The Role of Economic Space in Decision Making

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Abstract:

This paper analyzes the role that economic space plays in private and public decision making. Both geographic and characteristic space are considered. The choice of spatial location, whether it be a physical location or a product position, can have significant consequences for other economic decisions, and those secondary decisions are the focus of the paper. In particular, it is useful to have methods of measuring economic proximity in geographic and characteristic space and to have ways of determining the implications of that proximity for market outcomes. I summarize some of the methods that my coauthors and I have developed for estimating economic distance and for assessing its implications.

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1 Introduction

For centuries, geographic space has been a factor in economic models (e. g., von Thunen 1826). However, only after the seminal article by Hotelling (1929) did spatial models assume a central position in the discipline of industrial organization. Furthermore, Hotelling emphasized that the same framework that is used to analyze geographic space can also be used to analyze characteristic space.

My paper is concerned with economic space — both geographic and characteristic — and the role that it plays in private and public decision making. Once a spatial location has been chosen, it can have important consequences for other economic decisions, and I focus on those decisions. In particular, it is useful to have methods of measuring economic proximity, or its reciprocal distance, and to have ways of determining the implications of that proximity for market outcomes. To illustrate, although it is straight forward to measure geographic closeness in kilometers and one might therefore be tempted to use that measure, Euclidean distance might not be the relevant economic concept. Instead, a discrete measure such as sharing a market boundary might be more relevant. Unfortunately, it is often difficult to specify an appropriate concept *a priori*. In this paper, I summarize some of the methods that my coauthors and I have developed for choosing amongst measures of economic distance and for assessing the implications of those measures.

To clarify what the paper is about, it helps to state what it is not about. In particular, I do not deal with two important spatial topics: the choice of geographic location by firms and households (e.g., Ellison and Glaeser 1997) and the new economic geography (e.g., Krugman 1991). Both model the choice of geographic location and the forces that determine agglomeration, whereas I assume that those factors have been determined at an earlier stage that is not modeled. Furthermore, both deal solely with geographic space in one or two dimensions, whereas I consider more general measures of economic distance in geographic and characteristic space and possibly higher dimensions.

To motivate the discussion, I first illustrate the importance of space with four examples; two that involve location in geographic space and two that involve location in characteristic space. Consider first a retail outlet. Its geographic location determines not only who its customers will be but also the number and identity of its rivals. Indeed, since consumers view retailers of similar products that are located in close proximity to one another as close substitutes, the pattern of firm location will influence any market game. In particular, consider a pricing game. The slopes of a firm's best–reply function in that game are proportional to diversion ratios (i.e., to the number of buyers who move from product i to product j when the price of

i increases divided by the number of buyers who stop purchasing from i).[2] Moreover, both numerator and denominator of those ratios are influenced by location. In particular, a firm with more neighbors will lose more customers when it increases its price, and the customers that it looses will gravitate towards outlets that are located in close geographic proximity. Finally, since market equilibrium is determined by the intersection of the best–reply functions, market prices will be strongly influenced by location patterns.

A closely related issue is the delineation of geographic markets for antitrust purposes. Such markets must include rivals who can provide customers with highly substitutable products. Moreover, the diversion ratios that determine the slopes of firms' best–reply functions are closely related to own and cross–price elasticities of demand, our usual measures of substitutability.[3] Therefore, for the reasons noted above, geographic location is a principal determinant of the set of firms that are included in an antitrust market.

Now consider location in characteristic space. One can view products as bundles of characteristics, and when two products possess similar characteristics, they are apt to be viewed by consumers as close substitutes. Therefore, just as the locations of retail outlets in geographic space have strong implications for market games, the locations of differentiated products in characteristic space have similar implications. Indeed, in this context, diversion ratios are determined by the number of products in the group of substitutes and by the clustering of those products' characteristics.

Finally, it should be obvious that, just as an outlet's geographic location is essential for determining its geographic market for antitrust purposes, a product's location in characteristic space is essential for determining its antitrust product market. In particular, own and cross–price elasticities of demand are heavily influenced by product positioning.

The paper is organized as follows. The next section develops a fairly general parametric spatial econometric model in which choices (e.g., prices or quantities) are continuous. The distinguishing feature of that model is that dependent variables, regressors, and errors can be spatially related. Section three maintains the continuous–choice assumption but relaxes the parametric framework. In particular, the ways in which firms interact (for example, the diversion ratios) are estimated semiparametrically, and no assumptions are made concerning the distribution of the errors. Section four considers the situation in which choices are discrete variables such as entry, contract type, or drilling decisions. As with the continuous–choice model,

[2] This is true up to a second–order approximation.

[3] In a symmetric game, the ratio of the cross to the own–price elasticity is equal to the diversion ratio.

dependent variables, regressors, and errors can be spatially related. Finally, section five concludes with a discussion of applications. Most but not all of the applications involve problems from industrial organization. Like the more general discussions, the applications are divided into two classes — continuous and discrete choices. In both of those classes, some examples are based on work that has already been done, whereas others are suggestions of areas that seem fruitful for future research.

2 Spatial Econometrics: The Standard Case

The analysis of space in cross–section data bears a close resemblance to the analysis of time in time–series data. Nevertheless, there are at least four important differences. First, time is one dimensional, whereas geographic space is two dimensional and characteristic space can be of higher dimension. Second, time flows in one direction – from past to future, whereas, even in a linear spatial world, people can travel in two directions. Third, time–series observations are usually measured at equal frequencies (e.g., monthly, quarterly, or yearly), whereas economic activity is rarely organized at equally spaced points on a grid. Finally, spatial data are more apt to be nonstationary, where nonstationarity is used here to mean that joint distributions can depend on locations, not just on distances between locations, and not to denote a unit root. It is nevertheless useful to keep the analogy in mind.

Consider the following cross–sectional equation that includes a continuously–measured, spatially–lagged dependent variable, y, a spatially–related regressor, x,[4] and a spatially–dependent error, u,

$$y_i = \sum_{j \neq i} w_{ij} y_j + \sum_{j=1}^{n} b_{ij} x_j + u_i, \qquad u_i = \sum_{j \neq i} y_{ij} u_j + {}_i, \qquad i = 1, \ldots, n. \tag{1}$$

For concreteness, assume that y_i is agent i's choice, x_i is an observed characteristic of i, and u_i is an unobserved characteristic. Equation (1) says that choices are simultaneously determined and that individual decisions depend on the characteristics of all agents. For example, equation (1) might be a best–reply function in a pricing game with knowledge spillovers. y_i would then be the price that i chooses, which is determined simultaneously with all rival prices, and i's marginal cost would depend on the cost–lowering investments made by all firms. Finally, some of those investments, x, are observed, whereas others, u, are not.

This equation can be written in matrix notation as

[4] A single regressor is considered to simplify notation. The generalization to multiple regressors is straight forward.

$$y = Wy + Bx + u, \qquad u = \Gamma u + \ ,\tag{2}$$

where W, B, and Γ are spatial weighting matrices, and   is a zero–mean random variable that is spatially independent. Anselin (1988) considers estimation of this model in some detail. I therefore just summarize common practice. Researchers often specify the spatial weighting matrices up to a scale factor *a priori.* For example, one might assume that $W = \rho\tilde{W}$, where ρ is a parameter that must be estimated,[5] and the elements of the prespecified matrix $\tilde{W}$ are reciprocals of Euclidean distances between the geographic locations of i and j, $i \neq j$. In addition, it is common to make a parametric assumption concerning the distribution of . For example, one might assume that $\ \sim i.i.d.N(0, \sigma^2)$.

Under sufficiently restrictive assumptions, it is possible to estimate equation (2) by maximum–likelihood methods. To illustrate, if in addition B is a diagonal matrix with $b_{ii} = \beta$ for all i and $\Gamma = 0$ (no spillovers), the likelihood function is

$$l = -\frac{N}{2}\ln(2\pi\sigma^2) + \ln|I_n - \rho\tilde{W}| - \frac{1}{2\sigma^2}(y - \rho\tilde{W}y - x\beta)^T(y - \rho\tilde{W}y - x\beta),\tag{3}$$

where I_n is the identity matrix of size n. This function can be maximized to obtain efficient estimates of $\beta, \rho,$ and σ^2.

3 Estimation of Continuous–Choice Spatial Models by Semiparametric Methods

It is clearly desirable to place less structure on the estimation. In particular, the results obtained might be sensitive to assumptions concerning the form of the weighting matrices, and it is difficult to ascertain the appropriate form of spatial dependence *a priori.* On the other hand, with a cross section or short panel, one cannot simply estimate all of the parameters in equation (2) without placing some restrictions on the estimation. Indeed, a single weighting matrix contains approximately n^2 parameters, whereas a single cross section contains n observations. In addition, the matrices must satisfy some form of mixing condition. In other words, the influence of other locations must decay as the distance between locations increases. The goal of this section is to suggest how one might specify a parsimonious model that is at the same time flexible.

[5] This is the approach that is taken by Case (1991), who assumes that $\tilde{W} = 1$ if i and j share a market boundary and zero otherwise, and by Pinkse and Slade (1998), who experiment with several weighting matrices.

The route that my coauthors and I have taken involves estimating the spatial–weighting matrices W and B nonparametrically and correcting the errors u for heteroskedasticity and spatial correlation of an unknown form (see Pinkse, Slade, and Brett 2002, Pinkse and Slade 2002, and Slade 2003). This approach appears to work well. However, due to the need for parsimony, attention has been limited thus far to one matrix (i.e., either W or B) per application.

The basic idea is very simple. We assume that each element g_{ij} of a weighting matrix G is a common function of a vector of distances between locations i and j in some metrics, $g_{ij} = g(d_{ij})$, where G equals W or B, and the elements of d_{ij} are measures of distance (or its inverse, proximity) in geographic or characteristic space. It is up to the practitioner to specify the measures to include in d. However, since the estimation of g is nonparametric, the data determine the functional form of the relationship among those measures.

To illustrate, continuous proximity measures in geographic space can be constructed from Euclidean distance (e.g., the reciprocal of the Euclidean distance between two retail outlets) or from transport costs. However, measures can also be discrete, such as a zero/one variable that indicates whether two outlets are nearest neighbors, share a market boundary, or belong to the same geographic region (e.g., are in the same province).

Proximity measures in product–characteristic space can also be continuous or discrete. An example of the former is the reciprocal of the absolute value of the difference in the alcohol contents of two brands of beer, whereas an example of the latter is a zero/one variable that indicates whether two brands are of the same product type (both lagers) or of different product types (a lager and a stout).

More generally, given any characteristic that pertains to an individual product i, it is possible to construct a proximity measure from that characteristic (e.g., closeness in that characteristic). The proximity measure, however, pertains to two products, i and j. Measures can also be constructed based on $m \geq 2$ continuously–measured characteristics by, for example, taking the Euclidean distance between products in m–dimensional space or by determining if they are nearest neighbors in that space. This means that virtually any hypothesis concerning the nature of market interactions can be assessed in our framework.

For any notion of economic distance, there are many possible proximity measures. Indeed, one measure that is based on Euclidean distance is the reciprocal of that distance. However, one could also use the reciprocal of the logarithm of that distance or of any other monotonic transform of that variable. One might therefore worry that there are too many possibilities. An advantage of the nonparametric framework,

however, is that the functional form of the basic proximity measure isn't important.[6]

In the remainder of this and in the following section, I discuss general issues, assumptions, and techniques. Some of the issues are quite technical. However, my discussion is nontechnical, and everything is developed in the context of applications. Nevertheless, those readers who are interested only in applications might want to skip directly to section 5.

Since there are two matrices — W and B — and two types of space — geographic and characteristic — there are clearly four possibilities. However, I consider only two of those possibilities: estimation of W in a geographic context and estimation of B in a characteristic context.

3.1 Estimation of W in a Geographic Context

The discussion of some of the issues that arise in the estimation of an equation that contains a spatially lagged dependent variable is facilitated by focusing on an application. The application, which is taken from Pinkse, Slade, and Brett (2002), involves estimation of best–reply functions.

Suppose that there are two groups of agents in a market, buyers and sellers. Suppose furthermore that sellers are distinguished by their locations in geographic space. Finally, assume that sellers are imperfectly competitive firms and that buyers are price–taking firms or households.

I consider the simpler case first in which buyers are firms and discuss the derivation of first–order conditions for a market game with y (say price or quantity) as the upstream choice variable. Semiparametric methods can then be used to estimate the matrix of diversion ratios or reaction–function slopes. In particular, substitution patterns, and thus competitive responses, can be allowed to depend in a possibly nonlinear fashion on a vector of distance measures that have been proposed in the literature.

In order to derive the upstream firms' best–reply functions, one must assume something about the functional form for downstream profit functions. I assume that buyers have normalized–quadratic profit functions. Specifically, profits are quadratic functions of input and output prices that have been divided (or normalized) by a single price or price index (Berndt, Fuss, and Waverman 1977 and McFadden 1978). Under that assumption, derived demands are linear.

Let $X = (x_{ih}), i = 1, \ldots, n, h = 1, \ldots, H$, be a matrix of observed demand and cost variables. If, in addition, there are unobserved variables u, the system of best–

[6] By the basic measure, I mean the measure that is expanded.

reply functions can be written as[7]

$$y = R(y) = a + X\beta + Wy + u, \qquad u = \Gamma u + \ . \tag{4}$$

In the parametric part of (4), a is a vector of intercepts that can be treated as random effects, and β is a vector of parameters that must be estimated.

The elements of the matrix W that premultiplies the price vector are assumed to be a common function of measures of distance in geographic space, $W = [w(d_{ij})]$. One can use a series expansion to approximate w, and, as is standard, let the number of expansion terms that are estimated increase with the sample size. In particular, w can be written as a function of unknown parameters and a set of expansion terms e_k that form a basis of the function space to which w belongs. For example, if w has finite support, Fourier and polynomial approximations are appropriate.

Finally, the random variable u, which captures the influence of unobserved demand and cost variables, can be heteroskedastic and spatially correlated.

Since rival choices appear on the right–hand side of (4), an instrumental–variables (IV) approach is taken. Furthermore, since the number of right–hand–side variables must increase with the sample size, so must the the number of instruments. Additional instruments can be created by interacting the original instruments with each term in the series expansion. For example, if the expansion involves polynomials, exogenous regressors and other instruments can be interacted with powers of the continuous distance measures. Nevertheless, other than the fact that the number of columns of W increases with the sample size, the form of the semiparametric estimator is identical to that of the traditional IV estimator.

Pinkse, Slade, and Brett (2002) demonstrate that w and β are identified and that the IV estimator is consistent, and they derive the limiting distributions of $\hat{\beta}$ and $\hat{w}$. Finally, they show how the covariance matrix of $\hat{\beta}$ and $\hat{w}$ can be estimated.

The covariance–matrix estimator is similar to the one that is proposed in Newey and West (1987) in a time–series context.[8] In particular, observations that are close to one another are assumed to have nonzero covariances, where closeness is measured by one or more of the distance metrics. The estimator, however, which involves correlation in space rather than time, can be used when the errors are nonstationary, as is more apt to be the case in a spatial context.

As is standard, the instruments are assumed to be uncorrelated with the errors in the estimating equation. Unfortunately, in many applications the exogeneity assump-

[7] The fact that the best–reply functions are linear is a consequence of the assumption that upstream profit functions are quadratic.

[8] Conley (1999) also proposes a method of correcting standard errors for spatial correlation of an unknown form.

tion is questionable for some of the instruments. Furthermore, if new instruments are created from the original ones as the sample size grows, and if some of the original instruments are questionable, the new instruments will also be suspect. Pinkse and Slade (2002) therefore develop a test of exogeneity that is valid in the presence of heteroskedasticity and spatial correlation of an unknown form. Intuitively, the test involves assessing correlation between instruments and residuals, taking into account the fact that the residuals are not errors but are estimates of errors.[9]

Now consider the case where buyers are price–taking households. One can still use a normalized–quadratic functional form. With the new assumption, however, instead of being the functional form for buyer profit, it is the functional form for buyer indirect–utility. As before, however, since indirect–utility functions are quadratic, the brand–level demand equations are linear in normalized prices and income.[10]

The upstream–imperfect–competition/downstream–price–taking assumption is thus maintained. Nevertheless, there is a technical differences between the two situations. In particular, aggregation conditions are more stringent for price–taking individuals or households than for competitive firms. In both cases, it is necessary to sum the demands of individual buyers to obtain the aggregate demands that the upstream firms face. It is common practice to do this by assuming a functional form for the distribution of unobserved consumer or firm heterogeneity and integrating over that distribution to obtain aggregate demands (see, e.g., Berry, Levinsohn, and Pakes, 1994). The alternative is to require that aggregation be independent of the distribution of consumer heterogeneity (i.e., exact aggregation). My coauthors and I have taken the second approach.

The difference between the two cases arises because although there are no restrictions that competitive firms' profit functions must satisfy in order for aggregation to be exact,[11] when buyers are individuals, simplicity in aggregation is not obtained costlessly. Indeed, it requires that all consumers have the same constant marginal utility of income.[12] Fortunately, however, exact aggregation places no restrictions on individual preferences for brands of the differentiated product and thus on individual

[9] In most applications there are more moment conditions than there are unknown parameters, and the excess moment conditions can be employed to test the validity of the set of moment conditions by means of an overidentification test. Since this is a general test of specification, it provides an alternative method of assessing instrument validity. Indeed, as in the generalized–method–of–moments (GMM) literature, it is possible to use the $\mathcal{J}$ statistic for this purpose, where $\mathcal{J}$ is proportional to the minimized value of the GMM objective function.

[10] By Roy's identity, demands must be divided by the marginal utility of income, which is a price index here that differs over time but not by consumer. In a short time series, this number can be set equal to 1.

[11] See Bliss (1975). There are, of course, restrictions that must be satisfied by any profit function, such as positive linear homogeneity in prices and incomes.

[12] See Blackorby, Primont, and Russell (1978) for a discussion of the conditions that are required for consistent aggregation across households.

or aggregate substitution patterns.

3.2 Estimation of B in a Product–Characteristic Context

The discussion of some of the issues that arise in the estimation of an equation that contains a lagged regressor in product–characteristic space is also facilitated by focusing on an application. The second application, which is taken from Pinkse and Slade (2002), involves imperfectly competitive firms that sell a differentiated product to price–taking buyers. The equation that is estimated is the demand for a brand of the differentiated product, and the spatially related regressor is price. Finally, the elements of the matrix B that premultiplies the price vector is assumed to be a common function of measures of distance in product–characteristic space, $B = [b(d_{ij})]$. The estimating equation is thus

$$y = a + X\beta + Bp + u, \qquad u = \Gamma u + \ . \tag{5}$$

With minor modifications, the methods that are discussed in the previous subsection can be used to estimate B.

One important difference between the two applications arises however because, when the nonparametric expansion involves product characteristics, those characteristics can also enter the X part of the regression model. To illustrate, suppose that the reciprocal of the difference in alcohol content is one of the distance measures. Since alcohol content can also enter as an X regressor, it is not immediately obvious that the function b is identified, even by functional form (i.e., even if alcohol content only enters X linearly). However, provided that there is more than one market and that price distributions are different across regions and/or time periods but b is the same across regions and time periods, b can be identified.[13] In general, this procedure will not work well if price distributions and/or locations in taste space do not vary much across markets.

A second difference in the estimation stems from another difference between product–characteristic and geographic space. In particular, with geographic distance, as the sample size increases, one can assume that new sellers are located at farther distances from existing sellers and that the geographic size of the market increases.

[13] For instance, a regression with dependent variable $q_{i2t} - q_{i1t}$, where i, j, and t refer respectively to brand, region, and time period, eliminates the alcohol component of X as well as any time dummies, but not b. Combinations are also possible to simultaneously eliminate time and region fixed effects. The suggested procedure is not necessary if discrete distance measures (such as product groupings) are used in b, but no corresponding product dummies are included in X. The second suggestion will not work well, however, if price distributions and/or locations in taste space do not vary much across categories.

This procedure will not always work, however, when products are located in characteristic space. Indeed, some characteristics are constrained to lie between finite limits. For example, there are limits on the alcohol content of beers, which generally lie between 2% and 6%.

The issue of identification is therefore somewhat different from that in the first application. Indeed, one now has the potential for an infinite number of rivals (in the limit) at a finite distance from each brand.[14] If the number of rival brands at a finite distance increases without bound with the sample size, then the influence of each individual brand is assumed to be asymptotically negligible. In essence, one is then estimating a conditional mean, where the conditioning variable is the brand's location in taste space.

4 Estimation of Discrete–Choice Spatial Models

Discrete–choice models can also be spatial. For example, just as rival continuous choices (e.g., prices or quantities) are important determinants of own behavior in continuous strategic environments, rival discrete–choices (e.g., entry decisions) are important determinants of own behavior in discrete games. Moreover, when firms have more than one rival, the choices of 'similar' firms are more important than those of dissimilar firms. We therefore also need to assess notions of similarity in discrete contexts.

When space is introduced into discrete–choice models, problems arise that do not surface in continuous–choice environments. I illustrate this idea by considering a probit model with spatially dependent errors. The application of OLS to a linear model with dependent errors results in estimates that are consistent but not efficient. In a discrete–choice situation, if the errors are homoskedastic, the ordinary–probit estimator is also consistent but not efficient. However, if the errors are heteroskedastic, the probit estimator is inconsistent. Moreover, spatial dependence can introduce heteroskedasticity.

To see this, consider the latent–variable model with spatially correlated errors,

$$y^* = X\beta + u, \; u = \Gamma u + \; , \quad \sim N(0, I_n). \tag{6}$$

Under these assumptions, the variance–covariance matrix of u is

$$V[u] = \{(I_n - \Gamma)^T (I_n - \Gamma)\}^{-1}. \tag{7}$$

[14] Commonly used terminology is 'fill–in asymptotics' versus 'increasing–domain asymptotics'.

(7) shows that the u's are heteroskedastic and that the probit estimates are therefore inconsistent.

Estimation of a probit model with spatially dependent errors is not entirely straigth forward. For this reason, Pinkse and Slade (1998) develop a test of the null, $\Gamma = 0$, for the case where Γ consists of a prespecified weighting matrix that is multiplied by a parameter that must be estimated. Our test is based on the notion of a generalized residual. Diagnostic tests based on generalized residuals have proved useful in many discrete–choice applications (see, e.g., Gourieroux, *et. al.* 1987 and Pagan and Vella 1989). A generalized error is defined as $E[u_i|y_i]$, where y_i is the ith observed discrete choice, $y_i = I[y_i^* \geq 0]$, and I is the indicator function that equals one (zero) when its argument is true (false). The generalized residual is then the generalized error evaluated at the probit estimate, $\hat{\beta}^p$.

In the eventuality that the test fails, it is useful to have methods of estimating equation (6) with $\Gamma \neq 0$. Pinkse and Slade (1998) develop a generalized method of moments (GMM) estimator in which the moment conditions make use of the fact that suitably chosen instruments are orthogonal to appropriately modified generalized errors.[15] We prove that our estimates $\hat{\beta}$ and $\hat{\Gamma}$ are consistent and asymptotically normal, and we show how to estimate their covariance matrix.

Space only enters equation (6) through the errors. More generally, one can have a spatially lagged dependent variable and spatially dependent regressors. Moreover, if one has panel data, the latent-variable equation can be dynamic. In particular, one might want to include a temporally lagged dependent variable. Finally, just as it is often appropriate to include fixed effects in continuous–choice panel–data models, one may wish to include fixed effects in discrete–choice models.

With linear panel–data models, it is standard practice to difference the continuous–choice equation to remove the fixed effects. Due to the nonlinearity of discrete–choice models, however, this practice won't work, and a more complex procedure must be used. In the presence of fixed effects, researchers often limit attention to static logit models with independent errors and strictly exogenous regressors, see, e.g., Chamberlain (1982). Magnac (2002) generalizes this model to include dependent errors with arbitrary known marginal distribution. However, he maintains the logit, strict–exogeneity, and static assumptions. Pinkse, Shen, and Slade (2003), in contrast, develop an estimator that relaxes a different set of assumptions. In particular, we consider a probit and allow for arbitrary patterns of spatial dependence, endogenous regressors, measurement error, and a temporally lagged dependent variable. Unlike in previous work, however, our fixed effects are not included in the unobserved latent–

[15] The principal modification is that the ith element of the vector equation (6) must be divided by an estimate of the standard error of u_i. This modification purges the equation of heteroskedasticity.

variable equation (6). Instead, they are included in the equation that relates the observed binary–choice variable, y, to the regressors. Since our fixed effects enter linearly, they can be removed by differencing.

Specifically, we estimate the following dynamic panel–data model with fixed effects,

$$y_{it} = I(x'_{it1}\beta_0 - \ _{it1} \geq 0)y_{i,t-1} + I(x'_{it0}\beta_0 - \ _{it0} \geq 0)(1 - y_{i,t-1}) + \eta_i + u_{it}, \tag{8}$$

$$i = 1, \ldots, N, \quad t = 1, \ldots, T,$$

where y_{it} is the binary choice of firm i at time t, η_i is a fixed effect, the $\ _{itj}$'s and u_{it}'s are errors, β_0 is an unknown vector of regression coefficients, x_{it1} and x_{it0} are regressor vectors, and I is the indicator function.

The coefficients in equation (4) are allowed to vary by regime. Furthermore, our model encompasses a number of special cases including a static model ($x_{it1} = x_{it0}$). In addition, the regressors can be the same in the two components ($x_{it1} = [x'_{it}, 0']'$ and $x_{it0} = [0', x'_{it}]'$).

We use the continuous–updating estimator (CUE) of Hansen, Heaton, and Yaron (1996), which is a one–step GMM estimator, and we take an agnostic approach to the nature of spatial dependence. Our approach is similar to that in Conley (1999), where a two–step GMM estimator is developed that also allows for general patterns of spatial dependence in the errors. Our estimator is valid, however, when some of his assumptions (e.g., nonstationarity) are not satisfied. Finally, we show that our CUE is consistent and asymptotically normal, and we derive the standard errors of the estimated coefficients.

5 Applications

This section discusses actual and potential applications of the techniques that are described in sections 3 and 4. It is divided into subsections depending on whether the choice variable is continuous or discrete. The applications include the ones upon which the more technical discussions are based as well as additional examples that are meant to illustrate the range of possible problems that can be addressed within the general framework that has been laid out. Furthermore, many of the latter examples involve hypothetical rather than actual applications.

5.1 Continuous Choice

Spatial–Price Competition in Wholesale Gasoline Markets

With the first application, the procedure that is described in subsection 3.1 is used to assess the nature of spatial price competition in US wholesale–gasoline markets. Here, the imperfectly competitive upstream firms are sellers of refined products (oil companies). They produce a physically homogeneous product that is differentiated by its spatial location. Specifically, sellers are located at terminals that form the interface between pipelines or barges and major US cities. The competitive downstream buyers in this market are wholesalers or resellers who purchase petroleum products at terminals and transport them to retailers. The equation that is estimated is a best–reply function for a pricing game, price is the spatially lagged dependent variable, and the spatial–weighting matrix contains the diversion ratios that determine who competes with whom.

One research question is how big is the market? In order to answer that question, substitution patterns, and thus competitive responses, are allowed to depend in a possibly nonlinear fashion on a vector of distance measures that have been proposed in the literature, including the common-market-boundary measure used in Feenstra and Levinsohn (1995) and the Euclidean-distance measure used in Davis (1997). The objective is to determine the relevance of competing models of price competition for a particular market.

The estimated model of price competition reveals that, in this market, competition is highly localized. Indeed, although a number of notions of distance are used in the estimation, including being nearest neighbors, having markets with common boundaries, and being located a certain Euclidean distance apart, only the first is found to be a strong determinant of the strength of interterminal rivalry. In particular, direct rivalry appears to decay abruptly with distance, not in a more gradual manner as would be the case if competition in the market were global.[16] However, the results are stronger than mere rejection of global competition; wholesale–price competition is found to be even more local than in a typical Hotelling model, where firms compete directly with all competitors with whom they share a market boundary. In this market, therefore, economic distance can clearly not be equated with Euclidean distance or any smooth transformation of that distance. It would have been difficult, however, to anticipate this regularity *a priori*.

[16] Following Anderson, de Palma, and Thisse (1989), the expression local (global) competition is used to denote a situation in which most cross–price elasticities are zero (all cross-price elasticities are positive, albeit of varying sizes).

The Price Effects of UK Brewing Mergers

With the second application, the procedure that is described in subsection 3.2 is used to simulate mergers between UK brewers. Here, the imperfectly competitive upstream firms are brewers, and downstream buyers are individuals who frequent bars and other establishments where beer is sold on the premises. The equation that is estimated is a retail–demand equation that contains a spatially related regressor — price. The spatial–weighting matrix therefore contains the coefficients of own and rival prices, and those coefficients determine the own and cross–price elasticities of demand.

In evaluating a merger between firms that produce differentiated products, it is necessary to have good measures of product substitutability. Indeed, even when the merged firm has a large share of the differentiated–product market, the merger might have little effect on prices if the brands of the merging firms are not close substitutes. However, it is difficult to determine the strength of brand substitutability *a priori*. In our framework, it is possible to experiment with many hypotheses concerning the dimensions along which products compete and to select those dimensions (those proximity measures) that have strongest empirical support. This procedure allows the researcher to estimate a market model that is accurate and at the same time parsimonious.

Merger simulations are performed by calculating market equilibria under various assumptions about brand–ownership patterns (i.e., different assumptions about merger partners).[17] Not surprisingly, the simulations reveal that post–merger market shares are weak predictors of competitive harm, whereas brand–characteristic overlaps are more accurate.

Measuring Technological Spillovers

To my knowledge, no one has used flexible spatial techniques to model technology spillovers. Nevertheless, it seems like a natural application.[18]

Knowledge spillovers can raise a firm's efficiency and thus work through lowering costs, or they can increase the quality of a firm's products and thus work through enhancing demand. The research questions are to assess the magnitude and pervasiveness of spillovers as well as to determine who benefits from the knowledge of whom.

Suppose that a firm's costs depend on a vector of factor rices, v, its output, q, a vector of own and rival stocks of knowledge, k, and a random variable, u_1, $\mathsf{TC} =$

[17] Before this can be done, it is necessary to have measures of costs. In the application, engineering data on costs are used.

[18] This discussion is base loosely on Organhi (2003), who uses more standard techniques (i.e., who specifies spatial–weighting matrices *a priori*).

$C(v, q, B_1 k, u_1)$. In this equation, B_1 is a spatial–weighting matrix that determines who benefits from the knowledge of whom. We are thus dealing with a spatially related regressor. In addition, since stocks of knowledge are difficult to measure accurately, the error term, u_1, is likely to contain unmeasured knowledge factors. It is therefore likely that we are also dealing with spatially correlated errors.

In addition, suppose that demand for a firm's product depends on own and rival output prices, p, economic activity variables, Y, stocks of knowledge, k, and a random variable, u_2, $q = D(p, Y, B_2 k, u_2)$. As with costs, we therefore have a spatially related regressor and spatially correlated errors.

Suppose that there are n firms indexed by i. The conventional approach to this sort of problem is to create an aggregate rival–knowledge variable, K_i, for each firm i from the individual stocks, $k_j, j \neq i$. This is done by specifying B *a priori* and summing,

$$K_i = \sum_{j \neq i} b_{ij} k_j. \tag{9}$$

A number of interesting hypotheses concerning who benefits from whom surface in the literature, and it is difficult to know which is more relevant for a particular application. For example, B could be based on a discrete distance measure — same industrial sector — or on that measure modified to account for rival–firm size.[19] There are, however, equally plausible hypotheses concerning likely determinants of spillovers, such as that economic distance can be measured by patent citations (i.e., by the relative frequency with which firm i cites the patents of firm j) or by input/output flows (i.e., by the importance of firm j as a supplier of inputs to i or as a demander of output from i).

Given these competing hypotheses, it seems natural to construct a vector of distance measures, d_{ij}, with each element based on one of the hypotheses. The data can then be used to discriminate among hypotheses. In addition, the data will indicate if and how the competing measures interact in a possibly nonlinear fashion.

Promotional Activity: Market Expanding or Market Stealing?

Many firms, especially those in consumer–product industries, spend large sums on promotional activity, and there are a number of research questions that are related to those expenditures. For example, it is possible that each firm's activities affect only its own sales. When this is true, advertising is a pure private good. Just as there can be knowledge spillovers, however, there can also be promotional spillovers. When this is the case, a firm's activities affect the sales of other products that are in the

[19] This is the approach that Organhi (2003) takes.

same market. At one extreme, a dollar spent on promoting a particular product can affect the sales of all products in the market equally, and advertising can be a pure public good.

With pure private goods, promotional expenditures must expand the product market as a whole. Indeed, those activities enhance the sales of the firm that incurs the expenditures and have no impact on the sales of its rivals. In the presence of promotional spillovers, in contrast, things are less clear. In the extreme, it is possible that the size of the market remains unchanged and that advertising merely shifts customers from one firm to another.[20] When market stealing dominates, advertising presents the firms with a prisoner's dilemma. In particular, they would all be better off if no one advertised, but promoting is a dominant strategy. Furthermore, society would also be better off, since costs and prices would be lower.

These hypotheses can be tested by estimating a demand equation at the product or brand level that is similar to the demand equation in the technological–spillover example. Specifically, let p be a vector of product prices, a be a vector of promotional activities, Y be a vector of economic–activity variables, and u be a random variable. The demand equation is then $q = D(B_1 p, B_2 a, Y, u)$. This specification contains two spatial–weighting matrices, one for price and one for advertising, and there are many research questions that can be addressed in this framework.

This problem is similar to the one that is studied in Slade (1995) in a more conventional setting.[21] In particular, the spatial–weighting matrices there are specified *a priori* and are proportional to rival firms' market shares. This choice is perhaps sensible based on the fact that the product, a saltine cracker, is relatively homogeneous. With applications involving products that are more differentiated, however, one could construct weights based on similarity of product characteristics, as in the beer example, and one could estimate the spatial–weighting matrices nonparametrically.[22]

With the saltine–cracker application, I find that prices are strategic complements (best–reply functions slope up), advertising expenditures are strategic substitutes (best–reply functions slope down), and the two strategic variables are cross strategic substitutes (an increase in the use of one leads to diminished rivalry in the other dimension). Furthermore, I find evidence of promotional spillovers. Indeed, advertising by one firm causes own sales to rise and rival sales to fall. When all firms advertise simultaneously, however, the market as a whole expands, since the market–expansion

[20] The distinction between 'predatory' or market–stealing and 'cooperative' or market–expanding advertising was first made by Friedman (1983).

[21] That problem, however, is dynamic (a state–space game).

[22] No applications to date have dealt with two matrices simultaneously. For tractability, it might be necessary to concentrate on competition with one choice variable — price or advertising — and to treat the other more conventionally.

effect dominates the business–stealing effect. It would be interesting to know, however, if these findings persist when promotional spillovers are modeled more flexibly.

Municipal Tax Competition

The final continuous–choice application is taken from the area of public finance and involves the use of local tax rates to attract business entities. Local authorities have the power to tax both households and business establishments. Moreover, the property taxes that households and firms face determine to some extent who will reside in the community. If each community were an island, it would choose its tax rates based solely on within–jurisdiction considerations. However, high taxes in one area can lead households and firms to exit the area and move to another. Spillovers are therefore potentially present as, in the parlance of public finance, decision makers 'vote with their feet.'

The choice of municipal tax rates by local jurisdictions is similar to the choice of product prices by competing firms in an oligopoly. In particular, high taxes (prices) generate more revenue per establishment (per sale) but cause some establishments (some consumers) to leave the municipality (cease purchasing the brand). Moreover, just as oligopolists choose prices strategically, in the presence of spillovers there is scope for municipalities to choose tax rates strategically. Market equilibrium is therefore a fixed point in the space of tax–rate best–reply functions. As with price competition, an obvious tool for modeling tax competition is the best–reply function, and the problem involves a spatially–lagged dependent variable as well as spatially–correlated errors.

If municipalities are revenue or profit maximizers, the slopes of tax–rate best–reply functions are also proportional to diversion ratios.[23] However, those ratios now equal the number of establishments that move from municipality i to municipality j when the tax rate in i increases divided by the number of establishments that leave i. The techniques that were discussed in section 3 can be modified in obvious ways to handle this problem.

One can consider tax competition in geographic or characteristic space or a combination of the two. The first could use the geographic–distance measures that were described earlier, whereas the second could use measures that are based on the characteristics of municipalities. Obvious characteristics include some notion of size and some measure of the business services that are available in the community. These variables are clearly endogenous in a game with a longer time horizon. However, just as product characteristics can be considered predetermined in a short–run pricing game, characteristics of municipalities can be considered predetermined in a short–

[23] The usual caveat of a second–order approximation applies here as well.

run tax–competition game.

A tax–competition game is analyzed in Brett and Pinkse (2000), who model municipal business tax rates in British Columbia. Using a model with a prespecified weighting matrix, W, they find evidence that municipal governments do indeed respond to tax changes in neighboring jurisdictions. However, they find little evidence of fierce competition over tax base and conclude that the primary effect is not strategic.

5.2 Discrete Choice

Contracting in Retail Gasoline Markets

In many franchising situations, some outlets are owned and operated by the franchisor, whereas others are operated (and perhaps owned) by independent franchisees. The discrete choice that the upstream firms face is thus vertical integration or vertical separation. In the former situation, the company chooses the retail price, whereas in the latter, the retailer chooses price.

There are many factors that must be considered when a company decides whether to operate an outlet itself or to contract out.[24] I concentrate here on strategic motives for vertical separation, as in Slade (1998). In particular, I assume that both upstream and downstream markets are imperfectly competitive. There is therefore scope for double marginalization. Unlike the monopoly situation, however, since there are several upstream firms, if they can capture the rent, double marginalization leads to higher upstream profits.

A number of factors introduce spatial considerations into the contracting problem. First, consider the strategic choice of contract. Let S stand for separation (operator chooses price) and I stand for integration (company chooses price). With wholesale prices that are two–part tariffs,[25] it can be shown that $\pi^{II} < \pi^{SI} < \pi^{SS}$, where π stands for upstream profit and the first (second) superscript is own (rival) contract (see Rey and Stiglitz 1995). This relationship is due to the fact that prices are strategic complements in the retail–pricing game.

Absent costs, we would only see S contracts, since S is a dominant strategy. However, there are a number of reasons why costs are apt to be important. For example, costs can be associated with changing contracts (negotiation costs) or with relinquishing control over prices (delegation costs). Furthermore, the type of contract that is chosen is affected by the outlet's characteristics, which are also costly to

[24] For a detailed analysis of the vertical integration/vertical separation decision, see Lafontaine and Slade (2001).

[25] 'Wholesale prices' should be interpreted loosely. For example, the two–part tariff could consist of a franchise fee and a royalty rate.

change. When these sorts of contracting costs are introduced, since each firm's profit depends not only on its own contract type but also on the type chosen by its rival, the probability of vertical separation, $Prob[y_i = 1]$, will be affected by rival contract choice, y_j, where $j \neq i$ is a 'neighbor.' In other words, the contract–choice equation can contain a spatially lagged dependent variable.

Finally, prior to the choice of contract, outlet characteristics must be chosen. At this stage, firms face a tradeoff between enhancing their market power and increasing their market shares. In particular, if they choose characteristics that differentiate their outlets from those of neighboring outlets they can augment their market power. On the other hand, if they choose characteristics that are similar to those of neighboring outlets they can steal rival customers and augment their market shares. This means that, although the direction of the effect is not known *a priori*, outlet characteristics are apt be spatially related.

Formally, let C_i be the characteristics of outlet i, and suppose that some characteristics, x_i, are observed, whereas others, u_i, are not, $C_i^T = [x_i, u_i]$. Then, in the latent–variable model $y^* = Wy + Bx + u$, $u = \Gamma u +$, there are reasons why y, x, and u might all be spatially related.

Pinkse and Slade (1998) examine the strategic contracting decision in retail gasoline markets. In that application, upstream firms are oil companies and downstream firms are gasoline retailers. We use the estimation scheme for a probit model with spatially correlated errors that is developed in section (4) and find that, in this market, the market–share effect dominates the market–power effect. It would be possible, however, to extend the discrete–choice model to include a spatially lagged dependent variable and spatially related regressors.[26] One could then see if the earlier finding holds in a more general spatial context.

Exercising a Real Option

Some investment decisions involve continuous expenditures. Others, in contrast, are discrete. Furthermore, discrete investments usually require up-front expenditures that are at least partially irreversible. Such expenditures are often analyzed in a real–options framework (Brennan and Schwartz 1985).

For concreteness, consider a mine. Prices of mineral commodities are notoriously volatile, and when they fall precipitously, variable profits in mining can be negative. This does not necessarily mean, however, that the mine will close. Indeed, due to the presence of fixed closing and reopening costs, operators might prefer to weather the storm. Similarly, if the mine is closed and prices rise, the mine will not reopen as

[26] The inclusion of a spatially lagged dependent variable in a discrete–choice model is a subject of future research.

soon as variable profits are positive. This behavior gives rise to price thresholds for opening and closing, with the former greater than the latter.[27]

The theory of real options points to a number of factors that should affect when a mine will be active and when it will be inactive. For example it is more likely to be active when prices and reserves are high and when costs are low. However, those predictions can be derived from many other investment models. The theory also yields more unusual predictions — for example, that increased price volatility tends to delay investment decisions. In other words, high volatility causes the probability that a mine will be active to increase (decrease) if it was active (inactive) in the previous period. These are testable hypotheses that have received empirical support in a nonspatial setting (e.g., Moel and Tufano 2002).

Space can enter the problem in a number of ways. For example, most mineral–commodity markets are workably competitive. However, some are highly to moderately concentrated (e.g., diamonds, tin, and nickel). Moreover, a single mine can account for a significant fraction of world output. Under those circumstances, opening and closing decisions are apt to be strategic. In particular, all else equal, it is better to remain open (closed) if rival mines are closed (open). This sort of strategic behavior gives rise to spatially lagged dependent variables in discrete games.

In addition, although the use of time–period fixed effects in a discrete–choice equation removes the effect of aggregate supply and demand factors, such as energy prices and industrial production, cost conditions can differ across regions. Local variation can be due to common geological factors or to local factor prices, and not all local costs are observable. This means that the errors in the discrete–choice equation are apt to be spatially correlated.

Finally, with a real–options model, it is often appropriate to include a temporally lagged dependent variable among the regressors. Indeed, the status of a mine at the beginning of the period (open or closed) is an important determinant of whether that mine will operate during the period. Moreover, the coefficients of some of the regressors should be state dependent. For example, the effect of price volatility on the decision to operate depends on whether the mine was open or closed in the previous period. This means that a real–options application can require a dynamic nonlinear panel–data econometric model.

Pinkse, Shen, and Slade (2003) model opening and closing decisions for a panel of Canadian copper mines. Since the copper market is workably competitive, they estimate a discrete–choice model with spatially correlated errors, a temporally (but not spatially) lagged dependent variable, and mine fixed effects. They find that many

[27] These thresholds are usually called S, s thresholds, with $S > s$. With the model that I discuss, the S, s thresholds are functions of the state variables, which can include the status of rival mines.

of the factors that are predicted to influence the open/close decision are empirically important. However, their evidence is not supportive of a real–options model. In particular, the sign of the coefficient of volatility does not depend on the prior state.

Drilling for Oil

There are many other possible applications within the context of real–options. For example, when oil companies bid successfully on an off or onshore tract of land, they receive the option to explore for oil. Furthermore, successful drilling gives them the option to develop the property, and development gives them the option to produce.

In addition to the spatial factors that are noted above, the drilling decision is associated with an informational externality. Indeed, many wildcat tracts are dry, and little is known about the quantity of oil that is contained in those that are not.[28] Furthermore, nearby tracts tend to share common geological features. This means that there can be knowledge spillovers, and each firm can learn from the drilling activities of its neighbors. Moreover, in a strategic context, firms would like to wait to learn from the costly exploratory activities of rivals. In other words, the players are engaged in a game of 'chicken' in which each would like to be the last to drill. They cannot wait forever, however, because tracts that have not been explored within a fixed time frame revert to the government (the real option has a fixed expiry date).

A drilling equation should therefore contain temporally and spatially lagged dependent variables. Moreover, in this context, the elements of W are apt to be positive (i,e., firms are more apt to explore after rivals have explored), whereas in the above mining context, the coefficients are apt to be negative (i,e., firms are less apt to be active when rivals are active). Furthermore, with the drilling example, the weighting–matrix coefficients reveal who learns from whom, as in the technological–spillover example.

There are a number of possible hypotheses about the nature of spatial drilling competition. Perhaps the most obvious is that firms learn from the activities of rivals who own adjacent tracts. However, spillovers can be broader. For example, the reciprocal of Euclidean distance might be used as a proximity measure, or one might experiment with a discrete measure that indicates if rival activity occurs within the same geological basin.

Hendricks and Porter (1996) assess drilling decisions on US offshore oil and gas tracts. Using a prespecified weighting matrix, they find that the drilling activities of rivals are important determinants of own drilling behavior. Moreover, they conclude that firms behave strategically, that the game that they are engaged in is noncooperative, and that strategic delay is associated with considerable waste that could be

[28] A wildcat tract is a property that has not been explored.

alleviated by cooperation.

This completes my enumeration of actual and potential applications of spatial econometric techniques. Clearly, there are many others, both inside and outside of the field of industrial organization. However, I hope that this list has convinced the reader that there are many interesting spatial problems that can be addressed within the framework that my coauthors and I have developed.

References Cited

Anderson, S.P., de Palma, A., and Thisse, J-F (1989) "Demand for Differentiated Products, Discrete Choice Models, and the Characteristics Approach," *Review of Economic Studies*, 56, 21–35.

Anselin, L., (1988) *Spatial Econometrics: Methods and Models*, Dordrecht: Kluwer Academic Publishers.

Berndt, E.R., Fuss, M.A., and Waverman, L. (1977) "Dynamic Models of the Industrial Demand for Energy," Palo Alto, CA: Electric Power Institute, EA–580.

Berry, S., Levinsohn, J., and Pakes, A. (1995) "Automobile Prices in Market Equilibrium," *Econometrica*, 63, 841–890.

Blackorby, C., Primont, D., and Russell, R. (1978) *Duality, Separability, and Functional Structure: Theory and Economic Applications*, Amsterdam: North Holland.

Bliss, C.J. (1975) *Capital Theory and the Distribution of Income*, Amsterdam: North-Holland.

Brennan, M.J. and Schwartz, E.S. (1985) "Evaluating Natural-Resource Assets," *Journal of Business*, 58: 135-157.

Brett, C. and Pinkse, J. (2000) "The Determinants of Municipal Tax Rates in British Columbia," *Canadian Journal of Economics*, 33: 695–714.

Case, A. (1991) "Spatial Patterns in Household Demand," *Econometrica*, 59: 953–965.

Chamberlain, G. (1984) "Panel Data," in *Handbook of Econometrics*, volume II, Z. Griliches and M Intriligator (eds.) Amsterdam: North Holland.

Conley, T.G. (1999) "GMM Estimation with Cross Sectional Dependence," *Journal of Econometrics*, 92: 1–45.

Davis, P. (1997) "Spatial Competition in Retail Markets: Movie Theaters," Yale University mineo.

Ellison, G. and Glaeser, E.L. (1997) " Concentration in Manufacturing Industries: A Dartboard Approach," *Journal of Political Economy*, 105: 889–927.

Feenstra, R.C. and Levinsohn, J.A. (1995) "Estimating Markups and Market Conduct with Multidimensional Product Attributes," *Review of Economic Studies*, 62: 19–52.

Friedman, J. (1983) "Advertising and Oligopolistic Equilibrium," *Bell Journal of Economics*, 14: 464–473.

Gourieroux, C., Monfort, A., Renault, E., and Trognon, A. (1987) "Generalized Residuals," *Journal of Econometrics*, 34: 5–32.

Hansen, L.P., Heaton, J., and Yaron, A. (1996) "Finite–Sample Properties of Some Alternative GMM Estimators," *Journal of Business and Economic Statistics*, 14: 262–280.

Hendricks, K. and Porter, R.H. (1996) "The Timing and Incidence of Exploratory Drilling on Offshore Wildcat Tracts," *American Economic Review*, 86: 388–407.

Hotelling, H., (1929) "Stability in Competition", *Economic Journal*, 39: 41–57.

Krugman, P. (1991) "Increasing Returns and Economic Geography," *Journal of Political Economy*, 99: 483–499.

Lafontaine, F. and Slade, M.E. (2001) "Incentive Contracting and the Franchise Decision," in *Game Theory and Business Applications*, K. Chatterjee and W. Samuelson (eds.) Kluwer Academic Press, 133-188.

Magnac, T. (2002) "Binary Variables and Fixed Effects: Generalizing the Conditional Logit Model," INRA working paper, Paris.

McFadden, D. (1978) "The General Linear Profit Function," in *Production Economics: A Dual Approach to Theory and Applications*, Vol. 1, M. Fuss and D. McFadden (eds.) Amsterdam: North Holland, 269–286.

Moel, A. and Tufano, P. (2002) "When are Real Options Exercised? An Empirical Study of Mine Closings," *The Review of Financial Studies*, 15: 33–64.

Newey, W.K. and West, K., (1987) "A Simple Positive Semi–Definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55: 703–708.

Ornaghi, C. (2003) " Spillovers in Product and Process Innovation: Evidence from Manufacturing Firms," University of Carlos III mimeo.

Pagan, A. and Vella, F. (1989) "Diagnostic Tests for Models Based on Individual Data: A Survey," *Journal of Applied Econometrics*, 4: S29–S59.

Pinkse, J. and Slade, M.E. (1998) "Contracting in Space: An Application of Spatial Statistics to Discrete-Choice Models," *Journal of Econometrics*, 85: 125–154.

Pinkse, J., Slade, M.E., and Brett, C. (2002) "Spatial Price Competition: A Semiparametric Approach," *Econometrica*, 70: 1111–1155.

Pinkse, J. and Slade, M.E. (2002) "Mergers, Brand Competition, and the Price of a Pint," *European Economic Review*, forthcoming.

Pinkse, J., Shen, L., and Slade, M.E. (2003) "Dynamic Spatial Probit with Fixed Effects Using One–Step GMM: An Application to Mine Operating Decisions," Penn State mimeo.

Rey, P. and Stiglitz, J. (1995) "The Role of Exclusive Territories in Producers' Competition," *RAND Journal of Economics*, 26: 431–451.

Slade, M.E. (1995) "Product Rivalry with Multiple Strategic Weapons: An Analysis of Price and Advertising Competition," *Journal of Economics and Management Strategy*, 4: 445–476.

Slade, M.E. (1998) "Strategic Motives for Vertical Separation: Evidence from Retail Gasoline," *Journal of Law, Economics, and Organization*, 14: 84–113.

Slade, M.E. (2001) "Managing Projects Flexibly: An Application of Real-Option Theory to Mining Investments," *Journal of Environmental Economics and Management*, 41: 193-233.

Slade, M.E., (2003) "Market Power and Joint Dominance in the UK Brewing Industry," *Journal of Industrial Economics*, forthcoming.

von Thunen, J. (1826) *The Isolated State,* English edition, London: Pergamon.