Financial Intermediary Capital*

Adriano A. Rampini
Duke University

S. Viswanathan
Duke University

First draft: July 2010
This draft: June 2011

Abstract

We propose a dynamic theory of financial intermediaries as collateralization specialists that are better able to collateralize claims than households. Intermediaries require capital as they can borrow against their loans only to the extent that households themselves can collateralize the assets backing the loans. The net worth of financial intermediaries and the corporate sector are both state variables affecting the spread between intermediated and direct finance and the dynamics of real economic activity, such as investment, and financing. The accumulation of net worth of intermediaries is slow relative to that of the corporate sector. A credit crunch has persistent real effects and can result in a delayed or stalled recovery. We provide sufficient conditions for the comovement of the marginal value of firm and intermediary capital and discuss when incomplete risk management is optimal.

Keywords: Collateral; Financial intermediation; Financial constraints; Investment

*We thank Nittai Bergman and Nobu Kiyotaki and seminar participants at the IMF, the MIT theory lunch, Boston University, the Federal Reserve Bank of New York, the 2010 SED Annual Meeting, the 2010 Tel Aviv University Finance Conference, the 2011 Jackson Hole Finance Conference, the 2011 FIRS Annual Conference, and the 2011 WFA Annual Meeting for helpful comments. This paper subsumes the results on financial intermediation in our paper "Collateral, financial intermediation, and the distribution of debt capacity" (Rampini and Viswanathan (2007)), which is now titled "Collateral, risk management, and the distribution of debt capacity" (Rampini and Viswanathan (2010)). Address: Duke University, Fuqua School of Business, 100 Fuqua Drive, Durham, NC, 27708. Rampini: Phone: (919) 660-7797. Email: rampini@duke.edu. Viswanathan: Phone: (919) 660-7784. Email: viswanat@duke.edu.

1 Introduction

The capitalization of financial intermediaries is arguably critical for economic fluctuations and growth. We provide a dynamic model in which financial intermediaries are collateralization specialists and firms need to collateralize promises to pay with tangible assets. Financial intermediaries are modeled as lenders that are able to collateralize a larger fraction of tangible assets than households who lend to firms directly, that is, are better able to enforce their claims. Financial intermediaries require net worth as their ability to refinance their collateralized loans from households is limited, as they, too, need to collateralize their promises. The net worth of financial intermediaries is hence a state variable and affects the dynamics of the economy. Importantly, both firm and intermediary net worth play a role in our model and jointly affect the dynamics of firm investment, financing, and loan spreads. Spreads on intermediated finance are high when both firms and financial intermediaries are poorly capitalized and in particular when intermediaries are moreover poorly capitalized relative to firms. One of our main results is that intermediaries accumulate net worth more slowly than the corporate sector. This has important implications for economic dynamics. For example, a credit crunch, that is, a drop in intermediary net worth, has persistent real effects and can result in a delayed or stalled recovery.

In our model, firms can raise financing either from households or from financial intermediaries. Firms have to collateralize their promises to pay due to limited enforcement.[1] Both households and intermediaries extend collateralized loans, but financial intermediaries are better able to collateralize promises and hence are able to extend more financing per unit of tangible assets collateralizing their loans. Financial intermediaries in turn are able to borrow against their loans, but only to the extent that other lenders themselves can collateralize the assets backing the loans. Intermediaries thus need to finance the additional amount that they are able to lend out of their own nemoun

achieved with financial intermediaries, which cannot be achieve otherwise. Financial intermediaries have constant returns in our model and hence there is a representative financial intermediary. We first analyze the choice between intermediated and direct finance in the cross section of firms in a static environment. Taking the spread on intermediated finance as given, our model predicts that severely constrained firms borrow as much as possible from intermediaries while less constrained firms borrow less and dividend paying firms do not borrow from intermediaries at all. These implications are empirically plausible and similar to predictions in the literature. We then consider the equilibrium spread on intermediated finance when there is a representative firm. Importantly, the spread on intermediated finance critically depends on both firm and intermediary net worth. Given the (representative) firm's net worth, spreads are higher when the intermediary is less well capitalized. However, spreads are particularly high when firms are poorly capitalized, and intermediaries are poorly capitalized relative to firms at the same time. Poor capitalization of the corporate sector per se does not imply high spreads, as low firm net worth reduces the demand for loans from intermediaries. Given the net worth of the intermediary sector, a reduction in the net worth of the corporate sector may reduce spreads as the intermediaries can more easily accommodate the reduced loan demand.

Our model allows the analysis of the dynamics of intermediary capital. A main result of our model is that the accumulation of net worth of intermediaries is slow relative to that of the corporate sector. We first consider the deterministic dynamics of intermediary net worth and the spread on intermediated finance. In a deterministic steady state, intermediaries are essential, have positive net worth, and the spread on intermediated finance is positive. Dynamically, if firms and intermediaries are initially poorly capitalized, both firms and intermediaries accumulate net worth over time. Importantly, firms in our model accumulate net worth faster than financial intermediaries, because the marginal and in particular the average return on net worth for financially constrained firms is relatively high due to the high marginal product of capital. Financial intermediaries accumulate net worth at the interest rate earned on intermediated finance, which is at most the marginal return on net worth of the corporate sector and may be below when the collateral constraint for intermediated finance binds. Thus, intermediaries, with constant returns to scale, earn at most the marginal return on all their net worth, whereas firms, with decreasing returns to scale, earn the average return on their net worth.

Suppose that firms are initially poorly capitalized also relative to financial intermediaries. Then the dynamics of the spread on intermediated finance are as follows. Because the firms are poorly capitalized, the current demand for intermediated finance is low and

the spread on intermediated finance is zero. Intermediaries save net worth by lending to households to meet higher future corporate loan demand. As the firms accumulate more net worth, their demand for intermediated finance increases, and intermediary finance becomes scarce and the spread rises. The spread continues to rise as long as the firm's collateral constraint for intermediated finance binds. Once the spread gets so high that the collateral constraint is slack, the spread declines again as both firms and intermediaries accumulate net worth. As intermediary net worth accumulates more slowly, firms may temporarily accumulate more net worth and then later on re-lever as they switch to more intermediated finance when intermediaries become better capitalized. Eventually, the spread on intermediated finance declines to the steady state spread as intermediaries accumulate their steady state level of net worth.

A credit crunch, modeled as a drop in intermediary net worth, has persistent real effects in our model. While small drops to intermediary net worth can be absorbed by a cut in dividends, larger shocks reduce intermediary lending and raise the spread on intermediated finance. Real investment drops, and indeed drops even if the corporate sector is well capitalized, as the rise in the cost of intermediated finance raises firms' cost of capital. Remarkably, the recovery of investment after a credit crunch can be delayed, or stall, as the cost of intermediated finance only starts to fall once intermediaries have again accumulated sufficient net worth.

In a stochastic economy, we provide sufficient conditions for the marginal value of intermediary and firm net worth to comove. For example, if intermediary net worth is sufficiently low, these values comove and indeed move proportionally. Thus, the marginal value of intermediary net worth may be high exactly when the marginal value of firm net worth is high, too. We also show that incomplete risk management of both firms and intermediaries can be optimal. Indeed, if the risk is sufficiently small, neither firms nor intermediaries engage in risk management. More generally, we characterize the stochastic steady state dynamics of the shadow interest rates on intermediated financing in a stochastic economy in which intermediaries have no capital. When investment opportunities are constant, the shadow interest rates on intermediated finance are high when the corporate sector is poorly capitalized. When investment opportunities are stochastic, times when productivity is high feature small spreads and a well capitalized corporate sector while times when productivity is low feature large spreads and a poorly capitalized corporate sector.

Few extant theories of financial intermediaries provide a role for intermediary capital. Notable is in particular Holmström and Tirole (1997) who model intermediaries as monitors that cannot commit to monitoring and hence need to have their own capital at

stake to have incentives to monitor. Our static results mirror theirs. Diamond and Rajan (2001) and Diamond (2007) model intermediaries as lenders which are better able to enforce their claims due to their specific liquidation or monitoring ability in a similar spirit to our model. In contrast, the capitalization of financial intermediaries plays essentially no role in liquidity provision theories of financial intermediation (Diamond and Dybvig (1983)), in theories of financial intermediaries as delegated, diversified monitors (Diamond (1984), Ramakrishnan and Thakor (1984), and Williamson (1986)) or in coalition based theories (Townsend (1978) and Boyd and Prescott (1986)).

Dynamic models in which net worth plays a role, such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997a), typically consider the role of firm net worth only, although dynamic models in which intermediary net worth matters have recently been considered (see, for example, Gertler and Kiyotaki (2010), who also summarize the recent literature, and Brunnermeier and Sannikov (2010)). However, to the best of our knowledge, we are the first to consider a dynamic model in which both firm and intermediary net worth are critical and jointly affect the dynamics of financing, spreads, and economic activity.

In Section 2 we describe the model. The choice between intermediated and direct finance in a simplified static version of the model is analyzed in Section 3; we first analyze this choice in the cross section of firms, taking the spread on intermediated finance as given, and then study how the spread on intermediated finance varies with firm and intermediary net worth. The dynamics of intermediary capital are analyzed in Section 4. We first consider the deterministic steady state and dynamics of firm and intermediary capital, and the dynamic effects of a credit crunch. We then provide sufficient conditions for the comovement of the marginal value of intermediary and firm net worth in a stochastic economy, as well as a characterization of the stochastic steady state. Section 5 concludes.

2 Model

We consider a model in which promises to pay need to be collateralized due to limited enforcement. Time is discrete and the horizon infinite. There are three types of agents: agents that run firms, households, and financial intermediaries. We discuss these in turn.

2.1 Corporate sector

There is a representative firm which is risk neutral and subject to limited liability and discounts the future at rate $\beta \in (0, 1)$. The representative firm (which we at times refer

to simply as the firm or the corporate sector) has limited net worth w and has access to a standard neoclassical production technology $A'f(k)$ where $A' > 0$ is the stochastic total factor productivity, $f(\cdot)$ is the production function, and k is the amount of capital the firm deploys next period, which depreciates at the rate $\delta \in (0, 1)$. We assume that the production function $f(\cdot)$ is strictly increasing and strictly concave and satisfies the usual Inada condition. The firm can raise financing from both households and intermediaries by issuing one-period collateralized state-contingent claims b' to households and b'_i to intermediaries.

We assume that the exogenous state $s \in S$ follows a Markov chain with transition probability $\Pi(s, s')$, where S is a finite state space.[2] Total factor productivity A' next period depends on the exogenous state next period, that is, $A' \equiv A(s')$. We suppress the dependence on s' and use the short-hand A' throughout. The state of the economy $Z \equiv \{s, w, w_i\}$ includes the exogenous state s as well as two endogenous state variables, the net worth of the corporate sector w and the net worth of the intermediary sector w_i. The state-contingent interest rate on intermediated finance R'_i depends on the state Z of the economy, as shown below, but we again suppress the argument for notational simplicity.

We write the representative firm's problem recursively. The firm maximizes the discounted expected value of future dividends by choosing a dividend payout policy d, capital k, state-contingent promises b' and b'_i to households and intermediaries, and state-contingent net worth w' for the next period, taking the state-contingent interest rates on intermediated finance R'_i and their law of motion as given, to solve:

$$v(w, Z) = \max_{\{d,k,b',b'_i,w'\} \in \mathbb{R}^2_+ \times \mathbb{R}^S \times \mathbb{R}^{2S}_+} d + \beta E\left[v(w', Z')\right] \tag{1}$$

subject to the budget constraints

$$\begin{aligned} w + E\left[b' + b'_i\right] &\geq d + k, \qquad (2) \\ A'f(k) + k(1-\delta) &\geq w' + Rb' + R'_i b'_i, \qquad (3) \end{aligned}$$

and the collateral constraints

$$\begin{aligned} \theta k(1-\delta) &\geq Rb', \qquad (4) \\ (\theta_i - \theta)k(1-\delta) &\geq R'_i b'_i, \qquad (5) \end{aligned}$$

where θ is the fraction of tangible assets, that is, capital, that households can collateralize while θ_i is the fraction of tangible assets that intermediaries can collateralize. Since the

[2] In a slight abuse of notation, we denote the cardinality of S by S as well.

firm issues state-contingent claims to both households and intermediaries and pricing of the state-contingent loans is risk neutral, it is the expected value of the claims that enters the budget constraint in the current period, equation (2). Depending on the realized state next period, the firm repays Rb' to households and $R_i'b_i'$ to financial intermediaries as the budget constraint for the next period, equation (3), shows. We assume that the interest rate on direct finance R is constant as discussed below. Note moreover that the expectation operator $E[\cdot]$ denotes the expectation conditional on state Z, but the dependence on the state is again suppressed to simplify notation.

Importantly, to simplify the analysis we use notation that keeps track separately of the claims that are ultimately financed by households (b') and the claims that are financed by intermediaries out of their own net worth b_i'. In particular, whenever the firm borrows from financial intermediaries and issues strictly positive promises $R_i'b_i'$, the corresponding promises Rb' should be interpreted as being financed by the intermediary who in turn refinances them by issuing equivalent promises to households. Thus, we do not distinguish between claims financed by households directly, and claims financed by households indirectly by lending to financial intermediaries against collateral backing intermediaries' loans. This allows a simple formulation of the collateral constraints: firms can borrow up to fraction θ of the resale value of their capital by issuing claims to households (whether these are held directly or are indirectly finance via the intermediary) and can borrow up to the difference in collateralization rates, $\theta_i - \theta$, additionally by issuing claims which are financed by intermediaries out of their own net worth. We elaborate on the enforcement and settlement of claims further after the explicit discussion of households and the intermediaries' problem.[3]

The first order conditions, which are necessary and sufficient as the problem is well behaved, can be written as

$$\mu = 1 + \nu_d, \tag{6}$$

$$\mu = E\left[\beta\left(\mu'\left[A' f_k(k) + (1-\delta)\right] + \left[\lambda'\theta + \lambda_i'(\theta_i - \theta)\right](1-\delta)\right)\right], \tag{7}$$

$$\mu = R\beta\mu' + R\beta\lambda', \tag{8}$$

$$\mu = R_i'\beta\mu' + R_i'\beta\lambda_i' - R_i'\beta\nu_i', \tag{9}$$

$$\mu' = v_w(w', Z'), \tag{10}$$

where the multipliers on the constraints (2) through (5) are μ, $\Pi(Z,Z')\beta\mu'$, $\Pi(Z,Z')\beta\lambda'$, and $\Pi(Z,Z')\beta\lambda_i'$, and ν_d and $\Pi(Z,Z')R_i'\beta\nu_i'$ are the multipliers on the non-negativity con-

[3] A model with two types of collateral constraints is also studied by Caballero and Krishnamurthy (2001) who consider international financing in a model in which firms can raise funds from domestic and international financiers subject to separate collateral constraints.

straints on dividends and intermediated borrowing.[4] The envelope condition is $v_w(w, Z) = \mu$.

2.2 Households

There is a continuum of households (of measure 1) in the economy which are risk neutral and discount future payoffs at a rate R where $R^{-1} > \beta$, that is, are more patient than the agents who run firms. These lenders are assumed to have a large endowment of funds in all dates and states, and have a large amount of collateral and hence are not subject to enforcement problems but rather are able to commit to deliver on their promises. They are willing to provide any state-contingent claim at an expected rate of return R so long as such claims satisfy the firms' and intermediaries' collateral constraints.

2.3 Financial intermediaries as collateralization specialists

There is a continuum of financial intermediaries (of measure 1) which are risk neutral, subject to limited liability, and discount future payoffs at β_i where $\beta_i \in (\beta, R^{-1})$. Financial intermediaries are *collateralization specialists.* Intermediaries are able to seize up to fraction $\theta_i > \theta$ of the (resale value of) collateral backing promises issued to them. Financial intermediaries can in turn issue claims against such collateralized loans. Lenders to financial intermediaries can lend to intermediaries up to the amount of the collateral backing the intermediaries' loans that they themselves can seize. Consider the problem of a representative financial intermediary[5] with current net worth w_i and given the state variable Z. The intermediary maximizes the discounted value of future dividends by choosing a dividend payout policy d_i, state-contingent loans to households l', state-contingent intermediated loans to firms l'_i, and state-contingent net worth w'_i next period to solve

$$v_i(w_i, Z) = \max_{\{d_i, l', l'_i, w'_i\} \in \mathbb{R}_+^{1+3\#Z}} d_i + \beta_i E\left[v_i(w'_i, Z')\right] \quad (11)$$

[4] We use $\Pi(Z, Z')$ for the transition probability of the state of the economy in a slight abuse of notation. We ignore the constraints that $k \geq 0$ and $w' \geq 0$ as they are redundant, due to the Inada condition and the fact that the firms can never credibly promise their entire net worth next period (which can be seen by combining (3) at equality with (4) and (5).

[5] We consider a representative financial intermediary since intermediaries have constant returns to scale in our model and hence aggregation in the intermediation sector is straightforward. The distribution of intermediaries' net worth is hence irrelevant and only the aggregate capital of the intermediation sector matters.

subject to the budget constraints

$$w_i \geq d_i + E[l'] + E[l'_i], \tag{12}$$

$$Rl' + R'_i l'_i \geq w'_i. \tag{13}$$

Note that we state the intermediary's problem as if the intermediary only lends the additional amount it can collateralize. Again this simplifies the notation and analysis. In particular, we do not need to consider the intermediary's collateral constraint explicitly, as the firms' collateral constraint for financing ultimately provided by the households already ensures that this constraint is satisfied and hence renders the additional constraint redundant. However, whenever the intermediary is essential in the sense of the following definition, the interpretation is that the firms' claims are held by the intermediary and the intermediary in turn refinances the claims with households to the extent that they can collateralize the claims themselves.

Definition 1 (Essentiality of intermediation) *Intermediation is* ***essential*** *if an allocation can be supported with a financial intermediary but not without.*[6]

In contrast, we interpret financing which does not involve the intermediary as direct or unintermediated financing.

The first order conditions, which are necessary and sufficient as the problem is well behaved, can be written as

$$\mu_i = 1 + \eta_d, \tag{14}$$

$$\mu_i = R\beta_i \mu'_i + R\beta_i \eta', \tag{15}$$

$$\mu_i = R'_i \beta_i \mu'_i + R'_i \beta_i \eta'_i, \tag{16}$$

$$\mu'_i = v_{i,w}(w'_i, Z'), \tag{17}$$

where the multipliers on the constraints (12) through (13) are μ_i and $\Pi(Z, Z')\beta_i \mu'_i$, and η_d, $\Pi(Z, Z')R\beta_i \eta'$, and $\Pi(Z, Z')R'_i \beta_i \eta'_i$ are the multipliers on the non-negativity constraints on dividends and direct and intermediated lending. The envelope condition is $v_{i,w}(w_i, Z) = \mu_i$.

2.4 Enforcement and settlement

The borrowers' and intermediaries' collateral constraints can be derived from limited enforcement constraints. Rampini and Viswanathan (2010, 2011) study an economy with

[6]This definition is analogous to the definition of essentiality of money in monetary theory (see, e.g., Hahn (1973)).

limited enforcement and show that the optimal allocation can be implemented with complete markets in one period ahead Arrow securities subject to state-by-state collateral constraints. These are the collateral constraints we analyze here and are similar to the col-

each firm's problem in (1)-(5) and x_i solves the representative intermediary's problem (11)-(13) and (ii) the market for intermediated finance clears in all dates and states

$$l_i' = b_i'. \tag{18}$$

Note that equilibrium promises are default free, as the promises satisfy the collateral constraints (4) and (5), which ensures that neither firms nor financial intermediaries are able to issue promises on which it is not credible to deliver. While this is of course the implementation that we study throughout, we emphasize that the promises traded in our economy are contingent claims and that these contingent claims may be implemented in practice with noncontingent claims on which issuers are expected and in equilibrium indeed do default (see Kehoe and Levine (2006) for an implementation with equilibrium default in this spirit).

2.6 Endogenous minimum down payment requirement

Define the *minimum down payment requirement* $\wp$ when the firm borrows the maximum amount it can from households only as $\wp = 1 - R^{-1}\theta(1-\delta)$.[7] Similarly, define the minimum down payment requirement when the firm borrows the maximum amount it can from both households (at interest rate R) and intermediaries (at state-contingent interest rate R_i') as $\wp_i(R_i') = 1 - [R^{-1}\theta + E[(R_i')^{-1}](\theta_i - \theta)](1-\delta)$. Note that the minimum down payment requirement, at times referred to as the margin requirement, is endogenous in our model. The firm's investment Euler equation can then be written concisely as

$$1 \geq E\left[\beta \frac{\mu'}{\mu} \frac{A' f_k(k) + (1-\theta_i)(1-\delta)}{\wp_i(R_i')}\right]. \tag{19}$$

2.7 User cost of capital with intermediated finance

We can extend Jorgenson's (1963) definition of the user cost of capital to our model with intermediated finance. Define the premium on internal funds ρ as $1/(R+\rho) \equiv E[\beta\mu'/\mu]$ and the premium on intermediated finance ρ_i as $1/(R+\rho_i) \equiv E[(R_i')^{-1}]$. Using (7) through (9) the user cost of capital u is

$$u \equiv r + \delta + \frac{\rho}{R+\rho}(1-\theta_i)(1-\delta) + \frac{\rho_i}{R+\rho_i}(\theta_i - \theta)(1-\delta), \tag{20}$$

where $r + \delta$ is the frictionless user cost derived by Jorgenson (1963) and $1 + r \equiv R$. The user cost of capital exceeds the user cost in the frictionless model, because part of

[7] We use the character $\wp$, a fancy script p, for down *payment* (`\wp` in LaTeX and available under miscellaneous symbols).

investment needs to be financed with internal funds which are scarce and hence command a premium ρ (the second term on the right hand side) and part of investment is financed with intermediated finance which commands a premium ρ_i, as the funds of intermediaries are scarce as well (the last term on the right hand side).[8]

Internal funds and intermediated finance are both scarce in our model and command a premium as collateral constraints drive a wedge between the cost of different types of finance. The premium on internal finance is higher than the premium on intermediated finance, as the firm would never be willing to pay more for intermediated finance than the premium on internal funds.

Proposition 1 (Premia on internal and intermediated finance) *The premium on internal finance ρ (weakly) exceeds the premium on intermediated finance ρ_i*

$$\rho \geq \rho_i \geq 0,$$

and the two premia are equal, $\rho = \rho_i$, iff the collateral constraint for intermediated finance does not bind for any state next period, that is, $E[\lambda_i'] = 0$. Moreover, the premium on internal finance is strictly positive, $\rho > 0$, iff the collateral constraint for direct finance binds for some state next period, that is, $E[\lambda'] > 0$.

When all collateral constraints are slack, there is no premium on either type of finance, but often the inequalities are strict and both premia are strictly positive, with the premium on internal finance strictly exceeding the premium on intermediated finance.

3 Intermediated versus direct finance

In this section we study how the choice between intermediated and direct finance varies with firm and intermediary net worth in a static version of our model with one period. We further simplify but considering the deterministic case, although the results in this section do not depend on this assumption.[9] Taking the spread on intermediated finance

[8] Alternatively, the user cost can be written in a weighted average cost of capital representation as $u \equiv R/(R+\rho)(r_w+\delta)$ where the weighted average cost of capital r_w is defined as $r_w \equiv (r+\rho)\wp_i(R_i') + rR^{-1}\theta(1-\delta) + (r+\rho_i)(R+\rho_i)^{-1}(\theta_i-\theta)(1-\delta)$. The cost of capital r_w is a weighted average of the fraction of investment financed with internal funds which cost $r+\rho$ (first term on the right hand side), the fraction financed with households funds at rate r (second term), and the fraction financed with intermediated funds at rate $r+\rho_i$ (third term).

[9] With one period only, the interest rate on intermediated finance is independent of the state, as the marginal value of net worth next period for financial intermediaries and firms equals 1 for all states, that is, $\mu' = \mu_i' = 1$, rendering the model effectively deterministic.

as given, we first show that our model has plausible implications for the choice between intermediated and direct finance in the cross section of firms; sufficiently constrained firms borrow as much as possible from intermediaries while less constrained firms borrow less from intermediaries and dividend paying firms do not borrow from intermediaries at all. These cross-sectional results are similar to the ones in Holmström and Tirole (1997). We then analyze the equilibrium spread on intermediated finance when there is a representative firm. The spread depends on both firm and intermediary net worth. Given firm net worth, spreads are higher when the intermediary is less well capitalized. Importantly, the spread on intermediated finance depends on the relative capitalization of firms and intermediaries. Spreads are particularly high when firms are poorly capitalized and intermediaries are relatively poorly capitalized at the same time. Poor capitalization of the corporate sector does not per se imply high spreads, as firms' limited ability to pledge may result in a reduction in firms' loan demand which intermediaries with given net worth can more easily accommodate.

3.1 Intermediated versus direct finance in the cross section

Consider the firm's problem taking the interest rate on intermediated finance R_i' as given. For simplicity, consider a static (one period) environment without uncertainty. The firm solves

$$\max_{\{d,k,b',b_i',w'\}\in\mathbb{R}_+^2\times\mathbb{R}\times\mathbb{R}_+^2} d+\beta w' \tag{21}$$

subject to (2) through (5). More constrained firms borrow from financial intermediaries whereas less constrained firms and dividend paying firms borrow from households only, consistent with the cross sectional stylized facts.

Proposition 2 (Intermediated vs. direct finance across firms) *Suppose $R_i' > \beta^{-1}$.*[10] ***(i)*** *Firms with net worth $w \leq \underline{w}_l$ borrow as much as possible from intermediaries, firms with net worth $\underline{w}_l < w < \underline{w}_u$ borrow a positive amount from intermediaries but less than the maximal amount, and firms with net worth exceeding $\underline{w}_u$ do not borrow from intermediaries, where $0 < \underline{w}_l < \underline{w}_u$.* ***(ii)*** *Only firms with net worth exceeding $\bar{w}$ pay dividends at time 0, where $\underline{w}_u < \bar{w} < \infty$.* ***(iii)*** *Investment is increasing in w and strictly increasing for $w \leq \underline{w}_l$ and $\underline{w}_u < w < \bar{w}$.*

[10]We consider the case in which $R_i' > \beta^{-1}$ since, proceeding analogously as in the first part of the proof, one can show that $R_i' < \beta^{-1}$ would imply that $\lambda_i' > 0$ and thus the cross sectional financing implications would be trivial as all firms would borrow the maximal amount from intermediaries. When $R_i' = \beta^{-1}$, this would also be true without loss of generality.

Intermediated finance is costlier than direct finance. Indeed, under the conditions of the proposition, intermediated finance is costlier than the shadow cost of internal finance of well capitalized firms. Thus, well capitalized firms, which pay dividends, do not borrow from financial intermediaries. In contrast, firms with net worth below some threshold ($\underline{w}_u$) have a shadow cost of internal finance which is sufficiently high that they choose to borrow a positive amount from intermediaries. For severely constrained firms, with net worth below $\underline{w}_l$, the shadow cost of internal funds is so high that they borrow as much as they can from intermediaries, that is, their collateral constraint for intermediated finance binds. Moreover, more constrained firms have lower investment and are hence smaller.

Thus, our model has plausible implications for the choice between intermediated and direct finance in the cross section of firms. Smaller (and more constrained) firms borrow more from financial intermediaries and have higher costs of financing, while larger (and less constrained firms) borrow from households, for example in bond markets, and have lower financing costs.

3.2 Effect of intermediary net worth on spreads

We now analyze the factors determining the equilibrium spreads. For simplicity, we consider an economy with a representative firm and a representative financial intermediary. The representative firm's problem is the firm's problem stated above, (21) subject to (2) through (5).

In a static (one period) environment without uncertainty, the (representative) intermediary solves

$$\max_{\{d_i, l', l'_i, w'_i\} \in \mathbb{R}^4_+} d_i + \beta_i w'_i \tag{22}$$

subject to (12) through (13). An equilibrium is defined in Definition 2. In addition to the equilibrium allocation, the spread on intermediated finance, $R'_i - R$, is determined in equilibrium.

The following proposition summarizes the characterization of the equilibrium spread. Figures 1, 2, and 3 illustrate the results. The key insight is that the spread on intermediated finance depends on both the firm and intermediary net worth. Importantly, low capitalization of the corporate sector does not necessarily result in a high spread on intermediated finance. Indeed, it may reduce spreads. Similarly, while low capitalization of the intermediation sector raises spreads, spreads are substantial only when the corporate sector is poorly capitalized and intermediaries are poorly capitalized relative to the corporate sector at the same time.

Proposition 3 (Firm and intermediary net worth) ***(i)*** *For $w_i \geq w_i^*$, intermediaries are well capitalized and there is a minimum spread on intermediated finance $\beta_i^{-1} - R > 0$ for all levels of firm net worth.* ***(ii)*** *Otherwise, there is a threshold of firm net worth $\underline{w}(w_i)$ (which depends on w_i) such that intermediaries are well capitalized and the spread on intermediated finance is $\beta_i^{-1} - R > 0$ as long as $w \leq \underline{w}(w_i)$. For $w > \underline{w}(w_i)$, intermediated finance is scarce and spreads are higher. For $w_i \in [\bar{w}_i, w_i^*)$, spreads are increasing in w until w reaches $\hat{w}(w_i)$, at which point spreads stay constant at $\hat{R}_i'(w_i) - R \in (\beta_i^{-1} - R, \beta^{-1} - R]$. For $w_i \in (0, \bar{w}_i)$, spreads are increasing in w until w reaches $\hat{w}(w_i)$, then decreasing in w until $\bar{w}(w_i)$ is reached, at which point spreads stay constant at $\beta^{-1} - R$. As $w_i \to 0$, $\hat{w}(w_i) \to 0$.*

Figure 1 displays the cost of intermediated finance as a function of firm net worth (w) and intermediary net worth (w_i). Figure 2 displays the contours of the various areas in Figure 1. Figure 3 displays the cost of intermediated finance as a function of firm net worth for different levels of intermediary net worth, and is essentially a projection of Figure 1. When financial intermediaries are well capitalized the spread on intermediated finance is at its minimum, $\beta_i^{-1} - R > 0$. This is the case when financial intermediary net worth is high enough ($w_i \geq w_i^*$) so that they can accommodate the loan demand of even a well capitalized corporate sector or when corporate net worth is relatively low so that the financial intermediary sector is able to accommodate demand despite its low net worth ($w \leq \underline{w}(w_i)$). When intermediary capital is below w_i^* and the corporate sector is not too poorly capitalized ($w > \underline{w}(w_i)$), spreads on intermediated finance are higher. Indeed, when intermediary capital is in this range, higher firm net worth initially raises spreads as loan demand increases (until firm net worth reaches $\hat{w}(w_i)$). This effect can be substantial when $w_i < \bar{w}_i$. Indeed, interest rates in our example increase to around 200% when financial intermediary net worth is very low, albeit our example is not calibrated. If firm net worth is still higher, spreads decline as the marginal product of capital and hence firms' willingness to borrow at high interest rates declines. When corporate net worth exceeds $\bar{w}(w_i)$, the cost on intermediated finance is constant at β^{-1}, which equals the shadow cost of internal funds of well capitalized firms.

To sum up, spreads are determined by firm and intermediary net worth jointly. Spreads are higher when intermediary net worth is lower. But firm net worth affects both the demand for intermediated loans and, via investment, the collateral available to back such loans. When collateral constraints bind, lower firm net worth reduces spreads.

4 Dynamics of intermediary capital

Our model allows the analysis of the dynamics of intermediary capital and indeed the joint dynamics of the capitalization of the corporate and intermediary sector. We first characterize a deterministic steady state and then analyze the deterministic dynamics of firm and intermediary capitalization. Both firms and intermediaries accumulate capital over time, but the corporate sector initially accumulates net worth faster than the intermediary sector, which has important implications for the dynamics of spreads on intermediated finance. We also study the dynamic effects of a credit crunch, and show that the economy may be slow to recover. Moreover, we show that incomplete risk management by both firms and financial intermediaries may be optimal. Finally, we characterize the dynamics of shadow interest rates on intermediated financing in a stochastic steady state in which intermediaries have no capital. Spreads are high when the corporate sector is poorly capitalized.

4.1 Intermediaries are essential in a deterministic economy

We first show that intermediaries always have positive net worth, that is, they never choose to pay out their entire net worth as dividends if the economy is deterministic or eventually deterministic, that is, deterministic from some time $T < +\infty$ onward.

Proposition 4 (Positive intermediary net worth) *Financial intermediaries always have positive net worth in an equilibrium in a deterministic or eventually deterministic economy.*

Since intermediaries always have positive net worth, the interest rate on intermediated finance R_i' must in equilibrium be such that the representative firm never would want to lend at that interest rate, as the following lemma shows:

Lemma 1 *In any equilibrium, **(i)** the cost of intermediated funds (weakly) exceeds the cost of direct finance, that is, $R_i' \geq R$; **(ii)** the multiplier on the collateral constraint for direct finance (weakly) exceeds the multiplier on the collateral constraint for intermediated finance, that is, $\lambda' \geq \lambda_i'$; and **(iii)** the constraint that the representative firm cannot lend at R_i' never binds, that is, $\nu_i' = 0$ w.l.o.g. Moreover, in a deterministic economy, **(iv)** the constraint that the representative intermediary cannot borrow at R_i' never binds, that is, $\eta_i' = 0$; and **(v)** the collateral constraint for direct financing always binds, that is, $\lambda' > 0$.*

These results together imply that financial intermediaries must always be essential. First note that firms are always borrowing the maximal amount from households. If firms

moreover always borrow a positive amount from intermediaries, then they must achieve an allocation that would not otherwise be feasible. If $R'_i = R$, then the firm must be collateral constrained in terms of intermediated finance, too, that is, borrow a positive amount. If $R'_i > R$, then intermediaries lend all their funds to the corporate sector and hence in equilibrium firms must be borrowing from intermediaries. We have hence proved the following:

Proposition 5 (Essentiality of intermediaries) *In an equilibrium in a deterministic economy, financial intermediaries are always essential.*

4.2 Intermediary capitalization and spreads in a steady state

We define a deterministic steady state in the economy with an infinite horizon as follows:

Definition 3 (Steady state) *A deterministic steady state equilibrium is an equilibrium with constant allocations, that is,* $x^* \equiv [d^*, k^*, b'^*, b_i'^*, w'^*]$ *and* $x_i^* \equiv [d_i^*, l'^*, l_i'^*, w_i'^*]$.

In the deterministic steady state, intermediaries are essential, have positive capital, and spreads are positive.

Proposition 6 (Steady state) *In a steady state, intermediaries are essential, have positive net worth, and pay positive dividends. The spread on intermediated finance is* $R_i'^* - R = \beta_i^{-1} - R$. *Firms borrow the maximal amount from intermediaries. The relative (ex dividend) intermediary capitalization is*

$$\frac{w_i^*}{w^*} = \frac{\beta_i(\theta_i - \theta)(1-\delta)}{\wp_i(\beta_i^{-1})}.$$

Note that the relative (ex dividend) intermediary capitalization, that is, the ratio of the representative intermediary's net worth (ex dividend) relative to the representative firm's net worth (ex dividend), is the ratio of the intermediary's financing (per unit of capital) to the firm's down payment requirement (per unit of capital). In a steady state, the shadow cost of internal funds of the firm is $\beta^{-1} - 1$ while the shadow cost of internal funds of the intermediary is $\beta_i^{-1} - 1$ and equals the interest rate on intermediated finance $R_i'^* - 1$. Since $\beta_i < \beta$, intermediated finance is cheaper than internal funds for firms in the steady state, and firms borrow as much as they can. In a steady state equilibrium, financial intermediaries have positive capital and pay out the steady state interest income as dividends $d_i^* = (R_i'^* - 1)l_i'^*$. Note that in a steady state both firms and intermediaries have positive net worth despite the fact that their rates of time preference differ and despite the fact that both are less patient than households.

4.3 Deterministic dynamics of intermediary capital and spreads

Consider now the dynamics of both firm and intermediary capitalization in an equilibrium converging to the steady state. We show that the equilibrium dynamics evolve in two main phases, an initial one in which the corporate sector pays no dividends and a second one in which the corporate sector pays dividends. Intermediaries do not pay dividends until the steady state is reached, except that they may pay an initial dividend (at time 0), if they are well capitalized relative to the corporate sector at time 0. We first state these results formally and then provide an intuitive discussion of the equilibrium dynamics.

Proposition 7 (Deterministic dynamics) *Given w and w_i, there exists a unique deterministic dynamic equilibrium which converges to the steady state characterized by a no dividend region (ND) and a dividend region (D) (which is absorbing) as follows:*

Region ND *$w_i \leq w_i^*$ (w.l.o.g.) and $w < \bar{w}(w_i)$, and (i) $d = 0$ ($\mu > 1$), (ii) the cost of intermediated finance is*

$$R_i' = \max\left\{R, \min\left\{\frac{(\theta_i - \theta)(1-\delta)\left(\frac{w}{w_i}+1\right)}{\wp}, \frac{A' f_k\left(\frac{w+w_i}{\wp}\right) + (1-\theta)(1-\delta)}{\wp}\right\}\right\},$$

(iii) investment $k = (w + w_i)/\wp$ if $R_i' > R$ and $k = w/\wp_i(R)$ if $R_i' = R$, and (iv) $w'/w_i' > w/w_i$, that is, firm net worth increases faster than intermediary net worth.

Region D *$w \geq \bar{w}(w_i)$ and (i) $d > 0$ ($\mu = 1$). For $w_i \in (0, \bar{w}_i)$, (ii) $R_i' = \beta^{-1}$, (iii) $k = \bar{k}$ which solves $1 = \beta[A' f_k(\bar{k}) + (1-\theta)(1-\delta)]/\wp$, (iv) $w_{ex}'/w_i' < w_{ex}/w_i$, that is, firm net worth (ex dividend) increases more slowly than intermediary net worth, and (v) $\bar{w}(w_i) = \wp\bar{k} - w_i$. For $w_i \in [\bar{w}_i, w_i^*)$, (ii) $R_i' = (\theta_i - \theta)(1-\delta)k/w_i$, (iii) k solves $1 = \beta[A' f_k(k) + (1-\theta)(1-\delta)]/(\wp - w_i/k)$, (iv) $w_{ex}'/w_i' < w_{ex}/w_i$, that is, firm net worth (ex dividend) increases more slowly than intermediary net worth, and (v) $\bar{w}(w_i) = \wp_i(R_i')k$. For $w_i \geq w_i^*$, $\bar{w}(w_i) = w^*$ and the steady state of Proposition 6 is reached with $d = w - w^*$ and $d_i = w_i - w_i^*$.*

Figure 4 displays the contours of the two regions in terms of firm net worth w and intermediary net worth w_i and Figure 5 illustrates the dynamics of firm and intermediary net worth, the interest rate on intermediated finance, and investment over time.

Lemma 2 (Initial intermediary dividend) *The representative intermediary pays at most an initial dividend and no further dividends until the steady state is reached. If $w_i > w_i^*$, the initial dividend is strictly positive.*

To understand the intuition, suppose both firms and financial intermediaries are initially poorly capitalized, and assume moreover that firms are poorly capitalized even relative to financial intermediaries. The dynamics of financial intermediary net worth are relatively simple, since as long as no dividends are paid (which is the case until the steady state is reached, except possibly at time 0), the intermediaries' net worth evolves according to the law of motion $w_i' = R_i' w_i$, that is, intermediary net worth next period is simply intermediary net worth this period plus interest income. When no dividends are paid, intermediaries lend out all their funds at the interest rate R_i'. Of course, the interest rate R_i' needs to be determined in equilibrium.

Given our assumptions, the corporate sector's net worth, investment and loan demand, evolve in several phases, which are reflected in the dynamics of the equilibrium interest rate. If firms are initially poorly capitalized even relative to financial intermediaries, as we assume, loan demand is low and intermediaries are relatively well capitalized. In this case, intermediaries conserve net worth to meet future loan demand and spreads are zero, that is, $R_i' = R$. Corporate investment is then $k = w/\wp_i(R)$. Note that intermediaries accumulate net worth at rate R in this phase while the corporate sector accumulates net worth at a faster rate, given the high marginal product; thus, the net worth of the corporate sector rises relative to the net worth of intermediaries.

Eventually, the increased net worth of the corporate sector raises loan demand so that intermediated finance becomes scarce. The corporate sector then borrows all the funds intermediaries are able to lend and invests $k = (w + w_i)/\wp$. The interest rate on intermediated finance is determined by the collateral constraint, which is binding, and equals $R_i' = (\theta_i - \theta)(1 - \delta)\left(w/w_i + 1\right)/\wp$. Note that since corporate net worth increases faster than intermediary net worth, the interest rate on intermediated finance rises in this phase. As the corporate sector accumulates net worth, it can pledge more and the equilibrium interest rate rises.

As the net worth and investment of the corporate sector continues to rise faster than intermediary net worth, the increase in firms' collateral means that firms' ability to pledge no longer constrains their ability to raise intermediated finance. Intermediated finance is scarce in this phase because of limited intermediary net worth, however, and so spreads are high but declining. The law of motion of investment is as in the previous phase $k = (w+w_i)/\wp$, while the equilibrium interest rate on intermediated finance is determined by $R_i' = [A' f_k(k) + (1 - \theta)(1 - \delta)]/\wp$. Note that both firm and intermediary net worth continue to increase, and hence investment increases and the equilibrium interest rate on intermediated finance decreases.

Eventually, the interest rate on intermediated finance reaches β^{-1}, the shadow cost of internal funds of the corporate sector. At that point, corporate investment stays constant and firms start to pay dividends. Note however that intermediaries continue to accumulate net worth and the economy is not yet in steady state. As intermediaries accumulate net worth, the corporate sector reduces its net worth by paying dividends. Essentially, the corporate sector relevers as the supply of intermediated finance increases when financial intermediary net worth increases.

Once intermediary capital is sufficiently high to accommodate the entire loan demand of the corporate sector at an interest rate β^{-1}, the cost of intermediated funds decreases further. As the interest rate on intermediated finance is now below the shadow cost of internal funds of the corporate sector, the collateral constraint binds again. Investment increases due to the reduced cost of intermediated financing. Eventually, intermediaries accumulate their steady state level of net worth and the cost of intermediated finance reaches β_i^{-1}, the intermediaries' shadow cost of internal funds.

We emphasize two key aspects of the dynamics of intermediary capital, beyond the fact that intermediary and firm net worth affect the dynamics jointly. First, intermediary capital accumulates more slowly than corporate net worth in our model. Second, the interest rate on intermediated finance is low when intermediaries conserve net worth to meet the higher loan demand later on when the corporate sector is relatively poorly capitalized early on. And vice versa, the corporate sector accumulates additional net worth and spreads remain higher (and investment lower than in the steady state) as the corporate sector "waits" for intermediary net worth to rise and eventually reduce spreads, at which point firms relever. The second two observations of course are a reflection of the relatively slow pace of intermediary capital accumulation.

4.4 Dynamics of a credit crunch

Suppose the economy experiences a *credit crunch*, which we model here as an unanticipated one-time drop in intermediary net worth w_i. We assume that the economy is otherwise deterministic and is in steady state when the credit crunch hits. The effect of a credit crunch depends on its size. Intermediaries can absorb a small enough credit crunch simply by cutting dividends. But a larger drop in intermediary net worth results in a reduction in lending and an increase in the spread on intermediated finance. Moreover, the higher cost of intermediated finance increases the user cost of capital (20) (as the premium on internal finance is either unchanged or increases) and so investment drops. Thus, a credit crunch has real effects in our model. Remarkably, investment drops even if the corporate sector is still well capitalized (that is, even if $w'^* > \bar{w}$). The reason is that the

cost of capital increases even if the corporate sector is well capitalized, as intermediaries' capacity to extend relatively cheap financing is reduced. In that case, the credit crunch results in a jump in the interest rate on intermediated finance to $R'_i = \beta^{-1} > R'^*_i = \beta_i^{-1}$ and an immediate drop in investment (and capital, which drops to $\bar{k} < k^*$). The real effects in our model are moreover persistent, even if the corporate sector remains well capitalized. Indeed, the recovery of the real economy can be delayed. After a sufficiently large credit crunch, investment and capital remain constant at the lower level, and spreads remain constant at the elevated level, until the intermediary sector accumulates sufficient capital to meet the loan demand. At that point, intermediary interest rates start to fall and investment begins to recover, until the economy eventually recovers fully.

If the corporate sector is no longer well capitalized after the credit crunch, the spread on intermediated finance rises further and investment drops even more. Moreover, after an initial partially recovery, the recovery stalls in the sense that the interest rate on intermediated finance remains at $R'_i = \beta^{-1}$ and investment remains constant below its steady state level (in fact, capital remains constant at $\bar{k}$), until the intermediaries accumulate sufficient capital. The recovery then resumes.

If net worth of both the intermediaries and the corporate sector drop at the same time, for example, because of a one-time depreciation shock to capital, then investment and output fall more substantially. The dynamics of the recovery from such a downturn are as described in Section 4.3. It is noteworthy, though, that the spreads on intermediated finance may or may not go up in such a general downturn, and in fact may well go down despite the scarcity of intermediary capital. The point is that the lower net worth of the corporate sector reduces loan demand, possibly by more than the drop in intermediary net worth reduces loanable funds. If corporate loan demand drops sufficiently, intermediaries may pay a one time dividend when the downturn hits, and then cut dividends to zero until the economy recovers.

4.5 Comovement of firm and intermediary capital

Do the marginal value of firm and intermediary net worth comove? We consider this question in a stochastic economy which is deterministic from time 1 onward. Importantly, this allows both firms and intermediaries to engage in risk management at time 0 and hedge the net worth available to them in different states $s' \in S$ at time 1. We first show that the representative firm optimally engages in incomplete risk management, that is, the collateral constraint for direct finance against at least one state $s' \in S$ must bind. We then provide sufficient conditions for the marginal value of net worth of the representative firm and the representative intermediary to comove.

Proposition 8 (Comovement of the value of firm and intermediary capital) *In an economy which is deterministic from time* 1 *onward and has constant expected productivity,* ***(i)*** *the representative firm must be collateral constrained for direct finance against at least one state at time* 1*;* ***(ii)*** *the marginal value of firm and intermediary net worth comove, in fact* $\mu(s')/\mu(s'_+) = \mu_i(s')/\mu_i(s'_+)$, $\forall s', s'_+ \in S$, *if* $\lambda_i(s') = 0, \forall s' \in S$. ***(iii)*** *Suppose moreover that there are just two states, that is,* $S = \{\hat{s}', \check{s}'\}$*. If only one of the collateral constraints for direct finance binds,* $\lambda(\check{s}') > 0 = \lambda(\hat{s}')$*, then the marginal values must comove,* $\mu(\hat{s}') > \mu(\check{s}')$ *and* $\mu_i(\hat{s}') \geq \mu_i(\check{s}')$.

Proposition 8 implies that the marginal values of firm and intermediary net worth comove, for example, when the intermediary has very limited net worth and hence the collateral constraints for intermediated finance are slack for all states. They also comove if the firm hedges one of two possible states, as then the intermediary effectively must be hedging that state, too. Thus, the marginal value of intermediary net worth may be high exactly when the marginal value of firm net worth is high, too. The marginal values may however move in opposite directions, for example, if a high realization of productivity raises firm net worth substantially, which lowers the marginal product of capital and hence the marginal value of firm net worth, while it may raise loan demand substantially and hence raise the marginal value of intermediary net worth.

4.6 Intermediary risk management

We now show that limited risk management can be optimal for both firms and intermediaries. Indeed, we characterize a quasi-deterministic steady state, in which neither the corporate sector nor the financial intermediary sector engage in risk management. This result obtains for a perturbation of the economy around the deterministic steady state with small enough shocks. Recall from Proposition 6 that in a deterministic steady state both firms and intermediaries pay strictly positive dividends, and hence the marginal value of net worth of firms and intermediaries equals 1.

Suppose firms' productivity A' is stochastic and depends on the state Z. Suppose moreover that productivity is independent and identically distributed over time and has the same mean as in the steady state. As long as the variation in productivity is small enough, then the following quasi-deterministic steady state is still an equilibrium:

Proposition 9 (Quasi-deterministic steady state) *Suppose productivity* A' *is stochastic with* $|\min A' - E[A']| < \varepsilon$ *for sufficiently small* $\varepsilon > 0$*. Then there is a quasi-deterministic equilibrium in which the interest rate on intermediated finance is constant*

at the steady state value $R_i'^* = \beta_i^{-1}$*; firms invest a constant amount* k^* *and borrow a constant amount from households and financial intermediaries; intermediaries pay constant dividends* d_i^* *and intermediary (cum and ex dividend) net worth is constant; firms pay stochastic dividends* $d' = d^* + (A' - E[A'])f(k^*)$*, have stochastic cum dividend net worth* $w' = w'^* + (A' - E[A'])f(k^*)$*, and constant ex dividend net worth* w^**.*

To see this note that the allocation described in the proposition together with the values of all multipliers as in a deterministic steady state solve the first order conditions and constitute an equilibrium.

A similar result can be obtained for an economy with stochastic depreciation of capital, as long as the depreciation shocks are sufficiently small. In this case, the dividends and cum dividend net worth of financial intermediaries is stochastic as well. Optimally, neither firms nor intermediaries engage in risk management.

4.7 Dynamics of shadow spreads on intermediated finance

We can characterize the dynamics of the shadow interest rates on intermediated financing in a stochastic steady state in which intermediaries have no capital. When intermediaries have no capital, the analysis is simplified as the only state variable is the capital of the corporate sector, which is a problem studied in Rampini and Viswanathan (2011) and we build on the example provided there. The shadow interest rates are the interest rate that firms would be willing to pay if the financial intermediary would have an infinitesimal amount of capital. If the capital of the intermediary is infinitesimal, the firms' collateral constraint for intermediated finance cannot bind and we can read the shadow interest rate on intermediated finance off the firm's first order condition (9), that is, $R_i' = (\beta\mu'/\mu)^{-1}$. Figure 6 illustrates the results for a stochastic steady state. Panel A shows the shadow interest rates for the two states next period when investment opportunities are constant. Shadow interest rates and hence spreads are higher when the corporate sector is more poorly capitalized. Panel B shows the shadow interest rates for the two states next period when investment opportunities are stochastic because productivity is persistent. Shadow interest rates and hence spreads are higher when the corporate sector is more poorly capitalized and depend on the state of the economy. Since firms are more poorly capitalized when productivity is low, the spreads on intermediated finance are higher than.

5 Conclusion

We develop a dynamic theory of financial intermediation and show that the capital of both the financial intermediary and corporate sector affect real economic activity, such as firm investment, financing, and the spread between intermediated and direct finance. Financial intermediaries are modeled as collateralization specialists that are better able to collateralize claims than households themselves. Financial intermediaries require capital as their ability to borrow against their collateralized loans is limited by households' ability to collateralize the assets backing the loans themselves.

The spread on intermediated finance is high when both firms and intermediaries are poorly capitalized, and in particular when intermediaries are moreover poorly capitalized relative to firms. Intermediary capital in our model accumulates more slowly than the capital of firms, and thus spreads on intermediated finance may initially rise as loan demand increases more than loanable funds as the net worth of the corporate sector increases relative to the net worth of financial intermediaries. A credit crunch, that is, a drop in intermediary net worth results in a drop in intermediated finance, a rise in spreads on intermediated loans, and a drop in real activity. The recovery can be delayed, or stall, with real activity constant at a reduced level and persistently high spreads on intermediated finance, because it takes time for intermediaries to reaccumulate sufficient net worth. In the cross section, the model predicts that more constrained firms borrow from financial intermediaries, consistent with stylized facts. In addition, the model shows that the marginal value of intermediary and firm net worth may comove and that limited risk management by both firms and financial intermediaries may be optimal. Our model may provide a useful framework for the analysis of the dynamic interaction between financial structure and economic activity.

Appendix

Proof of Proposition 1. Using (9) and the fact that $\nu_i' = 0$ (proved below in Lemma 1, part (iii)), we have $(R_i')^{-1} = \beta\mu'/\mu + \beta\lambda_i'/\mu$ and, taking expectations,

$$\frac{1}{R+\rho_i} \equiv E\left[(R_i')^{-1}\right] = \frac{1}{R+\rho} + E\left[\beta\frac{\lambda_i'}{\mu}\right]$$

and hence $\rho \geq \rho_i$ with equality iff $E[\lambda_i'] = 0$. Moreover, since $R_i' \geq R$ (proved below in Lemma 1, part (i)), $\rho_i \geq 0$. Finally, using (8), we have $1/(R+\rho) \equiv E[\beta\mu'/\mu] = 1/R - E[\beta\lambda'/\mu]$, implying that $\rho > 0$ iff $E[\lambda'] > 0$. $\square$

Proof of Proposition 2. The first order conditions are (6)-(9) and $\mu' = 1 + \nu_d'$ where $\beta\nu_d'$ is the multiplier on the constraint $w' \geq 0$. By the Inada condition, (7) implies that $k > 0$ and using (3) at equality and (4) and (5) we have $d' \geq A'f(k) + k(1-\theta_i)(1-\delta) > 0$ and $\mu' = 1$. But (6) and (8) imply $1 \leq \mu = R\beta + R\beta\lambda'$ and thus $\lambda' > 0$ since $R\beta < 1$ by assumption; that is, all firms raise as much financing as possible from households.

Suppose the firm pays dividends at time 0. Then $\mu = \mu' = 1$ and (9) implies $0 > 1 - R_i'\beta = R_i'\beta\lambda_i' - R_i'\beta\nu_i'$ and thus $\nu_i' = 1 - (R_i'\beta)^{-1} > 0$, $b_i' = 0$, and $\lambda_i' = 0$; thus, the firm does not use intermediated finance. Note that the problem of maximizing (21) subject to (2) through (5) has a (weakly) concave objective and a convex constraint set and hence induces a (weakly) concave value function. Thus, μ is (weakly) decreasing in w and let $\bar{w}$ be the lowest value of net worth for which $\mu = 1$; by the Inada condition, such a $\bar{w} < +\infty$ exists. At $\bar{w}$, $d = 0$, $\bar{w} = \bar{k}\wp$ (using (2)), and $\bar{k}$ solves $1 = \beta[A'f_k(\bar{k}) + (1-\theta)(1-\delta)]/\wp$ (using (7)). For $w \geq \bar{w}$, $d = w - \bar{w}$ while the rest of the optimal policy is unchanged.

Suppose $\lambda_i' = 0$ and $\nu_i' = 0$. Then $\mu = R_i'\beta > 1$. Moreover, rearranging (7) we have $1 = \beta/(R_i'\beta)[A'f_k(\underline{k}) + (1-\theta)(1-\delta)]/\wp$ which defines $\underline{k} < \bar{k}$. Define $\underline{w}_u$ such that investment is $\underline{k}$ and $b_i' = 0$; then $\underline{w}_u = \underline{k}\wp$. Similarly, define $\underline{w}_l$ such that investment is $\underline{k}$ and $b_i' = (R_i')^{-1}(\theta_i - \theta)\underline{k}(1-\delta)$; then $\underline{w}_l = \underline{k}[\wp - (R_i')^{-1}(\theta_i - \theta)(1-\delta)]$. Note that $\underline{w}_l < \underline{w}_u < \bar{w}$. So firms below $\underline{w}_l$ raise as much financing as possible from intermediaries (since $\mu > R_i'\beta$ by concavity and hence $\lambda_i' > 0$). Firms with net worth between $\underline{w}_l$ and $\underline{w}_u$ pay down intermediary financing linearly. Firms with net worth above $\underline{w}_u$ do not borrow from intermediaries and scale up until $\bar{k}$ is reached at $\bar{w}$, at which point firms initiate dividends. $\square$

Proof of Proposition 3. First, consider the intermediary's problem. The first order conditions are (14)-(16) and $\mu_i' = 1 + \eta_d'$, where $\beta_i\eta_d'$ is the multiplier on the constraint $w_i' \geq 0$. Since (13) holds with equality, the non-negativity constraints on l' and l_i' render

the non-negativity constraint on w_i' redundant and hence $\mu_i' = 1$. Using (15) we have $\eta' = (R\beta_i)^{-1}\mu_i - 1 \geq (R\beta_i)^{-1} - 1 > 0$ (and $l' = 0$) and similarly using (16) $\eta_i' > 0$ as long as $R_i' < \beta_i^{-1}$. Therefore, for $l_i' > 0$ it is necessary that $R_i' \geq \beta_i^{-1}$. If $R_i' > \beta_i^{-1}$, then $\mu_i' > 1$ (and $l_i' = w_i$) while if $R_i' = \beta_i^{-1}$, $0 \leq l_i' \leq w_i$.

Now consider the representative firm's problem. The first order conditions are (14)-(16) and $\mu' = 1 + \nu_d'$, where $\beta\nu_d'$ is the multiplier on the constraint $w' \geq 0$. Proceeding as in the proof of Proposition 2 one can show that $\mu' = 1$. Suppose $\nu_i' > 0$ (and hence $b_i' = 0$). Since $k > 0$, (5) is slack and $\lambda_i' = 0$. Using (6) and (9) we have $1 \leq \mu < R_i'\beta$ which implies that $R_i' > \beta^{-1}$. But at such an interest rate on intermediated finance $l_i' = w_i > 0$, which is not an equilibrium as $b_i' = 0$. Therefore, $\nu_i' = 0$ and $R_i' \leq \beta^{-1}$. Moreover, if $R_i' < \beta^{-1}$, then $\lambda_i' = (R_i'\beta)^{-1}\mu - 1 > 0$ and hence $b_i' = (R_i')^{-1}(\theta_i - \theta)k(1-\delta) > 0$. Since $l_i' = 0$ if $R_i' < \beta_i^{-1}$, we have $R_i' \in [\beta_i^{-1}, \beta^{-1}]$ in equilibrium. The firm's investment Euler equation (19) simplifies to $1 = \beta(1/\mu)[A'f_k(k) + (1-\theta_i)(1-\delta)]/\wp_i(R_i')$. Given the interest rate on intermediated finance, the firm's problem induces a concave value function and thus μ (weakly) decreases in w, implying that k (weakly) increases.

We first show that intermediaries are well capitalized and there is a minimum spread on intermediated finance $\beta_i^{-1} - R > 0$ for all levels of firm net worth when $w_i \geq w_i^*$ and for levels of firm net worth $w \leq \underline{w}(w_i)$ when $w_i < w_i^*$. If $R_i' = \beta_i^{-1}$, a well capitalized firm invests k^* which solves (19) specialized to $1 = \beta[A'f_k(k^*) + (1-\theta_i)(1-\delta)]/\wp_i(\beta_i^{-1})$, while less well capitalized firms invests $k \leq k^*$. The intermediary can meet the required demand for intermediated finance for any level of firm net worth w if $w_i \geq w_i^* \equiv \beta_i(\theta_i - \theta)k^*(1-\delta)$. Suppose instead that $w_i < w_i^*$. In this case the intermediary is able to meet the firm's loan demand at $R_i' = \beta_i^{-1}$ only if the firm is sufficiently constrained; the constrained firm invests $k = w/\wp_i(\beta_i^{-1})$ using (2), (4), and (5) at equality, and thus $b_i' = \beta_i(\theta_i - \theta)k(1-\delta)$; the intermediary can meet this demand as long as $w \leq \underline{w}(w_i) \equiv \wp_i(\beta_i^{-1})/[\beta_i(\theta_i - \theta)(1-\delta)]w_i$.

Suppose now that $w_i < w_i^*$ and $w > \underline{w}(w_i)$ as defined above. First, consider $w_i \in [\bar{w}_i, w_i^*)$ where $\bar{w}_i \equiv \beta(\theta_i - \theta)\bar{k}(1-\delta)$ and $1 = \beta[A'f_k(\bar{k}) + (1-\theta)(1-\delta)]/\wp$, that is, $\bar{w}_i$ is the loan demand of the well capitalized firm when the cost of intermediated finance is $R_i' = \beta^{-1}$. Note that $R_i' < \beta^{-1}$ on $(\bar{w}_i, w_i^*)$ since the intermediary has more than enough net worth to accommodate the loan demand of the well capitalized firm (and thus any constrained firm) at $R_i' = \beta^{-1}$. Thus, the firm's collateral constraint binds, that is, $w_i = (R_i')^{-1}(\theta_i - \theta)k(1-\delta)$. If the firm is poorly capitalized, $d = 0$ and (2) implies $w + w_i = \wp k$, and $R_i' = (\theta_i - \theta)(1-\delta)(w/w_i + 1)$. If the firm is well capitalized, $\mu = 1$ and $\bar{k}(w_i)$ solves $1 = \beta[A'f_k(\bar{k}(w_i)) + (1-\theta_i)(1-\delta)]/[\wp - w_i/\bar{k}(w_i)]$. Moreover, $\bar{w}(w_i) \equiv \wp\bar{k}(w_i) - w_i$ and for $w \geq \bar{w}(w_i)$ the cost of intermediated finance is constant at $\bar{R}_i'(w_i) = (\theta_i - \theta)\bar{k}(w_i)(1-\delta)/w_i$. Note that $\bar{R}_i'(w_i^*) = \beta_i^{-1}$ and $\bar{w}(w_i^*) = \wp k^* - w_i^* =$

$\wp_i(\beta_i^{-1})k^* = \underline{w}(w_i^*)$, that is, the two boundaries coincide at w_i^*. In contrast, at $\bar{w}_i$ we have $\underline{w}(\bar{w}_i) = \wp_i(\beta_i^{-1})/[\beta_i(\theta_i - \theta)(1-\delta)]\bar{w}_i = \wp_i(\beta_i^{-1})\beta/\beta_i\bar{k} = \wp\bar{k}\beta/\beta_i - \bar{w}_i < \bar{w}(\bar{w}_i)$ and $\bar{R}_i'(\bar{w}_i) = \beta^{-1}$.

Finally, consider $w_i \in (0, \bar{w}_i)$ and $w > \underline{w}(w_i)$ as defined above. If the firm is well capitalized (9) implies $\lambda_i' = (R_i'\beta)^{-1} - 1 \geq 0$. Moreover, since $w_i < \bar{w}_i$ the intermediary cannot meet the well capitalized firm's loan demand at $R_i' = \beta^{-1}$ and thus the cost of intermediated finance is in fact β^{-1} and $\lambda_i' = 0$, that is, the collateral constraint for intermediated finance does not bind. Thus, the firm's investment Euler equation (19) simplifies to $1 = \beta[A'f_k(\bar{k}) + (1-\theta_i)(1-\delta)]/\wp_i(\beta^{-1})$ which is solved by $\bar{k}$ as defined earlier in the proof. Define $\bar{w}(w_i) \equiv \wp\bar{k} - w_i$; the firm is well capitalized for $w \geq \bar{w}(w_i)$. Suppose $w < \bar{w}(w_i)$ and hence $\mu > 1$. If the collateral constraint for intermediated finance does not bind, then (9) implies $R_i' = \beta^{-1}\mu > \beta^{-1}$ and (19) implies $R_i' = [A'f_k(k) + (1-\theta)(1-\delta)]/\wp$, while (2) yields $w + w_i = \wp k$. Observe that $k < \bar{k}$ and R_i' decreases in w. If instead the collateral constraint binds, then $R_i' = (\theta_i - \theta)k(1-\delta)/w_i$ and $w + w_i = \wp k$ (so long as $w > \underline{w}(w_i)$). Note that k and R_i' increase in w in this range. The collateral constrain is just binding at $\hat{w}(w_i) \equiv \wp\hat{k}(w_i) - w_i$ where $[A'f_k(\hat{k}(w_i)) + (1-\theta)(1-\delta)]/\wp = (\theta_i - \theta)\hat{k}(w_i)(1-\delta)/w_i$.

We now show that if the collateral constraint for intermediated finance binds at some $w < \bar{w}(w_i)$ then it binds for all $w^- < w$. Note that $d = 0$ in this range and $w + w_i = \wp k$. At w^-, either $b_i'^- < w_i$ and $R_i' = \beta_i^{-1}$ and hence $\lambda_i'^- = (\beta_i^{-1}\beta)^{-1}\mu^- - 1 > 0$ or $b_i'^- = w_i$ and $w^- + w_i = \wp k^-$, implying $k^- < k$. Suppose the collateral constraint for intermediated finance is slack at w^-. Then $R_i'^- b_i'^- < (\theta_i - \theta)k^-(1-\delta) < (\theta_i - \theta)k(1-\delta) = R_i'b_i'$ and since $b_i'^- = w_i$ and $b_i' \leq w_i$ by above $R_i'^- w_i < R_i'b_i' \leq R_i'w_i$ which implies $R_i'^- < R_i'$. But

$$R_i'^-\beta = \mu^- = \beta\frac{A'f_k(k^-) + (1-\theta_i)(1-\delta)}{\wp - (R_i'^-)^{-1}(\theta_i - \theta)(1-\delta)} > \beta\frac{A'f_k(k) + (1-\theta_i)(1-\delta)}{\wp - (R_i')^{-1}(\theta_i - \theta)(1-\delta)} = \mu > R_i'\beta$$

or $R_i'^- > R_i'$, a contradiction.

Moreover, $\underline{w}(w_i) < \hat{w}(w_i) < \bar{w}(w_i)$ on $w_i \in (0, \bar{w}_i)$. Suppose, by contradiction, that $\hat{w}(w_i) \leq \underline{w}(w_i)$ and recall that $\underline{w}(w_i) + w_i = \wp k$ and $\hat{w}(w_i) + w_i = \wp\hat{k}(w_i)$, so $\hat{k}(w_i) \leq k$. But $\hat{R}_i'(w_i) = (\theta_i - \theta)\hat{k}(w_i)(1-\delta)/w_i \leq (\theta_i - \theta)k(1-\delta)/w_i = \beta_i^{-1}$. But if $\hat{R}_i'(w_i) \leq \beta_i^{-1}$, then at $\hat{w}(w_i)$ we have $\mu = \hat{R}_i'(w_i)\beta < 1$ (since the collateral constraint is slack), a contradiction. Thus, $\underline{w}(w_i) < \hat{w}(w_i)$. Suppose, again by contradiction, that $\bar{w}(w_i) \leq \hat{w}(w_i)$ and hence $\bar{k} \leq \hat{k}(w_i)$. Recall that $\hat{k}(w_i)$ solves $[A'f_k(\hat{k}(w_i)) + (1-\theta)(1-\delta)]/\wp = (\theta_i - \theta)\hat{k}(w_i)(1-\delta)/w_i$. At $\bar{w}_i$ this equation is solved by $\bar{k}$ (and $\hat{R}_i'(\bar{w}_i) = \beta^{-1}$), but since $w_i < \bar{w}_i$, $\hat{k}(w_i) < \bar{k}$, a contradiction. Moreover, as $w_i \to 0$, $\hat{k}(w_i) \to 0$ and $\hat{w}(w_i) = \wp\hat{k}(w_i) - w_i \to 0$. $\square$

Proof of Proposition 4. Consider a deterministic economy. Suppose intermediaries pay out their entire net worth at some point. From that point on, the firm's problem is as if there is no intermediary. We first characterize the solution to this problem and then show that the solution implies shadow interest rates on intermediated finance at which it would not be optimal for intermediaries to exit.

To characterize the solution in the absence of intermediaries, consider a steady state at which $\mu = \mu' \equiv \bar{\mu}$ and note that (8) implies $\bar{\lambda}' = ((R\beta)^{-1} - 1)\bar{\mu} > 0$. The investment Euler equation (19) simplifies to $1 = \beta[A' f_k(k) + (1-\theta)(1-\delta)/\wp$ which defines $\bar{k}$. The firm's steady state net worth is $\bar{w}' = A' f(\bar{k}) + (1-\theta)\bar{k}(1-\delta)$ and the firm pays out

$$
\begin{aligned}
\bar{d} &= \bar{w}' - \wp\bar{k} = A' f(\bar{k}) - \bar{k}[1 - (R^{-1}\theta + (1-\theta))(1-\delta)] \\
&> A' f(\bar{k}) - \beta^{-1}\bar{k}[1 - (R^{-1}\theta + \beta(1-\theta))(1-\delta)] \\
&= \int_0^{\bar{k}} [A' f_k(k) - \beta^{-1}(1 - (R^{-1}\theta + \beta(1-\theta))(1-\delta))]dk > 0.
\end{aligned}
$$

Therefore, $\bar{\mu} = 1$. Investment $\bar{k}$ is feasible as long as $w \geq \bar{w} = \bar{w}' - \bar{d}$. Whenever $w < \bar{w}$, $k < \bar{k}$ and hence using (19) we have $\mu/\mu' = \beta[A' f_k(k) + (1-\theta)(1-\delta)]/\wp > 1$. The shadow interest rate on intermediated finance is $R_i' = \beta^{-1}\mu/\mu' \geq \beta^{-1}$ for all values of w. But then it cannot be optimal for intermediaries to pay out all their net worth in a deterministic economy as keeping $\varepsilon > 0$ net worth for one more period improves the objective by $(\beta_i R_i' - 1)\varepsilon > 0$.

Consider now an eventually deterministic economy. From time T onward, the economy is deterministic and the conclusion obtains by above as long as the intermediary has positive net worth in all states at time T. Suppose not, that is, suppose intermediary net worth is zero for some state. As before the discounted marginal value on an infinitesimal amount of intermediary net worth at time T lent out for one period is at least $\beta_i R_i' \geq \beta_i \beta^{-1} > 1$ since $R_i' \geq \beta^{-1}$. Lending for τ periods thus guarantees a discounted marginal value of $(\beta_i \beta)^\tau$. As $\tau \to \infty$, the marginal value grows without bound. (Note that since we consider an infinitesimal amount, the collateral constraint cannot be biding for any finite τ.) The expected marginal value of this lending policy at time 0 is at least $(\beta_i R)^T$ times the marginal value at time T and hence grows without bound as $\tau \to \infty$.

But the marginal value of intermediary net worth at time 0 is finite as either the intermediary pays dividends and the marginal value is one, or the intermediary saves into at least one state at R_i' and thus $\mu_i = R_i' \beta \mu_i'$ and R_i' is bounded above by (5) and otherwise $R_i' = R$. Furthermore, μ_i' is bounded by a similar argument going forward until dividends are paid at which point the marginal value is one. But then it cannot be an equilibrium for intermediaries to pay out all their net worth. □

Proof of Lemma 1. Part (i): If $R_i' < R$, then using (8) and (9) we have $0 < (R - R_i')\beta\mu' \leq R_i'\beta\lambda_i'$ and thus $b_i' > 0$. But (15) and (16) imply that $0 < (R - R_i')\beta\mu_i' \leq R_i'\beta\eta_i'$ and thus $l_i' = 0$, which is not an equilibrium.

Part (ii): Given $\nu_i' = 0$ (see part (iii)), (8) and (9) imply that $\lambda' = (R_i'/R - 1)\mu' + R_i'/R\lambda_i' \geq \lambda_i'$.

Part (iii): First, suppose to the contrary that $\nu_i' > 0$. Then $\lambda_i' = 0$ as $b_i' = 0 < (R_i')^{-1}(\theta_i - \theta)k(1-\delta)$ implies that (5) is slack. Using (9) and (8) we have $\beta\mu' R_i' > \mu \geq \beta\mu' R$ and thus $R_i' > R$. Equations (15) and (16) imply that $R\eta' - R_i'\eta_i' = (R_i' - R)\mu_i' > 0$ and thus $\eta' > 0$ and $l' = 0$. But if $w_i' > 0$, which is always true under the conditions of Proposition 4, we have $l_i' = (R_i')^{-1}w_i' > 0 = b_i'$, which is not an equilibrium. If instead $w_i' = 0$, then $l_i' = 0$ and we can set $R_i' = (\beta\mu'/\mu)^{-1}$ and $\eta_i' = 0$ w.l.o.g.

Part (iv): Suppose to the contrary that $\eta_i' > 0$ (and hence $l_i' = 0$). Since intermediaries never pay out all their net worth in a deterministic economy, equation (13) implies $0 < w_i' \leq Rl'$ and hence $\eta' = 0$. But then (15) and (16) imply $\beta_i\mu_i'/\mu_i R = 1 > \beta_i\mu_i'/\mu_i R_i'$ or $R > R_i'$ contradicting the result of part (i). Thus, $\eta_i' = 0$ and $\mu_i' = (\beta_i R_i')^{-1}\mu_i$.

Part (v): Suppose $\lambda' = 0$. Then (8) reduces to $1 = \beta\mu'/\mu R$ and thus $1 \leq \mu = \beta R\mu' < \mu'$ and $d' = 0$. By part (ii), $\lambda_i' = 0$ and using (9) we have $R_i' = R$, $\mu_i' = (\beta R)^{-1}\mu_i > 1$, and $d_i' = 0$. The investment k^{**} solves $R = [A'f_k(k^{**}) + (1 - \theta_i)(1 - \delta)]/\wp_i(R)$ or $R - 1 + \delta = A'f_k(k^{**})$; this is the first best investment when dividends are discounted at R and it can never be optimal to invest more than that. To see this use (19) and note $[A'f_k(k) + (1 - \theta_i)(1 - \delta)]/\wp_i(R_i') = \mu/(\beta\mu') \geq R = [A'f_k(k^{**}) + (1 - \theta_i)(1 - \delta)]/\wp_i(R)$, that is, $f_k(k) \geq f_k(k^{**})$. Note that the firm's net worth next period, using (3) and (19), is

$$
\begin{aligned}
w' &= A'f(k^{**}) + (1 - \theta_i)(1 - \delta)k^{**} - [Rb' - \theta(1 - \delta)k^{**}] - [Rb_i' - (\theta_i - \theta)(1 - \delta)k^{**}] \\
&> R\wp_i(R)k^{**} - [Rb' - \theta(1 - \delta)k^{**}] - [Rb_i' - (\theta_i - \theta)(1 - \delta)k^{**}] = R[k^{**} - b' - b_i'] \\
&= Rw_{ex}.
\end{aligned}
$$

Note that $d' = 0$, $d_i' = 0$, $k' \leq k^{**}$, and $w' > w_{ex}$, and from (2) next period, $k' = w' + b'' + b_i''$. If $R_i'' > R$, then $b_i'' = w_i'$ and $b'' = R^{-1}\theta(1 - \delta)k'$. Therefore, $\wp k' = w' + w_i'$, but using (2) we have $\wp k^{**} \leq k^{**} - b' = w_{ex} + b_i' < w' + w_i' = \wp k'$, a contradiction. If $R_i'' = R$, then $b'' + b_i'' = k' - w' < k^{**} - w_{ex} = b' + b_i'$, that is, the firm is paying down debt, and $w'' > w'$ and $w_i'' > w_i'$. But then w and w_i grow without bound unless the firm or the intermediary eventually pay a dividend. But since μ and μ_i are strictly increasing as long as $R_i' = R$, if either pays a dividend at some future date, then $\mu < 1$ or $\mu_i < 1$ currently, a contradiction. □

Proof of Proposition 6. First, note that $k^* > 0$ due to the Inada condition and hence $w'^* \geq A'f(k^*) + k^*(1-\theta_i)(1-\delta) > 0$. Moreover, $d^* > 0$ since otherwise the value would be zero which would be dominated by paying out all net worth. Hence, $\mu^* = \mu'^* = 1$. By Proposition 4 intermediary net worth is positive and hence $d_i^* > 0$ (arguing as above), which implies $\mu_i^* = \mu_i'^* = 1$. But then $\eta'^* = (R\beta_i)^{-1} - 1 > 0$ and $l_i'^* > 0$ (and $\eta_i'^* = 0$), since otherwise intermediary net worth would be 0 next period. Therefore, $R_i'^* = \beta_i^{-1}$, and thus $\lambda_i'^* = (\beta_i^{-1}\beta)^{-1} - 1 > 0$, that is, the firm's collateral constraint for intermediated finance binds. Moreover, k^* solves $1 = \beta[A'f_k(k^*) + (1-\theta_i)(1-\delta)]/\wp_i(\beta_i^{-1})$ and d'^*, b'^*, $b_i'^*$, and w'^* are determined by (2)-(5) at equality. Specifically, $d^* = A'f(k^*) + k^*(1-\theta_i)(1-\delta) - \wp_i(\beta_i^{-1})k^* > 0$ and $b_i'^* = \beta_i(\theta_i - \theta)k^*(1-\delta)$. The net worth of the firm after dividends is $w^* = \wp_i(\beta_i^{-1})k^*$. Finally, $l_i'^* = b_i'^* = w_i^*$ and $d_i^* = (\beta_i^{-1} - 1)w_i^*$. □

Proof of Proposition 7. Consider first region D and take $w \geq \bar{w}(w_i)$ (to be defined below) and $d > 0$ forever ($\mu = \mu' = 1$). The investment Euler equation then implies $1 = \beta[A'f_k(k) + (1-\theta_i)(1-\delta)]/\wp_i(R_i)$. If the collateral constraint for intermediated finance (5) does not bind, then $\mu = R_i'\beta\mu'$, that is, $R_i' = \beta^{-1}$, and investment is constant at $\bar{k}$ which solves $1 = \beta[A'f_k(\bar{k}) + (1-\theta_i)(1-\delta)]/\wp_i(\beta^{-1})$ or, equivalently, $1 = \beta[A'f_k(\bar{k}) + (1-\theta)(1-\delta)]/\wp$. Define $\bar{w}(w_i) \equiv \wp\bar{k} - w_i$ and $\bar{w}_i = \beta(\theta_i - \theta)\bar{k}(1-\delta)$. At $\bar{w}_i$, (5) is just binding. For $w_i \in (0, \bar{w}_i)$, (5) is slack. Moreover, $w_i' = \beta^{-1}w_i$ and, if $w_i' \in (0, \bar{w}_i)$, the ex dividend net worth is $w_{ex} = \bar{w}(w_i)$ both in the current and next period, and we have immediately $w_{ex}'/w_i' < w_{ex}/w_i$. Further, using (3) and (19) we have

$$w' = A'f(\bar{k}) + (1-\theta)\bar{k}(1-\delta) - R_i'w_i > [A'f_k(\bar{k}) + (1-\theta)(1-\delta)]\bar{k} - R_i'w_i = R_i'\bar{w}(w_i).$$

But $w_{ex}' = \bar{w}(w_i') < \bar{w}(w_i)w_i'/w_i = R_i'w_{ex}$, so $d' = w' - w_{ex}' > 0$. For $w_i \in [\bar{w}_i, w_i^*)$, (5) binds and $k(w_i)$ solves $1 = \beta[A'f_k(k(w_i)) + (1-\theta_i)(1-\delta)]/[\wp - w_i/k(w_i)]$ and $R_i' = (\theta_i - \theta)k(w_i)/w_i(1-\delta)$. Note that the last two equations imply that $k(w_i) \geq \bar{k}$, $w_i/k(w_i) \geq \bar{w}_i/\bar{k}$, and $R_i' \leq \beta^{-1}$ in this region. As before, define $\bar{w}(w_i) = \wp k(w_i) - w_i$ and note that the ex dividend net worth is $w_{ex} = \bar{w}(w_i)$. Suppose $w_i^+ > w_i$ then $k(w_i^+) > k(w_i)$, $k(w_i^+)/w_i^+ < k(w_i)/w_i$, and $w_{ex}^+/w_i^+ = \wp k(w_i^+)/w_i^+ - 1 < w_{ex}/w_i$. Moreover, $w_i' = R_i'w_i > w_i$ and hence k (strictly) increases and R_i' (strictly) decreases in this region. Proceeding as before,

$$\begin{aligned} w' &= A'f(k(w_i)) + (1-\theta_i)k(w_i)(1-\delta) > [A'f_k(k(w_i)) + (1-\theta_i)(1-\delta)]k(w_i) \\ &\geq R_i'\beta[A'f_k(k(w_i)) + (1-\theta_i)(1-\delta)]k(w_i) = R_i'\bar{w}(w_i). \end{aligned}$$

But $w_{ex}' = \bar{w}(w_i') < \bar{w}(w_i)w_i'/w_i = R_i'w_{ex}$, so $d' = w' - w_{ex}' > 0$. Finally, if $w_i \geq w_i^*$ and $w \geq \bar{w}(w_i) = w^*$, the steady state of Proposition 6 is reached.

We now show that the above policies are optimal for both the firm and the intermediary given the interest rate process in region D and hence constitute an equilibrium. Since $R_i' > \beta_i^{-1}$ before the steady state is reached, the intermediary lends its entire net worth to the firm, $l_i' = w_i$, and does not pay dividends until the steady state is reached. Hence, the intermediary's policy is optimal. To see that the firm's policy is optimal in region D, suppose that the firm follows the optimal policy from the next period onward but sets $\tilde{d} = 0$ in the current period. If the firm invests the additional amount, then $\tilde{k} = (w_i + w)/\wp > k$ and $\tilde{w}' > w'$ (and therefore $\tilde{\mu}' = 1$). The investment Euler equation requires $1 = \beta/\tilde{\mu}[A' f_k(\tilde{k}) + (1-\theta_i)(1-\delta)]/\wp_i(R_i')$, but since $f_k(\tilde{k}) < f_k(k)$ and k satisfies the investment Euler equation at $\mu = \mu' = 1$, this implies $\tilde{\mu} < 1$, a contradiction. Suppose the firm instead invests the same amount $\tilde{k} = k$ but borrows less $\tilde{b}_i' < b_i'$. Then $\tilde{w}' > w'$, $\tilde{\mu}' = 1$, and from (19) $\tilde{\mu} = 1$. If $R_i' < \beta^{-1}$, then (5) is binding, a contradiction. If $R_i' = \beta^{-1}$, then the firm is indifferent between paying dividends in the current period or in the next period. But in equilibrium $b_i' = w_i$ and hence $\tilde{d} = d > 0$ for the representative firm. By induction starting at the steady state and working backwards, the firm's policy is optimal in region D. Further, we show in Lemmata 3 and 4 that the equilibrium in region D is the unique equilibrium converging to the steady state.

Consider now region ND with $w_i \leq w_i^*$ (as Lemma 2 shows) and $w < \bar{w}(w_i)$ as defined in the characterization of region D above and $d = 0$. Denote the firm's ex dividend net worth by $w_{ex} \leq w$. There are 3 cases to consider: $w_{ex}/w_i > \bar{w}/\bar{w}_i$, $w_{ex}/w_i \in [w^*/w_i^*, \bar{w}/\bar{w}_i]$, and $w_{ex}/w_i < w^*/w_i^*$.

First, if $w_{ex}/w_i > \bar{w}/\bar{w}_i$, then $w_{ex} + w_i < \bar{w}(w_i) + w_i = \bar{w} + \bar{w}_i$ and $k \leq (w_{ex} + w_i)/\wp < (\bar{w} + \bar{w}_i)/\wp = \bar{k}$. Note that since $b_i' \leq w_i - d_i \leq w_i$, we have $w_{ex}/b_i' \geq w_{ex}/w_i > \bar{w}/\bar{w}_i$. If (5) binds, then $R_i' = (\theta_i - \theta)(1-\delta)(w_{ex}/b_i' + 1)/\wp > (\theta_i - \theta)(1-\delta)(\bar{w}/\bar{w}_i + 1)/\wp = \beta^{-1}$. If (5) does not bind, then $R_i' = [A' f_k(k) + (1-\theta)(1-\delta)]/\wp > [A' f_k(\bar{k}) + (1-\theta)(1-\delta)]/\wp = \beta^{-1}$. In either case, $R_i' > \beta^{-1}$, and hence $d = 0$, $d_i = 0$, and $b_i' = w_i$.

Second, consider $w_{ex}/w_i \in [w^*/w_i^*, \bar{w}/\bar{w}_i]$. If $w_{ex}/b_i' > \bar{w}/\bar{w}_i$ we are in the first region and hence $d_i = 0$ and $b_i' = w_i$, a contradiction. Hence, w.l.o.g. $w_{ex}/b_i' \in [w^*/w_i^*, \bar{w}/\bar{w}_i]$. Take $\tilde{w}_i$ such that $w_{ex}/b_i' = \bar{w}(\tilde{w}_i)/\tilde{w}_i$. Note that (5) binds at $\tilde{w}_i$ and $\bar{w}(\tilde{w}_i)$, and thus $b_i' + w_{ex} < \tilde{w}_i + \bar{w}(\tilde{w}_i)$ and moreover $k < \hat{k}(\tilde{w}_i)$. If (5) does not bind, then

$$\begin{aligned}
\hat{R}_i'(\tilde{w}_i) &= (\theta_i - \theta)(1-\delta)(\bar{w}(\tilde{w}_i)/\tilde{w}_i + 1)/\wp > (\theta_i - \theta)(1-\delta)(w_{ex}/b_i' + 1)/\wp > R_i' \\
&= [A' f_k(k) + (1-\theta)(1-\delta)]/\wp > [A' f_k(\hat{k}(\tilde{w})) + (1-\theta)(1-\delta)]/\wp.
\end{aligned}$$

But since (5) binds at $\tilde{w}_i$ and $\bar{w}(\tilde{w}_i)$, $\hat{R}_i'(\tilde{w}_i) < [A' f_k(\hat{k}(\tilde{w})) + (1-\theta)(1-\delta)]/\wp$, a contradiction. Therefore, (5) binds and $R_i' = \hat{R}_i'(\tilde{w}_i)$. From (19), $\beta\mu'/\mu[A' f_k(k) + (1-\theta_i)(1-\delta)]/\wp_i(R_i') = 1 = \beta[A' f_k(\hat{k}(\tilde{w}_i)) + (1-\theta_i)(1-\delta)]/\wp_i(\hat{R}_i'(\tilde{w}_i))$ and, since $k < \hat{k}(\tilde{w}_i)$,

$\mu > \mu' \geq 1$, that is, $d = 0$. Further, if $w_{ex}/w_i \in (w^*/w_i^*, \bar{w}/\bar{w}_i]$, then $R_i' \in (\beta_i^{-1}, \beta^{-1}]$, and thus $d_i = 0$ and $b_i' = w_i$. If $w_{ex}/w_i = w^*/w_i^*$, then either $d_i > 0$ or $b_i' < w_i$ yields $R_i' > \beta_i^{-1}$ and therefore $d_i = 0$ and $b_i' = w_i$ at such w_{ex} and w_i as well.

Third, consider $w_{ex}/w_i < w^*/w_i^*$. As before, w.l.o.g. $w_{ex}/b_i' < w^*/w_i^*$. Then from (5), $R_i' \leq (\theta_i - \theta)(1-\delta)(w_{ex}/b_i' + 1)/\wp < (\theta_i - \theta)(1-\delta)(w^*/w_i^* + 1)/\wp = \beta_i^{-1}$, that is, $R_i' < \beta_i^{-1}$. From (19), $\beta\mu'/\mu[A'f_k(k) + (1-\theta_i)(1-\delta)]/\wp_i(R_i') = 1 = \beta[A'f_k(k^*) + (1-\theta_i)(1-\delta)]/\wp_i(\beta_i^{-1})$ and, since $k < k^*$ and $R_i' < \beta_i^{-1}$, $\mu > \mu' \geq 1$, that is, $d = 0$. Moreover, (5) binds, since otherwise $\beta_i^{-1} > R_i' = [A'f_k(k) + (1-\theta)(1-\delta)]/\wp > [A'f_k(k^*) + (1-\theta)(1-\delta)]/\wp$, but since in the steady state (5) binds $\beta_i^{-1} < [A'f_k(k^*) + (1-\theta)(1-\delta)]/\wp$, a contradiction.

Thus, we conclude that $d = 0$, (property (i) in the statement of the proposition), $d_i = 0$ (except possibly in the first period (see Lemma 2), that R_i' satisfies the equation in property (ii) of the proposition), and that $b_i' = w_i$ and $k = (w + w_i)/\wp$ if $R_i' > R$ and $k = w/\wp_i(R)$ if $R_i' = R$ (property (iii)). Moreover, using (3) and (19) we have

$$\begin{aligned} w' &= A'f(k) + (1-\theta_i)(1-\delta)k - [R_i'b_i' - (\theta_i - \theta)(1-\delta)k] \\ &> R_i'\wp_i(R_i')k - [R_i'b_i' - (\theta_i - \theta)(1-\delta)k] \geq R_i'\wp k - R_i'b_i' = R_i'w, \end{aligned}$$

which, together with the fact that $w_i' = R_i'w_i$, implies that $w'/w_i' > w/w_i$ (property (iv)). Note that the equilibrium is thus unique in region ND as well. $\square$

Proof of Lemma 2. We first show that $d_i > 0$ when $w_i > w_i^*$. If $w \geq w^*$, the stationary state is reached and the result is immediate. Suppose hence that $w < w^*$. Suppose instead that $d_i = 0$. We claim that $R_i' < \beta_i^{-1}$ for such w_i and w. Either $R_i' = R$ and hence the claim is obviously true or $R_i' > R$, but then $b_i' = w_i$. Using (5) and (2) we have $R_i' \leq (\theta_i - \theta)(1-\delta)k/b_i' \leq (\theta_i - \theta)(1-\delta)(w/w_i + 1) < (\theta_i - \theta)(1-\delta)(w^*/w_i^* + 1) = \beta_i^{-1}$, that is, $R \leq R_i' < \beta_i^{-1}$. But as long as $d_i = 0$, $w_i' = R_i'w_i \geq Rw_i > w_i$, that is, intermediary net worth keeps rising. If eventually firm net worth exceeds w^*, then the steady state is reached and $\mu_i' = 1$ from then onward. But then $\mu_i = \beta_i R_i'\mu_i' = \beta_i R_i' < 1$, which is not possible. The intermediary must pay a dividend in the first period, because if it pays a dividend at any pont after that, an analogous argument would again imply that $\mu_i < 1$ in the first period, which is not possible. Similarly, if $w < w^*$ forever, then $w > w_i^*$ forever and the firm must eventually pay a dividend in this region, as never paying a dividend cannot be optimal. But by the same argument again then the dividend must be paid in the first period.

To see that at most an initial dividend is paid and no further dividends are paid until the steady state is reached, note that in equilibrium once $R_i' > \beta_i^{-1}$, then this is the case until the steady state is reached. But as long as $R_i' > \beta_i^{-1}$, the intermediary does not pay

a dividend (and this is true w.l.o.g. also at a point where $R_i' = \beta_i^{-1}$ before the steady state is reached). Before this region is reached, $R_i' < \beta_i^{-1}$, but then the intermediary would not postpone a dividend in this region, as other wise again $\mu_i = \beta_i R_i' \mu_i' = \beta_i R_i' < 1$, which is not possible. □

Lemma 3 *Consider an equilibrium with $R_i' \in [\beta_i^{-1}, \beta^{-1}]$ and $\mu = \mu' = 1$ and assume the equilibrium is unique from the next period onward. Consider another equilibrium interest rate $\tilde{R}_i'$, then $\tilde{k} \lesseqgtr k$ and $\tilde{R}_i' \lesseqgtr R_i'$ is impossible.*

Proof of Lemma 3. Using (19) at the two different equilibria, if $\tilde{k} \lesseqgtr k$ and $\tilde{R}_i' \lesseqgtr R_i'$, then

$$\frac{\tilde{\mu}}{\tilde{\mu}'} = \beta \frac{A' f_k(\tilde{k}) + (1-\theta_i)(1-\delta)}{\wp_i(\tilde{R}_i')} \gtreqless \beta \frac{A' f_k(k) + (1-\theta_i)(1-\delta)}{\wp_i(R_i')} = 1 \qquad (23)$$

If $\tilde{k} < k$ and $\tilde{R}_i' < R_i' = \beta_i^{-1}$, then by (23) $\tilde{\mu} > \tilde{\mu}'$. Thus, $\tilde{\mu} > \tilde{\mu}' \tilde{R}_i' \beta_i$ implying that (5) must be binding. But then the firm must pay a dividend and $1 = \tilde{\mu} > \tilde{\mu}'$, a contradiction.

If $\tilde{k} > k$ and $\tilde{R}_i' > R_i'$ and the collateral constraint binds at the original equilibrium, then $\tilde{w}' \geq A' f(\tilde{k}) + (1-\theta)i)(1-\delta)\tilde{k} > A' f(k) + (1-\theta_i)(1-\delta)k = w'$. Since $\tilde{w}' > w'$, $\mu' = 1$, and the equilibrium is unique , $\tilde{\mu}' = 1$. By (23), $\tilde{\mu} < \tilde{\mu}' = 1$, a contradiction.

If $\tilde{k} > k$ and $\tilde{R}_i' > R_i'$ and the collateral constraint does not bind at the original equilibrium, the $R_i' = \beta^{-1}$ (using (9)). But then $\tilde{\mu}/\tilde{\mu}' \geq \tilde{R}_i' \beta > 1$ while (23) implies $\tilde{\mu}/\tilde{\mu}' < 1$, a contradiction. □

Lemma 4 *The equilibrium in region D is the unique equilibrium converging to the steady state.*

Proof of Lemma 4. The proof is by induction. First, note that if $w \geq w^*$ and $w_i \geq w_i^*$, then the unique steady state is reached. Consider an equilibrium interest rate R_i' in region D and suppose the equilibrium is unique from the next period on. Suppose $R_i' \in [\beta_i^{-1}, \beta^{-1})$ and consider another equilibrium with $\tilde{R}_i'$. If the collateral constraint (5) binds at this equilibrium, then $\tilde{R}_i' = (\theta_i - \theta)(1-\delta)\tilde{k}/w_i \gtreqless (\theta_i - \theta)(1-\delta)k/w_i = R_i'$, which is impossible by Lemma (3). If the collateral constraint (5) does not bind at this equilibrium and $\tilde{k} < k$, then $\tilde{R}_i' < (\theta_i - \theta)(1-\delta)\tilde{k}/w_i < (\theta_i - \theta)(1-\delta)k/w_i = R_i'$, which is also impossible by Lemma (3). If the collateral constraint (5) does not bind at this equilibrium and $\tilde{k} > k$, by Lemma (3) $\tilde{R}_i' < R_i$. But then by (9) $\tilde{\mu}/\tilde{\mu}' = \beta \tilde{R}_i' < \beta R_i' < 1$. Since $\tilde{k} > k$ and the collateral constraint binds at R_i', $\tilde{w}' > w'$ implying $\tilde{\mu}' = 1$ and by above inequality $\tilde{\mu} < 1$, a contradiction. Thus for $R_i' \in [\beta_i^{-1}, \beta^{-1})$ the equilibrium is unique. Suppose $R_i' = \beta^{-1}$. By Lemma (3), we need only consider the two cases $\tilde{k} \gtreqless k$

and $\tilde{R}'_i \lesseqgtr R'_i = \beta^{-1}$. If $\tilde{k} < k$ and $\tilde{R}'_i > \beta^{-1}$, (9) implies that $\tilde{\mu} > 1$ and hence the firm does not pay a dividend. But then the firm must be borrowing less from intermediaries, which cannot be an equilibrium as $l'_i = w_i$ at this interest rate. If $\tilde{k} > k$ and $\tilde{R}'_i < R'_i = \beta^{-1}$, and if (5) binds at $\tilde{R}'_i$, $\tilde{R}'_i = (\theta_i - \theta)(1-\delta)\tilde{k}/w_i > (\theta_i - \theta)(1-\delta)k/w_i \geq R'_i$, a contradiction; if (5) instead does not bind at $\tilde{R}'_i$, $\tilde{\mu}/\tilde{\mu}' = \beta\tilde{R}'_i < 1$. Since $\tilde{k} > k$ and $\tilde{R}'_i\tilde{b}'_i \leq R'_i w_i$, $\tilde{w}' > w'$ implying $\tilde{\mu}' = 1$ and by above inequality $\tilde{\mu} < 1$, a contradiction. Therefore the equilibrium in region D is unique. □

Proof of Proposition 8. Part (i): By assumption the expected productivity in the first period equals the deterministic productivity from time 1 onward (denoted $\bar{A}'$ here), that is, $E[A'] = \bar{A}'$. Define the first best level of capital k_{fb} by $r + \delta = \bar{A}' f_k(k_{fb})$. Using the definition of the user cost of capital the investment Euler equation (19) for the deterministic case can be written as

$$r + \delta + \frac{\rho}{R+\rho}(1-\theta_i)(1-\delta) + \frac{\rho_i}{R+\rho_i}(\theta_i - \theta)(1-\delta) = R\beta\bar{A}' f_k(k^*) < \bar{A}' f_k(k^*)$$

and thus $k^* < k_{fb}$. Now suppose that $\lambda(s') = 0$, $\forall s' \in S$. Part (ii) of Lemma 1 then implies that $\lambda_i(s') = 0$, $\forall s' \in S$, and (8) and (9) simplify to $\mu = R\beta\mu'$ and $\mu = R'_i\beta\mu'$, implying that $R'_i = R$, $\forall s' \in S$, and that $d' = 0$, $\forall s' \in S$, as otherwise $\mu < 1$. Moreover, (16) simplifies to $\mu_i = R\beta_i\mu'_i$ and thus $d'_i = 0$, $\forall s' \in S$, as well since otherwise $\mu_i < 1$. Investment Euler equation (19) reduces to $r + \delta = \bar{A}' f_k(k_{fb})$, that is, investment must be k_{fb}. We now show that this implies that the sum of the net worth of the intermediary and the firm exceeds their steady state (cum dividend) net worth in at least one state, which in turn implies that at least one of them pays a dividend, a contradiction. To see this note that $w' = A' f(k_{fb}) + k_{fb}(1-\delta) - Rb' - R'_i b'_i$ and $w'_i = Rl' + R'_i l'_i \geq R'_i l'_i = R'_i b'_i$ and thus

$$w' + w'_i \geq A' f(k_{fb}) + k_{fb}(1-\delta) - Rb' \geq A' f(k_{fb}) + (1-\theta)k_{fb}(1-\delta) > A' f(k^*) + (1-\theta)k^*(1-\delta)$$

whereas $w'^* + w'^*_i = \bar{A}' f(k^*) + (1-\theta)k^*(1-\delta)$. For $A' > \bar{A}'$, $w' + w'_i > w'^* + w'^*_i$, and either the intermediary or the firm (or both) must pay a dividend, a contradiction.

Part (ii): If $\lambda_i(s') = 0$, $\forall s' \in S$, then $(\beta\mu'/\mu)^{-1} = R'_i = (\beta_i\mu'_i/\mu_i)^{-1}$ where the first equality uses (9) and the second equality uses (16) and the fact that part (iv) of Lemma 1 holds for an eventually deterministic economy.

Part (iii): Since $\lambda(\hat{s}') = 0$, $\lambda_i(\hat{s}') = 0$ by part (ii) of Lemma 1 and $R_i(\hat{s}') = R$. From (8), $\mu(\hat{s}') = \mu(\check{s}') + \lambda(\check{s}') > \mu(\check{s}')$. Using (16), $(\beta_i\mu_i(\hat{s}')/\mu_i)^{-1} = R \leq R_i(\check{s}') = (\beta_i\mu_i(\check{s}')/\mu_i)^{-1}$ and thus $\mu_i(\hat{s}') \geq \mu_i(\check{s}')$. □

References

Bernanke, Ben, and Mark Gertler, 1989, Agency costs, net worth and business fluctuations, *American Economic Review* 79, 14-31.

Boyd, John H., and Edward C. Prescott, 1986, Financial intermediary coalitions, *Journal of Economic Theory* 38, 211-232.

Brunnermeier, Markus K., and Yuliy Sannikov, 2010, A macroeconomic model with a financial sector, Working paper, Princeton University.

Caballero, Ricardo J., and Arvind Krishnamurthy, 2001, International and domestic collateral constraints in a model of emerging market crises, *Journal of Monetary Economics* 48, 513-548.

Diamond, Douglas W., 1984, Financial intermediation and delegated monitoring, *Review of Economic Studies* 51, 393-414.

Diamond, Douglas W., 2007, Legal systems, bank finance, and debt maturity, Working paper, University of Chicago.

Diamond, Douglas W., and Philip H. Dybvig, 1983, Bank runs, deposit insurance, and liquidity, *Journal of Political Economy* 91, 401-419.

Diamond, Douglas W., and Raghuram G. Rajan, 2000, A theory of bank capital, *Journal of Finance* 55, 2431-2465.

Gertler, Mark, and Nobuhiro Kiyotaki, 2010, Financial intermediation and credit policy in business cycle analysis, Working paper, NYU and Princeton.

Hahn, F.H., 1973, On the foundations of monetary theory, in: M. Parkin and A.R. Nobay, eds., Essays in Modern Economics, Barnes and Noble, New York, NY, 230-242.

Holmström, Bengt, and Jean Tirole, 1997, Financial intermediation, loanable funds, and the real sector, *Quarterly Journal of Economics* 112, 663-91.

Jorgenson, Dale W., 1963, Capital theory and investment behavior, *American Economic Review (Papers and Proceedings)* 53, 247-259.

Kehoe, Timothy J., and David K. Levine, 1993, Debt-constrained asset markets, *Review of Economic Studies* 60, 865-888.

Kehoe, Timothy J., and David K. Levine, 2006, Bankruptcy and collateral in debt constrained markets, Working paper, Federal Reserve Bank of Minneapolis.

Kiyotaki, Nobuhiro, and John Moore, 1997a, Credit cycles, *Journal of Political Economy* 105, 211-248.

Kiyotaki, Nobuhiro, and John Moore, 1997b, Credit chains, Working paper, University of Minnesota and LSE.

Ramakrishnan, Ram T., and Anjan V. Thakor, 1984, Information reliability and a theory of financial intermediation, *Review of Economic Studies* 51, 415-432.

Rampini, Adriano A., and S. Viswanathan, 2007, Collateral, financial intermediation, and the distribution of debt capacity, Working paper, Duke University.

Rampini, Adriano A., and S. Viswanathan, 2010, Collateral, risk management, and the distribution of debt capacity, *Journal of Finance* 65, 2293-2322.

Rampini, Adriano A., and S. Viswanathan, 2011, Collateral and capital structure, Working paper, Duke University.

Townsend, Robert M., 1978, Intermediation with costly bilateral exchange, *Review of Economic Studies* 45, 417-425.

Williamson, Stephen D., 1986, Costly monitoring, financial intermediation, and equilibrium credit rationing, *Journal of Monetary Economics* 18, 159-179.

Figure 1: Role of Firm and Financial Intermediary Net Worth

Interest rate on intermediated finance $R_i' - 1$ (percent) as a function of firm (w) and intermediary net worth (w_i). The parameter values are: $\beta = 0.90$, $R = 1.05$, $\beta_i = 0.94$, $\delta = 0.10$, $\theta = 0.80$, $\theta_i = 0.90$, $A' = 0.20$, and $\alpha = 0.333$.0210121022103210421052106210721082109211221005210121015210221025FirmnetworthIntermed

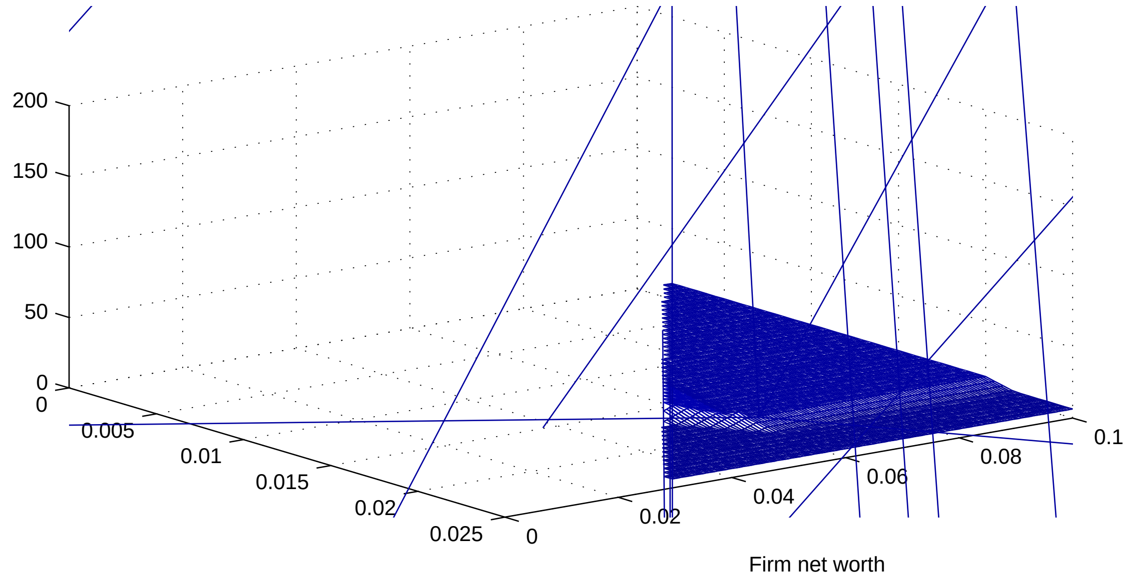


Figure 4: Dynamics of Firm and Financial Intermediary Net Worth

Contours of the regions describing the deterministic dynamics of firm and financial intermediary net worth (see Proposition 7). Region ND, in which firms pay no dividends, is to the left of the solid line and Region D, in which firms pay positive dividends, is to the right of (and including) the solid line. The point where the solid line reaches the dotted line is the deterministic steady state (w^*, w_i^*). The kink in the solid line is the point $(\bar{w}, \bar{w}_i)$ where $R_i' = \beta^{-1}$ and the collateral constraint just binds. The solid line segment between these two points is $\bar{w}(w_i) = \wp k(w_i) - w_i$ (with $R_i' \in (\beta_i^{-1}, \beta^{-1})$). The solid line segment sloping down is $\bar{w}(w_i) = \wp \bar{k} - w_i$ (with $R_i' = \beta^{-1}$). Region ND is dividend by two dash dotted lines: below the dash dotted line through $(\bar{w}, \bar{w}_i)$ $R_i' > \beta^{-1}$; between the two dash dotted lines $R_i' \in (\beta_i^{-1}, \beta^{-1})$; and above the dash dotted line through (w^*, w_i^*) $R_i' < \beta_i^{-1}$. The parameter values are as in Figure 1.

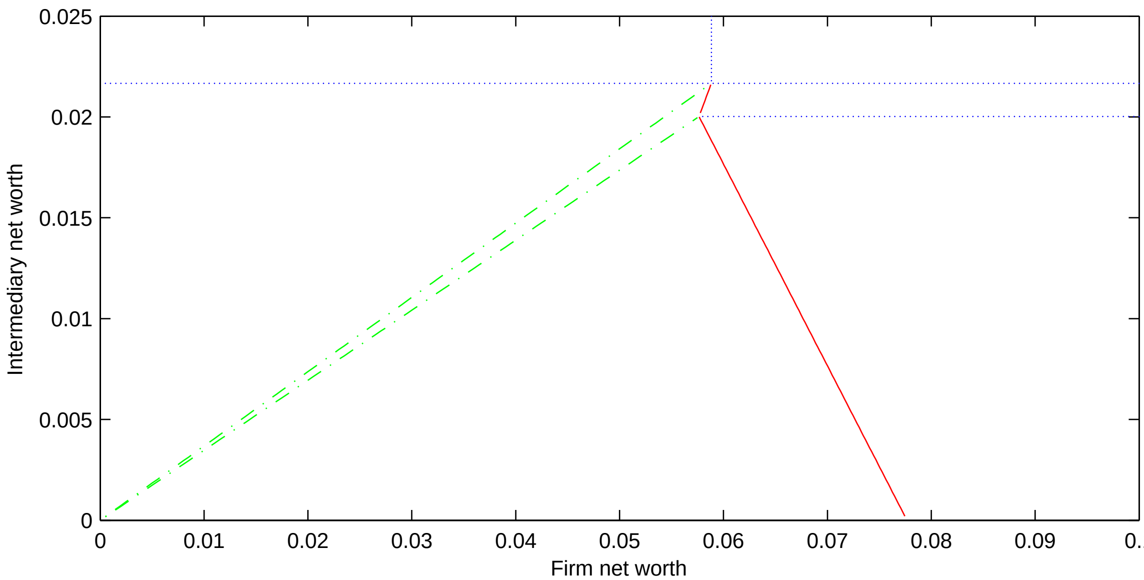


Figure 5: Dynamics of Firm and Financial Intermediary Net Worth

This figures illustrates the deterministic dynamics starting from initial values of from initial values $w = 0.0222$ and $w_i = 0.06$. Panel A traces out the path of firm and intermediary net worth in w vs. w_i space with the contours as in Figure 4. Panel B shows firm net worth (dashed) cum dividends (higher) and ex dividend (lower) and intermediary net worth (solid) cum dividends (higher) and ex dividend (lower) against time. Panel C shows the interest rate on intermediated finance against time. Panel D shows investment against time. The parameter values are as in Figure 1 except that $\alpha = 0.8$.

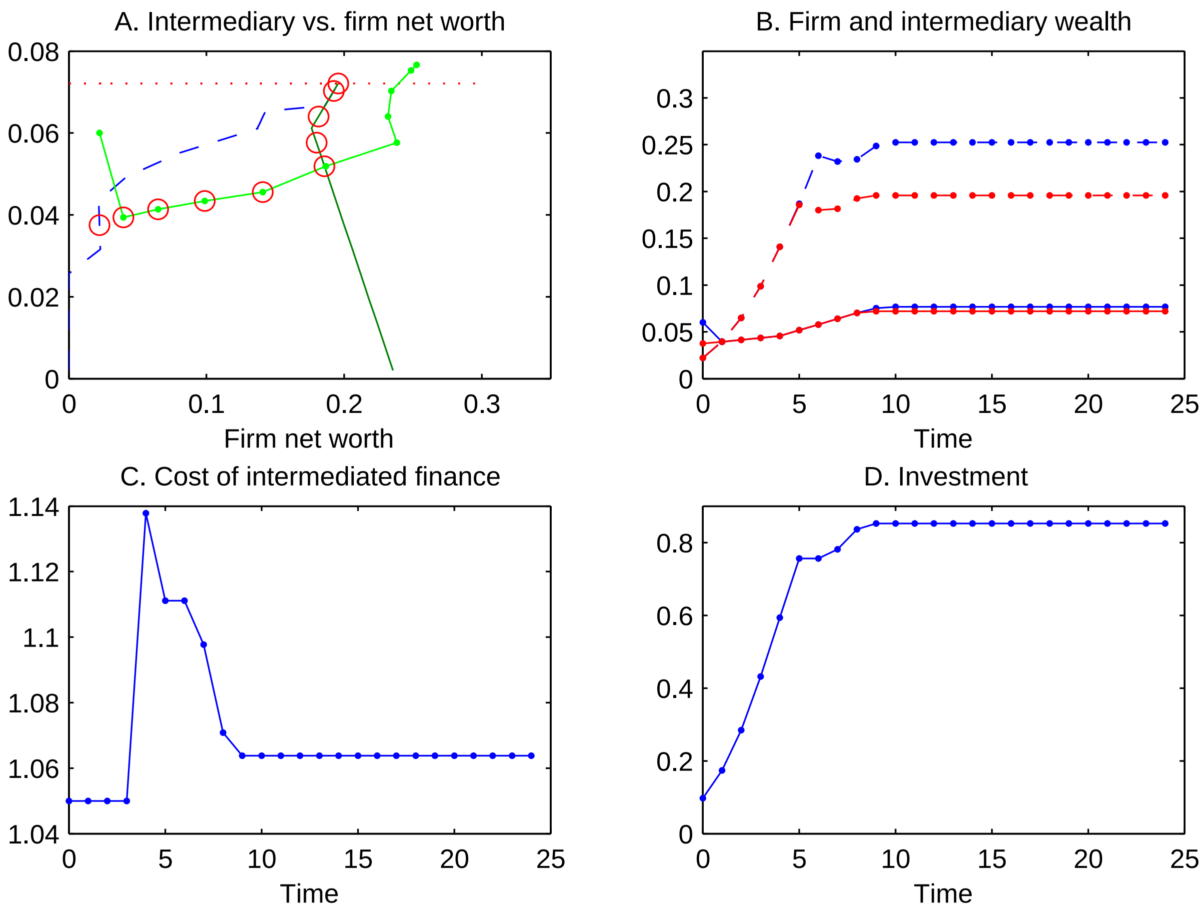
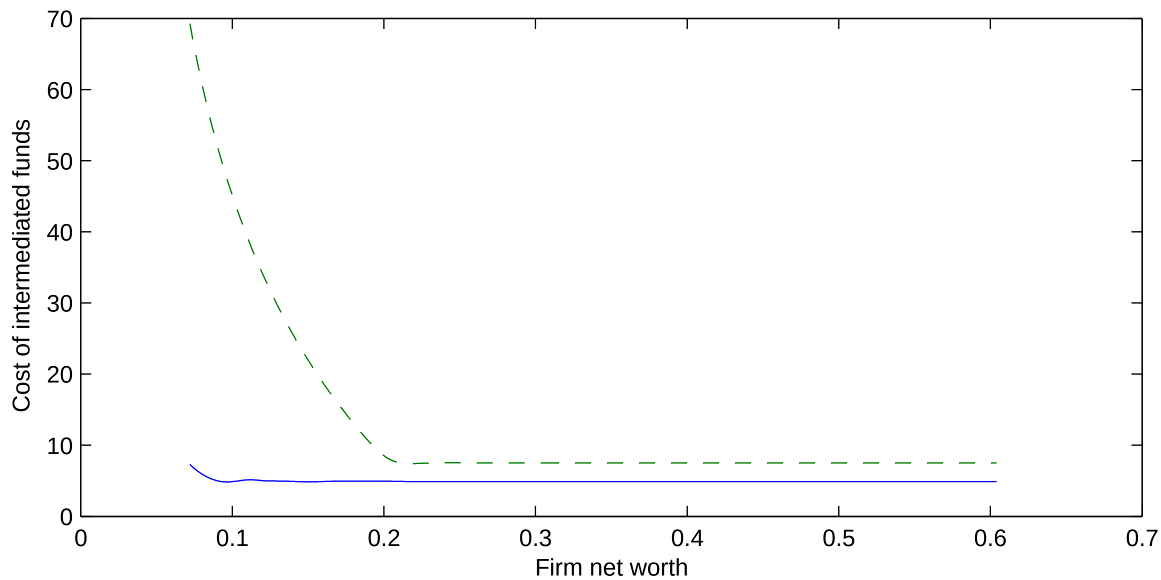


Figure 6: Interest Rates on Shadow Intermediary Capital

Shadow interest rates on intermediated financing against high state ($R_i(s'_h)$) (dashed) and low state ($R_i(s'_l)$) (solid) as a function of firm net worth (w) under the stationary distribution. In Panel A productivity is independent across time and the shadow interest rates do not depend on the current state of productivity. In Panel B productivity is persistent and the shadow interest rates depend on the current state of productivity as well (with the higher (lower) dashed and solid lines correspond to s_h and s_l respectively. The parameter values are: $\beta = 0.93$, $R = 1.05$, $\delta = 0.10$, $\theta = 0.80$, $A(s'_h) = 0.60$, $A(s'_l) = 0.05$, and $\alpha = 0.333$.

Panel A: Constant investment opportunities ($\Pi(s_h, s'_h) = \Pi(s_l, s'_l) = 0.5$)



Panel B: Stochastic investment opportunities ($\Pi(s_h, s'_h) = \Pi(s_l, s'_l) = 0.6$)