International Asset Pricing with Recursive Preferences

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Abstract

Focusing on US and UK, we document that both the Backus and Smith (1993) finding—concerning the low correlation between consumption differentials and exchange rates—and the forward-premium anomaly—concerning the tendency of high interest rate currencies to appreciate—have become more severe through time. After accounting for different capital mobility regimes, we show that these anomalies turn into general equilibrium regularities in a two-country and two-good economy with Epstein and Zin (1989) preferences, frictionless markets, and correlated long-run growth prospects.

JEL classification: C62; F31; G12.
First Draft: February 14, 2008. *This draft*: March 13, 2012.

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1 Introduction

Two of the best established empirical facts in international finance are the tendency of high interest rate currencies to appreciate (forward premium anomaly, Fama (1984)), and the lack of correlation between consumption growth differentials and exchange rate movements (Backus and Smith (1993) anomaly). The first one contradicts the logic of the basic Uncovered Interest Rate Parity (UIP) relationship, while the second one is at odds with the condition impelled by any model with standard time-additive preferences and complete markets.

A less well known aspect is the dramatic change in the behavior of these empirical regularities before and after the early 1970s. Focusing on real data from the US and UK, in the left panel of figure 1 we document that the UIP regression coefficient is remarkably close to unity in the pre-1970 sample and then it sharply falls right after 1970. In the right panel of figure 1, we depict the time-varying correlation of real consumption growth differentials and real exchange rate movements. Similarly to the UIP regression coefficient, this correlation experiences a sizable decline right after 1970.

These empirical findings raise at least two relevant questions. First, how can we simultaneously explain the almost complete lack of correlation between exchange rates and consumption differentials and the negative UIP regression coefficient in the post-1970 sample? Second, how do we justify the drop of these two statistics across the two sub-samples? Providing a coherent answer to these questions is the main challenge of our study.

To explain these dynamics, we propose a general equilibrium model featuring three crucial elements: (i) recursive preferences as in Epstein and Zin (1989); (ii) existence of highly correlated international long-run components in output, and (iii) a substantial increase in capital mobility going from the pre- to the post-1970 sample. After explaining in detail the role of each element for the dynamics of both international quantities and prices, we show that our economy replicates a wide set of international moments across the two samples. Our model, therefore, offers the first unified framework to address international quantity and price dynamics across capital mobility regimes. Our novel general equilibrium approach is suitable to study several

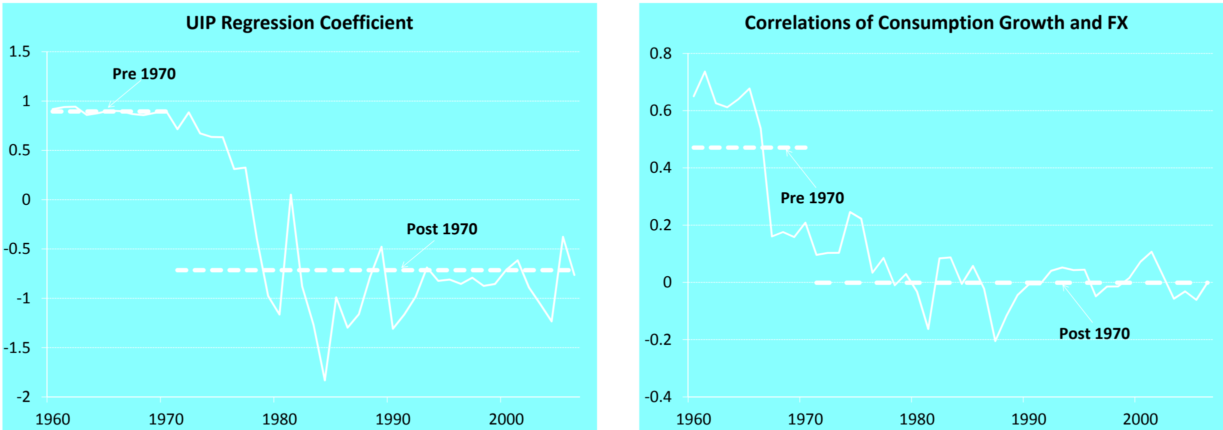

FIG. 1 - UIP coefficients and Backus and Smith correlation across regimes. The left panel reports the UIP coefficient estimated using a rolling sample of 30 years. The right panel shows the correlation between consumption growth differentials and exchange rate growth.

phenomena in international finance, ranging from the effects of financial integration (see for example Bekaert, Harvey and Lundblad (2005)) to currency risk premia (see for example Lustig, Roussanov and Verdelhan (2011a)).

In particular, we start by characterizing international trade and asset prices in a frictionless two country-two good economy in which markets are complete both domestically and internationally. We compare this setup to the data for the post-1970 sample, a period of substantial financial integration across all major industrialized countries (among others see Taylor (1996), Quinn (1997) and Obstfeld (1998)). This step is standard in the literature, except for the fact that our agents have recursive preferences and entertain a novel risk-sharing mechanism.

To characterize the trading arrangement of our agents, we generalize the methods developed by Colacito and Croce (2010) to the case in which the intertemporal elasticity of substitution (IES) differs from one. In this class of economies, investors care not only about expected utility, but also about utility variance (see equation 3 for the specific functional form). Specifically, agents dislike uncertainty about their future utility (or wealth), as long as the intertemporal elasticity of substitution exceeds the reciprocal of the coefficient of risk aversion. This additional channel introduces an endogenous trade-off between first and second moments, which is otherwise absent

in a model with standard preferences. An expected wealth mean-variance frontier arises in equilibrium. This frontier becomes steeper, hence more relevant, when the intertemporal elasticity of substitution increases and when endowments feature long-run uncertainty.

According to this endogenous trade-off, the country that is expected to grow more than the other is willing to reduce its share of world consumption, and hence its expected utility, in order to reduce the uncertainty about its future wealth. By doing so, this country becomes less risky going forward. This implication enables our model to account for the forward premium anomaly. In fact, the country with higher expected growth rate simultaneously features relatively higher interest rate and safer consumption profile. By no arbitrage, its currency becomes more valuable and is expected to appreciate, despite its higher interest rate.

Qualitatively, the mechanism described above arises just because of recursive preferences. Quantitatively, our expected wealth mean-variance trade off becomes relevant only when combined with long-run risks. Indeed, we show that in a regime of financial integration, the role of long-run risk is twofold. On the one hand it amplifies utility risk and risk-sharing motives. On the other hand, it breaks the perfect correlation between short-run consumption differentials and exchange rate growth and it contributes to the resolution of the Backus-Smith anomaly.

Let us focus, for example, on the realization of good long-run news to the supply of the home good. The continuation utility of the home country increases as she anticipates more consumption going forward. Since our agents are averse to utility risk in addition to consumption risk, the increase in future utility reduces the intertemporal marginal rate of substitution of the home country. Such decline has two effects: (i) by no arbitrage, the home currency depreciates in anticipation of a higher supply of the home good, and (ii) resources are optimally reallocated from the home country (low marginal utility country) to the foreign one (high marginal utility country). Since long-run news do not change short-run endowments, this reallocation produces an increase in foreign consumption growth and a decrease in home consumption. Hence, consumption growth differential and exchange rate growth move in opposite directions. If the IES is high enough this channel lowers the Backus-Smith correlation bringing it close to the data.

This novel theoretical result introduces a currency-based lower bound on the IES that complements the closed economy lower-bound found by Bansal and Yaron (2004). Interestingly, our model also sets an upper bound on this elasticity. As the IES gets larger, agents are so adverse to utility risk that they are willing to make the risk-sharing scheme more aggressive in order to bear less utility risk. In this case, the endogenous reduction of future utility risk promoted by trade dampens the impact of long-run endowment shocks on consumption growth, ultimately reducing the ability of the model to reproduce the Backus-Smith anomaly. We find that values of IES in a neighborhood of 1.5 can closely match the data.

When focusing on the pre-1970 period, we assume that the two countries trade only goods, but cannot exchange international assets and borrow from (lend to) each other. In the language of Cole and Obstfeld (1991), they are in a regime of financial autarky. In this setup, intertemporal risk-sharing cannot take place. Risk sharing consists only in smoothing short-run consumption by exchanging goods whose value is perfectly balanced in every period and state of the world. Since this implies that the share of world consumption of each country is fixed, the utility mean-variance trade-off which characterizes complete markets is inactive. In this regime, the model predicts that the UIP regression coefficient and the Backus-Smith correlation are in a neighborhood of one. When we compare the data in the pre- and post-1970 sample to our results with and without financial integration,we find that our model conforms well with our empirical evidence.

Furthermore, in financial autarky the exchange rate growth is just a reflection of the difference in the short-run growth rate of the endowments of the two goods. Since endowments are smooth, the exchange rate is smooth as well. With financial integration, instead, international trading of securities produces a substantial amount of pressure on currency making it twice as volatile. Overall, our model captures almost entirely the increase in the volatility of the real exchange rate observed in the post-1970 data.

Last but not least, we provide empirical evidence for the key mechanism of the model. Using a predictive regressions approach pioneered by Harvey (1986), and Harvey (1988) and subsequently used by Bansal, Kiku and Yaron (2010), and Colacito and Croce (2011), we estimate short- and long-run shocks to GDP growth rates for US and UK. We find that upon the realization of good long-run news to domestic output,

domestic consumption growth declines. This empirical finding, which appears to contradict the logic of the permanent income hypothesis, finds theoretical foundations in our model: the prospect of a more sustained long-term growth prompts a country to redistribute abroad part of its resources in exchange for a smoother wealth profile.

We also document that the relative riskiness of a country, as measured by the realized variance of its stock market's excess return, declines when good news to its output materialize. This effect is stronger for long-run news, consistent with our model. Additionally, news to output growth appear to have strong predictive power on the stock market excess returns spread across countries. The data support the model's prediction that countries receiving relatively better news to their output growth become less risky and experience a reduction in their local expected returns.

The paper is organized as follows. The next two sections describe preferences, endowments, and financial markets under autarky and perfect integration, respectively. We explains our novel risk sharing mechanism in section 4 and connect it to the forward premium anomaly and the Backus and Smith puzzle in section 5. Section 6 highlights the role of the IES. Section 7 documents our empirical investigation and section 8 concludes.

2 The economy: preferences and endowments

The economy consists of two countries, home (h) and foreign (f), and two goods, X and . Agents' preferences are defined over consumption aggregates of the two goods at each history as follows.

Consumption aggregate. Let ${}^i_t$ and y^i_t denote the consumption of good X and good in country $\in \{ \ , \ \}$ at date t. Let $\alpha \in (0, 1)$. The consumption aggregates in the two countries are:

$$C^h_t = \left({}^h_t \right)^\alpha \left(y^h_t \right)^{1-\alpha} \quad \text{and} \quad C^f_t = \left({}^f_t \right)^{1-\alpha} \left(y^f_t \right)^\alpha , \tag{1}$$

respectively. The parameter α captures the degree of bias of the consumption of each representative agent. In what follows we assume that the home country is endowed

with good X, while the foreign country is endowed with good . Following the international macro-finance literature surveyed by Lewis (2011), we assume that is larger than $1/2$. This allows us to build consumption home bias into the model.

Preferences. As in Epstein and Zin (1989), agents' preferences are recursive but non-time separable:

$$ {}^{i}_{t} = \left[(1-\delta)\cdot(C)^{1-1/\psi} + \delta \; {}_{t}\left[\left({}^{i}_{t+1}\right)^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \quad \forall \in \{ , \}. \tag{2}$$

The coefficients and measure the relative risk aversion (RRA) and the intertemporal elasticity of substitution (IES), respectively.

The main departure from the constant relative risk aversion case often analyzed in the literature lies in the fact that these preferences allow agents to be risk averse in future utility in addition to future consumption. The extent of such utility risk aversion depends on the preference for early resolution of uncertainty measured by $-1/ \quad 0$. To better highlight this feature of the preferences, we focus on the ordinally equivalent transformation

$$ {}_{t} = \frac{{}_{t}^{1-1/\psi}}{1-1/} $$

and document in Appendix B that

$$\begin{aligned} {}_{t} &= \frac{C^{1-1/\psi}}{1-1/} + \delta \; {}_{t}\left[{}^{1-\theta}_{t+1}\right]^{\frac{1}{1-\theta}} \\ &\approx \frac{C^{1-1/\psi}}{1-1/} + \delta \; {}_{t}\left[{}_{t+1}\right] - \frac{\delta}{2}\frac{\theta}{{}_{t}\left[{}_{t+1}\right]} \; r_{t}\left[{}_{t+1}\right], \end{aligned} \tag{3}$$

where $\theta \equiv \frac{\gamma-1/\psi}{1-1/\psi}$. Note that the sign of $\left(\frac{\theta}{E_t[V_{t+1}]}\right)$ depends on the sign of $(\; -1/ \;)$. When $=1/$, the agent is utility-risk neutral and preferences collapse to the standard time additive case. When the agent prefers early resolution of uncertainty, $1/$, the coefficient θ is positive: uncertainty about continuation utility reduces welfare and generates an incentive to trade-off future expected utility, ${}_{t}[\; {}_{t+1}]$, for future utility risk, $r_{t}[\; {}_{t+1}]$. This mean-variance trade-off is a very appealing feature

of these preferences and yet one that is otherwise absent when agents have standard time-additive preferences. This trade-off drives international allocations and exchange rate adjustments in our economy with complete markets and it represents the most important element of our analysis. Our study is the first one to fully characterize trade with Epstein and Zin (1989) preferences.

Since there is a one-to-one mapping between utility, $\ {}_t^i$, and life-time wealth, i.e., the value of a perpetual claim to consumption, $W_{c,t}^i$,

$$\ {}_t^i = \left[(1-\delta)(C + W_{c,t}^i)\right]^{\frac{1}{1-1/\psi}}, \quad \forall \ \in \{\ ,\ \}, \tag{4}$$

the optimal risk-sharing scheme can also be interpreted in terms of mean-variance trade-off of wealth. For this reason, in what follows we will use the terms 'wealth' and 'continuation utility' interchangeably.

Endowments. We choose to endow each country with the stochastic supply of its most preferred good (X for home and for foreign). Endowments are co-integrated processes and embody predictive variables as follows:

$$\begin{aligned} \log X_t &= \ {}_x + \log X_{t-1} + z_{1,t-1} + \tau \cdot (\log \ {}_{t-1} - \log X_{t-1}) + \varepsilon_{x,t} \\ \log \ {}_t &= \ {}_y + \log \ {}_{t-1} + z_{2,t-1} - \tau \cdot (\log \ {}_{t-1} - \log X_{t-1}) + \varepsilon_{y,t} \end{aligned} \tag{5}$$

where $\tau \in (0,1)$ determines the extent of co-integration, and z_1 and z_2 are modeled as highly persistent AR(1) processes,

$$z_{j,t} = \rho_j z_{j,t-1} + \varepsilon_{j,t}, \forall j \in \{1,2\}\,. \tag{6}$$

Throughout the paper, we shall refer to $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ as the long-run shocks, due to their long-lasting impact on the growth rates of the two endowments. Similarly, we will call $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ as the short-run shocks. Shocks are jointly log-normal: $\ {}_t \equiv \begin{bmatrix} \varepsilon_{1,t} & \varepsilon_{2,t} & \varepsilon_{x,t} & \varepsilon_{y,t} \end{bmatrix} \sim_{i.i.d.} \ (\mathbf{0}, \Sigma)$.

We abstract from exogenous time-varying volatility in endowments to better quantify the amount of endogenous consumption and asset prices volatility generated by our recursive risk-sharing mechanism with complete markets. By doing so, we provide

a general equilibrium foundation for the exogenous time-varying consumption risk examined by Colacito (2008) and Bansal and Shaliastovich (2010). We also provide a general equilibrium explanation of the findings obtained by Lustig and Verdelhan (2007), Lustig et al. (2011a), and Lustig, Roussanov and Verdelhan (2011b).

3 Markets' Structures

In order to understand the impact of market integration on the joint dynamics of international asset prices and quantities, we consider two markets' structures. We take the stand that the pre-1970 sample can be characterized as a regime of portfolio autarky, while the post-1970 sample features instead complete markets. We start by describing the equilibrium under portfolio autarky.

3.1 Portfolio Autarky

We follow Cole and Obstfeld (1991) in defining portfolio autarky as a ban on the trade of international securities. Trade in the goods' market is allowed and it has to be balanced in every period. Financial markets are complete only within each country. Let p_t denote the price of good in terms of good X. Since state-contingent securities are in zero net supply, the budget constraints are

$$\begin{aligned} {}_t^h + p_t y_t^h &= X_t \\ {}_t^f + p_t y_t^f &= p_t \ {}_t. \end{aligned} \tag{7}$$

for the home and foreign country respectively.

Allocations and Prices. In this setup, agents have no room for intertemporal consumption smoothing, because no international trade of securities can take place. In equilibrium, allocations behave as in the case of standard time-additive preferences:

$$\begin{aligned} {}_t^h = \alpha X_t, \qquad {}_t^f = (1-\alpha)X_t \\ y_t^h = (1-\alpha) \ {}_t, \qquad y_t^f = \alpha \ {}_t. \end{aligned} \tag{8}$$

Since international financial markets are shut down, the real exchange rate simply reflects the current relative supply of the two goods adjusted for the degree of home bias:

$$\Delta\ _t \quad = \quad (2\ - 1)\,(\Delta\ \ _t - \Delta y_t)\,. \tag{9}$$

3.2 Complete Markets

At each date, trade occurs in a complete set of one period ahead claims to state-contingent consumption. Financial and goods' markets are assumed to be frictionless. The budget constraints of the two agents can be written as:

$$\begin{aligned} {}^{h}_{t} + \ _t y_t^h + \int_{\zeta^{t+1}} A_{t+1}^h\left(\ ^{t+1}\right)\ _{t+1}(\ ^{t+1}) &= A_t^h + X_t \\ {}^{f}_{t} + \ _t y_t^f + \int_{\zeta^{t+1}} A_{t+1}^f\left(\ ^{t+1}\right)\ _{t+1}(\ ^{t+1}) &= A_t^f + \ _t\ \ _t, \end{aligned} \tag{10}$$

where $\ _t$ denotes the relative price of good and good X (the terms of trade), $A_t^i\,(\ ^t)$ denotes country 's claims to time t consumption of good X, and $\ ^h_{t+1}(\ ^{t+1})$ gives the price of one unit of time $t+1$ consumption of good X contingent on the realization of $\ ^{t+1}$ at time $t+1$. In equilibrium, the market for international state contingent claims clears, implying that $A_t^h + A_t^f = 0, \forall t$.

Allocations. Since markets are complete, we can compute efficient allocations by solving the associated Pareto problem. The planner attaches date 0 nonnegative Pareto weights $\ ^h =\ $ and $\ ^f = 1 -\ $ to the consumers and chooses the sequence of allocations $\left\{\ ^h_t,\ \ ^f_t, y_t^h, y_t^f\right\}_{t=0}^{+\infty}$ to maximize

$$\Lambda = \ \cdot\ \ ^h_0 + (1 -\ )\cdot\ \ ^f_0$$

subject to the sequence of economy wide feasibility constraints:

$$\begin{aligned} {}^h_t + \ ^f_t &= X_t \\ y_t^h + y_t^f &= \ \ _t, \quad \forall t \geq 0, \end{aligned}$$

where the state-dependent notation is omitted for the sake of clarity. In characteriz-

ing the equilibrium, we follow Anderson (2005) and formulate the problem using the ratio of time-varying pseudo-Pareto weights, $S_t = \lambda_t/(1-\lambda_t)$, as an additional state variable. This technique enables us to take into account the non-separability of the utility functions. We show in Appendix C that the first order necessary conditions imply the following allocations:

$$x_t^h = \alpha X_t \left[1 + \frac{(1-\alpha)(S_t - 1)}{1 - \alpha + \alpha S_t}\right], \qquad x_t^f = (1-\alpha) X_t \left[1 - \frac{\alpha(S_t - 1)}{1 - \alpha + \alpha S_t}\right] \tag{11}$$
$$y_t^h = (1-\alpha) Y_t \left[1 + \frac{\alpha(S_t - 1)}{\alpha + (1-\alpha) S_t}\right], \qquad y_t^f = \alpha Y_t \left[1 - \frac{(1-\alpha)(S_t - 1)}{\alpha + (1-\alpha) S_t}\right]$$

where

$$S_t = S_{t-1} \cdot \frac{M_t^h}{M_t^f} \cdot \left(\frac{C_{t+1}^h / C_t^h}{C_{t+1}^f / C_t^f}\right), \quad \forall t \geq 1 \tag{12}$$

and $S_0 = 1$, as we start the economy from an identical allocation of wealth and endowments. This is consistent with the ergodic distribution of the model, which implies that on average the two countries consume an identical share of world resources because of symmetry.

By comparing the system of equations (8) and (11), we can see that the allocations under portfolio autarky equal those under complete markets if $S_t = 1$. Financial integration hence differs from the portfolio autarky regime because it features a time-varying share of world resources which is measured by S_t and that evolves according to equation (12). This extra component changes the dynamics of consumption in at least three significant ways. First, it endogenously introduces an additional slowly moving predictive component of consumption growth, in addition to the two exogenous long-run risks. In section 4, we show that this endogenous component guarantees highly correlated pricing kernels even in the absence of international long-run risks.

Second, S_t introduces an endogenous time-varying volatility term in consumption growth, since allocations are non-linear functions of this component. In section 5.1 we further discuss the importance of this channel in the context of our explanation of the forward premium anomaly.

Third, consumption responds immediately to news about future long-run growth,

through the endogenous adjustment of S_t. This differs from the basic Bansal and Yaron (2004) model, in which there is only a lagged response. We show the properties of consumption in our model in section 5.2, and document that US and UK data provide strong support for our findings in section 7.

Prices. The stochastic discount factor that is used to discount future uncertain payoffs is

$$M^i_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U^{i1-\gamma}_{t+1}}{E_t \left[U^{i1-\gamma}_{t+1} \right]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{13}$$

Since markets are assumed to be complete, the log growth rate of the real exchange rate is:

$$\Delta q_t = \log M^f_t - \log M^h_t. \tag{14}$$

4 Equilibrium with complete markets: economic interpretation

The equilibrium with complete markets is non-standard because our agents have recursive preferences. In particular, equations (11)-(14) suggest that a complete analysis of the joint dynamics of allocations and asset prices requires a full characterization of the dynamics of S_t, the relative share of world consumption.

According to the system of equations (11), the home share of world consumption is monotonic in S_t, meaning that when S_t declines, the home country exports more and imports less. While this relation is intuitive, explaining how and why S_t moves requires further discussion.

How does S_t move? In figure 2, we depict the response of the logarithm of the share of world consumption, $s_t = \log(S_t)$, to positive short- and long-run shocks to the supply of good X. Assuming that no shock affects the supply of the foreign good, good news to the supply of good X imply a persistent reduction in the domestic share of world consumption. This countercyclical adjustment is consistent with equation (12): as relative good news to the supply of good X materialize, the home country experiences a drop in her marginal utility. Therefore, it is optimal to reallocate resources

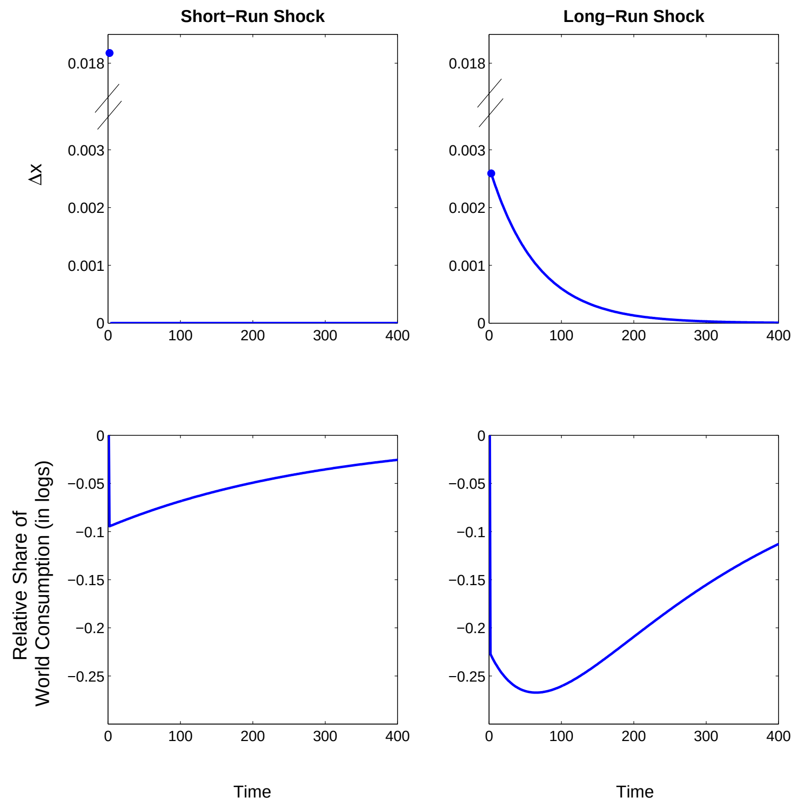

FIG. 2 - Ratio of Pareto Weights when IES = 1.5. This figure shows the impulse response function of the log-ratio of the Pareto weight, $s_t = \log \frac{\mu_t^h}{1-\mu_t^h}$. All parameters are calibrated to the values reported in Table 1 for specification (1). Shocks to the home country good endowment, Δ , materialize at time 1.

toward the foreign country. In the decentralized economy, the home country optimally substitutes part of her current consumption with exports toward her foreign trading partner.

By comparing the magnitude of the responses of s_t across the two shocks, we see that even very small long-run shocks can produce a substantial and near permanent adjustment in the ratio of the Pareto weights, indeed promoting significant risk-sharing for the long-run.

Why does S_t move? The dynamics of the model are driven by the tension between expected utility and utility variance that we have highlighted in equation (3). Agents with recursive preferences are willing to trade-off higher expected future utility level,

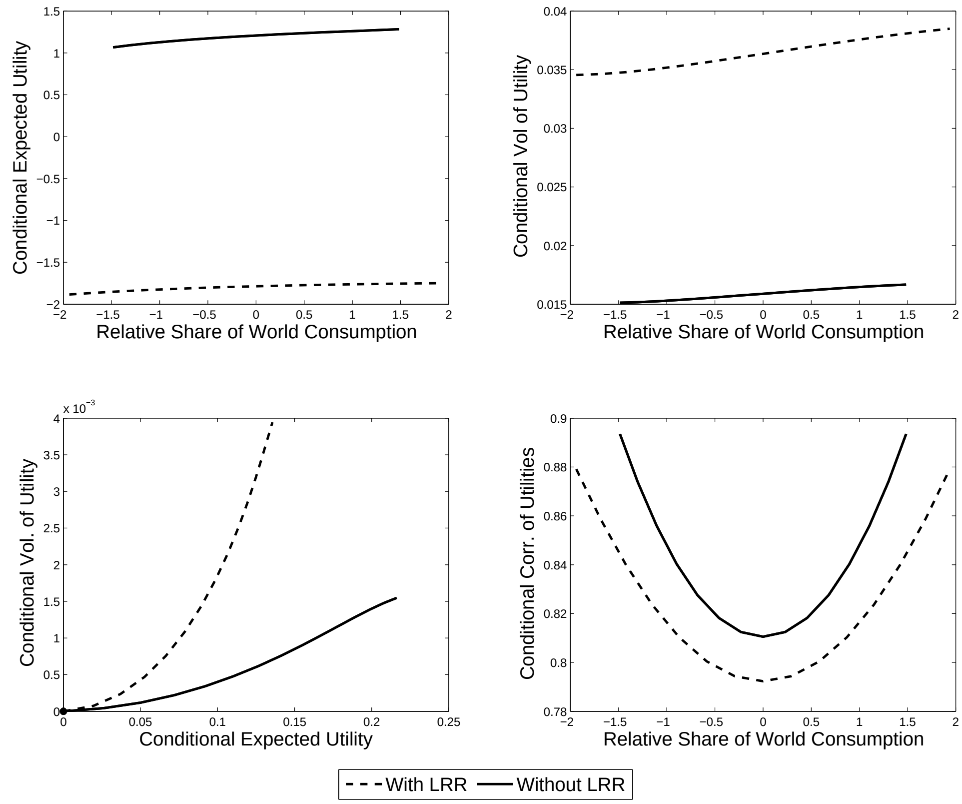

FIG. 3 **- Utility mean-variance frontier and long-run risk.** This figure shows the utility mean-variance frontier for the home country with and without long-run risks. By symmetry, a similar figure applies to the foreign country. The IES is set to 1.5, all other parameters are calibrated to the annual values reported in Table 1. All moments refer to the stationary process $U^h_{t+1}/(X^{\alpha}_t Y^{1-\alpha}_t)$. The relative share of World Consumption is measured by the log ratio of the Pareto weights, s_t.

$_t[\ \ _{t+1}]$, for lower expected future utility risk, $\boldsymbol{r}_t[\ \ _{t+1}]$. We show this trade-off with and without long-run risk in figure 3.

The top-left panel shows the conditional expected utility of the home country as a function the share of world consumption measured by s_t. The expected future utility is increasing in s, as a higher ratio of the Pareto weights corresponds to a higher share of world consumption. Adding long-run risk in the economy reduces welfare and it explains why the dashed line (model with long-run risk) lays below the solid one (no long-run risk case).

In the top-right panel, we depict the conditional variance of future utility as a function of s_t. The volatility is increasing in the ratio of the Pareto weights because a higher share of world consumption for the home country leaves smaller room for risk-sharing with the foreign partner, and hence implies higher exposure to aggregate risk. Not surprisingly, the model featuring both short- and long-run risks results in more utility-risk than the model without long-run risk (that is, the dashed line is above the thick line).

In the bottom-left panel, we combine the top two charts by plotting the equilibrium conditional expectations and volatilities of the utility over the domain of the ratio of the Pareto weights. To facilitate their comparison, all frontiers have been re-centered at zero (we removed all intercepts) so that they can be easily plotted on the same scale. Two key results emerge from this picture. First, when a relative good shock to the home good materializes, the decline in the current share of home consumption, $\mathcal{S}_t$, produces a beneficial reduction in future utility risk that compensates the agent for the decline in future expected utility. Second, in a model with long-run risk, the trade-off frontier becomes steeper, implying that a more pronounced reduction in utility risk is required to compensate the agent for a given reduction of future expected utility level. Long-run risk, therefore, amplifies the risk-sharing motive of agents with recursive utilities.

The bottom-right panel of figure 3 reports the cross-country conditional correlation of utilities. Our recursive risk-sharing model is capable of producing highly correlated utilities regardless of whether or not exogenously highly correlated long-run risks are present in the endowments. The reason is that under complete markets, marginal utilities need to be equalized across countries. Since with recursive preferences marginal utilities are mainly driven by continuation utilities, $corr_t({}^{f}_{t+1}, {}^{h}_{t+1})$ has to be close to one as well. We also point out that our consumption profiles feature a lower cross-country correlation due to our home bias assumption. This allows the model to produce highly correlated stochastic discount factors—consistent with international asset pricing data—despite a relatively low correlation of consumption growth rates—consistent with international consumption data.

The role of IES. We highlight the role of the IES in figure 4. In the top-left panel, it is possible to see that a higher IES implies a lower unconditional expected utility. This is explained by the fact that as the elasticity gets larger, the agent is more affected by

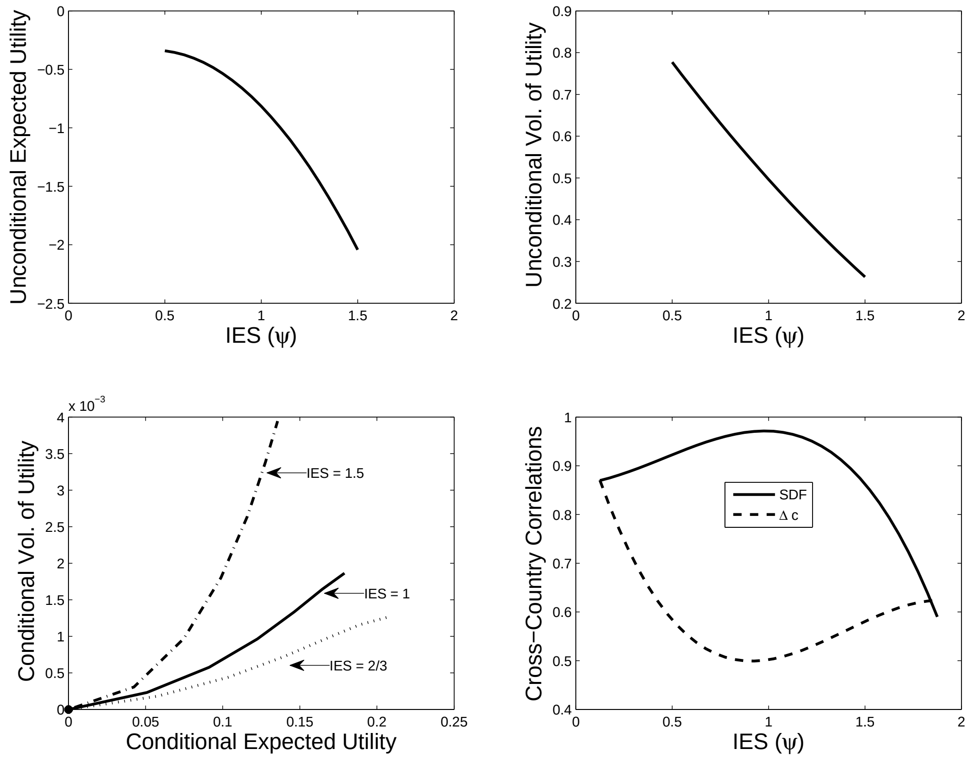

FIG. 4 - The utility mean-variance frontier for different levels of IES. This figure shows the utility mean-variance frontier for the home country across different levels of IES. All other parameters are calibrated to the values reported in Table 1. By symmetry, a similar figure applies to the foreign country. The home country utility is standardized as follows: $U_{t+1}^{h}/(X_t^{\alpha}Y_t^{1-\alpha})$. All other parameters are calibrated to the annual values reported in Table 1.

utility risk, hence her welfare declines.

Moreover, the top-right panel shows that the unconditional volatility of future utility declines, as the IES increases. This result is driven by a general equilibrium channel. As increases, a stronger risk-sharing incentive kicks in: agents become more and more averse to utility risk and are more willing to give up part of their current consumption (through international trade) in exchange for smoother future utility profiles. A higher IES, therefore, reduces the amount of utility volatility that agents are willing to bear in equilibrium.

In the bottom-left panel, we show the utility mean-variance frontier for three different levels of the IES. Consistent with our intuition, as the IES increases the frontier

becomes steeper. Specifically, at the equilibrium a more substantial decline in wealth risk is required to offset drops in expected wealth due to an adjustment in the share of world endowments.

In the bottom-right panel, we show that as long as the IES is in a neighborhood of one, the cross-country correlation of the stochastic discount factors, $corr(m_t^h, m_t^f)$, is substantial while the one of consumption growth rates, $corr(\Delta c_t^h, \Delta c_t^f)$, is moderate as in the data. It is important to analyze the reason why the spread between $corr(m_t^h, m_t^f)$ and $corr(\Delta c_t^h, \Delta c_t^f)$ is hump-shaped with respect to the IES because this implicitly introduces both a lower and an upper bound on the calibration of this preference parameter. This is one of the key aspects of our general equilibrium approach.

Equations (3) and (13) suggest that our mechanism is relevant when both the concern about utility risk, measured by $\;-1/\;$, and the amount of utility risk, measured by $\;r_t(\;_{t+1})$, are positive and sizable. When the IES is below one, the concern about utility risk declines (that is $\;-1/\;$ decreases) and eventually the preferences collapse to the standard time-additive case when $\;-1/\;=0$. In this setting, the cross-country correlation of the stochastic discount factors coincides with the one of consumption growth rates, with the latter being too high with respect to the data.

On the other hand, as the IES increases, the concern about utility risk rises (that is $\;-1/\;$ increases): investors are so sensitive to the risk of their future wealth profiles that they are willing to trade aggressively to substantially reduce their amount of utility risk (see figure 4, top-right panel). For values of the IES substantially larger than one, the general equilibrium channel on the volatility of continuation utilities dominates and international risk sharing manifests itself through a larger correlation of consumption growth rates and a lower correlation of continuation utilities.

5 General Equilibrium Explanation of International Finance Anomalies

In this section we focus on the implications of our model for the forward premium and the Backus and Smith (1993) anomalies, respectively. The forward premium anomaly refers to the empirical observation that high interest rate currencies tend to appre-

ciate, contrary to what predicted by the uncovered interest rate parity relationship. Fama (1984) finds that time-varying volatility is a necessary condition to replicate this anomaly. Colacito (2008) and Bansal and Shaliastovich (2010) document that exogenous time-varying volatility can resolve the forward premium puzzle in a long-run risk setting. We show that the forward premium anomaly is actually an endogenous equilibrium outcome in a risk-sharing scheme with recursive preferences, a novel result in the literature. Furthermore, we show that this anomaly vanishes in a setting with limited capital mobility.

The Backus and Smith (1993) anomaly refers to the almost complete lack of correlation between consumption differentials across countries and exchange rates. Colacito and Croce (2011) have already shown that international long-run components paired with Epstein and Zin (1989) preferences can resolve this puzzle. Our study differs from previous analysis in at least two dimensions. First, our resolution of the Backus-Smith puzzle is based on a contemporaneous endogenous adjustment of consumption differentials upon the arrival of long-run news, a mechanism novel to the literature. Given the relevance of this response, we also present new empirical evidence in favor of this channel in section 7. Second, we show that the transition from financial autarky to financial integration experienced by the US and the UK explains the structural change in the Backus-Smith correlation observed in the pre- and post-1970 data.

5.1 The Forward Premium Anomaly

Financial Autarky. We characterize the forward premium anomaly by looking at the regression coefficient obtained by projecting the growth rate of the exchange rate onto the pre-determined cross-country interest rate differential. We shall refer to this as the ${}_{UIP}$. Given equations (8)–(9), in autarky the following holds:

$$\begin{aligned} {}_t[\Delta\,{}_{t+1}] &= {}_t[\Delta c^h_{t+1} - \Delta c^h_{t+1}] \\ &= (r^h_t - r^f_t), \end{aligned} \tag{15}$$

which implies ${}^{aut}_{UIP} = \quad 0$. In financial autarky, therefore, obtaining a positive ${}_{UIP}$ should not come as a surprise. If we think of the pre-1970 sample as a period of

limited capital mobility, financial autarky rationalizes the positive $_{UIP}$ that we find in the data.

International Complete Markets. Assume that $\tilde{}^{h}_{t} \equiv {}^{h}_{t} / \left(X_t^{\alpha}\ {}_t^{1-\alpha}\right)$ and $\tilde{}^{f}_{t} \equiv {}^{f}_{t} / \left(X_t^{1-\alpha}\ {}_t^{\alpha}\right)$ are approximately log-normally distributed. Under our symmetric calibration we show in Appendix D that the difference of the risk-free rates is:

$$
\begin{aligned}
r_t^h - r_t^f \approx\ & \frac{1}{}\left({}_t\left[\Delta c^h_{t+1}\right] - {}_t\left[\Delta c^f_{t+1}\right]\right) \\
& -\frac{1}{2}\left(\frac{1}{}\right)^2 \cdot \left({}_t\left[\Delta c^h_{t+1}\right] - {}_t\left[\Delta c^f_{t+1}\right]\right) \\
& +\frac{1}{2}\underbrace{\left(1-\frac{1}{}\right)\left(\frac{1}{}-\right)}_{<0,\text{ if }\psi>1\text{ and }\psi>1/\gamma}\left({}_t\left[\log \tilde{}^{h}_{t+1}\right] - {}_t\left[\log \tilde{}^{f}_{t+1}\right]\right).
\end{aligned}
\tag{16}
$$

It is also possible to show that the expected growth of the real exchange rate takes the following form:

$$
\begin{aligned}
{}_t\left[\Delta_{t+1}\right] \approx\ & \frac{1}{}\left({}_t\left[\Delta c^h_{t+1}\right] - {}_t\left[\Delta c^f_{t+1}\right]\right) \\
& +\frac{1}{2}\underbrace{(1/\ -\ )(1-\ )}_{>0,\text{ if }\psi>1/\gamma\text{ and }\gamma>1}\cdot\left({}_t\left[\log \tilde{}^{h}_{t+1}\right] - {}_t\left[\log \tilde{}^{f}_{t+1}\right]\right)
\end{aligned}
\tag{17}
$$

Both expected exchange rate growth and interest rate differential depend on first and second moments of consumption growth rates and continuation utilities. Expected consumption growth differentials tend to produce a positive co-movement between ${}_t[\Delta_{t+1}]$ and $r_t^h - r_t^f$ as in the case of standard CRRA preferences. Without the endogenous time-varying second moments that arise in equilibrium, all other terms would be constant and the $_{UIP}$ would be exactly one. In what follows, we explain how endogenous time-varying risk helps us getting a low $_{UIP}$, provided that 1 and $1/$.

Consider the case of a positive shock to the growth rate of good X. As this happens, the home consumer faces a higher domestic risk-free rate for three reasons: (1) she is expected to consume relatively more in the future (higher ${}_t\left[\Delta c^h_{t+1}\right]$); (ii) she achieves a smoother future consumption profile by trading away part of its current consumption (lower ${}_t(\Delta c^h_{t+1})$), and (iii) she locks in a less risky wealth profile (lower

$_t(\log \tilde{\ }^h_{t+1})$).

Turning our attention to the real exchange rate, we notice that two forces are at work in opposite directions. On the one hand, the home currency is expected to depreciate as a consequence of the larger supply of the home good (higher $_t\left[\Delta c^h_{t+1}\right]$). On the other hand, there is also an appreciation pressure through the utility-risk channel. Upon the realization of relative good news to X, the home country locks in a safer utility profile (lower $_t(\Delta c^h_{t+1})$), implying that its entire future consumption stream is less risky. By no arbitrage, the domestic consumption bundle is more valuable than the foreign one and the exchange rate should appreciate. Depending on which effect dominates, the home currency will either appreciate or depreciate.

The role of the IES. The forward premium anomaly effectively puts both a lower and an upper bound on the IES. Indeed, a calibration with a large enough is needed to have a quantitatively relevant utility mean-variance trade-off (see figure 4). However, this preference parameter cannot be too large. As already mentioned, an increasing level of IES reduces the amount of volatility in the model and hence mitigates the contribution of the time-varying second moments in (16)–(17). In section 6.1, we quantify the intensity of these different forces by looking at simulated moments from the model and suggest 1.5 as benchmark value for the IES.

5.2 The Backus and Smith Puzzle

Financial Autarky. Combining equations (8) and (9), it follows immediately that:

$$\Delta c^h_t - \Delta c^f_t = (2\ - 1)(\Delta\ _t - \Delta y_t) = \Delta\ _t.$$

Hence, under financial autarky the correlation between consumption differentials and exchange rate is 1, a value that goes in the direction of our empirical findings for the pre-1970 sample.

Internationally Complete Markets. Under complete markets, we do not have closed form solutions for the dynamics of consumption. In order to inspect the implications of our model for the Backus and Smith correlation, we analyze the impulse response functions depicted in figure 5. We first consider the case of a positive short-run shock to the supply of good X (the most preferred good for the home country), and

then turn our attention to a positive long-run shock.

Following a positive short-run shock, the home country exports part of its abundant supply of good X. Because of home bias, there is only a limited amount of good X that the foreign country is willing to take. This means that consumption will increase in both countries, but relatively more at home (see the third panel on the left column of figure 5). Since good X is more abundant, the marginal utility of consumption is lower at home than in the foreign country. This leads to a depreciation of the home currency. Hence, short-run shocks produce a positive co-movement between consumption differentials and exchange rate.

Upon the realization of a positive long-run shock, in contrast, the exchange rate growth and the consumption differential experience a sizable adjustment in opposite directions (see right columns of figure 5). Even though on impact a long-run shock is an order of magnitude smaller than a short-run shock (top right panel), by its very definition it affects the growth rate of good X for a very long time and hence has an extremely large effect on the life-time utility value. Since with recursive preferences a rise in continuation utility corresponds to a decline in marginal utility, it is optimal to reduce the share of consumption allocated to the home country (see figure 2).

Contrary to short-run shocks, long-run shocks do not alter the current endowment of goods, as they only carry news about future supply conditions. The decline in the home share of world consumption, therefore, results in a decline (rise) in domestic (foreign) consumption, ultimately forcing the consumption growth differential to decrease and become negative. Despite the fall in home current-period consumption, the exchange rate growth increases on impact, as the expectation of a persistently larger future supply of the home good determines a depreciation of the home currency.

By combining the analyses of left and the right panels of figure 5, we can conclude that on average we are going to get $corr(\Delta e_t, \Delta c_t^H - \Delta c_t^f) < 1$, a necessary condition to replicate the Backus-Smith anomaly in the post-1970 sample. The final quantitative impact of long-run shocks depend on the calibration of the IES. In fact, similarly to the case of the forward premium anomaly, a large IES reduces the ability of the model to explain the Backus–Smith puzzle because the concern about utility risk becomes so large that wealth risk is substantially traded away in equilibrium.

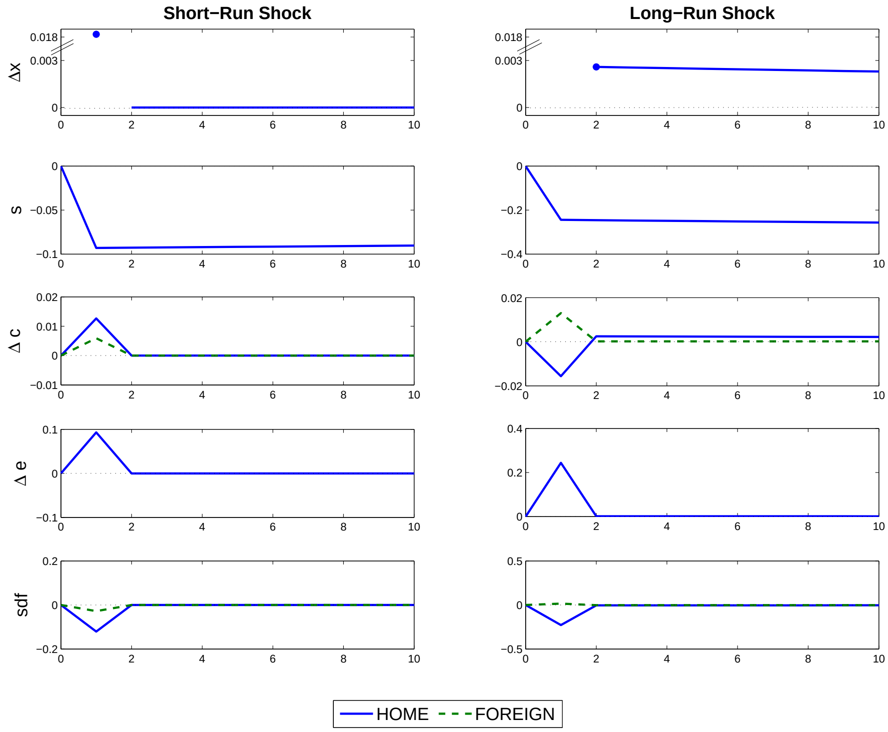

FIG. 5 - Impulse Response Functions when IES = 1.5. This figure shows the impulse response function of several variables of interest for both the home (solid line) and the foreign country (dashed line). All parameters are calibrated to the values reported in Table 1 and the IES is set to 1.5. Shocks materialize at time 1.

6 Simulated Moments

In this section we present the main predictions of our model and show that they conform well to the data. Our analysis focuses only on the US and the UK for a number of reasons both practical and historical. First of all, the identification of highly persistent time series, such as the long-run components analyzed in the model, requires as long a sample as possible. From this stand-point, the US and UK are probably the best examples of countries with a long history of accurate consumption, import, and export data. Indeed, our sample spans almost a century as it ranges from 1929 to 2006.

Second, the US and UK have become increasingly financially integrated starting from the late 1960s-early 1970s, as documented by Taylor (1996), Quinn (1997), and Ob-

stfeld (1998) among others. Taking seriously this observation, we think of the post–1970 sub-sample as a regime of complete markets between US and UK. In contrast, we consider the sub-sample 1945–1969 as a period of financial autarky, consistent with the limited capital mobility documented by Obstfeld (1998). This enables us to make a comparison between times of different degrees of financial integration.

6.1 Complete Markets

In table 1 we report moments from both our model and our dataset. Specification (1) is our preferred calibration as it features an IES of 1.5 and both short- and long-run risks, consistent with Bansal and Yaron (2004). In specification (2) we remove long-run risks to better asses their contribution in the context of our model. Specifications (3) and (4) show how the performance of the model changes for lower values of the IES. We calibrate common parameters as in Colacito and Croce (2010) and set τ to a small value to be consistent with the empirical weak co-integration between US and UK output time-series (see for example, Cheung and Pascual (2004)). This implies that our endowments' co-integration residual plays a quantitatively marginal role in our analysis.

Benchmark Calibration. Specification (1) perfectly reproduces the cross-country correlation and the autocorrelation of consumption growth rates observed in the post-1970 sample. Since our model cannot fully match the volatility of the bilateral net export ratio between US and UK, short-run consumption growth rates are only slightly smoother than endowment growth rates.

On the asset pricing side, our model performs well along several dimensions. In each country, the equity risk premium is 6.68%. Our risk-free rates are on average low and smooth, while our stochastic discount factors are volatile and capable to match the Hansen and Jagannathan (1991) bound. Despite the high volatility of the pricing kernels, the growth rate of the exchange rate is smooth as in Colacito and Croce (2011) thanks to the high correlation of the continuation utilities. The cross-country

TABLE 1: Results with Complete Markets

Panel A: calibration of common parameters.									
	σ	σ_x	ρ	ρ_{12}	ρ_{xy}	τ		δ	
2.00%	1.87%	4%σ	.985	.90	.05	.05%	.97	.98	8

Panel B: main moments.						
Specification	DATA	(1)	(2)	(3)	(4)	(5)
IES ( )		1.5	1.5	1	0.67	1/
		(with LRR)	(no LRR)		(with LRR)	
Std (Δc) /Std (Δy)	0.87	0.96	0.87	0.99	0.99	0.93
ACF$_1$ (Δc_t)	0.38	0.28	0.04	0.24	0.27	0.31
corr(Δc_t^h,Δc_t^f)	0.55	0.59	0.82	0.42	0.48	0.81
E[r_f]	1.25	1.81	2.93	3.18	5.38	16.46
Std[r_f]	1.15	0.68	0.00	1.15	1.54	9.17
corr($r_{f,t}^h$, $r_{f,t}^f$)	0.64	0.88	-1.00	0.92	0.92	0.97
Std[M]/E[M]		27.83	12.99	70.51	87.78	16.64
Std (Δe_t)	11.65	14.47	7.45	20.58	17.99	10.23
corr($\Delta c_t^h - \Delta c_t^f$, Δe_t)	-0.02	-0.02	1.00	-0.53	-0.34	1.00
$_{UIP}$	-0.72	-0.71	-155.69	-2.36	-1.59	1.01
E($r_{d,t}^{ex}$)	6.0	6.68	0.32	3.75	0.74	0.46
corr($r_{d,t}^{ex}$,Δe_t)	0.05	0.08	0.12	-0.14	-0.17	0.06
corr($r_{d,t}^{ex}$,$r_{FX,t}$)	-0.05	0.03	0.03	0.10	0.00	0.00

Notes - In panel A, we report our annual calibration for the post-Bretton-Wood sample. Our data sources are described in appendix A. The IES varies across different specifications. For specification (2), we impose $_x = 0$ and $_{xy} = 0.35$ so that the cross-country correlation of the output growth rates remains unchanged. The currency return is defined as $r_{FX,t+1} = \Delta e_{t+1} + r_{f,t}^f - r_{f,t}^h$. The equity excess return, $r_{d,t}^{ex}$, refers to the following cash-flow: $\Delta d_t^i = \lambda \Delta c_t^i + \varepsilon_t^i, \quad i \in \{h, f\}$, where $\lambda = 1.7$ and $\varepsilon_t^i \sim_{i.i.d.} N(0, .15^2)$.

correlation of the risk-free rates, however, is higher than that observed in the data because of the high cross-country correlation of the long-run components.

The model successfully replicates the almost lack of correlation between exchange rate and consumption growth differential, consistent with Backus and Smith (1993)'s findings. Additionally, the predicted $_{UIP}$ is negative and close in value to its empirical counterpart thanks to our endogenous volatility channel. Furthermore, both exchange rate growth and currency returns are almost uncorrelated with domestic equity excess returns, as in the data.

The role of long-run risk. In specification (2) we abstract away from international long-run risks and let output growth be driven only by short-run fluctuations. The model fails along several dimensions. First of all, in contrast to the data, the contemporaneous correlation between home and foreign consumption is almost one. This result is determined by the positive co-movement between home and foreign consumption induced by short-run supply shocks.

Second, the risk-free rates are excessively smooth and perfectly negatively correlated, which is at odds with the empirical evidence. This is due to the fact that they are driven mainly by the relative share of world resources, s_t. In an economy without long-run shocks, indeed, s_t introduces a very small amount of predictability in consumption growth, implying almost no volatility of the risk-free rates. Note also that in a two-country model the share of world resources is perfectly negatively correlated across countries as a higher share for the domestic country always implies a lower share for the foreign trading partner.

Third, exchange rate and pricing kernels are not volatile enough. In particular, upon removing long-run risks, the market price of risk declines by more than 50% and the equity risk premium becomes one order of magnitude smaller, implying that international long-run risks explain a significant share of the dynamics of international asset prices.

Fourth, the model without long-run risks cannot reproduce the Backus-Smith correlation, as exchange rate is just a reflection of short-run endowment dynamics. As observed in our analysis of the impulse response functions, short-run shocks generate a perfectly positive correlation between consumption differentials and exchange rate growth. Furthermore, the model produces an excessively negative UIP regression coefficient as a consequence of the interest rate differential becoming almost constant and creating a collinearity problem.

Finally, although we do not report them in table 1, we have computed the volatilities

of the conditional volatility (hereafter, vol-of-vol) of both the log stochastic discount factor, m^i_{t+1}, and consumption growth rate, Δc^i_{t+1}, for specifications (1) and (2). Specification (1) generates an annual vol-of-vol of m^i_{t+1} of approximately 12%, consistent with the findings in Lustig et al. (2011b). Simultaneously, the vol-of-vol of Δc^i_{t+1} is approximately .9%, i.e., a low value consistent with consumption data. In contrast, under specification (2), our model generates a of vol-of-vol for m^i_{t+1} of about .5%, i.e, one order of magnitude smaller than that suggested by data on the cross-section of currencies.

The role of the IES. Specifications (3) and (4) show the performance of the model with both short- and long-run risk for an IES equal to 1 and 0.67, respectively. We highlight three points. First, with a lower IES, agents are willing to accept more risk hence their stochastic discount factors become much more volatile. Given the higher volatility of the pricing kernels, the exchange rate variance increases as well reaching counterfactually high levels. Additionally, the risk-free rate tends to become on average too high and too volatile as the IES declines. Simultaneously, the equity premium declines and actually becomes one order of magnitude smaller.

Second, consistent with our previous analysis, the $_{UIP}$ coefficient and the Backus-Smith correlation are not monotonic with respect to the IES. On the one hand, if the IES is much smaller than one, the concern about utility risk is too moderate to substantially affect equilibrium dynamics. This implies that both the Backus-Smith correlation and the $_{UIP}$ coefficient tend to 1, a finding which is at odds with the data. On the other hand, as the IES becomes increasingly larger than one, agents trade to eliminate a substantial share of utility risk, again implying that both the Backus-Smith correlation and UIP regression coefficient tend to 1. Calibrating the IES to 1.5 enables the model to be just right about the magnitude of both the Backus-Smith correlation and the $_{UIP}$ coefficient.

Summarizing, the risk-free rate puzzle (Weil (1989)) and the exchange rate volatility

puzzle (Brandt, Cochrane and Santa-Clara (2006)) put a lower bound on the IES, suggesting that this coefficient should be calibrated to a value at least as high as 1. In addition, the forward premium anomaly and the Backus-Smith puzzle jointly put an upper bound on the IES demanding a value of approximately 1.5.

Finally, specification (5) features no concern about utility risk ($-1/\quad = 0$) and corresponds to the case of time-additive preferences. This calibration summarizes all the well-known counterfactual implications produced by standard preferences. Specifically, in contrast to our benchmark specification, the time-additive case fails in (1) solving the risk-free rate puzzle, (2) matching the Hansen and Jagannathan (1991) bound and hence the equity premium, and (3) addressing the Backus-Smith and forward premium anomalies.

6.2 Results across capital mobility regimes

When looking at the pre-1970 data, we alter our benchmark calibration by decreasing the international correlations of short- and long-run shocks to better match international correlations of endowments. After refining the calibration, our model with financial autarky is able to replicate several key features of the data, as illustrated in table 2. First of all, we document that the cross-country correlation of the consumption growth rates is close to zero due to the lack of intertemporal risk-sharing, consistent with the pre–1970 data.

Furthermore, consumption growth adjusts almost one-to-one with the growth of the home endowment, because of consumption home bias (see equation (8)). The extent of exchange rate's fluctuations is moderate, since in both countries the endowment growth rate has a low volatility, again consistent with empirical evidence.

In addition, since the share of world consumption is constrained to be constant, long-

TABLE 2: Results with Autarky

	Pre-1970			Post-1970 – Pre-1970		
	DATA	Model (with LRR)	Model (no LRR)	DATA	Model (with LRR)	Model (no LRR)
Std (Δc) /Std (Δy)	0.69	0.97	0.97	0.18	0.00	-0.10
ACF$_1$ (Δc_t)	0.41	0.39	-0.01	-0.03	-0.11	0.01
corr$(\Delta c_t^h, \Delta c_t^f)$	0.02	-0.06	-0.06	0.53	0.68	0.93
E$[r_f]$	0.61	1.75	2.89	0.65	0.06	0.04
Std$[r_f]$	1.72	0.85	0.00	-0.57	-0.17	0.01
corr$(r_{f,t}^h, r_{f,t}^f)$	0.46	0.59	–	0.18	0.29	–
Std[M]/E[M]		27.54	14.55		0.29	-1.56
Std (Δe_t)	5.59	3.53	2.81	6.06*	10.94	3.52
corr$(\Delta c_t^h - \Delta c_t^f, \Delta e_t)$	0.47	1.00	1.00	-0.49**	-1.02	0.00
β_{UIP}	0.94	1.50	–	-1.66*	-2.21	–

Notes - We adopt the calibration corresponding to specification (1) in table 1 and adapt it to the pre-1970 sample as follows: (1) in the model with long-run risk, we set $\rho_{12} = 0.50$ and $\rho_{12} = -0.50$; (2) in the model without long-run risk we set $\rho_{xy} = -0.12$. This enables us to match the lower correlation of the long-run components in the pre-1970 sample and keep the unconditional correlation of output growth constant across model specifications. The data sources are explained in detailed in section A. The last three columns refer to the change in the moments of interest across the post-1970 and pre-1970 sample for the data, and across the model with internationally complete markets and financial autarky. We test the null of no change in Std(Δe_t), corr$(\Delta c_t^h - \Delta c_t^f, \Delta e_t)$, and β_{UIP} across regimes and denote p-values smaller than 1% and 2% by *** and **, respectively.

run shocks no longer generate lack of co-movement between exchange rate and the difference of consumption growth rates. This implies that $corr(\Delta c_t^h - \Delta c_t^f, \Delta e_t)$ increases by an amount comparable to that observed in our sub-samples. Additionally,

Turning our attention to the last column of table 2, we can see that the model without long-run risk misses several key aspects of the transition occurred in the seventies. First of all, the model cannot replicate the higher volatility of the exchange rate in the post-1970 and the lower correlation between short-run consumption growth differentials and exchange rate movements. Furthermore, since the co-integration residual features a negligible amount of volatility, consumption growth rates are almost $. .d.$ and the risk-free rates are basically constant. This implies that there is no scope for predictability of exchange rate growth and for this reason we do not report any values for $_{UIP}$.

7 Empirical Evidence

In this section, we provide direct empirical evidence supporting the implications of our model for the response of consumption and asset prices to both short- and long-run endowment news. Specifically, we confront the predictions of our model under the complete markets regime with novel empirical evidence for US and UK in the post-1970 sample. We find that consumption growth, excess returns, expected returns and returns conditional volatility co-move with endowment shocks as prescribed by our recursive risk-sharing mechanism. In the pre-1970 sample, in contrast, these co-movements are absent, consistent with our predictions for the financial autarky regime.

In what follows, we describe in detail our empirical approach and report our results for the post-1970 sample in table 3. The results for the pre-1970 sample are available upon request. We describe in detail our data sources in Appendix A.

7.1 Identification of short- and long-run shocks.

We follow Colacito and Croce (2011), and Bansal et al. (2010) in identifying short- and long-run innovations to GDP growth by the means of predictive regressions. To study the robustness of our empirical results, we form the following three sets of variables commonly used in the long-run risk literature to identify long-run components:

$$
\begin{aligned}
{}^{i}_{1,t} &= \left[pd^i_t\right] \\
{}^{i}_{2,t} &= \left[pd^i_t, cy^i_t, \Delta \; y^i_t\right] \\
{}^{i}_{3,t} &= \left[pd^i_t, cy^i_t, \Delta \; y^i_t, r \; {}^{i}_{t}, d \; t^i_t\right], \quad \forall \; \in \{ \; , \; \}
\end{aligned}
$$

where pd, r , cy, and d t denote the price-dividend ratio, the risk-free rate, the consumption output ratio, and the default spread, respectively. The variable $\Delta \; y_t$ proxies for total endowment growth and is computed as the sum of consumption and net-exports for both US and UK. We exclude investment and government expenditure from our empirical measure to be consistent with our endowment economy in which we abstract from both physical investment and demand of public goods.

We identify short- and long-run shocks to US and UK endowments by estimating the system of equations (5) and (6) in conjunction with the following projection restrictions:

$$
\begin{aligned}
z_{1,t,j} &= {}_{1,j} \; {}^{US}_{j,t}, \\
z_{2,t,j} &= {}_{2,j} \; {}^{UK}_{j,t} \quad j = 1, 2, 3.
\end{aligned} \tag{18}
$$

This is consistent with our convention of treating the US as our home country and the UK as the foreign one.

7.2 Testable Implications

Consumption contemporaneous response. The model predicts that the difference between home and foreign consumption growth rates should respond negatively (positively) to home (foreign) long-run news and positively (negatively) to home (foreign) short-run news. We test this prediction by regressing consumption growth differentials between US and UK on the contemporaneous short-run and long-run shocks estimated according to the methodology described above. We summarize our main results in panel A of table 3.

First of all, across all our sets of predictive variables, based on the p-value of our -statistics we reject the null that all the coefficients are equal to zero. This implies that our short- and long-run shocks have indeed significant explanatory power on the contemporaneous response of consumption growth differentials, consistent with the very high R^2 of our regression.

Second, we compare the R^2 of the above regression with the R^2 of a regression that includes only the long-run shocks $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$. Our long-run news–based R^2 is orders of magnitude smaller then the total R^2, implying that the short-run consumption growth dynamics are mainly driven by short-run endowment shocks, consistent with our model's predictions.

Third and most importantly, we test the null that our estimated coefficients have the opposite sign of that predicted by our model. The low p-values of our t-statistics support our theory. In particular, the best results are obtained with the last two specifications in which we use both asset prices and quantities to filter the low frequency components of GDP growth. Overall, we regard these results as significantly in favor of our recursive risk-sharing mechanism.

Excess returns contemporaneous response. By no-arbitrage, we can price a claim to the endowment of the local good of each country. Qualitatively, our local

excess returns behave as in a Colacito and Croce (2011), i.e, they are monotonically increasing in both short- and long-run domestic shocks. According to our model, therefore, the spread between the US and UK market excess returns should increase (decline) upon the realization of good news to the US (UK) endowment.

We test this implication and summarize our main results in panel B. First of all, we note that the data support the responses predicted by our model. Across all our specifications, our t-statistics reject the null that long-run shocks produce responses going in the opposite direction of that predicted by our risk-sharing scheme. The results for the short-run shocks are less strong, but this should come as no surprise. Consistent with the long-run risk framework, in fact, the returns' spread is driven almost exclusively by long-run news, as documented by the high R^2 obtained when focusing only on long-run news.

Conditional volatility and risk premia of equity returns. A distinct implication of our recursive risk-sharing mechanism is that the conditional riskiness of US returns should decline (increase) relative to that of UK returns upon the realization of relative good news to the supply of the US (UK) good. This result is determined by the endogenous counter-cyclical variation in the volatility of both our consumption profiles and stochastic discount factors. By construction, our endowment cash-flows do not feature time-varying volatility; hence, time-varying returns' risk is just a reflection of the properties of the pricing kernel.

We apply our empirical procedure to the analysis of both conditional volatility (panel C) and expected value (panel D) of equity excess returns spreads. In both panels, our tests confirm that good news for the long-run for the home (foreign) good decreases (increases) conditional volatility and risk premia of the home country relative to the foreign country. The results for short-run shocks are less strong, consistent with our analysis of specification (2) in table 1: short-run shocks to endowment growth produce a very small amount of time-varying volatility that is difficult to detect especially in

small sample.

Overall, we regard the results in table 3 as strongly in favor of our recursive risk-sharing mechanism. Our empirical findings confirm that the relative riskiness of international securities in financially integrated markets is determined by long-horizon growth factors. Furthermore, our general equilibrium approach enables us to relate the no-arbitrage–based hypothesis of Lustig et al. (2011a) and Lustig et al. (2011b) to macroeconomic fundamentals, such as the international consumption dynamics.

8 Concluding remarks

In this paper we have developed a novel international recursive general equilibrium model with long-run risk that accounts for several quantitatively challenging facts of international finance. More precisely, the model generates series that agree with both international prices and quantities across different capital mobility regimes and addresses both the forward premium and the Backus-Smith anomalies. These are new findings in the international finance literature.

Under perfect capital mobility, our risk-sharing mechanism is based on a novel state-contingent wealth-mean-variance trade-off. This channel generates endogenous time-varying volatility—and hence predictability—in international asset prices. On the basis of our general equilibrium model, we propose new predictability tests and show that US and UK data support our recursive preferences-driven theory.

Given the potential of the model and the relatively simple solution technique adopted in this study, we believe that it could easily be extended to international real business cycles settings in order to study international investment flows, portfolio diversification, and diffusion of real regional shocks across countries and currencies.

Table 3: Testable Implications on Consumption and Returns

Contemporaneous Responses

A: Consumption Growth

$$\Delta c_t^{US} - \Delta c_t^{UK} = \mu_t + \beta_x \varepsilon_{x,t} + \beta_y \varepsilon_{y,t} + \beta_1 \varepsilon_{1,t} + \beta_2 \varepsilon_{2,t} + \epsilon_t$$

	pd only	pd,cy,dy	All
F-stat	0.000	0.000	0.000
Total R^2	0.834	0.875	0.848
Long-run news R^2	0.003	0.007	0.005
β_1 (H_0: $\beta_1 \geq 0$)	-0.013^{***}	-0.018^{***}	-0.003^{***}
β_2 (H_0: $\beta_2 \leq 0$)	0.003^{*}	0.007^{***}	0.005^{***}
β_x (H_0: $\beta_x \leq 0$)	0.005^{***}	0.012^{***}	0.004^{***}
β_y (H_0: $\beta_y \geq 0$)	-0.009^{***}	-0.008^{***}	-0.011^{***}

B: Excess Returns

$$r_{ex,t}^{US} - r_{ex,t}^{UK} = \mu_t + \beta_x \varepsilon_{x,t} + \beta_y \varepsilon_{y,t} + \beta_1 \varepsilon_{1,t} + \beta_2 \varepsilon_{2,t} + \epsilon_t$$

	pd only	pd,cy,dy	All
F-stat	0.000	0.000	0.000
Total R^2	0.874	0.455	0.493
Long-run news R^2	0.786	0.255	0.356
β_1 (H_0: $\beta_1 \leq 0$)	0.606^{***}	0.299^{***}	0.086^{***}
β_2 (H_0: $\beta_2 \geq 0$)	-0.262^{***}	-0.274^{***}	-0.173^{***}
β_x (H_0: $\beta_x \leq 0$)	0.004	-0.148	-0.003
β_y (H_0: $\beta_y \geq 0$)	0.009	-0.034^{***}	0.046

Equity Risk Premia

C: Realized Variance

$$(r_{ex,t}^{US})^2 - (r_{ex,t}^{UK})^2 = \mu_t + \beta_x \varepsilon_{x,t} + \beta_y \varepsilon_{y,t} + \beta_1 \varepsilon_{1,t} + \beta_2 \varepsilon_{2,t} + \epsilon_t$$

	pd only	pd,cy,dy	All
F-stat	0.092	0.201	0.234
Total R^2	0.467	0.396	0.341
Long-run news R^2	0.172	0.127	0.172
β_1 (H_0: $\beta_1 \geq 0$)	0.072	-0.091^{**}	-0.058^{**}
β_2 (H_0: $\beta_2 \leq 0$)	0.079^{**}	0.127^{***}	0.085^{**}
β_x (H_0: $\beta_x \geq 0$)	-0.020^{**}	0.025	-0.002
β_y (H_0: $\beta_y \leq 0$)	0.017^{**}	0.047^{***}	-0.001

D: Excess Returns (one period ahead)

$$r_{ex,t+1}^{US} - r_{ex,t+1}^{UK} = \mu_t + \beta_x \varepsilon_{x,t} + \beta_y \varepsilon_{y,t} + \beta_1 \varepsilon_{1,t} + \beta_2 \varepsilon_{2,t} + \epsilon_t$$

	pd only	pd,cy,dy	All
F-stat	0.001	0.007	0.000
Total R^2	0.169	0.128	0.174
Long-run news R^2	0.158	0.027	0.126
β_1 (H_0: $\beta_1 \leq 0$)	-0.371^{***}	-0.203^{**}	-0.070^{***}
β_2 (H_0: $\beta_2 \leq 0$)	0.172^{***}	0.197^{***}	0.169^{***}
β_x (H_0: $\beta_x \geq 0$)	-0.038^{***}	0.052	-0.033^{**}
β_y (H_0: $\beta_y \leq 0$)	0.007	0.041^{***}	-0.043

Notes - In all panels, US and UK short-run (long-run) endowment shocks are denoted as $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ ($\varepsilon_{1,t}$ and $\varepsilon_{2,t}$), respectively. These shocks are identified by estimating equations (5) –(6) and (18). For each regression, we report: (1) the p-value associated to the F-statistic on the null that all the coefficients are zero; (2) the total R^2 and the R^2 imputed to long-run news only; iii) the p-value associated to the null that the sign of $\beta_1, \beta_2, \beta_x$, and β_y is inconsistent with that predicted by our model. All regressions include a constant and the estimated predictive components as controls ($\mu_t = \mu + \beta_{z_1} z_{1,t-1} + \beta_{z_2} z_{2,t-1}$).

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Appendix

A: Data Sources

Data on US consumption of nondurables and services, gross domestic product, and population were collected from the NIPA tables of the Bureau of Economic Analysis. Value-weighted market returns, yields on 3-month T-bills, dividends, and dividend yields for the US are from CRSP. CPI inflation and total imports and exports were obtained from the website of the Federal Reserve of St. Louis. The UK consumption series for the years 1963-2006 were obtained from the UK Office of National Accounts. For earlier data, we referred to Mitchell (2007). Bilateral imports and exports are also reported in Mitchell (2007). Long time series for UK FTSE returns, yields on three-month T-bills, dividend yields, population, and CPI inflation were obtained from the website of Global Financial Data.

B: Mean-Variance Approximation of the Utility Function

Epstein and Zin preferences are defined as

$$U_t = \left\{(1-\delta)C_t^{(1-\gamma)/\rho} + \delta E_t[U_{t+1}^{1-\gamma}]^{1/\rho}\right\}^{\rho/(1-\gamma)}$$

where $\rho = (1-\gamma)/(1-1/\psi)$. Define $V_t = U_t^{1-1/\psi}/(1-1/\psi)$. It follows that

$$V_t = (1-\delta)\frac{C_t^{(1-\gamma)/\rho}}{1-1/\psi} + \delta E_t[V_{t+1}^{\rho}]^{1/\rho}$$

Taking a second order Taylor expansion of V_{t+1}^{ρ} about $E_t\left[V_{t+1}\right]$ yields

$$V_{t+1}^{\rho} \approx E_t\left[V_{t+1}\right]^{\rho} + \rho E_t\left[V_{t+1}\right]^{\rho-1}\left(V_{t+1} - E_t\left[V_{t+1}\right]\right) - \frac{\rho(1-\rho)}{2}E_t\left[V_{t+1}\right]^{\rho-2}\left(V_{t+1} - E_t\left[V_{t+1}\right]\right)^2$$

Using the above approximation for V_{t+1}^{ρ}, we take a first order Taylor expansion of $E_t[V_{t+1}^{\rho}]^{1/\rho}$ with respect to ρ evaluated at $\rho = 1$ and obtain what follows:

$$V_t \approx \frac{C_t^{1-1/\psi}}{1-1/\psi} + \delta E_t\left[V_{t+1}\right] + \frac{\delta}{2}\frac{\rho - 1}{E_t\left[V_{t+1}\right]}Var_t\left[V_{t+1}\right]$$

In the body of the paper, we report this approximation by defining [illegible] $= -(\rho - 1)$.

C: Allocation as a Function of Pareto-weights

Let $W_t^i = W(C_t^i, U_{t+1}^i)$ be the right-hand side of equation (2). If we denote the partial derivatives of the aggregator W^i as follows,

$$W_{1,t}^i := \frac{\partial W_t^i}{\partial C_t^i}, \quad W_{2,t}^i := \frac{\partial W_t^i}{\partial U_{t+1}^i},$$

the stochastic discount factor is equal to:

$$M_{t+1}^i = \frac{W_{2,t}^i W_{1,t+1}^i}{W_{1,t}^i} \quad \forall i = \{h, f\}. \tag{C.1}$$

The optimality condition for the allocation of good X_t for $t = 1, 2, ...$ in each possible state is:

$$\mu_0^h \cdot \left(\prod_{j=0}^{t-1} W_{2,j}^h \right) \cdot W_{1,t}^h C_t^h \frac{\alpha}{x_t^h} = \frac{(1-\alpha)}{x_t^f} C_t^f W_{1,t}^f \cdot \left(\prod_{j=0}^{t-1} W_{2,j}^f \right) \cdot \mu_0^f \tag{C.2}$$

Define the date t Pareto weights as:

$$\begin{aligned} \mu_t^i &= \mu_0^h \cdot \left(\prod_{j=0}^{t-1} W_{2,j}^i \right) \cdot W_{1,t}^i C_t^i \\ &= \mu_{t-1}^i \cdot W_{2,t-1}^i \cdot \frac{W_{1,t}^i}{W_{1,t-1}^i} \cdot \frac{C_t^i}{C_{t-1}^i} = \mu_{t-1}^i \cdot M_t^i \cdot \exp\left\{ \Delta c_t^i \right\}, \quad \forall i \in \{h, f\} \end{aligned}$$

It follows that equation (C.2) can be rewritten as:

$$\mu_t^h \cdot \frac{\alpha}{x_t^h} = \frac{(1-\alpha)}{x_t^f} \cdot \mu_t^f \tag{C.3}$$

Let $S_t := \mu_t^h / \mu_t^f$. Then the optimality condition in equation (C.3) can be represented by the following system of recursive equations:

$$\begin{aligned} S_t \frac{\alpha}{x_t^h} &= \frac{(1-\alpha)}{x_t^f} \\ S_t &= S_{t-1} \frac{M_j^h e^{\Delta c_t^h}}{M_j^f e^{\Delta c_t^f}}. \end{aligned} \tag{C.4}$$

Using a similar first-order condition with respect to the Y good and the resource constraints, we recover the system of equations (11).

We use perturbation methods to solve our system of equations (1)–(14). We compute our policy functions using the `dynare++4.2.1` package. All variables are in expressed in log-units.

D: Approximation of risk-free rates and expected exchange rate growth

Define $\widetilde{U}_t^h \equiv U_t^h / \left(X_t^{\alpha} Y_t^{1-\alpha}\right)$ and $\widetilde{U}_t^f \equiv U_t^f / \left(X_t^{1-\alpha} Y_t^{\alpha}\right)$. The log-stochastic discount factors are:

$$m_{t+1}^i = \log(\delta) - \frac{1}{\ }\Delta c_{t+1}^i + \left(\frac{1}{\ } - \gamma\right)\log \widetilde{U}_t^i - \frac{1/\ \ - \gamma}{1-\gamma}\log E_t\left[\exp\left\{(1-\gamma)\log \widetilde{U}_t^i\right\}\right],$$

$\forall i \in \{h, f\}$. If we assume that $\widetilde{U}_t^h$ and $\widetilde{U}_t^f$ are approximately log-normally distributed, then the stochastic discount factors can be approximated as:

$$\begin{aligned} m_{t+1}^i &= \log(\delta) - \frac{1}{\ }\Delta c_{t+1}^i + \left(\frac{1}{\ } - \gamma\right)\log \widetilde{U}_t^i - (1/\ \ - \gamma) E_t\left[\log \widetilde{U}_t^i\right] \\ &\quad -\frac{1}{2}(1-\gamma)\left(\frac{1}{\ } - \gamma\right) V_t\left[\log \widetilde{U}_t^i\right], \quad \forall i \in \{h, f\}. \end{aligned}$$

Then, under a symmetric calibration, the difference of the risk-free rates is:

$$\begin{aligned} r_t^h - r_t^f &= -\log E_t\left[\exp\left\{m_{t+1}^h\right\}\right] + \log E_t\left[\exp\left\{m_{t+1}^f\right\}\right] \\ &= \frac{1}{\ }\left(E_t\left[\Delta c_{t+1}^h\right] - E_t\left[\Delta c_{t+1}^f\right]\right) - \frac{1}{2}\left(\frac{1}{\ }\right)^2 \cdot \left(V_t\left[\Delta c_{t+1}^h\right] - V_t\left[\Delta c_{t+1}^f\right]\right) \\ &\quad + \left(\frac{1}{\ } - \gamma\right)\left(Cov_t\left[\Delta c_{t+1}^h, \log \widetilde{U}_{t+1}^h\right] - Cov_t\left[\Delta c_{t+1}^f, \log \widetilde{U}_{t+1}^f\right]\right) \\ &\quad + \frac{1}{2}\left(1 - \frac{1}{\ }\right)\left(\frac{1}{\ } - \gamma\right)\left(V_t\left[\log \widetilde{U}_{t+1}^h\right] - V_t\left[\log \widetilde{U}_{t+1}^f\right]\right). \end{aligned}$$

In the main text we omit the difference of covariance terms to facilitate the discussion. From our simulations, this term can be shown to reinforce our results. The expected growth of the exchange rate takes the following form:

$$\begin{aligned} E_t\left[\Delta e_{t+1}\right] &= E_t\left[m_{t+1}^f - m_{t+1}^h\right] \\ &= \frac{1}{\ }\left(E_t\left[\Delta c_{t+1}^h\right] - E_t\left[\Delta c_{t+1}^f\right]\right) + \frac{1}{2}(1/\ \ - \gamma)(1-\gamma) \cdot \left(V_t\left[\widetilde{U}_{t+1}^h\right] - V_t\left[\widetilde{U}_{t+1}^f\right]\right) \end{aligned}$$