Rational Price-Contingent Trading and Asset Price Dynamics*

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Abstract

We study the incentives of large, rational agents to pursue short-term trading strategies contingent only on past prices. It is often asserted that such trading should not occur in efficient markets populated by rational investors and must stem from behavioral biases or liquidity needs. We show instead that price-contingent trading is optimal in a rational-expectations framework. One assumption – not all market participants know whether large traders are informed – generates all our results. The reason is that large agents have superior knowledge of their own trades and market impact, and thus learn from past prices better than average market participants.

JEL classification: G12, G14, D82

Keywords: price-contingent trading, positive-feedback trading, rational expectations, momentum, excess volatility

1 Introduction

This paper studies the incentives of large, rational traders in financial markets to pursue short-term strategies that are contingent only on past price movements (i.e. buy/sell after prices have gone up/down). Although widespread and profitable in real-world financial markets[1], the dominant stance in the literature has been that such trading strategies should not occur in (weak form) efficient markets populated by rational traders. In general, if

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[1] E.g. Grinblatt et al. (1995), Griffin et al. (2003), Nofsinger and Sias (1999), Osler (2003), and Shu (2009) document in various settings the pervasive use of price-contingent trading by financial institutions.

prices already fully reflect publicly available information, then rational profit-maximizing uninformed agents cannot obtain returns in excess of the market risk premium over the risk-free rate, because they have no superior information. Thus, in markets with risk-neutral agents and zero risk-free rate, expected returns are zero, so uninformed agents should not trade at all. This has been highlighted by Easley and O'Hara (1991) who argue that in rational expectations models any price-contingent strategy would be neutralized by any risk-neutral trader who observes the same past prices. Consequently, the enormous literature that examines price-contingent strategies and their impact on asset prices has relied on behavioral biases, imperfect rationality, non-standard preferences, or institutional frictions.[2] In contrast, we show here that price-contingent strategies are in general optimal for fully-rational large traders, without relying on violations of market efficiency or institutional frictions.

As these strategies are most prominent among financial institutions, we focus on a setting where rational traders are also large, that is, their orders have market impact. A crucial assumption in this class of standard rational expectations models is that the types of all large traders are public information, that is, everybody knows whether and how many large traders are informed about the fundamental. Therefore, the market efficiency condition − price equals expected fundamental value − guarantees that any trader who is not informed cannot obtain excess returns from price-contingent strategies because he has no superior information compared to the rest of the market. The key innovation in our paper is relaxing the assumption that the types of all large traders are public information. Namely, we introduce a large trader who may or may not be informed about the fundamental and his type is not known to the market. We show that such large trader systematically profits from price-contingent trading strategies in states in which he is uninformed. This result arises because a large uninformed trader always knows to what extent past price movements reflect his own trading, or lack thereof. His superior information compared to the market arises solely from his superior information about his own type, which directly translates into a superior understanding of the information revealed in past asset prices.

More specifically, to capture short-term incentives and asset price dynamics, we focus on the simplest model with two trading rounds and one risky asset. In the spirit of Kyle (1985) and Holden and Subrahmanyan (1992), there is a number of large risk-neutral informed traders, which we call type K traders. To be directly comparable to these benchmark models, we abstract from a situation where the market is solely populated by noise traders and

[2] See e.g, DeLong et al. (1990), Barberis et al. (1998), Daniel et al. (1998), Hong and Stein (1999), Barberis and Shleifer (2003), Dybvig (1988), Krugman and Miller (1993), Grossman (1988), Brennan and Schwartz (1989), Genotte and Leland (1990), Frankel and Froot (1990), Easley and O'Hara (1991), Basak (1995), Grossman and Zhou (1996). Brunnemeier and Pedersen (2005), Dasgupta et al. (2008).

uninformed agents and we assume that prices can reflect at least some new information about the fundamental. Our main innovation is to introduce another large risk-neutral trader, trader P, who may be informed or uninformed, and we focus on his trading incentives.

During each trading round, all large traders and noise traders submit market orders before knowing the execution price. As standard in such rational expectations models, the market is efficient by construction. A hypothetical agent, called the Market, observes total order flow and sets prices equal to the expected value of the fundamental based on publicly available information. Publicly available information includes the prior distribution of the fundamental, the distribution of noise trading, and the probability that P is informed.

We show that during the first trading round, P only trades when he is informed. This is because the only information available to P when uninformed is the prior, which is publicly known. Our main result - price-contingent trading strategies are optimal - arises in the second trading round whereby P also observes period 1 equilibrium price. This is because P, despite being uninformed, obtains more accurate information from past prices than the Market. To understand this result, note that in addition to the prior there are two sources of information, the price and the (noisy) order flow. Because prices are in turn a function of the prior and of the order flow, in standard rational expectations models the price and the order flow have the same basic information content. Indeed, if P's type is public information then the distribution of the fundamental is the same, whether one conditions on the price or on the order flow, so that P's and the Market's expectations coincide. Crucially, these expectations endogenously differ in our setting.

For illustration, suppose that there is a positive order flow and therefore period 1 price is higher than the prior. There are two forces driving the differences between the Market's and uninformed P's expectations. First, by conditioning on period 1 price, uninformed P concludes that the fundamental is likely to be high. The reason for this is the following. P knows that there are fewer informed traders (i.e., he know he is uninformed), while the Market, who does not know P's type, weighs two possibilities, either more informed traders (i.e. including P), or fewer informed traders (i.e. excluding P). Fewer informed traders imply that a given order flow is more likely generated by a higher fundamental. Therefore the price signal that P observes indicates a higher fundamental than the order flow signal that the Market observes. This force pushes uninformed P to pursue a positive-feedback strategy (i.e. buy after prices have gone up).

Second, uninformed P knows that the price signal he observes is more noisy than the order flow signal observed by the Market. Indeed, if fewer traders are informed then the total order flow is likely to be lower, and a noise trading shock is going to have a bigger impact. Because the Market still weighs the possibility that there are more informed traders, when setting

period 1 price the Market overestimates the precision of the order flow signal in the state in which P is uninformed. As a result, uninformed P relies relatively more on the prior when updating his expectations. This force tends to make P's conditional expectations of the fundamental based on all his information to be lower than period 1 price. This force pushes uninformed P to pursue a contrarian strategy (i.e., sell after prices have gone up).

In general, due to these two forces, the Market's expectations of the fundamental based on the prior and the order flow and P's expectations based on the prior and the price are different. The Market tends to underestimate the level of the fundamental and to overestimate the precision of the order flow signal compared to P. Whether P expects the fundamental to be higher or lower than period 1 price (and the direction of his optimal trading strategy) depends on which of these two forces dominates. We investigate this in different settings.

We start by analyzing a basic model and we make two strong assumptions. First, we assume that all large traders in period 1 trade under the assumption that they will not have an opportunity to trade in period 2. The additional trading opportunity in date 2 arrives as a surprise to them. Second, the Market does not update his beliefs about P's type after observing the order flow. This setting is useful as it allows us to characterize the main effects and obtain a simple analytical solution. We find that the first force described above always dominates and the optimal price-contingent strategy is positive-feedback in this setting.

We ask next, is positive-feedback trading stabilizing or destabilizing ex post? The answer is that it depends on the nature of the shock, because positive-feedback trading makes any exogenous shock more persistent. Thus, in case of a shock to fundamentals, positive-feedback trading helps prices converge faster to fundamentals, and is therefore stabilizing ex post. However, in the case of a noise trade shock, positive-feedback trading causes prices to stray further away from fundamentals and is thus destabilizing. We find that the overall ex ante pure effect of equilibrium positive-feedback trading is destabilizing, and generates excess volatility in asset prices.

We then relax the restrictive assumptions of the basic setting. First, we allow the Market to update his beliefs about P's type after observing the order flow. In general, the order flow not only reveals information about the fundamental, but also on P's type, because a smaller order flow is more likely associated with a state where P is uninformed. We show that allowing updating does not change our main findings qualitatively, and has only a negligible quantitative effect on prices and quantities traded. While this setting is not analytically solvable, we approximate the solution around an arbitrarily small order flow where such updating would reduce uninformed P's information advantage the most. We find that all our findings are almost identical to those in the basic setting. We also evaluate the differences in approximated prices with the Market's expectations of the fundamental

given strategies and confirm that this simplifying assumption does not drive our results. Furthermore, assuming that the Market does not update his beliefs about traders' types allows us to focus on the pure short-run effects of price-contingent trading that do not rely on incentives to build reputation.

Second, we consider a forward-looking setting where large traders know in period 1 that they will also have an opportunity to trade in period 2. We find that when there is always Cournot competition among informed traders (i.e., there are always at least two informed traders) the optimal price contingent strategy remains positive-feedback and all our findings from the basic model continue to hold. There is one important exception. When there is only one type K trader, then the optimal price-contingent strategy for uninformed P is contrarian. This is because a large information monopolist has an incentive to strategically hide his information and trade little in an early trading round as shown by Kyle (1985). As shown by Holden and Subrahmanyan (1992), even two informed traders compete aggressively enough to make it difficult for informed traders to hide information. As a result P knows that prices reveal very noisy information while the Market expects more aggressive competition among traders and much better quality of information revealed by the order flow. In this particular case, the second force discussed above dominates. While theoretically interesting, the case where more than one trader can be informed, but there is exactly one informed trader, seems to be the least realistic.[3] Overall, in Cournot competitive environments we find a tendency to pursue optimal positive-feedback strategies, and our most general and robust finding − the optimality of price-contingent strategies in rational expectations setting with large traders − continues to hold.

Our paper relates to the literature on stock price manipulation, that is the idea that rational traders may have an incentive to trade against their private information. Provided manipulation is followed by some (exogenously assumed) price-contingent trading, short run losses can be more than offset by long term gains (e.g. De Long et al. 1990, Jarrow 1992, Kumar and Seppi 1992, Allen and Gale 1992, Brunnermeier 2005). Chakraborty and Yilmaz (2004a, 2004b, CY henceforth) study the effect of uncertainty about whether there is an informed trader in the market, or the market is solely populated my noise traders. If there is an informed trader, he may trade against his information in order to obtain a reputation as a noise trader. As a result, the informed can earn long-term profits that offset the short-term

[3] Private information in Kyle (1985) is best interpreted as pure inside information, that is, a particular type of information that several traders do not have access to. By contrast, in our model private information is something that several large traders have a potential to access. Large financial institutions may have a superior ability to understand the fundamentals. Also, it would endogenously arise in a setting with multiple assets, where investors specialize on a set of assets to be informed about given limited capacity to process information (see Van Nieuwerburgh and Veldkamp 2008, 2009). In such case there would be uncertainty about how many large traders are informed about a specific asset.

losses from noise trading. There are crucial differences with our paper. First and foremost, in contrast to our main focus, CY do not study the trading incentives of rational agents when they are uninformed. In their setting, incentives to trade against private information are purely driven by informed traders' desire to mimick their own behaviour when uninformed, which CY impose to be always noise trading. Second, while their mechanism is about long-term gains from reputation, we show that there exist immediate short-term profits for uninformed traders from price-contingent strategies that do not rely on reputation building – if anything, reputation building is likely to have a small effect compared to gains from price-contingent strategies, at least for rational uninformed traders. Finally, CY emphasize the long-term benefits for informed traders to gain a reputation as noise traders. In sharp contrast, we emphasize a novel mechanism, where uninformed traders obtain short-term benefits from a reputation of being seen as informed.

Two interesting recent papers study forms of stock-price manipulation that are closely linked to particular instances of uninformed trading. Goldstein and Guembel (2008) show that if stock prices affect real activity then a form of trade-based manipulation such as short-sales by uninformed speculators can be profitable insofar as it causes firms to cancel positive NPV projects. Such manipulation is possible because there is uncertainty about whether speculators are informed. Brunnermeier and Pedersen (2005) study a model where an institutional friction (the inability to meet margin calls) can trigger price-contingent trading by large traders that are financially constrained. As a result, rational speculators can (short-)sell the asset, to trigger further sales and then buy later at a lower price, with a riskless profit. In these papers, uninformed trading and successful stock price manipulation stem from the same source. In Goldstein and Guembel (2008) it is the feedback effect between stock prices and real activity; in Brunnermeier and Pedersen (2005) it is the presence of margin calls. By contrast, our paper highlights that uninformed trading and stock price manipulation need not be inextricably intertwined. While we explicitly allow for the possibility of price manipulation, we show that it is not an equilibrium in our setting with rational unconstrained investors, while uninformed trading itself is rational and profitable. More broadly, our results demonstrate that price-contingent trading does not make uninformed investors the inevitable prey of (potentially informed) speculators. On the other hand, to the extent that stock-price manipulation is to succeed, our results highlight that it does need additional features such as for example feedback effects or institutional frictions, as shown in the aforementioned papers.

More broadly, our paper relates to the literature on the incentives to trade in financial assets (see O'Hara 1995, Brunnermeier 2001 and Vives 2008 for reviews). In particular, a subset of this literature studies whether past prices contain useful information for a rational

trader (e.g. Brown and Jennings 1989, Kyle and Xiong 2001). In these papers there are no excess returns from uninformed trading or gains require additional features of preferences, beliefs or other frictions that are not necessary in our setup. We highlight that in addition to learning from prices, a sufficient condition that leads to profitable price-contingent trading is uncertainty about whether large traders that have market power are informed.

Finally, Grossman (1988), Gennotte and Leland (1990), Jacklin et al. (1992) and Romer (1993) address the crash of October 1987. A common theme in this literature is that market participants are assumed to have strongly underestimated the extent of positive-feedback trading (and specifically of portfolio insurance) in the market. While our model is compatible with the above features of price-destabilizing speculation, the main difference is that in this literature positive-feedback trading is exogenously assumed while we derive it endogenously.

2 Setup

The model is in the spirit of Kyle (1985) and Holden and Subrahmanyan (1992), where a risky asset is traded by large risk-neutral agents who internalize their market impact. The liquidation value of the risky asset, θ, is drawn from a prior distribution $\theta \sim N\left(p_0, \sigma^2\right)$. To capture traders' short-run incentives, we assume that there are two trading rounds, in date 1 and date 2 such that the equilibrium prices in the first trading round can reveal information in the second trading round. The liquidation value, θ, is realized in date 3.

Large traders are rational and place market orders before knowing the execution price. To keep our setting directly comparable to the benchmark models in this literature, we focus on a setting where there are always some large traders with private information present in the market. Namely, we assume that there are $N-1$ large traders, who as in Kyle (1985), always learn the value of θ before date 1, where $N \geq 2$ and finite. We call these traders type "K" traders. As it will be shown in Section 5, whether $N=2$ or $N>2$ affects some of our results.

We are particularly interested in the trading incentives of other large traders, who may or may not be informed. Without loss of generality, we assume that there is one such trader, trader "P", who becomes informed about the value of the fundamental, θ, before date 1 with probability η and remains uninformed about it with probability $1-\eta$. All agents know η. Before date 1 P observes the realization of his type (the state), $R \in \{I, U\}$, where "I" ("U") indicates that P is informed (uninformed).

We are particularly interested in P's trading strategy in case he turns out to be uninformed, i.e., $R=U$. As he does not observe θ, he can only base his trading decision on the prior and on the information revealed in past asset prices in that state. As it will become

clear shortly, P will never trade in date 1 when he only knows the prior. However, in date 1 he learns new information from p_1 and can in principle set up a price-contingent strategy for date 2. The main questions we address are whether and when any such strategy can be profitable in a rational setting, and what is the impact of such strategies on asset prices.

In addition to rational investors, there are noise traders who demand a random quantity $s_1 \sim \mathcal{N}(0, \sigma^2)$ and $s_2 \sim \mathcal{N}(0, \sigma^2)$, uncorrelated with the noise in the prior, in both date 1 and 2.[4] The presence of noise traders is necessary to prevent the order flow from fully revealing the fundamental.[5]

As in Kyle (1985), the market is efficient by construction. To implement the market efficiency condition, we introduce a hypothetical agent, called the Market[6], who observes only publicly available information. Namely, he cannot tell how much of the total order flow, y for $t \in \{1, 2\}$, comes from each type of investor: K, P and noise traders. Furthermore, the Market does not know if P is informed or not, but knows the ex ante probability η of P's type. This assumption is crucial and captures the fact that an average market participant does not know whether all large traders are informed. The market efficiency condition is

$$p \ = \mathbb{E}\left[\theta|\Omega \quad \right], \tag{1}$$

where Ω is the information set available to the Market in $t \in \{1, 2\}$, specified shortly.

To stack the cards against price-contingent strategies, we assume that type K traders know whether P is informed or not. There are a number of reasons to make that assumption. First, it avoids the situation in which P would in any state have superior information compared to other large traders[7]. This allows us to identify clearly that the source of P's gains from price-contingent strategies is his information advantage compared to the Market and not necessarily compared to the other large traders. Second, this assumption allows for the presence of one or more agents, who in principle can take an opposite position to the price-contingent strategy, thereby potentially neutralizing it. This is realistic for some price-contingent strategies (e.g. stop-loss and take profit rules) that are executed through

[4]The normality assumption is standard in the rational expectations literature as it keeps the model analytically tractable and the simplest.

[5]This is equivalent to the argument in Grossman and Stiglitz (1980) about the necessity of noise for the existence of an informationally efficient asset market. Exogenous noise trade is just a convenient technical assumption and can be easily endogenized in various ways, however without additional insights for our goal.

[6]In models based on Kyle (1985), this agent is frequently referred to as the "market maker". We prefer to call him the Market to emphasize that he proxies for the information observed by the whole market, as opposed to any individual broker. The market efficiency condition (1) can therefore also be interpreted as the equilibrium outcome of a large number of small agents, potentially playing mixed strategies.

[7]When P is informed, he knows both θ and the market structure (exacty N informed traders). If type K traders would not know P's type, P would be the most informed agent in the market.

a broker who knows that the order is price-contingent. Finally, it simplifies the solution without altering any of our main results.[8]

More formally, let us denote the n'th type K trader as K . In state $R \in \{I, U\}$, a trader $J \in \{K_1, ..., K \quad _{-1}, P\}$

$$\max_{\substack{RJ \\ 1}} \mathbb{E}\left[h_1 \quad (\theta - p_1) + \tilde{1} \cdot h_2 \quad (\theta - p_2) \,|\Omega_1 \quad \right] \tag{2}$$

$$\max_{\substack{RJ \\ 2}} \mathbb{E}\left[h_2 \quad (\theta - p_2) \,|\Omega_2 \quad \right],$$

where p is price in date $t \in \{1, 2\}$, and h and Ω are the demand and information set of trader J in state R in date $t \in \{1, 2\}$, respectively. Additionally, $\tilde{1}$ is an indicator function that take value zero or one: $\tilde{1} = 0$ if large traders are not aware in date 1 that they will have a trading opportunity also in date 2 and $\tilde{1} = 1$ if large traders know in date 1 that they will have a trading opportunity in date 2. In the basic setting of Section 3, we consider $\tilde{1} = 0$ and in the forward-looking case of Section 5, we consider $\tilde{1} = 1$.

If P is informed, then both type K and P traders have the same information set, i.e., $\Omega_1 \quad ^n = \Omega_1 \quad = \{\theta, p_0, \sigma \ , \sigma \ , R = I, \eta\}$ and $\Omega_2 \quad ^n = \Omega_2 \quad = \{\theta, p_0, p_1, \sigma \ , \sigma \ , R = I, \eta\}$ for $n \in \{1, ..., N-1\}$. If P is uninformed, then only type K traders know the fundamental, i.e., $\Omega_1 \quad ^n = \{\theta, p_0, \sigma \ , \sigma \ , R = U, \eta\}$ and $\Omega_2 \quad ^n = \{\theta, p_0, p_1, \sigma \ , \sigma \ , R = U, \eta\}$ for $n \in \{1, ..., N-1\}$ and $\Omega_1 \quad = \{p_0, \sigma \ , \sigma \ , R = U, \eta\}$ and $\Omega_2 \quad = \{p_0, p_1, \sigma \ , \sigma \ , R = U, \eta\}$.

The Market does not know the fundamental and P's type. He nevertheless obtains information about both by observing the total order flow submitted by all types of traders in date $t \in \{1, 2\}$,

$$y \ = \begin{cases} h \quad + h \quad + s & \text{if } R = I \\ h \quad + h \quad + s & \text{if } R = U \end{cases} . \tag{3}$$

Information set available to the Market in date 1 and 2 respectively is $\Omega_1 \quad = \{p_0, y_1, \sigma \ , \sigma \ , \eta\}$ and $\Omega_2 \quad = \{p_0, y_1, y_2, \sigma \ , \sigma \ , \eta\}$. We can then use the law of total expectations to express the market efficiency condition (1) as

$$p \ = Q \ \mathbb{E}\left[\theta|\Omega \quad , R = I\right] + (1 - Q \ ) \, \mathbb{E}\left[\theta|\Omega \quad , R = U\right], \tag{4}$$

where $Q \ = \Pr\left(R = I|\Omega \quad \right)$. Figure 1 illustrates the timing of events.

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[8] We have solved the basic model and the fully rational setting in the case where type K traders do not know P's type. We find that the results remain robust for every $0 \leq \eta < 1$. These results are available upon request.

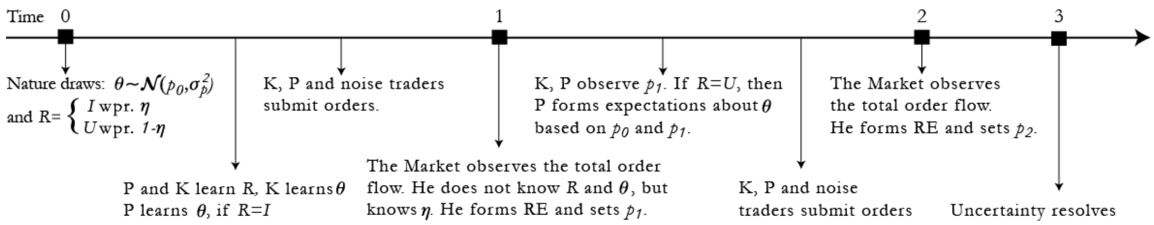

Figure 1: Timing of events

3 Basic model

We start by analysing a simplified setting under two additional assumptions compared to Section 2.

Assumption 1: The Market updates his beliefs about the fundamental (θ) based on the total order flow, but does not update his beliefs about P's type (R), i.e., he takes $Q_1 = Q_2 = \eta$.

Assumption 2: Type K traders and P submit orders in date 1 without knowing that they also have an opportunity to trade in date 2, i.e., $\tilde{1} = 0$.

This basic model is analytically tractable and allows us to illustrate the main effects in the simplest possible setting. The full solution is presented in Appendix A. There are some important issues to note regarding these assumptions.

3.1 Assumption 1 and linearity of equilibrium

Under Assumption 1 we can solve for the unique linear rational expectations equilibrium in pure strategies, as it is standard in this literature. The only non-standard assumption in our setting is that the type (U or I) of one trader is not known with certainty. If his type were known, the linearity of equilibrium would simply follow from the fact that all relevant random variables are normally distributed, as in Kyle (1985).

In our setting there are two possible states, $R \in \{I, U\}$, and in general the total order flow potentially reveals information about both, the fundamental, θ, and P's type, R. The market clearing condition (4) would generally lead to prices that are non-linear in total order flows, y , because both Q and $\mathbb{E}\left[\theta|\Omega \quad, R\right]$ depend on the total order flow. As a result, traders' strategies would become non-linear in random variables and the problem would not be analytically solvable.

3.2 Assumption 2 and dynamic incentives

Assumption 2 eliminates the strategic motives of large traders to hide private information from uninformed P or the Market, or to attempt to confuse them by trading in the opposite direction from the fundamental in the first trading period to gain in the second trading period. We relax this assumption in Section 5.

We consider only the extreme possibilities of $\tilde{1} = 0$ and $\tilde{1} = 1$ in (2) mainly for clarity of argument. A more realistic setting with trading costs, e.g., uncertainty about future trading opportunities, would deliver results in between these two extreme cases.

3.3 Optimal price-contingent trading

Under Assumptions 1 and 2 we find that uninformed P optimally pursues a positive-feedback strategy.

Proposition 1 If P is uninformed, i.e. $R = U$, but the Market does not know whether he is informed or not and attaches a probability $\eta > 0$ on him being informed, then P's optimal demand in date 1 is zero, $h_1 \quad = 0$ and he trades in the direction of past price movements in date 2

$$h_2 \quad = \frac{-}{P_2}\left(p_1 - p_0\right), \text{ where } e - c > 0 \text{ and } \partial\left(e - c\right)/\partial\eta > 0. \tag{7}$$

The equilibrium prices are

$$p_1 = p_0 + \frac{-1}{}\left(\theta - p_0\right) + \lambda_1 s_1 \text{ and } p_2 = p_0 + \frac{N-1}{N}\frac{N+\nu}{N}\left(\theta - p_0\right) + \frac{\nu}{N}\lambda_1 s_1 + \lambda_2 s_2 \tag{8}$$

where $\lambda_1 \equiv \sqrt{\frac{(\quad -1)}{2}\frac{e_p}{e_s}\frac{1}{\sqrt{N}}}$, $\lambda_2 \equiv \sqrt{\frac{(\quad -1)}{3}\frac{e_p}{e_s}\frac{1}{\sqrt{R}}}$, $\kappa > 1$, $\mu > 1$, $\partial\kappa/\partial\eta > 0$, $\partial\mu/\partial\eta > 0$, $e = \frac{N}{1+N(\quad -1)} > 1$, $0 < c \leq 1$ and $\nu \equiv \frac{\quad + \quad}{\quad +1} > 1$. All constants κ, μ, e, c and ν are uniquely determined by η and it holds that $\lim_{K \longrightarrow 1} \kappa = \frac{(\quad -1)(\quad +1)^2}{3} > 0$ and $\lim_{K \longrightarrow 1} \mu = \frac{(1+\quad)^2\left(\quad^2 -1\right)}{4} > 1$.

Proof. See Appendix B. ■

The most striking implication of Proposition 1 is equation (7) that shows that P always trades in the direction of the past price movement, $p_1 - p_0$, provided he is uninformed. This result is driven by the assumption that P's type is not known with certainty. In particular, we can contrast it to standard rational expectations models, where the types of all traders are known.

Corollary 2 If $R = U$ and $\eta = 0$, optimal demand by the uninformed trader P is $h_1 = h_2 = 0$ and equilibrium prices are

$$p_1 = p_0 + \frac{-1}{}(\theta - p_0) + \hat{\lambda}_1 s_1 \text{ and } p_2 = p_0 + \frac{N^2 - 1}{N^2}(\theta - p_0) + \frac{1}{N}\hat{\lambda}_1 s_1 + \hat{\lambda}_2 s_2, \qquad (9)$$

where $\hat{\lambda}_1 \equiv \sqrt{\frac{(\ -1)}{2}\frac{e_p}{e_s}}$, $\hat{\lambda}_2 \equiv \sqrt{\frac{(\ -1)}{3}\frac{e_p}{e_s}}$.

Proof. Appendix A shows that if $\eta = 0$ then $\kappa = 1$, $\mu = 1$ and $c = 1$. As a result, $e = \frac{}{1+(\ -1)} = 1$ and $\nu \equiv \frac{+}{+1} = 1$. Replacing these into (8) we obtain (9). ■

Contrasting Proposition 1 to Corollary 2 highlights that the profitability of P's price-contingent strategy arises because there is uncertainty about P's type. Without such uncertainty P we obtain the same "no trade" result as Easley and O'Hara (1991).

In particular, P pursues a positive-feedback strategy, because the the Market sets the sensitivity to the order flow too low, i.e., $\lambda_1 < \hat{\lambda}_1$ because $\kappa > 1$. Due to his superior information about his own type, P's expectations differ from the Market's expectations, $\mathbb{E}\left[\theta|\Omega_1\ \right] \neq \mathbb{E}\left[\theta|\Omega_2\ \ \right]$. In particular, note that by the efficient market assumption period 1 price reflects the Markets' expectations based of the information revealed by the order flow, i.e., $p_1 = \mathbb{E}\left[\theta|\Omega_1\ \right]$. In period two P obtains information from two sources: he knows the prior, $\theta \sim N\left(p_0, \sigma^2\right)$ and observes a price signal $\tilde{p}_1 \equiv \frac{}{-1}(p_1 - p_0) + p_0$, where $\tilde{p}_1|\theta \sim N\left(\theta, \left(\frac{}{-1}\right)^2 \lambda_1^2 \sigma^2\right) \sim N\left(\theta, \frac{1}{-1}\sigma^2 \frac{1}{N}\right)$. Using Bayes' rule we find that $\mathbb{E}\left[\theta|\Omega_2\ \ \right] = p_0 + \frac{}{-1}\frac{(\ -1)N}{e_p^2}\left(\frac{1}{e_p^2} + \frac{(\ -1)N}{e_p^2}\right)^{-1}(p_1 - p_0) = p_0 + \frac{N}{1+(\ -1)N}(p_1 - p_0) = p_0 + e\,(p_1 - p_0) > p_1 = \mathbb{E}\left[\theta|\Omega_1\ \right]$. The last inequality also follows from $\kappa > 1$ (which implies $e > 1$). As P expects the Market to systematically underreact to the fundamental, he expects to systematically profit from price-contingent trading.

We can analyse this further and distinguish the two driving forces discussed in the Introduction. This is easiest to see in the limit where $\eta \longrightarrow 1$, when the Market expects P to be almost certainly informed. In such a case, he believes that the total order flow is driven by N informed traders who each demand $\frac{1}{(\ +1)P_1}(\theta - p_0)$ and noise trading (see Appendix A). Hence, his signal from the total order flow is $\tilde{y}_1 \equiv \frac{+1}{}\lambda_1 y_1 + p_0$, where $\tilde{y}_1|\theta \sim \mathcal{N}\left(\theta, \left(\frac{+1}{}\right)^2 \lambda_1^2 \sigma^2\right)$. Given (6), we can write the price signal that P observes as $\tilde{p}_1 \equiv \frac{}{-1}\lambda_1 y_1 + p_0$ and $\tilde{p}_1|\theta \sim \mathcal{N}\left(\theta, \left(\frac{}{-1}\right)^2 \lambda_1^2 \sigma^2\right)$. In addition to these signals, uninformed P and the Market observe the same prior, hence differences in their expectations are driven by their differences about interpreting the order flow/price signal. Furthermore, both the Market's and P's expectations of the fundamental are weighted average of the prior and the order flow and price signal, respectively. As the latter signals have different mean and variance, P's and the Market's expectations differ because of the value of the signals as well

as because of the weight they put on the prior as compared to the order flow/price signal. This highlights the main forces. First, the same change in prices indicates a higher fundamental to uninformed P than the corresponding order flow does to the Market, i.e., $\tilde{p}_1 > \tilde{y}_1$ because $\frac{\;^{+1}}{} < \frac{}{\;_{-1}}$. This force pushes P towards expecting the fundamental to be higher than the period 1 price and thus towards pursuing a positive feedback strategy. Second, the Market overestimates the precision of the information revealed by the order flow, i.e., $\mathrm{Var}\left(\tilde{y}_1|\theta\right) < \mathrm{Var}\left(\tilde{p}_1|\theta\right)$ because $\frac{\;^{+1}}{} < \frac{}{\;_{-1}}$. As a result, the Market tends to put too much weight on the order flow signal when forming his expectations. This force pushes P towards expecting a lower fundamental when forming his expectations, and thus to trade in contrarian manner. In the basic setting, the first force dominates and leads to a positive-feedback trading.

From this argument, it is also easy to see why there is no profitable uninformed trading if P's type is known. In such a case P and the Market have exactly the same information and their expectations coincide.

While we emphasise P's trading incentives when he is uninformed, it is clear that he also earns excess returns if he is informed. In such a case both K and P traders earn positive excess returns, while the Market tends to set λ_1 to be too high.

Finally, a question might arise, why do K traders not neutralize the price contingent trading by P? After all, they perfectly know P's type and trade. The main reason is that K has conflicting incentives. On the one hand, K has an incentive to trade against the uninformed P. On the other hand, K has still an incentive to trade in the direction of the fundamental. Therefore, if the change $p_1 - p_0$ was due to noise trading only (i.e. $\theta = p_0$, $s_1 \neq 0$), K takes an opposite position against P. By contrast, if the change $p_1 - p_0$ was due to the fundamental (i.e. $\theta \neq p_0$, $s_1 = 0$), K has an incentive to trade in the same direction as uniformed P. The latter effect limits K's ability to always neutralize P's trade. We ask next whether this strategy is stabilizing or not for asset prices.

3.4 Do price-contingent trading rules stabilize prices?

Given that there is uncertainty about P's type, it should be incorporated when addressing the pure impact of a price-contingent trading rule. To address this, suppose that P trades only if he is informed and does not trade if he is uninformed. While it is not optimal as shown in the previous Section, it provides a useful benchmark. This benchmark keeps the uncertainty in the economy exactly the same in both the basic model and the benchmark.

Proposition 3 If P is uninformed, i.e. $R = U$

P being informed, and P does not trade when uninformed, then benchmark prices are

$$p_1 = p_0 + \frac{\quad -1}{\ } (\theta - p_0) + \lambda_1 s_1 \text{ and } p_2 = p_0 + \frac{\quad -1}{\ }\frac{\quad +\bar{U}}{\ } (\theta - p_0) + \frac{\bar{\nu}}{N}\lambda_1 s_1 + \lambda_2 s_2, \tag{10}$$

where $\lambda_1 \equiv \sqrt{\frac{(\quad -1)}{2}\frac{e_p}{e_s}\frac{1}{\sqrt{N}}}$, $\lambda_2 \equiv \sqrt{\frac{(\quad -1)}{3}\frac{e_p}{e_s}\frac{1}{\sqrt{R}}}$, $1 \leq \bar{\nu} < \nu$ for $\eta \in (0, 1]$ and $\bar{\nu} = \nu = 1$ for $\eta = 0$.

Proof. See Appendix C. ∎

In date 1, the prices in the basic setting and in the benchmark case are the same. This is straightforward, because P's demand in date 1 is zero in both cases. In order to assess whether the positive feedback strategy is stabilizing (destabilizing), we look at whether prices are closer to (further away from) the fundamental in period 2.

First, suppose that noise trading is at its mean, $s_1 = s_2 = 0$, and the fundamental differs from the prior, $\theta \neq p_0$. Comparing Proposition 1 and Proposition 3 shows that the optimal positive-feedback trading strategy brings date 2 prices closer to the fundamentals ($\bar{\nu} < \nu$). The reason for this is higher competition between large traders who all trade in the direction of the fundamental in the absence of a noise trading shock. We can conclude that positive-feedback trading is stabilizing if there are shocks to fundamental.

Second, suppose that $\theta = p_0$, but there is a noise trading shock in date 1, $s_1 \neq 0$. We can see that a noise trading shock in date 1 has more persistence because of the positive-feedback trading ($\bar{\nu}\lambda_1 < \nu\lambda_1$). As a result, following a pure noise trading shock, prices stray further away from fundamentals. This suggests that positive-feedback trading strategies are destabilizing when there is a noise trading shock. Given that the overall uncertainty is the same in the basic model and the benchmark and there are only two trading dates, a date 2 noise trading shock has the same impact in both cases.

To assess the overall impact of the trading rule, we look at the unconditional mean squared error of the difference between asset price and fundamental. From (8) and (10)

$$\mathbb{E}\left[(p_2 - \theta)^2\right] = \mathrm{Var}\,(p_2 - \theta) = \frac{(q-2)^2}{N^4}\sigma^2 + \frac{q^2}{N^4\kappa}\sigma^2 + \frac{1}{N^3\mu}\sigma^2, \text{ where } q \in \{\bar{\nu}, \nu\}.$$

From here, we find that $\mathbb{E}\left[(p_2 - \theta)^2\right]\Big|_{\ =U} > \mathbb{E}\left[(p_2 - \theta)^2\right]\Big|_{\ =\bar{U}}$ and the mean squared error is higher in the basic setting than in the benchmark case.[10] This finding implies that the pure

[10] Notice that $\partial\mathbb{E}\left[(p_2 - \theta)^2\right]/\partial q = -\frac{\sigma_p^2}{N^3}\left(2 - q\left(\frac{1}{\kappa} + 1\right)\right) < 0$ if $\frac{\kappa N}{1+\kappa(N-1)} > q$. Given the definition of $e = \frac{\kappa N}{1+\kappa(N-1)}$, (see Proposition 1), this condition can be written as $q < e$. Given that $q \in \{\bar{\nu}, \nu\}$ and $\nu > \bar{\nu}$, a sufficient condition for $\partial\mathbb{E}\left[(p_2 - \theta)^2\right]/\partial q < 0$ is $\nu < e$. From Proposition 1, $\nu = \frac{e+Nc}{N+1}$. Using this, $\nu < e$ iff $c < e$ that holds by Proposition 1 for any $\eta > 0$.

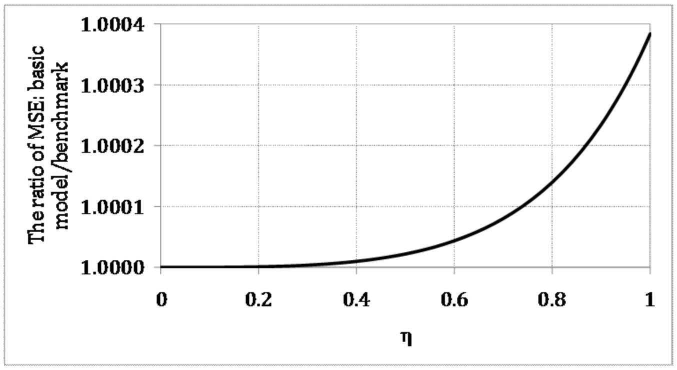

Figure 2: The ratio of mean squared error (MSE), $\mathbb{E}\left[(p_2-\theta)^2\right]$, in the basic model to MSE in the benchmark as a fundtion of η.

overall effect of optimal positive-feedback trading is destabilizing. On average, the stabilizing effect from faster convergence to the fundamentals is more than offset by the destabilizing effect of over-reaction to the noise trading shock.

Figure 2 plots $\frac{\mathbb{E}\left[(\ _2-L)^2\right]\big|_{\nu=\nu_\eta}}{\mathbb{E}\left[(\ _2-L)^2\right]\big|_{\nu=\bar{\nu}_\eta}}$ for all values of $\eta=[0,1]$. It is clear that the difference in the mean-squared error between the basic model and the benchmark increases in the probability that P is informed, η. Intuitively, the less widespread the Market perceives the positive-feedback trading to be, the more destabilizing it is at times when it is implemented.

4 Updating P's type

In this Section we relax Assumption 1 and allow the Market to update P's type. While this problem is not analytically solvable, we can still explore some of its theoretical properties. Furthermore, by focusing on a situation where updating should have the largest effect, we can assess whether and how much allowing updating alters our results. In this section we maintain Assumption 2.

4.1 Some theoretical properties of the model with updating.

The main finding in our basic model, price-contingent trading is profitable for a large uninformed trader, arises because of two related factors. First, P's type remains unknown, by Assumption 1. Second, because of that P can interpret information that is revealed in period 1 order flow better than the Market. (While P learns from the price, there is a one-to-one relationship between p_1 and y_1). The reason is that given the fundamental, θ, the total demand by informed traders in period 1 is always different in states $R=I$ and $R=U$. This

generates an information advantage for any trader who knows the true state R. Because P's superior information in period 2 follows directly from the equilibrium outcome of the first trading round, in this section we focus on period 1 equilibrium.

While the problem does not have a closed form analytical solution, we prove the basic properties of the equilibrium. Without loss of generality, we normalize the prior mean, $p_0 = 0$, to save notation.

Proposition 4 When the Market does not know P's type, but updates his beliefs about P's type, there exists a rational expectations equilibrium in period 1 with the following properties
1) the Market does not learn P's type perfectly,
2) only informed traders trade and follow strategy $g_1 \ (\theta)$ in state $R = U$, where $g_1' \ > 0$ and $-g_1 \ (-\theta) = g_1 \ (\theta)$, and strategy $g_1 \ (\theta)$ in state $R = I$, where $g_1' \ > 0$ and $-g_1 \ (-\theta) = g_1 \ (\theta)$,
3) for any nonzero value of θ, the total order ‡ow is always di¤erent in states $R = I$ and $R = U$,
4) the equilibrium price is an increasing function of the order ‡ow, i.e., $p_1 = \Lambda(y_1)$, where $\Lambda' > 0$, and $-\Lambda(-y_1) = \Lambda(y_1)$.

Proof. See Appendix D ■

The Proposition 4 establishes that the main properties of the equilibrium we derived under Assumption 1 are all maintained once we relax Assumption 1. Crucially the Market cannot perfectly learn P's type. Essentially he updates the probability that P is informed based on the order flow. The reason why the order flow is informative about P's type (state R) is that the distribution of the order flow is different in different states (i.e., it has a different variance). Because the order flow is a monotonic function of its underlying random variables (θ and s_1) that are drawn from distributions with infinite support, then it follows that also y_1 is drawn from a distribution with infinite support. Therefore, it only gives a very noisy signal about the state R. Note also that learning about the state does not need to be always in the "correct direction". This is because any learning depends solely on the Market's knowlegde of the shape of the distribution of y_1 in the different states. Therefore, the more "unusual" realizations of θ would make the Market update the probability that $R = I$ away from the true one. Intuitively, our findings in the basic setting suggest that a higher order flow in absolute value always indicates a higher probability that there are two informed traders (in the basic setting $y_1|R = I$ has always higher variance than $y_1|R = U$). Hence, the higher is θ in absolute value, the more likely is the Market to update the probability that P is informed upwards.

Parts 2 and 4 of Proposition 4 state that the main properties of traders' strategies and the pricing rule of the Market remain unaltered. While both are non-linear, they remain monotonic functions of θ and y_1, respectively, and symmetric around the prior mean (which we normalized to zero).

Most crucially, part 3 establishes that the informativeness of the order flow depends on the state R. Therefore, by knowing the state P continues to have an information advantage compared to the Market.

We can now turn to the quantitative properties of the model with updating.

4.2 Quantitative effects of updating P's type.

While we can derive and prove some theoretical properties of the equilibrium, it is not analytically solvable beyond that. Therefore, in order to gain additional insights about the effect of updating, we need to look at it numerically. We are particularly interested in the situation where updating is most likely to have a substantial effect and compare it to the basic model.

In order to focus on the case that is the least favourable to us, we consider the case where there is only one type K trader, i.e., $N = 2$. This is because any effect that P has on the equilibrium outcomes is highest when his market power is the highest. In Appendix D, we also derive that the probability that P is informed conditional on all the information that the Market has is

$$Q_1 = \frac{\eta f\left(y_1 | R = I\right)}{\eta f\left(y_1 | R = I\right) + (1 - \eta) f\left(y_1 | R = U\right)}.$$

It is immediate that at the limits $\eta \to 0$ and $\eta \to 1$, there is no updating and the equilibrium is the same as in the basic model (and linear). Therefore, any effects from updating must be largest when η is at some intermediate value and we focus on the case where the prior probability that $R = U$ is $\eta = 1/2$. Finally, because updating P's type is purely based of the total order flow, it is also clear that updating must be easier when the noise trading variance is low. This is because the total order flow is the sum of orders by K and P and noise traders. The latter is the same irrespectively of whether P is informed of not. So the differences in the distributions $f\left(y_1 | R = I\right)$ and $f\left(y_1 | R = U\right)$ must be largest if there is little noise trading. We will therefore look explicitly at how low noise trading variance affects the results.

We base our analysis on the expressions derived in Appendix ???. The expression for the equilibrium price is a function of informed traders trading strategies $g_1\ \ (\theta)$ and $g_1\ \ (\theta)$ (see Proposition 4). To obtain any numerical solution we need to approximate these strategies with a polynomial. A good starting point is to consider the linear strategies as in the basic

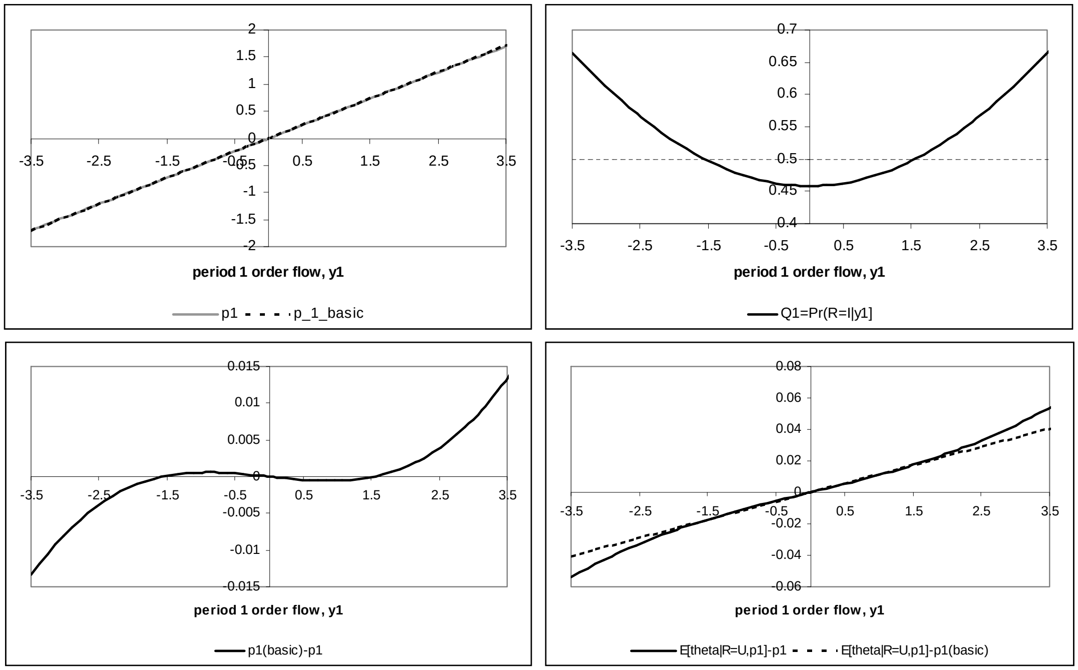

Figure 3: Comparison of equilibrium prices and uninformed P's expectations in basic setting and in the model with updating assuming noise trading variance $\sigma^2 = 1$.

setting. If the resulting price function $p_1 = \Lambda(y_1)$ appears close to linear then also the true equilibrium strategies are likely to be close to linear. Note that informed traders strategies mostly rely on the fundamental itself and drive most of the order flow, while P's trading stategy in period 2 is driven by more subtle effects.

In Figure 3 we consider the case where the informed traders strategy is the same as in the basic model. We assume that $\eta = 0.5$, $\sigma^2 = 1$ and $\sigma^2 = 1$. The figure on top left plots the resulting price against the order flow, i.e. $p_1 = \Lambda(y_1)$. We can see that the relationship between the price and the order flow is very close to linear and very similar to the one we obtain in the basic setting for the same parameter values. Looking at the difference between the price in the case with updating and the price in the basic model we see that there are small differences and the price is nonlinear due to updating. Nevertheless as the informed traders' strategy is driven by information about θ and they trade relatively large quantities, their optimal strategy would remain approximately the same as in the basic setting.[11]

The top right plot on Figure 3 plots the updated probability of $R = I$, Q_1. We can see that if the order flow is small in absolute value then the Market updates the probability that P is informed downwards, while if it is large he updates it upwards. That highlights

[11] Approximating informed traders optimal strategies with higher order polynomial lead to coeficients of θ^2 and higher to be very close to zero.

that not only there is never an order flow that leads to the Market to perfectly learn that P is uninformed, in the case that the order flow is large the Market updates P's type in the direction that is favourable to P's price-contingent trading strategy. This is particularly important in the case where the high order flow is driven by a high fundamental. Intuitively, this strengthens our findings in the basic setting in the case that P is uninformed and there is

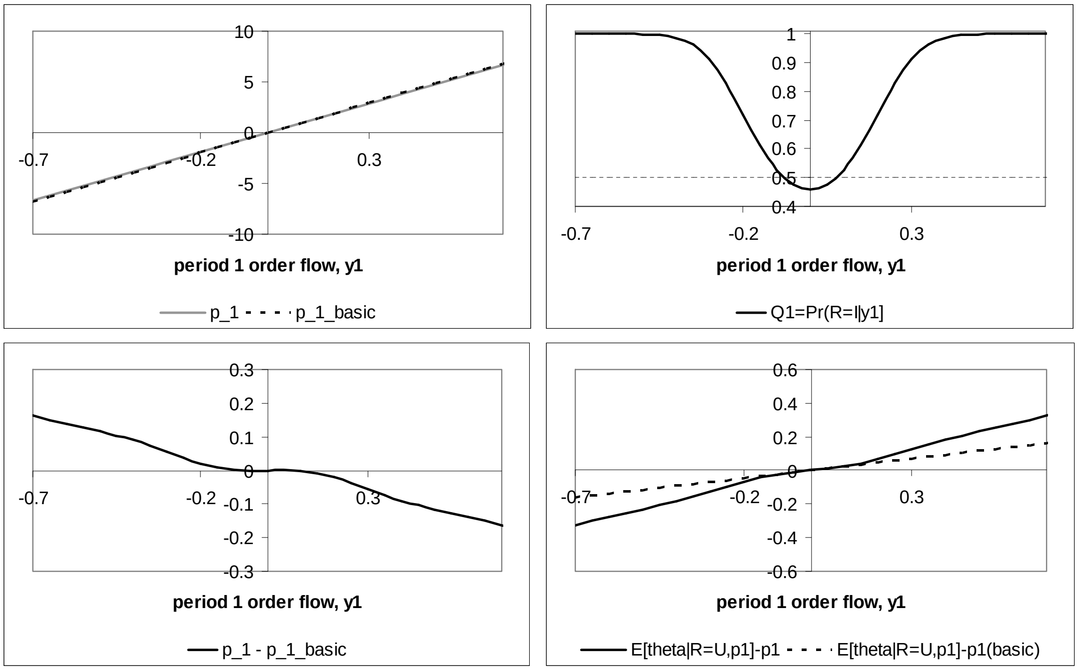

Figure 4: Comparison of equilibrium prices and uninformed P's expectations in basic setting and in the model with updating assuming noise trading variance $\sigma^2 = 0.0025$

of the model is presented in Appendix E. This setting is more directly comparable to Kyle (1985) than the basic setting in Section 3. As argued by Kyle (1985), a monopolistic informed trader has an incentive to hide his information and trade a smaller quantity in the earlier period. In contrast, Holden and Subrahmanyan (1992) show in a similar dynamic setting with Cournot competition between (even two) informed traders, that the ability of informed traders to hide information is limited and they trade more aggressively in early trading periods. Whether there is an information monopolist and Cournot competition between informed traders turns out to be crucial for the direction of price-contingent trading.

Nevertheless the main finding that the price-contingent trading is optimal continues to hold also in the forward-looking case

Proposition 5 If P is known to be uninformed with certainty, i.e., $R = U$ and $\eta = 0$ then P demands zero in both trading periods. If there is uncertainty about P's type, i.e., $\eta > 0$ and P is uninformed $R = U$ then $\mathbb{E}\left[\theta|\Omega_1\right] \neq \mathbb{E}\left[\theta|\Omega_2\right]$ an P pursues an optimal price-contingent strategy.

Proof. See Appendix E.1. ■

Proposition 5 confirms the main result in this paper - price-contingent trading strategies are optimal in an environment where the types of large traders are not known with certainty.

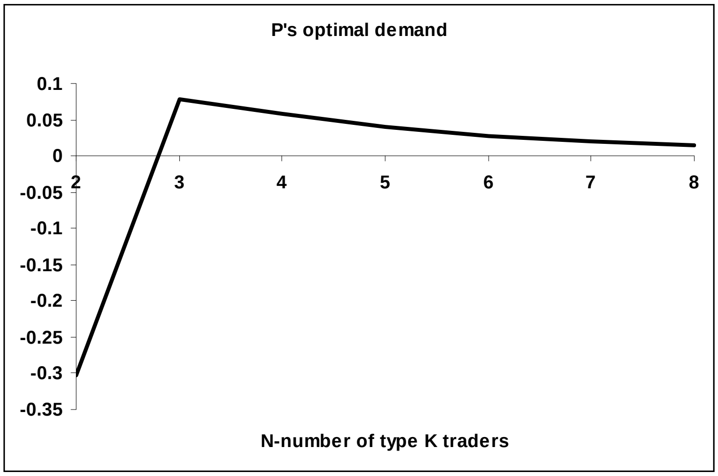

Figure 5: P's optimal demand at limit $\eta \rightarrow 1$

This gives P an information advantage compared to the Market as he can intepret the information revealed in prices more accurately than the Market (or small traders who do not have a price impact or know the types of large traders).

In contrast to the basic setting in Section 3 the price-contingent strategy is here no longer always positive-feedback. It crucially depends on whether there is a large information monopolist or Cournot competition among informed traders. To illustrate the dependence of the direction of optimal price-contingent strategy on the number of informed (type K) traders, Appendix E.2 focuses on the limit $\eta \rightarrow 1$, where the Market believes that P is almost certainly informed while P is actually uninformed. We find that if there is only one informed trader and P, i.e., $N = 2$ then P's optimal strategy is contrarian, while for any $N > 2$ the optimal strategy is positive-feedback as in Section 3. Figure 5 plots P's optimal demand in period 2[13] when $p_1 - p_0 = 1$.

To understand this result recall the discussion about the two forces that drive the differences in the Market's and P's expectations. The same two forces are present in this setting. First, P knows that the price signal that he observes has a higher mean than the order flow signal that the market observes. This force dominates for $N > 2$ as in the basic setting and pushes P towards pursuing an optimal positive-feedback strategy. Second, P knows that the Market underestimates the noise in the order flow signal and thus the market puts too much weight on the information revealed by the order flow when forming his expectations. This force dominates in $N = 2$ and in this case P's optimal strategy is contrarian.

The fact that the results are dramatically different in the presence of a forward-looking

[13] The figure plots $h_2^{UP} = \frac{(e-1)(p_1-p_0)}{(N+1)\lambda_2}$, where $\lambda_2 = L_2 \frac{\sigma_p}{\sigma_s}$ and e and L_2 are uniquely determined by N for $\eta \longrightarrow 1$. We normalize $\frac{\sigma_p}{\sigma_s} = 1$.

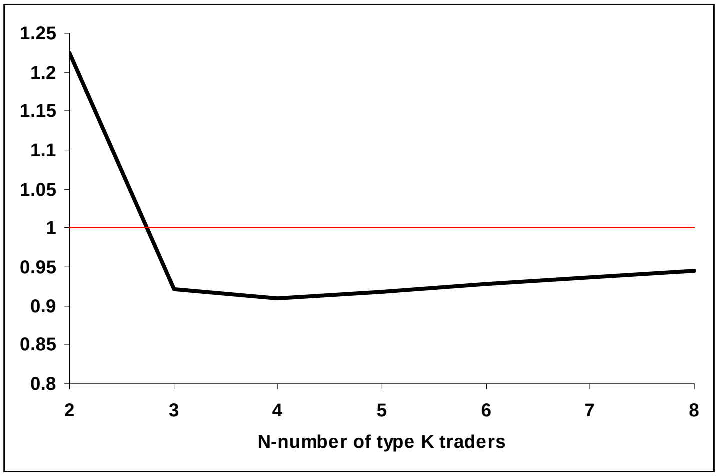

Figure 6: The ratio $\lambda_1/\hat{\lambda}_1$, where λ_1 is period 1 sensitivity of prices to order flow and $\hat{\lambda}_1$ is the sensitivity of prices to order flow when P is known to be uninformed ($\eta = 0$)

information monopolist is not surprising in the light of Kyle (1985) and Holden and Subrahmanyan (1992). An information monopolist has a strategic incentive to trade little to hide his information. By doing so, the information revealed by the order flow is very noisy. At the same time the Market weights a possibility of an aggressive competition between two informed traders and reacts too much on the order flow. Indeed Figure 6 shows that the Market sets the sensitivity of prices to order flow λ_1 too high if $N = 2$, while he sets it too low as in the basic setting for $N > 2$. When N increases, then the effect of uncertainty about P's type on the sensitivity of prices to the order flow as well as P's trading volume decrease. This is a straightforward outcome of increasing competition among large traders that limits any individual trader's profit opportunities.

These results remain robust to any value of η. Figure 6 plots P's optimal trading strategy for $N = 2$ and $N = 3$ as a function of η. As before, we consider a unit increase in prices $p_1 - p_0 = 1$. P's optimal strategy remains contrarian for $N = 2$ and positive-feedback for $N = 3$.

It is also worth pointing out that despite the fact that uninformed P's price-contingent trades are known to K traders, these traders never trade against their private information in period one to trigger a price contingent trade. This is due to the rational expectations framework - P's optimal trading stategy would change accordinly. As in the basic setting K traders do take an opposite position against P, prices change due to noise trading shock. Only in the case where there is an information monopolist, he strategically hides information from both the Market and P.

Finally we analyze here a setting where P can only become informed in period one.

Allowing P to become informed in period two and K not to know P's future type does not affect any of the results qualitatively.[14]

6 Conclusion

We have showed that price-contingent trading is in general the optimal strategy for large uninformed investors. This holds as long as there is uncertainty in the market about whether large traders are informed. We derive conditions under which the optimal trading strategy of uninformed large investors is positive-feedback or contrarian.

The explanation for our results is that a large uninformed investor has an information advantage with respect to the average market participants, because he always knows to what extent past price changes reflect his own trading, or lack thereoff. As a result, on observing a price increase, the large uninformed investor knows that it reflects the trades of few informed traders and therefore partial adjustment toward the fundamental. By contrast, average market participants still weigh the possibility that all large traders are informed, and the signal that average market participants can extract from past prices is less precise. Therefore, the large uninformed investor has an incentive to submit a price-contingent, positive-feedback trade, and earn a positive excess return.

At the same time, a large informed investor is also aware that past price movements are more likely prone to noise trading shocks, precisely because there are fewer informed traders, while average market participants still weigh the possibility of more informed traders and therefore overestimate the precision of past price movements. Therefore, the large uninformed investor has an incentive to submit a price-contingent, contrarian trade, and also earn a positive excess return.

The first effect prevails in almost all settings, with the exception of the case in which there is an information monopolist who can hide private information. In equilibrium, returns feature excess volatility, expected autocorrelation in returns is zero.

We show how uncertainty about market structure – whether and how many large traders are informed – generates incentives to trade in financial markets because it implies that different market participants endogenously interpret the same public signal differently. Our mechanism can represent a microfoundation, in a rational expectations framework, of the existence and persistence of differences in beliefs between large and small investors. Our mechanism is also an example of a more general idea that sophisticated trading strategies can be driven by superior knowledge about the market environment rather than about the

[14] Extensions where P can become informed in date 2 and some K traders to not know P's type are available upon request.

fundamental.

A Solution of the basic model

From (2), Assumption 2 and (6), we find that in state $R = U$ large rational traders demand

$$
\begin{aligned}
h_1 \quad {}^{n} &= \frac{\theta - p_0}{N\lambda_1} - \frac{h_1}{N} \text{ for } n \in \{1, .., N-1\} \\
h_1 \quad &= \frac{\mathbb{E}\left[\theta | \Omega_1 \quad\right] - p_0}{2\lambda_1} - \frac{(N-1)\,\mathbb{E}\left[h_1 \quad {}^{n} | \Omega_1 \quad\right]}{2}
\end{aligned}
$$

Given that the only relevant information available for P is the prior, we find that $\mathbb{E}\left[\theta | \Omega_1 \quad\right] = p_0$ and $\mathbb{E}\left[h_1 \quad {}^{i} | \Omega_1 \quad\right] = \frac{\mathbb{E}\left[L | \Omega_1^{UP}\right] - \ \ _0}{P_1} - \frac{\ \ ^{UP}_1}{} = -\frac{\ \ ^{UP}_1}{}$, which implies that

$$
\begin{aligned}
h_1 \quad {}^{n} &= \frac{\theta - p_0}{N\lambda_1} \text{ for } n \in \{1, .., N-1\} \\
h_1 \quad &= 0
\end{aligned}
\tag{11}
$$

Using again (2), Assumption 2 and (6), we find that in state $R = I$ large rational traders demand

$$
h_1 \ {}^{n} = h_1 \quad = \frac{\theta - p_0}{(N+1)\,\lambda_1} \text{ for } n \in \{1, .., N-1\}\,. \tag{12}
$$

In date 2, let us make an additional conjecture that uninformed P's expectations are linear in prices, i.e.,

$$
\mathbb{E}\left[\theta | \Omega_1 \quad\right] = p_0 + e\,(p_1 - p_0)\,, \tag{13}
$$

where e is a constant to be determined in equilibrium. Using (2), Assumption 2, (6) and (13) we find that

$$
\begin{aligned}
h_2 \quad {}^{n} &= \frac{\theta - p_0 - c\,(p_1 - p_0)}{N\lambda_2} - \frac{h_2}{N} \text{ for } n \in \{1, .., N-1\} \\
h_2 \quad &= \frac{(e - c)\,(p_1 - p_0)}{(N+1)\,\lambda_2} \\
h_2 \ {}^{n} = h_2 \quad &= \frac{\theta - p_0 - c\,(p_1 - p_0)}{(N+1)\,\lambda_2} \text{ for } n \in \{1, .., N-1\}\,.
\end{aligned}
\tag{14}
$$

We can now find equilibrium prices. From (11) and (12) the total order flow in period 1

is

$$y_1 = \begin{cases} \frac{-1}{} \frac{L- \ _0}{P_1} + s_1 \text{ if } R = U \\ \frac{}{+1} \frac{L- \ _0}{P_1} + s_1 \text{ if } R = I \end{cases} .$$

We can define the Market's signal from the order flow in state $R = U$ as $\tilde{y}_1 \equiv \frac{}{-1}\lambda_1 y_1 + p_0$, where $\tilde{y}_1 \, |\theta \sim \mathcal{N}\left(\theta, \left(\frac{}{-1}\right)^2 \lambda_1^2 \sigma^2\right)$ and in state $R = I$ as $\tilde{y}_1 \equiv \frac{+1}{}\lambda_1 y_1 + p_0$, where $\tilde{y}_1|\theta \sim \mathcal{N}\left(\theta, \left(\frac{+1}{}\right)^2 \lambda_1^2 \sigma^2\right)$. In addition to these signals the market knows the prior that is also normally distributed, i.e., $p_0|\theta \sim \mathcal{N}\left(\theta, \sigma^2\right)$. Therefore, conditional on a state the signals are normally distributed. This implies that conditional on a state Market's expectations remain linear in the order flow and prior. Namely using Bayes' rule, we find that

$$\mathbb{E}\left[\theta|\Omega_1 \ , R = I\right] = \frac{\frac{1}{e_p^2} p_0 + \frac{1}{\left(\frac{N+1}{N}\right)^2 P_1^2 e_s^2} \tilde{y}_1}{\frac{1}{e_p^2} + \frac{1}{\left(\frac{N+1}{N}\right)^2 P_1^2 e_s^2}} = p_0 + \frac{\frac{1}{\left(\frac{N+1}{N}\right)^2 P_1^2 e_s^2} \frac{+1}{}}{\frac{1}{e_p^2} + \frac{1}{\left(\frac{N+1}{N}\right)^2 P_1^2 e_s^2}} \lambda_1 y_1 \tag{15}$$

$$\mathbb{E}\left[\theta|\Omega_1 \ , R = U\right] = \frac{\frac{1}{e_p^2} p_0 + \frac{1}{\left(\frac{N}{N-1}\right)^2 P_1^2 e_s^2} \tilde{y}_1}{\frac{1}{e_p^2} + \frac{1}{\left(\frac{N}{N-1}\right)^2 P_1^2 e_s^2}} = p_0 + \frac{\frac{1}{\left(\frac{N}{N-1}\right)^2 P_1^2 e_s^2} \frac{}{-1}}{\frac{1}{e_p^2} + \frac{1}{\left(\frac{N}{N-1}\right)^2 P_1^2 e_s^2}} \lambda_1 y_1 .$$

As under Assumption 1, the market efficiency condition is (5), and we confirm that in Basic setting the equilibrium price in date 1 remains linear as conjectured in (6). Replacing (15) into (5) and equating the coefficients with those in (6) we find that $\lambda_1^2 = \frac{-1}{2} \frac{e_p^2}{N e_s^2}$, where κ solves

$$\eta \left(\frac{\kappa \frac{3}{2-1}}{1 + \kappa \frac{4}{(\ +1)(\ ^2-1)}} - 1 \right) + (1 - \eta) \left(\frac{\kappa N}{1 + \kappa (N - 1)} - 1 \right) = 0. \tag{16}$$

Note that it must hold that $\kappa > 0$ and equation (16) **has only one real positive solution**.[15] Therefore, κ is uniquely determined by η and N. It holds that $\lim_{K \to 0} \kappa = 1$ and $\lim_{K \to 1} \kappa = \frac{(\ -1)(\ +1)^2}{3} = 1 + \left(N - \frac{1-\sqrt{5}}{2}\right)\left(N - \frac{1+\sqrt{5}}{2}\right) > 1$.

Claim 6 It also holds that $\kappa > 1$ for any $0 < \eta < 1$ and $\frac{N}{K} > 0$.

Proof. Let us prove that by contradiction. Suppose that $\kappa \leq 1$. We write (16) as $\eta K \ + (1 - \eta) K \ = 0$, where $K \ \equiv \left(\frac{N \frac{N^3}{N^2-1}}{1 + N \frac{N^4}{(N+1)(N^2-1)}} - 1 \right)$ and $K \ \equiv \left(\frac{N}{1 + N(\ -1)} - 1 \right)$. For $0 < \eta < 1$ it must hold that $\{K \ > 0 \text{ and } K \ < 0\}$ or $\{K \ = 0 \text{ and } K \ = 0\}$ or $\{K \ < 0 \text{ and } K \ > 0\}$.

[15] Equation (16) is a quadratic equation. From quadratic formula, it is clear that a quadratic equation in form $a_\kappa \kappa^2 + b_\kappa \kappa + c_\kappa = 0$ has a one real positive solution if $a_\kappa > 0$, $b_\kappa < 0$ and $c_\kappa < 0$. Simplifying (16) we find that $a_\kappa = N^3 (N - \eta) > 0$, $b_\kappa = -N(1 + N)\left((N-1)^2 - \eta\right) - 1 < 0$ and $c_\kappa = -(N+1)^2 (N-1) < 0$ for $0 < \eta < 1$ because $N \geq 2$. If $\eta = 0$ then $\kappa = 1$ and if $\eta = 1$ then $\kappa = \frac{(N-1)(N+1)^2}{N^3} > 1$.

If $\kappa < 1$ then $K \quad < 0$. Hence it must hold that $K \quad > 0 \Longrightarrow \kappa \frac{3}{2-1} - 1 - \kappa \frac{4}{(\ +1)(\ 2-1)} > 0$ $\Longrightarrow \kappa > \frac{(\ 2-1)(\ +1)}{3}$. As $\frac{(\ 2-1)(\ +1)}{3} > 1$ this contradicts $\kappa < 1$. Similarly if If $\kappa = 1$ then $K \quad = 0$ and it must hold that $K \quad = 0 \Longrightarrow \kappa > \frac{(\ 2-1)(\ +1)}{3}$. As $\frac{(\ 2-1)(\ +1)}{3} \neq 1$ it contradicts $\kappa = 1$. Hence it must hold that $\kappa > 1$ and thus $K \quad > 0$ and $K \quad < 0$. To prove that $\frac{N}{K} > 0$ let us take a total derivative of (16) with respect to κ and η. Using the definitions of K and K , we find that $\frac{N}{K} = \frac{(\ U - \ I)}{\left(K\frac{\partial K^I}{\partial \kappa} + K\frac{\partial K^U}{\partial \kappa}\right)}$. Because $\frac{I}{N} = \frac{\frac{N^3}{(N^2-1)}}{\left(1+N\frac{N^4}{(N+1)(N^2-1)}\right)^2} > 0$, $\frac{U}{N} = \frac{}{(1+N(\ -1))^2} > 0$ and $K \quad < 0 < K \quad$ it holds that $\frac{N}{K} > 0$. ■

After solving for the date 1 prices let us derive the information revealed to uninformed P from date 1 prices. Unlike the Market uninformed P knows that the state is $R = U$ and period 1 order flow is $y_1 = \frac{-1}{} \frac{L - \ 0}{P_1} + s_1$. Because the equilibrium price from the guess (6) that we have verified $p_1 = p_0 + \lambda_1 y_1$, it holds that $p_1 = p_0 + \lambda_1 y_1 = p_0 + \frac{-1}{} (\theta - p_0) + \lambda_1 s_1$, where $\lambda_1 = \sqrt{\frac{-1}{2} \frac{e_p^2}{N e_s^2}}$. Uninformed P obtains a price signal $\tilde{p}_1 \equiv \frac{}{-1} (p_1 - p_0) + p_0$, where $\tilde{p}_1 | \theta \sim \mathcal{N}\left(\theta, \left(\frac{}{-1}\right)^2 \lambda_1^2 \sigma^2\right)$. Using the equilibrium value of λ_1, we obtain that $\tilde{p}_1 | \theta \sim \mathcal{N}\left(\theta, \frac{1}{-1} \frac{e_p^2}{N}\right)$. He also know the prior $p_0 | \theta \sim \mathcal{N}\left(\theta, \sigma^2\right)$, using the Bayes' rule, we obtain that

$$\mathbb{E}\left[\theta | \Omega_1 \quad \right] = \frac{\frac{1}{e_p^2} p_0 + \frac{N(\ -1)}{e_p^2} \tilde{p}_1}{\frac{1}{e_p^2} + \frac{N(\ -1)}{e_p^2}} = p_0 + \frac{\kappa N}{1 + \kappa (N - 1)} (p_1 - p_0).$$

This confirms conjecture (13), where $e = \frac{N}{1+N(\ -1)}$. From $\kappa > 1$, it follows that $e > 1$ for any $\eta > 0$.

Finally, we need to solve for date 2 prices. From (14) the total order flow in date 2 is

$$y_2 = \begin{cases} \frac{-1}{} \frac{L - \ 0}{P_2} - \frac{(\ -1) - \frac{1}{N+1}(\ - \ )}{P_2} (p_1 - p_0) + s_2 \text{ if } R = U \\ \frac{}{+1} \frac{L - \ 0 - (\ 1 - \ 0)}{(\ +1) P_2} + s_2 \text{ if } R = I \end{cases}. \tag{17}$$

Now the Market's signal from the order flow in state $R = U$ is $\tilde{y}_2 \equiv \frac{}{-1} \lambda_2 y_2 + p_0 + \left(c - \frac{-}{2-1}\right)(p_1 - p_0)$, where $\tilde{y}_2 | \theta \sim \mathcal{N}\left(\theta, \left(\frac{}{-1}\right)^2 \lambda_2^2 \sigma^2\right)$ and in state $R = I$ as $\tilde{y}_2 \equiv \frac{+1}{} \lambda_2 y_2 + p_0 + c(p_1 - p_0)$, where $\tilde{y}_2 | \theta \sim \mathcal{N}\left(\theta, \left(\frac{+1}{}\right)^2 \lambda_2^2 \sigma^2\right)$. Using Bayes' rule, we find

that

$$\mathbb{E}\left[\theta|\Omega_2 \ , R = I\right] = \frac{\frac{1}{e_p^2} p_0 + \frac{1}{\left(\frac{N+1}{N}\right)^2 P_1^2 e_s^2} \tilde{y}_1 + \frac{1}{\left(\frac{N+1}{N}\right)^2 P_2^2 e_s^2} \tilde{y}_2}{\frac{1}{e_p^2} + \frac{1}{\left(\frac{N+1}{N}\right)^2 P_1^2 e_s^2} + \frac{1}{\left(\frac{N+1}{N}\right)^2 P_2^2 e_s^2}}$$

$$\mathbb{E}\left[\theta|\Omega \quad , R = U\right] = \frac{\frac{1}{e_p^2} p_0 + \frac{1}{\left(\frac{N}{N-1}\right)^2 P_1^2 e_s^2} \tilde{y}_1 \ + \frac{1}{\left(\frac{N}{N-1}\right)^2 P_2^2 e_s^2} \tilde{y}_2}{\frac{1}{e_p^2} + \frac{1}{\left(\frac{N}{N-1}\right)^2 P_1^2 e_s^2} + \frac{1}{\left(\frac{N}{N-1}\right)^2 P_2^2 e_s^2}}.$$

Using these and $\tilde{y}_1$, $\tilde{y}_1$, $\tilde{y}_2$, $\tilde{y}_2$, $p_1 = p_0 + \lambda_1 y_1$ in (5) confirms the linear guess (6) also in date 2. Equating the coefficients and using λ_1 we find the equilibrium coefficients. Equating the coefficients of y_2 we find that $\lambda_2 = \sqrt{\frac{e_p^2}{e_s^2} \frac{-1}{R \quad ^3}}$, where μ solves

$$\eta \left(\frac{\mu \frac{^4}{^2-1}}{1 + \kappa \frac{^4}{(\ +1)(\ ^2-1)} + \mu \frac{^5}{(\ +1)(\ ^2-1)}} - 1 \right) + (1-\eta) \left(\frac{\mu N^2}{1 + \kappa (N-1) + \mu N (N-1)} - 1 \right) = 0. \tag{18}$$

There is a unique positive solution and μ is determined by η and N (as also κ is uniquely determined by η and N).[16] We can also find the limits as $\lim_{K \to 0} \mu = 1$ and $\lim_{K \to 1} \mu = \frac{(1+ \ )^2 \left(\ ^2-1 \right)}{^4} > 1$.

Second, equating the coefficients of $(p_1 - p_0)$ and using 18, we find that

$$c = (N+1) \frac{\kappa}{\mu N} - (1-\eta) \frac{\mu N e}{1 + \kappa (N-1) + \mu N (N-1)},$$

where $e = \frac{N}{1+N(\ -1)}$. Therefore also c is uniquely determined by η and N. When $\eta \to 1$, then $c = (N+1) \frac{^4(\ -1)(\ +1)^2}{(1+ \ )^2(\ ^2-1) \quad ^3} = 1$. Also when $\eta \to 1$, then $c = (N+1) \frac{1}{\ } - \frac{\ }{1+(\ -1)+ \ (\ -1)} = \frac{\ +1}{\ } - \frac{1}{\ } = 1$. Using (6), (17) and $p_1 = p_0 + \frac{\ -1}{\ } (\theta - p_0) + \lambda_1 s_1$ we then find if $R = U$ that

$$\begin{aligned} p_2 &= p_0 + c (p_1 - p_0) + \lambda_2 y_2 = \\ &= p_0 + \frac{N-1}{N} (\theta - p_0) + \frac{\nu}{N} (p_1 - p_0) + \lambda_2 s_2 = \\ &= p_0 + \frac{N-1}{N} \frac{N+\nu}{N} (\theta - p_0) + \frac{\nu}{N} \lambda_1 s_1 + \lambda_2 s_2, \end{aligned}$$

where $\nu \equiv \frac{\ + \ }{\ +1}$.

[16] Equation (18) is a quadratic equation and can be written in the form $a_\mu \mu^2 + b_\mu \mu + c_\mu = 0$. There is one real positive solution if $a_\mu > 0$, $b_\mu < 0$ and $c_\mu < 0$. Using the facts that $N \geq 2$, $0 < \eta < 1$ and $\kappa > 1$ and simplifying (18) we find that $a_\mu = N^5 (N - \eta) > 0$, $b_\mu = -\mu N D_\mu < 0$ as $D_\mu \equiv N (N+1) \left((N-1)^2 - \eta \right) + \kappa N^3 (\eta + N (N-2)) + 1 > 0$, and $c_\mu = -\left((N-1)(N+1)^2 + N^4 \kappa \right) (\kappa (N-1) + 1) < 0$

B Proof of Proposition 1

The main results are in Appendix A where we derive the equilibrium. What remains to be shown is that $e-c>0$ always holds, $\frac{(\ -\ )}{K}>0$ and $\nu=\frac{+}{+1}>1$.

Note that $\lim\limits_{K\to 0} e-c=0$ and $\lim\limits_{K\to 1} e-c=e-1>0$, because we proved in Appendix A that $e>0$ for any $\eta>0$. Hence for $e-c>0$ it is sufficient to show that $\frac{(\ -\ )}{K}>0$.

Using the results in Appendix A we find that $e-c=1+\frac{R\ \ (1-K)}{N\ +R\ \ (\ \ -1)}e^2-\frac{+1}{}\frac{N}{R}=1+\frac{(1-K)}{+(\ \ -1)}e^2-\frac{+1}{}x$, where $x\equiv\frac{N}{R}$. We obtain that $\frac{-}{K}=\frac{}{K}(1-\eta)\,e\frac{1}{(\ +(\ \ -1)\ )^2}\left(2x+(N-1)\,e\right)-\frac{}{K}\left(\frac{(1-K)}{(\ +(\ \ -1)\ )^2}e^2+\frac{+1}{}\right)$. Because $\frac{}{K}>0$, $(1-\eta)\,e\frac{1}{(\ +(\ \ -1)\ )^2}\left(2x+(N-1)\,e\right)>0$ and $\left(\frac{(1-K)}{(\ +(\ \ -1)\ )^2}e^2+\frac{+1}{}\right)$, we need to show that $\frac{}{K}=\frac{\frac{\kappa}{\mu}}{K}<0$. After some algebra (18) reduces to

$$
\begin{aligned}
&G(\kappa,\mu,\eta;N)\equiv\eta\left\{N^4\left[1+\kappa(N-1)-\mu N\right]-N(N+1)^2(N-1)^2\right\}\\
&+(1-\eta)\left\{N^5[\kappa-1]+N(N+1)^2(N-1)\right\}+\mu N^6-\frac{(N+1)^2(N-1)}{\mu}\\
&-\frac{\kappa}{\mu}\left[(N+1)^2(N-1)^2+N^4\right]-\kappa N^5(N-1)-\frac{\kappa^2}{\mu}N^4(N-1)
\end{aligned}
$$

By the implicit function theorem

$$\frac{dx}{d\eta}=-\frac{\overline{}_{K}}{\overline{}}$$

The derivative $\frac{}{K}$ is straightforward

$$\frac{\partial G(\kappa,\mu,\eta;N)}{\partial\eta}=N^4\left[1+\kappa(N-1)-\mu N\right]-N(N+1)^2(N-1)^2-N^5[\kappa-1]-N(N+1)^2(N-1)$$

Moreover, $\frac{(NRK\ \ )}{K}<0$ as

$$N^4+N^5\kappa-N^4\kappa-\mu N^5-N^5\kappa+N^5<N(N+1)^2(N-1)^2+N(N+1)^2(N-1)$$

which becomes

$$N^4(1-\kappa)+N^5(1-\mu)<N(N+1)^2(N-1)(N-1+1)$$

and finally

$$N^2(1-\kappa)+N^3(1-\mu)<(N+1)^2(N-1)$$

Now, we need to compute $\frac{}{}$, where $x \equiv \frac{N}{R}$ and $dx = (d\kappa - d\mu \cdot \mu)/\mu$. Let us hold μ constant (so $d\mu = 0$), then take derivatives of all κ terms, and then substitute $d\kappa = dy \cdot \mu$. We can rewrite $G(\kappa, \mu, \eta; N)$ using the fact that $\kappa = x \cdot \mu$

$$G(\kappa, \mu, \eta; N) = G(x, \mu, \eta; N)$$

$$\begin{aligned} G(x, \mu, \eta; N) &\equiv \eta \left\{ N^4 \left[1 + x\mu (N-1) - \mu N \right] - N (N+1)^2 (N-1)^2 \right\} \\ &+ (1-\eta) \left\{ N^5 \left[x\mu - 1 \right] + N (N+1)^2 (N-1) \right\} + \mu N^6 - \frac{(N+1)^2 (N-1)}{\mu} \\ &- x \left[(N+1)^2 (N-1)^2 + N^4 \right] - x\mu N^5 (N-1) - x^2 \mu N^4 (N-1) \end{aligned}$$

and

$$\begin{aligned} &\frac{\partial G(x, \mu, \eta; N)}{\partial x} \\ &= \eta N^4 (N-1) \mu + (1-\eta) N^5 \mu - (N+1)^2 (N-1)^2 - N^4 - \mu N^5 (N-1) - 2x\mu N^4 (N-1) \end{aligned}$$

which after substituting back $x = \frac{N}{R}$ becomes

$$\begin{aligned} &\frac{\partial G(\kappa, \mu, \eta; N)}{\partial x} \\ &= \eta N^4 (N-1) \mu + (1-\eta) N^5 \mu - (N+1)^2 (N-1)^2 - N^4 - \mu N^5 (N-1) - 2\kappa N^4 (N-1) \end{aligned}$$

and finally

$$\frac{\partial G(\kappa, \mu, \eta; N)}{\partial x} = -N^4 \left(N^2 - 2N + \eta \right) \mu - 2\kappa N^4 (N-1) - 2N^2 \left(2N^2 - 1 \right) - 1 < 0$$

Hence

$$\frac{dx}{d\eta} = -\frac{\overline{}_K}{} = -\frac{N^2 (\kappa - 1) + N^3 (\mu - 1) + (N+1)^2 (N-1)}{N^4 (N^2 - 2N + \eta) \mu + 2\kappa N^4 (N-1) + 2N^2 (2N^2 - 1) + 1} < 0.$$

C Proof of Proposition 3

Let us guess similarly to (6), that $p_1 = p_0 + \bar{\lambda}_1 y_1$ and $p_2 = p_0 + \bar{c}(p_1 - p_0) + \bar{\lambda}_2 y_2$. We can use the intermediate results from Appendix A and impose that $h_1 = h_2 = 0$. With probability $1-\eta$, the Market observes $\tilde{y}_1 | \theta \equiv \frac{}{-1} \bar{\lambda}_1 y_1 + p_0$, where $\tilde{y}_1 | \theta \sim \mathcal{N}\left(\theta, \left(\frac{}{-1} \right)^2 \bar{\lambda}_1^2 \sigma^2 \right)$

and $\tilde{y}_2 \equiv \frac{}{-1}\bar{\lambda}_2 y_2 + p_0 + \bar{c}\,(p_1 - p_0)$, where $\tilde{y}_2 \sim \mathcal{N}\left(\theta, \left(\frac{}{-1}\right)^2 \bar{\lambda}_2^2\sigma^2\right)$). With probability η, the Market observes $\tilde{y}_1 \equiv \frac{+1}{}\bar{\lambda}_1 y_1 + p_0$, where $\tilde{y}_1|\theta \sim \mathcal{N}\left(\theta, \left(\frac{+1}{}\right)^2 \bar{\lambda}_1^2\sigma^2\right)$ and $\tilde{y}_2 \equiv \frac{+1}{}\bar{\lambda}_2 y_2 + p_0 + \bar{c}\,(p_1 - p_0)$, where $\tilde{y}_2|\theta \sim \mathcal{N}\left(\theta, \left(\frac{+1}{}\right)^2 \lambda_2^2\sigma^2\right)$. Using this in (5), we obtain that $\bar{\lambda}_1^2 = \lambda_1^2 = \frac{-1}{2}\frac{e_p^2}{Ne_s^2}$ and $\bar{\lambda}_1^2 = \lambda_2^2 = \frac{-1}{3}\frac{e_p^2}{Re_s^2}$, where κ and μ solve (16) and (18) respectively, as in the basic setting. This implies that p_1 is the same as in (8).

From Appendix A, (6) and (3) $h_2 \quad = 0$ and λ_2 the price is $p_2 = p_0 + \frac{L - _0}{} + \frac{U(\ _1 - \ _0)}{} + \sqrt{\frac{-1}{3}\frac{1}{\sqrt{R}}\frac{e_p^2}{e_s^2}}\,s_2$, where $\bar{\nu} \equiv \bar{c}$. Comparing it with (8), it is clear that the pure effect of price-contingent trading can be found by comparing $\bar{\nu}$ with ν. Equating coefficients in (5) and in the conjectured prices (6) and using $\lambda_1^2 = \frac{-1}{2}\frac{e_p^2}{Ne_s^2}$, $\lambda_2^2 = \frac{-1}{3}\frac{e_p^2}{Re_s^2}$, (16) and (18), we find that $\bar{\nu} = \frac{+1}{}\frac{N}{R}\frac{1+N(\ -1)+R\ (\ -1)}{1+N(\ -1)+R\ (\ -1)+R\ (1-K)}$ and $\lim_{K\longrightarrow 0}\bar{\nu} = \lim_{K\longrightarrow 1}\bar{\nu} = 1$. From the definition of $\nu \equiv \frac{+}{+1}$, $e = \frac{N}{1+N(\ -1)}$, and $\lim_{K\Leftarrow 0}\nu = 1$ and $\lim_{K\Leftarrow 1}\nu = \frac{\left(2\ ^2 - \ + \ ^3 - 1\right)\ ^2}{(\ ^3 - 2\ ^2 + \ ^4 + 1)(\ +1)} > 1$. It is clear that $\nu > \bar{\nu}$ if the probability of P being uninformed is very low. It can be shown that $\bar{\nu} < \nu$ at any value of $\eta > 0$.

C.1 Proof of Proposition 4

We conjecture the existence of an equilibrium strategies with the following properties:

1. uninformed P's demand is $h_1 \quad = 0$
2. informed trader's demand is $h_1\ ^n = g_1\ (\theta)$ if $R = U$ and $h_1\ ^n = h_1 \quad = g_1\ (\theta)$ if $R = I$ for every $n \in \{1, ..., N-1\}$
3. it holds that $g_1'\ , g_1'\ > 0$, $-g_1\ (-\theta) = g_1\ (\theta)$ and $-g_1\ (-\theta) = g_1\ (\theta)$.

We have to derive the equilibrium strategies given these properties and show that traders do not want to deviate from the conjectured strategies. To save notation, we include to information set only the relevant variables, i.e., θ, y_1, y_2 and R

By the market efficiency condition

$p_1 = Q_1 E\left[\theta|y_1, R = I\right] + (1 - Q_1)\, E\left[\theta|y_1, R = U\right]$

Using Bayes' rule

$Q_1 = \Pr\left(R = I|y_1\right) = \frac{(\ _1|\ =\ )\Pr(\ =\ )}{(\ _1)} = \frac{(\ _1|\ =\ )\Pr(\ =\ )}{(\ _1|\ =\ )\Pr(\ =\ ) + (\ _1|\ =\ )\Pr(\ =\ )}$

Given that $\Pr\left(R = I\right) = \eta$, we obtain

$$Q_1 = \frac{\eta f\left(y_1|R = I\right)}{\eta f\left(y_1|R = I\right) + (1-\eta)\, f\left(y_1|R = U\right)} \tag{19}$$

Using again the Bayes' rule $f(\theta|y_1,R) = \frac{(\ _1|L\ \ )\ (L|\ )}{(\ _1|\ \ )}$. As the prior distribution of θ does not depend on R, we can write $f(\theta|R) = f(\theta)$.

Replacing this and Q_1 into the market efficiency condition, we obtain

$$p_1 = \frac{\int_{-\infty}^{\infty} \theta\eta f(y_1|\theta,R)\, f(\theta)\, d\theta + \int_{-\infty}^{\infty} \theta\eta f(y_1|\theta,R)\, f(\theta)\, d\theta}{\eta f(y_1|R=I) + (1-\eta)\, f(y_1|R=U)} \tag{20}$$

Let us now investigate the distributions. The prior distribution is normal and thus $f(\theta) = \frac{1}{\sqrt{2c}\, e_p^2} \exp\left(-\frac{1}{2}\frac{L^2}{e_p^2}\right)$.

Given the assumed strategies, the order flow

$$y_1 = \begin{cases} (N-1)\, g_1\ \ (\theta) + s_1 \text{ if } R=U \\ Ng_1\ \ (\theta) + s_1 \text{if } R=I \end{cases}$$

From here we can see that the Market never learn P's type perfectly under these strategies. Assume that the state is $R=U$. Suppose that the Market learns P's type, i.e., $Q_1 = 0$. From this to be the case, it must hold that $f(y_1|R=I) = 0$ or $f(y_1|R=U) \to \infty$,note that as $g_1\ \ (\theta)$, $g_1\ \ (\theta)$ and s_1 are all continuous variables with infinite support and thereore neither $f(y_1|R=I) = 0$ or $f(y_1|R=U) \to \infty$ can be true.

It is straightforward that the distributions conditional on R and θ remain normal, i.e., $f(y_1|\theta, R=U) = \frac{1}{\sqrt{2c}\, e_s} \exp\left(-\frac{1}{2}\frac{(\ _1-(\ \ -1)\ _{1U}(L))^2}{e_s^2}\right)$ and $f(y_1|\theta, R=U) = \frac{1}{\sqrt{2c}\, e_s} \exp\left(-\frac{1}{2}\frac{(\ _1-\ \ \ _{1I}(L))^2}{e_s^2}\right)$.

What makes the problem non-linear is the presence of $f(y_1|R)$ that is not does not remain normal when that strategies are not linear in θ that is likely the case in this general problem.

Nevertheless, because we assume $g_1\ \ (\theta)$ and $g_1\ \ (\theta)$ to be monotonic functions of θ they also have support $[-\infty, \infty]$. Because θ is continuous random variable with probability density function $f_L(\theta)$ and g_1 and g_1 are monotonically increasing in θ, we obtain that p.d.f-s of $l_1\ \ (\theta) \equiv (N-1)\, g_1\ (\theta)$ and $l_1\ \ (\theta) \equiv Ng_1\, (\theta)$ are respectively

$$f_{\ _{1U}(L)}\left(l_1\ \ (\theta)\right) = f_L(\theta)\, \frac{d\left(l_1^{-1}\left(l_1\ \ (\theta)\right)\right)}{dl_1\ \ (\theta)} = f_L(\theta)\, \frac{d\theta}{dl\ _1(\theta)}$$

$$f_{\ _{1I}(L)}\left(l_1\ \ (\theta)\right) = f_L(\theta)\, \frac{d\left(l_1^{-1}\left(l\ _1(\theta)\right)\right)}{dl\ _1(\theta)} = f_L(\theta)\, \frac{d\theta}{dl\ _1(\theta)}$$

Also, note that θ is independent of s_1. Therefore also the random variables that are functions of θ are independent of s_1. The distribution of sum of two continuous independent

variables is the convolution $f_{\ 1R(L)}(g_1 \ \ (\theta))$ and $f_{\ 1}(s_1)$ for $R = \{U, I\}$. Therefore

$$\begin{aligned} f(y_1|R=U) &= \int_{-\infty}^{\infty} f_{\ 1U(L)}(l_1 \ \ (\theta))\, f_{\ 1}(y_1 - l_1 \ \ (\theta))\, dl_1 \ \ (\theta) = \\ &= \int_{-\infty}^{\infty} f_L(\theta) \frac{d\theta}{dg \ \ _1(\theta)} f_{\ 1}(y_1 - l_1 \ \ (\theta))\, lg_1 \ \ (\theta) = \\ &= \int_{-\infty}^{\infty} f_L(\theta)\, f_{\ 1}(y_1 - l_1 \ \ (\theta))\, d\theta = \\ &= \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma \ \ \sigma} \exp\left(-\frac{\theta^2}{2\sigma^2} - \frac{(y_1 - l_1 \ \ (\theta))^2}{2\sigma^2} \right) d\theta = \\ &= \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma \ \ \sigma} \exp\left(-\frac{\theta^2}{2\sigma^2} - \frac{\left(y_1 - (N-1)\, g_1 \ \ (\theta)\right)^2}{2\sigma^2} \right) d\theta, \end{aligned}$$

where we used also that $f_{\ 1}(s_1) = \frac{1}{\sqrt{2c}\, e_s^2} \exp\left(-\frac{1}{2}\frac{\ _1^2}{e_s^2}\right)$ and $f_L(\theta) = \frac{1}{\sqrt{2c}\, e_p^2} \exp\left(-\frac{1}{2}\frac{L^2}{e_p^2}\right)$. Similarly, we obtain that

$$f(y_1|R=I) = \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma \ \ \sigma} \exp\left(-\frac{\theta^2}{2\sigma^2} - \frac{\left(y_1 - N g_1(\theta)\right)^2}{2\sigma^2} \right) d\theta$$

Replacing the distributions into (20), we obtain that

$$p_1 = \frac{\int_{-\infty}^{\infty} \theta \left(\eta \exp\left(-\frac{L^2}{2e_p^2} - \frac{\left(\ _1 - \ \ _1^I(L)\right)^2}{2e_s^2} \right) + (1-\eta) \exp\left(-\frac{L^2}{2e_p^2} - \frac{\left(\ _1 - (\ \ -1) \ _1^U(L)\right)^2}{2e_s^2} \right) \right) d\theta}{\int_{-\infty}^{\infty} \left(\eta \exp\left(-\frac{L^2}{2e_p^2} - \frac{\left(\ _1 - \ \ _1^I(L)\right)^2}{2e_s^2} \right) + (1-\eta) \exp\left(-\frac{L^2}{2e_p^2} - \frac{\left(\ _1 - (\ \ -1) \ _1^U(L)\right)^2}{2e_s^2} \right) \right) d\theta} \tag{21}$$

It is clear that p_1 is a function of y_1. As we integrate over θ in numerator and denominator. So we can write that $p_1 = \Lambda_1(y_1)$.

We can now address the other the basic properties of the pricing rule.

First, let us slow that given the assumed strategies, the equilibrium price is symmetric around zero, i.e., $-\Lambda_1(-y_1) = \Lambda_1(y_1)$. We can write

$$-\Lambda(-y_1) = -\frac{\int_{-\infty}^{\infty} L \left(K \exp\left(-\frac{\theta^2}{2\sigma_p^2} - \frac{(-y_1 - N g_{1I}(\theta))^2}{2\sigma_s^2} \right) + (1-K) \exp\left(-\frac{\theta^2}{2\sigma_p^2} - \frac{(-y_1 - (N-1) g_{1U}(\theta))^2}{2\sigma_s^2} \right) \right) \ L}{\int_{-\infty}^{\infty} \left(K \exp\left(-\frac{\theta^2}{2\sigma_p^2} - \frac{(-y_1 - N g_{1I}(\theta))^2}{2\sigma_s^2} \right) + (1-K) \exp\left(-\frac{\theta^2}{2\sigma_p^2} - \frac{(-y_1 - (N-1) g_{1U}(\theta))^2}{2\sigma_s^2} \right) \right) \ L}$$

Let us then change the variable $\theta = -\hat{\theta}$. Given that $d\theta = -d\hat{\theta}$ and by symmetry of strategies around zero $g_1\left(-\hat{\theta}\right) = -g_1\left(\hat{\theta}\right)$ and $g_1\left(-\hat{\theta}\right) = -g_1\left(\hat{\theta}\right)$, we obtain

$$
\begin{aligned}
&-\Lambda_1(-y_1) = \\
&-\frac{\int_{-\infty}^{\infty}(-\hat{L})\left(K\exp\left(-\frac{\hat{\theta}^2}{2\sigma_p^2}-\frac{(-y_1+Ng_{1I}(\hat{\theta}))^2}{2\sigma_s^2}\right)+(1-K)\exp\left(-\frac{\hat{\theta}^2}{2\sigma_p^2}-\frac{(-y_1+(N-1)g_{1U}(\hat{\theta}))^2}{2\sigma_s^2}\right)\right)(-\ \hat{L})}{\int_{-\infty}^{\infty}\left(K\exp\left(-\frac{\hat{\theta}^2}{2\sigma_p^2}-\frac{(-y_1+Ng_{1I}(\hat{\theta}))^2}{2\sigma_s^2}\right)+(1-K)\exp\left(-\frac{\hat{\theta}^2}{2\sigma_p^2}-\frac{(-y_1+(N-1)g_{1U}(\hat{\theta}))^2}{2\sigma_s^2}\right)\right)(-\ \hat{L})} = \\
&= \frac{\int_{-\infty}^{\infty}\hat{L}\left(K\exp\left(-\frac{\hat{\theta}^2}{2\sigma_p^2}-\frac{(y_1-Ng_{1I}(\hat{\theta}))^2}{2\sigma_s^2}\right)+(1-K)\exp\left(-\frac{\hat{\theta}^2}{2\sigma_p^2}-\frac{(y_1-(N-1)g_{1U}(\hat{\theta}))^2}{2\sigma_s^2}\right)\right)\ \hat{L}}{\int_{-\infty}^{\infty}\left(K\exp\left(-\frac{\hat{\theta}^2}{2\sigma_p^2}-\frac{(y_1-Ng_{1I}(\hat{\theta}))^2}{2\sigma_s^2}\right)+(1-K)\exp\left(-\frac{\hat{\theta}^2}{2\sigma_p^2}-\frac{(y_1-(N-1)g_{1U}(\hat{\theta}))^2}{2\sigma_s^2}\right)\right)\ \hat{L}} = \Lambda_1(y_1)
\end{aligned}
$$

This confirms that symmetric strategies around zero lead to the price as the function of the order flow to be symmetric around zero as well.

Next we need to show that price is an increasing function of the order flow. Differentiating p_1 in (21) with respect to the order flow and simplifying, we obtain

$$
\frac{\partial p_1}{\partial y_1} = \frac{\int_{-\infty}^{\infty}(L-\ _1)\left(K\exp\left(-\frac{L^2}{2e_p^2}-\frac{\left(\ _1-\ \ _1^I(L)\right)^2}{2e_s^2}\right)\frac{Ng_1^I(\theta)}{\sigma_s^2}+(1-K)\exp\left(-\frac{\theta^2}{2\sigma_p^2}-\frac{\left(y_1-(N-1)g_1^U(\theta)\right)^2}{2\sigma_s^2}\right)\frac{(N-1)g_1^U(\theta)}{\sigma_s^2}\right)\ L}{\int_{-\infty}^{\infty}\left(K\exp\left(-\frac{\theta^2}{2\sigma_p^2}-\frac{\left(y_1-Ng_1^I(\theta)\right)^2}{2\sigma_s^2}\right)+(1-K)\exp\left(-\frac{\theta^2}{2\sigma_p^2}-\frac{\left(y_1-(N-1)g_1^U(\theta)\right)^2}{2\sigma_s^2}\right)\right)\ L} \tag{22}
$$

The denominator is clearly positive, because $0 < \eta < 1$ and exponential function is always positive. We need to show that also the numerator is positive. We can write the numerator as

$$
\begin{aligned}
&\int_0^{\infty}(\theta-p_1)\exp\left(\frac{-L^2}{2e_p^2}\right)\left(\eta\exp\left(\frac{-\left(\ _1-\ \ _1^I(L)\right)^2}{2e_s^2}\right)\frac{\ _1^I(L)}{e_s^2}+(1-\eta)\exp\left(\frac{-\left(\ _1-(\ \ -1)\ \ _1^U(L)\right)^2}{2e_s^2}\right)\frac{(\ \ -1)\ \ _1^U(L)}{e_s^2}\right)d\theta+ \\
&\int_{-\infty}^{0}(\theta-p_1)\exp\left(\frac{-L^2}{2e_p^2}\right)\left(\eta\exp\left(\frac{-\left(\ _1-\ \ _1^I(L)\right)^2}{2e_s^2}\right)\frac{\ _1^I(L)}{e_s^2}+(1-\eta)\exp\left(\frac{-\left(\ _1-(\ \ -1)\ \ _1^U(L)\right)^2}{2e_s^2}\right)\frac{(\ \ -1)\ \ _1^U(L)}{e_s^2}\right)d\theta= \\
&\int_0^{\infty}\theta\exp\left(\frac{-L^2}{2e_p^2}\right)\left(\eta\exp\left(\frac{-\left(\ _1-\ \ _1^I(L)\right)^2}{2e_s^2}\right)\frac{\ _1^I(L)}{e_s^2}+(1-\eta)\exp\left(\frac{-\left(\ _1-(\ \ -1)\ \ _1^U(L)\right)^2}{2e_s^2}\right)\frac{(\ \ -1)\ \ _1^U(L)}{e_s^2}\right)d\theta+ \\
&\int_0^{\infty}\hat{\theta}\exp\left(\frac{-\hat{L}^2}{2e_p^2}\right)\left(\eta\exp\left(\frac{-\left(\ _1-\ \ _1^I(\hat{L})\right)^2}{2e_s^2}\right)\frac{\ _1^I(\hat{L})}{e_s^2}+(1-\eta)\exp\left(\frac{-\left(\ _1-(\ \ -1)\ \ _1^U(\hat{L})\right)^2}{2e_s^2}\right)\frac{(\ \ -1)\ \ _1^U(\hat{L})}{e_s^2}\right)d\hat{\theta} > 0
\end{aligned}
$$

The last uses again a change of variable $\theta = -\hat{\theta}$, and the property that $g_1\left(-\hat{\theta}\right) = -g_1\left(\hat{\theta}\right)$ and $g_1\left(-\hat{\theta}\right) = -g_1\left(\hat{\theta}\right)$. The terms with p_1 cancel out. The numerator is positive as all terms inside the intergals are positive in interval $[0, \infty)$. This proves that prices are always increasing in the order flow, i.e., $\frac{\ _1}{\ _1} > 0$.

We can now verify the strategies.

First, suppose that the Market and informed traders have equilibrium strategies and consider whether uninformed P wants to deviate and trade h_1' .. Before period 1 prices are known, the only signal that P has about the fundamental is the prior. His maximization problem is $\max_{{}_1'^{UP}} h_1' \quad (E[\theta] - E[p_1])$, where $p_1 = \Lambda_1(y_1')$.and $y_1' = (N-1)\, g_1\ (\theta) + h_1' \quad + s_1$

P's first order condition is $E[\theta] - E[p_1] = h_1' \quad \frac{[\Lambda_1(\ _1')]}{{}_1'^{UP}} =$

$h_1' \quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\Lambda_1(\ _1')}{{}_1'} \frac{{}_1'}{{}_1'^{UP}} f s_1(s_1)\, f_L(\theta)\, ds_1 d\theta$, where $f s_1(s_1)$ is the density of s_1 and $f_L(\theta)$ is the density of θ.. As $\frac{{}_1'}{{}_1'^{UP}} = 1$ and $\frac{\Lambda_1(\ _1')}{{}_1'} > 1$ it is clear that the left hand side of the first order condition is increasing in h_1' . It also holds that $E[\theta] = p_0 = 0$ and by the market efficiency condition $p_1 = E[\theta|y_1]$ and law of iterated expectations, it holds that $E[p_1] = E[E[\theta|y_1]] = E[\theta] = 0$. We obtain the uninformed P's equilibrium strategy in period 1 is indeed $h_1' \quad = h_1 \quad = 0$.

Second, consider K 's deviating strategy is state $R = U$. Suppose that the Market, other informed traders as P follow the equilibrium strategies and K demands h_1' . K's objective is $\max_{\tilde{}_1^{UP}} h_1' \quad (\theta - E[p_1|\theta])$, where $p_1 = \Lambda_1(y_1')$.and $y_1' = (N-2)\, g_1\ (\theta) + h_1' \quad {}^n + s_1$.

We obtain the first order condition $\theta = E[\Lambda_1(y_1')\,|\theta] + h_1' \quad {}^n \frac{[\Lambda_1(\ ')|I]}{{}_1'^{UP}}$.

This can be written as

$$\theta = \int_{-\infty}^{\infty} \left(\Lambda_1 \left((N-2)\, g_1\ (\theta) + h_1' \quad {}^n + s_1 \right) + h_1 \quad {}^n \frac{\partial \Lambda(y_1')}{\partial y_1'} \cdot 1 \right) f s_1(s_1)\, ds_1$$

In equilibrium, $y_1' \quad {}^n = h_1 \quad {}^n = g_1\ (\theta)$ and $y_1' = y_1 = (N-1)\, h_1 \quad {}^n + s_1$. Note that all type K traders are identical and have the same first order condition. Therefore, $h_1 \quad {}^n = h_1$ for any n and

$$\theta = \int_{-\infty}^{\infty} (\Lambda(y_1\ ) + h_1 \quad \Lambda'(y_1\ ))\, \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2} \frac{s_1^2}{\sigma^2} \right) ds_1 \equiv G_1 \quad (h_1 \quad), \tag{23}$$

where $y_1 \ = (N-1)\, h_1 \quad + s_1$

Because the right hand side of this is monotonically increasing in θ as $\frac{\Lambda(\ _1)}{{}_1} = \frac{{}_1}{{}_1} > 0$, we obtain that $G_1' \quad > 0$. Inverting it we confirm that $h_1 \quad = G_1^{-1}(\theta) = g_1 \quad (\theta)$, where $g_1' \quad > 0$. In order to show that $g_1 \quad (\theta) = -g_1 \quad (-\theta)$, note that this is equivalent to showing that $G\ _1(h_1 \quad) = -G\ _1(-h_1 \quad)$, because $\theta = -G\ _1(-h_1 \quad) \Leftrightarrow \theta = -G\ _1(-h_1 \quad) \Leftrightarrow -\theta = G\ _1(-h_1 \quad) \Leftrightarrow G_1^{-1}(-\theta) = G_1^{-1}(G\ _1(-h_1 \quad)) = -h_1 \quad \Leftrightarrow g_1 \quad (-\theta) = -g_1 \quad (\theta)$.

Using (23)

$$
\begin{aligned}
&- G_1 \quad (-h_1 \quad) = \\
&- \int_{-\infty}^{\infty} \left(\Lambda \left(- (N-1) h_1 \quad + s_1 \right) - h_1 \quad \Lambda' \left(- (N-1) h_1 \quad + s_1 \right) \right) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2} \frac{s_1^2}{\sigma^2} \right) ds_1 \\
&- \int_{-\infty}^{\infty} \left(\Lambda \left(- (N-1) h_1 \quad - \hat{s}_1 \right) - h_1 \quad \Lambda' \left(- (N-1) h_1 \quad - \hat{s}_1 \right) \right) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2} \frac{\hat{s}_1^2}{\sigma^2} \right) d\hat{s}_1 \\
&\int_{-\infty}^{\infty} \left(\Lambda \left((N-1) h_1 \quad + \hat{s}_1 \right) + h_1 \quad \Lambda' \left((N-1) h_1 \quad + \hat{s}_1 \right) \right) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2} \frac{\hat{s}_1^2}{\sigma^2} \right) d\hat{s}_1 \\
&= G_1 \quad \left(h_1 \quad ^{n} \right)
\end{aligned}
$$

where the second line uses a change of variable as $\hat{s}_1 = -\hat{s}_1$ and the last line uses the symmetry $\Lambda(y_1) = -\Lambda(-y_1)$ which also implies that $\Lambda'(y_1) = \Lambda'(-y_1)$.

Third, we have to consider K 's (or informed P's) deviating strategy is state $R = I$. The logic of derivation is the same as in the case of state $R = I$. We obtain $h_1 \quad ^{n} = h_1 \quad = h_1$ for any n and

$$
\theta = \int_{-\infty}^{\infty} \left(\Lambda(y_1) + h_1 \Lambda'(y_1) \right) \tfrac{1}{\sqrt{2c}\, e_s} \exp\left(-\tfrac{1}{2} \tfrac{\ {}^{2}_{1}}{e_s^2} \right) ds_1 \equiv G_1 \quad (h_1), \tag{24}
$$

where $y_1 = Nh_1 + s_1$

Following the same steps as before, we proves that $G_1' \quad > 0$, $g_1' \quad > 0$, $G \ _1 (h_1) = -G \ _1 (-h_1)$ and $g_1 \quad (-\theta) = -g_1 \quad (\theta)$. Therefore there exists a rational expectations equilibrium with the conjectured properties.

It also holds that for a given realization of θ, $(N-1) g_1 \quad (\theta) < N g_1 \ (\theta)$. For a given θ, (23) and (24) imply that

$$
\begin{aligned}
&\int_{-\infty}^{\infty} \tfrac{1}{\ } \left(\Lambda(y_1) + h_1 \Lambda'(y_1) \right) \tfrac{1}{\sqrt{2c}\, e_s} \exp\left(-\tfrac{1}{2} \tfrac{\ {}^{2}_{1}}{e_s^2} \right) ds_1 = \\
&\int_{-\infty}^{\infty} \tfrac{1}{\ -1} \left(\Lambda(y_1 \ ) + h_1 \ \Lambda'(y_1 \ ) \right) \tfrac{1}{\sqrt{2c}\, e_s} \exp\left(-\tfrac{1}{2} \tfrac{\ {}^{2}_{1}}{e_s^2} \right) ds_1 \\
&\int_{-\infty}^{\infty} \left(\tfrac{1}{\ } \Lambda(y_1) + \tfrac{1}{\ } h_1 \Lambda'(y_1) - \tfrac{1}{\ -1} \Lambda(y_1 \ ) - \tfrac{1}{\ -1} h_1 \ \Lambda'(y_1 \ ) \right) \tfrac{1}{\sqrt{2c}\, e_s} \exp\left(-\tfrac{1}{2} \tfrac{\ {}^{2}_{1}}{e_s^2} \right) ds_1 = 0.
\end{aligned}
$$

From here it is immediately clear that $(N-1) g_1 \quad (\theta) \neq N g_1 \ (\theta)$. Let us prove this by contradiction. Suppose that $(N-1) g_1 \quad (\theta) = N g_1 \ (\theta)$ holds for every θ. As $y_1 = Nh_1 + s_1 = N g_1 \ (\theta) + s_1$ and $y_1 \ = (N-1) g_1 \quad (\theta) + s_1$, it holds that $y_1 = y_1 \ $. Using this above, we obtain $-\frac{1}{(\ -1)} \int_{-\infty}^{\infty} \left(\Lambda(y_1) + h_1 \Lambda'(y_1) \right) \frac{1}{\sqrt{2c}\, e_s} \exp\left(-\frac{1}{2} \frac{\ {}^{2}_{1}}{e_s^2} \right) ds_1 = 0$. But then, by (24) we have $-\frac{1}{(\ -1)} \theta = 0 \Longrightarrow \theta = 0$, which clearly contradicts that $(N-1) g_1 \quad (\theta) = N g_1 \ (\theta)$

holds for every θ.

D Solution of the forward looking model

We now assume that $\tilde{1} = 1$ in (2). We continue to assume that Assumption 1 holds and thus we can guess that equilibrium prices take a liner for as in (6). Given that there are only two trading periods, the asset demand by K and P traders remains unchanged in period 2 and is given by (14). In order to find period 1 demand we first need to find $\mathbb{E}\left[h_2 \quad (\theta - p_2)\,|\Omega_1 \quad\right]$.

Let us start from the case where P is informed, i.e., $R = I$. From (6), (3) and (14) we obtain that

$$\mathbb{E}\left[h_2 \quad (\theta - p_2)\,|\Omega_1 \quad\right] = \mathbb{E}\left[h_2 \quad \left(\theta - p_0 - c\,(p_1 - p_2) - \lambda_2 \sum_{=1}^{-1} \left(h_2 \quad ^{n}\right) - \lambda_2 h_2 \quad - \lambda_2 s_2\right) |\Omega_1 \quad\right].$$

From $h_2 \quad ^{n} = h_2$ for any $n \in \{1, .., N-1\}$, $\mathbb{E}\left[s_2|\Omega_1 \quad\right] = 0$, (14), (6) and (3), we obtain

$$\mathbb{E}\left[h_2 \quad (\theta - p_2)\,|\Omega_1 \quad\right] = \frac{1}{(N+1)^2 \lambda_2} \mathbb{E}\left[(\theta - p_0 - c\,(p_1 - p_0))^2\,|\Omega_1 \quad\right] =$$

$$= \frac{1}{(N+1)^2 \lambda_2} \left(\theta - p_0 - c\lambda_1 \sum_{=1}^{-1} h_1 \quad ^{n} - c\lambda_1 h_1 \quad\right)^2 + \frac{(c\lambda_2)^2}{(N+1)^2 \lambda_2} \mathbb{E}\left[s_1^2|\Omega_1 \quad\right]$$

Using this, (6) and (3) in (2) we find the period 1 demand as

$$h_1 \quad ^{n} = h_1 \quad = \frac{(\theta - p_0)}{N\lambda_1 \Gamma} \text{ for } n \in \{1, .., N-1\},$$

where $\Gamma \quad \equiv \left(1 - \frac{2}{(\quad +1)^2}\frac{P_1}{P_2}\right)^{-1}\left(\frac{\quad +1}{} - \frac{2\ \ 2}{(\quad +1)^2}\frac{P_1}{P_2}\right)$.

Similarly, using (6), (14) and 3) we obtain that

$$\mathbb{E}\left[h_2 \quad ^{n}\,(\theta - p_2)\,|\Omega_1 \quad ^{n}\right] =$$

$$\frac{1}{N^2 \lambda_2} \left(\theta - p_0 - \frac{Nc + e}{N+1}\lambda_1 \sum_{=1}^{-1} h_1 \quad ^{n} - \frac{Nc+e}{N+1}\lambda_1 h_1 \quad\right)^2 + \frac{(Nc+e)^2}{N^2 (N+1)^2 \lambda_2} \mathbb{E}\left[s_1^2|\Omega_1 \quad ^{n}\right]$$

for $n \in \{1, .., N-1\}$ and

$$\mathbb{E}\left[h_2 \quad (\theta - p_2) |\Omega_1 \quad \right] =$$

$$\frac{1}{\lambda_2}\mathbb{E}\left[\frac{(e-c)\lambda_1 \left(\sum \,_{=1}^{-1} h_1 \quad ^n + h_1 \quad\right)}{(N+1)}\left(\frac{\theta - p_0}{N} - \frac{Nc + e}{N(N+1)}\lambda_1\left(\sum_{=1}^{-1} h_1 \quad ^n + h_1 \quad\right)\right)|\Omega_1 \quad \right].$$

Using this, (6) and (3) in (2) we find the period 1 demand as

$$\begin{aligned} h_1 \quad &= 0 \\ h_1 \quad ^n &= \frac{(\theta - p_0)}{(N-1)\lambda_1 \Gamma} \text{ for } n \in \{1, .., N-1\}, \end{aligned}$$

where $\Gamma \quad \equiv \left(1 - \frac{2}{2} \frac{+}{+1} \frac{P_1}{P_2}\right)^{-1} \left(\frac{}{-1} - \frac{2}{2}\left(\frac{+}{+1}\right)^2 \frac{P_1}{P_2}\right)$.

Uninformed P knows that period type K traders demand $h_2 \quad ^n \; h_1 \quad ^n$. Hence he observes price signal $\tilde{p}_1 = p_0 + \Gamma \quad (p_1 - p_0)$, where $\tilde{p}|\theta \sim \mathcal{N}(\theta, \Gamma^2 \,_1 \lambda_1^2 \sigma^2)$. Using Bayes rule, we obtain that

$$\mathbb{E}\left[\theta|\Omega_2 \quad \right] = p_0 + e\,(p_1 - p_0), \text{ where } e = \frac{\frac{1}{\Gamma_U P_1^2 e_s^2}}{\frac{1}{e_p^2} + \frac{1}{\Gamma_U^2 P_1^2 e_s^2}} \tag{25}$$

Conditional on state $R = I$ the Market observes order flow signal $\tilde{y}_1 = \lambda_1 \Gamma \; y_1 + p_0$, where $\tilde{y}_1|\theta \sim \mathcal{N}(\theta, \Gamma^2 \lambda_1^2 \sigma^2)$. Conditional on state $R = U$, we observes $\tilde{y}_1 \; = \Gamma \; \lambda_1 y_1 + p_0$, where $\tilde{y}_1 \,|\theta \sim \mathcal{N}(\theta, \Gamma^2 \; \lambda_1^2 \sigma^2)$ in date 1. In date 2 the signals conditional on state $R = I$ and $R = U$ the signals are as in Appendix A respectively $\tilde{y}_2 \; \equiv \frac{}{-1}\lambda_2 y_2 + p_0 + \left(c - \frac{-}{2-1}\right)(p_1 - p_0)$, where $\tilde{y}_2 \,|\theta \; \sim \; \mathcal{N}\left(\theta, \left(\frac{}{-1}\right)^2 \lambda_2^2 \sigma^2\right)$ and $\tilde{y}_2 \; \equiv \; \frac{+1}{}\lambda_2 y_2 + p_0 + c\,(p_1 - p_0)$, where $\tilde{y}_2 \,|\theta \; \sim \mathcal{N}\left(\theta, \left(\frac{+1}{}\right)^2 \lambda_2^2 \sigma^2\right)$.

Let us define $L_1 \equiv \lambda_1 \frac{e_s}{e_p}$ and $L_2 \equiv \lambda_2 \frac{e_s}{e_p}$. Using the Bayes rule to update the Market's expectations in states $R = I$ and $R = U$ we find $\mathbb{E}\left[\theta|\Omega \quad , R = I\right]$ and $\mathbb{E}\left[\theta|\Omega \quad , R = U\right]$ for $t = \{1, 2\}$. Using these in the equation (5), i.e., the market efficiency condition under Assumption 1, we verify the price guess (6).

Equating the coeficients, the equilibrium coeficients L_1, L_2, c and e solve the following

system

$$\begin{cases} \eta \frac{\frac{1}{\Gamma_I}}{\frac{2}{1}+\frac{1}{\Gamma_I^2}} + (1-\eta)\frac{\frac{1}{\Gamma_U}}{\frac{2}{1}+\frac{1}{\Gamma_U^2}} = 1 \\ \eta \frac{\frac{N}{N+1}}{\frac{2}{2}+\frac{1}{\Gamma_I^2}\left(\frac{L_2}{L_1}\right)^2+\left(\frac{N}{N+1}\right)^2} + (1-\eta)\frac{\frac{N-1}{N}}{\frac{2}{2}+\frac{1}{\Gamma_U^2}\left(\frac{L_2}{L_1}\right)^2+\left(\frac{N-1}{N}\right)^2} = 1 \\ c\frac{1}{+1} = \eta \frac{\frac{1}{\Gamma_I}\left(\frac{L_2}{L_1}\right)^2}{\frac{2}{2}+\frac{1}{\Gamma_I^2}\left(\frac{L_2}{L_1}\right)^2+\left(\frac{N}{N+1}\right)^2} + (1-\eta)\frac{\frac{1}{\Gamma_U}\left(\frac{L_2}{L_1}\right)^2-\frac{(N-1)}{N^2(N+1)}}{\frac{2}{2}+\frac{1}{\Gamma_U^2}\left(\frac{L_2}{L_1}\right)^2+\left(\frac{N-1}{N}\right)^2} \\ e = \frac{\frac{1}{\Gamma_U}}{\frac{2}{1}+\frac{1}{\Gamma_U^2}} \end{cases}, \tag{26}$$

where $\Gamma \equiv \left(1-\frac{2}{(\ +1)^2}\frac{1}{2}\right)^{-1}\left(\frac{}{+1}-\frac{2\ 2}{(\ +1)^2}\frac{1}{2}\right)$ and
$\Gamma \equiv \left(1-\frac{2}{2}\frac{+}{+1}\frac{1}{2}\right)^{-1}\left(\frac{}{-1}-\frac{2}{2}\left(\frac{+}{+1}\right)^2\frac{1}{2}\right)$

D.1 Proof of Proposition 5

Assume that P's type is known to be uninformed, i.e., $\eta = 0$. From (26) we then obtain that $L_1^2 = \frac{\Gamma_U-1}{\Gamma_U^2}$ and $L_2^2 = \frac{-1}{2}\frac{\Gamma_U-1}{\Gamma_U}$. From here $\frac{\frac{2}{2}}{1} = \frac{-1}{2}\Gamma$. Given L_1^2, it is clear that $e = 1$. Furthermore, using these results in the expression for c,we obtain that $c = 1$. As uninformed P's strategy is from (14) $h_2 = \frac{(\ -\ )(\ _1-\ _0)}{(\ +1)P_2}$, it $e = c = 1$ proves that P's optimal trading strategy is zero also in period 2, if he is known to be uninformed with certainty. As P's expectations are given by (25), it holds that $\mathbb{E}\left[\theta|\Omega_2\ \right]|_{K=0} = p_0 + e\,(p_1 - p_0)$

If P's type is not known with certainty, P's and the Market's expectations will be different. We can prove it as follows. Note that by the market efficiency condition period 1 price equals the $p_1 = \mathbb{E}\left[\theta|\Omega_1\ \right]$, while by (25) uninformed P's expectations are $\mathbb{E}\left[\theta|\Omega_2\ \right] = p_0 + e\,(p_1 - p_0)$. P's expectations of the fundamental will be different than the Market's expectations if and only iff $e \neq 1$. We can prove that this does not hold for $\eta > 0$ by contradiction.

Assume that $e = 1$ for $\eta > 0$. From the last equation in (14), we obtain that $e = 1 \Longrightarrow L_1^2 = \frac{\Gamma_U-1}{\Gamma_U^2}$. Given this, the first equation in (14) becomes $\eta\frac{\frac{1}{\Gamma_I}}{\frac{2}{1}+\frac{1}{\Gamma_I^2}} + (1-\eta) = 1 \Longrightarrow L_1^2 = \frac{\Gamma_I-1}{\Gamma_I^2}$. This implies that $\frac{\Gamma_U-1}{\Gamma_U^2} = \frac{\Gamma_I-1}{\Gamma_I^2}$. Using these in the second and equation of (14), we obtain that $\eta\left(\frac{\frac{N}{(N+1)^2}-\frac{2}{2}\left(\frac{\Gamma_I}{\Gamma_I-1}\right)}{\frac{2}{2}\left(\frac{\Gamma_I}{\Gamma_I-1}\right)+\left(\frac{N}{N+1}\right)^2}\right) + (1-\eta)\left(\frac{\frac{N-1}{N^2}-\frac{2}{2}\left(\frac{\Gamma_U}{\Gamma_U-1}\right)}{\frac{2}{2}\left(\frac{\Gamma_U}{\Gamma_U-1}\right)+\left(\frac{N-1}{N}\right)^2}\right) = 0$. For $0 < \eta < 1$, it must hold that $\frac{\frac{N}{(N+1)^2}-\frac{2}{2}\left(\frac{\Gamma_I}{\Gamma_I-1}\right)}{\frac{2}{2}\left(\frac{\Gamma_I}{\Gamma_I-1}\right)+\left(\frac{N}{N+1}\right)^2} > 0 > \frac{\frac{N-1}{N^2}-\frac{2}{2}\left(\frac{\Gamma_U}{\Gamma_U-1}\right)}{\frac{2}{2}\left(\frac{\Gamma_U}{\Gamma_U-1}\right)+\left(\frac{N-1}{N}\right)^2}$ or $\frac{\frac{N}{(N+1)^2}-\frac{2}{2}\left(\frac{\Gamma_I}{\Gamma_I-1}\right)}{\frac{2}{2}\left(\frac{\Gamma_I}{\Gamma_I-1}\right)+\left(\frac{N}{N+1}\right)^2} < 0 < \frac{\frac{N-1}{N^2}-\frac{2}{2}\left(\frac{\Gamma_U}{\Gamma_U-1}\right)}{\frac{2}{2}\left(\frac{\Gamma_U}{\Gamma_U-1}\right)+\left(\frac{N-1}{N}\right)^2}$ or $\frac{\frac{N}{(N+1)^2}-\frac{2}{2}\left(\frac{\Gamma_I}{\Gamma_I-1}\right)}{\frac{2}{2}\left(\frac{\Gamma_I}{\Gamma_I-1}\right)+\left(\frac{N}{N+1}\right)^2} = \frac{\frac{N-1}{N^2}-\frac{2}{2}\left(\frac{\Gamma_U}{\Gamma_U-1}\right)}{\frac{2}{2}\left(\frac{\Gamma_U}{\Gamma_U-1}\right)+\left(\frac{N-1}{N}\right)^2} = 0$. Simplifying we obtain that that the first two can never hold, because it implies $0 > 0$ or $0 < 0$. Therefore it must hold that $L_2^2 = \frac{}{(\ +1)^2}\frac{\Gamma\ -1}{\Gamma_I} = \frac{-1}{2}\frac{\Gamma_U-1}{\Gamma_U}$. We then obtain that $\frac{\frac{2}{2}}{1} = \frac{}{(\ +1)^2}\Gamma\ = \frac{-1}{2}\Gamma$. Using this in the third equation of

(14) we obtain that $c = 1$. Also from $\frac{}{(\ +1)^2}\Gamma\ = \frac{-1}{2}\Gamma$ and $\frac{\Gamma_U-1}{\Gamma_U^2} = \frac{\Gamma_I-1}{\Gamma_I^2}$ we obtain that $\Gamma\ = \frac{^3+(\ +1)^2(\ -1)}{^3}$ and $\frac{^2_2}{^2_1} = \frac{^3+(\ +1)^2(\ -1)}{^2(\ +1)^2}$. But then using these results in the definition of Γ , we obtain that $\Gamma\ = \left(\frac{}{+1} - \frac{2}{(\ +1)^2}\sqrt{\frac{^3+(\ +1)^2(\ -1)}{^2(\ +1)^2}}\right)\left(1 - \frac{2}{(\ +1)^2}\sqrt{\frac{^3+(\ +1)^2(\ -1)}{^2(\ +1)^2}}\right)^{-1}$, which contradicts Γ we found before. Therefore it must hold that $e \neq 1$ if for $\eta > 0$ and there is price-contingent trading.

D.2 Solution for $\eta \to 1$ case

We now consider the case where $\eta \to 1$. Then (14) implies that $L_1^2 = \frac{\Gamma_I-1}{\Gamma_I^2}$ and $L_2^2 = \frac{}{(\ +1)^2}\frac{\Gamma_I-1}{\Gamma_I}$ and $\frac{^2_2}{^2_1} = \frac{}{(\ +1)^2}\Gamma$. We also obtain that $c = 1$ and $e = \frac{\frac{1}{\Gamma_U}}{\frac{\Gamma_I-1}{\Gamma_I^2}+\frac{1}{\Gamma_U^2}} = \frac{\Gamma_I^2\Gamma_U}{\Gamma_U^2(\Gamma_I-1)+\Gamma_I^2}$. From here $e > 1$ if $\Gamma^2(\Gamma\ -1) > \Gamma^2(\Gamma\ -1)$. This can also be written as $L_1^2 < \frac{\Gamma_U-1}{\Gamma_U^2} = \hat{L}_1^2$, where $\hat{L}_1$ is the solution of the case where P is known to be uninformed (see Appendix E.1). Using the definition of $\Gamma\ = \left(1 - \frac{2}{(\ +1)^2}\frac{1}{2}\right)^{-1}\left(\frac{}{+1} - \frac{2}{(\ +1)^2}\frac{1}{2}\right)$, we can solve $\left(1 - \frac{2}{(\ +1)^2}\frac{1}{2}\right) = \left(\frac{1}{+1} - \frac{2}{(\ +1)^4}\frac{1}{2}\right)\left(\frac{1}{2}\right)^2$ for $\frac{1}{2}$. We can then find $L_1 = \sqrt{\frac{\Gamma_I-1}{\Gamma_I^2}}$ and $L_2 = \sqrt{\frac{}{(\ +1)^2}\frac{\Gamma_I-1}{\Gamma_I}}$ and solve for e using that $e = \frac{\frac{1}{\Gamma_U}}{^2_1+\frac{1}{\Gamma_U^2}}$, where

$\Gamma\ = \left(1 - \frac{2}{2}\frac{+}{+1}\frac{1}{2}\right)^{-1}\left(\frac{}{-1} - \frac{2}{2}\left(\frac{+}{+1}\right)^2\frac{1}{2}\right)$ and $e = \frac{\frac{1}{\Gamma_U}}{^2_1+\frac{1}{\Gamma_U^2}}$. It is clear that $e > 1$ imples that P's optimal trading startegy is positive feedback as $h_2\ = \frac{(\ -1)(\ _1-\ _0)}{(\ +1)P_2}$ in this case.

Here are the results for different values of N.

N	$\frac{1}{2}$	L_1	L_2	$\hat{L}_1$	e
2	1.593	0.496	0.311	0.405	0.717
3	1.925	0.460	0.239	0.499	1.074
4	2.190	0.422	0.193	0.464	1.057
5	2.418	0.390	0.161	0.425	1.038
6	2.623	0.364	0.139	0.392	1.027
7	2.812	0.342	0.122	0.365	1.019
8	2.986	0.324	0.108	0.343	1.015

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