Constraints on Own- and Cross-Price Elasticities Obtained from Random Coefficient Multinomial-Logit Models

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Comments and Suggestions Extremely Welcome

Abstract: In this paper I explore the properties of own- and cross-price elasticities derived from the random coefficient multinomial logit (RC-MNL) model of demand. First, I show that, in the version of this model typically estimated, own and cross price elasticities can be bounded systematically. Second, I demonstrate that the basic version of the RC-MNL model typically estimated imposes Slutsky Symmetry type restrictions on the slope of demand. There are good reasons not to expect such restrictions hold in aggregate demand models. Third, I show that a 'selection' effect in the basic model appears to impose strong restrictions on the properties of estimated elasticities of demand as prices become large. Each of these results are attempts understand the way in which the basic version of the RC-MNL produces estimates of demand elasticities whose variation across products are determined in important ways by observed prices and market shares. Overall, the results suggest that in practical settings such as merger analysis, researchers working with variants of the RC-MNL model must remain vigilant to the dangers that (1) the model is constraining estimated price-elasticities rather than the data and (2) price elasticities are being identified by other than price variation.

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I. Introduction

The Random Coefficient or Mixed Multinomial Logit model of demand (RC-MNL) has recently been extensively estimated following the seminal works of Mcfadden(1981), Berry (1994) and Berry, Levinsohn and Pakes (1995,1999). Recent applications include Nevo (2002), Petrin(2002) and Davis(2006). See for example, Davis (2000) and Nevo (2000) for an overview and Berry, Linton, Pakes (2004) for the econometric theory required for estimation.

In this paper, I have a single simple aim: to explore the properties of own- and cross-price elasticities of demand that are obtained from the versions of RC-MNL models typically estimated. I provide a number of results. First I show that in the version typically taken to data, the estimated own- and cross-price elasticities of demand can be shown to be bounded by functions of prices and market shares in a fashion somewhat reminiscent of the constraints known to be imposed by the MNL and nested MNL models. Second I show that estimated cross-price elasticities often impose 'Slutsky Symmetry' style restrictions, which will be familiar to many readers as generally unreasonable restrictions to impose on aggregate demand models – indeed they are restrictions that more usually arise in representative agent models. Third, I show that a 'selection' effect in the basic model appears to impose strong restrictions on the properties of estimated elasticities of demand as prices become large.

The results help us understand the kinds of restrictions that the version of the RC-MNL model typically implemented in practice places on implied demand elasticities and, in particular, on the variation in own- and cross-price elasticities of demand across products. These estimated elasticities are central for activities such as testing conduct and performing merger simulation.

My aim in this paper is not in anyway to discourage use of these models. Rather my hope is to raise some issues with the way in which they are currently applied. By exploration, I discover some practical methods for relaxing some of the problems that emerge in the basic formulation of the model. Overall, however these results do suggest that in practical settings like merger

analysis, researchers must remain vigilant to the danger that the model is constraining estimated price-elasticities rather than the data, even when estimating the RC-MNL model and despite the fact that a rich enough specification of the RC-MNL model can be a flexible functional form; see Mcfadden and Train(2000).

The paper proceeds by first introducing some notation and a brief overview to the class of RC-MNL models in Section II and discus the analytic form of own-and cross-price elasticities derived from such models. In section III I discus a particularly popular version of the RC-MNL model and provide expressions bounding the own-and cross-price elasticities obtained from such models. In Section IV I discus the 'slutsky symmetry' style restrictions that estimated RC-MNL models often implicitly impose. In section V I discus a 'selection' effect and its implications for cross-sectional predicted own-price elasticities, and most importantly for the way in which predicted mark-ups will vary across products. Section VI provides a numerical example based on BLP's(1999) paper. Section VII concludes.

II. The Random Coefficient Multinomial Logit Model

Let be a vector of observed product characteristics for product j and let ξ be a product characteristic that is observed by the consumer but unobserved by the econometrician. Each consumer *i* is assumed to solve $\max_{=\{0,1..,\ \}} \ (\ ,\xi \ ,\upsilon \ ,\varepsilon \ ;\theta)$ where

$(\ ,\xi \ ,\upsilon \ ,\varepsilon \ ;\theta) = \overline{\ } (\ ,\xi \ ,\upsilon \ ;\theta) + \varepsilon$ with $\theta = (\beta,\sigma)$ and where ε is an additive random term specific to both consumer and product. In this model, the vector, $= (\ _1,.., \ ,\varepsilon_0,..,\varepsilon \ )$ describes the consumer's 'type' and we assume that types are distributed in the population according to a distribution, $(\ _1,.., \ ,\varepsilon_0,..,\varepsilon \ )$. Aggregate market shares for product j are then: $(\ ,\xi, \ ;\theta) = \int_{\{ \ | \ (\ ) > \max_{\neq} (\ )\}} (\ )$.

Adding the assumption that ε is independently and identically distributed across products and individuals with a type-I extreme value distribution, and the assumption that the $(\ {}_1,..,\ ,\varepsilon_0,..,\varepsilon\ ) = \ (\ {}_1,..,\ )\ (\varepsilon_0,..,\varepsilon\ )$, it is well known that:

$(\ ,\xi,\ ;\theta) = \int\ (\ ,\ ,\xi,\ ;\theta)\ (\ )$ where $= (\ {}_1,..,\ )$ and where

$(\ ,\ ,\xi,\ ;\theta) = \dfrac{{}^{-(\ ,\xi\ ,\upsilon;\theta)}}{1+\sum_{=1} {}^{-(\ ,\xi\ ,\upsilon;\theta)}}$. Note also that I distinguish between the 'individual' logit share functions $(\ ,\ ,\xi,\ ;\theta)$ and the aggregate share share functions $(\ ,\xi,\ ;\theta)$ by including the vector argument, in the former. By definition, $(\ ,\xi,\ ;\theta) = \ \left[\ (\ ,\ ,\xi,\ ;\theta)\right]$ with the expectation indicating it is taken with respect to the vector argument .

Own- and cross-price elasticities from the RC-MNL model are:

$$\eta\ \equiv \frac{}{(\ ,\xi,\ ;\theta)}\frac{\partial\ (\ ,\xi,\ ;\theta)}{\partial} = \frac{}{(\ ,\xi,\ ;\theta)}\int\frac{\partial\ (\ ,\ ,\xi,\ ;\theta)}{\partial}\ (\nu) = \frac{}{\left[\ (\ ,\ ,\xi,\ ;\theta)\right]}\ \left[\frac{\partial\ (\ ,\ ,\xi,\ ;\theta)}{\partial}\right] \quad (1)$$

where $\dfrac{\partial\ (\ ,\ ,\xi,\ ;\theta)}{\partial} = \dfrac{\partial\ \overline{\ }\ (\ ,\xi\ ,\upsilon,\ ;\theta)}{\partial}\ (\ ,\ ,\xi,\ ;\theta)\big(1(\ =\ ) - \ (\ ,\ ,\xi,\ ;\theta)\big)$.

Throughout, I shall assume that we are considering a class of models, following Berry (1994) and BLP(1995), in assuming that ξ is chosen as the solution to the Jx1 vector of equations $\left[\ (\ ,\ ,\xi,\ ;\theta))\right] =$ for j=1,..,J, wherein predicted market shares are set equal to observed market shares, so we can write:

$\eta\ \equiv \dfrac{}{}\ \left[\dfrac{\partial\ \overline{\ }\ (\ ,\xi\ ,\upsilon;\theta)}{\partial}\ (\ ,\ ,\xi,\ ;\theta)\big(1(\ =\ ) - \ (\ ,\ ,\xi,\ ;\theta)\big)\right]$. Doing so makes clear that, if we are to understand the nature of own-and cross-price elasticities in this model we are really interested in just three objects: (i) , (ii) and (iii) $\left[\dfrac{\partial\ \overline{\ }\ (\ ,\xi\ ,\upsilon;\theta)}{\partial}\ (\ ,\ ,\xi,\ ;\theta)\big(1(\ =\ ) - \ (\ ,\ ,\xi,\ ;\theta)\big)\right]$. Since we would not, in general want the level of price or the level of market share per-se to be driving predicted elasticities, the study of

predicted elasticities in this class of models boils down to the properties of this expectation. Evidently, the elasticities obtained will depend on (i) the assumptions made regarding the distribution of v, (ii) the assumptions made about the form of the derivative function, $\frac{\partial \overline{\ } (\ ,\xi \ ,\upsilon;\theta)}{\partial}$ and (iii) the shape of the MNL market share functions.

In the MNL model with no income effects, the formula collapses to the well known form: $\eta \quad = \frac{}{} \beta \quad \left(1(\ = \ ) - \quad \right) = \beta \quad \left(1(\ = \ ) - \quad \right)$ which numerous authors in this literature have argued is unrealistic because the substitutability between any pair of goods depends only on prices, the market share of good k and not, for example, on the similarity of the goods in terms of their characteristics.

Multiplying the expression for the own- and cross-elasticities by $(-1)^{1(\ = \ )}$ gives the useful expression $|\eta \quad | = (-1)^{1(\ = \ )}\eta \quad \equiv \frac{}{} \left[\frac{\partial \overline{\ } (\ ,\xi \ ,\upsilon;\theta)}{\partial} \quad (\ , \ ,\xi, \ ;\theta) \right]$, where $(\ , \ ,\xi, \ ;\theta) = (-1)^{1(\ = \ )} \quad (\ , \ ,\xi, \ ;\theta)\left(1(\ = \ ) - \quad (\ , \ ,\xi, \ ;\theta)\right) = \begin{cases} -1 \quad (\cdot)(1 - \quad (\cdot)) & = \\ -1 \quad (\cdot) \quad (\cdot) & \neq \end{cases}$, which is always non-positive.[2] Whenever consumers dislike high prices $\frac{\partial \overline{\ } (\ ,\xi \ ,\upsilon;\theta)}{\partial} \leq 0$, we will have $\left[\frac{\partial \overline{\ } (\ ,\xi \ ,\upsilon;\theta)}{\partial} \quad (\ , \ ,\xi, \ ;\theta) \right] \geq 0$. The advantage of working with this formulation is that we can simultaneously provide bounds on both the own- and the cross-price elasticities. To do so, we attempt to derive statements of the form, $\left[\frac{\partial \overline{\ } (\ ,\xi \ ,\upsilon;\theta)}{\partial} \quad (\ , \ ,\xi, \ ;\theta) \right] \leq \eta$ where $\eta \geq 0$. One way to help ensure such

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[2] Note that there is an abuse of notation here since the equality $|\eta \quad | = (-1)^{1(\ = \ )}\eta$ in truth only holds when the model takes on the correct signs for own and cross elasticiies. Unfortunately it is perfectly possible for the predicted own-price elasticities to be positive (ie the wrong sign.) If that is the case, formally throughout the text one should replace all statements using $|\eta \quad |$ with $(-1)^{1(\ = \ )}\eta$.

bounds are satisfied will involve keeping $(\ , \ ,\xi, \ ;\theta)$ small in magnitude, i.e., bigger than some negative number.

In that direction, Lemma 1 provides a number of bounds on the function $(\ , \ ,\xi, \ ;\theta)$ which will be useful in describing the properties of this expectation.

Lemma 1:

(i) $\begin{cases} (\ , \ ,\xi, \ ;\theta) \geq (-1)\min\left\{\frac{1}{4}, \ (\ , \ ,\xi, \ ;\theta), 1- \ (\ , \ ,\xi, \ ;\theta), 1\right\} & = \\ (\ , \ ,\xi, \ ;\theta) \geq -\min\{1, \ (\ , \ ,\xi, \ ;\theta), \ (\ , \ ,\xi, \ ;\theta)\} & \neq \end{cases}$

(ii) $(\ , \ ,\xi, \ ;\theta) \geq (-1)\exp\{\overline{\ } \ (\ , \ , \ ,\xi) + 1(\ \neq \ )\overline{\ } \ (\ , \ , \ ,\xi)\}$ for all j,k.

Proof to Lemma 1:

Part (i) follows because $(\ , \ ,\xi, \ ;\theta) \leq 1$, $1- \ (\ , \ ,\xi, \ ;\theta) \leq 1$ and $(\ , \ ,\xi, \ ;\theta)(1- \ (\ , \ ,\xi, \ ;\theta)) \leq 1/4$. Part (ii) follows from part (i) and the observation that since $1+\sum_{=1} \exp\{\overline{\ } \ (\ ,\xi \ ,\upsilon \ ;\theta)\} \geq 1$, we have that $(\ , \ ,\xi, \ ;\theta) = \frac{\exp\{\overline{\ } \ (\ ,\xi \ ,\upsilon \ ;\theta)\}}{1+\sum_{=1} \exp\{\overline{\ } \ (\ ,\xi \ ,\upsilon \ ;\theta)\}} \leq \exp\{\overline{\ } \ (\ ,\xi \ ,\upsilon \ ;\theta)\}$. That means we can write $(\ , \ ,\xi, \ ;\theta) \geq \begin{cases} (-1)\exp\{\overline{\ } \ (\ , \ , \ ,\xi \ )\} & = \\ (-1)\exp\{\overline{\ } \ (\ , \ , \ ,\xi \ ) + \overline{\ } \ (\ , \ , \ ,\xi \ )\} & \neq \end{cases}$, which part (ii) just expresses more compactly. ■

A theorem that proves useful in providing bounds of the expectation of products of functions is the Rogers-Hölder inequality. Let $(\ ,\Im,\mu)$ be a measure space, $0< \ <\infty$ and let $(\ ,\Im,\mu)$ be the set of all measurable functions f on X such that $\left(\int| \ | \ \mu\right)^{1/} <\infty$ and the values of *f* are real numbers except possibly on a set of measure 0, where f may be undefined or infinite.

Theorem *(Rogers-Hölder inequality; see for example Dudley(1989, page 117)) For* $1< \ <\infty$, $\frac{1}{\ }+\frac{1}{\ }=1$, $\in \ (\ ,\Im,\mu)$ *and* $\in \ (\ ,\Im,\mu)$ *then* $\int| \ | \ \mu \in {}^{1}(\ ,\Im,\mu)$ *and* $\left|\int \ \mu\right| \leq \int| \ | \ \mu \leq \left(\int| \ | \ \mu\right)^{1/} \left(\int| \ | \ \mu\right)^{1/}$.

We can now state our first result, which describes some properties of the expectation, $\left[\ (\ , \ ,\xi, \ ;\theta)\right]$.

Lemma 2: $\lfloor \quad (\ , \ ,\xi, \ ;\theta)\rfloor \geq$

(i) $\begin{cases} (-1)\min\left\{\frac{1}{4}, \quad ,1- \quad ,1, \quad (1- \quad)\right\} & = \\ -\min\{1, \quad , \quad \} & \neq \end{cases}$

(ii) $(-1) \quad \left[\exp\left\{\overline{\ } \ (\ , \quad , \quad ,\xi \ )+1(\ \neq \ )\,\overline{\ } \ (\ , \quad , \quad ,\xi \ )\right\}\right].$

(iii) $(-1)\min_{>1}\left(\quad \right)^{1/} \left(\left(\quad \right)^{1-1/}\right)^{1(\ \neq \ )}$

Proof: See Appendix.

One interesting, general, feature of the result provided in part (ii) is that the cross-elasticities are bounded by functions that involve the summation of utilities. Instead, one might expect that a model of substitution effects would generate cross-elasticities that depend on the difference in utilities, capturing the idea that when products are very different they should have small cross elasticitities and vice versa. The additive form that emerges here comes directly from using the MNL model as the 'base' consumer model in the RC-MNL model.

III. The Additively Separable Case

In the (BLP style) additively separable case, we define the 'mean' conditional indirect utility associated with option j as $\overline{\ } \ (\ ,\xi \ ,\upsilon;\theta) = \ ' \beta + \sum_{=1} \sigma \quad (\ ) \quad + \xi \ .$ Defining $\delta \ = \ ' \beta + \xi \ ,$ and $\alpha' = \left(\sigma_1 \quad _1, \ldots, \sigma_{-1} \quad _{-1}, \sigma \quad \right)$ we can write $\overline{\ } \ (\ ,\xi \ ,\upsilon;\theta) = \delta \ + \alpha' \ (\ )$ where $(\ )$ denotes the Lx1 vector of univariate functions with l^{th} element, $(\ )$.[3] For notational convenience, I will assume w.l.o.g. that the price variable

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[3] For many cases it suffices to consider $(\ ) =$. However, the BLP specification in their VER paper, the specification we examine below, takes the price parameter to be individual specific $\alpha \ = \frac{\alpha}{\ }$ which we can put in this framework by setting $\sigma \quad = \alpha \ ,$ $\quad = \quad$ and $(\quad) = \frac{-1}{\ }$.

takes the Lth position in the (Lx1) vector $'$ and define $\beta \equiv \beta$, $\sigma \equiv \sigma$ and $(\ ) \equiv \ (\ )$

with $\equiv$. While this duplicates a little notation, it aids transparency. For example, it allows us

to write: $\frac{\partial \bar{\ } (\ , \xi \ , \upsilon ; \theta)}{\partial} = \left(\beta \ + \sigma \quad (\ )\right)$. Notice that the additive separability assumptions

on the form $\bar{\ } (\ , \xi \ , \upsilon \ ; \theta)$ translate directly into a simplification of the function

$\frac{\partial \bar{\ } (\ , \xi \ , \upsilon ; \theta)}{\partial}$, a central component of the own- and cross-price elasticities that will emerge

from the model. In particular, notice that it depends only on one random coefficient - σ - rather than the L of them included in the utility specification. Thus, however many random coefficients are in fact included in the model all J^2 own- and cross-price elasticities will in large part be determined by just one. We will see this is a very important restriction implicitly imposed in this style of model.

With this model we can write:

$$| \eta \ | = (-1)^{1(\ = \ )} \eta \ \equiv \frac{}{} \left[\frac{\partial \bar{\ } (\ , \xi \ , \upsilon ; \theta)}{\partial} \quad (\ , \ , \xi , \ ; \theta) \right] = \frac{}{} [\beta \quad [\quad (\ , \ , \xi , \ ; \theta)] + \sigma \quad [\ (\ ) \quad (\ , \ , \xi , \ ; \theta)]] .$$

And so we will be concerned with the two expectations on which these elasticities depend, namely (i) $\lfloor \ (\ , \ , \xi , \ ; \theta) \rfloor$ and (ii) $\lfloor \ (\ ) \quad (\ , \ , \xi , \ ; \theta) \rfloor$. Notice that to provide bounds of the form, $\lfloor (\beta \ + \sigma \quad (\ )) \quad (\ , \ , \xi , \ ; \theta) \rfloor \leq \eta$, where $\eta \geq 0$ and $\beta \ \leq 0$, it will help to develop bounds of the form (i) $\left[\ (\ , \ , \xi , \ ; \theta) \right] \geq \eta^1$ for some $\eta^1 \leq 0$ so that $\beta \quad \left[\ (\ , \ , \xi , \ ; \theta) \right] \leq \beta \ \eta^1$ where $\beta \ \eta^1 \geq 0$ and (ii) $\left[\ (\ ) \quad (\ , \ , \xi , \ ; \theta) \right] \leq \eta^2$, so that we can argue that $\left[(\beta \ + \sigma \quad (\ )) \quad (\ , \ , \xi , \ ; \theta) \right] \leq \beta \ \eta^1 + \eta^2$.

Corollary to Lemma 2. If $\beta\ \ < 0$, $\beta\ \ \ \ \lfloor\ \ (\ ,\ \ ,\xi,\ \ ;\theta)\rfloor \leq$

(i) $\begin{cases} -\beta\ \ \min\left\{\frac{1}{4},\ \ \ \ ,1-\ \ \ \ ,1,\ \ \ (1-\ \ \ \ )\right\} & = \\ -\beta\ \ \min\{1,\ \ \ ,\ \ \ \} & \neq \end{cases}$

(ii) $(-1)\beta\ \ \exp\{\delta\ \ +1(\ \ \neq\ \ )\delta\ \}\ \ [\exp\{(\alpha\ \ +1(\ \ \neq\ \ )\alpha\ \ )'\ \ (\ \ )\}]$.

(iii) $(-\beta\ \ )\min_{>1}(\ \ \ \ )^{1/}\ ((\ \ \ \ )^{1-1/}\ )^{1(\ \ \neq\ \ )}$

Proof: Each component follows immediately from Lemma 2. Part (ii) for example follows by substituting $\overline{\ \ }\ (\ \ ,\xi\ ,\upsilon;\theta) = \delta\ \ +\alpha'$ into the expression,

$(-1)\ \ \ [\exp\{\overline{\ \ }\ (\ ,\ \ \ ,\ \ \ ,\xi\ )+1(\ \ \neq\ \ )\overline{\ \ }\ (\ ,\ \ \ ,\ \ \ ,\xi\ )\}]$.

At this point, I note that, in the commonly estimated special case wherein $(\ \ ) =$

(iii) $\min_{>1,\ >1} \leq \left(\int_{\{\ |\ (\ )\leq 0\}} \left(-\quad (\ )\right) \quad (\nu) \right)^{1/} \left(\left(\quad \right)^{1/} \right)^{1-1/} \left(\left(-1\right)^{1(\ \neq\ )} \left(1(\ =\ )-\quad \right) \right)^{1-1/} \Big)^{1-1/}$

(iv) $\left[\ (\ )\exp\left\{\overline{\ }\ (\ ,\ ,\ \ ,\ ,\xi\ )+1(\ \neq\ )\overline{\ }\ (\ ,\ ,\ \ ,\ ,\xi\ )\right\}\right]$

(v) $\min_{>1}\left(\int_{\{\ |\ (\ )\leq 0\}} (-\quad (\ ))\quad (\nu\ ) \right)^{1/} \exp\{\delta\ +1(\ =\ )\delta\ \}\left(\ \left[\exp\{(\alpha\ +1(\ =\ )\alpha\ )'\ (\ )\}\right]\right)^{1-1/}$

where $\alpha' = (\left(\frac{\ }{-1}\right)\ {}_1\sigma_1,\ldots,\left(\frac{\ }{-1}\right)\ \sigma\ )$.

Proof: See Appendix.

In this section I combine the results above to present the first of our main lemmas, showing that own- and cross-price elasticities in the RC-MNL model are bounded below by a function of prices, market shares, the *two* parameters $(\beta\ ,\sigma\ )$ and a term which depends on the assumptions made about the distribution of consumer heterogeneity through the terms, $\int_{\{\ |\ (\ )\leq 0}(-\quad (\ ))\quad (\ )$ or $\left(\int_{\{\ |\ (\ )\leq 0} (-\quad (\ ))\quad (\ ) \right)^{1/}$ for some m>1.

Corollary to Lemma 3 $\frac{\sigma}{\ }\quad\left[\ (\ )\quad (\ ,\ ,\xi,\ ;\theta)\right]\leq$

(i) $\dfrac{\sigma\quad \left(1(\ =\ )\frac{1}{4}+1(\ \neq\ )\right)}{\ } \int_{\{\ |\ (\ )\leq 0\}} (-\quad (\ ))\quad (\nu\ )$

(ii) $\frac{\sigma}{\ }\min_{>1}\left(\int_{\{\nu|\ (\ )\leq 0\}} \left(-\quad (\ )\right)\quad (\ ) \right)^{1/} \left(\ \left[(-1)\quad (\ ,\ ,\xi,\ ;\theta)\right]\right)^{1-1/}$

(iii) $\sigma\quad \min_{>1,\ >1}\left(\int_{\{\ |\ (\ )\leq 0\}} \left(-\quad (\ )\right)\quad (\nu) \right)^{1/} \left(\ \right)^{(1-1/\ )/\ -1} \left(\left(-1\right)^{1(\ \neq\ )} \left(1(\ =\ )-\quad \right) \right)^{1-1/} \Big)^{1-1/}$

(iv) $\frac{\sigma}{\ }\quad\left[\ (\ )\exp\left\{\overline{\ }\ (\ ,\ ,\ \ ,\ ,\xi\ )+1(\ \neq\ )\overline{\ }\ (\ ,\ ,\ \ ,\ ,\xi\ )\right\}\right]$

(vi) $\frac{\sigma}{\quad} \min_{>1} \left(\int_{\{ \mid (\ ) \leq 0\}} (- \ (\ )) \quad (\nu\ ) \right)^{1/} \exp\{\delta + 1(\ = \ )\delta\}(\ [\exp\{(\alpha + 1(\ = \ )\alpha\ )'\ (\ )\}])^{1-1/}$

where

IV. High Priced Goods and the Selection Effect

In this section, I consider what happens in this class of models to own- and cross- price elasticities in a cross section of goods. In particular, I hope to understand if patterns in the price elasticities we estimate are likely to be driven by the properties of the model rather than the particular dataset under consideration. Understanding such properties of the RC-MNL model may be important if it can help us understand how the model's construction implies a particular cross sectional pattern calculated own- and cross-price elasticities and thereby a particular pattern of predicted mark-ups. We will see that the RC-MNL appears to impose a particular pattern on estimated price elasticities.

To illustrate the kinds of pattern that I have noted in a number of applications of this class of model, consider Figure 1 below which reports estimated own-price elasticities for a RC-MNL model with additively separable structure. Specifically, the figure shows a cross-sectional graph of implied own-price elasticity against own-price for a simulated DGP with 200 products. The DGP is an additively separable conditional indirect utility function with $\sim \quad (0,1)$ for l=1,2,3 and $(\sigma_1, \sigma_2, \sigma \ ) = (1,1,1)$, $(\beta_1, \beta_2, \beta \ ) = (1,1,-1)$ $' = (1 \quad _2 \quad)$ where $_2 \sim \ (0,1)$ $\ln \quad \sim \quad (0,1)$, $\xi \sim \ (0,1)$. Price is assumed log-normal in order to ensure it is always positive.

The own-price elasticities first fall and then increase in the level of price. The bounds type results suggest that own-price elasticities never become too negative – and that is bourne out in the graph. However, there is a second pattern – the fact that predicted own-price elasticities turn upward, getting smaller in magnitude as we move to the high priced goods. In fact, in this example, which uses a normally distributed distaste of price parameter, high enough priced

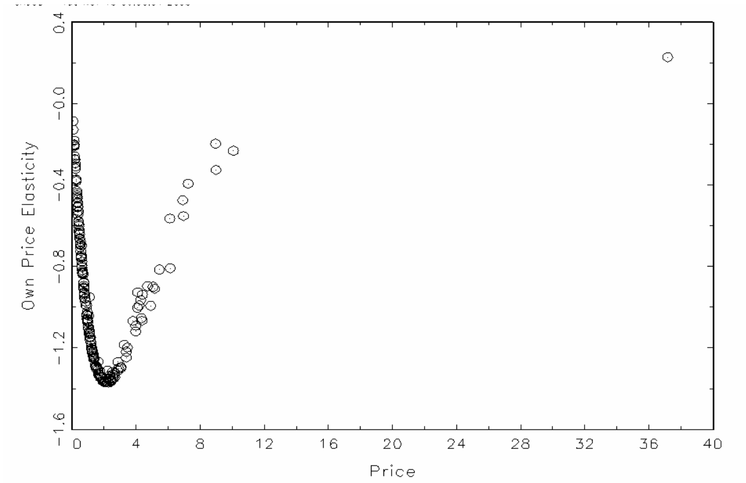

Figure 1: Plotting the cross-sectional variation in own-price elasticities.

Recall that in the additive separability case: $|\eta \quad | = \text{——} \quad \left[(\beta \ + \sigma \quad (\ )) \quad (\ , \ , \xi, \ ; \theta)\right]$ where $(\ , \ , \xi, \ ; \theta) = (-1) \quad (\ , \ , \xi, \ ; \theta)\big(1(\ = \ ) - \ (\ , \ , \xi, \ ; \theta)\big)$. Notice that when making the elasticities large we require that there be at least some consumer types, , who have non-trivial demand for a product, j. For any given *v* we will tend to get low market shares for any fixed consumer for high priced good. Indeed, as the price of any good j increases, $(\ , \ , \xi, \ ; \theta)$ decreases, converging down towards zero, and hence $(\ , \ , \xi, \ ; \theta)$ also converges towards zero. ($\lim_{\to\infty} \ (\ , \ , \xi, \ ; \theta) = 0$ provided that $\lim_{\to\infty} \overline{\ } \ (\ , \ , \xi \ , \ ; \theta) \to -\infty$ which happens in a linear price model for any consumer with $(\beta \ + \sigma \quad (\ ) \leq 0$.)

One way to build intuition is to note that we can think of the expectation $\left[(\beta \ + \sigma \quad (\ )) \quad (\ , \ , \xi, \ ; \theta)\right]$ as reporting a 'weighted average' marginal disutility of \$1, $(\beta \ + \sigma \quad (\ ))$, with weights equal to $(\ , \ , \xi, \ ; \theta)(1 - \ (\ , \ , \xi, \ ; \theta)) \quad (\ )$. These weights are increasing in $(\ , \ , \xi, \ ; \theta)$ for any *v* with $(\ , \ , \xi, \ ; \theta) < \frac{1}{2}$. Intuitively then, as prices increase, the only consumer types who will allocate a significant share to buying good *j* will be those with sufficiently small in magnitude disutility of paying more (or if there are consumer types v who like to pay more), ie., those for whom $\frac{\partial \overline{\ } \ (\ , \xi \ , \upsilon; \theta)}{\partial} = \big(\beta \ + \sigma \quad (\ )\big)$ is either negative but small in magnitude or actively positive (if the distributional assumption on means it is possible to get positive values; eg if $\big(\ \big) =$ and is standard normal.) This 'selection

effect' works to make the implied own-price elasticities as large as possible, i.e., either close to zero or actually positive, when prices of goods become large. For this reason, the additively separable RC-MNL model appears to .systematically generate low or even positive own-price effects for high-price goods. Thus this formulation of the RC-MNL model tends to impose on our predictions the conclusion that margins increase in the quality spectrum.

V. Symmetry Restrictions in the RC-MNL model

Next I show that a wide class of RC-MNL models, including all of the famous examples in the recent industrial organization literature, impose a form of symmetry restriction which should not be expected to hold in most datasets. Specifically, the specifications estimated in Berry, Levinsohn and Pakes (1995), Nevo (2001) and Petrin (2002) each implicitly impose symmetry while Petrin and Goolsbee (2004) explicitly impose it.

Consider the slope of demand function in the RC-MNL model $\frac{\partial}{\partial} = \left[\frac{\partial \bar{\ } (\ ,\xi\ ,\upsilon;\theta)}{\partial} \ (\ ,\ ,\xi,\ ;\theta)\big(1(\ =\ ) - (\ ,\ ,\xi,\ ;\theta)\big)\right]$. Notice that in cases where $\frac{\partial \bar{\ } (\ ,\xi\ ,\upsilon;\theta)}{\partial}$ is independent of k, such as in the additively separable model where $\frac{\partial \bar{\ } (\ ,\xi\ ,\upsilon;\theta)}{\partial} = \big(\beta + \sigma \quad (\ )\big)$, we can change the indexes j and k and the expression for the slope of demand remains identical, thus from such a model we will *always* find that the model predicts $\frac{\partial}{\partial} = \frac{\partial}{\partial}$. Notice that this will be true no matter, for example, how many random coefficients we include in the model – there could be hundreds. Lemma 5 states this result slightly more generally, allowing to enter the form of $\frac{\partial \bar{\ } (\ ,\xi\ ,\upsilon;\theta)}{\partial}$ so that we can cover models where, for example, utility varies with $\ln(\ )$.

Lemma 5 *All members of the RC-MNL model with* $(\quad ,\xi \,, \quad , \quad ,\varepsilon \;) = \overline{} \;(\quad ,\xi \,, \quad , \quad) + \varepsilon$ *and* $\overline{} \;(\quad ,\xi \,, \quad , \quad) = \quad (\quad ,\xi \,, \quad) - \quad (\quad , \quad)$, *ie., that satisfy an additive separability condition in* $(\quad ,\xi \;)$ and $\quad$, *impose the symmetry restriction* $\frac{\partial}{\partial} = \frac{\partial}{\partial}$*.* for some choice of units of demand.[4]

Proof (See Appendix.)

Notice that such a restriction, imposed by the model, would only not lead to distortions if the true data generating process does in fact satisfy the restriction $\left(\frac{\partial}{\partial} \right)^{*} = \left(\frac{\partial}{\partial} \right)^{*}$. Unfortunately, such a restriction embodies a very strong assumption, albeit one that is very familiar from representative agent models as 'Slutsky Symmetry with zero income effects.' Unfortunately, such a restriction should not typically be expected to hold in aggregate demand models. For example, consider a market where at current prices two firms have a 90% and a 10% market share respectively. When firm 1, the larger firm, increases its price a full 90% of the consumers in the market are deciding whether to continue to buy the good or to switch. On the other hand when firm 2 increases its price only 10% of the consumers then decide whether to switch. Clearly, we should be testing the validity of such restrictions in any given context. Doing so here, as the lemma suggests, will involve specifying model forms that relax the additive separability constraint between prices and other goods characteristics so that $\frac{\partial \overline{} \;(\quad ,\xi \;,\upsilon;\theta)}{\partial}$ varies across choices.

[4] Let me also comment on the role of the statement for 'some choice of the units of demand.' To do so, consider the case wherein utility involves a random coefficient on the log of prices so that we get $\frac{\partial \overline{} \;(\quad ,\xi \;,\upsilon;\theta)}{\partial} = \left(\beta \;\; + \sigma \quad (\quad)\right)\frac{1}{}$. In that case, the estimated RC-MNL model usually will not obviously exhibit symmetry properties in the reported estimates. However, non-symmetries will only arise from the choice of the units of quantities. To see why, recall that we can always choose the units of each good and doing so simply changes the units of prices (to see why consider the budget constraint - that $\sum_{=1} \quad = \quad$.) In a continuous choice demand model normalizing all the prices and income to 1 is standard practice for proving theoretical properties of models – for example to establish a demand system is 'flexible' the literature notes that we can, without loss of generality, only prove flexibility at a point in price and income space with all prices and income normalized to one. See for example Diewert (1974). In a discrete choice setting, the logic is admittedly less obvious, but nonetheless carries across. Specifically, instead of associating our discrete choice as $\quad = 1$ and having a price per unit of $\quad$ we may equally well from a theoretical perspective have considered the choice of j to be associated with $\tilde{} = 5$ which will cost $\tilde{} = \quad /5$ per 'unit' of consumption (where each unit is one fifth of the previous unit.) If you will, buying a single car can equally well be conceived of as buying five one-fifths of a car.

VI. An Illustration

Berry, Levinsohn and Pakes (1999) use their random coefficient demand model to evaluate the impact of Voluntary Export Restraints (VERs) on the US Automobile market.[5] The programs and dataset have very kindly been made available by those authors. Their demand specification involves setting the utility of the inside goods equal to:

$$(\quad , \xi \; , \upsilon \; , \quad , \varepsilon \; ; \theta) = \; ' \beta + \sum_{=1} \sigma \qquad + \xi \; + \varepsilon$$

In BLP's specification, the authors denote the individual specific price coefficient α and assume that it is inversely related to income so that $\alpha \; = \frac{\alpha}{\;}$ where α is a constant parameter. This specification of the utility model can be mapped to the results in the lemmas developed above by defining $\beta \;\; = 0$, $\alpha = \sigma$ and $(\quad) = -(\quad)^{-1}$, $\nu \;\; = \;$.

Substituting provides the formula for the elasticity of demand which, since $\beta \;\; = 0$, collapses to:

$$| \qquad | = \frac{\;}{\;} \int (\quad) \quad (\;,\; ,\; , \xi, \; ; \theta) \quad (\nu)$$

the difference in levels of the slopes (the units on the y-axes.) Evidently, in this case, the analytic bounds I have derived are nothing like tight bounds in the sample range of market shares. Strikingly however, the graph does show the same qualitative features – that the bounds increase systematically with market share of product j. The qualitative prediction of the bounds, that small slopes are associated with small market shares, is demonstrated clearly. This suggests that we may be able to derive better bounds on these slopes analytically. Essentially we would like to derive a bound which multiplies the current one by something like .

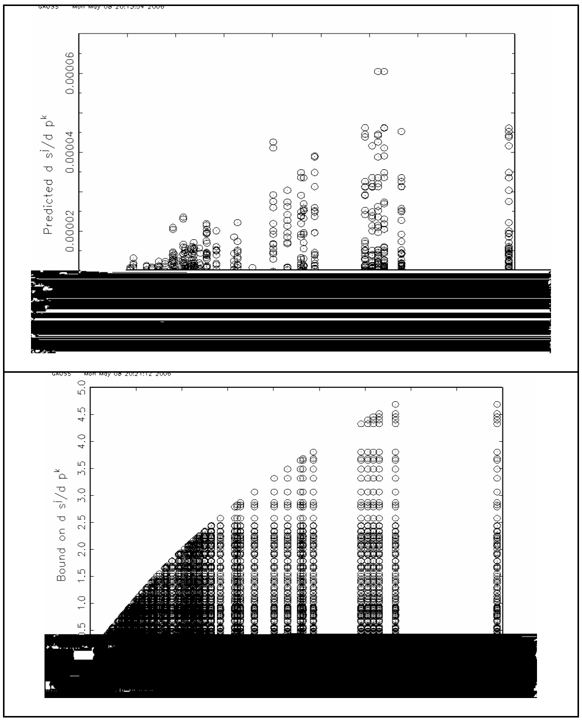

Figure 2: The top panel of the figure plots BLP's VER model's prediction of $\partial \quad / \partial$ against the observed market share, . The bottom panel reports the analogous plot but uses bounds on the slope of demand calculated for the BLP paper.

Next I turn to the 'selection' effect, which suggested that as prices get large, own-price elasticities will tend to become large. Figure 3 plots the calculated own price elasticity of demand against the model's own price, again using the BLP (1999) VER model, dataset and parameter values. Clearly, the graph shows the expected tendency of the BLP model to generate low own-price elasticities for high price goods. These in turn will correspond to predictions of high margins on high end goods.

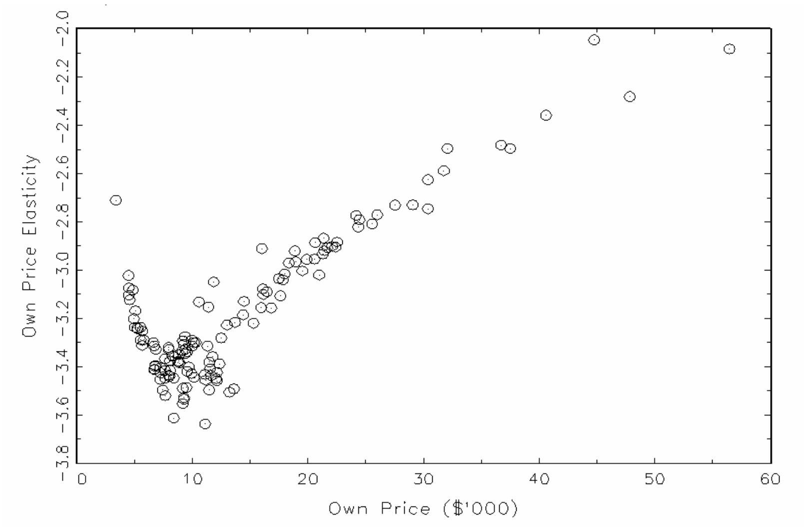

Figure 3 Own price elasticity versus own price for the BLP (1999) VER model, dataset and parameter values.

Finally, I turn to the symmetry restriction. This can be clearly seen with the BLP data/model, simply by printing out the terms $\frac{\partial}{\partial}$ and $\frac{\partial}{\partial}$ for their default set of cars. Here I report a subset of their default set in order to conserve space; the point is evident – the matrix is entirely symmetric!

	Mazda 323	Nissan Sentra	Ford Escort	Chevy Cavalier	Honda Accord	Ford Taurus	Buick Century	Nissan Maxima
Mazda 323	-156.	0.854	5.164	5.088	3.403	2.405	0.784	0.341
Nissan Sentra	0.854	-306	10.175	10.090	7.505	5.346	1.765	0.834
Ford Escort	5.164	10.175	-1758	60.438	44.191	31.198	10.388	4.849
Chevy Cavalier	5.088	10.090	60.438	-1775.	46.089	33.534	10.861	5.073
Honda Accord	3.403	7.505	44.191	46.089	-1574.	45.22	15.618	10.558
Ford Taurus	2.405	5.346	31.198	33.534	45.221	-1168.	11.77	8.045
Buick Century	0.784	1.765	10.388	10.861	15.618	11.770	-404.	3.232
Nissan Maxima	0.341	0.834	4.849	5.073	10.558	8.045	3.232	-249

Table 1 This table reports 10,000 times the slope of demand, $\partial \ / \partial$ **,** obtained from the BLP VER dataset at their parameter values. The table illustrates the symmetry restriction implicitly imposed by the model.

VII. Discussion

Recall that in the oft-criticised MNL model, the own and cross-price elasticities are given by:

$\eta \quad = \beta \quad \left(1(\quad = \quad) - \quad \right)$. These MNL elasticities have a number of features that are correctly considered undesirable in the literature. First while the true own- price elasticities among J products might take on up to J arbitrarily different values, in the MNL model the price elasticities are heavily constrained to vary systematically as a function of prices and market

shares, $\eta \quad = \beta \quad \left(1 - \quad\right)$. Second for a given product k, the cross-price elasticities do not vary across rival products, $\eta \quad = -\beta$ for $\neq$ does not depend on j at all. Thus, while the true cross-price elasticities among J products might take on up to J(J-1) arbitrarily different values in a flexible model (in the sense of Diewert(1974)), in the MNL model these cross-price elasticities can only take on a maximum of J values – one for each value of k - and moreover those values are heavily constrained to vary systematically with prices and market shares across k. Such is the cost of having only one parameter to fit a JxJ matrix of own- and cross-price elasticities of demand.

For the random coefficient model typically taken to data, the additively separable RC-MNL model the situation is somewhat less restrictive. However, the results described above aim to make clear that adding even large numbers of additional random coefficients need not improve the situation sufficiently; a point made most clear via the symmetry assumptions. With a random coefficient on price and additive separability, even if we have large numbers of random coefficients on other characteristics, we effectively have only one additional parameter to fit a JxJ matrix of own- and cross-price elasticities.

Finally, I note that since the bounds depend on market shares, econometric identification of the parameters (β, σ) in this class of models will be aided by exogenous variation in market shares - whatever generates that variation in market shares. As Ackerberg and Rysman (2005) argue, it is not obvious that drivers of market share dynamics such as changes in the number of products on sale should be used by researchers to identify parameters that control price elasticities among a fixed set of products.

VIII. Conclusions

In this paper I have shown that the version of the Random Coefficient Multinomial Logit (RC-MNL) model typically taken to data substantially constrain estimated own- and particularly cross-price elasticities.

Most concretely, I demonstrate that both the symmetry restriction and the ‘selection’ effect which drives own-price elasticities to be smaller in magnitude for high price goods, appear directly in the well known, indeed canonical, empirical example BLP (1999). The results strongly suggest that our models are constraining the patterns in elasticities that we’re ‘predicting’ rather than the datasets which underly the estimated parameters. Such a situation is worrying. The good news is that at least with symmetry, it becomes clear from the analysis that a relatively straightforward solution is available – introducing interactions between prices and characteristics so that the term $\frac{\partial \bar{\ } \ (\ \ , \xi \ , \upsilon ; \theta)}{\partial}$ is a function of the characteristics of good k.

In addition, I derived results which suggest that own- and cross-price elasticities can be bounded by functions of market shares, prices and properties of the assumed distribution of consumer heterogeneity. Unfortunately, the current versions of these bounds do not appear to be very sharp. Nonetheless, the qualitiative predictions of the bounds on own and cross price elasticiites do appear to be bourn out in the BLP dataset. An important question for future research will be to establish tighter bounds for this class of models; For the present it is purely conjecture, but I expect such tighter bounds to have similar qualitative features. In addition, future research should examine the circumstances under which more general specifications of the RC-MNL model do manage to provide a flexible method for estimating price elasticities of demand. The results in Mcfadden and Train(2000) suggest they exist, but at present we do not really know what practical implementations of such models look like. Alternative approaches are those followed recently by Berry and Pakes (1999) and Benkard and Bajari (2001) who each suggest dropping the logit style error completely and Davis (2001) who suggests moving to a richer member of the class of Multivariate Extreme Value models instead.

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Appendix: Proofs

Proof of Lemma 2.
Parts (i) and (ii) follow immediately from Lemma 1 except in establishing that $[\ (\ ,\ ,\xi,\ ;\theta)] \geq (-1)\quad (1-\quad)$. To do so, notice that

$$[\ (\ ,\ ,\xi,\ ;\theta)] = -\int\ (\ ,\ ,\xi,\ ;\theta)(1-\ (\ ,\ ,\xi,\ ;\theta))\quad(\nu)$$

$$= -\ (\ ,\xi,\ ;\theta) + \left(\int\ (\ ,\ ,\xi,\ ;\theta)^2\quad(\nu)\right)$$

$$\geq -\ (\ ,\xi,\ ;\theta) + \left(\int\ (\ ,\ ,\xi,\ ;\theta)\quad(\nu)\right)^2 = -\ (\ ,\xi,\ ;\theta) +\ (\ ,\xi,\ ;\theta)^2$$

where the inequality follows by Jensen's inequality since the quadratic function is convex and for any convex function ϕ, $\left(\int \phi(\ (\ ))\quad(\ )\right) \geq \phi\left(\int\ (\ )\quad(\ )\right)$.

Part (iii) uses the Rogers-Hölder inequality, applied to the product function, $(-1)\quad(\ ,\ ,\xi,\ ;\theta) = (-1)(-1)^{1(\ =\ )}\ (\ ,\ ,\xi,\ ;\theta)\big(1(\ =\ ) -\ (\ ,\ ,\xi,\ ;\theta)\big) = (-1)^{1(\ \neq\ )}\ (\ ,\ ,\xi,\ ;\theta)\big(1(\ =\ ) -\ (\ ,\ ,\xi,\ ;\theta)\big)$ which is everywhere non-negative. For any m>1, we can write $[(-1)\quad(\ ,\ ,\xi,\ ;\theta)] \leq \big(\ [\ (\ ,\ ,\xi,\ ;\theta)\ ]\big)^{1/}\ \big(\ [((-1)^{\neq}(1(\ =\ ) -\ (\ ,\ ,\xi,\ ;\theta)))^{1/(1-1/\ )}]\big)^{(1-1/\ )}$.

Moreover, $\left(\int\ (\ ,\ ,\xi,\ ;\theta)\quad(\nu)\right)^{1/} \leq \left(\int\ (\ ,\ ,\xi,\ ;\theta)^{1}\quad(\nu)\right)^{1/} = \big(\quad\big)^{1/}$ for m>1

and analogously when $\neq$, $\neq$

$$\big(\ [((-1)^{\neq}(1(\ =\ ) -\ (\ ,\ ,\xi,\ ;\theta)))^{1/(1-1/\ )}]\big)^{(1-1/\ )} = \left(\int\ (\ ,\ ,\xi,\ ;\theta)^{1/(1-1/\ )}\quad(\nu)\right)^{1-1/} \leq \big(\quad\big)^{1-1/}$$

. If j=k then $[((-1)^{\neq}(1(\ =\ ) -\ (\ ,\ ,\xi,\ ;\theta)))^{1/(1-1/\ )}] = \int (1-\ (\ ,\ ,\xi,\ ;\theta))^{1/(1-1/\ )}\quad(\nu) \leq 1$ so that

$$\big(\ [((-1)^{\neq}(1(\ =\ ) -\ (\ ,\ ,\xi,\ ;\theta)))^{1/(1-1/\ )}]\big)^{(1-1/\ )} \leq 1$$

Combining each of these results we get: $[(-1)\quad(\ ,\ ,\xi,\ ;\theta)] \leq \big(\quad\big)^{1/}\ \Big[\big(\quad\big)^{1-1/}\Big]^{1(\ \neq\ )}$

Since this is true for any m>1, we can tighten the inequality by minimizing over all possible m>1, viz., $[(-1)\quad(\ ,\ ,\xi,\ ;\theta)] \leq \min_{>1}\big(\quad\big)^{1/}\ \Big[\big(\quad\big)^{1-1/}\Big]^{1(\ \neq\ )}$. Multiplying by (-1) changes the inequality's direction.■

Proof to Lemma 3:

First we use the fact that we can write

$$\int \quad (\ ) \quad (\ ,\ ,\xi,\ ;\theta) \quad (\nu\ ) = \int_{\{\ |\ (\ )\le 0\}} (-\ (\ ))(-\ (\ ,\ ,\xi,\ ;\theta)) \quad (\nu) + \int_{\{\ |\ (\ )\ge 0\}} (\ ) \quad (\ ,\ ,\xi,\ ;\theta) \quad (\nu)$$

$$\le \int_{\{\ |\ (\ )\le 0\}} (-\ (\ ))(-\ (\ ,\ ,\xi,\ ;\theta)) \quad (\nu)$$

For part (i) we can then apply the result from Lemma 1 which established $(-\ (\ ,\ ,\xi,\ ;\theta)) \le \begin{cases} 1/4 & = \\ 1 & \ne \end{cases}$, so that

$$[\ (\ ) \quad (\ ,\ ,\ ,\xi,\ ;\theta)] \le \int_{\{\ |\ (\ )\le 0\}} (-\ (\ ))(-\ (\ ,\ ,\xi,\ ;\theta)) \quad (\nu) \le \begin{cases} \frac{1}{4} \int_{\{\ |\ (\ )\le 0\}} (-\ (\ )) \quad (\nu\ ) & = \\ \int_{\{\ |\ (\ )\le 0\}} (-\ (\ )) \quad (\nu\ ) & \ne \end{cases}$$

and where the integral collapses down to just a univariate integral (since $\int (-\ (\ ,\ ,\xi,\ ;\theta)) \quad (\nu\,|\,\nu\ ) \le 1(\ \ne\ ) + \frac{1}{4} 1(\ =\ )$.)

For part (ii) we apply the Rogers-Hölder inequality twice:

$$[\ (\ ) \quad (\ ,\ ,\xi,\ ;\theta)] \le \int_{\{\ |\ (\ )\le 0\}} (-\ (\ ))(-\ (\ ,\ ,\xi,\ ;\theta)) \quad (\nu)$$

$$\le \left(\int_{\{\ |\ (\ )\le 0\}} (-\ (\ )) \quad (\nu\ ) \right)^{1/} \left(\int_{\{\ |\ (\ )\le 0\}} (-\ (\ ,\ ,\xi,\ ;\theta))^{1/(1-1/\ )} \quad (\nu) \right)^{(1-1/\ )}$$

Next notice that $\left(\int_{\{\ |\ (\ )\le 0\}} (-\ (\ ,\ ,\xi,\ ;\theta))^{1/(1-1/\ )} \quad (\nu) \right)^{(1-1/\ )} \le \left(\ [(-\ (\ ,\ ,\xi,\ ;\theta))] \right)^{(1-1/\ )}$ since $-\ (\ ,\ ,\xi,\ ;\theta) \le 1$ and $1/(1-1/\ ) = \frac{}{-1} \ge 1$.

For part (iii) we apply Lemma 2 with $\beta\ = -1$ so that

$$[\ (\ ) \quad (\ ,\ ,\xi,\ ;\theta)]$$

$$\le \left(\int_{\{\ |\ (\ )\le 0\}} (-\ (\ )) \quad (\nu) \right)^{1/} \left(\left(\int_{\{\ |\ (\ )\le 0\}} (\ (\ ))^{/(1-1/\ )} \quad (\nu) \right)^{1/} \left(\int_{\{\ |\ (\ )\le 0\}} \left((-1)^{1(\ \ne\ )} (1(\ =\ ) -\ (\ )) \right)^{1/((1-1/\ )(1-1/\ ))} \quad (\nu) \right)^{1-1/} \right)^{1-1/}$$

And then noting $/(1-1/\ ) > 1$ (since >1 and >1 implies that $1/(1-1/\ ) > 1$), since $(\ ,\ ,\xi,\ ;\theta) \le 1$ we have

$$\left(\int_{\{\ |\ (\ )\le 0\}} (\ (\ ))^{/(1-1/\ )} \quad (\nu) \right)^{1/} \le \left(\int_{\{\ |\ (\ )\le 0\}} (\ (\ ))^{1} \quad (\nu) \right)^{1/} \le (\ )^{1/}$$

. And

$$\left(\int_{\{ | \ (\ ) \le 0\}} \left((-1)^{1(\ \ne \ )}(1(\ = \ ) - \ (\ ))\right)^{1/((1-1/ \ )(1-1/ \ ))} \ (\nu) \right)^{1-1/} \le \left(\int_{\{ | \ (\ ) \le 0\}} (-1)^{1(\ \ne \ )}\left(1(\ = \ ) - \ (\ )\right) \ (\nu) \right)^{1-1/}$$

$$\le \left(-1\right)^{1(\ \ne \ )}\left(1(\ = \ ) - \ \right)^{1-1/}$$

So we can write

$$\left[\ (\ ) \ (\ , \ , \xi, \ ; \theta) \right] \le \left(\int_{\{ | \ (\ ) \le 0\}} \left(- \ (\ )\right) \ (\nu) \right)^{1/} \left(\left(\ \right)^{1/} \right)^{1-1/} \left(\left(-1\right)^{1(\ \ne \ )}\left(1(\ = \ ) - \ \right)^{1-1/} \right)^{1-1/}$$

which holds for all m>1 and q>1.

Part (iv) uses the Rogers-Hölder inequality and the result from Lemma 1 that $(-1) \ (\ , \ , \xi, \ ; \theta) \le \exp\left\{ \overline{\ } \ (\ , \ , \ , \xi) + 1(\ \ne \ ) \overline{\ } \ (\ , \ , \ , \xi) \right\}$, so that we can write

$$\left[\ (\ ) \ (\ , \ , \xi, \ ; \theta) \right]$$

$$\le \left(\int_{\{ | \ (\ ) \le 0\}} (- \ (\ )) \ (\nu \ ) \right)^{1/} \left(\int_{\{ | \ (\ ) \le 0\}} \left(\exp\left\{ \overline{\ } \ (\ , \ , \ , \xi) + 1(\ \ne \ ) \overline{\ } \ (\ , \ , \ , \xi) \right\}\right)^{1/(1-1/ \ )} \ (\nu) \right)^{(1-1/ \ )}$$

$$\le \left(\int_{\{ | \ (\ ) \le 0\}} (- \ (\ )) \ (\nu \ ) \right)^{1/} \left(\int_{\{ | \ (\ ) \le 0\}} \left(\exp\left\{ (1/(1-1/ \ ))\left(\overline{\ } \ (\ , \ , \ , \xi) + 1(\ \ne \ ) \overline{\ } \ (\ , \ , \ , \xi) \right)\right\}\right) \ (\nu) \right)^{(1-1/ \ )}$$

Now under additive separability, $\overline{\ } \ (\ , \xi \ , \upsilon; \theta) = \delta \ + \alpha^{'} \ (\ )$, so that

$$\int_{\{ | \ (\ ) \le 0\}} \left(\exp\left\{ (1/(1-1/ \ ))\left(\overline{\ } \ (\ , \ , \ , \xi) + 1(\ \ne \ ) \overline{\ } \ (\ , \ , \ , \xi) \right)\right\}\right) \ (\nu)$$

$$\le \exp\left\{ \left(\frac{\ }{-1} \right)\left(\delta \ + 1(\ = \ ) \delta \ \right) \right\} \ \left[\exp\left\{ (\alpha \ + 1(\ = \ ) \alpha \ )' \ (\ ) \right\} \right]$$

where $\alpha^{'} = (\left(\frac{\ }{-1} \right) \ {}_1\sigma_1, \ldots, \left(\frac{\ }{-1} \right) \ \sigma \ )$.

$$\left[\ (\ ) \ (\ , \ , \xi, \ ; \theta) \right] \le \min_{>1} \left(\int_{\{ | \ (\ ) \le 0\}} (- \ (\ )) \ (\nu \ ) \right)^{1/} \left(\exp\left\{ \left(\frac{\ }{-1} \right)\left(\delta \ + 1(\ = \ ) \delta \ \right) \right\} \ \left[\exp\left\{ (\alpha \ + 1(\ = \ ) \alpha \ )' \ (\ ) \right\} \right] \right)^{1-1/}$$

, which simplifies slightly to

$$\left[\ (\ ) \ (\ , \ , \xi, \ ; \theta) \right] \le \min_{>1} \left(\int_{\{ | \ (\ ) \le 0\}} (- \ (\ )) \ (\nu \ ) \right)^{1/} \ \exp\left\{ \delta \ + 1(\ = \ ) \delta \ \right\} \left(\ \left[\exp\left\{ (\alpha \ + 1(\ = \ ) \alpha \ )' \ (\ ) \right\} \right] \right)^{1-1/}$$

■

Proof to Lemma 5.

To see this note that for some general distribution of consumer types, $(\;)$ for the additively separable RC-MNL model we can write for any $\neq$:

$$\frac{\partial}{\partial} = -\iint \frac{\partial \overline{\;} \;(\;,\xi\;,\;\;,\;)}{\partial} \;(\;,\xi,\;\;,\;)\;\;(\;,\xi,\;\;,\;)\;\;(\;)$$

$$= \iint \frac{\partial\;(\;\;,\;)}{\partial} \;(\;,\xi,\;\;,\;)\;\;(\;,\xi,\;\;,\;)\;\;(\;)$$

So that $\frac{\partial}{\partial} = \frac{\partial}{\partial}$ in general, only if $\frac{\partial \overline{\;} \;(\;,\xi\;,\;\;,\;)}{\partial}$ is independent of the option k. Notice that the only way in which $\frac{\partial\;(\;\;,\;)}{\partial}$ can possibly depend on k, if at all, is through . Since we can choose the units in which each good k is measured we can, without loss of generality, always normalize so that $=1$ for all k (see the discussion in footnote 4). Thus, for all k>0 (and using the appropriate normalizations) we can write $\frac{\partial\;(\;\;,\;)}{\partial} = \frac{\partial\;(\;,\;)}{\partial}\Big|_{\;=1}$ which is independent of k.

Lemma (MGF bounds.)

(i) $\left[\left(\;(\;,\;,\xi,\;;\theta)\right)^{1-1/}\;\right] \leq \exp\left\{\left(1-\frac{1}{\;}\right)\delta\;\right\}\;\;\left[\exp\left\{\;'\;\right\}\right]$ where $' = \left(\left(1-\frac{1}{\;}\right)\;{}_1\sigma_1,\ldots,\left(1-\frac{1}{\;}\right)\;\;\sigma\;\right)$ and where $\left[\exp\left\{\;'\;\right\}\right]$ is the moment generating function of the multivariate distribution, .

(ii) If is a multivariate normal with mean $[\upsilon\;]$ and variance $[\upsilon\;]$, then: $\left[\exp\left\{\;'\;\right\}\right] = \exp\left\{\;'\;\;[\upsilon\;] + \frac{1}{2}\;'\;\;\;[\upsilon\;]\;\;\right\}$.

(iii) If is a multivariate normal with mean $[\upsilon\;]$ and variance $[\upsilon\;]$, $\left[\left(\;(\;,\;,\xi,\;;\theta)\;\;(\;,\;,\xi,\;;\theta)\right)^{1-1/}\;\right] \leq \;\;\left[\left(\exp\left\{(\;\;+\;\;)'\;\right\}\right)\right]$