Mixing Media with Two-Part Tariffs*

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Abstract

We consider a media market where consumers mix content offered by different firms and firms charge two-part tariffs. As compared to pure linear pricing (pay-per-view), firms make higher profits, while consumers are worse off and the allocation is not first-best. We also consider flat subscription fees and show that they make mixing unattractive. Both two-part tariffs and pay-per-view Pareto-dominate flat fees.

Keywords: Two-part tariffs, pay-per-view, flat fees, combinable products

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1 Introduction

The Hotelling location model is widely used in Industrial Organization to describe imperfect competition among suppliers that are located in some product or characteristic space. A feature of the standard model is that each consumer buys exclusively from one single firm. However, in many market situations, consumers are free to mix the characteristics embodied in different goods. Anderson and Neven (1989) allow consumers to combine various products, instead of buying exclusively from a single supplier. They study a location-price game and find the remarkable result that first-best allocations are reached in equilibrium. This is in contrast with the Hotelling location model that typically delivers inefficient (excessive) differentiation (see d'Aspremont *et al.*, 1979).

In this paper we consider again the problem of combinable goods, but we allow for more general pricing strategies than Anderson and Neven (1989), who study only the case of linear pricing. Our study is motivated by a recent and growing literature on media markets.[1] This literature, among other topics, models the behavior of viewers/listeners of media programmes as a main ingredient of the analysis. In most models it is assumed that audiences make a discrete choice of which broadcaster to watch; typically, the standard Hotelling model with exclusivity is adopted. This is appropriate if one thinks that, at any given point in time, a viewer must choose one, and only one, channel. For instance, if two movies start at about the same time, a viewer will watch only one movie, not a mix of the two. On the other hand, there are some media markets where mixing is more appropriate. For instance, a listener may want to spend some time listening to classical music and some time listening to jazz. Similarly, a viewer may want to mix between a sport channel and a movie channel over a particular time period, say a month. In this case, an alternative approach would be to follow the mixing model of Anderson and Neven (1989). This is done for instance in Gal-Or and Dukes (2003) and Gabszewicz *et al.* (2004), where viewers/listeners are able to diversify their viewing/listening experience, obtaining a mix of programs to match their preferences. The resulting aggregate demand in the mixing model is the same one as the demand derived in the Hotelling model if audiences, under the mixing model, pay only for the proportion of time they spend with a broadcaster (pay-per-view).

Linear pricing (pay-per-view) seems to be rather restrictive and is not widely adopted in practice. Instead, media stations typically charge subscription fees as well as extra charges if a viewer wants to watch particular

[1] See Anderson and Gabszewicz (2005) for a survey.

movies or programmes. Still, viewers are allowed to mix among different broadcasters if they are willing to do so, i.e., exclusivity of viewers cannot be imposed by any broadcaster. We study a model where viewers can mix in principle, but firms charge two-part tariffs (e.g., both a subscription fee and a pay-per-view fee in the media example). We show how this leads to significant departures from the linear pricing setting of Anderson and Neven (1989). The introduction of subscription fees induces some viewers to buy from one firm only, implying that sub-optimal mixing occurs in equilibrium. We also show, contrary to the mixing model with linear pricing which always produces equilibrium profits identical to the Hotelling profits, that when firms compete in two-part tariffs they make more profits in equilibrium. This increase in profits does not come from exclusive customers but from mixing ones that must now pay the subscription fixed fee twice.

We also study a version of the model with flat fees only. Again, this is motivated by the media industry where flat subscription pricing is quite common. This could be due, for instance, to costly monitoring technologies of usage, which makes it too expensive to charge per viewing time. A true pay-per-view system, such as video-on-demand, necessitates sophisticated (two-way) broadcasting technologies, where a viewer can download a particular movie when she wants it. This may not be possible, and simpler (one-way) standard broadcasting could be the only option available. This makes it much more difficult (or even irrelevant) to charge for usage, while subscription fees are easier to administer. We show that competition in flat fees produces a very striking inefficiency result where no single customer mixes but everybody buys from a single supplier instead. Therefore making two-part tariffs possible leads to a Pareto improvement where consumers who start mixing increase their utility, while at the same time firms increase their profits.

Implicit in our results is the firms' non-cooperative choice between linear tariffs, flat rates and two-part tariffs. A linear tariff (respectively, a flat tariff) can be interpreted as a two-part tariff with zero subscription price (respectively, with a zero variable price). The choice of a two-part tariff with non-zero subscription and variable prices means that the two-part tariff *individually* dominates both of these.

The rest of the paper is organized as follows: Section 2 sets out the model and derives the demand system. Section 3 derives the equilibrium two-part tariffs and location choice, while Section 4 considers flat rates. Finally, Section 5 concludes.

2 A mixing model with nonlinear tariffs

2.1 The model

There are two firms $i = 1, 2$, located along the Hotelling unit line at locations $0 \leq a_1 \leq a_2 \leq 1$. Firm i charges a two-part tariff $T_i(q_i) = F_i + p_i q_i$. In the media example, F_i is the fixed subscription fee, while p_i is price per-view of a quantity q_i of programs. This formulation allows to consider the extreme cases of pure pay-per-view ($F_i = 0$; this is the case analyzed by Anderson and Neven, 1989), and pure subscription fee for unlimited viewing ($p_i = 0$).

Each firm incurs a constant marginal cost $c \geq 0$ per unit supplied. Firms play a two-stage game. First, they choose their locations, then they compete in two-part tariffs. We will consider subgame-perfect equilibria of this game.

Consumers buy a unit total quantity $q = 1$. This can be thought as the total viewing time devoted to media entertainment, which is normalized to 1. Consumers are uniformly located between 0 and 1. They can decide whether to buy only from firm 1, only from firm 2, or to combine products to obtain a mix of their characteristics. To continue with the media example, if viewers subscribe to both media platforms, they can decide the share of their viewing time on each channel. A consumer located at x who combines the two products with a share $q_1 = \lambda \leq 1$ of product 1 and a share $q_2 = (1-\lambda)$ of product 2 incurs a quadratic transport cost $t(\lambda a_1 + (1-\lambda) a_2 - x)^2$, where t is the unit transportation cost, with $q_1 + q_2 = 1$. Consumers also derive a fixed utility v from buying from any firm, which is assumed to be high enough such that the market is always "covered" in equilibrium. This fixed utility represents the utility of simply being able to consume, while the gain in utility due to more variety under mixing is represented by the transport cost specification.

To summarize, the utility of a customer who buys only good 1 is:

$$U_1(x) = v - F_1 - p_1 - t(a_1 - x)^2 \tag{1}$$

Similarly, the utility if only good 2 is bought is:

$$U_2(x) = v - F_2 - p_2 - t(a_2 - x)^2 \tag{2}$$

Finally, the utility if a customer buys some mixture λ of both goods is:

$$V(x, \lambda) = v - F_1 - F_2 - \lambda p_1 - (1-\lambda) p_2 - t(\lambda a_1 + (1-\lambda) a_2 - x)^2 \tag{3}$$

In the latter case, the optimal mixture is endogenous and is found minimizing total cost (pay-per-view plus transportation cost):

$$\min_{\lambda} \lambda p_1 + (1-\lambda) p_2 + t(\lambda a_1 + (1-\lambda) a_2 - x)^2 \tag{4}$$

The first-order condition can be rewritten as:

$$\lambda(x) = \frac{1}{2}\frac{p_2 - p_1 + 2t\left(a_2 - a_1\right)\left(a_2 - x\right)}{t\left(a_2 - a_1\right)^2} = \frac{a_2 - x - R}{a_2 - a_1} \tag{5}$$

with $R = \frac{p_1 - p_2}{2t(a_2 - a_1)}$. Notice that, for mixing to occur, we must have $0 \leq \lambda \leq 1$, which can be restated as:

$$a_1 - R \leq x \leq a_2 - R \tag{6}$$

The previous inequalities represent a necessary condition for an interior choice of λ. Actual mixing will be smaller in the presence of fixed payments, as some customers may decide to buy only from one firm and avoid paying two subscription fees. To determine this, we first need to consider the net utility, conditional on buying both goods and mixing them:

$$\begin{aligned} U_{12}(x) &= V\left(x, \lambda(x)\right) \\ &= v - F_1 - F_2 - \lambda(x)\, p_1 - \left(1 - \lambda(x)\right) p_2 - tR^2 \\ &= v - F_1 - F_2 - p_2 - 2t\left(a_2 - x\right) R + tR^2 \\ &= v - F_1 - F_2 - p_1 + 2t\left(x - a_1\right) R + tR^2 \end{aligned}$$

A consumer located at x chooses to buy only from firm 1 either if $x < a_1 - R$ (thus $\lambda = 1$), or if $x \geq a_1 - R$ and $U_1(x) > U_{12}(x)$, which leads to

$$F_2 > t\left(x - \left(a_1 - R\right)\right)^2$$

or

$$x < x_l = \left(a_1 - R\right) + \sqrt{F_2/t} \tag{7}$$

In a similar fashion, the consumer chooses to buy only from firm 2 if

$$x > x_h = \left(a_2 - R\right) - \sqrt{F_1/t} \tag{8}$$

It is clear that the last two inequalities (7) and (8) for mixing are more stringent than (6) required for $0 \leq \lambda \leq 1$. They boil down to the same inequalities only when there are no fixed subscription fees imposed by either firm. Only consumers in (x_l, x_h) mix. That it to say, only customers located in between the two firms (e.g., near the center of the line if firms choose extreme locations) eventually combine the two products. These are the customers that benefit most from mixing, as they can "create" their ideal product by combining the two products that are more or less equally distanced. On the other hand, a customer that is already located close to one of the two firms

is already enjoying a product that is very close to her ideal, and thus has less benefits from mixing. Consumers in $[0, x_l]$ buy exclusively from firm 1, and consumers in $[x_h, 1]$ buy exclusively from firm 2.

Note that the intervals of exclusive customers shrink with the level of the *rival's* subscription fee. The own subscription fee does not matter since it is paid either way, while the benefits from mixing are reduced if a customer has to pay an additional fixed fee. In the presence of positive fixed fees, $\lambda(x_h) > 0$ (similarly, $\lambda(x_l) < 1$). This means that the very first viewer that already subscribes to firm 2 and decides to buy also from firm 1 (this consumer is located at x_h) buys a strictly positive amount of product from firm 1. All the viewers to the right of x_h suddenly drop firm 1 altogether to avoid paying F_1. Also note that $x_h - x_l = a_2 - a_1 - \sqrt{F_1/t} - \sqrt{F_2/t} \leq a_2 - a_1 \leq 1$, that is to say, in the presence of fixed subscription fee, there will always be customers who prefer to buy from a single firm even if these have chosen extreme differentiation, $a_1 = 0$, $a_2 = 1$.

2.2 Definition of demands

Let D_1 denote the quantity demand of firm 1. Since we assumed the market is always covered, demand of firm 2 is $D_2 = 1 - D_1$. If customers were buying exclusively from one firm only, demand for each firm would be equivalent to the number of its buyers. However, because of the possibility of mixing, it is indeed possible that a buyer is a customer of both firms. Thus we need to determine subscription demands B_i as well, with $B_1 + B_2 > 1$ whenever there is mixing.

Several cases should be considered, according to the values taken by x_l and x_h. These are relegated to the Appendix. We report here only the case with $0 < x_l < x_h < 1$, which is a natural candidate to emerge in equilibrium. In this case customers located close to the extremities buy exclusively from one firm, while the customers close to the middle buy both products and obtain a mix.

$$\begin{aligned} D_1(p_1, F_1, p_2, F_2) &= x_l + \int_{x_l}^{x_h} \lambda(x)\, dx \\ &= \frac{a_1 + a_2}{2} + \frac{(F_2 + p_2) - (F_1 + p_1)}{2t(a_2 - a_1)} \qquad (9) \\ B_1(p_1, F_1, p_2, F_2) &= x_h, \quad B_2(p_1, F_1, p_2, F_2) = 1 - x_l \end{aligned}$$

Aggregate quantity demand D_i for each firm (9) coincides with aggregate demand in the classic Hotelling model, where buyers cannot mix but buy products exclusively from a single firm (given unit demand, what matters in

this case is only the total price paid to the producer, $F_i + p_i$). This important remark explains the results of Anderson and Neven (1989). In their model, buyers pay only a fee linear in the amount they purchase, thus $p_i > 0$ and $F_i = 0$ in our notation. As revenues come only from D_i and not from B_i, and D_i is the same as in the Hotelling model, indeed they find that prices and locations in equilibrium are the same ones as in the Hotelling model. However, once fixed subscription fees are introduced and can be charged to buyers, and $B_i \neq D_i$, the formal equivalence between these models breaks down.

3 Price equilibrium with two-part tariffs

3.1 Tariff Choice

Given locations (a_1, a_2), profits for firm 1 are:

$$\begin{aligned}\pi_1(p_1, F_1, p_2, F_2) &= (p_1 - c) D_1(p_1, F_1, p_2, F_2) + F_1 B_1(p_1, F_1, p_2, F_2) \\ &= (p_1 - c)\left(\frac{a_1 + a_2}{2} + \frac{F_2 + p_2 - F_1 - p_1}{2t(a_2 - a_1)}\right) \\ &\quad + F_1\left(a_2 - \frac{p_1 - p_2}{2t(a_2 - a_1)} - \sqrt{\frac{F_1}{t}}\right)\end{aligned} \tag{10}$$

Proposition 1 *In the second stage of the game, there is a unique Nash equilibrium in two-part tariffs for each pair of locations $0 \leq a_1 \leq a_2 \leq 1$. Firms choose the following subscription fees and variable prices:*

$$\begin{aligned}p_1 = c + \frac{1}{6}t(4 + 23a_1 - 19a_2)(a_2 - a_1) + \frac{3\sqrt{5}}{2}t(a_2 - a_1)^2 \\ F_1 = \frac{7 - 3\sqrt{5}}{2}t(a_2 - a_1)^2 \\ p_2 = c + \frac{1}{6}t(8 + 19a_1 - 23a_2)(a_2 - a_1) + \frac{3\sqrt{5}}{2}t(a_2 - a_1)^2 \\ F_2 = \frac{7 - 3\sqrt{5}}{2}t(a_2 - a_1)^2\end{aligned} \tag{11}$$

Proof. The first-order condition with respect to price p_1 is:

$$\frac{\partial \pi_1}{\partial p_1} = \frac{a_1 + a_2}{2} + \frac{F_2 + p_2 - 2(F_1 + p_1) + c}{2t(a_2 - a_1)} = 0,$$

which can be re-written as:

$$F_1 + p_1 = \frac{1}{2}\left(t\left(a_2^2 - a_1^2\right) + c + (F_2 + p_2)\right) \tag{12}$$

The derivative with respect to the fixed fee F_1 is:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial F_1} &= a_2 + \frac{p_2 - 2p_1 + c}{2t\left(a_2 - a_1\right)} - \frac{3}{2}\sqrt{F_1/t} = 0 \\
F_1 &= t\left(\frac{2}{3}a_2 + \frac{p_2 - 2p_1 + c}{3t\left(a_2 - a_1\right)}\right)^2
\end{aligned}$$

The Hessian is negative semidefinite iff $F_1 \leq \frac{9t(a_2-a_1)^2}{16}$. Similar expressions can be obtained for firm 2, leading to a system of equations in p_1, F_1, p_2, F_2, which can be solved to obtain the candidate solution. It is easy to check that the conditions $0 < x_l < x_h < 1$ are satisfied by the candidate solution, which also satisfies the condition for the Hessian. In the Appendix we show that possible deviations to any other branch of the demand system cannot upset this equilibrium candidate, therefore it is a Nash equilibrium. We also show that there cannot be any equilibrium involving other branches, therefore this Nash equilibrium is unique. QED

The equilibrium subscription fee is always positive. This means that it is not profit-maximizing for firms to charge on based on usage only, nor that in equilibrium they would subsidize subscription. That firms use both components of their two-part tariffs in equilibrium should come as no surprise. Imagine F_i was zero instead. By decreasing p_i and increasing F_i by the same amount, a firm does not gain anything from exclusive customers, but it strictly gains from infra-marginal mixing customers. This is because the increase in the fixed fee is paid in full by mixing customers, while the decrease in the linear price is "diluted" by the mixing share λ.

It can also be shown that variable prices do not fall below marginal cost, $p_i \geq c$ for $i = 1, 2$. Therefore flat fees would not be chosen, either. We have $p_i = c$ if and only if both firms are located at the same spot, $a_1 = a_2$. In this case competition brings variable prices down to marginal costs, and subscription fees fall to zero.

Notice that in the general case, at equilibrium, the sum of the prices paid to firm 1 is

$$p_{Tot} = p_1 + F_1 = c + \frac{1}{3}t\left(a_2 - a_1\right)\left(2 + a_1 + a_2\right) \tag{13}$$

which is the *same* expression that emerges in a standard Hotelling model under exclusivity. Hence customers who still prefer to buy exclusively from

a firm, despite having the opportunity to mix, pay the same price, and firms make the same profits from these customers as in the Hotelling model. However, there is a group of customers who do mix. These pay the fixed fee F_i to each firm while the price p_i is only paid for the amount bought (the viewing time in the media example), which is necessarily less than 1 for mixing customers. Thus it must be the case that firms earn less per mixing customer with two-part pricing than in a Hotelling model with exclusivity. Still, this does not imply that profitability is decreased with two-part pricing since there is also a demand effect: Firms sell to more customers than under exclusivity. The overall effect is analyzed in the next proposition.

Proposition 2 *For any given locations $0 \leq a_1 < a_2 \leq 1$, Nash equilibrium profits are higher with two-part tariffs than in the classic Hotelling duopoly.*

Proof. In the standard Hotelling duopoly, profits are:

$$\pi_1(Hotelling) = \frac{t}{18}\left(a_2 - a_1\right)\left(2 + a_1 + a_2\right)^2$$

After substituting the equilibrium prices, profits with two-part tariffs are:

$$\begin{aligned} \pi_1(2\ part) &= \left(p_1 - c\right) D_1 + F_1 B_1 \\ &= \frac{t}{18}\left(a_2 - a_1\right)\left(2 + a_1 + a_2\right)^2 + \frac{13\sqrt{5} - 29}{4} t \left(a_2 - a_1\right)^3 \end{aligned}$$

Taking the difference, one obtains:

$$\begin{aligned} \pi_1(2\ part) - \pi_1(Hotelling) &= \frac{13\sqrt{5} - 29}{4} t \left(a_2 - a_1\right)^3 \\ &\simeq 0.017 t \left(a_2 - a_1\right)^3 > 0. \text{ QED} \end{aligned}$$

While we have seen that it is intuitive that in equilibrium each firm uses its two-part tariff, it is perhaps more intriguing that the net effect results in higher profits than in the Hotelling model (or, equivalently, the model with mixing and linear tariffs only). Clearly, nothing changes with respect to customers that decide to buy exclusively from one firm. The increase in profits then must come from mixing customers. These are customers in the middle, who benefit most from mixing. Their mixing choices are dictated only by the linear prices, not by the fixed fees. Without fixed fees, they would have paid p_{Tot} in total, λp_{Tot} accruing to firm 1 and the remaining to

firm 2. With fixed fees, a customer pays $\lambda p_1 + F_1$ to firm 1 and $(1-\lambda)\, p_2 + F_2$ to firm 2. The total payment exceeds p_{Tot} because this customer pays two subscription fees. When comparing our results to the mixing model with linear tariffs only, the presence of fixed fees increases the mass of customers that buy exclusively, but at the same time mixing consumers pay a higher total bill. The latter effect always prevails.

3.2 Location choice

In the first stage, firms choose their locations. Given the pricing equilibrium in the second stage, the market shares that delimit the mixing area are:

$$x_l = \frac{2}{3}a_1 + \frac{1}{3}(1-a_2) + \frac{3-\sqrt{5}}{2}(a_2 - a_1) \tag{14}$$

$$x_h = \frac{2}{3}a_2 + \frac{1}{3}(1-a_1) - \frac{3-\sqrt{5}}{2}(a_2 - a_1) \tag{15}$$

Proposition 3 *Firms locate at the end point of the Hotelling line in the first stage of the game.*

Proof. Profits for firm 1 are made of two additive terms as expressed in the proof of Proposition 2:

$$\pi_1 = \frac{t}{18}(a_2 - a_1)(2 + a_1 + a_2)^2 + \frac{13\sqrt{5}-29}{4} t\,(a_2 - a_1)^3$$

The first term is the same as in the standard Hotelling model with quadratic transportation costs. As shown by d'Aspremont et al. (1979), firm 1 would choose to locate at the extreme $a_1 = 0$ in this case. The second term is strictly decreasing in a_1. Thus the maximum of the overall profit π_1 is at $a_1 = 0$. Similarly, the maximum for firm 2 is at $a_2 = 1$. QED

Given the extreme location equilibrium, $a_1 = 0$, $a_2 = 1$, prices in equilibrium simplify to:

$$p_1 = p_2 = c + \frac{3\sqrt{5}-5}{2} t \simeq c + 0.854t \tag{16}$$

$$F_1 = F_2 = \frac{7-3\sqrt{5}}{2} t \simeq 0.146t \tag{17}$$

The mixing area is delimited by

$$x_l = \frac{3-\sqrt{5}}{2} \simeq 0.382,\ x_h = \frac{\sqrt{5}-1}{2} \simeq 0.618, \tag{18}$$

that is, a fraction $x_h - x_l = \sqrt{5} - 2 \simeq 0.236$ of the customers mix. Profits are:

$$\pi_1 = \pi_2 = \frac{13\sqrt{5} - 27}{4} t \simeq 0.517t. \tag{19}$$

Turning to consumers, a mixing consumer in the area $x_l \leq x \leq x_h$ has a net utility of

$$U_{12}(x) = \bar{U}_{12} = v - c - \frac{9 - 3\sqrt{5}}{2} t \simeq v - c - 1.146t, \tag{20}$$

while a non-mixing customer in $0 \leq x < x_l$ who buys exclusively from firm 1 derives a net utility:

$$U_1(x) = v - c - \left(1 + x^2\right) t. \tag{21}$$

These are equal at $x = \frac{3-\sqrt{5}}{2} = x_l$, as it should be. Total consumer surplus can be found as

$$\begin{aligned} CS &= \int_0^{x_l} U_1(x)\, dx + (x_h - x_l)\, \bar{U}_{12} + \int_{x_h}^1 U_2(x)\, dx \\ &= v - c - \frac{23\sqrt{5} - 45}{6} t \simeq v - c - 1.072t \end{aligned}$$

We end this section by comparing our results with the model of Anderson and Neven (1989).[2] This is the only legitimate comparison once locations are endogenized and consumers can mix, as the standard Hotelling model would have different implications in terms of socially optimal locations. While the equilibrium in Anderson and Neven with linear prices achieves the first-best allocation where *all* consumers mix, this is no longer true once two-part tariffs are allowed. This indicates that the first-best efficiency result of Anderson and Neven holds only conditional on the assumption that no subscription fees can be charged.

Proposition 4 *Competition with mixing and two-part tariffs leads to inefficient outcomes. As compared to linear pricing both firms benefit, while all consumers suffer (both mixing and non mixing) and overall welfare decreases.*

[2] In our analysis we have assumed that the location space is [0, 1]. However, it is conceivable in media applications that media platforms can offer more extreme content, outside the [0, 1]-interval. This corresponds to polarization of content that is not matched by the heterogeneity of viewers' tastes. If we allow for these extreme locations, the equilibrium locations would become $a_1 = -0.686$ and $a_2 = 1.686$, while in Anderson and Neven these values were -0.25 and 1.25, respectively. That is, with two-part tariffs networks would seek more differentiation (polarization of content).

Proof. Under linear pricing, firms choose extreme locations. All consumers mix optimally and pay a price $p_{Tot} = c + t$, thus their net utility is $v - c - t$, which is always greater than either $U_1(x)$ or $U_{12}(x)$ defined above. The result on profits is a re-statement of Proposition 2. Finally, welfare must decrease because customers in $0 \leq x < x_l$ and $x_h < x \leq 1$ buy exclusively, rather than mixing optimally, and thus incur in higher transportation costs which is socially wasteful. QED

The two models produce no difference with respect to location choices: there is always extreme differentiation, which is efficient when consumers can mix. However, the pricing format introduces inefficiencies since it induces some customers to buy exclusively from a single firm, so that benefits from mixing are lost. As concerns customers:

- Mixing customers mix optimally but pay more overall to firms. They lose out because of the price effect. There is no welfare loss from these customers, but a redistribution of rents occurs;
- Exclusive customers pay the same to the firms as before, but incur more transport costs because they no longer mix, which is a social loss. There is no additional profit made from these customers (some are exclusive, but some are lost to the rival, the effects cancel out).

4 Flat Rates

We have shown above that tariffs consisting only of a subscription payment (flat rates) will not arise in equilibrium if firms have the possibility to charge for usage. As mentioned in the Introduction, there may be situations where charging for usage is not possible or needs large investments in monitoring technology. In this case firms must compete in subscriptions only.

Proposition 5 *If firms can only compete in flat rates, then for any pair of locations $0 \leq a_1 \leq a_2 \leq 1$ there is a unique Nash equilibrium, with subscription fees*

$$F_1^* = c + \frac{1}{3} t (2 + a_1 + a_2)(a_2 - a_1), \tag{22}$$

$$F_2^* = c + \frac{1}{3} t (4 - a_1 - a_2)(a_2 - a_1). \tag{23}$$

In equilibrium no consumer mixes.

Proof: The relevant demand system is identical to the one described in the Appendix, with $p_1 = p_2 = 0$. First we show that the case with $0 < x_l < x_h < 1$, where some consumers mix, does not contain a Nash equilibrium (this is denoted as case IIc in the Appendix). Firm 1's profits are

$$\begin{aligned} \pi_1(F_1, F_2) &= F_1 B_1(F_1, F_2) - cD_1(F_1, F_2) \\ &= F_1\left(a_2 - \sqrt{F_1/t}\right) - c\left(\frac{a_1 + a_2}{2} + \frac{F_2 - F_1}{2t(a_2 - a_1)}\right), \end{aligned}$$

with equilibrium candidate $F_1 = t\left(\frac{2}{3}a_2 + \frac{c}{3t(a_2-a_1)}\right)^2$. In the same manner one finds $F_2 = t\left(\frac{2}{3}(1 - a_1) + \frac{c}{3t(a_2-a_1)}\right)^2$. It is straightforward to show that these values violate the condition delimiting case IIc.

Our candidate case is the case where $x_l \geq x_h$, i.e., no consumer mixes (this is denoted as case I in the Appendix; the other subcases of case II lead to deviations as before). Demands are

$$B_1(F_1, F_2) = D_1(F_1, F_2) = \frac{a_1 + a_2}{2} + \frac{F_2 - F_1}{2t(a_2 - a_1)},$$

leading to standard Hotelling payoffs for both firms:

$$\begin{aligned} \pi_1 &= (F_1 - c)\left(\frac{a_1 + a_2}{2} + \frac{F_2 - F_1}{2t(a_2 - a_1)}\right) \\ \pi_2 &= (F_2 - c)\left(1 - \frac{a_1 + a_2}{2} - \frac{F_2 - F_1}{2t(a_2 - a_1)}\right) \end{aligned}$$

These payoffs are concave, and the equilibrium candidate is given by (22) and (23) stated in the proposition. These values are contained in case I since

$$\begin{aligned} F_1^*/t &\geq \frac{1}{3}(2 + a_1 + a_2)(a_2 - a_1) \geq (a_2 - a_1)^2, \\ F_2^*/t &\geq \frac{1}{3}(4 - a_1 - a_2)(a_2 - a_1) \geq (a_2 - a_1)^2, \end{aligned}$$

and therefore $\sqrt{F_1^*/t} + \sqrt{F_2^*/t} \geq 2(a_2 - a_1)$. Given F_2^*, any deviation of firm 1 to one of the subcases of case II involves the choice of F_1 such that $\sqrt{F_1/t} < a_2 - a_1 - \sqrt{F_2^*/t}$. Since the right-hand side is negative there is no profitable deviation by firm 1 that leads to case II. Therefore our candidate is a Nash equilibrium, and there is no other Nash equilibrium in case I (nor in the other subcases of case II). QED

The remarkable feature of this equilibrium is that the outcome is identical to the standard Hotelling model with unit demands. No consumer mixes, even though all of them could do so. The intuition is simple: Since there are no usage charges, both firms have an incentive to charge higher subscription fees, and consumers end up subscribing to one firm only. This result highlights therefore the role of usage charges in limiting the level of subscription fees.[3]

Given the above result, the firms' choice of locations leads to the familiar maximum differentiation result of $a_1 = 0$ and $a_2 = 1$. Equilibrium subscription fees are $F_1 = F_2 = c + t$, and profits are equal to $\pi_1 = \pi_2 = t/2$. These profits are lower than under two-part tariffs. Consumer surplus of firm 1's subscribers is $U_1(x) = v - c - (1 + x^2)\,t$, equal to the non-mixing consumers' utility (21) under two-part tariffs (a corresponding results holds for subscribers to firm 2). Therefore all consumers in (x_l, x_h) are strictly worse off under competition in flat rates, and total consumer surplus $CS = 2\int_0^{1/2} U_1(x)\,dx = v - c - \frac{13}{12}t \simeq v - c - 1.083t$ is lower.

The outcome is inefficient in a circular sense: First, given that consumers do not mix, firms are too far apart: the socially optimal locations are $a_1 = \frac{1}{4}$ and $a_2 = \frac{3}{4}$; second, given firms' locations consumers should be mixing.

Let us summarize:

Proposition 6 *If firms compete in flat rates then the equilibrium market outcome is as if consumers were not able to mix at all. Consumer surplus and profits are identical to the standard Hotelling model, and lower than under two-part tariffs. The outcome is inefficient because no mixing occurs.*

5 Conclusions

This paper has considered the problem of location choice with combinable goods, which applies to many media markets where audiences can subscribe to more than one broadcaster and mix among different content. We have extended the model of Anderson and Neven (1989, AN) with linear pricing to the case of two-part tariffs. In a restricted version, we have also considered the case with purely flat subscription fees. Both the pricing structures studied in this paper have practical relevance. The change in pricing structures has no impact on location choices: Firms always choose extreme differentiation

[3] In a media model, Peitz and Valletti (2005) conjecture that no mixing might occur with flat tariffs if media content is sufficiently differentiated (i.e., transportation costs are sufficiently high). We have shown here that this result is more general as it does not depend on the magnitude of transportation costs.

in all the versions studied. Notwithstanding this robust result, the change in pricing produces significant changes on a different type of allocation, namely on the choice of subscribers whether to mix or buy exclusively instead. We can summarize our results in the following table:

Pricing	Who mixes	CS	Profits	Welfare
Linear (AN)	everybody	$v - c - 1.000t$	$0.500t$	$v - c - 0.500t$
2 part (this paper)	middle	$v - c - 1.072t$	$0.517t$	$v - c - 0.554t$
Flat (this paper)	nobody	$v - c - 1.083t$	$0.500t$	$v - c - 0.583t$

Compared to linear pricing, two-part tariffs achieve higher equilibrium profits for firms but are welfare-reducing. With flat fees, firms make the same profits as under linear pricing, but consumers are worse off than even under two-part tariffs. Even though two-part tariffs do not achieve the first best, contrary to the linear case, making them possible leads to a Pareto improvement in the market outcome as compared to flat rates: both consumers and firms benefit.

Even though in our model each consumer may buy a variable quantity from each firm, we have assumed inelastic *total* demand, as is common in the literature. Future research will relax this assumption and allow for elastic total demand. Our intuition is that, with elastic demand, two-part tariffs may lead to higher total welfare as compared to linear tariffs, because the lower marginal price may well induce sufficiently more additional demand as to outweigh the loss due to fewer mixing consumers.

Furthermore, in this paper we have concentrated the analysis on competition for viewers only, while we have neglected other sources of revenues for broadcasters, such as revenues from advertisers that want to reach viewers. As a simple extension of our model, consider the case where the number of commercials aired does not impose any nuisance on viewers, and broadcasters are able to earn a fixed amount per customer from advertisers. If advertisers pay broadcasters only per minute of viewing time, then our model with two-part tariffs would be solved in a straightforward way. All advertising revenues would be passed on to customers via a reduction in the pay-per-view price, while the subscription fee would be unaffected. In equilibrium, profits would be neutral to the presence of advertising revenues. On the contrary, if advertisers pay broadcasters per subscriber (independently of viewing time), then there would not be a full pass-through of advertising revenues anymore. Advertising revenues would lower also the subscription fee, which implies that more viewers would mix than without advertising revenues. It would be

interesting to analyze in detail how our results with mixing viewers interact with the two-sidedness of these markets.

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Appendix

Proof of Proposition 1.

We first start with all the possible cases of demand.

Case I: $x_l \geq x_h$. There will be no mixing if

$$x_l = (a_1 - R) + \sqrt{F_2/t} \geq x_h = (a_2 - R) - \sqrt{F_1/t}$$

or

$$a_2 - a_1 \leq \sqrt{F_1/t} + \sqrt{F_2/t}.$$

In this case, demands will be defined by the choice between the two varieties, with indifferent consumer at

$$v - F_1 - p_1 - t\,(a_1 - x)^2 = v - F_2 - p_2 - t\,(a_2 - x)^2$$

or

$$x_m = \frac{a_1 + a_2}{2} + \frac{(F_2 + p_2) - (F_1 + p_1)}{2t\,(a_2 - a_1)}$$

This corresponds to the standard Hotelling model, and demands are

$$\begin{aligned} D_1(p_1, F_1, p_2, F_2) &= \begin{cases} 0 & if \quad x_m < 0 & \begin{array}{c}(F_1 + p_1) > t\,(a_2^2 - a_1^2) \\ +(F_2 + p_2)\end{array} \\ x_m & if \quad 0 \leq x_m \leq 1 & \\ 1 & if \quad x_m > 1 & \begin{array}{c}(F_1 + p_1) < t\,(a_2^2 - a_1^2) \\ -2t\,(a_2 - a_1) + (F_2 + p_2)\end{array} \end{cases} \\ B_1(p_1, F_1, p_2, F_2) &= D_1(p_1, F_1, p_2, F_2) \end{aligned}$$

Case II: $0 < x_h - x_l$ or $0 < a_2 - a_1 - \sqrt{F_1/t} - \sqrt{F_2/t}$

IIa) $1 \leq x_l < x_h$: all consumers prefer variety 1 to mixing or variety 2.

$$D_1(p_1, F_1, p_2, F_2) = B_1(p_1, F_1, p_2, F_2) = 1, B_2(p_1, F_1, p_2, F_2) = 0.$$

IIb) $0 < x_l < 1 \leq x_h$: variety 2 is only bought by mixers.

$$\begin{aligned} D_1(p_1, F_1, p_2, F_2) &= x_l + \int_{x_l}^{1} \lambda(x)\,dx \\ &= \left((a_1 - R) + \sqrt{F_2/t}\right) + \int_{(a_1-R)+\sqrt{F_2/t}}^{1} \frac{a_2 - x - R}{a_2 - a_1} dx \\ &= 1 - \frac{1}{2\,(a_2 - a_1)} \left(1 - a_1 + \frac{p_1 - p_2}{2t\,(a_2 - a_1)}\right)^2 + \frac{1}{2} \frac{F_2}{t\,(a_2 - a_1)} \end{aligned}$$

and

$$B_1(p_1, F_1, p_2, F_2) = 1, B_2(p_1, F_1, p_2, F_2) = 1 - x_l.$$

IIc) $0 < x_l < x_h < 1$: both varieties are bought alone and mixed. This is the case considered in the main text.

$$\begin{aligned} D_1(p_1, F_1, p_2, F_2) &= x_l + \int_{x_l}^{x_h} \lambda(x)\, dx \\ &= \frac{a_1 + a_2}{2} + \frac{(F_2 + p_2) - (F_1 + p_1)}{2t(a_2 - a_1)} \\ B_1(p_1, F_1, p_2, F_2) &= x_h, B_2(p_1, F_1, p_2, F_2) = 1 - x_l. \end{aligned}$$

IId) $x_l \leq 0 < x_h < 1$): variety 1 is only bought by mixers.

$$\begin{aligned} D_1(p_1, F_1, p_2, F_2) &= \int_0^{x_h} \lambda(x)\, dx \\ &= \frac{1}{2(a_2 - a_1)} \left(\frac{p_1 - p_2}{2t(a_2 - a_1)} - a_2 \right)^2 - \frac{1}{2} \frac{F_1}{t(a_2 - a_1)} \\ B_1(p_1, F_1, p_2, F_2) &= x_h, B_2(p_1, F_1, p_2, F_2) = 1. \end{aligned}$$

IIe) $x_l < x_h \leq 0$: all consumers only buy variety 2.

$$D_1(p_1, F_1, p_2, F_2) = B_1(p_1, F_1, p_2, F_2) = 0, B_2(p_1, F_1, p_2, F_2) = 1.$$

Both D_i and B_i are not differentiable in F_i at the borders, and B_i is not differentiable in p_i at these points.

Given the candidate solution (eq. (10)), we now have to check for deviations to the other cases.

A lower pair of prices (F_1, p_1) could violate $[x_h < 1]$, so demand would fall under case IIb) $0 < x_l < 1 \leq x_h$. Profits would be:

$$\pi_1 = (p_1 - c) \left(1 - \frac{1}{2(a_2 - a_1)} \left(1 - a_1 + \frac{p_1 - p_2}{2t(a_2 - a_1)} \right)^2 + \frac{1}{2} \frac{F_2}{t(a_2 - a_1)} \right) + F_1.$$

This expression is increasing in F_1, thus it would always collapse either to case IIc) or to case I.

Take now case IIa): $D_1(p_1, F_1, p_2, F_2) = B_1(p_1, F_1, p_2, F_2) = 1$. Profits are simply

$$\pi_1 = (p_1 - c) + F_1$$

which always collapses to case IIb).

A higher p_1 would violate $x_l > 0$, so demand would move to case IId, with $x_l \leq 0 < x_h < 1$. Profits would be:

$$\begin{aligned} \pi_1 &= (p_1 - c)\left(\frac{1}{2(a_2 - a_1)}\left(\frac{p_1 - p_2}{2t(a_2 - a_1)} - a_2\right)^2 - \frac{F_1}{2t(a_2 - a_1)}\right) \\ &\quad + F_1\left(a_2 - \frac{p_1 - p_2}{2t(a_2 - a_1)} - \sqrt{\frac{F_1}{t}}\right) \end{aligned}$$

(which becomes zero at $x_h = 0$ or $p_1 = p_2 + 2t(a_2 - a_1)\left(a_2 - \sqrt{F_1/t}\right)$). This is the same function of F_1 as in case IIc) in the main text, thus we need only look at p_1. A complication arises as D_1 is not differentiable in p_1 at the border point if $F_2 > 0$. The border price is: $p_1 = p_2 + 2t(a_2 - a_1)\left(a_1 + \sqrt{F_2/t}\right)$, $\frac{\partial \pi_1}{\partial p_1}$ is negative at border price with (p_2^*, F_2^*), independently of the value of F_1:

$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1} &= \frac{1}{2(a_2 - a_1)}\left(\frac{3p_1 - p_2 - 2c}{2t(a_2 - a_1)} - a_2\right)\left(\frac{p_1 - p_2}{2t(a_2 - a_1)} - a_2\right) - \frac{F_1}{t(a_2 - a_1)} \\ &= -\frac{\sqrt{5} - 1}{12}(5a_1 - a_2 + 4) - \frac{F_1}{t(a_2 - a_1)} < 0. \end{aligned}$$

Since π_1 is third order in p_1 with positive coefficient on p_1^3, the border point is on the falling middle segment, and the local maximum is outside interval IId). The upper limit at $x_h = 0$ has value zero, therefore there is no profitable deviation in p_1.

Finally, consider case I. A higher F_1 would move to case I, $x_h \leq x_l$. Profits would be

$$\pi_1 = (p_1 + F_1 - c)\,x_m = (p_1 + F_1 - c)\left(\frac{a_1 + a_2}{2} + \frac{(F_2 + p_2) - (F_1 + p_1)}{2t(a_2 - a_1)}\right).$$

The best response in $P_1 = p_1 + F_1$, given $P_2 = p_2 + F_2$, is $P_1 = \frac{1}{2}t\left(a_2^2 - a_1^2\right) + \frac{1}{2}(P_2 + c)$. However, this is already true at the equilibrium candidate in IIc), therefore there is no profitable deviation to case I.

Let us now consider uniqueness. First of all, there is at most one equilibrium in case IIc). Furthermore, with the above arguments we can rule out equilibria in cases I), IIa) and IIb), and therefore also in cases IId) and IIe). Thus the Nash equilibrium is unique. QED