Patent Office in Innovation Policy: Nobody's Perfect

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Preliminary draft

Abstract

Recent concerns about "low-quality patents" in new technologies like software and business-methods in the US have given rise to attacks against the US patent office. Because of the fast growing number of patent applications, patent examiners lack time and resources to study and evaluate efficiently all the relevant prior art on each application. As a result, the patent office suffers from an overload problem which leads examiners to make mistakes, issuing "bad" patents. This model analyzes the overload problem faced by the patent office. We show the existence of multiple equilibria in firms' application strategies and R&D activity. We also investigate the examination process, introducing a specific examination effort that increases the probability of finding existing prior art on an application. After specifying the patent office's objective function, we study its optimal policy.

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1 Introduction

Often seen as a rubber-stamp body, the patent office is usually given little attention by economists. Still, a number of issues have arisen from the recent debate over patents delivered by the US Patent and Trademark Office (USPTO), dealing mainly with the efficiency of its examination process and its economic impact.

Recent concerns about "low-quality patents" in new technologies like software and business-methods in the US have given rise to attacks against the USPTO, accused to do a bad job examining patents and to grant "bad" patents (patents that should not have been granted).

The role of a Patent Office is to implement patentability standards: in the US, to be patentable, an invention must be novel, non obvious and have utility.[1] In particular, the invention cannot already exist or be described in the relevant prior art. Patent examiners grant or reject patents on applications according to those patentability standards (an application is rejected if it is proven unpatentable).

The USPTO is accused of weakening its patentability standards. According to the critics, the USPTO is becoming much laxer with patent grants than its foreign counterparts: for the period 1993-1998, Quillen and Webster (2001) estimate the grant rate (the number of grants divided by the number of applications) to more than 85% in the US, whereas it is less than 67% in Europe and 64% in Japan. Moreover, the number of litigated patents has dramatically risen in the last decade: the yearly number of lawsuits filed in the US rose almost 100%, from 1,162 in 1989 up to 2,120 in 1998.[2] According to Allison and Lemley (1998), 46% of challenged patents are currently invalidated in court for not satisfying patentability requirements, specially novelty and non obviousness.[3] Those figures reveal errors made by the USPTO and issuances of "bad" patents, *i.e.* patents on inventions that are not truly new or non obvious.

Several explanations can be provided to the weakened quality of patents currently

[1] European patentability requirements are: novelty, inventive step and industrial application.

[2] Sources: Business Journal, July 2, 1999; Trends in Patent Infringement Lawsuits 1990-1999.

[3] Invalidation is possible if the plaintiff brings the proof of existing prior art that invalidates the challenged patent.

granted by the USPTO, mainly linked to law changes in the last two decades, like the broadening of patent subject matter to new technologies (GMOs, software and business methods),[4] and the creation of the Court of Appeals of the Federal Circuit (CAFC) in 1982.[5] Those changes have been followed by a boom in patent applications: between 1985 and 2000, the yearly number of applications rose from 100,000 to almost 300,000 (Allison and al. [2003]).

As patent examiners face more and more demands, they lack time and resources to study and evaluate efficiently all the relevant prior art on each application. This is emphasized in new technologies like software and business methods, that became patentable subject matter only recently (so that most of prior art is unpatented). As a result, the patent office suffers from an overload problem that is unlikely to disappear, because of the fast growing number of applications (Merges [1999]). Today the examination process lasts for 26 month on average, whereas 85% of issuance decisions were made in less than 2 years in 1990. During that patent pending period, an examiner spends only 18 hours on average studying the application. There are currently 500,000 patent applications in the US that have not been examined yet (Lemley and Shapiro [2004]).

With no proof of existing prior art that shows an invention is not truly new or non obvious, a patent must be issued, provided it meets the other standards. As they are unable to cover the whole prior art for each application, examiners are likely to make mistakes of judgement and to grant "bad" patents. Such patents can be socially costly: as they protect invention that already exist, they involve a social deadweight loss and no social benefit on innovation.

As they anticipate a bad examination process, firms can ask for patents on inventions that lack patentability requirements: they know the overload problem will lead examiners to make mistakes, issuing patents on their "bad" applications.

Those mistakes can create a vicious circle (an issue raised by Caillaud [2003] and by Hall [2003]): issuances of "bad" patents imply a high probability of obtaining a patent, which

[4] **These technologies became patentable after court decisions in the 1980's.**

[5] **The CAFC invalidated PTO decisions by allowing patents previously rejected by the PTO for lack of novelty.**

increases incentives to apply for a patent; it increases the number of patent applications, which increases examiners' workload and forces them to make a weaker examination effort on each application (in search of prior art); as a result, more "bad" patents are being issued.

This model studies the overload problem faced by the patent office and analyzes its impact on R&D activity, according to firms' patent application strategies. We consider an industry with n firms, each owning an invention of a given quality (novelty compared to prior art) that is the firm's private information. A firm can file a costly patent application on its invention, or keep it secret. Keeping secrecy is potentially less profitable than owning a patent, but it avoids possible disclosure of information on the invention content during the patent examination process.

The patent office must find out existing prior art, if any, for all patent applications (a patent is granted if no prior art is found). The examination process is imperfect because of the overload problem, so patents can be granted to applications that are not truly new or non obvious. We assume the probability of granting a patent increases with the number of applications (as the probability to find prior art decreases). A firm's decision to apply or not for a patent depends on its anticipation on the total number of applications.

We show the existence of multiple equilibria (when there is no coordination between firms) in the number of patent applications. Locally, a reform of the application process (like a change the application fee) has a limited impact on firms' strategies. We also study the impact of the overload problem on R&D activity, according to the anticipated number of patent applications in the equilibrium. Then we investigate the patent examination process, introducing a specific examination effort that increases the probability of finding existing prior art on an application. After specifying the patent office's objective function, we study its optimal policy regarding grants and rejections.

2 Link to the literature

Most of academic research on patents addresses issues once a patent is delivered and considers the patent office and its examination process as a black box. However, recent

attacks against the USPTO have led to a growing number of works by legal scholars, followed by economists.

Several recent legal articles analyze the overload problem currently faced by the USPTO. According to Merges (1999), US patent examiners are paid in a way that encourages them to grant patents rather than to reject them. Moreover, Merges (1999) and Kesan (2002) propose solutions to improve the procedure. Those authors suggest the USPTO should be more severe regarding its non obviousness requirement, and should introduce an *ex post* reexamination system (where a third party can bring the proof of existing prior art to challenge a patent application). However, Lemley (2000) says that the costs of improving the examination process and the quality of issued patents overrides the cost of mistakes currently made (the patent office is "rationally ignorant").

Little attention in the economic literature on patents is paid to the patent office, and its examination process has not yet been formalized, except by Langinier and Marcoul (2003). Those authors address the problem faced by examiners who lack information on the relevant prior art. Their model describes a situation where neither the PTO nor the innovator observes whether an innovation is "good" or "bad". The innovation field can be a rich or a poor prior art field (for instance software has poor prior art). In a sequential game, the innovator first makes a costly effort to search for prior art (he then obtains an amount of information and a signal on the quality of his innovation), and then decides whether to apply or not for a patent (and on the amount of information he reveals if he applies). There is a moral hazard problem on his search of prior art and an adverse selection problem on what information he reveals in the application. Second, the patent office makes a costly effort to search for extra information before deciding whether to issue a patent or not. The authors compare the case where the examiner exerts the same effort on all applications to the case where he examines only a part of those applications.

Hunt (1999) and O'Donoghue (1998) formalize the non obviousness standard in order to find its socially optimal threshold. R&D activity depends on how severe is the standard. Both authors analyze the case of sequential innovations, where a patentee enjoys a monopoly position until it is replaced by a patented improvement. A higher standard

implies a lower likelihood of obtaining a patent, which hampers R&D (short term effect), but the replacement effect will be slower, which encourages R&D (long term effect). In a very innovative industry, the replacement effect dominates the short term effect, so a higher standard encourages R&D.

This model addresses the issue of the potential impact of a reform in the application process on firms' submission strategies and on R&D activity. It does not aim at analyzing an incentive system of patent examiners.

3 The Model

Consider an innovating sector with n R&D firms and a patent office.

Firms

The n firms first invest in R&D in order to innovate, and second they decide simultaneously whether to apply for a patent or not (but nothing is public before the end of the second stage). A R&D effort π costs $\gamma(\pi) = \frac{\gamma\pi^2}{2}$ to the firm and yields a successful innovation with probability π. So there are two types of firms ($i \in \{0, 1\}$): those who have succeeded in innovating ($i = 1$) and those who have failed ($i = 0$). Projects of type 1 are of good quality according to patentability standards (specially novelty), whereas projects of type 0 are of bad quality (they are not truly new compared to prior art). We call a firm of type 1 an "innovator" and a firm of type 0 an "imitator".[6] Firm i's type is private information to firm i: Efforts are not observed, and so innovation or imitation are not observed either.

Suppose that all projects are characterized by the following (intertemporal) profitability: $\Pi = v_i$ if the project i gets protection from a patent, $\Pi = \lambda_i$ if it does not, $\Pi = \mu_i$ is a patent has been asked but has been refused.

Assumption 1. $\mu_i \leq \lambda_i \leq v_i$: *profits are larger when patent protection is granted. When a patent request has been filed, there are chances that information leaks out of the procedure, undermining the protection by secrecy.*

[6] **With probability $1 - \pi$ the firm fails but is still able to understand the state of the art and to come up with an "imitation".**

Assumption 2. $v_1 - f \geq \lambda_0$*: an "innovator" who owns a patent has a higher net profit than an "imitator" who keeps secrecy.*

Patent Office

The patent office must find out existing prior art, if any, for all patent applications (a patent is granted if no prior art is found).

Two different situations can arise, according to whether the patent office faces an overload problem or not. It can depend on its budget or capacity (and on the number of examiners it can hire).

With no overload problem,[7] the office's technology is characterized by p_i, the (constant) probability that no prior art is found out for a firm of type $i \in \{0, 1\}$.

With an overload problem,[8] the total number of applications received changes the examination process: the more applications the office receives, the less resources are affected to each application, the lower is the probability to find prior art.[9] The impact of the volume of applications on the examination process is captured in the office's technology, characterized by $p_i(j)$, the probability that no prior art is found out for a firm of type $i \in \{0, 1\}$ when the office faces j patent applications.

Whatever the situation faced by the patent office, we assume there exists no anteriority on an "innovation":

Assumption 3. $p_1 = 1$ *and* $p_1(j) = 1 \forall j \in [1, n]$*: no prior art can be found on an innovation.*

Assumption 3 implies that we focus on type 1 errors, i.e. when the patent office can grant patents to "imitators". Indeed, since the USPTO examiners' policy is to grant a patent if no prior art has been found, "innovators" are assured to obtain a patent if asked

[7] This is the case for instance when the patent o□ce is able to adapt its resources to the volume of applications (hiring more examiners when the number of patent applications rises).

[8] This is the case when the patent o□ce is constrained: for instance it may not be able to hire more examiners as the volume of patent applications rises.

[9] In other words, the more applications are received, the more di□cult it is for an examiner to study the whole technological field of each application, the less likely he is to find prior art, the more likely the application will be granted a patent.

(by definition an innovation has no prior art). The case where $p_1(j) \leq 1$ is analyzed in the appendix.

Assumption 4. $\frac{\partial p_0(j)}{\partial j} \geq 0$*: the more applications the patent office receives, the higher the probability of obtaining a patent.*

Assumption 5. *For all* j, $1 > p_0(j)$*: an innovator is more likely to obtain a patent than an imitator for a given amount of applications.*

Submitting a patent request costs a fixed fee f to the submitting firm, which is sunk whatever the examination result.

The simple submission game that is considered is the following: all firms decide simultaneously both their R&D effort and their submission strategy (whether to apply for a patent or not), which determines the number of patent applications and grants, and therefore how efficient the review process of the office is.

Before analyzing the second situation where firms' submission strategies depend on the volume of applications received by the patent office because of its overload problem, we investigate the first situation as a benchmark. All proofs are given in the appendix.

4 Benchmark: no overload

We first assume that the patent office faces no overload problem, so that a project of type i obtains a patent with probability p_i (with $p_i \leq 1$). A firm submits a patent application when it gets a higher expected utility than with secrecy:

$$p_i v_i + (1 - p_i)\mu_i - f \geq \lambda_i \Leftrightarrow p_i \geq \frac{\lambda_i + f - \mu_i}{v_i - \mu_i}.$$

Let $\rho_i \equiv \frac{\lambda_i + f - \mu_i}{v_i - \mu_i}$ be the probability of obtaining a patent that leaves a firm of type i indifferent between making an application or not. Rewriting ρ_i as $\frac{\lambda_i - (\mu_i - f)}{(v_i - f) - (\mu_i - f)}$ shows that it measures the relative profit of secrecy for firm i.

Note that since $p_1(j) = 1 \forall j$, an "innovator" asks for a patent if $\rho_1 \leq 1$. Moreover, $\rho_0 \leq 1$ if and only if $\lambda_0 \leq v_0 - f$, *i.e.* if an "imitator" prefers to own a patent than to keep secrecy.

If the patent office sets a submission fee f such that $\rho_1 \leq 1 \Leftrightarrow f \leq v_1 - \lambda_1$ and $p_0 \leq \rho_0$, then it receives only good applications: firms with good projects (of type $i = 1$) apply for a patent, whereas firms with bad projects (of type $i = 0$) keep secrecy.[10] When it is able to invest as many resources as needed to make the examinations process efficient, the patent office can easily implement a submission fee that selects only good applications and excludes bad ones.

Let us now consider the situation where the patent office is not able to efficiently examine all applications received. We investigate the impact of an overload problem on firms' submission and R&D strategies.

5 Overload: equilibrium R&D activity and volume of applications

In this section the patent office is constrained so the examination process depends upon the number of applications received.

We look for a global symmetric equilibrium, characterized by an effort π^* and a continuation equilibrium. Note that the application strategy for a deviating firm that has invested $\pi \neq \pi^*$ (but is the only one who knows it) is the same as in the candidate equilibrium path, since deviation is not observed.

5.1 Submission strategies

We first consider pure strategy equilibria. Firm i's strategy is $s_i \in \{0, 1\}$ where $s_i = 1$ means that the firm i asks for patent protection, and $s_i = 0$ means that it does not. Firm i takes the equilibrium number of the $n - 1$ other firms as given. The expected utility of a R&D firm i with decision s_i is given by:

$$U_i(s_i, k) = (1 - s_i)\lambda_i + s_i[\mu_i + (v_i - \mu_i)E(p_i(k + 1)) - f].$$

As in the benchmark, an "innovator" asks for a patent if the submission fee is such

[10] **Note that it's possible only if $v_1 - \lambda_1 \geq p_0 v_0 + (1 - p_0)\mu_0 - \lambda_0$, i.e. if the relative profitability of asking for a patent compared to keeping secrecy is higher for an "innovator" than for an "imitator".**

that $\rho_1 \leq 1 \Leftrightarrow v_1 - f \geq \lambda_1$. An "imitator" asks for a patent if its expected probability of obtaining a patent, given other firms' strategies, exceeds ρ_0:

$$E(p_0(k+1) \geq \frac{\lambda_0 + f - \mu_0}{v_0 - \mu_0}.$$

We focus on symmetric equilibria. *We restrict the analysis to the case where* $v_1 - f \geq \lambda_1$, *so an innovator always applies for a patent.*[11] Such a restriction doesn't modify the analysis and allows us to exclude meaningless equilibria (which could not be the result of a patent office's policy). However a general analysis can be found in the appendix. The strategy of an imitator (of type 0) depends upon the total number of applications received by the patent office.

If all $n-1$ other firms apply for a patent, the total number of applications is $k = n-1$, and an imitator obtains a patent with probability $p_0(n)$. So an imitator applies if and only if $p_0(n) \geq \rho_0$, as it anticipates that facing many applications, the office will be inefficient in looking for prior art. As a result, there exists a pooling equilibrium where all firms ask for a patent if $\rho_0 \leq p_0(n)$.

If among the $n-1$ other firms only innovators ask for a patent, an imitator obtains a patent with the following probability:[12] $H_0(\pi, n) = \sum_{k=0}^{n-1} C_{n-1}^k \pi^k (1-\pi)^{n-1-k} p_0(k+1)$.[13] Then a firm $i = 0$ keeps secrecy if $H_0(\pi, n) \leq \rho_0$, as it anticipates that the office will receive few applications and therefore will be likely to refuse patents on imitations. There exists a separating equilibrium where only innovators ask for a patent if $\rho_0 \geq H_0(\pi, n)$.

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The function $H_0(\pi, n, \alpha)$ is increasing in α[14], from $H_0(\pi, n) = \sum_{k=0}^{n-1} C_{n-1}^k \pi^k (1 - \pi)^{n-1-k} p_0(k+1)$ to $p_0(n)$ as α goes from 0 to 1.

As a result, when $H_0(\pi, n) \leq \rho_0 \leq p_0(n)$, there exists a mixed strategies equilibrium where all innovators ask for a patent and imitators do so with probability α^*, defined by $\rho_0 = H_0(\pi, n, \alpha^*)$ (which lets an imitator indifferent between applying or not for a patent, given other firms' strategy).

5.2 R&D strategies

f the firm anticipates the continuation equilibrium is the separating equilibrium, it will ask for a patent only with a good project. Its objective is thus

$$\max_{\pi} \Pi_S = \pi(v_1 - f) + (1 - \pi)\lambda_0 - \frac{\gamma\pi^2}{2},$$

which yields the optimal choice: $\pi_S^* = \frac{1}{\gamma}[v_1 - f - \lambda_0]$. Given assumption 2, that effort is positive: $\pi_S^* \geq 0$.

If the firm anticipates the continuation equilibrium is the pooling equilibrium, it will ask for a patent whatever its type. Its objective is thus

$$\max_{\pi} \Pi_P = \pi[v_1 - f] + (1 - \pi)[p_0(n)v_0 + (1 - p_0(n))\mu_0 - f] - \frac{\gamma\pi^2}{2},$$

which yields the optimal choice: $\pi_P^* = \frac{1}{\gamma}[v_1 - p_0(n)v_0 - (1 - p_0(n))\mu_0]$. So $\pi_P^* \geq 0$ if $v_1 \geq p_0(n)v_0 + (1 - p_0(n))\mu_0$, i.e. if an "innovator" gains more profit from a patent than an "imitator". Otherwise there is no R&D in the pooling equilibrium ($\pi_P^* = 0$)

Proposition 1 characterizes firms' R&D efforts and their submission strategies according to the value of the relative profit of secrecy for imitators ρ_0.

Proposition 1. Pure strategies equilibria

- *When the relative profit of secrecy for imitators is high, such that $\rho_0 > p_0(n)$, the only possible equilibrium is a separating equilibrium characterized by a R&D effort π_S^* where only innovators apply for a patent.*

[14] **The proof is the same as in proof of footnote 13.**

- *When the relative profit of secrecy for imitators is low, such that $H_0(\pi_S^*, n) > \rho_0$, the only possible equilibrium is a pooling equilibrium characterized by a R&D effort π_P^* where all firms ask for a patent.*

- *Finally, when the relative profit of secrecy for imitators is such that $H_0(\pi_S^*, n) \leq \rho_0 \leq p_0(n)$, both equilibria are possible: a separating equilibrium characterized by a R&D effort π_S^* and a pooling equilibrium characterized by a R&D effort π_P^*.*

Proof.

- If $\rho \leq p_0(n)$, there are potentially two continuation equilibria that could prevail:
  - The equilibrium with separating continuation leads to π_S^*. But if $H_0(\pi_S^*, n) > \rho$, then we hit a contradiction. So, there exists an equilibrium with separating continuation if $H_0(\pi_S^*, n) \leq \rho$, in which case $\pi = \pi_S^*$ is chosen by all firms.
  - In the equilibrium with pooling continuation the firm's optimal choice is: π_P^*.
- If $\rho > p_0(n)$, the firm anticipates the only continuation equilibrium is the separating equilibrium. Its optimal choice is then π_S^*.

$\square$

Moreover, imitators may have a mixed submission strategy, asking for a patent with probability α^*. Proposition 2, characterizes the mixed strategy equilibrium.

Proposition 2. Mixed strategies equilibrium

When the relative profit of secrecy for imitators is high, such that $H_0(\pi_S^*, n) \leq \rho_0 \leq p_0(n)$, there exists a mixed strategies equilibrium characterized by a R&D effort π_S^*, where all innovators ask for a patent and imitators do so with probability α^* (such that $H_0(\pi_S^*, n, \alpha^*) = \rho_0$).

Proof.

The mixed strategies equilibrium is such that an "imitator" is indifferent between asking for a patent and keeping secrecy: $H_0(\pi_S^*, n, \alpha^*) = \rho_0$. A firm's objective is then: $\max_{\pi} \pi(v_1 - f) + (1-\pi)\lambda_0 - \frac{\gamma\pi^2}{2}$, which yields the optimal R&D effort $\pi = \pi_S^*$. $\square$

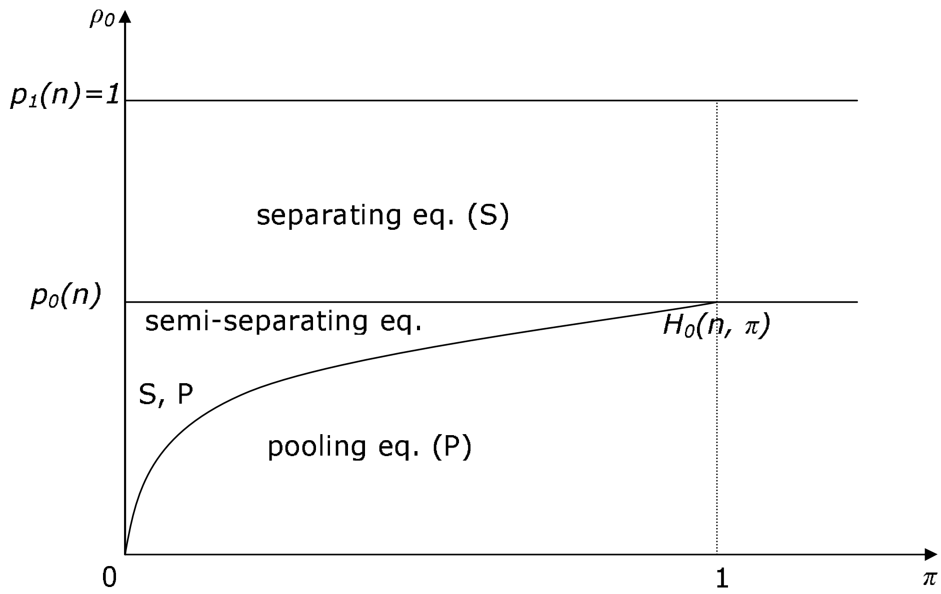

Figure 1:

All equilibria are represented in figure 1.

Comparative static analysis on Propositions 1 and 2 leads to several results. The following corollaries compare the separating equilibrium and the pooling equilibrium when they both coexist (i.e. when $H_0(\pi, n) \leq \rho_0 \leq p_0(n)$).

First, corollary 1 compares equilibrium profits when pure strategies and mixed strategies continuation equilibria coexist.

Corollary 1.

When both equilibria coexist, firms' profit is higher in the pooling equilibrium than in the separating equilibrium: $\Pi_P^* \geq \Pi_S^*$.

Indeed, $\Pi_S^* = \frac{(v_1 - f - \lambda_0)^2}{2\gamma} + \lambda_0$, and $\Pi_P^* = \frac{(v_1 - p_0(n)v_0 - (1-p_0(n))\mu_0)^2}{2\gamma} + p_0(n)v_0 + (1 - p_0(n))\mu_0 - f$. So, $\Pi_P^* - \Pi_S^* = \frac{1}{2}(v_0 - \mu_0)(p_0(n) - \rho_0)[2 - (\pi_S^* + \pi_P^*)]$, which is positive when $p_0(n) \geq \rho_0$, *i.e.* when both equilibria coexist.

So firms are better off when they all apply for a patent. However, because there is no coordination between firms, the separating equilibrium is also possible.

Second, corollary 2 compares R&D equilibrium efforts when pure strategies and mixed strategies continuation equilibria coexist.

Corollary 2.

When $H_0(\pi_S^*, n) \leq \rho_0 \leq p_0(n)$, both pure strategies equilibria and mixed strategies equilibrium coexist, and R&D efforts are such that $\pi_P^* < \pi_S^*$.

Indeed, $\pi_S^* - \pi_P^* = \frac{1}{\gamma}(p_0(n) - \rho_0)(v_0 - \mu_0)$, which implies that when $\rho_0 \leq p_0(n)$, $\pi_P^* < \pi_S^*$. So R&D is obviously less intensive than when only innovators apply for a patent. In the "all apply" equilibrium, an "imitator" applies for a patent and gets $p_0(n)(1-\mu_0)+\mu_0-f$, whereas in the separating equilibrium an imitator keeps the secret and gets λ_0. Since $\rho_0 \leq p_0(n)$, $p_0(n)(1-\mu_0)+\mu_0-f \geq \lambda_0$, so a firm has more incentive to be an imitator in the pooling equilibrium than in the separating equilibrium, so it makes a lower R&D effort.

So, when $H_0(\pi_S^*, n) \leq \rho_0 \leq p_0(n)$, there is a pooling equilibrium with a R&D activity and many applications, and a separating equilibrium with a high R&D activity and few applications. In that case, the patent office overload has a negative impact on R&D activity: as examiners receive too many demands, the probability to obtain a patent on an "imitation" increases, so incentives to innovate decrease and firms make a weaker R&D effort. As a result, more "bad" patents are issued and there are less "innovators" in the industry. As a consequence, in addition to being socially costly, issuance of "bad" patents slows down R&D activity and induces less innovation.

Third, corollary 3 compares the expected number of patents issued in each equilibrium, when both pure strategies equilibria coexist. The expected number of patent grants is $n\pi_S^*$ in the separating equilibrium and $n\pi_P^* + n(1-\pi_P^*)p_0(n)$ in the pooling equilibrium.

Corollary 3.

When both equilibria coexist, the expected number of patent issuances is higher in the pooling equilibrium (characterized by a weaker R&D effort and more patent applications)

- *for high values of the R&D cost, such that $\gamma \geq v_0 - \mu_0 + v_1 - \mu_0$, or*
- *for high or small values of $p_0(n)$.*

Indeed, when $p_0(n)$ is high (*i.e.* n is high or the patent office isn't strict), there are many imitators in the pooling equilibrium, and they all ask for a patent and obtain it with a high probability. So the expected number of delivered patents is higher than in the

separating equilibrium. Moreover, when the cost of R&D is high, or when $p_0(n)$ is small ($i.e.$ n is small or the patent office is strict), there are almost as many "innovators" in the pooling equilibrium as in the separating equilibrium, and still there are some imitators who obtain a patent in the pooling equilibrium. So the overall expected number of delivered patents is higher than in the separating equilibrium. In these cases, both equilibria coexist: a pooling equilibrium with a weak R&D activity and many patents issued, and a separating equilibrium with a high R&D effort and few patents issued.

Third, corollary 4 describes the impact of the submission fee f on R&D equilibrium efforts.

Corollary 4.

An increase in the submission fee f decreases the separating equilibrium effort π_S^* has no impact on the pooling equilibrium effort π_P^*.

Indeed, $\frac{\partial \pi_S^*}{\partial f} = -\frac{1}{\gamma}$, whereas $\frac{\partial \pi_P^*}{\partial f} = 0$, since in the pooling equilibrium the submission fee is spent in any case.

The previous analysis emphasizes several different situations according to the value of the submission fee f. A situation where the only possible equilibrium is a separating equilibrium where the office receives only "good" applications arises when it sets a high submission fee f such that $\rho_0 > p_0(n)$ (but still with $v_1 - f \geq \lambda_1$). However, the R&D effort π_S^* is then a decreasing function of f. An other situation where both pooling and separating equilibria are possible arises if the patent office sets a lower submission fee f such that $H_0(\pi_S^*, n) \leq \rho_0 \leq p_0(n)$. It is worth noting that in such a situation R&D efforts π_S^* and π_P^* are higher than in the previous one,[15] with $\pi_S^* \geq \pi_P^*$, and π_S^* is maximal for a minimal fee (such that $H_0(\pi_S^*, n) = \rho_0$). So from a R&D incentives viewpoint, the situation where both equilibria coexist is more efficient than the situation where the only possible equilibrium is a pooling equilibrium. However, when both pooling and separating equilibria are then possible, f is still not a sufficient tool to ensure the highest R&D effort (π_S^*). The existence of multiple equilibria (because firms do not coordinate) implies that local variations of f have no impact on submission strategies, so the patent office is not

[15] **Indeed, π_S^* is decreasing in f, such that $\pi_S^* \geq \pi_P^*$ if $\rho_0 \leq p_0(n)$, and $\pi_S^* \leq \pi_P^*$ otherwise.**

able to select only good applications through f. So a change in the submission fee is not a sufficient tool to solve the overload problem of the patent office and to encourage R&D activity.

However, the patent office may set an different fee according to whether it grants the patent or not: the firm would pay f if the patent is granted and $\hat{f} > f$ if it is rejected. In other words, the patent office could set a penalty *ex post* if prior art has been found on the application. Having an additional tool ($\hat{f}$), the patent office could set fees in order to receive only good applications, or to encourage R&D efforts.[16]

However, the patent office policy is not limited to the submission fee: the examination process has also an impact on firms' submission strategies. The next section investigates that issue with an endogenous examination process.

6 Endogenous patent examination process

In this section, we explain how the number of patent applications decreases the likelihood of finding prior art, through a costly examination effort made by examiners on each file. That examination effort consists in studying the application and searching for prior art: a higher effort increases the likelihood of finding existing prior art and thus reduces the probability of patent issuance.

At t=0 (stage 1), the patent office announces the submission fee f and commits itself to grant a patent when no prior art has been found. Its objective is to maximize the net social value of innovations, i.e. their social value less the social cost of patents (measured by the number of delivered patents plus the cost of patent examination process). Let W_1 be the social value of an innovation, and $\Delta(\pi, e)$ the social cost of patents. The patent office's objective is then:

$$\max_f n\pi W_1 - \Delta(\pi, e).$$

At t=1 (stage 2), firms choose simultaneously their R&D strategy (π) and their submission strategy (ask for patent or not). Simultaneously, the patent office chooses a costly

[16] **We are investigating that possibility right now.**

examination effort per demand e, that determines the probability of not finding prior art on a project i, given the number of patent submissions k: $r_i(e)$. We assume that no prior art can be found on a good project (it is truly new), so $r_1(e) = 1 \;\forall\; e$, whereas $r_0(e) \leq 1 \;\forall\; e$ (with $r_0'(e) \leq 0$, $r_0''(e) \geq 0$ and $r_0(0) = 1$). When the office receives k applications, the examination process costs $c(ke)$, assumed to be increasing and convex.[17] Once all applications received, its objective is to minimize the cost of patents. Since at this stage R&D is done, all patents are costly, so the cost is measured by the total number of patents granted:

$$\min_e \Delta(\pi, e) = E_j[j + (k-j)r_0(e) + c(ke)].$$

Assumption 6. *In this section we assume that $\rho_0(f) \leq \rho_1(f)$: the relative profit of secrecy is higher for an innovator than for an imitator.*

In order to find the optimal fee f, we determine the subgame perfect equilibrium. Several equilibria can emerge.

Lemma 1 investigates the case where $1 < \rho_1(f)$.[18]

Lemma 1.

If the relative profit of secrecy is higher than the probability of getting a patent for innovators, i.e. $\rho_1(f) > 1$, the optimal policy for the patent office is to reject all applications, which leads to an equilibrium where all firms keep secrecy and make a R&D effort $\pi_0 = \frac{\lambda_1-\lambda_0}{\gamma}$, and where the social value of innovations is $n\frac{\lambda_1-\lambda_0}{\gamma}W_1$.

Indeed, when $1 < \rho_1(f)$, innovators keep secrecy, and the expected payoff of innovation is λ_1. If $\rho_0(f) < 1 < \rho_1(f)$, the expected payoff of innovation is λ_1, whereas the expected payoff of imitation is at least λ_0. The R&D effort is less or equal to π_0, and the patent office bears the cost of examination. The net social value of innovations is then less or equal to $n\frac{\lambda_1-\lambda_0}{\gamma}W_1$. So excluding good applications (setting $1 < \rho_1(f)$) leads the patent office to exclude all applications. A firm's objective function is then: $\max\limits_{\pi} \pi\lambda_1 + (1-\pi)\lambda_0 - \frac{\gamma\pi^2}{2}$, which yields the optimal R&D effort $\pi_0 = \frac{\lambda_1-\lambda_0}{\gamma}$. Lemma 1 an equilibrium with no

[17] Let $c'(ke)$ be the derivative with respect to the global effort ke (the derivative with respect to the individual effort e equals to $kc'(ke)$).

[18] Recall that $r_1(e) = 1 \;\forall\; e$, so the probability of getting a patent is 1 for innovators.

patents, where R&D activity is motivated by secrecy. Such a situation avoids the social cost of patents and of an examination process.

From now on, we will study the case where $1 \geq \rho_1(f)$, so all innovators apply for a patent.

Proposition 3.

The optimal policy of the patent office cannot lead to a separating equilibrium where only innovators apply for a patent.

Proof.

A separating equilibrium where only innovators apply, characterized by efforts (π, e), is possible if $r_0(e) \leq \rho_0$. The number of application is then $k = i$ (the number of good projects), so the objective function of the patent office is: $\min_e E_i[i + c(ie)]$. That function is increasing in e: since all applications are of type $i = 1$, for which no prior art can be found, the patent office makes no examination effort: $e = 0$. An imitator then obtains a patent with probability $r_0(0) = 1$. Since we assumed that $1 \geq \rho_1(f) \geq \rho_0(f)$, we hit a contradiction, except if $e = 0$ and $\rho_1(f) = \rho_0(f) = 1$: all firms are then indifferent between secrecy and patent submission; they make a R&D effort $\pi = 0$; such a situation generates no social surplus and is dominated by a "all secret" policy with $1 < \rho_0(f)$ (see lemma 1 for such a policy). $\square$

According to proposition 3, there is no equilibrium where the patent office receives only "good" applications. Since an "innovation" receives a patent whatever the examination effort, the office does not examine applications in the separating equilibrium, which encourages imitators to apply for a patent. In order to use patents as a tool to encourage R&D activity, the office must attract "good" applications, but it implies to receive also "bad" applications. In other words, the overload is a necessary evil in a world where patents give incentives to innovate.

Let the functions $\tilde{e}(\pi)$ and $\tilde{\pi}(e)$ be such that:

$$\begin{cases} (1-\pi)r_0'(\tilde{e}(\pi)) + c'(n\tilde{e}(\pi)) = 0, \\ \gamma\tilde{\pi}(e) = v_1 - \mu_0 - r_0(e)(v_0 - \mu_0), \end{cases}$$

Lemma 2 shows there exists a unique equilibrium for those functions.

Lemma 2.

$\tilde{e}(\pi)$ is decreasing on $[0,1]$, with $\tilde{e}(1)=0$, and $\tilde{\pi}(e)$ is increasing when $e \geq 0$. There exists a unique equilibrium (e_P, π_P) such that $e_P = \tilde{e}(\pi_P)$ and $\pi_P = \tilde{\pi}(e_P)$.

Proposition 4 characterizes the pure strategies equilibrium for submission strategies, R&D efforts and examination effort when the patent office policy is such that $\rho_1(f) \leq 1$.

Proposition 4.

Under a policy such that $\rho_1(f) \leq 1$, there exists a unique equilibrium where all firms ask for a patent if and only if $r_0(e_P) \geq \rho_0(f)$. Such an equilibrium is characterized by (e_P, π_P) and is unique in the set of pooling equilibria.

Proof.

If $r_0(e) \geq \rho_0(f)$, there exists a pooling equilibrium where all firms apply for a patent and the office receives n patent applications. its objective is then : $\min_e \Delta(\pi, e) = n[\pi + (1-\pi)r_0(e)] + c(ne)$, which yields an optimal examination effort $\tilde{e}(\pi)$. Firms' objective is $\max_\pi \pi(v_1 + (1-\pi)(r_0(e)v_0 + (1-r_0(e))\mu_0) - f - \gamma\frac{\pi^2}{2}$, which yields an optimal R&D effort $\tilde{\pi}(e)$. The unique possible equilibrium is then (e_P, π_P), and it is possible if and only if $r_0(e_P) \geq \rho_0(f)$. $\square$

Comparative static analysis on proposition 4 leads to several results.

According to lemma 2, an increase in the examination effort e increases the R&D effort ($\tilde{\pi}'(e) \geq 0$): if e increases, $r_0(e)$ decreases, which encourages firms to innovate. An increase in the R&D effort π decreases the examination effort ($\tilde{e}'(\pi) \leq 0$): if π increases, the fraction of "good" demands increases, which encourages the patent office to make a weaker examination effort, since examining a good project is costly and useless. So the fraction of issued patents $\pi + (1-\pi)r_0(e)$ increases with the R&D effort: a high fraction of patent applications should be issued in highly innovative industries. Moreover, the R&D effort decreases as the R&D cost γ increases,which increases the examination effort.

Corollary 5 describes how the number of firms n in the industry modifies the equilibrium R&D effort.

Corollary 5.

In the pooling equilibrium, both efforts π_P et e_P decrease as n increases.

An increase in n has a negative impact on the R&D equilibrium effort through the examination process: facing more applications, the patent office has to weaken the examination effort on each file. Firms anticipate more examination errors, and thus have weaker incentives to innovate, so they reduce their R&D effort. The patent office undergoes a vicious circle: more applications create an overload, which damages the examination process and leads to a weaker R&D activity and a higher proportion of "bad" applications.

The examination effort $\tilde{e}(\pi)$ is defined by $(1-\pi)r_0'(\tilde{e}(\pi))+c'(n\tilde{e}(\pi)) = 0$. If n sufficiently high, $c'(n\tilde{e}(\pi)) > -(1-\pi)r_0'(\tilde{e}(\pi))$ $\forall e$ and the optimal examination effort becomes $e = 0$, which leads to a probability $r_0(0) = 1$. The the pooling equilibrium always exists ($\rho_0(f) \leq 1$). In other words, in an industry with many firms, all firms apply for a patent, which can lead to an overload of the patent office. Moreover, if n is sufficiently high so that the pooling equilibrium always exists, there is no other possible equilibrium. Indeed, a semi-separating equilibrium where imitators ask for a patent with probability α does not exist in such a situation (such an equilibrium is characterized in the appendix). So, in weakly concentrated industries (with a high number of firms), it is impossible for the patent office to filter some "bad" applications, and all firms ask for a patent.

One can study the impact of a degradation in the examination process, due to a new patentable subject matter for instance, which would make the prior art more difficult to search. Such a change has a positive impact on $r_0(e)$, so the R&D effort $\tilde{\pi}(e)$ decreases (firms have less incentives to innovate since they are more likely to get a patent on an "imitation"). The examination effort $\tilde{e}(\pi)$ also decreases if its inefficiency is reinforced by the change (for instance, if the degradation is represented by α, it means that $\frac{\partial^2 r_0}{\partial e \partial \alpha}(e, \alpha) \leq 0$). A degradation in the examination process can also be reflected in the global cost of examination $c(ne_P)$. With a quadratic cost $c(ne) = \frac{c(ne)^2}{2}$, an increase in c decreases the examination effort $\tilde{e}(\pi)$ and has no direct effect on $\tilde{\pi}(e)$. So, the examination effort e_P decreases. There is no direct effect and a negative indirect effect on π_P: firms have less incentives to innovate and make a weaker R&D effort. To sum up, a lower examination

process weakens R&D activity and the examination effort, which in turn deteriorates the examination process and can lead to a vicious circle. On the contrary, an improved examination process encourages R&D activity and increases the examination effort: there are more innovators in the industry and fewer "bad" patents.

According to proposition 4, if $\rho_1(f) \leq 1$, the patent office has the following objective:

$$\begin{cases} \max_f n\pi_P W_1 - \Delta(e_P, \pi_P), \\ s.c.\ 1 \geq \rho_1(f) \end{cases}$$

where $\Delta(e_P, \pi_P)$ is the minimized social cost of patents in the pooling equilibrium: $\Delta(e_P, \pi_P) = n[\pi_P + r_0(e_P)(1 - \pi_P)] + c(ne_P)$.

Proposition 5.

When the patent office policy leads to a pooling equilibrium, the optimal submission fee is $f^* = 0$.

Proof.
As f decreases, $\rho_1(f)$ decreases, which relaxes the constraint does not modify the objective function. □

The comparison of both possible optimal policies (a policy which leads to a "all secret" equilibrium and a policy leading to a "all apply" equilibrium) shows that the most socially desirable policy depends on the social value of innovations W_1:

- If W_1 sufficiently high compared to the minimal social cost of patents $\Delta(\pi_P, e_P)$, *i.e.* if $n(\pi_P - \frac{\lambda_1 - \lambda_0}{\gamma})W_1 > \Delta(e_P, \pi_P)$, the optimal policy is $f = 0$ (which leads to $1 > \rho_1(f)$) and all firms ask for a patent.

- Otherwise, the optimal policy is to reject all applications, and all firms to keep secrecy.

The submission cost f has no impact on equilibrium efforts (π_P, e_P), as long as the pooling equilibrium exists (*i.e.* as long as $1 \geq \rho_1(f)$, such that f does not modify firms' submission strategies). The submission fee is only a tool to switch from a "all apply"

equilibrium to a "all secret" equilibrium, and is not a sufficient tool to attract only "good" applications.

A "all secret" policy is socially better if the social cost of patents is very high or if secrecy is very profitable for "innovators" (it is not the case for instance in the pharmaceutic industry where the information necessary to imitate a new product is revealed as the product appears on the market).

It is worth noting that such an analysis is restricted to pure strategies equilibria. But other kinds of equilibria can emerge, like a semi-separating equilibrium (where all innovators ask for a patent and imitators apply with probability α).

7 Conclusion

Despite the fact that firms' R&D and submission strategies depend upon the examination process, it is impossible for the patent office to select only "good" applications. Moreover, the submission fee has no impact on the equilibrium volume of applications, so it is not a sufficient tool to eliminate the overload of the patent office and to encourage R&D activity. However, the patent office could encourage R&D through a penalty *ex post* when prior art has been found (that would lower μ_0 in the model).

The overload problem raises other issues concerning the examination process and optimal policy of the patent office. Examiners are obviously less informed than firms on prior art in their technological field. Improving the examination process can be very costly, and it may be better to let examiners make errors, so firms bear *ex post* the costs of litigation which would invalidate errors in patent issuances.

Figures on the US patent office show that the overload problem is getting worse as the volume of non examined applications increase each year. That problem can modify firms' submission strategies, which can prefer to keep secrecy if the waiting period is too long. That could partially solve the overload problem. So the patent office could deliberately leave a stock of non examined applications. Facing an increase in the number of applications, the patent office may trade off between a fast examination process which leaves few non examined applications but increases the number of issuance errors, and a

slow examination process which avoids errors but leaves many non examined applications.

Finally, the model distinguishes "good" projects from "bad" ones according to novelty compared to prior art. It would be interesting to make another distinction, according to their expected profitability: a very innovative project may be not profitable even if patented, whereas a weakly innovative project may be very profitable if patented, as it is the case of many business methods. Then some innovators could prefer to keep secrecy rather than to ask for a patent, while some imitators could choose to apply.

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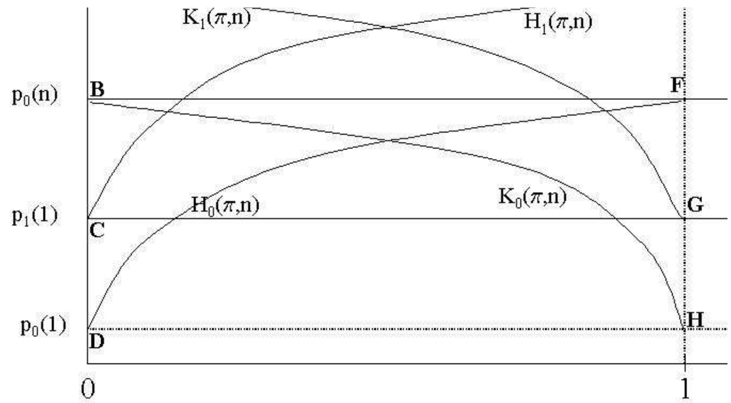

Figure 2: General case analysis

9 Appendix

9.1 General analysis on ρ_i

Consider a firm of type i, which takes the $n-1$ other firms' strategies as given.

If all other firms apply for a patent, firm i obtains a patent with probability $p_i(n)$.

If only innovators (of type $i=1$) ask for a patent, firm i obtains a patent with probability: $H_i(\pi,n) = \sum_{k=0}^{n-1} C_{n-1}^k \pi^k (1-\pi)^{n-1-k} p_i(k+1)$. Note that $H_i(\pi,n)$ is increasing in π. Indeed, $\sum_{k=0}^{n-1} C_{n-1}^k \pi^k (1-\pi)^{n-1-k} = 1$, so its derivative $\sum_{k=0}^{n-1} C_{n-1}^k \pi^{k-1} (1-\pi)^{n-2-k} (k-(n-1)\pi)$ is equal to 0, *i.e.* as π increases from 0 to 1, it contains negative terms followed by positive terms which perfectly compensate each others. Moreover, $\frac{\partial H_i}{\partial \pi} = \sum_{k=0}^{n-1} C_{n-1}^k \pi^{k-1} (1-\pi)^{n-2-k} (k-(n-1)\pi) p_i(k+1)$. By assumption, $p_i(k+1)$ is increasing in k, so the positive terms have more weight than negative ones, so $\frac{\partial H_i}{\partial \pi} \geq 0$, with $H_i(0,n) = p_i(1)$ and $H_i(1,n) = p_i(n)$.

If only imitators (of type $i=0$) ask for a patent, firm i obtains a patent with probability: $K_i(\pi,n) = \sum_{k=0}^{n-1} C_{n-1}^k \pi^k (1-\pi)^{n-1-k} p_i(n-k)$. Note that $K_i(\pi,n)$ is decreasing in π. Indeed, $\sum_{k=0}^{n-1} C_{n-1}^k \pi^k (1-\pi)^{n-1-k} = 1$, so its derivative $\sum_{k=0}^{n-1} C_{n-1}^k \pi^{k-1} (1-\pi)^{n-2-k} (k-(n-1)\pi)$ is equal to 0, *i.e.* as π increases from 0 to 1, it contains negative terms followed by positive terms which perfectly compensate each others. Moreover, $\frac{\partial K_i}{\partial \pi} = \sum_{k=0}^{n-1} C_{n-1}^k \pi^{k-1} (1-\pi)^{n-2-k} (k-(n-1)\pi) p_i(n-k)$. By assumption, $p_i(n-k)$ is decreasing in k, so the negative terms have more weight than positive ones, so $\frac{\partial K_i}{\partial \pi} \leq 0$, with $K_i(0,n) = p_i(n)$ and $K_i(1,n) = p_i(1)$.

Functions $H_i(\pi,n)$ and $K_i(\pi,n)$ are represented in figure 2.

Several cases are possible:

- There exists an equilibrium where no firm asks for a patent if $\rho_i \geq p_i(1) \forall i \in \{0,1\}$.
- There exists an equilibrium where all firms ask for a patent if $\rho_i \leq p_i(n) \forall i \in \{0,1\}$.
- There exists an equilibrium where only innovators ask for a patent if $\rho_0 \geq H_0(\pi, n)$ and $\rho_1 \leq H_1(\pi, n)$.
- There exists an equilibrium where only imitators ask for a patent if $\rho_0 \leq K_0(\pi, n)$ and $\rho_1 \geq K_1(\pi, n)$.

On figure 2, taking π as given:

- if ρ_1 is in the area AEG and ρ_0 is in the area BHD, then an equilibrium where no firm asks for a patent, an equilibrium where all firms ask for a patent and a separating equilibrium where only imitators ask for a patent coexist.
- if ρ_1 is in the area CEG and ρ_0 is in the area BFD, then then an equilibrium where no firm asks for a patent, an equilibrium where all firms ask for a patent and a separating equilibrium where only innovators ask for a patent coexist.

So, the separating equilibrium where only innovators ask for a patent and the separating equilibrium where only imitators ask for a patent cannot coexist. Moreover, the patent office will not adopt a policy which attracts only bad demands (of type $i = 0$). So, the separating equilibrium where only imitators ask for a patent can be excluded with no loss of generality.

9.2 proof of corollary 3

The expected number of issued patents is higher in the pooling equilibrium than in the separating equilibrium if: $n\pi_S^* \leq n\pi_P^* + n(1-\pi_P^*)p_0(n)$. Replacing π_S^* and π_P^* allows us to rewrite that condition as: $p_0(n)^2 - p_0(n)[1 + \frac{v_1 - \mu_0}{v_0 - \mu_0} - \frac{\gamma}{v_0 - \mu_0}] + \rho_0 \geq 0$.

- If $1 + \frac{v_1 - \mu_0}{v_0 - \mu_0} - \frac{\gamma}{v_0 - \mu_0} \leq 0$, *i.e.* if $\gamma \geq v_0 - \mu_0 + v_1 - \mu_0$, then the condition is always true, *i.e.* $n\pi_S^* \leq n\pi_P^* + n(1-\pi_P^*)p_0(n)$.
- Otherwise, it is true for $p_0(n) \in [\rho_0, \underline{p}] \cup [\overline{p}, 1]$, where $\underline{p} = \frac{1}{2}[(1 + \frac{v_1 - \mu_0}{v_0 - \mu_0} - \frac{\gamma}{v_0 - \mu_0}) - \sqrt{\Delta}]$ and $\overline{p} = \frac{1}{2}[(1 + \frac{v_1 - \mu_0}{v_0 - \mu_0} - \frac{\gamma}{v_0 - \mu_0}) + \sqrt{\Delta}]$, with $\Delta = (1 + \frac{v_1 - \mu_0}{v_0 - \mu_0} - \frac{\gamma}{v_0 - \mu_0})^2 - 4\rho_0$. Note that $\underline{p} \geq \rho_0$, since $1 - \pi_S^* \geq 0$

9.3 Proof of corollary 5

Following the equation which defines $\tilde{\pi}(e)$, $\frac{\partial \tilde{\pi}(e)}{\partial n} = 0$. Moreover, differentiating the equation which defines $\tilde{e}(\pi)$ according to n, since the patent office objective is convex, $\frac{d\tilde{e}(\pi)}{dn}$ has the sign of

$-\tilde{e}(\pi)c''(n\tilde{e}(\pi)) \geq 0$, so $\frac{\partial \tilde{e}(\pi)}{\partial n} \leq 0$. So, at the equilibrium, as n increases, π_P and e_P decrease.

9.4 Proof of lemma 2

Following the equation which defines $\tilde{e}(\pi)$:

$$\tilde{e}'(\pi)[c''(n\tilde{e}(\pi)) + (1-\pi)r_0''(\tilde{e}(\pi))] = r_0'(\tilde{e}(\pi)).$$

By assumption, $c''(ne) \geq 0$, $r_0'(e) \leq 0$ and $r_0''(e) \geq 0$, so $\tilde{e}'(\pi) \leq 0$. Moreover, $r_0'(e) \geq 0$, so $\tilde{\pi}'(e) \geq 0$.

9.5 Characterization of the semi-separating equilibrium (section 6)

Here we characterize a semi-separating equilibrium where all innovators apply for a patent, and imitators apply with probability α. In such an equilibrium, imitators are indifferent between asking for a patent and keeping secrecy.

Firms make the R&D effort that maximizes their expected payoff:

$$\max_{\pi} \pi(v_1 - f) + (1-\pi)\lambda_0 - \gamma\frac{\pi^2}{2},$$

which yields the optimal effort $\pi = \frac{1}{\gamma}[\lambda_1 - \lambda_0 + (v_1 - \mu_1)(1-\rho_1)] \equiv \pi_M$, which does not depend upon e and α. That effort decreases as f increases: applying for a patent is less attractive, so the incentives to innovate and the R&D effort are weaker. An increase in λ_0 makes secrecy more attractive for imitators, which lowers the R&D effort.

The office makes the examination effort that minimizes the expected social cost of patents $\Delta(\pi, e, \alpha)$. Its objective is the following:

$$\min_{e} \sum_{k=0}^{n} C_n^k \pi^k (1-\pi)^{n-k} \sum_{h=0}^{n-k} C_{n-k}^h \alpha^h (1-\alpha)^{n-k-h}[k + hr_0(e) + c((k+h)e)],$$

which leads to the following first order condition:

$$\sum_{k=0}^{n} \binom{n}{k} \pi^k (1-\pi)^{n-k} \sum_{h=0}^{n-k} \binom{n-k}{h} \alpha^h (1-\alpha)^{n-k-h}[h(r_0'(e) + (k+h)c'((k+h)e)] = 0$$

That condition can be rewrote as $F(\pi, e, \alpha) = 0$, where

$$F(\pi, e, \alpha) = n\alpha(1-\pi)r_0'(e) + \sum_{k=0}^{n} C_n^k \pi^k (1-\pi)^{n-k} \sum_{h=0}^{n-k} C_{n-k}^h \alpha^h (1-\alpha)^{n-k-h}[(k+h)c'((k+h)e)].$$

We have:

$\frac{\partial F}{\partial \alpha}(\pi, e, \alpha) = n(1-\pi)r_0'(e) + \sum_{k=0}^{n} C_n^k \pi^k (1-\pi)^{n-k} \sum_{h=0}^{n-k} C_{n-k}^h \alpha^{h-1}(1-\alpha)^{n-k-h-1}(h - \alpha(n - k))[(k+h)c'((k+h)e)]$. the first term is negative, and the second term is positive[19]. So, it is not possible to conclude *a priori* on the monotonicity of $F(\pi, e, \alpha)$. However, $F(\pi, e, 0)$ is equal to $\sum_{k=0}^{n} C_n^k \pi^k (1-\pi)^{n-k} kc'(ke)$, which is increasing in e, so $\alpha = 0$ leads to an effort $e = 0$ on the one hand, and $F(\pi_M, e, 1)$ is equal to $n[p_0'(e)(1-\pi_M) + c'(ne)]$, so $\alpha = 1$ leads to an effort $e = \tilde{e}(\pi_M)$ on the other hand.

To sum up, the separating equilibrium, if exists, is defined by two functions that determine $\alpha(e)$ and $e(\alpha)$: $e = r_0^{-1}(\rho_0)$, and $F(\alpha, \pi_M, e) = 0$. A sufficient condition of existence is then: $\tilde{e}(\pi_M) > r_0^{-1}(\rho_0)$. That condition can be rewrote as: $\rho_0 > r_0(\tilde{e}(\pi_M))$.

If the patent office policy is such thtat $1 \geq \rho_1 \geq \rho_0$ and $r_0(\tilde{e}(\pi_M)) \leq \rho_0$, then it can lead to a semi-separating equilibrium (non degenerated).

It is a sufficient condition of existence, so a semi-separating equilibrium could exist even if that condition is not true.

When the patent office policy cannot lead to a pooling equilibrium, *i.e.* when $\rho_0 > r_0(e)$, comparison of π_P and π_M shows that $\pi_P > \pi_M$. Since $\tilde{e}(\pi)$ is decreasing (see lemma 2), it leads to $\tilde{e}(\pi_M) > \tilde{e}(\pi_P)$, which implies $r_0(\tilde{e}(\pi_M)) < r_0(\tilde{e}(\pi_P)) < \rho_0$, so the sufficient condition of existence of the semi-separating equilibrium is true. So, when the patent office has a policy which cannot lead to the pooling equilibrium where all firms ask for a patent, its policy leads to a semi-separating equilibrium where imitators ask for a patent with probability α.

It is worth noting that when n is sufficiently high, $\tilde{e}(\pi_M)$ tends to 0, so the function $F(\pi, e, \alpha)$ goes from $e = 0$ to $e = 0$, and the sufficient condition of existence of the semi-separating equilibrium is not true anymore.

[19] **There are negative terms followed by positive ones, with more weight on positive terms (since** $(k + h)c'((k+h)e)$ **is increasing in** h**)**