

Dynamic Debt Runs and Financial Fragility: Evidence from the 2007 ABCP Crisis*

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ABSTRACT

We use the 2007 asset-backed commercial paper (ABCP) crisis as a laboratory to study the determinants of debt runs. Our model features dilution risk: maturing short-term lenders demand higher yields in compensation for being diluted by future lenders, making runs more likely. The model explains the ten-fold increase in yield spreads leading to runs and the positive relation between yield spreads and future runs. Results from structural estimation show that runs are very sensitive to leverage and asset liquidity, but less sensitive to the degree of maturity mismatch, the strength of credit guarantees, and the asset's volatility and growth rate.

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Debt runs played a central role in the financial crisis of 2007-2008. Investors ran on asset-backed commercial paper (ABCP) starting in July, 2007; on repo starting in September, 2007; and on money market mutual funds in September, 2008. Investors also ran on large banks such as Northern Rock (September, 2007) and Bear Stearns (March, 2008).¹

These runs have reignited the debate about what causes runs and how we can prevent them. We contribute to this debate by measuring the sensitivity of runs to several contributing factors, including maturity mismatch, leverage, asset volatility and liquidity, and the strength of credit guarantees. The results help answer four questions that are vital to policy makers, regulators, bankers, and investors: How fragile are financial intermediaries? How can we design financial intermediaries *ex ante* to control the risk of future runs? What are the warning signs that a run is imminent? Finally, which interventions best prevent runs *ex post* once conditions have started deteriorating?

We address these questions by estimating a structural model of debt runs using data from the 2007 ABCP crisis. ABCP issuers, commonly referred to as conduits, are off-balance sheet investment vehicles that banks structure to invest in pools of medium- and long-term assets such as trade receivables and mortgage-backed securities.² A conduit finances these investments by issuing short-term debt to dispersed creditors and rolling over its debt until it chooses to stop investing. The bank sponsoring the conduit provides some form of credit guarantee in the event that the conduit cannot roll over its debt.

The amount of ABCP outstanding in the U.S. contracted by roughly \$400 billion (one third)

¹Brunnermeier (2008) and Krishnamurthy (2010) summarize the events of 2007-2008. We discuss the literature on ABCP below. Gorton and Metrick (2010b) and Krishnamurthy, Nagel, and Orlov (2012) empirically investigate the run on repo. Martin, Skeie, and Von Thadden (2012) provide a model of repo runs. Kacperczyk and Schnabl (2012) examine the run on money market funds.

²The prevalent view is that ABCP conduits were essentially a way for sponsoring banks to take on systemic risk beyond regulations, without transferring the risk to ABCP investors. See Acharya and Richardson (2009), Acharya and Schnabl (2009), Acharya, Schnabl and Suarez (2010), Brunnermeier (2009) and Shin (2009).

between July and December of 2007. Several authors have interpreted this event as a run on debt.³ In a debt run, creditors refuse to roll over their debt from an insolvent borrower, or even from a solvent borrower, if they fear that other creditors will refuse to roll over. In the case of ABCP, roughly half of conduits had stopped rolling over maturing debt by the end of 2007.

ABCP provides a useful laboratory to study financial fragility for four reasons. First, since ABCP conduits perform maturity transformation, they are representative of many other financial intermediaries. Second, the simple balance sheet and operating structure of ABCP conduits lends itself to modelling. Third, we have detailed data on the yield, maturity, size, and issuer’s identity for all U.S. ABCP transactions in 2007. Because yields adjust at each maturity date, their time series measures the conduit’s health continuously and can potentially be an important lead indicator of runs. Finally, as Krishnamurthy, Nagel, and Orlov (2012) argue, the run on ABCP was important in itself:

“[These] data suggest that ABCP played a more significant role than the repo market in supporting both the expansion and contraction of the shadow banking sector. The repo market is significant, but it is a sideshow compared to the happenings in ABCP.”

In fact, runs on ABCP may have had a broad effect on financial intermediation through two channels. First, runs impaired ABCP conduits’ ability to fund assets such as trade receivables or student loan receivables. Second, runs imposed losses on the banks that provided guarantees to ABCP conduits, which impaired lending to nonfinancial firms and ultimately harmed economic activity (Irani (2011)).

Our model of ABCP conduits is based on He and Xiong (2011). A conduit finances a long-term asset using short-term, dispersed debt with overlapping maturities. Creditors track the asset’s

³See, for instance, Covitz, Liang, and Suarez (2012), Acharya, Schnabl, and Suarez (2012), Gorton and Metrick (2010b), and Krishnamurthy, Nagel, and Orlov (2012).

value and optimally run as soon as the conduit’s leverage crosses above an endogenous threshold. A creditor’s decision to run depends on changing expectations that other creditors will run. Like He and Xiong’s (2011) model, ours features an equilibrium where creditors run even when the conduit is fundamentally solvent. We extend their model so that debt yields are not fixed but instead vary endogenously over time, so as to make lenders indifferent between rolling over or not. This extension is necessary: we show empirically that yields on ABCP forecast runs, and yields increase exponentially leading up to runs. To have any chance of fitting these data, the model must make predictions about the time series of yields.

We estimate the model’s parameters, which include the debt’s maturity; the perceived strength of the sponsor’s credit guarantee; and the asset’s volatility, growth rate, maturity, and liquidation value in the event of default. We observe four parameters directly in the data, and we estimate the others using the simulated method of moments (SMM).

We find three main results. First, we show that runs are very sensitive to leverage and asset liquidity, but are less sensitive to the degree of maturity mismatch, asset volatility, and credit guarantee strength. We measure these sensitivities by comparing simulated run probabilities between our estimated model and a counterfactual model with altered parameter values. We measure the sensitivity of runs to these fundamentals both before crises as well as at different stages of a crisis. Before the crisis starts, increasing the asset’s default recovery rate by 1% (from an estimated 83.2% to 84.0%) lowers the probability of a run within a year from 0.23 to 0.16. The same change to the recovery rate can still significantly reduce the one-year run probability from 0.82 to 0.61 when a crisis is underway, i.e., when yield spreads reach 53 basis points. Reducing the conduit’s leverage (debt-to-assets) ratio by 1% (from 0.937 to 0.928) reduces the probability of a run in a year by almost the same amount. By contrast, one percent changes in either the asset’s volatility or excess growth rate, in the maturity of the asset or debt, in the credit guarantee’s strength, or in the risk-free rate change the simulated probability of a run by less than 3 percentage points.

These results imply that regulators and bankers should focus especially on asset liquidity and conduit leverage when managing the risk of runs. For example, policies that improve asset liquidity (e.g., purchasing distressed assets) or reduce leverage (e.g., injecting equity) are most effective in preventing runs, both during crises and when forming new conduits. Increases in leverage or deteriorations in asset liquidity, both of which manifest as increases in ABCP yields, are warning signs that a run is imminent. The model provides a quantitative mapping between these warning signs and the likelihood of a run. For instance, the model predicts that as soon as yield spreads reach 20 basis points, the probability of a run within the next three months is roughly 35%. Of course, there are important caveats. As in any empirical exercise, our results do not necessarily extrapolate beyond the sample we use. However, our model could apply, with different parameter values, to different time periods or markets, e.g., money market funds. Additionally, we do not address the feasibility or the cost of policy interventions, nor do we analyze how changing one fundamental (e.g., liquidity) may affect another (e.g., debt maturity).

The second main result is that the model can fit several features of the 2007 ABCP crisis. For conduits offering weak credit guarantees to investors ('SIV' and 'Extendible Notes') the model comes remarkably close to fitting the magnitude and timing of the run-up in yields before runs, the overall level of ABCP yield volatility, the positive relation between yield volatility, the yield level, and the likelihood that conduits recover from a run. In both simulated and actual data, the current yield level helps forecast whether a run will occur. The model's main shortcoming is that, for conduits offering strong credit guarantees ('Full Credit' or 'Full liquidity'), it underpredicts the yield volatility and overpredicts runs when yields are high.

Our third result is theoretical. We show that introducing time-varying yields into the model makes runs more likely, relative to He and Xiong's (2011) model with constant, exogenous yields. Using He and Xiong's (2011) calibrated parameter values, we find that runs are 2 to 11 times more likely in our model than in theirs. The reason, as He and Xiong (2011) conjecture, is that the

conduit must offer high yields to induce rollover when conditions deteriorate. These high yields dilute all outstanding debt that matures later. Creditors preemptively demand higher yields to compensate them for the risk of future dilution. These higher yields in turn make leverage build up faster, which makes runs more likely. This new risk, which we call ‘dilution risk,’ can be an important driver of yields and runs.

Several papers measure the determinants of runs using a reduced-form approach. Covitz, Liang, and Suarez (2012) show that runs on ABCP conduits are negatively related to the strength of their credit guarantees. Calomiris and Mason (1997, 2003) show that bank runs during the Great Depression are correlated with measures of bank solvency and shocks to the aggregate, regional, and local economies. Using data on an Indian bank, Iyer and Puri (2011) show that runs are positively related to weaker deposit insurance, a shorter or shallower relationship with the bank, and runs by one’s peers. Chen, Goldstein, and Jiang (2010) provide evidence of strategic complementarities by showing that mutual funds with more illiquid assets exhibit a stronger relation between fund outflows and performance.

We depart from the existing empirical literature by taking a structural estimation approach. The structural approach complements the reduced-form approach by overcoming certain data limitations and by imposing different identifying assumptions. The reduced-form approach requires data on the determinants of runs, many of which are difficult to obtain in the ABCP setting.⁴ We overcome this limitation by structurally estimating these quantities. The reduced-form approach also requires a dataset with sufficient variation in the determinants of runs. Finding variation is potentially a challenge in the ABCP setting, because ABCP conduits resemble each other on many dimensions. The structural approach requires no heterogeneity in these determinants, as we use counterfactual analysis to measure the sensitivity of runs to their various determinants. Both approaches impose

⁴Data on conduit leverage are not publicly available. Asset holdings are opaque. Even if asset holdings were known, measuring asset liquidity is difficult. Even though we have data on credit guarantees’ types, we cannot measure their perceived strength.

strong identifying assumptions. The reduced-form approach assumes we have exogenous variation in the determinants of runs, which is difficult to satisfy. The structural approach assumes that the model is correct, but it allows us to jointly test predictions about runs, recoveries, and pricing. Also, it allows us to test the theory’s quantitative as well as directional predictions. The paper therefore takes a step toward providing a quantitative model of financial fragility, which is crucial for guiding the management and regulation of financial intermediaries.

The paper is structured as follows. Section I describes the model, its assumptions, and presents the solution. Section II discusses its predictions regarding yields and the likelihood of runs. Section III describes the data, and Section IV discusses the estimation method. Section V describes our empirical results. Section VI contains the sensitivity analysis, Section VII discusses policy implications, and Section VIII concludes.

I. The model

We extend the model of He and Xiong (2011) by allowing yields on short-term debt to change over time with the value of the underlying asset. All assumptions below are shared with He and Xiong (2011) unless otherwise noted.

The model includes several features of ABCP conduits. The conduit finances a long-term asset using short-term, dispersed debt with overlapping maturities. The conduit must roll over this debt several times before the program ends, so the conduit faces rollover risk. The conduit’s sponsor provides imperfect credit support if the program cannot roll over its paper. The yield on newly issued debt adjusts over time in response to changes in fundamentals.

A. Assumptions

A.1. Asset

At time zero an ABCP conduit, also referred to as the ‘firm’ or ‘program,’ borrows \$1 to purchase a long-horizon asset. In reality, the ABCP conduit holds a pool of assets from different classes, the largest being trade receivables (14%), credit cards (12%), auto loans (11%), and mortgage related assets (9%).⁵ The firm reinvests any interim cash flows from the asset. For example, the firm may buy new trade receivables using the payouts from maturing receivables. The firm therefore makes no net interim payouts to investors.⁶ The asset produces a single net payout when the firm matures, which is when the conduit winds down. The firm matures randomly and independently at a time R_ϕ which arrives according to a Poisson process with intensity W so the firm’s expected time until maturity is always $1/W$. At maturity, the asset produces a payout τ_ϕ where τ_ϕ follows a geometric Brownian motion

$$\frac{\tau_\phi}{\tau_\phi} = C + M \quad (1)$$

Agents observe τ_ϕ at all times. All agents in the economy are risk neutral and have discount rate L so the asset’s value at time t is

$$V(t) \equiv \tau_\phi \left[-(\tau_\phi - t)^\rho \right] = \frac{W}{L + W - C} \tau_\phi \quad (2)$$

⁵Table II shows a breakdown of ABCP assets by type.

⁶This assumption is made for parsimony. In He and Xiong (2011), the asset pays a fixed dividend, rdt , which is paid out in full to creditors as a coupon on the bond. Here, the face value of the zero-coupon debt can be converted into a fixed-coupon debt payment at any time, without loss of generality. In practice, money market mutual funds, the main ABCP investors, value their holdings of commercial paper using amortized cost accounting, which effectively imputes a "coupon" payment to a zero-coupon security.

A.2. Debt financing

The firm finances the asset by borrowing \$1 from a continuum of short-term creditors. The firm issues zero coupon debt with endogenous face value \bar{F}_t ⁷. Each debt contract matures randomly and independently with probability λ in the interval $[t, t + \Delta t]$ implying that a debt contract's average remaining maturity always equals $1/\lambda$. This modeling device, which follows Calvo (1983), Blanchard (1985), and Leland (1998), reflects that ABCP programs deliberately spread their debt maturities over time to reduce funding liquidity risk.

A.3. Runs, liquidation, and credit guarantees

As payment for a maturing loan, lenders accept a new loan with a potentially different face value. If lenders choose not to roll over, we say that they run. We assume lenders roll over if they are indifferent between rolling and running. If lenders run and the firm cannot raise funds to pay off maturing lenders, then the firm defaults. In default, the firm sells the asset at a fraction σ of its fair market price, which yields

$$V_t = \sigma \bar{F}_t$$

Parameter σ measures the asset's liquidity in the run state.⁸ Consistent with industry practice, the firm distributes bankruptcy proceeds V_t to outstanding creditors in proportion to their face value.

An ABCP program's sponsor typically provides a credit guarantee that helps the program pay maturing lenders if the program is unable to issue new paper. Acharya, Schnabl, and Suarez (2012) show that the type and strength of credit guarantee varies across ABCP programs, and

⁷In contrast, debt contracts in He and Xiong (2011) have face value normalized to one and offer exogenous interest rate r . In practice, commercial paper is a discount security that pays face value at maturity with no interim coupons.

⁸Note that we are not assuming that the asset liquidity is constant. We are assuming that the creditors' expected asset liquidation value *in the run state*, i.e., if the whole asset is sold to pay off running creditors, is constant.

Covitz, Liang, and Suarez (2012) find that programs with weaker credit guarantees were more likely to experience runs in 2007.

We follow He and Xiong (2011) by modeling credit guarantees as an imperfect credit line from the sponsor. If the firm experiences a run, it pays off maturing paper by borrowing from the program sponsor at the prevailing rollover yield and maturity. The credit line therefore allows the firm to potentially survive a run long enough for the program to recover and begin issuing paper again. We assume the credit line fails independently, causing default, each instant with probability λ . Once a run starts, the credit line is expected to last for $1/\lambda$ ($>$) years, so firms with higher values of λ have weaker credit guarantees.

B. ABCP pricing

The firm issues debt with face value W_t per dollar loaned at time t . We can convert face values W_t (in units of dollars) to yields r_t (in units of fraction per year) using the relation⁹

$$r_t = (W_t - 1) \times (W_t + 1) \quad (3)$$

Unlike in He and Xiong (2011), debt is priced in a competitive market so that creditors exactly break even. Specifically, the program sets its rollover face value W_t so that if creditors loan the firm \$1 at time t , they receive a debt contract worth \$1. Intuitively, if times are bad, the firm must issue paper at a high yield r_t to make creditors break even. If times are good, the new paper is almost risk free, and the new rollover yield will be close to the risk-free rate.

⁹The yield r_t is the interest rate that delivers the same value as a zero-coupon bond with face value R_t , under the assumption that both bonds are paid back in full at $\tau = \min(\tau_\delta, \tau_\phi)$. Equation (3) follows from the condition

$$E_t \left\{ \int_t^\tau e^{-\rho(s-t)} r_t ds + e^{-\rho(\tau-t)} \right\} = E_t \left\{ e^{-\rho(\tau-t)} R_t \right\}.$$

We assume the firm cannot or will not issue debt with face value above an exogenous cap, \bar{F} . Equivalently, rollover yields cannot exceed a cap \bar{r} (equation (3)). This is a critical assumption, which, as we show later, implies that creditors run exactly when rollover yields hit this cap. There are a few rationales for assuming yields are capped.

One rationale relates to clientele. The main investors in ABCP are money market funds, which are required to invest mainly in assets with very high ratings (A1 by S&P or P1 by Moody's).¹⁰ As an ABCP program's health declines and its rollover yields rise, eventually the program will lose its A1/P1 rating and its creditors will be unable to roll over its paper. Effectively, the program will be unable to roll over paper once yields reach a certain level, which we call a cap. According to this interpretation, it is the investors who walk away from the conduit in a run.

In contrast, in the next two rationales it is the sponsor who walks away from the conduit in a run. The yield cap \bar{r} may reflect the sponsor's borrowing costs. The sponsor compares the costs of rolling over paper at the market rate versus triggering the credit guarantee, bringing the conduit back onto the sponsor's balance sheet, and paying off maturing lenders by borrowing at the sponsor's own rate. For example, a highly levered sponsor may let rollover yields go to a higher yield level \bar{r} before triggering the credit guarantee, because the sponsor's own borrowing costs are higher.

The third rationale also relates to the sponsor's incentives. If conditions worsen enough, rollover yields become so high that the ABCP program's equity is almost wiped out. Without enough equity, the sponsors have little incentive to keep running the program, especially if the credit guarantee is so weak that runs impose few costs on the sponsor.

One of the goals of this paper is to estimate the (unobservable) yield cap. This estimate measures

¹⁰At the time of the 2007 crisis, rule 2a-7 under the Investment Company Act limited the portfolio share that registered money market mutual funds can invest in eligible securities not rated A1/P1 to 5% of the fund portfolio (these securities are typically rated at least A2/P2).

the minimum among the different caps implied by each of the reasons above. Future research could establish which one was the binding constraint. The answer would help understand whether runs were the result of conduits regarding short-term funding as too expensive or of investors considering the investment in ABCP as too risky.

The last rationale is that without a cap on yields we cannot find an equilibrium with runs. Intuitively, no matter how bad conditions are we can always make an infinitesimally small lender break even by promising him an extremely high face value. The high face value effectively transfers the entire firm to the lender by diluting previous lenders to nearly zero. Since the firm's value is always strictly positive, and since maturing lenders are infinitesimally small, we can always induce rollover by letting rollover yields go to infinity. This result is an artifact of the continuous-time setup. We suspect that a yield cap would arise endogenously in a more realistic model with non-infinitesimal lenders. The reason is that, once conditions get very bad, not even an infinite yield can make a larger lender break even.

C. Solution

First we solve for the dynamics of the firm's debt. Then we examine lenders' payoffs to derive the firm's value function. Next we solve for the dynamics of the state variable. We then find the condition that determines when lenders run. Finally, we solve for the equilibrium numerically. Details on the solution are in the Appendix.

C.1. Debt dynamics

The total face value outstanding at time t is given by:

$$F_t = F_0 e^{-\delta t} + \int_0^t F_s e^{-\delta(t-s)} ds. \quad (4)$$

This total face value includes paper issued at different past dates and yields. The first term reflects

that the firm borrows \$1 at time zero promises face value $\frac{1}{1 + r}$ per dollar borrowed, and a fraction $\exp(-\lambda \Delta t)$ of that initial debt has not yet matured as of time t . Since a fraction $\lambda \Delta t$ of face value $\frac{1}{1 + r}$ matures at instant t the firm must pay maturing lenders the dollar amount $\lambda \Delta t$ which it obtains by issuing new debt with total face value $(\frac{1}{1 + r} - \lambda \Delta t)$. A fraction $\exp(-\lambda (\Delta t - t))$ of the debt from time t is still outstanding at time T . The properties of the Poisson process imply that all debt is equally likely to roll over in the next instant, regardless of when the debt originated. Therefore, $\lambda \Delta t$ is also the average face value of debt rolling over at time t .

Taking derivatives of equation (4), the change in total face value at time t equals

$$\frac{dF}{dt} = \lambda F - \lambda F = 0 \quad (5)$$

The first term reflects that the firm is issuing new debt, and the second term reflects that the firm is retiring some of the old face value. The level of debt cannot decline, since debt is zero coupon and is rolled over. Face values F_t are higher in bad times, which makes the firm's debt level rise more quickly.

C.2. Lenders' payoffs

A lender receives a payout at time R which is the earliest of three events: program maturity, contract rollover, or default due to the failure of backup credit lines. That is,

$$R \equiv \min(R_\phi, R_\psi, R_\theta)$$

There are three possible scenarios a lender of vintage $\tau \leq R$ could find itself in:

1. The program matures at time $R = R_\phi$. The total proceeds are $\frac{1}{1 + r}$ so that an amount $\min(\frac{1}{1 + r} - \tau, \frac{1}{1 + r})$ is divided between the creditors. A lender with face value $\frac{1}{1 + r}$ will receive a fraction $\frac{\frac{1}{1 + r} - \tau}{\frac{1}{1 + r}}$ of this amount, since lenders are paid in proportion to their face value.

Therefore, for each dollar loaned at time t and not yet matured, the lender will receive a dollar amount

$$\frac{s}{\tau_\phi} \min(\tau_\phi - \tau_\phi) = s \min\left(1 - \frac{\tau_\phi}{\tau_\phi}\right) \quad (6)$$

2. The firm defaults at time $R = R_\theta$ after other creditors run and backup credit lines fail. The asset is sold for

$$(\tau_\theta) = \sigma \frac{W}{L + W - C} \tau_\theta \equiv \tau_\theta \quad (7)$$

For each dollar loaned at time t and not yet matured, the lender will receive

$$\frac{s}{\tau_\theta} \min(\tau_\theta - \tau_\theta) = s \min\left(1 - \frac{\tau_\theta}{\tau_\theta}\right) \quad (8)$$

3. The debt contract matures at time $R = R_\theta$. The lender chooses whether to roll over or run. As in He and Xiong (2011), a lender who runs gets paid back in full, because the amount of debt maturing at each instant is infinitesimally small and the firm's value is strictly positive. If the lender rolls over, the old loan is retired and a new loan is issued with face value τ_δ . The value in time R of one dollar loaned at time $t \leq R$ is denoted $(\tau_\theta - \tau_\theta - s; *)$ where $*$ denotes creditors' running strategy. At $R = R_\theta$, the lender takes $*$ as given, compares the value from running $(-s)$ to the value of rolling over $(-s)$, and solves

$$\max_{\text{roll over or run}} \{s - (\tau_\delta - \tau_\delta - \tau_\delta; *) - s\} = s \max_{\text{roll over or run}} \{(\cdot) - 1\}$$

C.3. Value function

In the Appendix we derive the value function and show that the lender's problem can be expressed in terms of just one state variable, $t \equiv t - t$. Loosely speaking, t measures the inverse of firm leverage. This result implies that rollover yields depend on leverage but not on the asset value (t) or the debt level (t) individually, which is intuitive. Also, the model exhibits hysteresis: even if two firms started with the same initial asset value ϕ_0 and share the same current asset value t the firm that experienced lower intermediate realizations of s for some 0 , will have

higher debt and hence higher yields and a higher probability of a run. Most importantly, investors choose whether to run by comparing the current inverse leverage ℓ_t to a threshold ℓ^* which we derive next.

C.4. Equilibrium debt prices and run threshold

Next we characterize the equilibrium properties of the face value, ℓ_t . Investors break even if for every \$1 invested in the firm at time t they receive a loan worth \$1. Formally, breaking even implies

$$1 = \ell_t \cdot (\ell_t; \ell^*) \quad (9)$$

where ℓ_t is the present value of \$1 of face value. Since face values cannot exceed the cap, ℓ^* the rollover face value is

$$\ell_t = \min [\ell^*, (\ell_t; \ell^*)^{-1}]$$

Following He and Xiong (2011), we focus on symmetric monotone equilibria: if all other investors use run threshold ℓ^* then an investor's optimal response is to use that same threshold. The following Proposition describes how to find this threshold.

Proposition 1 *Let $\ell_t \equiv \min [\ell^*, (\ell_t; \ell^*)^{-1}]$. Then*

$$\ell_t = \begin{cases} (\ell_t; \ell^*)^{-1} & \text{if } \ell_t < \ell^* \\ \ell^* & \text{if } \ell_t = \ell^* \\ \ell^* & \text{if } \ell_t > \ell^* \end{cases}$$

The proof to the Proposition is in the Appendix. The Proposition states that runs will not occur at face values ℓ_t below the cap ℓ^* . The reason is that face values can still increase if they are below their cap, potentially inducing creditors to roll over their debt. Proposition 1 characterizes

the equilibrium threshold, \bar{v}^* as the point $v_t = \bar{v}^*$ where investors break even at the capped face value, i.e., where

$$\bar{v}^* = (v^*; \bar{v}^*)^{-1}$$

The Appendix also describes the dynamics of inverse leverage v_t the Hamilton-Jacobi-Bellman equation for this problem, and the numerical procedure for finding debt prices \bar{v}_t and the equilibrium run threshold \bar{v}^* .

D. Discussion

This model assumes that the only variable that changes exogenously over time is the asset's value. Runs occur when investors expect other investors to run as a consequence of a drop in the asset's value below a threshold but not, for example, as a result of suddenly weaker credit guarantees or a jump in volatility. These and other model's parameters affect the run threshold but they remain constant.

Our assumption is strongly supported by Figure 1, which plots the time series of the price indices of the most important asset classes held by ABCP conduits in 2007 and the proportion of conduits experiencing a run each week. The figure shows that run activity intensifies between August and October of 2007, i.e., two months after the ABX index of mortgage related securities, which constitute 9% of the average conduit's portfolio, started a 20 percentage point drop.

We cannot rule out that some of the model's parameters changed suddenly in mid 2007. However, as we shall see below, the model fits the data remarkably well without these additional assumptions. Moreover, our parameter estimates are forward looking: we recover the parameter values consistent with yields and run intensities well into the crisis.

II. Model predictions

We highlight three predictions that illustrate how the model works and how our predictions differ from those in He and Xiong (2011). Since we lack closed-form solutions, we do not state model predictions as formal propositions. Instead, we illustrate predictions for specific parameter values.

A. Yields and leverage around runs

Figure 2 illustrates how leverage and yields adjust over time. The top panel plots the time series of inverse leverage (λ_t) for two simulated conduits with the same initial fundamentals but different outcomes. The flat dotted line represents λ^* the predicted run threshold. The dashed line depicts a firm whose assets grow steadily, so the firm never experiences a run. The solid line represents a firm that experiences two runs when its inverse leverage falls below λ^* . During the first run, credit lines survive long enough for the firm to recover and begin issuing paper again. Credit lines fail in the second run, causing the conduit to default and liquidate assets.

The bottom panel shows the corresponding rollover yields for those same simulated conduits. Since the conduit represented by the dashed line remains healthy, its yield remains at or near the risk-free rate, $L = 5\%$. The yields of the firm represented by the solid line spike up and become more volatile as a run becomes imminent, eventually reaching the cap \bar{y} when the run begins. As soon as this firm recovers from its first run, yields drop below the cap.

B. Runs and solvency

Like He and Xiong (2011), we find that creditors run on solvent firms but not on “super-solvent” firms. Solvent firms are those where the asset’s market value, λ_t exceeds the amount owed to creditors, λ_t . Super-solvent firms are those where the asset’s fire-sale value, $\sigma(\lambda_t)$, exceeds

¹¹ In other words, creditors will not run on a firm that has enough assets to pay off all lenders in the event that the firm defaults and has to sell the asset at a fire sale discount. In sum, extending He and Xiong's (2011) model to allow time-varying yields does not significantly affect the relation between runs and solvency.

C. Flexible pricing, dilution risk, and the likelihood of runs

In contrast, allowing time-varying yields makes runs significantly more likely, relative to He and Xiong's (2011) model with constant yields. We compare predicted run probabilities in our model to those in He and Xiong's (2011) model (henceforth, HX). The following assumptions make the models comparable. First, we ul

– values ¹⁴

The results of comparing the two models are in Table I. Panel A shows the fraction of simulated firms that experience a run within one year in our model, divided by the same fraction in HX’s model. We find that runs are between 1.93 and 11.16 times more likely in our model than in HX. Runs are especially more likely in our model if the exogenous interest rate r is higher in HX, because investors are less willing to run in HX if debt offers a higher interest rate. Runs are also relatively more likely in our model if we use a lower yield cap \bar{r} because a lower cap leaves less room for adjustment when conditions worsen. Of course, we do not claim that these results hold for all possible parameter values.

Next we explain the mechanics and then the intuition behind this result. Panels B and C help explain why runs are more likely when yields are flexible. In both models, the likelihood of a run depends on where the run threshold is, where the state variable starts, and the state variable’s dynamics. We examine each channel in turn.

First, Panel B in Table I shows that the run threshold in our model is between 1.56 and 2 times larger than in HX. All else equal, this higher threshold will tend to make runs more likely in our model.

Second, Panel C in Table I compares the firm’s initial market leverage (market value of debt divided by market value of asset) in the two models. Initial leverage is only 71-86% as high in our model as in HX. Although the initial asset value is the same in both models, debt initially has a lower market value in our model: lenders initially borrow \$1 at a low market interest rate in our model, whereas they borrow at the higher exogenous rate r in HX. The lower initial leverage means

¹⁴Our values of r are centered at the calibrated value of He and Xiong (2011). We choose values of \bar{r} much higher than r , because higher values of \bar{r} make runs *less* likely in our model, all else equal. We find that despite these high \bar{r} values, runs are still *more* likely in our model than in He and Xiong (2011).

our state variable's initial value is higher,¹⁵ which by itself will make runs less likely in our model. Since we find overall that runs are more likely in our model, it must be that this initial leverage effect is outweighed by the other two effects.

Third, state variable dynamics (i.e., leverage dynamics) tend to make runs more likely in our model. Comparing formulas for state variable dynamics in the two models, one can show that inverse book leverage has exactly the same volatility in both models, but the drift rate is lower in our model.¹⁶ The lower drift rate pulls the firm toward the run threshold, making runs more likely in our model.

Intuitively, flexible yields make runs more likely because they introduce a new risk, which we call “dilution risk,” on top of rollover risk and insolvency risk. If conditions deteriorate for a firm, the firm will have to offer higher yields to induce rollover. These higher yields increase the firm's debt by more, which dilutes the stakes of other lenders. This effect depends strongly on the assumption that bankruptcy proceeds are distributed in proportion to face value, consistent with the typical liquidation process in the ABCP market. A lender deciding whether to roll over in our model anticipates the possibility of being diluted in the future if conditions worsen. The lender therefore preemptively demands a higher yield to compensate him for dilution risk. These higher yields make the firm's leverage increase faster, pushing the firm closer to the run threshold. The higher yields also result in a higher run threshold, because yields hit their cap at higher values of δ , i.e., at a lower leverage.

¹⁵More formally, if the firm initially borrows at the risk-free rate ρ in our model, then we can show that

$$x_0 = \frac{y_0^{HX} + \frac{r}{\phi} \left(1 - \frac{\mu}{\rho + \phi}\right)}{1 + \frac{\rho}{\delta + \phi}},$$

where x_0 is the initial state variable in our model, and y_0^{HX} is the initial state variable in HX.

¹⁶The state variable x_t in our model follows equation (14). The state variable in HX follows our equation (1).

III. Data

We obtain data on all issuance transactions in the U.S. ABCP market from 2001-2010 from the Depository Trust and Clearing Corporation (DTCC). We observe the distribution of maturities and yields each time a conduit issues ABCP. We also use Barclay’s price index data on the main asset classes that ABCP programs purchase.

We obtain data on each conduit’s credit guarantee type from Moody’s Investors Service. ABCP conduits are structured with one of four possible types of guarantees (Acharya, Schnabl, and Suarez (2010)). First, in conduits structured with a full credit guarantee, the sponsor provides a line that can be drawn regardless of asset defaults. Second, in conduits with full liquidity guarantee, the sponsor provides a line that can be drawn as long as the assets are not in default. Third, in structured investment vehicle (SIV) guarantees, only a portion of conduit liabilities are covered by the line. Finally, in conduits created to issue extendible notes, issuers have the option of extending the maturity of the paper at a pre-specified penalty rate, exposing investors to asset defaults during the extension period. From the point of view of investors, full credit and full liquidity guarantees offer relatively stronger protection.

Table II shows the proportion of ABCP programs’ assets in different classes as of August 2007, at the onset of the run on the ABCP market. The largest asset classes were trade receivables (14%), credit card receivables (12%), auto loans (11%), and “securities” (11%). Mortgages and mortgage securities made up 9% of ABCP assets, although the “securities” category may contain mortgage-related securities.

We use the method of Covitz, Liang, and Suarez (2012) to identify runs in the data. We say that program i is in a run in week t if either (1) more than 10% of the program’s outstanding paper is scheduled to mature, yet the program does not issue new paper; or (2) the program was in a run in week $t - 1$ and the program does not issue new paper in week t . A program that was in a run

in week $t - 1$ recovers from the run in week t if it issues paper in week t .

We measure each program’s rollover spread as the dollar-weighted average annual yield for paper issued on Thursday of week t minus the prevailing federal funds rate.¹⁷ If the program did not issue paper on Thursday, we move one day ahead until finding an issuance transaction in week t .

Covitz, Liang, and Suarez (2012) show that the total amount of ABCP outstanding peaked at \$1.2 trillion in late July, 2007. At that time there were 339 ABCP programs operating. Yield spreads averaged 5 basis points in the first half of the year. In August 2007, the amount of debt outstanding plunged by \$190 billion and average spreads increased to 74 basis points.¹⁸ Roughly 25% of ABCP programs experienced a run in August, according to our measure. By the end of the year, 40% of ABCP programs were experiencing a run, and the total amount of ABCP outstanding was 30% below its peak. Rollover yields remained high and volatile in the second half of 2007. Although many ABCP programs experienced runs in 2007, investors took losses at only 6 ABCP programs (Acharya, Schnabl, and Suarez (2012)). The vast majority of programs were able to rely on credit guarantees to pay off maturing paper in full. It is unclear whether investors expected credit guarantees to be so strong.

In our initial analysis we use data on runs and spreads from 2007 only. We face the trade-off that a larger sample provides more precise estimates, but it is harder to argue that model parameters are constant over a longer period. Year 2007 is an ideal sample because it contains many runs and also several months of pre-run data. Moreover, yield spreads remained flat and low in 2006, which makes them uninformative about changes in run probabilities.

¹⁷We choose Thursday because amounts outstanding are measured at the end of Wednesday each week.

¹⁸Important events in early August 2007 include American Home Mortgage’s declaration of bankruptcy (August 6), the suspension of three subprime mortgage funds at BNP Paribas (August 9), emergency liquidity provision by the ECB (August 9) and the Federal Reserve (August 10), and the granting of an emergency loan to Countrywide Financial (August 15).

IV. Estimation method

We estimate the model in two subsamples.¹⁹ The first contains the 191 conduits with either a full credit or full liquidity guarantee. 45% of these conduits experienced a run in 2007. The second subsample contains the 90 conduits with either an SIV guarantee or extendible paper. 83% of these conduits experienced a run in 2007.

First we explain how we measure four model parameters directly from the data. Next we describe the structural estimation of the four remaining parameters.

A. Observable parameters

A.1. Risk-free rate

Investors' discount rate L is also the risk-free interest rate. We set L to 4.9%, the annualized yield of one-month T-bills at the beginning of 2007.

A.2. Debt maturity

The average debt maturity in our model is $1/\tau$. We set $1/\tau$ to 37 days, the average maturity of ABCP as of March of 2007. The assumption that τ is constant may be problematic given that most programs experienced a rat-race whereby they offered shorter rollover maturities to prevent creditors from running (Brunnermeier and Ohmke (2012)). However, average maturities were shortened to a minimum of about one month, on average, at least 25 weeks before runs occurred. In fact, maturities stabilized before yields started to adjust upwards (results available on request).

¹⁹Ideally we would estimate the model in even finer-grained subsamples, but the small number of conduits prevents us from doing so.

A.3. Asset duration

The expected program lifespan, which corresponds to the asset's duration, is $1/W$. If we add the assumption that new ABCP programs are created at a constant rate, then the model predicts that the average age of programs alive at any time equals $1/W$. The average age of ABCP programs as of July, 2011 is 9.7 years.²⁰ Therefore, we set W to $1/9.7 = 0.103$.

A.4. Asset growth rate

Equation (2) implies that C , the asset's growth rate, is also the asset's expected return. We set C to the estimated expected return on the average portfolio of ABCP assets.²¹ We estimate the expected return for each asset class ABCP programs hold (e.g., credit card receivables), then compute a weighted average using Société Générale's portfolio holdings for the ABCP industry in August 2007.²² We estimate each asset class's expected return following the method of Fama and French (1993). Specifically, for each asset class we estimate a time-series regression of the asset class's realized monthly excess return (from Barclay's) on three risk factors.²³ An asset class's expected return is then the risk-free rate plus its risk premium, which is the sum of its factor loadings times their respective risk premia. Estimated factor loadings, risk premia, and portfolio weights are in Table II. Our estimate of C is 6.1%, which is 1.2 percentage points above the risk-free rate.

²⁰We need to confirm that the average age at the beginning of our sample was stable over time.

²¹Eventually we hope to measure program-specific expected returns, since we know each program's portfolio by asset type.

²²"The ABC's of ABCP," Société Générale (2008).

²³The risk factors are $TERM$, DEF_HY , and DEF_IG . $TERM$ is the difference between the long-term government bond return and the monthly T-bill rate, both from CRSP. DEF_HY (DEF_IG) is the difference between the return on Barclay's U.S. long-term high yield (investment grade) corporate bond portfolio and the long-term government bond return.

B. Identification of remaining parameters

The remaining parameters to estimate are M (fundamental volatility), γ (the weakness of credit guarantees), σ (asset liquidity), and \bar{y} (the cap on yield spreads). We estimate parameters using the simulated method of moments (SMM). This estimator chooses parameter values that minimize the distance between moments generated by the model and their sample analogs. Details are in Appendix 3. Below we define the 61 moments used in SMM estimation. We also provide intuition for how the moments help identify each parameter. All moments depend on all parameters, but we highlight the moment to which each parameter is most sensitive.

B.1. Strength of credit guarantees

The moments most informative about γ (the weakness of credit guarantees) are $\pi_1(R)$ defined as the fraction of runs that are followed by a recovery (i.e., debt issuance) within R weeks after the run's start, for $R = 1 \dots 8$. Lowering γ increases the predicted probability of recovering from a run. Intuitively, a stronger credit guarantee (i.e. lower γ) buys the conduit time for fundamentals to improve so that the conduit can exit the run before defaulting.

B.2. Asset volatility

Fundamental volatility (M) is mainly identified off yield volatility. Predicted yield volatility depends directly on M

$$V_t(y) = \left(y_t - \bar{y} \right)^2 M^2 \quad (10)$$

The first term in (10) increases in the level of yields, so the model predicts that yield volatility is high when the level of yields is high.²⁴ The model therefore produces time-varying volatility in yields, even though fundamental volatility is constant.

²⁴In our numerical exercises we find that as x decreases, the increase in slope more than offsets the decrease in x , so that indeed yield volatility increases as x drops.

σ_{it}^2 is the moment we use to measure yield volatility. Specifically, σ_{it}^2 is a 3×1 vector containing the variance of one-week changes in yield spreads $(y_{it} - y_{ft})$ conditional on the current yield spread:

$$\sigma_{it}^2(\mathbf{y}_{ik}) \equiv [(y_{it+1} - y_{ft+1}) - (y_{it} - y_{ft}) | (y_{it} - y_{ft}) \in \mathbf{y}_{ik}] \quad \text{for } \mathbf{y}_{ik} = [y_{i1} \ y_{i2} \ y_{i3}]$$

We assign each conduit/week observation to one of three bins \mathbf{y}_{ik} depending on whether the yield spread is low, medium, or high. We create bins in both simulated and actual data by first computing y_i for each conduit. For conduits with runs, y_i is defined as the conduit's maximum yield in the sample. For conduits with no runs, y_i is the larger of the conduit's maximum yield spread in the sample, and the average y_j across conduits that experienced runs. We then define the bins as²⁵

$$\begin{aligned} \mathbf{y}_{i1} &= [10\% \ 40\%) \text{ of } y_i \\ \mathbf{y}_{i2} &= [40\% \ 70\%) \text{ of } y_i \\ \mathbf{y}_{i3} &= [70\% \ 100\%] \text{ of } y_i \end{aligned}$$

B.3. Yield caps

The yield cap (\bar{y}) is identified off the yield level immediately before runs begin. As discussed in Section I.C.4., the model predicts that a run begins the instant yields hit \bar{y} . We cannot observe \bar{y} directly since we do not have continuous time data on rollover yields. However, the yield levels leading up to runs allow the model to infer the value of \bar{y} . The moment we use to measure yield levels leading up to runs is \mathbf{y}_{i3} a vector containing the average yield spread in each of the 26 weeks before runs begin.

²⁵We choose these cutoffs so the number of observations is roughly equal across bins. We exclude observations where yields are less than 10% of $MAX R_i$, because yield volatility for these observations is sensitive to the choice of initial condition x_0 in our simulations, and we want moments that do not depend on x_0 .

B.4. Asset liquidity

Having identified $\gamma > M$ and γ^- we identify asset liquidity (σ) off of conditional run probabilities. Moment μ_4 is a 3×8 matrix containing the probability of a run within R weeks ($R=1 \dots 8$), conditional on conduit i 's current yield spread being low, medium, or high (we use the bins $\{\gamma_{ik}\}$ defined above). This moment helps identify σ because run probabilities are very sensitive to liquidity. The reason is that higher asset liquidity reduces yields: a higher σ means recovery rates in default are higher, so lenders can break even with relatively lower yields. Lower yields, in turn, reduce the growth in leverage over time and also induce yields to hit their cap at a higher leverage level, both of which make runs less likely.

V. Empirical results

We start by evaluating how well the model fits the data. Then we present and discuss our structural parameter estimates.

A. Model fit

Figure 3 compares actual and simulated values of $\mu_1(R)$, the fraction of runs that experience a recovery within R weeks of the run's start. The model fits these moments well: for all values of R and in both subsamples, simulated values are within the empirical 95% confidence interval, given by the shaded areas in the figure. Recovery rates are significantly higher in the subsample with stronger credit guarantees, consistent with the model's prediction.²⁶

Figure 4 plots μ_2 which measures the time-series volatility of ABCP yields conditional on the yield level. The model fits these moments quite well in the SIV/extendible subsample (left panel), capturing both the overall level of yield volatility and the positive relation between the yield level

²⁶The t -statistic for the difference in $M_1(8)$ across subsamples is 2.3.

and yield volatility. The fit is not as good in the full credit/liquidity subsample (right panel), producing simulated volatility much lower than empirical volatility. The estimation procedure sacrifices fitting this moment so that it can fit other moments better.

Figure 5 shows that the model does a good job at fitting average yield spreads in event time before runs (

These results contribute to the debate over what causes runs. The literature has been divided in two groups (see Goldstein (2011) for a survey). The first group proposes that a run’s causes are unobservable, so it is impossible to predict or assign probabilities to runs.²⁹ The second group, motivated by Gorton (1988), proposes that runs are caused by deteriorating fundamentals, hence we can predict and assign probabilities to runs. Our model, along with several others, belongs to this second group.³⁰ The results above show that a model of fundamental-driven runs fits the data quite well.

B. Parameter estimates

Table III contains parameter estimates along with their standard errors.³¹ The estimate of γ is higher in the subsample with full credit/liquidity guarantees. This difference is statistically significant (with a t -statistic of 3.4). This result provides a useful consistency check: we obtain a lower estimate of γ indicating a stronger credit guarantee, in the subsample with stronger credit guarantees. Estimates imply that investors expected conduits with full credit/liquidity (SIV/extendible) guarantees to survive 122 (38) days in a run before the credit guarantee failed.³²

The asset’s estimated volatility (σ) is roughly 4.5% per year in both subsamples. For comparison, the volatility of the ABX mortgage index in the first half of 2007 was 5.7%.³³ The other asset categories ABCP conduits hold, such as trade receivables and credit card receivables, are likely less

²⁹This group follows Bryant (1980) and Diamond and Dybvig (1983). The unobservable is sometimes labelled a sunspot or just “panic.”

³⁰Goldstein and Pauzner (2005), He and Xiong (2011), Morris and Shin (1998), and Vives (2011) also belong to this group.

³¹Standard errors depend on the 61×61 covariance matrix for the empirical moments. We estimate this matrix using the method of seemingly unrelated regressions, taking into account time-series autocorrelation as well cross-conduit correlation, both within moments and across moments.

³²Once a run starts, the average time until default is $1/(\theta\delta)$.

³³The ABX index we use tracks the cost of insurance against default on MBS backed by AAA tranches of mortgages originated in the second half of 2006.

volatile than the ABX was in 2007, so an estimate for M slightly below 5.7% seems reasonable.

The estimated cap on yield spreads, $\bar{\gamma}$ is 86 basis points per year for full credit/liquidity conduits and 107 basis points for SIV/extendible conduits. The difference in $\bar{\gamma}$ across subsamples is significant at the 10% but not the 5% confidence level (t -statistic of 1.76). A higher $\bar{\gamma}$ in SIV/extendible conduits makes sense if yield caps reflect the sponsor's own borrowing costs, as discussed in Section I.B. The sponsors of SIVs and conduits with extendible notes were typically nonbanking financial institutions. Unlike large commercial banks, which have ample access to diversified sources of short-term funding (including central bank borrowing), nonbanking financial institutions tend face higher borrowing costs.

Our estimate of σ the asset's liquidity or recovery rate in default, is 92% for full credit/liquidity conduits and 83% for SIV/extendible conduits. The difference across subsamples is highly significant (t -statistic of 5.1). SIVs held mostly bonds and other tradable securities, most notably asset-backed securities. Prior to the subprime crisis, these assets were considered liquid. However, during the financial crisis, securitization markets became severely impaired, effectively making these assets very illiquid. Similarly, many of the conduits structured with extendible notes had substantial exposures to mortgage markets, most significantly some conduits that specialized in warehousing mortgages prior to their sale to pools of mortgage-backed securities.

We estimate a lower σ for SIV/extendible conduits because their run rates were higher, conditional on yield levels (Figure 6). These conduits' weaker guarantees (i.e., higher γ) are not sufficient for explaining the higher run rates.³⁴ Our estimates of σ are within the literature's range of esti-

³⁴One potential concern is that the higher run rates simply resulted from asset values y dropping more in the SIV/extendible subsample, and we mistakenly infer a difference in asset liquidity. This concern cannot explain our result, however, because we only use moments that are conditional on yield levels or runs. For example, if there were fewer runs in the full credit/liquidity subsample because their asset values did not drop much, then we should not see high yields in that subsample, so we should not necessarily find a difference across subsamples in run rates *conditional on yields being high*. Empirically, we do find large differences in run rates conditional on yield levels

estimated default recovery rates and fire sale discounts. For corporate defaults, Hennessy and Whited (2007) measure recovery rates of around 90%, and Andrade and Kaplan (1998) estimate recovery rates of roughly 80-90%. In the fire-sale discount literature, Coval and Stafford (2007) find a 7.9% discount in fire sales of stocks, and Ellul, Jotikashira, and Lundblad (2010) report a 6.8% discount on fire sales of corporate bonds.

To summarize, our estimates offer two reasons why the extendible/SIV conduits experienced more runs than the full credit/liquidity conduits. First, their credit guarantees were weaker, consistent with Covitz, Liang, and Suarez (2012). Second, they held less liquid assets. Below we explore which of these reasons is more powerful. A third potential reason, which we do not test, is that extendible/SIV conduits owned assets that dropped more in value in 2007.

VI. Sensitivity analysis

All of the eight parameters estimated above, as well as the conduits current leverage, potentially affect the likelihood of runs. In this Section we measure the sensitivity of run probabilities to each of these contributing factors. In the next Section we discuss implications for regulators, banks, and investors.

We measure these sensitivities by computing the (counterfactual) run probability in response to a 1% change in each of the model’s parameters at a time. We use the parameter estimates for the SIV/extendible conduits as our base-case values, since the model has the best fit for that subsample. The results are shown in Table IV.

We compute these sensitivities at different states of a funding crisis. Panel A of Table IV describes the four selected intervention points. Column (1) depicts a state before the crisis, where yield spreads have reached only 1% of the estimated yield cap (1.07 basis points) and the current

(our M_4) across subsamples, from which the model infers a large difference in α .

debt-to-assets proportion is 90%. Columns (2), (3), and (4) represents states of increasing severity during the crisis, with yield spreads at 10% (92.2% leverage), 50% (93.7% leverage), and 90% of the cap (94.2% leverage), respectively.

A. Sensitivity or runs before a crisis

Panel B of Table IV shows that, given the estimated parameter values and a yield spread of 1.07 basis points, the baseline run probability within three months is a low 0.029 (column (1)). At this stage, runs are most sensitive to the conduit's leverage and the asset's liquidity. A 1% decrease in initial leverage (0.90 percentage points) decreases the three-month run probability to 0.007. A 1% increase in the asset's liquidity, σ , from the estimated 83.2% to 84.0%, reduces the run probabilities by almost the same amount.

The one-year run probabilities are also highly sensitive to these two parameters before the crisis: a 1% decrease in leverage, or a 1% increase in asset liquidity, decrease the one-year run probability by almost a third of its value, i.e., from 0.236 to just below 0.16 (Panel C, column (1)).

Table IV also shows the results of buying assets with higher growth rates or higher volatility. A 1% increase in the asset's excess growth rate, $C - L$, while keeping the volatility constant, will reduce the three-month run probability by 17%, to 0.025, and the one-year run probability by 5%, to 0.224. The sensitivity of run probabilities to a 1% increase in the asset's volatility, while keeping the growth rate constant, is very low.

Surprisingly, an increase in the asset's expected maturity makes runs less likely: a 1% increase in W which corresponds to just over a month, decreases the 3-month run probability by 0.005, and the one-year run probability by 0.012. An increase in asset maturity while keeping debt maturity fixed worsens the mismatch between assets and liabilities. In this model, however, because creditors always have the option to run, then longer maturities of assets relative to liabilities increase the

creditor’s upside in high states. Consistent with this interpretation, we would expect similar effects from a 1% shortening of debt maturity. However, the effect is negligible before the crisis.

Finally, the risk of a run is insensitive to 1% changes in the risk free rate, the cap on rollover yields, or the strength of the credit guarantee when yield spreads reach only 1% of the cap. Our estimates across subsamples of conduits with different types of credit guarantees show that large differences in the expected credit lines are associated with large differences in the run intensities. However, as opposed to asset liquidity or leverage, small differences in the perceived credit line strength have no effect at all on the probability of runs.

B. Sensitivity of runs during a crisis

The three-month run probabilities increase quickly after yield spreads rise above 10% of their cap. When yield spreads reach 50% of the cap, a run can occur within three-months with almost 0.7 probability (Panel B of Table IV). At this state, a run is very likely to occur within a year: the probability exceeds 0.8 (Panel C). However, a small capital injection has a large predicted effect on the run probability whether the crisis is still distant or imminent. For example, reducing leverage by 1% when a run is imminent reduces the three-month run probability by 41%, from 0.90 to 0.53 (column (4) of Panel B).

A 1% increase in asset liquidity (σ) affects run probabilities almost equally. The main intuition for this result is that the ultimate reason why creditors run is the fear that the asset has to be liquidated at a discount before they roll over their debt. An increase in σ effectively acts as deposit insurance for ABCP creditors. The impact of liquidity is even more striking given that a 1% change in σ amounts to a 4.6% ($1\% \times 0.82 / (1 - 0.82)$) change in the illiquidity discount, which is still small compared to the estimated discount variation in the literature. For example, Coval and Stafford (2007) report a standard deviation of 9.72% in the fire sale discount of stocks. The estimates in Ellul, Jotikashira, and Lundblad (2010) imply a standard deviation of almost 25% for fire sales of

corporate bonds.

Panels B and C in Table IV also show that an increase in the yield cap, \bar{y} has little effect on the run probabilities regardless of the state of the crisis. We obtain this result because an increase in \bar{y} has two opposing effects in the model. On one hand, it increases the distance to the yield cap, delaying the run. On the other hand, it increases the amount of dilution risk, which induces investors to run sooner. Finally, three-month and one-year run probabilities are fairly insensitive to all other parameters, especially at the worse stages of the crisis.

VII. Policy discussion

There are several reasons why regulators may want to prevent runs. Runs on financial institutions may disrupt the flow of credit to nonfinancial firms that rely on intermediated finance to fund investment and operations and, thus, ultimately harm economic activity. The view that bank runs hamper economic activity is supported by evidence from banking crises in the United States (Friedman and Schwartz (1963), Bernanke (1983), Calomiris and Mason (2003), Ramirez and Shively (2012)) and cross-country studies (Kaminsky and Reinhard (1999), Dell’Aricia, Detragiache, and Rajan (2008)). Also, a run on one part of the financial system may trigger runs on other parts, amplifying the run’s costs. For example, a run on ABCP could trigger a run on money market funds (the main investors in ABCP) or a run on the large banks sponsoring ABCP conduits.

Before discussing how regulators might prevent runs, we describe the warning signs that regulators, banks, and investors can use to gauge the probability of a future run. Figure IV plots the simulated probability of a run within 3, 6, and 12 months as a function of the ABCP conduit’s current leverage (top panel) and rollover yield spread (bottom panel). The top panel shows that the probability of a future run is strongly increasing in the conduit’s current leverage. An increase in leverage from 87% to 88% increases the probability of a run within 3 months from roughly 20% to 45%. While this figure is useful to a conduit’s sponsor, it is less useful to regulators or investors,

who currently do not have access to data on conduit leverage. The bottom panel, however, is useful to all parties. It shows that the current rollover yield strongly predicts runs. For example, an increase in rollover spreads from 20 to 40 basis points signals an increase from 35% to 55% in the probability of a run within 3 months.

In the previous Section we perturbed parameter values by 1% and measured the resulting changes in run probabilities. As in Rochet and Vives (2004) and Vives (2011), we interpret these perturbations as interventions by regulators or conduit sponsors. The estimated sensitivities are important for regulators interested in controlling the risk of runs on ABCP conduits and similar intermediaries. These sensitivities can also help sponsoring banks control the risk of runs when managing existing conduits or designing new ones.

Our analysis shows that runs are very sensitive to small changes in leverage ($1 - \lambda_t$). This result implies that conduits' sponsors can significantly reduce the probability of future runs by including more equity in new conduits' capital structure. Regulators can achieve the same effect by placing restrictions on new conduits' leverage. Our result also suggests that once a crisis is underway, modest equity injections by either program sponsors or regulators can make runs significantly less likely.

The sensitivity analysis also shows that runs are very sensitive to σ the asset's liquidity, i.e., expected recovery rate in default. It is less clear how regulators or sponsoring banks can improve liquidity. One possibility is that regulators make a market in distressed assets or purchase them outright.

We interpret reducing the asset's volatility (M) or increasing its growth rate (C) as buying higher quality assets. Sponsors can clearly influence asset quality when creating new conduits, but probably not once a crisis is underway. Regulators could influence asset quality by placing credit-rating restrictions on the assets conduits buy, similar to the restrictions on money market funds.

Our results show that an effective control of the run probabilities would require large changes in asset quality.

Increasing W corresponds to the conduit buying shorter-term assets, and increasing γ corresponds to issuing shorter debt maturities. Reducing β corresponds to a strengthening the conduit's credit guarantee, which, to be credible, may not only require a strengthening of its legal terms, but also an improvement of the sponsor's own financial health. Reducing L corresponds to reducing the Fed funds rate. If the cap on yields (\bar{r}) reflects the sponsor's own borrowing cost (Section I.B.), then increasing \bar{r} corresponds to interventions that improve the sponsor's health. Alternatively, if the yield cap results from restrictions on money market funds, then regulators could increase \bar{r} by loosening the requirement that money market funds mainly invest in A1/P1-rated assets. Our sensitivity analysis implies that interventions targeting these channels will have much smaller effects on the likelihood of runs, unless the interventions can change parameters by a large amount.

These policy implications have to be interpreted with caution. First, our analysis does not consider how policy interventions directed at ABCP conduits may spill over to other markets. To wit, an increase in the rollover yield caps may delay runs on ABCP conduits at the expense of weakening their sponsor's balance sheet, or at the expense of making money market funds take on more risk. Second, we do not address how policy interventions may affect future crises via moral hazard. For instance, an intervention in one crisis may make managers expect interventions in the future, leading to more risk-taking behavior and increased risk of future crises. Third, our sensitivity analysis is subject to the Lucas critique, as it does not consider how changing one parameter may affect other parameters. For example, Cheng and Milbradt (2011) endogenize the choice of the asset's growth rate and volatility of a short-term financed firm, such as an ABCP conduit, and find that a lengthening of debt maturities may lead to more risk-shifting. A comprehensive policy analysis would need to incorporate the reaction of ABCP conduit managers and investors to any interventions. We leave this analysis to future research.

VIII. Conclusions

We estimate a dynamic model of debt runs using data from the 2007 crisis in asset-backed commercial paper. The model allows yields to change over time, which introduces dilution risk: the conduit must offer higher yields to induce rollover if conditions worsen, which dilutes the claims of other lenders. Introducing dilution risk into the model can make runs up to 11 times more likely. Our model of fundamental-driven runs fits several features of the data, including the dramatic increase in yields on ABCP leading up to runs, the high probability of recovery once a run starts, the positive relation between yields and the probability of future runs, the overall level of volatility in ABCP yields, and the positive relation between yield volatility and the yield level. The model fits much better in the subsample of conduits with the weakest credit guarantees. We find that runs are very sensitive to conduit leverage and expected asset liquidation costs. Runs are much less sensitive to the degree of maturity mismatch, the strength of credit guarantees, and the asset's volatility and growth rate. These sensitivities are useful inputs to regulators and banks attempting to control the risk of runs.

Our analysis can be extended and improved in three main directions. To keep the estimation tractable, we assume that yields cannot exceed an exogenous cap. It would be interesting to explore the determinants of this cap. Second, we have taken debt maturity as given. Brunnermeier and Ohmke (2012) show that rollover risk can produce a rat race in which debt maturity unravels to its shortest possible value. Empirically, we see that the maturity rat race occurs *before* yields adjust: the average maturity drops but then levels off around 25 weeks before runs occur, after which the yields start adjusting upwards. Why maturities and yields adjust sequentially instead of simultaneously is an important question that we also leave for future research. Finally, the dynamic debt runs framework, and the estimation method we propose here, can be used to study the runs on money market funds in late 2008, potentially shedding more light on what causes runs.

Appendix 1: Proofs and Derivations

Value function

In this Appendix we derive the value function. Each creditor's strategy consists of choosing τ^* , a boundary for fundamental τ below which they will run. The value at time t to a creditor who last loaned one dollar at time $s \leq t$ equals

$$\begin{aligned} (V_t(\tau; \tau^*)) = & \int_t^\infty \left\{ -\rho(\tau-t) \int_s^\infty \min\left(1, \frac{\tau}{\tau}\right) \mathbf{1}_{\{\tau=\tau_\phi\}} \right\} + \\ & \int_t^\infty \left\{ -\rho(\tau-t) \int_s^\infty \min\left(1, \frac{\tau}{\tau}\right) \mathbf{1}_{\{\tau=\tau_\theta\}} \right\} + \\ & \int_t^\infty \left\{ -\rho(\tau-t) \max_{\text{rollover or run}} \left\{ \int_{\tau_\delta}^\infty \int_{\tau_\delta}^\infty (V_{\tau_\delta}(\tau; \tau^*) - 1) \mathbf{1}_{\{\tau=\tau_\delta\}} \right\} \right\} \end{aligned} \quad (11)$$

We introduce the notation $V_t \equiv V_t(\tau; \tau^*)$. Loosely speaking, V_t measures the inverse of firm leverage. Simplifying equation (11) yields

$$(V_t(\tau; \tau^*)) = \int_s^\infty (V_{\tau}(\tau; \tau^*)) \quad (12)$$

$$\begin{aligned} (V_{\tau}(\tau; \tau^*)) = & \int_t^\infty \left\{ -\rho(\tau-t) \min(1, \tau) \mathbf{1}_{\{\tau=\tau_\phi\}} \right\} + \\ & \int_t^\infty \left\{ -\rho(\tau-t) \min(1, \tau) \mathbf{1}_{\{\tau=\tau_\theta\}} \right\} + \\ & \int_t^\infty \left\{ -\rho(\tau-t) \max_{\text{rollover or run}} \left\{ \int_{\tau}^\infty (V_{\tau}(\tau; \tau^*) - 1) \mathbf{1}_{\{\tau=\tau_\delta\}} \right\} \right\} \end{aligned} \quad (13)$$

The new function $(V_t(\tau; \tau^*))$ is the value at time t to a creditor with one dollar of face value. This value does not depend on when the creditor last rolled over, due to the memoryless properties of the exponential distribution.

Applying Ito's Lemma and equation (5), it is straightforward to show that inverse leverage follows

$$\frac{dV_t}{V_t} = [C - \rho(V_t - 1)] \frac{dt}{V_t} + M \frac{d\tau}{V_t} \quad (14)$$

Since the value function (13) and the dynamics of π_t are both functions of π_t only, then π_t is the only state variable of the problem.

Proof of Proposition 1. Note first that any creditor's continuation payoff must be equal to 1. By definition, for any π_t the payoffs are

$$\begin{aligned} \max_{\text{run or roll over}} \{1 - \pi_t (\pi_t^*)^{-1}\} &= \max_{\text{run or roll over}} \{1 - \min [\bar{\pi} - (\pi_t^*)^{-1}] - (\pi_t^*)\} \\ &= \max_{\text{run or roll over}} \{1 - \min [\bar{\pi} - (\pi_t^*) - 1]\} = 1 \end{aligned}$$

First we show $\pi_t = \bar{\pi}$ if $\pi_t < \pi_t^*$. If $\pi_t < \pi_t^*$ creditors will refuse to roll over their loan at maturity. Because running gives them a payoff of 1, rolling over must give them a strictly lower payoff, i.e., $\pi_t - (\pi_t^*)^{-1} < 1$. By definition of π_t , this inequality becomes

$$\min [\bar{\pi} - (\pi_t^*)^{-1}] \times (\pi_t^*) < 1$$

Since $(\pi_t^*)^{-1} \times (\pi_t^*) = 1$ it must be that $\min [\bar{\pi} - (\pi_t^*)^{-1}] = \bar{\pi}$. Therefore, $\pi_t = \bar{\pi}$.

Suppose that $\pi_t \geq \pi_t^*$. In this case, creditors choose to roll over. If they do so, their payoff must be at least as high as running, which pays 1. Because their payoffs are bounded above by 1, then rolling over must always pay 1. Therefore, for $\pi_t \geq \pi_t^*$

$$\begin{aligned} \min [\bar{\pi} - (\pi_t^*)^{-1}] \times (\pi_t^*) &= 1 \\ \Rightarrow \min [\bar{\pi} - (\pi_t^*) - 1] &= 1 \end{aligned}$$

The previous equality holds if either $\bar{\pi} - (\pi_t^*)^{-1} = 1$ for every $\pi_t \geq \pi_t^*$ or if there exist some $\pi_t' \in [\pi_t^*, \infty)$ where $\bar{\pi} - (\pi_t')^{-1} = 1$ and $\bar{\pi} - (\pi_t^*)^{-1} < 1$ for all other $\pi_t \neq \pi_t'$. Because $(\pi_t^*)^{-1}$ is strictly increasing in π_t then π_t' is unique. Moreover, because $\bar{\pi} - (\pi_t')^{-1} = 1$ is a minimum, then $\pi_t' = \pi_t^*$ i.e., the lowest point in the support. In summary, then either

$$\pi_t = \begin{cases} \bar{\pi} - (\pi_t^*)^{-1} & \text{for all } \pi_t \geq \pi_t^* \\ \bar{\pi} - (\pi_t^*)^{-1} & \text{if } \pi_t = \pi_t^* \end{cases} \quad [\text{case (i)}]$$

or

$$t = \begin{cases} (t - \tau^*)^{-1} & \text{if } t < \tau^* \\ - & \text{if } t = \tau^* \\ - & \text{if } t > \tau^* \end{cases} \quad [\text{case (ii)}].$$

Next we show that case (i) cannot be true, arguing by contradiction. In case (i) we have

$$\tau^* \equiv (\tau^* - \tau^*)^{-1} -$$

exactly at the run boundary. Hence we have

$$1 = \tau^* (\tau^* - \tau^*) - (\tau^* - \tau^*) \quad (15)$$

The equality above is from the definition of τ^* and the inequality is from $\tau^* > 0$ and $\tau^* - \tau^* < 0$. By the assumed continuity of $(\tau^* - \tau^*)$ at $\tau^* = \tau^*$ there exists a $E > 0$ such that for all $\tau' \in (\tau^* - E, \tau^*)$ $\tau^* - (\tau' - \tau^*) > 1$. We therefore have a contradiction: At $\tau' < \tau^*$ the investor runs (since we assume runs happen below τ^*), but at τ' it is not optimal to run (since $\tau^* - (\tau' - \tau^*) > 1$ the payoff from rolling over at $\tau' = \tau^*$ is strictly greater than 1, the payoff from running). ■

Limits of the value function

The numerical procedure below relies on the limit of debt prices τ when inverse leverage becomes large. In this limit, there is effectively no chance of default or runs, so τ simplifies to

$$\begin{aligned} \lim_{x \rightarrow \infty} (\tau; \tau^*) &= \tau \left\{ -\rho(\tau - t) \left[\mathbf{1}_{\{\tau = \tau_\phi\}} + \mathbf{1}_{\{\tau = \tau_\delta\}} \right] \right\} \\ &= \frac{W_+}{L + W_+}. \end{aligned}$$

Analytical solution to the ODE for $(\tau; \tau^*)$ below the run threshold

Using equations (13) and (14), we can write the general Hamiltonian-Jacobi-Bellman (HJB)

equation:

$$\begin{aligned}
L(\cdot; \cdot^*) &= [C - \cdot(\cdot - 1)] \cdot_t x(\cdot) + \frac{M}{2} \cdot_t^2 x x(\cdot) \\
&+ W[\min(1 - \cdot_t) - \cdot(\cdot)] \\
&+ \mathbb{1}_{\{x_t < x^*\}} [\min(1 - \cdot_t) - \cdot(\cdot)] \\
&+ \cdot \left[\max_{\text{rollover or run}} \{ \cdot_t - (\cdot; \cdot^*) - 1 \} - \cdot(\cdot) \right]
\end{aligned} \tag{16}$$

Since $\cdot_t - (\cdot; \cdot^*) \leq 1$ the HJB equation simplifies to

$$\begin{aligned}
L(\cdot; \cdot^*) &= [C - \cdot(\cdot - 1)] \cdot_t x(\cdot) + \frac{M}{2} \cdot_t^2 x x(\cdot) \\
&+ W \min(1 - \cdot_t) + \mathbb{1}_{\{x_t < x^*\}} \min(1 - \cdot_t) \\
&- (W + \mathbb{1}_{\{x_t < x^*\}} + \cdot) \cdot(\cdot) + \cdot.
\end{aligned} \tag{17}$$

For a given threshold \cdot^* the HJB equation can be solved analytically for $\cdot_t = \cdot^* \iff \cdot_t = \cdot^* - (\cdot_t - \cdot^*)^{-1}$. We rely on this analytical solution in our numerical procedure for finding \cdot^* . The method follows He and Xiong (2011).

When $\cdot_t = \cdot^*$ the HJB simplifies to

$$\begin{aligned}
0 &= [C - \cdot(\cdot - 1)] \cdot_t x + \frac{M}{2} \cdot_t^2 x x \\
&+ W \min(1 - \cdot_t) + \mathbb{1}_{\{x_t < x^*\}} \min(1 - \cdot_t) \\
&- (L + W + \mathbb{1}_{\{x_t < x^*\}} + \cdot) \cdot(\cdot) + \cdot.
\end{aligned} \tag{18}$$

The exact solution as

$$\begin{aligned}
(\cdot^*) &= \cdot_2^\eta + \cdot_3^{-\gamma} - \frac{5}{3} - \frac{4}{3 + \cdot_1} \\
/ &\equiv \frac{1}{2 \cdot_2} \left(\cdot_2 - \cdot_1 + \sqrt{(\cdot_2 - \cdot_1)^2 - 4 \cdot_3 \cdot_2} \right) = 0 \\
-\phi &\equiv \frac{1}{2 \cdot_2} \left(\cdot_2 - \cdot_1 - \sqrt{(\cdot_2 - \cdot_1)^2 - 4 \cdot_3 \cdot_2} \right) = 0
\end{aligned}$$

$$\begin{aligned}
c_1 &= (C + \beta - \beta) \\
c_2 &= \frac{M}{2} - 0 \\
c_3 &= -(W + L + \beta + \beta) - 0 \\
c_4 &= \beta \mathbf{1}_{\{x \leq 1/l\}} + W \mathbf{1}_{\{x \leq 1\}} \geq 0 \\
c_5 &= \beta + \beta \mathbf{1}_{\{x \geq 1/l\}} + W \mathbf{1}_{\{x \geq 1\}} - 0
\end{aligned}$$

and coefficients c_2 and c_3 are determined by boundary conditions, value matching, and smooth pasting. Next we examine the cases where $\beta \leq \beta^*$ and either $\beta \leq 1$ $1 \leq \beta \leq 1$ or $\beta \geq 1$. Of course, some of these cases are irrelevant if, for instance, $\beta^* = 1$.

Case 1: $\beta \leq 1$

The solution is

$$(\beta^*) = \eta - \frac{5}{3} - \frac{4}{3 + \beta} \quad \text{for } \beta \leq 1$$

where

$$\begin{aligned}
c_4 &= \beta + W \\
c_5 &= \beta.
\end{aligned}$$

Following He and Xiong (2011), we eliminate the term with $\beta^{-\gamma}$ so that the solution does not explode as β approaches zero.

If $\beta^* = 1$ then we can already solve for β as a function of β^* . Value matching and Proposition 1 imply that

$$\begin{aligned}
(\beta^*) &= (\beta^*)^\eta - \frac{5}{3} - \frac{4}{3 + \beta} (\beta^*) = \frac{1}{\beta^*} \\
&= \left[\frac{1}{\beta^*} + \frac{5}{3} \right] (\beta^*)^{-\eta} + \frac{4}{3 + \beta} (\beta^*)^{1-\eta}
\end{aligned}$$

Case 2: $1 \leq \beta \leq 1$

The solution is

$$(\beta^*) = \beta_1^\eta + \beta_2^{-\gamma} - \frac{5}{3} - \frac{4}{3 + \beta}$$

where

$$\begin{aligned}
4 &= \succ \\
5 &= . + W \\
1 &= + \frac{W}{\phi + /} \left[\frac{\phi}{3} - \frac{\phi + 1}{3 + 1} \right] \\
2 &= \frac{W}{\phi + /} \left[\frac{(1 - /)}{3 + 1} + \frac{/}{3} \right] \\
&= \left(\frac{1}{=} + \frac{5}{3} \right) (\ast)^{-\eta} + \frac{4}{3 + 1} (\ast)^{1-\eta} \\
&\quad - 2 (\ast)^{-\gamma-\eta} - \frac{W}{\phi + /} \left[\frac{\phi}{3} - \frac{\phi + 1}{3 + 1} \right]
\end{aligned}$$

Case 3: 1

$$(\ast) = 1^{-\eta} + 2^{-\gamma} - \frac{5}{3} - \frac{4}{3 + 1}$$

where

$$\begin{aligned}
5 &= . + \succ + W \\
4 &= 0 \\
1 &= 1 + \eta \succ \frac{\phi}{\phi + /} \left[-\frac{1}{3} - \frac{1}{1 + 3} \left(1 + \frac{1}{\phi} \right) \right] \\
2 &= 2 + ^{-\gamma} \succ \frac{/}{\phi + /} \left[-\frac{1}{3} - \frac{1}{1 + 3} \left(1 - \frac{1}{/} \right) \right]
\end{aligned}$$

Formulas for 1 and 2 are above. The expression for is now

$$\begin{aligned}
&= \left(\frac{1}{=} + \frac{5}{3} \right) (\ast)^{-\eta} + \frac{4}{3 + 1} (\ast)^{1-\eta} - 2 (\ast)^{-\gamma-\eta} \\
&\quad - \eta \succ \frac{\phi}{\phi + /} \left[-\frac{1}{3} - \frac{1}{1 + 3} \left(1 + \frac{1}{\phi} \right) \right] \\
&\quad - \frac{W}{\phi + /} \left[\frac{\phi}{3} - \frac{\phi + 1}{3 + 1} \right]
\end{aligned}$$

Restrictions on parameter values

We impose the following necessary restrictions on the parameter values. To prevent the firm's fundamental value from exploding or becoming negative, equation (2) requires

$$C \leq L + W$$

Second, we limit σ the recovery rate in liquidation, to

$$\sigma \leq \frac{L + W - C}{W}$$

so that $\sigma \frac{\phi}{\phi + \rho - \mu} \leq 1$ i.e., the asset liquidation value $\sigma (V_t)$ is not enough to pay off all lenders when the firm's maturity value V_t drops below the total book value of outstanding debt, D_t

Appendix 2: Numerical solution of value function and run threshold

This Appendix describes the algorithm we use to solve numerically for the value function $V(x; \theta^*)$ and the run threshold x^* . The solution for V satisfies the HJB equation, value matching and smooth pasting for x everywhere (including at $x = x^*$), the limit condition $\lim_{x \rightarrow \infty} V(x; \theta^*) = 0$ and the condition $V(x^*; \theta^*) = 1 - \theta^*$. The algorithm follows the following steps:

1. Guess a value for θ^* .
2. Solve the HJB for $x \leq x^*$. The analytical solution is in the Appendix above.
3. Solve V numerically for $x > x^*$ as follows:
 - (a) Using the standard method, reduce the order of the ODE by introducing a new variable z :

$$z \equiv \frac{1}{V} \frac{dV}{dx} \quad (19)$$

$$\begin{aligned} z &= -\frac{1}{V} \frac{d^2V}{dx^2} \\ &= -\frac{2[C + \frac{1}{2}]}{M^2} + \frac{2}{M^2} - \frac{2W_{\min}(1 - \theta^*)}{M^2} + \\ &\quad \frac{2(L + W + \frac{1}{2})}{M^2} - \frac{2}{M^2} \frac{1}{z^2} \end{aligned} \quad (20)$$

- (b) Solve analytically for $z(x; \theta^*)$ and $V(x; \theta^*) = V(x; \theta^*)$ using the solutions for z in Appendix 1.
 - (c) Using the initial conditions in step (b), numerically integrate the system of ODEs in step (a) for $x \in [x^*, \infty]$ where ∞ is a very large value of x that approximates $x = \infty$.
4. Check whether the numerical solution for $V(x; \theta^*)$ is sufficiently close to its known limit, derived in Appendix 1. If so, we have found the equilibrium threshold x^* . If not, return to step 1.

Appendix 3: Details on SMM estimation

The SMM estimator is

$$\hat{\Theta} \equiv \arg \min_{\Theta} \left(\widehat{\mathbf{M}} - \widehat{\mathbf{m}}(\Theta) \right)' \mathbf{W} \left(\widehat{\mathbf{M}} - \widehat{\mathbf{m}}(\Theta) \right) \quad (21)$$

$\widehat{\mathbf{M}}$ is a vector of moments estimated from the actual data, and $\widehat{\mathbf{m}}(\Theta)$ is the corresponding vector of model-implied moments. The hat on $\widehat{\mathbf{m}}$ indicates that model-implied moments are estimated by simulation. For these simulations, we use parameter values Θ to simulate a sample many times larger than the empirical sample, then we compute the moment from simulated data in the same way we compute the empirical moment. \mathbf{W} is a positive definite weighting matrix. The efficient weighting matrix is the inverse of the estimated covariance of moments \mathbf{M} . Since our 61×61 covariance is estimated with considerable noise, we use only its diagonal elements to compute a weighting matrix, and we divided the diagonal elements by the number of elements in each group of moments to apply roughly equal weight to our four sets of moments. For instance, we divide the eight elements of \mathbf{W} corresponding to β_1 by eight.

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Table I: The Effect of Flexible Prices on Runs

This table compares the predictions from our model, in which yields change over time, to the predictions of He and Xiong (2011), which we denote by HX, where yields are constant and set to r . Parameter values are from HX: $\rho = 1.5\%$, $\phi=0.077$, $\alpha = 55\%$, $\sigma = 20\%$, $\mu = 1.5\%$, $y_0 = 1.4$, $\delta = 10$, and $\theta = 5$. Panel A shows the fraction of simulated firms that experience a run in our model within one year, divided by the same fraction from HX. Panel B shows the run threshold in our model (x^*) divided by the run threshold in HX (y^*). Panel C shows the firm's initial market leverage in our model ($= R(x_0; x^*)/y_0$), divided by initial market leverage in HX ($= V(y_0; y^*)/y_0$). \bar{r} is the yield cap in our model.

**Panel A: Ratio of the probability of a run in one year
in our model to He and Xiong (2011)**

	$r = 5\%$	$r = 7\%$	$r = 9\%$
$\bar{r} = 15\%$	1.97	3.89	11.16
$\bar{r} = 20\%$	1.93	3.80	10.87
$\bar{r} = 25\%$	1.90	3.72	10.61

**Panel B: Ratio of the run threshold (assets / debt)
in our model to He and Xiong (2011)**

	$r = 5\%$	$r = 7\%$	$r = 9\%$
$\bar{r} = 15\%$	1.58	1.77	2.00
$\bar{r} = 20\%$	1.57	1.75	1.98
$\bar{r} = 25\%$	1.56	1.74	1.97

**Panel C: Ratio of the initial market leverage (debt / assets)
in our model to He and Xiong (2011)**

	$r = 5\%$	$r = 7\%$	$r = 9\%$
	0.856	0.779	0.706

Table II: Estimating the Expected Return on ABCP Assets

For each category of ABCP assets reported by Moody's Investors Service with a matched portfolio in Barclays we estimate a time series regression of excess monthly returns on excess returns on three bond risk factors, TERM (long government bonds minus the T-bill rate), DEF_IG (long U.S. corporate investment grade bonds minus long-term government bonds), and DEF_HY (long U.S. corporate high yield bonds minus long-term government bonds). In all cases we use as many months of data as possible. Panel A shows the risk premium for each factor, estimated as the average excess return over the longest period of data available. Panel B shows the estimated risk premium for each asset class, estimated as the sum of factor loadings times factor risk premia. Their t -statistics are shown underneath, in brackets. All risk premia are in units of fraction per month. Fraction of total CP outstanding is measured on August 31, 2007 and is from Societe Generale. Panel C shows the calculations used to estimate the expected return on ABCP assets.

Panel A: Estimated factor risk premia (fraction per month)							
		TERM	DEF_IG	DEF_HY			
Average		0.27%	0.01%	0.19%			
Standard deviation		2.87%	1.63%	3.82%			
Standard error		0.13%	0.08%	0.21%			
Months used		12/1979 - 12/2010	12/1973 - 12/2010	7/1983 - 12/2010			
Panel B: Estimated factor loadings and risk premia for ABCP asset classes							
Moody's category	Barclay's index	TERM	Factor loadings (t statistics)		R^2	Risk premium	Fraction of total CP outstanding
			DEF_IG	DEF_HY			
Trade receivables	N/A						0.14
Credit cards	US ABS Credit Card	0.33 (9.05)	0.11 (1.65)	0.12 (3.08)	0.32	0.11%	0.12
Auto loans	US ABS Autos	0.18 (8.30)	0.08 (1.84)	0.08 (3.52)	0.32	0.07%	0.11
Securities	US Securitized	0.34 (22.35)	0.03 (1.13)	0.07 (4.48)	0.68	0.10%	0.11
Commercial loans	N/A						0.10
Other mortgages	N/A						0.02
Student loans	US ABS Floating Rate: Student Loans	0.14 (1.85)	-0.02 (-0.16)	0.23 (3.25)	0.38	0.08%	0.07
Residential mortgages	ABX (from Markit)						0.05
Auto leases	US ABS Autos	0.18 (8.30)	0.08 (1.84)	0.08 (3.52)	0.32	0.07%	0.04
CBO & CLO	N/A						0.04
Consumer loans	N/A						0.03
Commercial mortgage loans	US CMBS	0.71 (7.68)	0.10 (0.63)	0.44 (4.70)	0.42	0.28%	0.02
Equipment leases	N/A						0.02
Floorplan	US ABS Autos	0.18 (8.30)	0.08 (1.84)	0.08 (3.52)	0.32	0.07%	0.02
Other mortgages	US MBS	0.32 (20.38)	0.00 (-0.02)	0.05 (3.28)	0.65	0.10%	0.01
Equipment loans	N/A						0.01
Govt guaranteed loans	US Agencies Government Guaranteed	0.54 (19.49)	-0.04 (-0.73)	0.06 (2.23)	0.67	0.16%	0.01
Insurance premiums	N/A						0.01
Others	N/A						0.08
Panel C: Estimation of expected return on ABCP assets (μ)							
Fraction of ABCP with non-missing risk premium			0.520				
Weighted average risk premium (per month)			0.098%				
Weighted average risk premium (per year)			1.180%				
Annualized 1-month T-bill rate on 12/29/2006			4.910%				
μ = T-bill rate + risk premium =			6.090%				

Table III: Structural Parameter Estimates

This table reports the estimates of the model's structural parameters, with standard errors in parentheses. Estimation is done by the simulated method of moments (SMM), which chooses parameter estimates that minimize the distance between actual and simulated moments. Section I. describes the model used to simulate moments. Standard errors account for time-series autocorrelation and correlation across asset-backed commercial paper (ABCP) conduits, both within and across the 61 moments used in estimation. The data is for all issues of ABCP in 2007. There are 191 conduits offering full credit or full liquidity guarantees and 90 offering SIV or extendible notes guarantees. The χ^2 statistic is for the test of over-identifying restrictions, where the null hypothesis is that the distance between the model-simulated and empirical vectors or moments is zero. Its p -value is in parentheses.

Sample	Weakness of credit guarantee θ	Asset volatility (% per year) σ	Cap on yield spreads (b.p. per year) \bar{r}	Asset liquidity (proportion) α	χ^2 (p -value)
Full credit / liquidity guarantee	0.304 (0.150)	4.606 (0.026)	86.0 (2.1)	0.923 (0.006)	98,708 0.000
SIV / Extendible Notes guarantee	0.972 (0.126)	4.459 (0.028)	107.1 (11.8)	0.832 (0.017)	70,209 0.000

Table IV: Sensitivity of Run Probabilities to Model Parameters

This table shows the effect on run probabilities of changing model parameters by 1% from their estimated values. Panel A describes the four intervention points from which the sensitivity analysis starts. These points corresponds to yield spreads of 1%, 10%, 50%, and 90% of the capped value. Panel B (C) shows simulated 3-month (1-year) run probabilities. The first rows of Panels B and C show simulated run probabilities when parameters are at their estimated value for the SIV/extendible subsample (estimates in Table III). Each following row shows run probabilities using counterfactual parameter values. In each row, one parameter changes at a time from its estimated value by 1%, in the direction shown. The last column of Panel B shows the predicted run threshold x^* predicted for the corresponding set of parameter values.

Panel A: Description of intervention points				
Intervention point				
	(1)	(2)	(3)	(4)
Initial yield spread (basis points per year)	1.07	10.71	53.55	96.39
Leverage (debt-to-assets)	0.903	0.923	0.937	0.942

Panel B: 3-month run probabilities								
Perturbed parameter	Estimated value	Direction of change	Probability of a run within 3 months			Run threshold (x^*)		
			(1)	(2)	(3)		(4)	
None		None	Baseline case	0.029	0.244	0.683	0.901	1.0602
$1/x_t$	Panel A	-	Lower initial leverage	0.007	0.101	0.363	0.531	1.0602
α	0.832	+	Higher asset liquidity	0.008	0.102	0.366	0.534	1.0497
$\mu - \rho$	0.012	+	Higher excess growth rate	0.025	0.218	0.636	0.850	1.0588
ϕ	0.103	-	Longer asset maturity	0.025	0.219	0.636	0.850	1.0588
σ	0.045	+	Higher volatility	0.030	0.250	0.688	0.905	1.0603
δ	9.872	+	Shorter debt maturity	0.029	0.242	0.679	0.896	1.0601
ρ	0.049	-	Lower risk-free rate	0.029	0.245	0.685	0.903	1.0603
$\bar{\tau}$	0.011	+	Higher yield cap	0.029	0.243	0.682	0.900	1.0602
θ	0.972	-	Stronger credit guarantee	0.029	0.244	0.682	0.901	1.0602

(continues)

Table IV: continued

Panel C: 1-year run probabilities								
Perturbed parameter	Estimated value	Direction of change	Interpretation	Probability of a run within 1 year				Run threshold (x^*)
				(1)	(2)	(3)	(4)	
None		None	Baseline case	0.236	0.516	0.817	0.944	1.0602
$1/x_t$	Panel A	-	Lower initial leverage	0.158	0.368	0.610	0.724	1.0602
α	0.832	+	Higher asset liquidity	0.159	0.369	0.612	0.726	1.0497
$\mu - \rho$	0.012	+	Higher excess growth rate	0.224	0.493	0.788	0.915	1.0588
ϕ	0.103	-	Longer asset maturity	0.224	0.494	0.788	0.915	1.0588
σ	0.045	+	Higher volatility	0.241	0.522	0.820	0.946	1.0603
δ	9.872	+	Shorter debt maturity	0.236	0.514	0.815	0.941	1.0601
ρ	0.049	-	Lower risk-free rate	0.237	0.517	0.817	0.945	1.0603
\bar{r}	0.011	+	Higher yield cap	0.236	0.516	0.816	0.944	1.0602
θ	0.972	-	Stronger credit guarantee	0.236	0.516	0.817	0.944	1.0602

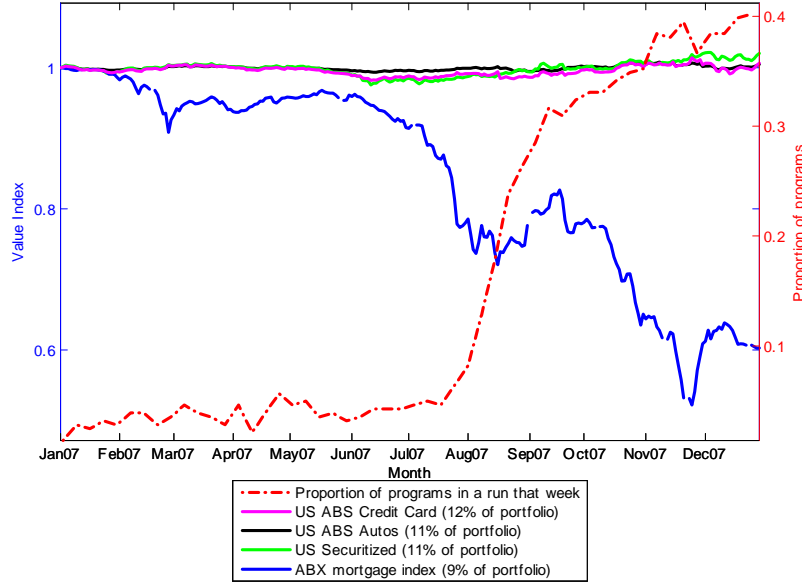


FIGURE 1:

This figure shows the time series of the price indices of the top four asset categories in the portfolio of asset-backed commercial paper (ABCP) programs in 2007, as well as the proportion of ABCP programs experiencing runs in a given week. Data for the prices is from the Barclay's bond indices matching the ABCP investments, as reported by Moody's. Data for the proportion of runs is from the DTCC data base on all issues by ABCP programs, where a run is defined as in Covitz, Liang, and Suarez (2012): an ABCP program experiences a run in a given week if either (1) more than 10 program's outstanding paper is scheduled to mature, yet the program does not issue new paper; or (2) the program was in a run the previous week and it does not issue new paper in the current week.

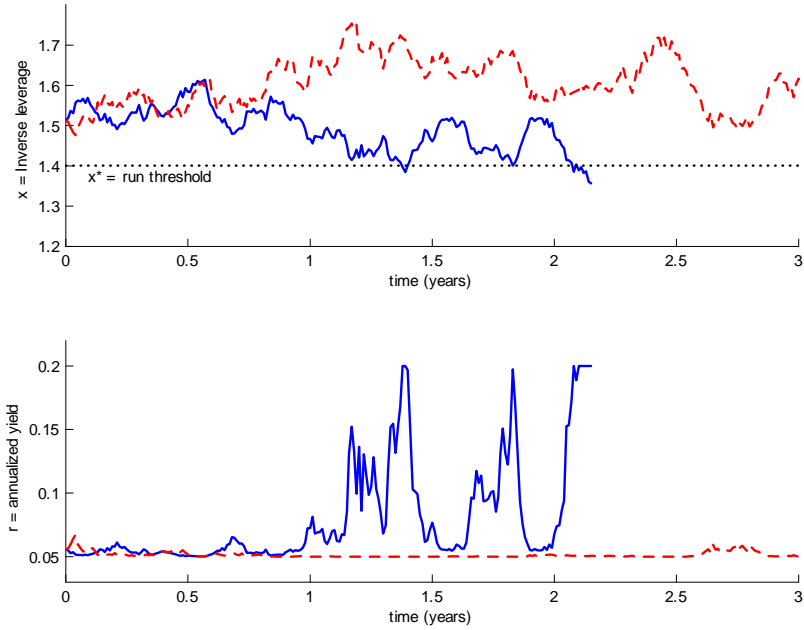


FIGURE 2:

This figure shows two possible simulated paths for a program of a given initial leverage and parameter values. The top panel shows simulated values of x_t , inverse leverage. The dotted line denotes the run threshold. The bottom panel shows simulated paths of annual yields at rollover for the same two programs. The risk-free rate is 5% and the cap on the rollover yield is 20%.

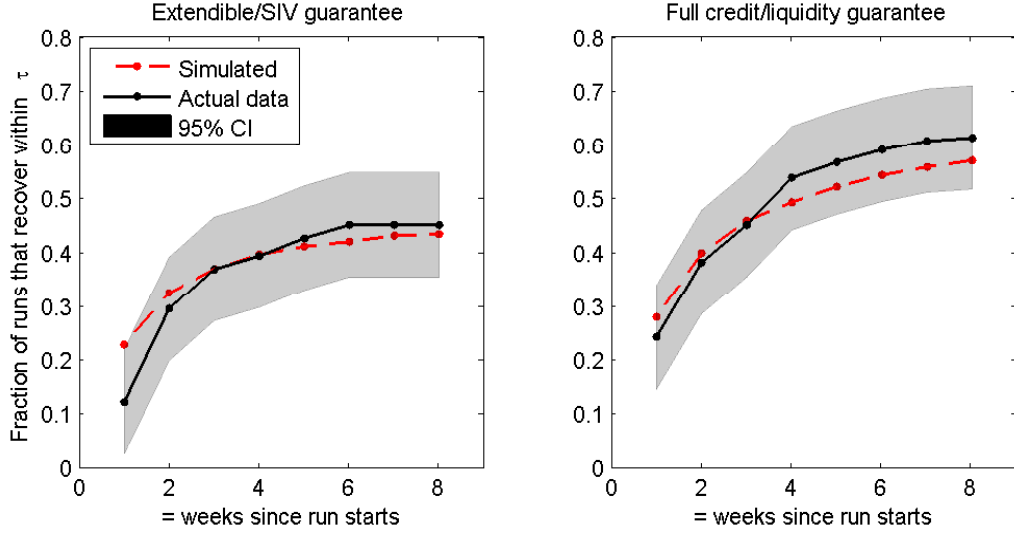


FIGURE 3:

This figure shows $\hat{\mu}_1$, the first set of moments used in simulated method of moments (SMM) estimation. $\hat{\mu}_1(R)$ is the fraction of runs that experience a recovery (i.e., the conduit reissues paper at least once) within R weeks of the run's start. The solid (dashed) line shows the empirical (simulated) values. The shaded area denotes the 95% confidence interval for empirical moments. Simulations use the parameter estimates in Table III.

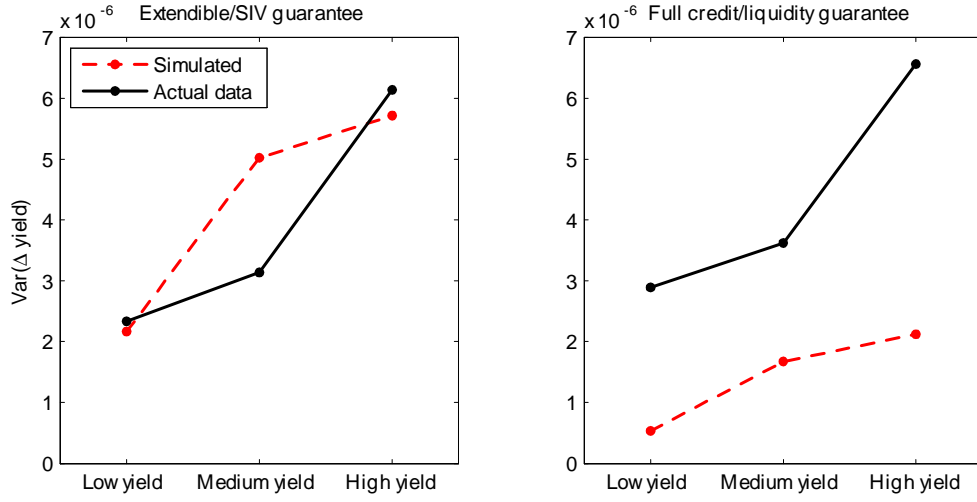


FIGURE 4:

This figure shows σ_2 , the second set of moments used in simulated method of moments (SMM) estimation. σ_2 is the variance of one-week changes in yield spreads, conditional on the yield spread's current value. "Low yield" includes conduit/week observations where the current yield spread is 10-40% of σ_1 , defined in Section IV.B.2. Cutoffs for "Medium yield" and "High yield" are 40-70% and 70%+ of σ_1 . The solid (dashed) line shows the empirical (simulated) values. Simulations use the parameter estimates in Table III.

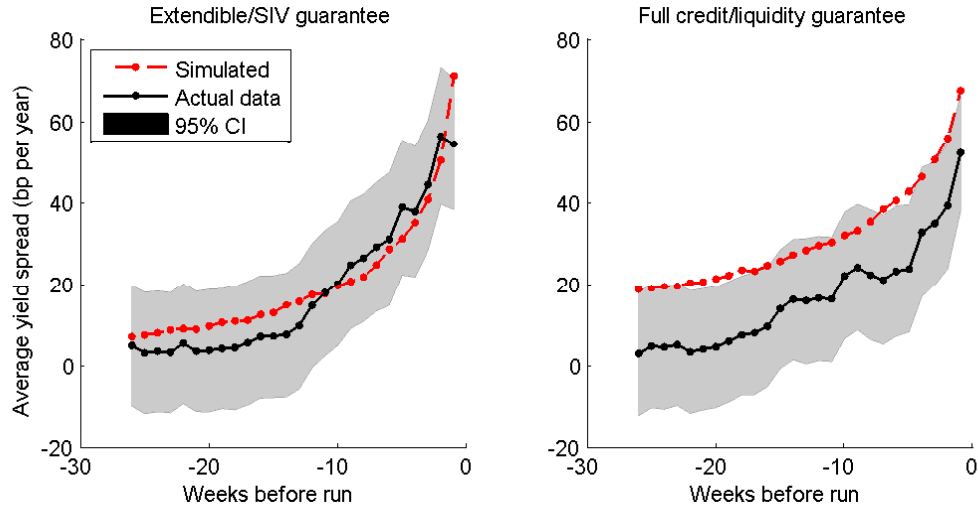


FIGURE 5:

This figure shows μ_3 , the third set of moments used in simulated method of moments (SMM) estimation. μ_3 is the average yield spread in event time before runs. The solid (dashed) line shows the empirical (simulated) values. The shaded area denotes the 95% confidence interval for empirical moments. Simulations use the parameter estimates in Table III.

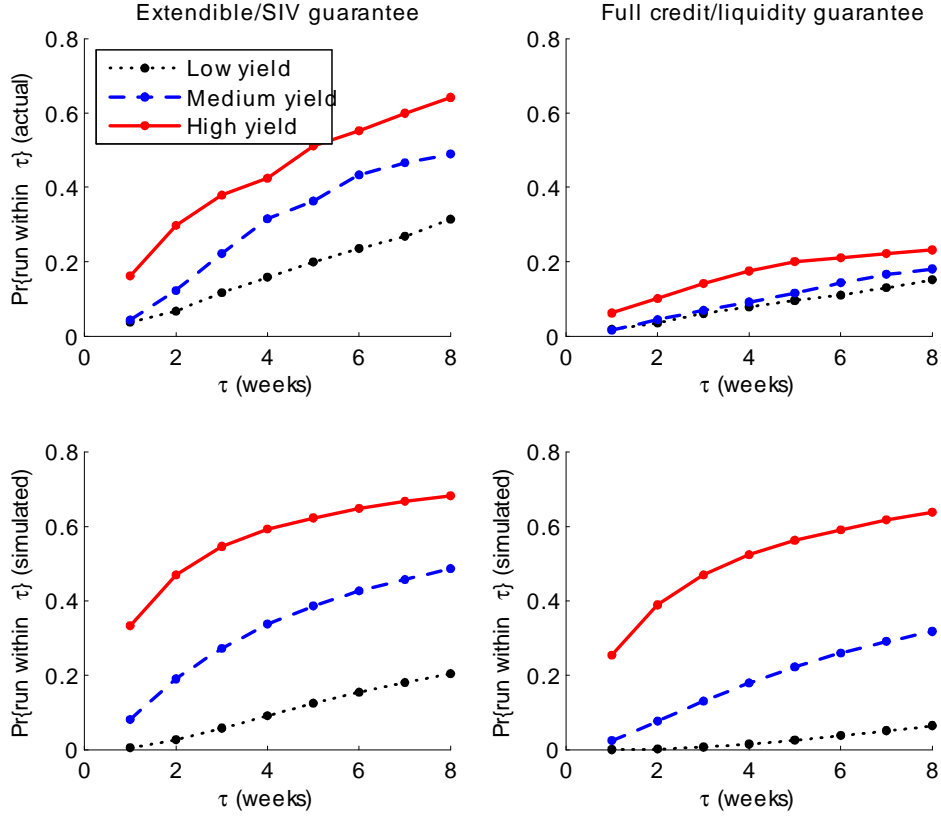


FIGURE 6:

This figure shows M_4 , the fourth set of moments used in simulated method of moments (SMM) estimation. M_4 is the probability of experiencing a run within the next τ weeks, conditional on the current yield level. "Low yield" includes observations where the current yield spread is 10-40% of $MAX R_i$, defined in Section IV.B.2. Cutoffs for "Medium yield" and "High yield" are 40-70% and 70%+ of $MAX R_i$. The top panels show the actual empirical values and the bottom panels the simulated values. The solid line represents the moments for initially "High yield," the dashed line represents the moments for initially "Medium yield," and the dotted line represents the moments for initially "Low yield." Simulations use the parameter estimates in Table III.

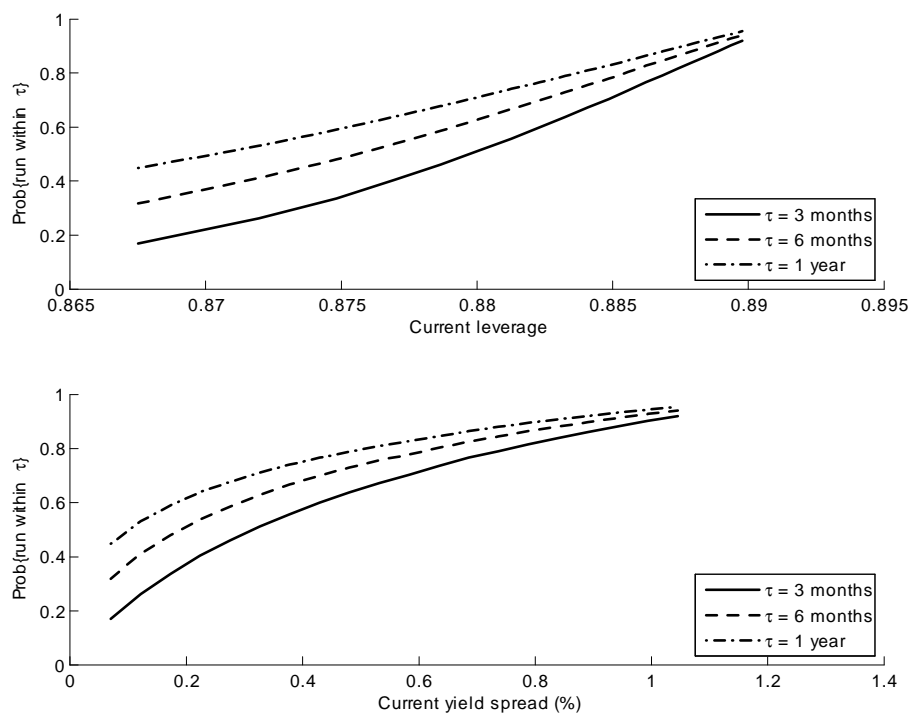


FIGURE 7:

Panel A plots the relation between the firm's current leverage and the probability of a run within the next 3, 6, and 12 months. Panel B shows the relation between the firm's current yield spread and the probability of a future run. Results are from model simulations using the parameter estimates for the structured investment vehicle (SIV) and extendible notes-guaranteed subsample of asset-backed commercial paper (ABCP) conduits in Table III.