth rate becomes considerably more autocorrelated in the model, 43% and 27% for the stockout motive and inventory as an input model, respectively, than in the data, where the autocorrelation is only 3% per annum.

On the real quantity side, the simulation results shows that stockout costs in inventories together with adjustment costs in both capital and inventory are important to match the cross-sectional facts associated with real quantities. In particular, the existence of stockout costs is important to endogenously generate a smooth and high inventory-to-sales ratio of around 10%. A model in which inventory is an input factor cannot match this fact because, for reasonable parameter values, the share of inventory in the production function is too small (for example, Christiano, 1988, estimates that the share of inventory stock in output is less than 1% in the U.S. economy). Thus, in this model, firms hold inventory-to-sales ratio that are too small compared with the data. Finally, we find that when inventory adjustment costs are lower than physical capital adjustment costs, all models considered here can generate firm level inventory investment rates that are more volatile than physical capital investment rates, consistent with the data.

Related literature: The work most closely related to ours is Jones and Tuzel (2009) who also investigate the link between inventory investment and the cost of capital. Using industry level data and different proxies of aggregate risk premia, they establish that inventory investment respond to changes in the cost of capital. They qualitatively explain the fact using a general equilibrium model with a representative aggregate firm and representative consumer. The key feature that differentiates our work is that our analysis is at the firm level and thus we focus on the cross-section. More important, we provide a quantitative assessment of the ability of several macroeconomic models of inventory investment to explain the large risk dispersion associated with inventory growth. Examining the quantitative predictions of the

model is important because, as we show here, the strong link between inventory growth and firms' risk is a quantitative, not qualitative, puzzle.

The work in this paper is also closely related to a large macroeconomic literature on aggregate inventory behavior and business cycles. For example, Blinder and Maccini (1991) and Ramey and West (1999) emphasize the production smoothing motive for holding inventories, while Kahn (1987, 1992), Bils and Kahn (1998) focus on the stockout avoidance motive for the firm to hold inventory. More recently, Fisher and Hornstein (2000) and Khan and Thomas (2007b) stress the importance of (S,s) rule in characterizing the inventory investment over business cycles. Our paper contributes to this strand of literature by examining the ability of existing macroeconomic models to simultaneously match the real quantity and asset price cross-sectional facts that we document in the data. In addition, we investigate the importance of inventory adjustment costs for improving the fit of these models. This paper also relates to the literature of investment-based asset pricing with multiple capital goods (e.g., Bazdresch, Belo and Lin, 2009, and Lin 2009, etc). The difference is that we also examine a class of macroeconomic models in which inventory is not a factor input in the production process (the stockout motive model). Thus here, the link between inventory and firms' risk is fundamentally different from the link with capital or labor, which are factor inputs in the production process.

Finally, because inventory is part of accruals, the empirical findings in this paper are also related to a large literature on the accruals anomaly, initiated with Sloan (1996). This anomaly is the fact that firms with high accrual earn abnormally lower returns on average than firms with low accruals. Thomas and Zhang (2002) show that the negative relationship between accruals and future abnormal returns is due mainly to inventory changes. Wu, Zhang and Zhang (2009) hypothesize, based on standard q-theory of investment, that firms optimally adjust their accruals in response to discount rate changes, and they argue that this channels can explain the negative relationship between accruals are expected returns observed in the data. Our work differs because we provide a quantitative assessment of

this hypothesis in a simulation economy, and we examine both empirically and theoretically the link between inventory growth and stock returns controlling for firms' physical capital investment rate. This allows us to establish the marginal response of inventory investment to firms' cost of capital which is important to quantitatively examine the fit of the alternative models in the simulations.

The paper proceeds as follows. Section 2 documents the empirical facts regarding the relationship between physical capital investment, inventory growth and stock returns in the cross-section. Section 3 presents a general production-based model that nests alternative macroeconomic models of inventory behavior as a special case. Section 4 discusses the properties of the numerical solution of each model. Section 5 reports the cross-sectional moments from the simulation of each model that we use to examine the ability of the alternative macroeconomic models to match the data. Finally, Section 6 concludes.

2 Capital, Inventories and Stock Returns: the Facts

In this section we examine the empirical link between inventory growth, physical capital investment, and stock returns in the cross-section of U.S. publicly traded firms. The analysis of the real quantity facts in this section complements the analysis in Khan and Thomas (2007a) for the aggregate U.S. economy (see their Table 1). The analysis of the asset pricing facts complements the analysis in Thomas and Zhang (2002) and Tuzel and Jones (2009). The novel feature of our approach is that we examine the link between inventory growth and stock returns at the firm level and controlling for the firm's physical investment rate. This is important since capital investment and inventory growth are naturally correlated in the data. In turn, this approach allows us to investigate the marginal link between these two firm characteristics and firms' total risk.

2.1 Data

Monthly stock returns are from the Center for Research in Security Prices (CRSP) and accounting information is from the CRSP/COMPUSTAT Merged Annual Industrial Files. The sample is from July 1965 to June 2006. The two key variables for the empirical work are the firm level physical capital and inventory investment rates. We construct these variables as follows. Firm level capital investment (I_t) is given by COMPUSTAT data item 128 (capital expenditures). The capital stock (K_t) is given by the data item 8 (Property, Plant and Equipment). Total inventory stock (N_t), which includes raw materials, finished goods and work-in-progress, is given by data item 3. Net inventory investment (H_t) is given by the change in the stock of total inventories in year t from year t-1 ($H_t = N_t - N_{t-1}$). The physical capital investment rate is given by the ratio of physical investment to the beginning of the period capital stock ($IK_t = I_t / K_{t-1}$), as in Xing (2009) and Gala (2005), and the inventory investment rate is given by the ratio of the change in the stock of total inventories to the beginning of the period stock of inventories ($HN_t = H_t / N_{t-1}$). Thus our inventory investment rate is effectively the net growth rate of total inventories of the firm. We winsorize the top 1% of the physical capital and the top and bottom 1% of the inventory investment distribution to reduce the influence of outliers observed in our sample, which are likely to reflect mergers and acquisitions. Firms that never report positive inventory holdings are excluded from the sample since the theory in this paper does not apply to those firms. The sample is representative. The total market capitalization of the firms included in our sample represents on average about 64% of the market capitalization of all firms in CRSP. Appendix A-1 provides a detailed description of the additional data used as well as additional sample selection criteria.

2.2 Business Cycle Facts

Table 1 reports summary statistics of firm level (Panel A) and cross-sectional (Panel B) moments of the firm level physical capital investment rate (IK), inventory growth (HN), the

inventory-to-(net) sales ratio (NS) and the real sales growth (SG) variables in our sample. For comparison, the table also reports the summary statistics of the book-to-market ratio, a characteristics that is known to be related to differences in average returns across firms. In computing the firm level moments in Panel A, we require a firm to have at least 15 (annual) observations in order to compute the firm level moments with sufficient precision. All variables are reported at annual frequency. We use the moments reported in Table 1 to evaluate the ability of the theoretical models to match the real quantity facts in the cross-section.

[Insert Table 1 here]

Both capital investment and inventory investment are very volatile: at the firm level, the standard deviation of the investment rate and the inventory rate is 19% and 34% per annum, respectively. Both capital and inventory investment are procyclical, since the correlation with sales growth is positive (30% and 51% respectively). The inventory-to-(net) sales ratio is around 11% and is smooth, with a standard deviation of 4% on annual data. These summary statistics are roughly consistent with those reported in Khan and Thomas (2007a) for the U.S. economy at the aggregate level.

The mean correlation between the firm's physical capital investment and inventory investment rate is around 32%. To the extent that this relatively low correlation is not only due to measurement error in these variables, this low correlation suggest that these two firm's characteristics carry potentially different information about firms' expected stock return and hence about firms' level of risk. In turn, this fact provides support for the approach of jointly investigating the role of physical capital and inventory investment as measures of firms' risk and hence of predictors of stock returns.

2.3 Asset Pricing Facts

We follow two completementary approaches to investigate the link between the firm level physical capital investment, inventory growth and stock return. In the first approach, we construct portfolios sorted on the firm level capital and inventory investment and investigate the properties of post formation average stock returns across these portfolios. The spread in the average returns (risk premia) across these portfolios naturally provide information about the risk associated with the two sorting characteristics. In the second approach, we run standard Fama and MacBeth (1973) (henceforth FMB) cross sectional regressions of firm level returns on lagged firm level physical capital and inventory investment rates as well as other control variables. The estimates of the slope coefficients in these regressions allows us to quantify the link between each characteristic and the firm's expected stock return, and thus to the firm's risk. The two approaches allow us to cross-check the results and establish the robustness of the findings.

2.3.1 Portfolio Level Evidence

We construct nine portfolios double sorted on physical capital and inventory investment rates. Following Fama and French (1993), in each June of year t, we first sort the universe of common stocks into three portfolios based on the firm's inventory investment rate (cutoffs at the 33^{th} and 66^{th} percentile) at the end of year $t-1$. Then, each one of these three inventory portfolios are equally sorted into three portfolios based on their capital investment rate (cutoffs at the 33^{th} and 66^{th} percentile) at the end of year $t-1$. Once the portfolios are formed, their value- and equally-weighted returns are tracked from July of year t to June of year $t+1$. The procedure is repeated in June of year $t+1$.

[Insert Table 2 here]

Table 2, Panel A, reports the mean equity returns and Panel B reports the mean characteristics of each portfolio. Except for returns, all characteristics are measured at the time

of the portfolio formation. Consistent with the results in Thomas and Zhang (2002), both the equally-weighted (rEW) and the value-weighted (rVW) average excess returns of these portfolios are decreasing in the inventory investment rate within each physical investment rate bin. Our new finding is that both the physical capital investment and the inventory investment rate seem to have marginal predictive power for stock returns. The spread in the average excess returns of these portfolios within the physical capital investment or the inventory investment dimensions is large, and even more impressive across the two characteristics together. Controlling for firms' physical capital investment rate, the spread in returns between the low versus high inventory growth rate portfolios (L-H) is as high as 6.9% per annum across value-weighted portfolios, and even higher, 8.3% per annum, across equally-weighted portfolios. We label this spread as the inventory growth risk premium. The Patton and Timmermann (2008) monotonic relation test, strongly rejects the hypothesis that the average returns of these portfolios are all equal against the hypothesis that they are decreasing in both the investment and inventory investment rate dimensions, with a p-value of 2.2%.

The characteristics of these portfolios shows that firms with lower physical capital and inventory investment rates have higher leverage (LEV) ratios, and hence their returns are more levered. This suggests that difference in leverage might potentially explain the difference in the equity returns of these portfolios. That is not the case, however. To show this, we compute unlevered equity returns of each portfolios using the average leverage ratio of each firm and bond excess return data (rBOND) computed following the procedure in Blume, Lim and Mackinlay (1998) (see Appendix A-1 for a detailed description of how the firm level bond return data is obtained). The unlevered value weighted portfolios excess returns (rVW^{unl}) (or total firm return) are computed using the standard weighted average cost of capital formula:

$$\text{rVW}^{\text{unl}} = (1\text{-LEV}) \times \text{rVW} + \text{LEV} \times \text{rBOND}.$$

According to Table 2, the spread in unlevered returns across the portfolios is somewhat smaller, but its magnitude is still substantial. Controlling for firms' investment rate, the spread in value weighted unlevered returns between the low versus high inventory growth rate portfolios (L-H) is as high as 4.9% per annum (a decrease of two percentage points relative to the spread in levered returns).

The characteristics of these portfolios also reveal that the book-to-market ratio (BM) is significantly negatively correlated with the average physical investment and inventory investment rate of these portfolios. This fact is consistent with q-theory models of inventory investment, since, in general, the book-to-market ratio can be shown to be a decreasing function of firms' investment rate. The size characteristics does not exhibit a systematic pattern across these portfolios, but the portfolio with the Mid characteristics tends to have larger firms.

2.3.2 Firm Level Evidence

To examine the marginal predictive power of the firm's capital investment or inventory investment for stock returns, and hence identify the individual contribution of each characteristic in capturing information about changes in the firm's risk, we run standard FMB cross sectional regressions of firm level returns on lagged firm level characteristics that include the firm level capital and inventory investment as explanatory variables. We perform this additional analysis because in the portfolio approach it is difficult to assess the marginal predictive power of the inventory investment rate relative to the capital investment rate when the sorting characteristics are correlated, which is the case here. In addition, the sorting procedure requires the specification of arbitrary breakpoints to construct the portfolios which is not required in the FMB regressions (see, for example, Fama and French, 2008, for a discussion on the advantages and disadvantages of the FMB and the portfolio approach in the context of stock return predictability).

[Insert Table 3 here]

We consider four different empirical specifications of the FMB cross-sectional regressions. In the first two specifications, we use either the physical capital investment rate (IK) or the inventory investment rate (HN) separately as explanatory variables and in the third specification we include both variables together. In the last specification we control for the firms' leverage ratio, which, as shown above, varies systematically with the firms' physical investment and inventory rates. The results in Table 3, confirm the qualitative results from the portfolio analysis. Across all specifications, the regression produces negative and statistically significant average slopes associated with the physical capital and inventory investment rates, even when both characteristics are included together or controlling for the firm's leverage ratio. Thus both characteristics seem to have marginal forecasting power for stock returns. In unreported results, we confirm that the results are robust to the exclusion of micro cap firms (i.e. firms with market capitalization in the bottom 20^{th} percentile of the corresponding cross-sectional distribution) from the sample, as suggested in Fama-French (2008) (results available upon request).

To interpret the economic magnitude of the slope coefficients in Table 3 we examine the implications of the estimates of the slope coefficient for the premium on an inventory growth Low minus High portfolio, which is an alternative way of computing the inventory growth risk premium. This inventory growth risk premium is computed from the regression implied expected excess return on a portfolio that goes long one dollar on firms with a inventory growth rate that is on the 10^{th} percentile of the cross sectional distribution and short one dollar on firms with a inventory growth rate that is on 90^{th} percentile of the cross sectional distribution. Here, we use the values of the 10^{th} and 90^{th} percentiles of the inventory growth rate observed the data, which are reported in Table 1. Holding the physical capital investment rate constant, the annualized implied inventory growth risk premium on this portfolio is computed as

$$E[\mathrm{R}_t^e]_{\text{Low minus High}}^{\text{Inventory}} = 12 \times \mathrm{b}_{\mathrm{HN}} \times (\mathrm{HN}_{10^{\text{th}}} - \mathrm{HN}_{90^{\text{th}}}) \tag{1}$$

where b_{HN} is the estimated slope coefficient from the FMB cross-sectional regressions and $HN_{i^{th}}$ is the i^{th} percentile of the inventory growth cross sectional distribution.[2] An analogous procedure is used to compute the implied investment rate risk premium. Note that this analysis is different from the portfolio level analysis in the previous section because it only uses the FMB slope coefficients, which are obtained in regressions that control for the effect of other variables.

The implied risk premium associated with both characteristics is substantial, as reported in the last two columns in Table 3. The inventory growth risk premium is 3.9% per annum, holding IK constant, and the physical capital investment rate risk premium is about 2.9% per annum, holding the inventory growth constant. Controlling for leverage has a very small effect on the magnitude of the estimated physical capital and inventory investment slope FMB coefficients, and hence on the implied risk premium, as reported in the last specification.

3 A Production-Based Model with Inventory Holdings

In this section we specify a general production-based asset pricing model in order to understand the empirical evidence presented in the previous section. By studying the producers' optimal production decisions, the model establishes an endogenous link between the firm's capital and inventory investment rates, and the firm's level of risk and expected stock return.

The general model considered here conveniently nests alternative representations of the

[2]This result is obtained as follows. Holding the investment rate constant arbitrarily at its mean value ($\overline{\text{IK}}$) (the exact value does not play any role here), the implied expected return on the low investment growth portfolio composed of firms that have an inventory growth rate on the 10^{th} percentile of the inventory growth rate distribution ($\text{HN}_{10^{\text{th}}}$) is given by

$$E[R_t]^{\text{Inventory}}_{\text{Low}} = \text{Int} + b_{\text{IK}} \times \overline{\text{IK}} + b_{\text{HN}} \times \text{HN}_{10^{\text{th}}}.$$

The expected return on the high inventory growth rate portfolio $E[R_t]^{\text{Inventory}}_{\text{High}}$ composed of firms that have a inventory growth rate on the 90^{th} percentile is computed similarly. Then, the premium on the Low minus High portfolio is just $E[R^e_t]^{\text{Inventoy}}_{\text{Low minus High}} = E[R_t]^{\text{Inventory}}_{\text{Low}} - E[R_t]^{\text{Inventory}}_{\text{High}}$. Multiplying this by 12 to get annualized values yields the inventory growth risk premium in equation (1).

firm's technology that have been previously proposed in the macroeconomics literature to explain the dynamics of inventory behavior in the data. In particular it nests two leading macroeconomic models of inventory behavior discussed in the introduction (also see introduction for references): the stockout and holding cost model, henceforth referred to as stockout motive model for short, and the inventory as an input factor model. In addition, we extend the previous macroeconomic models by allowing for the possibility of convex adjustment costs in capital investment and convex and non-convex adjustment costs in inventory investment, in the spirit of applications of the standard q-theory of investment in asset pricing.

3.1 Economic Environment

The economy is composed of a large number of firms that produce a homogeneous good. Firms are competitive and take as given the market-determined stochastic discount factor $M_{t,t+1}$, used to value the cash-flows arriving in period $t+1$. The existence of a strictly positive stochastic discount factor is guaranteed by a well-known existence theorem if there are no arbitrage opportunities in the market (see for example, Cochrane, 2001, chapter 4.2).

3.1.1 Technology

We focus on the optimal production decision problem of one firm in the economy (we suppress any firm specific subscript to save on notation). The firm uses capital inputs K_t, and potentially, inventory inputs, N_t, to produce output, Y_t, according to the technology

$$Y_t = e^{x_t + z_t} K_t^{\alpha_k} N_t^{\alpha_n}, \tag{2}$$

where $\alpha_k > 0$, $\alpha_n \geq 0$ and $\alpha_k + \alpha_n < 1$, x_t is an aggregate profitablity shock, and z_t is the firm's specific idiosyncratic profitablity shock. The profitability shock includes both productivity and demand shocks. The combination of perfect competition with decreasing returns to scale can be shown to be equivalent to that of a monopolist facing a downward

slopping demand curve for its output so that our assumptions are not too restrictive.

The aggregate profitablity shock follows the process

$$x_{t+1} = \bar{x}(1 - \rho_x) + \rho_x x_t + \sigma_x \varepsilon_{t+1}^x, \tag{3}$$

where ε_{t+1}^x is an independently and identically distributed (i.i.d.) standard normal shock. The idiosyncratic profitablity shock follows the process

$$z_{t+1} = \rho_z z_t + \sigma_z \varepsilon_{t+1}^z, \tag{4}$$

where ε_{t+1}^z is an i.i.d standard normal shock that is uncorrelated across all firms in the economy, and ε_{t+1}^x is independent of ε_{t+1}^z for each firm. In the model, the aggregate profitablity shock is the driving force of economic fluctuations and systematic risk, and the idiosyncratic profitablity shock is the driving force of firm heterogeneity.

In every period t, the capital stock K_t depreciates at rate δ_k and is increased (or decreased) by gross investment I_t. The law of motion of the capital stock is given by

$$K_{t+1} = (1 - \delta_k)K_t + I_t \ , \quad 0 < \delta_k < 1. \tag{5}$$

Similarly, the firm's inventory stock N_t depreciates at rate δ_n and is increased (or decreased) by gross inventory investment H_t. The law of motion of the inventory stock is given by

$$N_{t+1} = (1 - \delta_n)N_t + H_t \ , \quad 0 < \delta_n < 1. \tag{6}$$

Following Bils and Kahn (1998) and Ramey and West (1999), net sales are measured as

$$\text{Sales}_t = Y_t - H_t,$$

that is, net sales are specified by total output minus the gross investment in the inventory

stock.

3.1.2 Stockout and Adjustment Costs

The firm incurs in possibly three types of costs when choosing the optimal level of physical capital and inventory investment: stockout and holding costs, as well as capital and inventory adjustment costs.

Following Ramey and West (1999), stockout costs are specified by the function:

$$\text{Stockout}_t \equiv \frac{d}{2}\left(N_t - e \cdot \text{Sales}_t\right)^2, \tag{7}$$

where $d > 0$ and $e \geq 0$ are constants. To understand this specification, consider first the case $e = 0$, so that the term becomes $\frac{d}{2}\left(N_t\right)^2$. In this case, the costs can be interpreted as the second order term in a quadratic approximation to an arbitrary convex inventory holding cost function. When $e \neq 0$, the term is intended to reflect both stockout as well as inventory holding costs. Stockout costs arise when sales exceed the stock on hand, either due to lost sales or because of delayed payment if orders instead are backlogged. Ceteris paribus, the higher the stock of inventories, the less likely is a stockout and the lower are stockout costs. On the other hand, higher stocks entail higher inventory holding costs. This quadratic term approximates the trade-off between the two costs, with e falling as stockout costs fall relative to backlog costs.

Capital adjustment costs are specified by the following convex adjustment cost function

$$K\text{adj}_t \equiv \frac{c_k}{v_k}\left(\frac{I_t}{K_t}\right)^{v_k} K_t, \tag{8}$$

where $c_k > 0$ and $v_k > 0$ are constants. As standard from the q-theory of investment literature, capital adjustment costs include planning and installation costs, learning the use of new equipment, or the fact that production is temporarily interrupted.

Finally, in the specification of inventory adjustment costs, we consider two possible al-

ternative specifications: convex and non-convex adjustment costs. If adjustment costs are convex, the functional form is specified as

$$N\text{adj}_t^{\text{convex}} = \frac{c_n}{v_n} \left(\frac{N_{t+1} - N_t}{N_t} \right)^{v_n} N_t, \tag{9}$$

where $c_n > 0$ and $v_n > 0$ are constants. If adjustment costs are non-convex, the functional form is specified as

$$N\text{adj}_t^{\text{non-convex}} = \begin{cases} a^+ & \text{if} \quad H_t > 0 \\ 0 & \text{if} \quad H_t = 0 \\ a^- & \text{if} \quad H_t < 0 \end{cases} , \tag{10}$$

where $a^- > a^+ > 0$ are constants.

The convex adjustment cost specification in equation (9) follows from, Kogan and Yogo (2009), and this is a standard specification used in standard q-theory of investment. The non-convex adjustment cost specification for inventories was first proposed in Scarf (1960) and captures a fixed cost incurred when the firm adjusts its stock of inventories. The fixed cost includes, for example, the number of labor hours that the firm hire to undertake inventory investment, irrespective of the size of the investment.

3.1.3 Preferences

Following Berk, Green and Naik (1999) and Zhang (2005), we directly specify the stochastic discount factor (henceforth SDF) without explicitly modeling the consumer's problem. The stochastic discount factor is given by

$$\log M_{t,t+1} = \log \beta + \gamma_t (x_t - x_{t+1}) \tag{11}$$

$$\gamma_t = \gamma_0 + \gamma_1 (x_t - \bar{x}) , \tag{12}$$

where $M_{t,t+1}$ denotes the stochastic discount factor from time t to $t+1$. The parameters $\{\beta, \gamma_0, \gamma_1\}$ are constants satisfying $1 > \beta > 0$, $\gamma_0 > 0$ and $\gamma_1 < 0$.

Equation (11) can be motivated as a reduced-form representation of the intertemporal marginal rate of substitution for a fictitious representative consumer or the equilibrium marginal rate of transformation as in Cochrane (1993) and Belo (2010). Following Zhang (2005), we assume in equation (12) that γ_t is time-varying and decreases in the demeaned aggregate profitability shock $x_t - \bar{x}$ to capture the well documented countercyclical price of risk with $\gamma_1 < 0$. The precise economic mechanism driving the countercyclical price of risk is, e.g., time-varying risk aversion as in Campbell and Cochrane (1999) or time-varying economic uncertainty as in Bansal and Yaron (2004).

Having an exogenous SDF is a desirable feature of our approach because it allows us to focus on the production-side of the economy and consider an interesting cross-section of firms in a tractable way. More important, this feature maximizes the ability of the macroeconomic models considered here to match the firm level asset pricing facts, which is the novel dimension included in our study. As it is well known, it is difficult to generate even a high aggregate equity premium in general equilibrium models with production (see, for example, Cochrane, 2007, for a discussion). By having an exogenous SDF that, by construction, has properties that are reasonably consistent with aggregate asset prices, provide the models considered here with the maximum ability to fit the cross-sectional asset pricing facts. In turn, to the extent that the models fail in matching those facts, that failure can be interpreted as an inability of these models to internally generate the appropriate risk dispersion across firms, and not by the fact that in equilibrium, the endogenous SDF cannot account for asset prices.

3.2 Firm Value, Risk and Return

All firms in the economy are assumed to be all-equity financed, so we define

$$D_t = \text{Sales}_t - I_t - \text{Stockout}_t - N\text{adj}_t^{\text{i}} - K\text{adj}_t - f, \tag{13}$$

to be the dividends distributed by the firm to the shareholders. Here, f is a fixed cost of production, I_t is the total investment in capital and i =convex or non-convex. A negative dividend is considered as equity issuance.

Define the vector of state variables as $s_t = (K_t, N_t, x_t, z_t)$ and let $\text{V}^{\text{cum}}(s_t)$ be the cum-dividend market value of the firm in period t. The firm makes capital investment I_t and inventory investment H_t decisions to maximize its cum-dividend market value by solving the problem:

$$V^{\text{cum}}(s_t) = \max_{I_{t+j}, H_{t+j}, j=0..\infty} \left\{ \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} D_{t+j} \right] \right\}, \tag{14}$$

subject to the capital and inventory accumulation equations (5) and (6) and the flow of funds constraint (13) for all dates t. The operator $\mathbb{E}_t[.]$ represents the expectation over all states of nature given all the information available at time t.

In the model, risk and expected stock returns are determined endogenously along with the firm's optimal production decisions. To make the link explicit, we can evaluate the value function in equation (14) at the optimum,

$$V^{\text{cum}}(s_t) = D_t + \mathbb{E}_t \left[M_{t,t+1} V^{\text{cum}}(s_{t+1}) \right] \tag{15}$$

$$\Rightarrow \quad 1 = \mathbb{E}_t \left[M_{t,t+1} R^s_{t+1} \right], \tag{16}$$

where equation (15) is the Bellman equation for the value function and equation (16) follows from the standard formula for stock return $R^s_{t+1} = V^{\text{cum}}(s_{t+1}) / \left[V^{\text{cum}}(s_t) - D_t \right]$. Note that if we define $P^s_t \equiv V^{\text{cum}}(s_t) - D_t$ as the *ex*-dividend market value of equity, R^s_{t+1} reduces to

the usual definition, $R^s_{t+1} \equiv \left(P^s_{t+1} + D_{t+1}\right)/P^s_t$. Equation (16) can then be written in the standard expected return-beta form. Following Cochrane (2001 p. 19):

$$\mathbb{E}_t\left[R^s_{t+1}\right] = R_{ft} + \beta_t \lambda_{mt}, \tag{17}$$

where $R_{ft} \equiv \frac{1}{\mathbb{E}_t[M_{t;t+1}]}$ is the real (gross) interest rate, and β_t is the firm's risk defined as:

$$\beta_t \equiv \frac{-Cov_t\left[R^s_{t+1}, M_{t,t+1}\right]}{Var_t\left[M_{t,t+1}\right]} \tag{18}$$

and λ_{mt} is the price of risk defined as

$$\lambda_{mt} \equiv \frac{Var_t\left[M_{t,t+1}\right]}{E_t\left[M_{t,t+1}\right]}.$$

3.3 Alternative Models

The general production-based model nests alternative macroeconomic models of inventory behavior as a special case. Here, we present the alternative specifications. We then contrast the business cycle and cross-sectional asset pricing implications of the alternative models in Section 5 below.

Table 4 summarizes the parameters that vary across the alternative specifications. In the stockout motive model, the elasticity of output with respect to inventories in the production function in (2) is specified to be $\alpha_n = 0$, since here the production of output does not directly require the use of inventories. Within this model, we consider three alternative specifications: (1) no inventory adjustment costs; (2) convex inventory adjustment costs; and (3) non-convex inventory adjustment costs.

In the inventory as an input model, the elasticity of output with respect to inventories in the production function in (2) is specified to be $\alpha_n > 0$, since here inventories are assumed to be essential to generate sales. Similarly to the stockout motive model, we consider

three alternative specifications of this model: (4) no inventory adjustment costs; (5) convex inventory adjustment costs and (6) non-convex inventory adjustment costs. Specification number four is closely related to the inventory model proposed in Jones and Tuzel (2009). Specification (5) is closely related to the Q-theory model proposed (but not solved) in Wu, Zhang and Zhang (2009) to explain the accruals anomaly. For tractability, all specifications have convex physical capital adjustment costs, as in equation (8). We do not investigate alternative adjustment cost functions for physical capital since our main focus is on inventory investment.

4 Properties of Model Solutions

All the endogenous variables in the model, including risk and expected return, are functions of the state variables. Because the functional forms are not available analytically, in this section we solve for these functions numerically and study its properties. Appendix A-2 provides a description of the solution algorithm and of the numerical implementation of each model.

4.1 Calibration

The general model is calibrated at annual frequency using the parameter values reported in Table 5, Panels A and B. Panel A reports the parameters that are constant across all the alternative specifications of the general model. The first set of parameters specifies the technology of the firm. The second set of parameters describes the exogenous stochastic processes that the firm faces, including the aggregate and idiosyncratic profitablity shock, and the stochastic discount factor. Panel B reports the parameters of the firm's technology that vary across specifications.

[Insert Table 5 here]

The choice of the parameter values is based on the parameter values reported in previous studies whenever possible, or by matching known *aggregate* asset pricing facts, as well as key firm level moments reported in Table 1.

Firm's technology: We set the elasticity of output with respect to capital in the production function (2) to be α_k =0.7, roughly consistent with the average of the estimates in Cooper and Ejarque (2001, 2003) and Cooper and Haltiwanger (2006). In the stockout motive model, the elasticity of output with respect to inventories is set to $\alpha_n = 0$. In the inventory as an input model, this elasticity is set to $\alpha_n = 0.01$, which is the highest possible value that one can use (e.g., Christiano, 1988, estimates the share of inventory in aggregate output to be less than 1%). The capital depreciation rate δ_k is set as 10% per annum as in Jermann (1998). The depreciation rate of inventory is set at $\delta_n = 20\%$ per annum, following Jones and Tuzel (2009) who convincingly argue that the depreciation rate for inventory is higher than that of physical capital.

The slope coefficients c_k and the curvature parameter v_k in the capital adjustment cost function (8) control the importance of adjustment costs of capital. Empirical estimates of these parameters vary substantially across studies and so they are difficult to calibrate. Estimates of c_k in the investment literature range from 20 in Summers (1980), to 2 in Whited (1993) to not significantly different from 0 in Hall (2004). Similarly, the slope coefficients c_k and the curvature parameter v_k in the convex inventory adjustment cost function (7) are not readily available. Through some experimentation, we set the slope parameters in the convex inventory adjustment cost functions to $c_k = 2$, the curvature parameters to $v_k = 3$ and $v_n = 3$ in all related cases (1, 2, 4 and 5), and $c_n = 0.8$ in the stockout motive model and $c_n = 0.15$ in the inventory as an input model, in order to match as close as possible the firm level investment rate and net inventory investment rate volatilities of around 19% and 34%, respectively. In the non-convex inventory adjustment cost function (10), we set the parameters $a^+ = 0.01$ and $a^- = 0.02$ in order to match as close as possible the volatility of net inventory investment rate and restrict the total adjustment cost to output to be

within a reasonable range. We assess the reasonability of these parameter values below, by investigating the implied magnitude of the total investment and inventory investment adjustment costs.

We calibrate the parameters in the stockout costs function in equation (7) to be $d = 0.5$ and $e = 0.2$, to match the mean and volatility of firm level inventory to sales ratio of 10% and 4%, respectively. We set the fixed cost f to roughly match the average firm level book equity to market equity ratio of 0.96.

Stochastic processes: We set the persistence of the aggregate profitablity shock at $\rho_x = 0.98^4$ and its conditional volatility at $\sigma_x = 0.007 \times 2$, which roughly corresponds to the quarterly estimates in King and Rebelo (1999) (for productivity shocks). The long-run average level of aggregate profitablity shock, $\bar{x}$, is a scaling variable. Following Zhang (2005), we set the average long-run capital in the economy at one, which implies a different set values for the aggregate profitablity shock. To calibrate the persistence parameter ρ_z and the conditional volatility parameter σ_z of the firm-specific profitability shock, we follow Zhang (2005) and restrict these two parameters using their implications on the degree of dispersion in the cross-sectional distribution of firms' stock return volatilities. Thus we set $\rho_z = 0.78$, and $\sigma_z = 0.35$, which implies an average annual volatility of individual stock returns of $24\% - 30\%$ across different model specifications, approximately the average of 25% reported by Campbell at al (2001).

Following Zhang (2005), we pin down the three parameters governing the stochastic discount factor, β, γ_0, and γ_1 in equation (11) and (12), by matching three aggregate return moments: the average real interest rate, the volatility of the real interest rate, and the average maximum levered Sharpe ratio in the U.S economy (approximately 0.4). This procedure yields $\beta = 0.94$, $\gamma_0 = 28$, and $\gamma_1 = -300$.

[Insert Table 6 here]

To evaluate the economic magnitude of the calibrated parameters, Panel A in Table 6 reports key moments of aggregate asset prices and measures of adjustment costs both in the

real data and in the artificial data generated by all models. All models do a reasonable job matching these key aggregate moments. All models quantitatively match reasonably well the first two moments of the aggregate unlevered market return and risk free rate. The unlevered Sharpe ratio implied by each model is only slightly lower than in the data. The magnitude of the adjustment costs in the model is in general reasonable. The fraction of output that is lost due to capital and inventory adjustment costs is around $3\% - 4\%$. These values are within the lower end of the empirical estimates surveyed in Hamermesh and Pfann (1996).

4.2 The Firm's Value Function, Policy Functions and Risk

Figures 1, 2, and 3 plots the firm's value function, policy functions (gross investment and inventory investment rates) and risk, as measured by the firm's conditional beta β_t in equation (18), obtained from the numerical solution of each model. These functions depend on the four state variables, the capital stock K_t, the inventory stock N_t, the aggregate profitability x_t, and the idiosyncratic profitability z_t. Because the focus of this paper is on the cross-sectional heterogeneity, we fix the aggregate profitability at its long-run average, $x_t = \bar{x}$ and we plot each function at several different values of the idiosyncratic profitability z_t. The top panels in each figure plot each function against K_t and z_t, with N_t and x_t fixed at their long-run average levels $\bar{n}$ and $\bar{x}$. The bottom panels in each figure plot each function against N_t and z_t, with K_t and x_t fixed at their long-run average level $\bar{k}$ and $\bar{x}$.

[Insert Figure 1 here]

The top panels in Figure 1 shows that the firm's market value is increasing in the firm's profitability and capital stock. The bottom panels show that, for the stockout motive models (models one to three), the relationship between the firm's value and inventory stock is hump-shaped: the firm value is increasing in inventory stock when inventory stock is small, but is decreasing in inventory stock when inventory stock gets large. This reflects the property of quadratic stockout/backlog costs which reduces cash flow more when inventory stock is

lower or higher than the optimal level.

[Insert Figure 2 here]

Figure 2 shows that gross investment and net inventory investment are increasing in the firm's profitability. Thus more profitable/high productivity firms invest and accumulate more inventory than less profitable/low productivity firms, all else being equal. For investment, this result is consistent with the evidence documented in Fama and French (1995). Gross investment and net inventory investment are also decreasing in the capital stock and inventory stock, respectively. Thus small firms with less capital and inventory, invest and accumulate more inventory (and thus grow faster) than big firms with more capital and more inventory. This result is consistent with the evidence provided by Evans (1987) and Hall (1987).

[Insert Figure 3 here]

The top panels in Figure 3 shows that the firm's risk is decreasing in the firm's profitability and capital stock. This result is consistent with Zhang (2005), who shows that less profitable/low productivity firms are riskier than more profitable/high productivity firms, and is also consistent with Li, Livdan and Zhang (2009) who show that small firms with less capital are more risky than big firms with more capital. More important, in the bottom panels, the relationship between the firm's risk and the firm's inventory stock is essentially flat for all models (models 1 and 4) without convex inventory adjustment costs and slightly kinked in models 3 and 6 with non-convex adjustment costs. The relationship is slightly negative for the models with convex adjustment costs (models 2 and 5). Thus convex adjustment costs in capital and inventory stock play a key role in generating the negative relation between risk and capital investment and inventory investment. In the model, the risk of a firm is negatively related to its flexibility in adjusting capital and inventory stock to smooth its dividends when facing aggregate uncertainty. Adjustment costs are the offsetting forces of

dividend smoothing mechanism. With convex adjustment costs, high capital investment and high inventory investment firms are more flexible in dividend smoothing, and hence are less risky than low capital investment and low inventory investment firms.

5 Model Implications for the Cross-Section

In this section, we contrast the alternative specifications of the general model in terms of their fit in matching the cross-sectional empirical facts reported in Section 2. In order to compute each model implied cross-sectional moments for asset prices and quantities, we use each version of the calibrated theoretical model to simulate 100 artificial panels, each of which has 3600 firms and 200 annual observations. We then replicate the empirical procedures on the artificial data simulated by each specification of the model and report the cross-sample average results.

5.1 Matching Business Cycle Facts

Table 6, Panel B reports selected firm level moments (real quantities) of the firm level physical capital investment rate, inventory growth, inventory-to-sales ratio, and sales growth.

Overall, the calibration of each model does a reasonable job matching several firm level quantity moments, but there are some important differences across models. All models considered here match reasonably well the second moment of the firm level physical capital investment and net inventory growth rate (the mean of the investment and inventory investment rate is pinned down by the depreciation rates of capital and inventory so they are less interesting to analyze). All models generate a firm level sales growth volatility that is slightly higher than in the data (we do not match the mean of sales growth since there is no growth in our model). In addition, across all models, the autocorrelation of the investment rate is also higher than in the data, a result that can be partially explained by the absence of measurement error in the simulated data. The autocorrelation of the inventory growth

is reasonably close to the data for all models, except in the case with convex inventory adjustment costs (Cases 2 and 5). Here, the firm smooths inventory investment to avoid the adjustment costs, which results in an inventory growth that is highly autocorrelated (43% and 27% in Cases 2 and 5 respectively), in contrast with what is observed in the data, where the autocorrelation of inventory growth is 3%. Finally, the correlation between inventory growth, physical capital investment rate, inventory growth and sales growth are qualitatively consistent with the data.

The fit across the inventory-to-sales ratio varies considerably across the two types of models. The stockout motive models (Cases 1 to 3) can match both the level and the volatility of the inventory-to-sales ratio, but the inventory as an input models (Cases 4 to 6) cannot match neither the mean nor the volatility of the inventory-to-sales ratio. In these models, firms hold too few inventories in equilibrium (inventory-to-sales ratio of 4% versus 11% in the data). This result lends support to Bils and Kahn (1998) and Ramey and West (1999) who argue that stockout costs are important in explaining firms' inventory behavior.

The results with non-convex inventory adjustment costs (Cases 3 and 6) are quantitatively very similar to the results without inventory adjustment costs. So nonconvex adjustment costs (assuming, as we do here, that the cost of adjusting inventories (delivery cost) is constant over time) does not seem to improve the matching of the general model on the quantity dimension.

Taken together, the alternative models considered here do a reasonable job matching the firm level real quantity moments.

5.2 Matching Asset Pricing Facts

Turning to the analysis of the asset pricing implications of the alternative models, Tables 7 and 8 contains our main findings. Table 7 reports the value-weighted average returns (averages across the simulated 100 samples) of the nine physical capital and inventory growth portfolios for each model. We replicate the construction of these portfolios on the simulated

data following the procedure used in the real data, as described in Section 2.3. Table 8 reports the FMB firm level regression results obtained using the simulated data. It reports the time series averages of each cross-sectional regression loading on the firm level physical capital investment rate and inventory growth rate along with its time-series t-statistic. The results from this table are the simulated counterpart of the empirical results presented in Table 3. We report results for the six alternative specifications of the model.

5.2.1 Portfolio Level Evidence

As reported in Table 2, across all models, the portfolio procedure generates a pattern of average excess returns that is qualitatively similar to the pattern of average excess returns in the data. The value-weighted average excess returns of these portfolios are decreasing in both the physical capital investment rate and inventory growth rate. The spread in returns across the inventory growth dimension (column low minus high, L-H), however, is too small compared with the spread observed in the data. None of the models can quantitatively match the observed high inventory growth risk premium. Across all the models without inventory convex adjustment costs (Cases 1, 3, 4 and 6) the maximum spread is around 1% per annum, which is much smaller than the maximum spread of around 7% for levered returns, or 4.9% for unlevered returns, observed in the data . Adding convex adjustment costs slightly improves the maximum spread to around 2.4% in the stockout avoidance model (Case 2), and to around 1.65% in the inventory as an input model (Case 5), but this spread is still too small. The spread along the physical capital dimension is relatively close to the data for levered returns, except within the high inventory growth portfolios. For unlevered returns, the simulated spread across the physical investment rate dimension is in general slightly higher than in the data.

[Insert Table 7 here]

5.2.2 Firm Level Evidence

The results from the Fama-MacBeth cross-sectional regressions on the simulated data reported in Table 8, confirms the portfolio results. Regarding the physical capital investment characteristic, the FMB regression produces negative and significant average slopes coefficients, consistent with the data. The magnitude of the implied investment rate premium is also consistent with the data, especially across the stockout motive models (Cases 1 to 3). Regarding the FMB slope coefficient associated with the inventory growth characteristic, the fit of all models is not satisfactory. Without convex inventory adjustment costs (Cases 1, 3, 4 and 6), the FMB slope coefficient associated with inventory growth is essentially zero across all specifications. Thus, controlling for firms' physical capital investment rate, the firm level inventory growth does not provide any additional information about firms' risk. The implied inventory growth risk premium is essentially zero, a result that is at odds with the empirical facts documents in Table 3. With convex inventory adjustment costs, the inventory growth FMB slope coefficient becomes slightly negative and statistically significant, but the magnitude its too small relative to the data. The implied inventory growth risk premium is around 0.5% for the stockout motive model (Case 2) and around 0.35% for the inventory as input model (Case 3), which is much smaller than the implied inventory growth risk premium of 3.9% observed in the data.

[Insert Table 8 here]

Taken together, the results reported here show that some variations of the models considered here can match the real quantity facts in the data reasonably well, but all models have difficulty in matching the asset pricing facts with reasonable parameter values. Adding convex inventory adjustment costs slightly improves the fit of these models along the asset pricing dimension, but it deteriorates the fit along the real quantities dimension.

5.3 Additional Robustness Checks

In order to establish the robustness of the findings, we simulate additional alternative specifications of each model (results are available from the authors upon request).

First, we examine a more general specification of the firm's production technology. Instead of a Cobb-Douglas production function in physical capital and inventories, we specify a CES technology and investigated a wide range of elasticities of substitution between the two inputs. This specification does not improve the fit of any of the models considered here along the asset pricing dimension. Consistent with the results reported here, the link between inventory growth and risk premia with this specification is still too small compared with the data.

Second, in the spirit of Khan and Thomas (2007b), we extend the non-convex inventory adjustment costs specification to allow these costs to vary with either aggregate or firm level profitability. That is, we include idiosyncratic stochastic (delivery) costs of inventories. This specification captures the fact that it might be harder for the firm to adjust its inventory stocks in bad times, thus increasing the risk of the firms in those periods. This specification also does not improve the fit of the model along the asset pricing dimension.

6 Conclusion

Using firm level cross-sectional data and controlling for the firm's physical capital investment rate, there is a high inventory growth risk premium. We find that two leading macroeconomic models of inventory behavior in which inventory investment and risk premia are simultaneously and endogenously determined, fail to quantitatively explain the large magnitude of this inventory growth risk premium. Adding non-convex and especially convex adjustment costs in inventory investment slightly increases the magnitude of the risk premium in these models, but it deteriorates the fit of the model along the real quantity dimension. We conclude that the strong link between inventory growth and firms' risk documented here is a

quantitative puzzle for existing macroeconomic models of inventory behavior.

Our findings suggest that there are probably important mechanisms that link inventory growth and firms' risk that need to be identified. Identifying this mechanism might be useful for improving the specification of current macroeconomic models of inventory behavior, which are already successful at matching real quantities. An improved specification of these models along the asset pricing dimension might shed new light on the economic link between inventory investment, the business cycle and variation in risk premia over the business cycle. We leave this interesting question for future research.

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A-1 Data

Monthly stock returns are from the Center for Research in Security Prices (CRSP) and accounting information is from the CRSP/COMPUSTAT Merged Annual Industrial Files. The sample is from July 1965 to June 2006. We exclude from the sample any firm-year observation with missing data or for which total assets or the gross capital stock are either zero or negative. In addition, as standard, we omit firms whose primary SIC classification is between 4900 and 4999 (regulated firms) or between 6000 and 6999 (financial firms). We require a firm to have fiscal-year end between June and December in order to align the accounting data across firms. Since most firms have a December fiscal-year end, this selection requirement does not bias the representativeness of the sample. Following Fama and French (1993), we also require that each firm must have at least two years of data to be included in the sample.

Following Fama and French (1993), we define book value of equity as the COMPUSTAT book value of common equity (data item 60) plus balance-sheet deferred taxes (data item 74) and investment tax credits (data item 208), minus the book value of preferred stock. Depending on availability, we use the redemption (data item 56), liquidation (data item 10), or par value (data item 130) of preferred stock. When data item 60 is not available, the liquidation value of common equity (data item 235) is used. COMPUSTAT data item 128 is used for capital investment, i; the net book value of property, plant, and equipment (data item 8) is used for the capital, k; data item 3 is used for inventories stock, n. We follow Hou and Robinson (2006) and Fama-French (2008) in defining the main variables that we use in our analysis.

BM: Book-to-market equity, the ratio of the book value of equity to the market value of equity. Market equity is price times shares outstanding at the end of December of t, from CRSP.

IK: Investment-to-capital ratio, capital expenditures (128) in year t divided by property,

plant & equipment (8) in year t.

HN: Inventory growth rate, the ratio of the change in the stock of total inventories (3) from year t to year t-1 to the stock of inventories at year t-1.

NS: Inventory-to-sales ratio, the total inventories (3) in year t divided by sales (12) in year t.

SG: Sales growth rate, the ratio of the change in the sales (12) from year t to year t-1 to the sales in year t-1, deflated by the CPI.

SIZE: the price times shares outstanding at the end of June of year t, from CRSP.

LEV: leverage, book liabilities (total assets (6) minus book value of equity) in year t divided by the market value of the firm (SIZE plus total assets (6) minus book value of equity).

rBOND: one period real bond excess return. We first follow Blume, Lim, and Mackinlay (1998) to impute the credit ratings for firms with no rating data from Compustat, then assign the corporate bond return for a given credit rating (from Ibbotson Associates) to all firms with the same rating to obtain the corporate bond returns, minus the riskfree rate, for each firm from July of t to June of year $t+1$.

A-2 Numerical Algorithm

To solve the model numerically, we use the value function iteration procedure to solve the firm's maximization problem. The value function and the optimal decision rule are solved on a grid in a discrete state space. We use a multi-grid algorithm in which the maximum number of points is 50 in each dimension. In each iteration we specify a grid of points for capital and inventory, respectively with upper bounds $\bar{k}$ and $\bar{n}$ that are large enough to be non-binding. The grids for capital and inventory stocks are constructed recursively, following McGrattan (1999), that is, $k_i = k_{i-1} + c_{k1} \exp(c_{k2}(i-2))$, where $i = 1,\ldots,50$ is the index of

grids points and c_{k1} and c_{k2} are two constants chosen to provide the desired number of grid points and two upper bounds $\bar{k}$ and $\bar{n}$, given two pre-specified lower bounds $\underline{k}$ and $\underline{n}$. The advantage of this recursive construction is that more grid points are assigned around $\bar{k}$ and $\bar{n}$, where the value function has most of its curvature.

The state variable x has continuous support in the theoretical model, but it has to be transformed into discrete state space for the numerical implementation. The popular method of Tauchen and Hussey (1991) does not work well when the persistence level is above 0.9. Because both the aggregate and idiosyncratic profitability processes are highly persistent, we use the method described in Rouwenhorst (1995) for a quadrature of the Gaussian shocks. We use 5 grid points for the x process and 7 grid points for the z process. In all cases the results are robust to finer grids as well. Once the discrete state space is available, the conditional expectation can be carried out simply as a matrix multiplication. Linear interpolation is used extensively to obtain optimal investment and inventory investment which do not lie directly on the grid points. Finally, we use a simple discrete, global search routine in maximizing the firm's problem.

Table 1
Firm Level and Cross Sectional Moments of Selected Variables

This table reports summary statistics of firm level physical capital investment rate (IK), inventory growth rate (HN), the inventory-to-(net) sales ratio (NS), real sales growth rate (SG) as well the firm level book-to-market ratio (BM) for comparison. Panel A reports mean values across firms with at least fifteen observations of firm level characteristics: it reports the mean, median, standard deviation (std), autocorrelation (AC1), and the 10^{th} and 90^{th} percentile of selected variables as well as their correlations. Panel B reports mean values across time of cross-sectional moments computed in June of each year. It reports the cross-sectional mean, cross-sectional median, cross-sectional standard deviation (std) and the cross-sectional 10^{th} and 90^{th} percentile of selected variables as well as their cross-sectional correlations. The data are annual and the sample is July 1965 to June 2006.

Panel A: Mean Across Firms of Firm Level Moments

					Percentile		Correlation			
	Mean	Median	std	AC1	10^{th}	90^{th}	HN	NS	SG	BM
IK	0:28	0:23	0:19	0:34	0:11	0:53	0:32	−0:09	0:30	−0:14
HN	0:14	0:08	0:34	0:03	−0:18	0:53	1	0:10	0:51	−0:12
NS	0:11	0:10	0:04	0:72	0:06	0:16	0:10	1	−0:16	0:08
SG	0:08	0:05	0:22	0:17	−0:13	0:33	0:51	−0:16	1	−0:21
BM	0:98	0:86	0:55	0:59	0:43	1:71	−0:12	0:08	−0:21	1

Panel B: Mean Across Time of Cross-Sectional Moments

					Percentile		Correlation			
	Mean	Median	std	AC1	10^{th}	90^{th}	HN	NS	SG	BM
IK	0:32	0:22	0:32	–	0:08	0:64	0:29	0:03	0:32	−0:17
HN	0:18	0:08	0:53	–	−0:21	0:59	1	0:11	0:44	−0:12
NS	0:10	0:09	0:09	–	0:02	0:19	0:11	1	−0:02	0:04
SG	0:13	0:06	0:39	–	−0:15	0:41	0:44	−0:02	1	−0:15
BM	0:98	0:78	1:01	–	0:27	1:82	−0:12	0:04	−0:15	1

Table 2
Characteristics of Physical Capital and Inventory Growth Portfolios

This table reports average characteristics of selected variables of nine portfolios double sorted on physical capital and inventory growth rates. All characteristics (except returns) are measured at the time of the portfolio formation. Inventory-L, M and Inventory-H stands for the portfolio with low (L), medium (M) and high (H) cross sectional inventory growth rates respectively, and IK-L, MED and IK-High stand for the portfolio with low, medium and high cross sectional physical investment rates respectively. L-H stands for low minus high and it reports the difference between the mean of the characteristic of the corresponding low and high portfolios. The difference in returns reported in column L-H is labelled the inventory growth risk premium. Panel A in the table reports the mean of the following variables: rVW is the value weighted excess return (in excess of the risk-free rate), rEW is the equally weighted excess return, rVW^{unl} is the unlevered equity value weighted excess return. Panel B reports the mean of the following variables: HN is the net inventory growth rate (%), IK is the gross investment rate (%), SIZE is market equity, BM is the ratio of book equity to market equity, LEV is the leverage ratio and rBOND is the one period corporate bond excess return (in excess of the risk-free rate). The returns are annualized, and the accounting data is annual and the sample is from July 1965 to June 2006.

	Inventory Growth				Inventory Growth				Inventory Growth			
	Low	Mid	High	L-H	Low	Mid	High	L-H	Low	Mid	High	L-H
					Panel A: Equity Returns							
	rVW				rEW				rVW^{unl}			
IK-Low	9:6	6:9	6:8	2:9	14:6	11:8	10:7	3:9	8:5	7:3	6:2	2:3
IK-Mid	8:8	7:3	5:3	3:6	12:6	11:0	8:8	3:8	8:5	7:2	5:3	3:2
IK-High	8:5	4:6	1:6	6:9	12:7	10:3	4:4	8:3	8:1	7:6	3:2	4:9
L-H	1:1	2:3	5:2	8:0	1:9	1:5	6:2	10:2	0:4	−0:3	3:0	5:3
					Panel B: Characteristics							
	HN				IK				SIZE			
IK-Low	−20	7	46		7	11	12		3:9	4:7	4:5	
IK-Mid	−15	7	46		16	20	25		4:8	5:6	5:0	
IK-High	−19	7	71		44	43	74		4:4	5:2	4:7	
	BM				LEV				rBOND			
IK-Low	1:4	1:2	1:1		0:47	0:45	0:42		1:59	0:96	1:24	
IK-Mid	1:1	0:9	0:9		0:43	0:39	0:36		1:27	0:41	0:91	
IK-High	0:9	0:8	0:6		0:37	0:35	0:30		1:01	0:57	1:04	

Table 3
Fama-MacBeth Cross-Sectional Regressions of Firm Level Monthly Returns

The table shows average slopes and their t-statistics from monthly cross-sectional regressions to predict stock returns. Int is the average regression intercept, IK is the firm level physical capital investment rate and HN is the firm level inventory growth. The average regression R^2 is adjusted for degrees of freedom and NFirms is the time series average number of firms in each cross-sectional regression. Implied premium (%) is the implied physical investment annual risk premium (IK) or inventory growth risk premium (HN) implied by the corresponding FMB estimated slope coefficient as described in the text (see equation (1)). The t-statistics (in parenthesis) for the average regression slopes is computed using the standard deviation of the time-series monthly regression slopes (computed as in Newey-West with 4 lags). The returns data is monthly, the accounting data (IK and HN) is annual and the sample is from July 1965 to June 2006.

	Slope						Implied Premium (%)	
Specification	Int	IK	HN	LEV	R^2	NFirms	IK	HN
#1	1:58	−0:64			0:44	2161	4:3	
	[5:72]	[−4:42]						
#2	1:46		−0:49		0:21	1889		4:7
	[4:89]		[−8:28]					
#3	1:56	−0:42	−0:41		0:56	1857	2:8	3:9
	[5:73]	[−2:77]	[−7:21]					
#4	1:33	−0:35	−0:40	0:52	0:52	1856	2:4	3:8
	[5:17]	[−2:40]	[−7:42]	[1:79]				

Table 4
Specification of the Alternative Models

Specification	Elasticity of Output wrt Inventories	Inventory Adjustment Costs	Stockout Costs
A: Stockout motive model			
1 : no inventory ad. costs	$_n = 0$	No	Yes, eq. (7)
2 : convex inventory ad. costs	$_n = 0$	Yes, eq. (9)	Yes, eq. (7)
3 : non-convex inventory ad. costs	$_n = 0$	Yes, eq. (10)	Yes, eq. (7)
B: Inventory as an input model			
4 : no inventory adjustment costs	$_n > 0$	No	No
5 : convex inventory adjustment costs	$_n > 0$	Yes, eq. (9)	No
6 : non-convex inventory adjustment costs	$_n > 0$	Yes, eq. (10)	No

Table 5
Parameter Values

This table presents the calibrated parameter values of each specification of the model.

Panel A: Parameters Constant Across Specifications

Parameter	Symbol	Value
Technology		
Elasticity of output with respect to capital	$_k$	0:7
Rate of depreciation for capital	$_k$	0:10
Rate of depreciation for inventory	$_n$	0:20
Slope parameter in capital adjustment cost	c_k	2
Curvature parameter in capital adjustment cost	v_k	3
Curvature parameter in inventory adjustment cost	v_n	3
Stochastic Processes		
Persistence coefficient of aggregate productivity	$_x$	$0{:}98^4$
Conditional volatility of aggregate productivity	$_x$	0:014
Persistence coefficient of firm-specific productivity	$_z$	0:78
Conditional volatility of firm-specific productivity	$_z$	0:35
Time-preference coefficient		0:94
Constant price of risk	$_0$	28
Time-varying price of risk	$_1$	−300

Panel B: Parameters for Alternative Specifications

Parameter	Symbol	Case 1	2	3	4	5	6
		Value					
Technology							
Elasticity of output with respect to inventory	$_n$	0	0	0	:01	:01	:01
Slope parameter in inventory adjustment cost	c_n	0	0:8	0	0	0:15	0
Slope parameter in inventory stockout cost	d	0:5	0:5	0:5	0	0	0
Parameter for stockout (backlog) cost	e	0:2	0:2	0:2	0	0	0
Fixed cost in production	f	:55	:45	:10	:35	:35	:10
Parameter for nonconvex inventory adjustment cost	a^+	0	0	:01	0	0	:01
	a^-	0	0	:02	0	0	:02
Long run average of aggregate productivity	$\bar{x}$	−0:4	−0:8	−0:4	−0:6	−0:6	−0:3

Table 6
Data versus Model Implied Moments Across Several Calibrations

This table presents selected moments in the data and implied by the simulation of the model under alternative calibrations (Case 1 to Case 6). The reported statistics are averages from 100 samples of simulated data, each with 3600 firms and 200 annual observations, with the corresponding standard deviation across samples in parenthesis. Panel A reports moments of aggregate asset prices and measures of total investment and inventory investment adjustment costs. It reports the mean and the standard deviation, denoted by (:); of the market unlevered equity premium R_s^e, net real riskfree rate R_f, the Sharpe ratio on the (unlevered) aggregate stock market. Panel A also reports measures of adjustment costs. It reports the percentage of the firm's output that is lost due to total adjustment costs (Adj Cost to Output- the figure for this variable in the data is from the evidence provided in Hamermesh and Pfann, 1996). Panel B reports firm level moments - real quantities. It reports the mean, the standard deviation, autocorrelation and correlation of the firm level investment rate (IK), net inventory growth rate (HN), the inventory-to-(net) sales ratio (NS) and real sales growth (SG).

Panel A: Aggregate Asset Pricing Moments and Adjustment Costs

	Aggregate Asset Prices					Adj Costs
	Means		Std		Sharpe	Adj Cost
Case	R_s^e	R_f	(R_s^e)	(R_f)	Ratio	to Output
Data						
−	4:5	1:8	15:2	3	0:30	0:01 − 0:20
Stockout motive model						
1	4:57	2:16	21:54	4:85	0:22	0:03
	[0:63]	[1:21]	[4:35]	[1:22]	[0:06]	[0:01]
Stockout motive model with convex inventory adjustment costs						
2	4:89	1:74	23:17	5:06	0:22	0:03
	[0:63]	[1:48]	[5:42]	[0:95]	[0:05]	[0:00]
Stockout motive model with nonconvex inventory adjustment costs						
3	4:53	2:16	21:91	5:20	0:22	0:03
	[0:62]	[1:81]	[4:62]	[0:98]	[0:06]	[0:01]
Inventory as an input model						
4	5:01	1:81	22:05	5:06	0:24	0:03
	[0:49]	[1:55]	[4:88]	[1:01]	[0:06]	[0:01]
Inventory as an input model and convex inventory adjustment costs						
5	5:02	1:67	21:81	4:93	0:24	0:04
	[0:61]	[1:64]	[5:20]	[1:13]	[0:07]	[0:01]
Inventory as an input model and nonconvex inventory adjustment costs						
6	4:89	1:84	21:38	5:07	0:24	0:04
	[0:54]	[1:56]	[4:78]	[1:01]	[0:06]	[0:01]

Table 6 (cont)

Panel B: Firm Level Moments - Real Quantities

	Means				Std				Autocorrelation		Correlation		
Case	IK	HN	NS	SG	(IK)	(HN)	(NS)	(SG)	AC(IK)	AC(HN)	(IK,HN)	(IK,SG)	(HN,SG)
Data													
–	0:28	0:13	0:10	0:08	0:19	0:34	0:04	0:22	0:35	0:03	0:32	0:44	0:53
Stockout motive model													
1	0:11	0:02	0:10	0:02	0:16	0:40	0:05	0:31	0:60	−0:11	0:18	0:64	0:17
	[0:02]	[0:02]	[0:03]	[0:01]	[0:02]	[0:03]	[0:01]	[0:04]	[0:04]	[0:09]	[0:06]	[0:03]	[0:07]
Stockout motive model with convex inventory adjustment costs													
2	0:11	0:00	0:10	0:00	0:15	0:34	0:05	0:34	0:61	0:43	0:55	0:58	0:14
	[0:00]	[0:00]	[0:03]	[0:01]	[0:02]	[0:03]	[0:01]	[0:05]	[0:05]	[0:07]	[0:04]	[0:03]	[0:03]
Stockout motive model with nonconvex inventory adjustment costs													
3	0:11	0:01	0:10	0:01	0:14	0:41	0:03	0:32	0:58	−0:10	0:19	0:64	0:18
	[0:00]	[0:00]	[0:01]	[0:01]	[0:00]	[0:01]	[0:00]	[0:07]	[0:05]	[0:08]	[0:07]	[0:04]	[0:08]
Inventory as an input model													
4	0:11	0:02	0:04	0:02	0:16	0:39	0:01	0:36	0:62	−0:00	0:43	0:57	0:13
	[0:00]	[0:01]	[0:01]	[0:01]	[0:02]	[0:02]	[0:00]	[0:05]	[0:04]	[0:02]	[0:03]	[0:02]	[0:03]
Inventory as an input model and convex inventory adjustment costs													
5	0:11	0:00	0:04	0:01	0:15	0:27	0:01	0:36	0:61	0:27	0:52	0:57	0:12
	[0:00]	[0:00]	[0:02]	[0:01]	[0:02]	[0:02]	[0:00]	[0:05]	[0:04]	[0:03]	[0:04]	[0:03]	[0:05]
Inventory as an input model and nonconvex inventory adjustment costs													
6	0:11	0:01	0:04	0:00	0:16	0:39	0:01	0:37	0:62	0:00	0:43	0:57	0:12
	[0:00]	[0:00]	[0:00]	[0:01]	[0:02]	[0:02]	[0:00]	[0:07]	[0:04]	[0:03]	[0:03]	[0:02]	[0:03]

Table 7

Equity Returns of Physical Capital and Inventory Growth Portfolios on Simulated Data

This table reports mean value-weighted returns of nine portfolios double sorted on investment and inventory growth rate constructed on data simulated by six alternative specifications of the model (Case 1 to Case 6). As in the real data, the nine portfolios are first sorted on net inventory growth rate (cutoffs at the 33^{th} and 66^{th} percentile), and then, within each inventory growth rate bin, all firms are sorted on gross capital investment rate (cutoffs at the 33^{th} and 66^{th} percentile). Inv-L, M and Inv-H stands for the portfolio with low (L), medium (M) and high (H) cross sectional investment rates respectively, while Inventory Investment-Low, Inventory Investment-Med and Inventory Investment-High stand for the portfolio with low, medium and high cross sectional inventory growth rates respectively. L-H stands for low minus high and it reports the difference between the mean of the characteristic of the corresponding low and high portfolio. The difference in returns reported in column L-H is labeled the inventory growth risk premium. The reported statistics are averages from 100 samples of simulated data, each with 3600 firms and 200 annual observations.

	Inventory Growth				Inventory Growth				Inventory Growth			
	Low	Mid	High	L-H	Low	Mid	High	L-H	Low	Mid	High	L-H
					Stockout Motive Models							
	Case 1				Case 2				Case 3			
IK-Low	6:32	5:56	5:6	0:72	7:15	7:08	5:06	2:09	6:19	5:52	5:55	0:64
IK-Mid	5:55	4:19	4:36	1:2	6:13	5:35	3:74	2:39	5:46	4:21	4:4	1:05
IK-High	4:23	3:36	3:32	0:92	4:93	4:56	3:22	1:71	4:26	3:49	3:47	0:78
L-H	2:09	2:2	2:28	3:01	2:21	2:51	1:83	3:92	1:94	2:03	2:08	2:72
					Inventory as an Input Models							
	Case 4				Case 5				Case 6			
IK-Low	6:75	6:1	5:64	1:1	6:81	6:09	5:34	1:48	6:37	5:9	5:46	0:91
IK-Mid	5:87	4:71	4:55	1:33	5:97	4:89	4:32	1:65	5:64	4:63	4:49	1:15
IK-High	4:96	4:16	3:9	1:06	5:08	4:41	3:83	1:25	4:81	4:15	3:85	0:96
L-H	1:79	1:94	1:74	2:85	1:73	1:68	1:51	2:98	1:56	1:75	1:61	2:52

Table 8
Fama-MacBeth Cross-Sectional Regressions of Firm Level Monthly Returns on Simulated Data

The table reports average slopes and their t-statistics from monthly cross-section regressions to predict stock returns on the simulated data generated by six alternative specifications of the model (Case 1 to Case 6). Int is the average regression intercept, IK is the firm level physical capital investment rate and HN is the firm level inventory growth. The average regression R^2 is adjusted for degrees of freedom and NFirms is the time series average number of firms in each cross-sectional regression. Implied premium (%) is the implied physical investment annual risk premium (IK) or inventory growth risk premium (HN) implied by the corresponding FMB estimated slope coefficient as described in the text (see equation (1)). The t-statistics (in parenthesis) for the average regression slopes is computed using the standard deviation of the time-series monthly regression slopes (computed as in Newey-West with 4 lags). The reported statistics are averages from 100 samples of simulated data, each with 3600 firms and 200 annual observations.

	Slope		Implied Premium (%)	
Case	IK	HN	IK	HN
Data				
-	−0:42	−0:41	2:80	3:90
	[−2:77]	[−7:21]		
Stockout motive model				
1	−0:75	−0:00	2:25	0:05
	[0:05]	[0:00]		
Stockout motive model with convex inventory adj. costs				
2	−0:74	−0:06	2:07	0:49
	[0:07]	[0:02]		
Stockout motive model with nonconvex inventory adj. costs				
3	−0:70	−0:00	2:08	0:11
	[0:05]	[0:00]		
Inventory as an input model				
4	−0:68	−0:00	1:92	0:13
	[0:06]	[0:01]		
Inventory as an input model and convex inventory adj. costs				
5	−0:64	−0:04	1:82	0:35
	[0:07]	[0:01]		
Inventory as an input model and nonconvex inventory adj. costs				
6	−0:63	−0:01	1:78	0:18
	[0:06]	[0:01]		

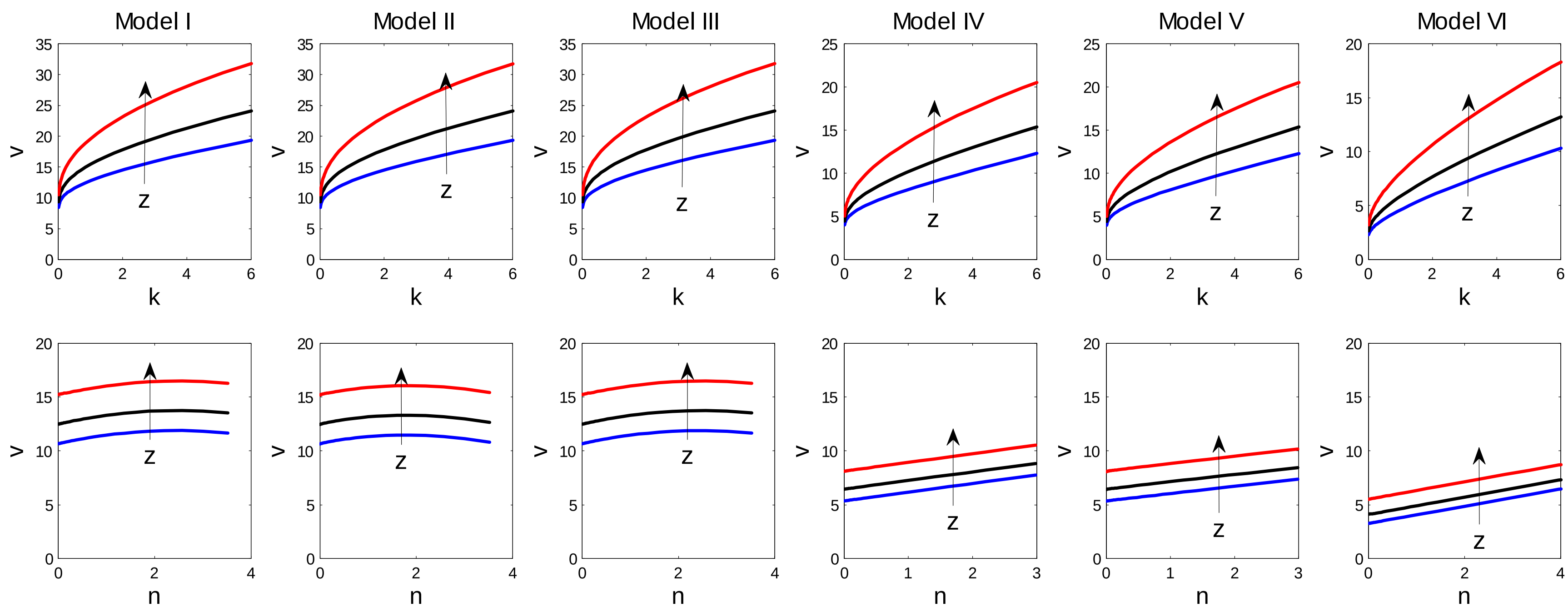

Figure 1: **The Value Functions of the Alternative Models against the Underlying States**

This figure plots the value functions $(v(k_t^j, n_t^j, x_t, z_t^j))$ against the four state variables in the six alternative models. We fix the aggregate profitability x_t at its long-run average level of $\bar{x}$ in all the panels. Each one of these panels has a set of curves corresponding to different values of z_t^j and the arrow in each panel indicates the direction along which z_t^j increases.

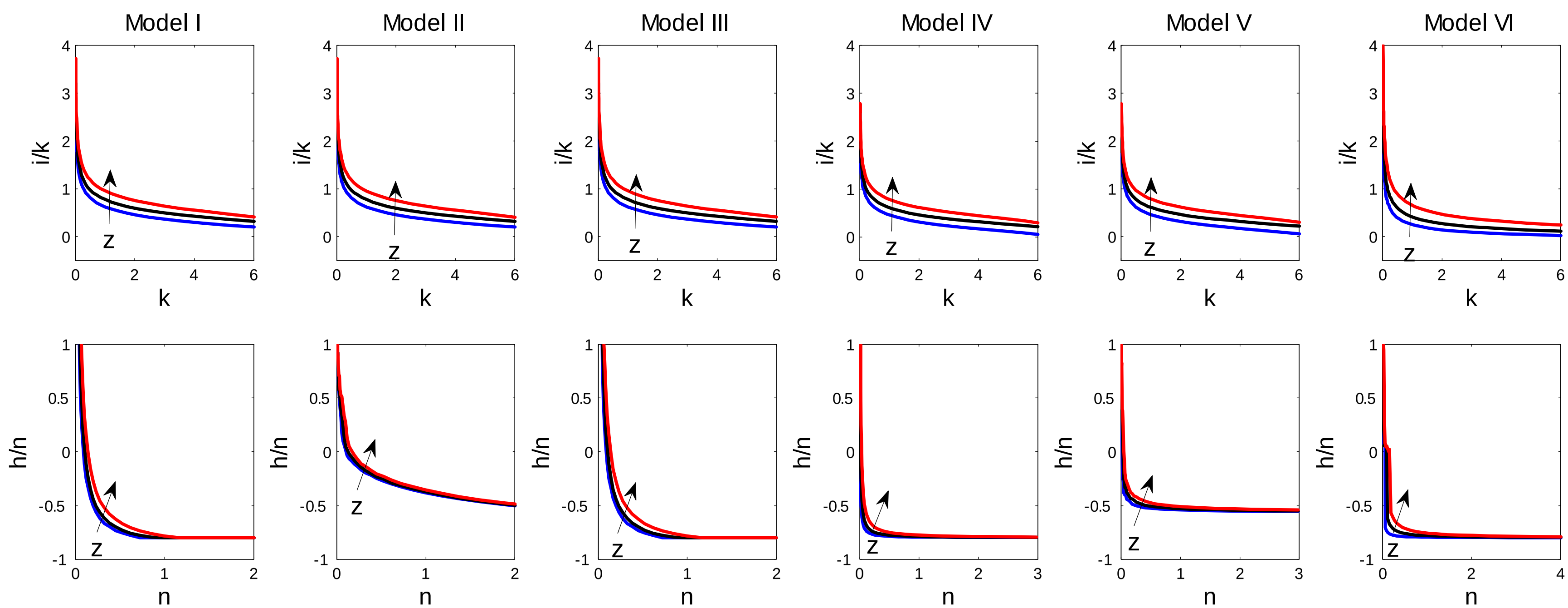

Figure 2: **The Policy Functions of the Alternative Models against the Underlying States**
This figure plots the policy functions $(i/k(k_t^j, n_t^j, x_t, z_t^j)$ and $h/n(k_t^j, n_t^j, x_t, z_t^j))$ against the four state variables in the six alternative models. We fix the aggregate profitability x_t at its long-run average level of $\bar{x}$ in all the panels. Each one of these panels has a set of curves corresponding to different values of z_t^j and the arrow in each panel indicates the direction along which z_t^j increases.

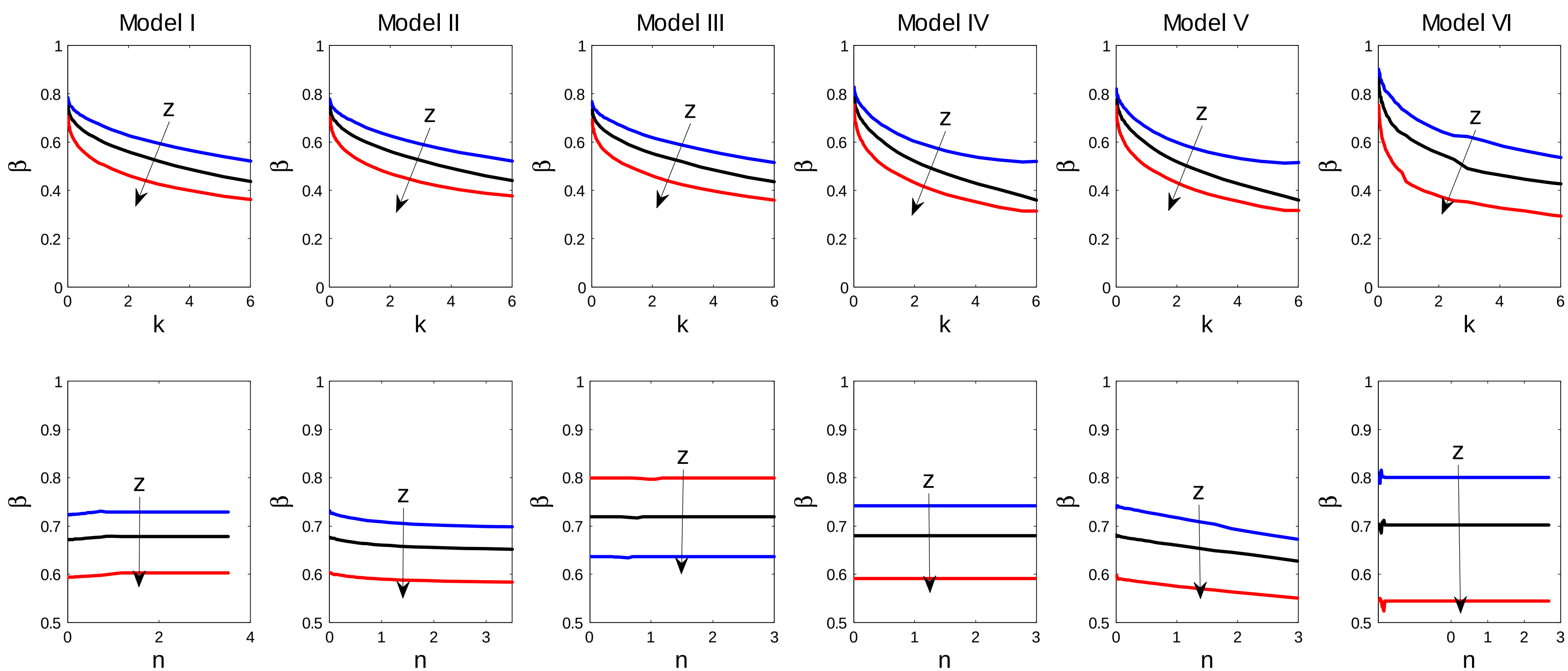

Figure 3: **The Conditional Betas of the Alternative Models against the Underlying States**
This figure plots the conditional betas $(\beta(k_t^j, n_t^j, x_t, z_t^j))$ against four state variables in the six alternative models. We fix the aggregate profitability x_t at its long-run average level of $\bar{x}$ in all the panels. Each one of these panels has a set of curves corresponding to different values of z_t^j and the arrow in each panel indicates the direction along which z_t^j increases.