Competition for Scarce Resources

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May 1, 2006
PRELIMINARY AND INCOMPLETE

Abstract

In this paper we consider the implications for downstream competition of the scarcity of inputs to the production process. We show that, even though downstream firms may have symmetric production technologies and access to a centralized input market, input purchases and downstream production will be asymmetric. One firm will appear to be a "fat cat", apparently purchasing too much input and using it very inefficiently; whereas the other, smaller firms will appear "lean and mean", making high rates of profit despite their low input purchases. Even as the number of downstream firms becomes large, downstream production is both allocatively and productively inefficient. Paradoxically, social welfare will ultimately fall as the supply of the valuable and scarce input is increased. A testable implication of our model is that firm size and Tobin's Q are negatively related.

1 Introduction

In this paper we consider the implications for downstream competition of the scarcity of inputs to the production process. With some notable exceptions, economists considering strategic competition among firms in output markets have ignored the potential for strategic behavior in input markets. To the extent that "buyer power" has been modelled, this has typically been done in a context divorced from downstream competition between firms. We show that this simplification is potentially misleading: there is an important interaction between upstream purchases and downstream competition that leads to some unexpected outcomes.

For example, modelling input and output competition between symmetric firms separately will typically give *symmetric outcomes*. In contrast, we show that this may not the case when one allows firms to compete both on input and on output markets. In particular, we show that, when the allocation of the production capacity (which is sufficiently abundant) is centralized in an "efficient auction" process,[1] there is no symmetric equilibrium. We assume that the firms have the same technology with increasing marginal costs, compete downstream à la Cournot, and yet downstream asymmmetry may arise endogenously. When the amount of total capacity is sufficiently large, one large firm buys up capacity and hoards it in order to keep up the market price of output, whilst other firms remain small and their production capacity-constrained. As the number of downstream firms becomes large, the resulting equilibrium resembles the textbook model of a dominant firm constrained by a competitive fringe but there are subtle differences. Even as the number of downstream firms goes to infinity, downstream output does not converge to competitive levels in our model, and the one-firm concentration ratio remains bounded away from zero.[2]

The capacity auction may transform the downstream market into a natural oligopoly with endogenous asymmetries. This is so even though all firms have access to the same technology and an efficient market for capacities. One firm

[1] An auction is called *efficient* if it allocates each unit of the good to the buyer that values it the most. In the context of our model, a Vickrey-Clarke-Groves mechanism with bids that are contingent on the entire allocation constitutes an efficient auction. This and other auction formats that may yield the same capacity allocation are discussed in Sections 2 and 3.4.

[2] There are also some technical differences. In the dominant firm model, the dominant firm sets price, taking as given the supply curve of the competitive fringe.

will appear to be a "fat cat", apparently purchasing too much capacity and using it very inefficiently; whereas the other, smaller firms, will appear "lean and mean", making high rates of profit despite their low capacity purchases. Viewed in isolation, this kind of outcome may seem to be a puzzle: why don't the small, productive firms expand to steal the downstream market from the large, inefficient firm? Observing such a situation might lead one to suspect that some unobserved regulation, illegal anti-competitive behavior or political influence protect the large firm from its more efficient rivals. The model helps us understand that this is not necessarily the case: the asymmetric outcome may simply be the result of standard non-cooperative behavior. The reason that the small firms' rate of profit is so high is precisely because the large firm is buying up input to make it unprofitable for them to expand. If the large firm were not so fat, the small firms would expand output and their rate of profit would decline.[3]

This asymmetric distribution of input results in two sorts of inefficiencies. The first is that output is too low; when firms compete in quantities downstream, and purchase their inputs via an efficient auction, output will be lower than the Cournot outcome that might be expected from modelling the downstream market alone. In fact, output will maximize the joint surplus of the firms subject to the incentive compatibility constraint that *one* firm's output choice is that firm's best reponse to the output choices of all other firms. Second, and conceptually distinct from this allocative inefficiency, is a productive inefficiency: for a given level of output, the asymmetric distribution of production means that costs are not minimized; the marginal physical product of the input is not equalized across firms.

The asymmetric (and doubly inefficient) distribution of capacities in the upstream auction arises when the total available capacity is is greater than a certain threshold level. When the total capacity to be distributed is less than this threshold, the outcome of the auction is symmetric: Each firm gets the

[3]The small firms may even benefit from the ineffiency of the large firm, making more profits than they would if, by fiat, input were distributed symmetrically (resulting in a symmetric Cournot outcome). Indeed, depending upon parameters, the large firm may be providing a public good in buying up the excess input to reduce the supply of output: not only is its rate of profit necessarily lower than that of the small firms, it may be that its absolute level of profit is lower too.

same capacity, and fully uses it in the downstream market. In this case, the second source of inefficiency (the production inefficiency due to the asymmetry of the firms' output levels) disappears.

Perhaps the most interesting result of our model is that as the total available capacity increases around the capacity threshold, and the capacity distribution changes from symmetric to asymmetric, the total surplus of the economy (which includes the firms' total profit and the downstream consumer surplus) falls by a discrete amount. That is, there is a certain discontinuity at the capacity level where the regime change occurs, and society is strictly worse off when more capacity is distributed among the firms. This is so because the firms' total profit is continuous at the capacity-threshold (they are indifferent between the symmetric and asymmetric allocation), however, total output (and hence consumer surplus) falls discontinuously due to the introduction of production inefficiencies as production becomes asymmetric.

Our results are interesting not only because they may help understand the size distribution of firms in some industries, but also from a normative perspective. To the extent that economists have recently been interested in the design of markets for the allocation of inputs,[4] the typical prescription has been that old rigid structures of bilateral contracts and vertical integration should be replaced by centralized auction markets for the input. The idea was that if all input is brought to a centralised market, then one can ensure that those that value the input most highly are able to purchase it, resulting in efficient production. Our results suggest that this intuition is misplaced in a context where downstream firms compete. It is not entirely surprising that an efficient auction, since it maximizes the bidders' surplus, may allocate input in a way which results in a lack of downstream competition.[5] Perhaps more surprising is that, in the presence of diseconomies of scale or complementarities between

[4] Examples include the sale of electicity by electricity generating companies to electricity retailers; the sale of licences for mobile phone operators; the sale of oil tracts to oil production companies; the sale of forestry tracts to logging companies...

[5] This idea can be found in Jehiel and Moldovanu (2000, 2003). However, the result is not completely evident. For example, McAfee (1999) argues that when large and small incumbents compete in an auction to purchase an additional unit of capacity, a small (constrained) firm will win the auction if there are at least two large (unconstrained) firms. McAfee does not consider the full dynamic game in which capacity is acquired over time. We show that, when inputs are allocated simultaneously, the symmetric outcome which he identifies is no longer an equilibrium.

inputs, an "efficient" auction will result in production inefficiencies. This suggests that allocating input by some more decentralised means such as bilateral contracting, might, contrary to intuition, actually be better for consumers.

We discuss in more depth the role of the "efficient capacity auction" for obtaining our results. First, we show that the same allocative and productive inefficiencies that appear when capacity is allocated via a Vickrey-Clarke-Groves mechanism may also arise in more practical auction formats, such as a uniform-price share auction. (Indeed, we show that the efficient allocation can always arise in an equilibrium of the uniform-price auction.) Secondly, we point out that other auction formats, where, for example, each unit of the capacity is allocated separately consecutively, in a dynamic procedure, fare better in terms of total social surplus (which includes the surplus of downstream consumers) than an "efficient" auction.

The paper is organized as follows. In Section 2, we outline the model and derive preliminary results. In Section 3, we derive the unique equilibrium of our game (Cournot competition following the joint-profit maximizing allocation of production capacities), and characterize several of its properties. We analyze the limiting case as the number of firms grows infinitely large, and discuss welfare. In Section 4, we show how the joint-profit maximizing allocation of production capacities can be implemented through various auction formats. In Section 5, we describe certain testable predictions of our model (e.g., on the relationship between Tobin's Q and firm size). In Section 6, we show that our main results continue to hold when competition between firms is (differentiated-goods) Bertrand rather than (homogeneous-goods) Cournot. Section 7 concludes.

2 Model and Preliminary Results

In the first stage of the game, n ex-ante identical firms are allocated *production capacities* via an efficient auction (such as a VCG-mechanism). Alternatively, we may assume that production capacities are (re-)allocated through efficient Coasian bargaining between these firms. Then, in the second stage, the same firms compete—à la Cournot and subject to their capacity constraints—in a market for a homogenous good. The firms' production technologies exhibit increasing marginal costs, and the market demand is downward sloping. The

participants have no private information, everything is commonly known.

In this section we introduce notation that formally describes this model, and perform some preliminary analysis (e.g., we show that the second-period subgame, Cournot competition with capacity constraints, has a unique equilibrium). In the next section we derive the equilibrium market structure and discuss its properties.

2.1 Notation and Assumptions

Denote the total available capacity by K, and the capacities of the firms, determined in the first-period auction (or through efficient Coasian bargaining), by k_i, $i = 1, \ldots, n$, where $\sum_i k_i = K$.

Denote the inverse demand function in the downstream market by $P(Q)$, where Q is the total production. We assume that P is twice differentiable, and that both $P(Q)$ and $P'(Q)Q$ are strictly decreasing for all $Q > 0$. Firm i's cost of producing $q_i \leq k_i$ units is $c(q_i)$, while its cost of producing more than k_i units is infinity. We assume that c is twice differentiable, strictly increasing, and strictly convex. Finally, we assume that producing a limited amount of the good is socially desirable: $P(Q) - c'(Q)$ is positive for $Q = 0$, and negative as $Q \to \infty$.

We can write the profit of firm i in the downstream market for quantity $q_i \leq k_i$ and total output from firms other than i, Q_{-i}, as

$$\pi_i(q_i, Q_{-i}) = P(Q_{-i} + q_i)q_i - c(q_i). \tag{1}$$

The marginal profit of firm i given the other firms' total production is $\partial \pi_i / \partial q_i = P'(Q_{-i} + q_i)q_i + P(Q_{-i} + q_i) - c'(q_i)$.

The assumptions on the market demand and individual cost functions made above are standard in the literature. They ensure that π_i is concave in q_i, and that the quantities are strategic substitutes,

$$\frac{\partial^2 \pi_i}{\partial q_i \partial Q_{-i}} \equiv P''(Q_{-i} + q_i)q_i + P'(Q_{-i} + q_i) < 0,$$

where the inequality follows from $d\left[P'(Q)Q\right]/dQ = P''(Q)Q + P'(Q) < 0$, $P'(Q) < 0$, and $q_i \in [0, Q_{-i} + q_i]$.

The assumptions are also known to imply that in the Cournot game without capacity constraints, there exists a unique equilibrium. The per-firm output in the unconstrained Cournot equilibrium, denoted by q^*, satisfies

$$\frac{\partial \pi_i(q^*, (n-1)q^*)}{\partial q_i} = P'(nq^*)q^* + P(nq^*) - c'(q^*) = 0. \tag{2}$$

This is just the first-order condition of maximizing π_i in q_i given Q_{-i}, and using $Q_{-i} = (n-1)q^*$. It is easy to see that under our assumptions, there is a unique q^* that solves (2): Consider the left-hand side of (2). At $q^* = 0$, it is positive, while as $q^* \to \infty$, it becomes negative by assumption. Its derivative in q^* is $nP''(nq^*)q^* + P'(nq^*)(n+1) - c''(q^*) < 0$. Therefore, there exists a unique q^* at which it equals zero, and so (2) is satisfied.

We will use $r_i(Q_{-i})$ to refer to the best response of firm i to the total production of the other firms, Q_{-i}, when firm i does not face a binding capacity constraint. That is, $r_i(Q_{-i}) = \arg\max_{q_i} \pi_i(q_i, Q_{-i})$. Equivalently, $r_i(Q_{-i})$ can be characterized by the first-order condition of this maximization, that is,

$$\frac{\partial \pi_i(r_i(Q_{-i}), Q_{-i})}{\partial q_i} = P'(Q)r_i(Q_{-i}) + P(Q) - c'(r_i(Q_{-i})) \equiv 0, \tag{3}$$

where $Q = Q_{-i} + r_i(Q_{-i})$. By totally differentiating this identity with respect to Q_{-i} and rearranging, we find that

$$r_i'(Q_{-i}) = -\frac{\partial^2 \pi_i / \partial q_i \partial Q_{-i}}{\partial^2 \pi_i / \partial q_i^2} = -\frac{P''(Q)r_i(Q_{-i}) + P'(Q)}{P''(Q)r_i(Q_{-i}) + 2P'(Q) - c''(r_i(Q_{-i}))} \in (-1, 0).$$

The unconstrained Cournot equilibrium satisfies $q^* = r_i((n-1)q^*)$. To ease notation, we drop the reference to firm i's identity when referring to the best response function because the best-response functions are identical across the firms.

The are two other industry-structure benchmarks besides the unconstrained symmetric Cournot outcome (where the production is q^* per firm and nq^* total) that will come up in later in the section. The *monopoly production* in the downstream market is denoted by Q^M, where $Q^M = \arg\max_Q P(Q)Q - c(Q)$, that is,

$$P'(Q^M)Q^M + P(Q^M) = c'(Q^M). \tag{4}$$

The other market structure that will turn out to be important for us is that of a perfectly coordinated *symmetric cartel*. By definition, the symmetric cartel output maximizes the firms' joint profits while each firm produces one-n^{th} of the total output. That is, the total output in the cartel, Q^C, maximizies $P(Q)Q - nc(Q/n)$, and so

$$P'(Q^C)Q^C + P(Q^C) = c'(Q^C/n). \tag{5}$$

Note that the monopoly output equals the total production of the symmetric cartel if and only if $c'(Q^M) = c'(Q^C/n)$.

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If firm i's capacity constraint is less than $q^U(Q)$ then it produces k_i. Note that all firms whose capacity constraints are slack produce the same output, $q^U(Q)$.

The function $q^U(Q)$ is continuous, and by the Implicit Function Theorem its derivative is

$$\frac{dq^U(Q)}{dQ} = -\frac{P''(Q)q^U(Q) + P'(Q)}{P'(Q) - c''(q^U(Q))}.$$

If $q^U(Q) \leq Q$ then $P''(Q)q^U(Q) + P'(Q) < 0$ by assumption. This, combined with $P' < 0$ and $c'' \geq 0$, implies that $dq^U(Q)/dQ < 0$ whenever $q^U(Q) \leq Q$.

Define

$$h(Q) = \sum_{i=1}^{n} \min\left\{k_i, q^U(Q)\right\} - Q. \tag{8}$$

Clearly, $Q^* \in [0, K]$ and $h(Q^*) = 0$ if and only if Q^* is the total production in a capacity-constrained Cournot equilibrium.

We claim that there exists a unique $Q^* \in [0, K]$ that satisfies $h(Q^*) = 0$. To see this, first note that $q^U(0) > 0$ by equation (7), hence $h(0) > 0$ by equation (8). If $Q \geq K \equiv \sum_i k_i$ then equation (8) yields $h(Q) \leq 0$. Since $q^U(Q)$ is continuous, $h(Q)$ is continuous as well. Therefore, by the Intermediate Value Theorem, there exists $Q^* \in (0, K]$ such that $h(Q^*) = 0$. If $Q < K$ then, by (8), $h(Q) \leq 0$ implies that $q^U(Q) < k_i$ for some i, and therefore $q^U(Q) \leq Q$. As a result, $q^U(Q)$ is strictly decreasing, and so is $h(Q)$, for all $Q \in [Q^*, K)$. Since $h(Q^*) = 0$, we have $h(Q) < 0$ for all $Q \in (Q^*, K]$. Therefore, any $Q^* \in [0, K]$ such that $h(Q^*) = 0$ is unique. ■

Denote the capacity-constrained Cournot equilibrium given capacity allocation $(k_1, \ldots, k_n)$ by $(q_i^e(k_1, \ldots, k_n))_{i=1}^n$, and let the indirect profit function of firm i be

$$\Pi_i(k_1, \ldots, k_n) = P\left(\textstyle\sum_i q_i^e(k_1, \ldots, k_n)\right) q_i^e(k_1, \ldots, k_n) - c\left(q_i^e(k_1, \ldots, k_n)\right).$$

An interesting feature of our model is that the buyers' (firms') marginal valuations for an additional unit of capacity *may not be monotonic* in the amount of capacity that they receive. This can be seen, at the level of intuition, for two firms as follows. When firm 1 is relatively small (has little capacity, which is a binding constraint in the downstream Cournot competition) then the marginal value of an additional unit of capacity is positive but decreasing because expand-

ing the firm's production generates a positive yet decreasing marginal profit in the downstream market. However, if the firm is relatively large, so much so that its capacity constraint is slack while its opponent's constraint is binding in the downstream Cournot game, then the marginal value of additional capacity is increasing. This is so because by buying more capacity the firm tightens the other firm's capacity constraint, and the returns on this activity are increasing for our firm.[6] Therefore, the marginal value of capacity for firm i is U-shaped in the capacity of the firm, as shown in Figure 1.

The initial capacity auction is called *efficient* if it leads to an allocation $(k_1, \ldots k_n)$, $\sum_i k_i = K$, that maximizes $\sum_i \Pi_i(k_1, \ldots, k_n)$. Such an allocation is obviously only "efficient" with respect to the welfare of the competing firms, and it ignores the consumer surplus in the downstream market. However, it *is* the outcome of the capacity auction if the sale of the K units of capacity is organized as a so-called Vickrey-Clark-Groves auction, and it may be the outcome (the focal equilibrium) if a practical share-auction is used; see Section.

3 Results

We turn to the analysis of the equilibrium market structure in the model of Section 2, and show that it may be qualitatively different depending on the amount of capacity sold in the auction. If the total capacity that is auctioned off is relatively little then the firms behave symmetrically, while if it is large then the only equilibrium is asymmetric, in which exactly one firm ends up with excess capacity and produces a larger quantity, while the other firms produce less while being constrained by their insufficient capacities. The "regime change" (from a symmetric to an asymmetric outcome) happens at a certain capacity threshold and makes the total production drop discontinuously as the total capacity increases (see Section 3.1). In Section 3.2 we discuss how the market structure in our model differs from the unconstrained Cournot outcome,monopoly, and the structure of a perfectly coordinated symmetric cartel. In Section 3.3, we extend the comparison for an infinite number of firms as well.

[6] These verbal statements are true and may be verified by direct calculation in the case of two firms.

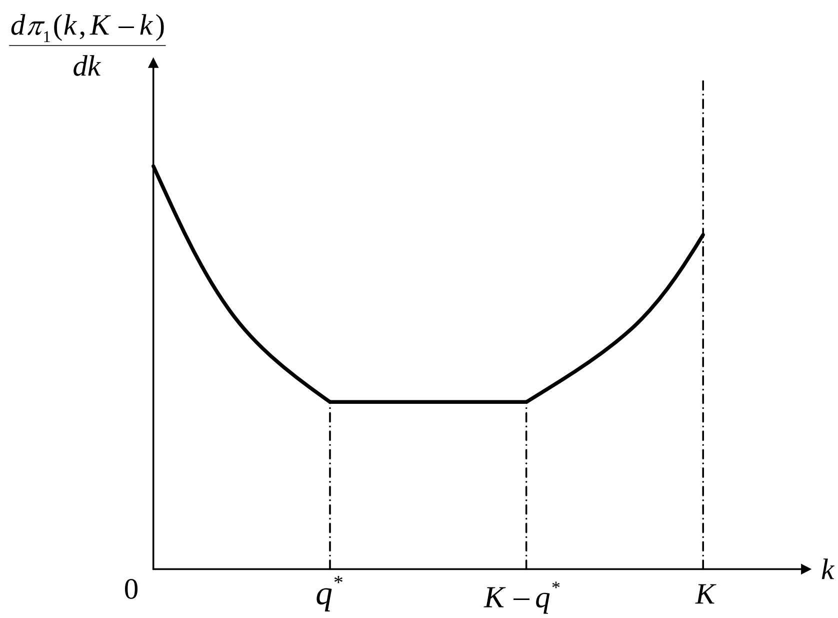

Figure 1: Firm i's marginal value of capacity when $n = 2$ and $K > 2q^*$.

3.1 Industry Structure in the Downstream Market

Interestingly, the qualitative results regarding the downstream market structure depend on the total available capacity, K. If the total available capacity is relatively low then the efficient capacity allocation is symmetric, and all firms end up producing at their capacity constraints in the downstream market. However, if K is large—exceeding a threshold that is strictly lower than the total capacity needed to produce the symmetric, unconstrained Cournot output—then the industry structure becomes asymmetric. All but one firm gets the same, low capacity and operates at full capacity, while one firm gets the remaining capacity (which is a bigger share of the total than the share of any other firm), and operates strictly within its capacity constraint.

As a preparation for stating the results formally, we first describe the asymmetric industry structure that prevails when K is sufficiently large. In the efficient capacity auction, each "small" firm buys a capacity $k_1 = \ldots = k_{n-1} = k^*$, while the "large" firm receives the rest, $k_n = K - (n-1)k^*$. The capacity level of each of the small firms, k^*, maximizes

$$P\left((n-1)k + r((n-1)k)\right)\left[(n-1)k + r((n-1)k)\right] \\ - (n-1)c(k) - c(r((n-1)k)). \quad (9)$$

This is the total industry profit given that $(n-1)$ firms produce k and one firm produces the unconstrained best reply, $r((n-1)k)$. The optimal level of k^* is characterized by the first-order condition of the maximization,

$$\left[P'(Q^*)Q^* + P(Q^*)\right]\left[1 + r'((n-1)k^*)\right] \\ = c'(k^*) + r'((n-1)k^*)c'(r((n-1)k^*)), \quad (10)$$

where $Q^* = (n-1)k^* + r((n-1)k^*)$. Note that k^* does not vary with K.

The following lemma states that if *at least one firm is allocated excess capacity* in the efficient auction then the capacity allocation must be the asymmetric one described above. We will later see that a sufficient condition for there being at least one firm with slack capacity is that the total capacity exceed the amount needed for producing the unconstrained Cournot equilibrium outcome, i.e., $K > nq^*$. In the statement and proof of the lemma recall that the firms'

indices are ordered so that $k_1 \leq k_2 \leq \ldots \leq k_n$.

Lemma 1 *Suppose that $(k_1, \ldots, k_n)$ is the capacity allocation in the efficient auction preceding the capacity-constrained Cournot game. If at least one firm's capacity constraint is slack, i.e., $k_n > q_n^e(k_1, \ldots k_n)$, then $k_1 = \ldots = k_{n-1} = k^*$ and $k_n = K - (n-1)k^*$.*

Proof. We will argue that if some firm or firms have excess capacity and $(k_1, \ldots, k_n)$ differs from the proposed asymmetric capacity allocation, then there exists some perturbation that increases the total industry profit thereby contradicting the efficiency of $(k_1, \ldots, k_n)$.

First, we show that under the hypothesis of the lemma, there is *at least one* firm whose capacity constraint is binding in the downstream market. Suppose towards contradiction that all firms are unconstrained. Then they each produce q^*, where $q^* < k_i$. Redistribute capacities so that for all $i < n$, $k_i = q^*$, and $k_n = K - (n-1)q^*$. This change does not affect the downstream equilibrium production of any firm. Then, carry out the following perturbation: Reduce the capacity of each firm except firm n by an infinitesimal amount, dq, and increase k_n by $(n-1)dq$. As a result, the total production changes: Firm n gains $dq_n = r^{\theta}((n-1)q^*)(n-1)dq$, while the other firms lose a combined $dQ_{-n} = (n-1)dq$. Since $r^{\theta} > -1$, the change in total production is negative, that is, $dq_n + dQ_{-n} < 0$. The change in the total industry profit is,

$$\begin{aligned} d\Pi &= \frac{\partial \pi_n(q^*, (n-1)q^*)}{\partial q_n} dq_n + \frac{\partial \pi_n(q^*, (n-1)q^*)}{\partial Q_{-n}} dQ_{-n} \\ &\quad + \sum_{i=1}^{n-1} \left[\frac{\partial \pi_i(q^*, (n-1)q^*)}{\partial q_i} dq + \frac{\partial \pi_i(q^*, (n-1)q^*)}{\partial Q_{-i}} \left(dq_n + \frac{n-2}{n-1} dQ_{-n} \right) \right]. \end{aligned}$$

q^* is the unconstrained Cournot equilibrium production, therefore $\partial \pi_i(q^*, (n-1)q^*)/\partial q_i = 0$ for all i. By symmetry,

$$\frac{\partial \pi_i(q^*, (n-1)q^*)}{\partial Q_{-i}} = \frac{\partial \pi_j(q^*, (n-1)q^*)}{\partial Q_{-j}} \quad \text{for all } i, j = 1, \ldots, n.$$

Using these facts, the expression for $d\Pi$ simplifies to

$$\begin{aligned} d\Pi &= \frac{\partial \pi_n(q^*, (n-1)q^*)}{\partial Q_{-n}} dQ_{-n} + \sum_{i=1}^{n-1} \frac{\partial \pi_i(q^*, (n-1)q^*)}{\partial Q_{-i}} \left(dq_n + \frac{n-2}{n-1} dQ_{-n} \right) \\ &= \frac{\partial \pi_n(q^*, (n-1)q^*)}{\partial Q_{-n}} (n-1) \left(dQ_{-n} + dq_n \right). \end{aligned}$$

By $\partial \pi_n / \partial Q_{-n} < 0$ and $dq_n + dQ_{-n} < 0$, the change in total industry profit is positive, that is, $d\Pi > 0$. The perturbation of capacities increases the firms' total profit, hence the original distribution of capacities was not efficient, which is a contradiction.

For $n = 2$, the previous argument establishes that *exactly one firm* has excess capacity. We now prove that the same is true for $n > 2$ as well. Suppose towards contradiction that more than one firm has excess capacity, i.e., due to the way firms are indexed, $q^e_{n-1}(k_1, \ldots, k_n) < k_{n-1}$. Note that the capacity of firm 1 is binding, therefore $q^e_1(k_1, \ldots, k_n) = k_1 < q_{n-1}$. Redistribute all excess capacity from firms 2 through $n-1$ to firm n; this obviously does not change the production levels. Denote the new capacity levels by $(\tilde{k}_1, \ldots, \tilde{k}_n)$. Now decrease $\tilde{k}_{n-1} = q^e_{n-1}(k_1, \ldots, k_n)$ by dq and increase $\tilde{k}_1 = k_1$ by dq. Since firm 1's capacity is a binding constraint for its producton, q^e_1 increases by dq as well. As a result, the total production of all firms is unchanged. However, as the cost functions are strictly convex and the distribution of production among the firms has become less asymmetrical (we have increased q^e_1, decreased q^e_{n-1}, and $q^e_1 < q^e_{n-1}$ at the initial capacity levels), the total industry profit increases. The original allocation of capacities was not maximizing the total industry profit, which is a contradiction.

We conclude that if the capacity auction is efficient and there is a firm with excess capacity in the downstream market then it is firm n (i.e., there can only be one firm with slack capacity). Due to symmetry, the allocation of capacities that maximizes the total downstream industry profit subject to the constraint that firm n best-responds to the joint production of the other firms is the same for firms 1 through $n-1$, that is, $k_1 = \ldots = k_{n-1} = k^*$. The capacity-constrained firms each produce k^*, while the unconstrained firm produces $r((n-1)k^*)$. The optimal capacity constraint, k^*, maximizes the total industry profit, (9). ■

The result of Lemma 1 is remarkable because it pins down the industry structure in our model whenever there is any slack capacity in the downstream market. Note that the asymmetric allocation of capacities ($k_1 = \ldots k_{n-1} = k^*$, $k_n = K - (n-1)k^*$) and the corresponding asymmetric production ($q_1^e = \ldots = q_{n-1}^e = k^*$, $q_n^e = r((n-1)k^*)$) do not depend on the total amount of available capacity, K. Observe that in this outcome, the small constrained firms indeed produce less than the unconstrained big firm as $k^* < r((n-1)k^*)$.

The only possibility that we have not considered is that *no firm has slack capacity* in the Cournot game that follows the efficient capacity auction. In this case, since the production technologies are symmetric and exhibit stritly decreasing returns, the efficient capacity allocation must be symmetric. (By distributing the total capacity K, we essentially distribute a fixed total production among the firms because all capacity is fully used. The most efficient way to produce a fixed quantity is by spreading it evenly across the firms.)

The conclusion of this discussion is that the efficient capacity allocation is either asymmetric with exactly one firm receiving excess capacity and the others all receiving k^*, or symmetric with all capacities binding in the downstream market. Note that the asymmetric outcome can only arise as the solution when it is feasible, that is, when $K \geq Q^* \equiv (n-1)k^* + r((n-1)k^*)$. If $K < Q^*$ then we know the efficient capacity allocation is symmetric, $k_i = K/n$ for all i.

The following proposition states the main result of this subsection: There exists a threshold level of total capacity, $\hat{K}$, such that the efficient capacity allocation is symmetric for $K < \hat{K}$ and asymmetric for $K > \hat{K}$. The threshold $\hat{K}$ falls strictly in between Q^* and nq^*.

Proposition 2 *Define $\hat{K}$ such that*

$$P(\hat{K})\hat{K} - nc(\hat{K}/n) = P(Q^*)Q^* - (n-1)c(k^*) - c(r((n-1)k^*)), \tag{11}$$

where k^* satisfies (10) with $Q^* = (n-1)k^* + r((n-1)k^*)$.

(a) If $K < \hat{K}$ then the efficient capacity allocation is symmetric, that is, each firm receives capacity K/n.

(b) If $K > \hat{K}$ then the efficient capacity allocation is such that all but one firm gets capacity $k^* < q^*$, while exactly one firm gets capacity $K - (n-1)k^*$.

Proof. We already know that the efficient capacity allocation is either symmet-

ric, where $k_i = K/n$ for all i and all capacity constraints bind, or asymmetric as in Lemma 1, where $k_i = k^*$ for $i < n$ and $k_n = K - (n-1)k^*$. Note that the former allocation is the efficient one when the latter is not feasible, that is, $K \leq Q^*$.

Recall that we say that the capacity allocation is efficient when it maximizes the total industry profit in the capacity-constrained Cournot game. In the downstream market following the symmetric capacity allocation the total industry profit is $P(K)K - nc(K/n)$, which is strictly concave in K. Moreover,

$$P(Q^*)Q^* - nc(Q^*/n) > P(Q^*)Q$$

the consumer surplus in the downstream market (payments to the auctioneer cancel). The total surplus is continuous and strictly increasing for $K < \hat{K}$ because K is allocated symmetrically (which is socially desirable), all capacity is fully used in production, and the total production is lower than the Cournot output ($K < \hat{K} < nq^*$). However, the total surplus *discretely falls* as K exceeds $\hat{K}$. This is so because the firms' total profit is continuous at $K = \hat{K}$ by equation (11), but the consumer surplus falls discontinuously together with the total output in the downstream market. (The total surplus stays constant for all $K > \hat{K}$.) The policy consequence is that a social planner should restrict the quantity sold in the capacity auction to $\hat{K}$ whenever K exceeds $\hat{K}$.

3.2 Comparison with Benchmarks

Now we turn to the comparison of the industry structure in our model to certain benchmarks: (i) the symmetric unconstrained Cournot outcome, (ii) the monopoly, and (iii) the perfectly coordinated collusive cartel. We also discuss the limiting case of the model as the cost function becomes affine (constant returns to scale).

First, by symmetry, $r' \in (-1, 0)$, and the fact that firms 1 through $n-1$ are capacity constrained while firm n is not, it follows that

$$q_1^e = k^* < q^* < r((n-1)k^*) = q_n^e.$$

The small firms each produce less than the per-firm Cournot output while the unconstrained firm produces more than that.

Second, we claim that $k^* > 0$, that is, the outcome of our model always differs from that of a monopoly. To see this,differentiate (9), the total industry profit in the asymmetric solution, in k to get

$$(n-1)\left\{(1 + r'((n-1)k))\left[P'(Q)\,Q + P(Q)\right] - c'(k) - r'((n-1)k)c'\left(r((n-1)k)\right)\right\},$$

where $Q = (n-1)k + r((n-1)k$. For $k = 0$ and $n < \infty$, this simplifies to

$$\begin{aligned}(n-1)\left\{\left[P'(Q^M)Q^M + P(Q^M)\right](1 + r'(0)) - c'(0) - r'(0)c'(Q^M)\right\}\\ = (n-1)\left[c'(Q^M) - c'(0)\right],\end{aligned}$$

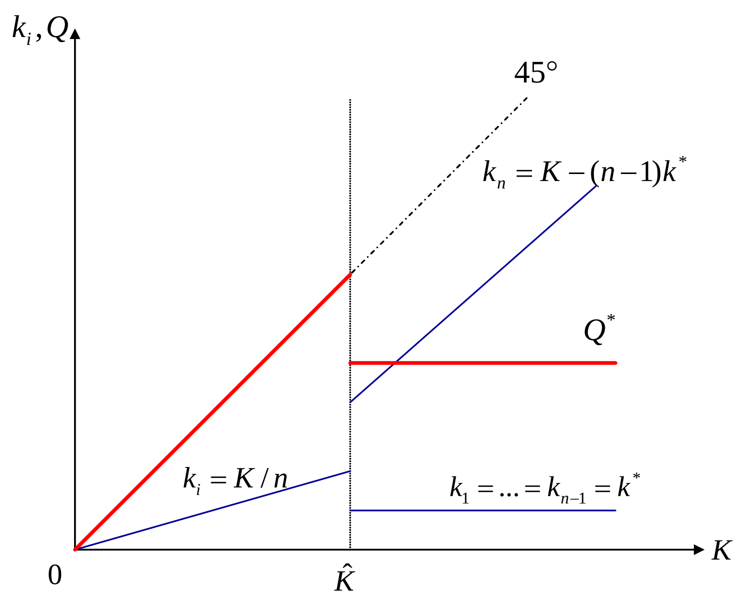

Figure 2: Capacities and total downstream production as a function of K

where $Q^M \equiv r(0)$ is the monopoly output, and on the second line the first-order condition of profit maximization by a monopoly, equation (4), is used. Since $c'(Q^M) > c'(0)$ by the strict convexity of c, the total industry profit is strictly increasing at $k = 0$. Hence $k^* > 0$.

Third, the outcome of our model is different from that of a collusive cartel unless $K = Q^C$ where Q^C is given by equation (5). This is so because our outcome is asymmetric for $K > \hat{K}$ (hence it cannot coincide with that of the cartel, which is symmetric), and it is symmetric but the total downstream production equals K (not Q^C) for $K < \hat{K}$.

Finally, it may be instructive to consider a limiting case of our model, when the production technology exhibits constant returns, that is, c is affine. While this case is ruled out by our assumption that c is strictly convex—which has been used in the proof of Lemma 1, for example—it is easy to check that Proposition 1 goes through with constant marginal costs as well. Therefore, for any capacity allocation, there exists a unique equilibrium in the follow-up Cournot game. It is interesting to note, by comparing equations (4) and (5), that when c' is constant, the monopoly and cartel outputs are equal, $Q^M = Q^C$.

When c is affine, the efficient allocation of capacities depends on the total available capacity as follows. If K is less than Q^M then *all capacity allocations* are efficient. To see this, note that firm i's unconstrained best response to the other firms' joint production is at least $Q^M - Q_{-i}$ (this is so because the best response would be Q^M for $Q_{-i} = 0$ and $r' > -1$). Since $Q^M > K$ and $Q_{-i} \leq \sum_{j \neq i} k_j$, the unconstrained best response is not feasible: $Q^M - Q_{-i} > K - \sum_{j \neq i} k_j \equiv k_i$. Hence firm i maximizes its profit by producing k_i. Since each firm operates at full capacity, the total industry output and profits are the same no matter how the capacities are allocated. Therefore, all allocations are equally efficient. On the other hand, if K is at least as large as Q^M then the efficient capacity allocation is such that *one firm gets all the capacity*. This follows because for any initial allocation of capacities, the production that maximizes the firms' joint profits is Q^M. However, if more than one firm is allocated a positive capacity then the joint production in the Cournot game exceeds Q^M. The cartel's profit is maximized by shutting down all firms but one. This is the outcome for $K > \hat{K} = Q^M$, therefore, the outcome of our model is monopoly, which can be interpreted as perfect collusion among the firms.

3.3 Market Structure with an Infinite Number of Firms

Our preceding analysis of the market structure is valid for any finite number of firms. In this subsection, we investigate what happens to the market structure as the *number of firms becomes infinitely large.* In particular, we are interested in knowing whether the market structure of our model collapses into monopoly, perfect collusion, or perhaps perfect competition, in the limit as $n \to \infty$.

If the marginal cost is constant between zero and Q^M, then obviously, for all finite n and in the limit as $n \to \infty$, the outcome of our model is monopoly, which can be interpreted as a perfectly coordinated cartel. Therefore, in what follows, we again do not consider this special limiting case of the model.

In the analysis of the prevailing market structure with an inifinite number of firms we will assume that an infinite amount of good can only be sold at zero price, and that the marginal cost of producing the first unit is positive. These two assumptions ensure that as $n \to \infty$, the unconstrained Cournot equilibrium converges to "perfect competition" in the sense that the per-firm production converges to zero, and the total output converges to a quantity where the market's willingness to pay equals the marginal cost of any single infinitesimal firm. To see this, recall that the per-firm output in the unconstrained Cournot equilibrium satisfies $P'(nq^*)q^* + P(nq^*) = c'(q^*)$. As $n \to \infty$, q^* has to go to zero, otherwise $\lim_{n\to\infty} P(nq^*) = 0$, and $\lim_{n\to\infty} P'(nq^*)q^* \leq 0 < \lim_{n\to\infty} c'(q^*)$ yields a contradiction. If $q^* \to 0$ then $\lim_{n\to\infty} P(nq^*) = c'(0)$. We will continue to assume that there is sufficient total capacity to produce the unconstrained Cournot output, that is, $\lim_{n\to\infty} nq^* < K$.

Proposition 3 *Suppose that* $\lim_{Q\to\infty} P(Q) = 0$, $0 < c'(0) < c'(Q^M)$, *and that* $\lim_{n\to\infty} nq^* < K$. *In our model, as* $n \to \infty$, k^* *converges to zero, however,* $(n-1)k^*$ *tends to a positive number which is less than the limit of the total industry production. The market structure remains different from monopoly, unconstrained Cournot competition, and perfect collusion even as* $n \to \infty$.

Proof. Under these assumptions, the per-firm Cournot output converges to zero as the number of firms goes to infinity. Since k^* is less than q^* for any given n, it must also converge to zero.

We claim that $(n-1)k^*$ cannot converge to zero as $n \to \infty$. If it did then

$\lim_{n\to\infty} r((n-1)k^*) = Q^M$. By equation (4),

$$\begin{aligned}\left[P'(Q^M)Q^M + P(Q^M)\right][1 + r'(0)] &= c'(Q^M)\,[1 + r'(0)] \\ &> c'(0) + r'(0)c'(Q^M),\end{aligned}$$

where $c'(0) < c'(Q^M)$ is used on the second line. The strict inequality contradicts (10), the first-order condition characterizing k^*, for n sufficiently large.

Finally, we claim that if the total industry production converges to $\bar{Q}^*$ as n goes to infinity then $\lim_{n\to\infty}(n-1)k^* < \bar{Q}^*$. In other words, the output of the unconstrained firm does not shrink to zero as the number of firms grows large. (Its output is greater than q^* for any finite n, but q^* goes to zero as n goes to infinity.) Suppose towards contradiction that $r(\bar{Q}^*) = 0$. By the definition of the best-response function, equation (3), $P(\bar{Q}^*) = c'(0)$. This contradicts the first-order condition that defines k^* for n sufficiently large, because as $n \to \infty$, by (10), $P'(\bar{Q}^*)\bar{Q}^* + P(\bar{Q}^*) = c'(0)$, and hence $P(\bar{Q}^*) > c'(0)$. ■

4 The First-Period Allocation of Capacity

The mechanism that determines the allocation of firms' capacities in the first period so far has been treated as a "black box": We posited that it allocates each unit of capacity to the firm that values it the most. We may think of this as the outcome of efficient Coasian bargaining amongst firms. In this section, we open the black box and discuss two auction formats that give rise to such an efficient outcome.

4.1 The Vickrey-Clarke-Groves Auction

Vickrey (1961) described a general method for designing efficient mechanisms, according to which all so-called Vickrey-Clarke-Groves mechanisms are devised. All participants are requested to submit their monetary valuations for all possible allocations of the goods. The auctioneer chooses the allocation that maximizes the sum of the buyers' valuations. Each buyer pays the difference between the other buyers' total valuation in the hypothetical case that the goods were allocated efficiently only among them (i.e., excluding him) and in the allocation actually selected by the auctioneer. The rules induce all participants to submit

their valuations for every allocation honestly, and the outcome of the auction is efficient.

Under certain assumptions on the valuations of the buyers, the efficient outcome and the Vickrey payments can be implemented by simple auctions. For example, for the sale of a single object, the English auction is efficient under general conditions. Ausubel (2004) designed an ascending-price auction for the sale of multiple identical goods with decreasing private marginal valuations. An ascending-price auction can be designed even more generally (see Schummer et al, 2005). Vickrey's methodology can be extended to interdependent (i.e., not private) valuations as long as the participants signals are uni-dimensional.[7] Such an extension was done by Dasgupta and Maskin (2000), and Perry and Reny (2002), (2005).

The VCG mechanism is originally meant to solve social choice problems where all parties affected by the allocation of goods have an opportunity to submit bids. Therefore, it may be questionable to use such mechanisms for the sale of inputs or capacities that the buyers will use in a downstream market. The outcome of the VCG auction in this case is still "efficient," but that just means that it maximizes the joint profits of the bidders. While the mechanism can be amended, and made *socially* efficient, by requiring to get "bids" for all allocations on behalf of downstream customers and other affected parties as well, this is usually overlooked in practice. In what follows, when we refer to the mechanism as the "efficient capacity auction" we mean a VCG auction where the buyers' joint payoffs are maximized.

The actual rules of the VCG auction in our model are as follows.

1. Each buyer-firm is required to submit a valuation for all capacity allocations, $(k_1, \ldots, k_n)$ with $\sum_{i=1}^{n} k_i = K$, denoted by $b_i(k_1, \ldots, k_n)$.

2. The auctioneer computes the allocation that maximizes $\sum_{i=1}^{n} b_i(k_1, \ldots, k_n)$, which we denote by $(k_1^*, \ldots, k_n^*)$.

3. Firm i is required to pay the difference between the value of the efficient allocation that would be obtained in i's absence, $\sum_{j \neq i} b_j(k_1^{\text{o}}, \ldots, k_n^{\text{o}})$, where $(k_1^{\text{o}}, \ldots, k_n^{\text{o}})$ maximizes $\sum_{j \neq i} b_j(k_1, \ldots, k_n)$ subject to $\sum_{j \neq i} k_j = K$, and the value of the efficient allocation for the other firms, $\sum_{j \neq i} b_j(k_1^*, \ldots, k_n^*)$.

[7] For multidimensional signals, see Jehiel and Moldovanu's (2001) impossibility theorem

It is routine to check that the above rules induce each firm to submit $b_i(k_1, \ldots, k_n) = \Pi_i(k_1, \ldots, k_n)$, i.e., all firms bid honestly. Each firm that gets something in the efficient capacity allocation pays a positive price. Finally, each firm has an incentive to participate in the auction. The last claim can be seen by noting that firm i's payoff in the auction is

$$\Pi_i(k_1^*, \ldots, k_n^*) - \left[\max\left\{\sum_{j \neq i} \Pi_j(k_1, \ldots, k_n) \middle| \sum_{j \neq i} k_j = K\right\} - \sum_{j \neq i} \Pi_j(k_1^*, \ldots, k_n^*)\right],$$

which is non-negative since

$$\Pi_i(k_1^*, \ldots, k_n^*) + \sum_{j \neq i} \Pi_j(k_1^*, \ldots, k_n^*) = \max\left\{\sum_{j=1}^n \Pi_j(k_1, \ldots, k_n) \middle| \sum_{j=1}^n k_j = K\right\}$$

is at least as large as $\max\left\{\sum_{j \neq i} \Pi_j(k_1, \ldots, k_n) \middle| \sum_{j \neq i} k_j = K\right\}$.

There are other auctions that may yield the same capacity allocation, which are in fact more widely used in practice. In the following, we discuss such auction formats.

4.2 Alternative Rules for the Capacity Auction

In the previous subsection, we demonstrated how to implement the "efficient" capacity allocation via a VCG auction. In this subsection we first show that the *uniform-price share auction* (first analyzed by Wilson (1979)) may result in the same outcome. Then, we discuss other mechanisms (including a dynamic auction) whose equilibrium is socially preferable to the "efficient" one. Throughout this subsection, to keep the notation and the arguments simpler, we assume $n = 2$.

In the uniform-price share auction for K units of capacity each firm i is required to submit an inverse demand schedule, $p_i(k_i)$, $k_i \in [0, K]$, which specifies the highest unit price firm i is willing to pay in exchange for k_i units of capacity. The auctioneer aggregates the demands and computes a market clearing price. A price level, say p^*, is called market clearing if there exists a capacity vector (k_1, k_2) such that $k_1 + k_2 = K$ and $p_i(k_i) = p^*$ for $i = 1, 2$. Each firm i is then required to buy k_i units of capacity at unit price p^*.

Proposition 4 *Assume $n = 2$. There exists an equilibrium in the uniform-price share auction that implements the efficient capacity allocation.*

Proof. Denote the efficient capacity allocation by $(k^*, K - k^*)$, and pick a positive p^* such that $p^*k^* \leq \Pi_1(k^*, K - k^*)$ and $p^*(K - k^*) \leq \Pi_2(k^*, K - k^*)$. We claim that the following strategies constitute an equilibrium of the share auction:

$$\begin{aligned} p_1(k_1) &= \frac{1}{K - k_1}\left[\Pi_2(k_1, K - k_1) - \Pi_2(k^*, K - k^*) + (K - k^*)p^*\right], \\ p_2(k_2) &= \frac{1}{K - k_2}\left[\Pi_1(K - k_2, k_2) - \Pi_1(k^*, K - k^*) + k^*p^*\right]. \end{aligned}$$

To see this, first note that if both firms submit these inverse demand schedules then the market clearing price is p^* with $k_1 = k^*$, $k_2 = K - k^*$ because $p_1(k^*) = p_2(K - k^*) = p^*$.

Second, notice that if firm 1 uses the proposed bid function p_1, then by the definition of p_1 and $k_1 + k_2 = K$, for all levels of k_2 that firm 2 may be able to induce by choosing an appropriate bid function, we have

$$\Pi_2(K - k_2, k_2) - p_1(K - k_2)k_2 = \Pi_2(k^*, K - k^*) - (K - k^*)p^*.$$

Therefore, firm 2 is indifferent between inducing the capacity allocations $(K - k_2, k_2)$ and $(k^*, K - k^*)$. Hence firm 2 has no profitable deviation from the equilibrium. Similarly, if firm 1 is able to induce an allocation $(k_1, K - k_1)$ given that firm 2 bids according to p_2, then by the definition of p_2 and $k_1 + k_2 = K$,

$$\Pi_1(k_1, K - k_1) - p_2(k_2)k_1 = \Pi_1(k^*, K - k^*) - k^*p^*.$$

So firm 1 has no incentive to deviate from the proposed equilibrium either. We conclude that p_1 and p_2 constitute an equilibrium of the uniform price share auction. ■

Remark 1 *From the proof of Proposition 2 it is clear that for $n = 2$, any capacity allocation can be supported in an equilibrium of the uniform-price share auction. When $n > 2$, this is not the case. However, the efficient capacity allocation can be supported in equilibrium for any n.*

It is well known that the uniform price share auction exhibits multiple equilibria. Wilson (1979) showed that there may be several unit prices that may be supported in an equilibrium. We are not concerned about the multiplicity of

p^* because our focus is not the revenue generated by the auction, but the the capacity allocation. As we remarked above, there is a multiplicity of equilibria with respect to the capacity allocation as well. A straightforward argument as to why we would expect the capacity allocation $(k^*, K - k^*)$ to emerge as the focal equilibrium is that this allocation is efficient, that is, it maximizes the joint profits of the two firms.

The difficulty of extending Proposition 4 to $n > 2$ stems from the observation that in the uniform-price auction, each firm submits a bid that only depends on the capacity that it receives, not the entire allocation. Therefore, the bids in the uniform-price share auction with $n > 2$ cannot reflect as much information as the bids in the VCG auction do. However, this difficulty can be resolved because in the efficient capacity allocation all "small" firms are symmetric (receive capacity k^* each), and the single "big" firm gets the left-over capacity. As a result, an equilibrium supporting the efficient capacity allocation similar to the equilibrium exhibited in the proof of Proposition 4 can be constructed even if $n > 2$.

In light of the finding that the "efficient capacity auction" yields an asymmetric and socially undesirable outcome in the downstream market, it is perhaps more interesting to find out whether some other auctions (which are not "efficient" from the perspective of the capacity buyers) could yield socially better outcomes. One interesting "twist" on the rules of the auction that we investigate in the rest of the subsection is where each unit of the capacity is auctioned off separately over time, in a dynamic auction.

The rules of the dynamic auction are as follows. The total capacity to be sold is divided into small units (i.e., the K physical units are divided to make T "units for sale", where T may be much larger than K). At each point in time, $t = 1, \ldots, T$, one "unit" of capacity is sold at a second-price auction. This dynamic process is carried out fast enough so that discounting between periods can be assumed away.

At first glance, it may seem surprising that this dynamic auction does not yield the same "efficient" result as the VCG auction. In fact, we now show that if K is sufficiently large and the cost function exhibits constant returns to scale, then the dynamic auction proposed above is socially more desirable than the VCG auction.

Recall that under constant returns to scale, if K is sufficiently large (greater than the monopoly output in the downstream market), then the outcome of our model with an "efficient capacity auction" (such as the VCG capacity auction) is *monopoly*—one firm gets all the capacity, and all the other firms get no capacity at all. If this were the case in the dynamic capacity auction as well then in period T (the last period), the large firm would have to outbid every single other firm for the last unit of capacity. As a result, the large firm would have to pay a price (for the last unit of capacity) that equals the marginal value of one unit of capacity for a firm with zero capacity. The marginal value (or profit) of an additional unit capacity for a firm with zero capacity is quite high, but it decreases as the capacity of the small firm increases (at least locally, at zero initial capacity). Therefore, the large firm may find it more desirable to "give up" a unit of capacity earlier in the dynamic game, in order to "soften" the competition it will face for the last unit of capacity in the final period. Indeed, such a move is profitable for the large firm because its profit is "flat" in its capacity. (As the large firm produces at the monopoly level, a small decrease in production has a second-order effect on its profit, while the reduction in the price of the last unit of capacity is of the first order). Therefore, the large firm will not attempt to buy up all the capacity in the dynamic auction. The resulting downstream output exceeds the monopoly output, hence the dynamic auction yields a socially more desirable outcome than the "efficient" VCG mechanism.

5 Tobin's Q, Firm Size, and Demand Cycles

Our model makes a testable prediction on the relationship between firm size and Tobin's Q: We predict that Tobin's Q and firm size are negatively related. This implication holds no matter whether the firms buy their capacities at a non-discriminatory unit price (as in the equilibrium of the uniform-price share auction), or if they make Vickrey payments for their purchase in the capacity auction (as they would in a VCG mechanism). In the first two subsections we consider these two cases separately. Then, at the end of the section, we investigate the comparative statics of our model with respect to demand fluctuations. In particular, we show that a small slump in demand may lead to a relatively large drop in the downstream production, and that the industry

becomes more asymmetrical and concentrated during a contraction than it is during a demand-driven expansion.

5.1 Tobin's Q and Firm Size under Vickrey Payments

Suppose that each firm is required to make the so-called "Vickrey payments" in the capacity auction. Vickrey payments are the payments made in the VCG auction.

Denote Π_{-1}^{eff} the total profit of a subset of $n-1$ firms when the total capacity K is allocated *only among them* (i.e., excluding one firm) in a VCG auction. The allocation that the VCG auction implements for $n-1$ firms is $k_1 = \ldots k_{n-2} = k_{-1}^*$ and $k_n = K - (n-1)k_{-1}^*$ where k_{-1}^* solves

$$\left[P^{\prime}(Q_{-1}^*)Q_{-1}^* + P(Q_{-1}^*)\right]\left[1 + r^{\prime}((n-2)k_{-1}^*)\right] \\ = c^{\prime}(k_{-1}^*) + r^{\prime}((n-2)k_{-1}^*)c^{\prime}(r((n-2)k_{-1}^*)),$$

with $Q_{-1}^* = (n-2)k_{-1}^*$. This condition is just (10) for $n-1$ firms instead of n.

Denote the profit of firm $i = 1, \ldots, n$ in the allocation implemented by the VCG auction when *all firms* participate by Π_i^*. Firms $1, \ldots, n-1$ are small firms (with capacities k^* each), while firm n is the large firm (with capacity $K - (n-1)k^*$).

The Vickrey payment that a small firm is required to make in the VCG auction is

$$V_1(k^*) = \Pi_{-1}^{eff} - (n-2)\Pi_1^* - \Pi_n^*$$

while the payment that the large firm makes is

$$V_n(K - (n-1)k^*) = \Pi_{-1}^{eff} - (n-1)\Pi_1^*.$$

Tobin's Q is defined as the ratio of the firm's market value (here, its downstream profit) divided by its book value (here, book value of the capacity, i.e., the Vickrey payment). That is,

$$Q_i = \frac{\Pi_i^*}{V_i^*}.$$

Proposition 5 *If the firms are required to make Vickrey payments in the ca-*

pacity auction then firm size and Tobin's Q are negatively related, that is,

$$Q_1 > Q_n.$$

Proof. $Q_1 > Q_n$ is equivalent to

$$\frac{\Pi_1^*}{\Pi_{-1}^{eff} - (n-2)\Pi_1^* - \Pi_n^*} > \frac{\Pi_n^*}{\Pi_{-1}^{eff} - (n-1)\Pi_1^*}.$$

Cross-multiplying and rearranging yields, equivalently,

$$\Pi_n^{*2} + (n-2)\Pi_n^*\Pi_1^* - (n-1)\Pi_1^{*2} > \Pi_n^*\Pi_{-1}^{eff} - \Pi_1^*\Pi_{-1}^{eff}.$$

Factoring out $(\Pi_n^* - \Pi_1^*)$ yields

$$(\Pi_n^* - \Pi_1^*)\left[\Pi_n^* + (n-1)\Pi_1^*\right] > (\Pi_n^* - \Pi_1^*)\,\Pi_{-1}^{eff}.$$

Since $\Pi_n^* > \Pi_1^*$, this is equivalent to

$$\Pi_n^* + (n-1)\Pi_1^* > \Pi_{-1}^{eff},$$

which holds because, by definition, the VCG allocation is efficient for the firms.
■

5.2 Tobin's Q and Firm Size under Uniform Price for Capacity

Now assume that the (uniform) equilibrium price of a unit of capacity is $p^* > 0$. Let χ_i denote firm i's Tobin's Q after having been allocated capacity k_i; that is,

$$\chi_i \equiv \frac{P(Q)q_i - c(q_i)}{p^* k_i},$$

where the numerator is firm i's market value and the denominator firm i's book value.

Proposition 6 *Assume that $K > \widehat{K}$ so that the efficient auction induces the asymmetric allocation $(k^*, ..., k^*, K-(n-1)k^*)$ of capacity. In equilibrium, $\chi_1 = ... = \chi_{n-1} > \chi_n$, and so there is a negative relationship between firm size*

(as measured by either book or market value, capacity, output, or sales) and Tobin's Q.

Proof. Note first that if $K > \widehat{K}$, then one firm, say firm n, is allocated capacity $K-(n-1)k^*$ through the efficient auction, while each other firms is allocated capacity $k^* < K-(n-1)k^*$. Firms 1 to $n-1$ will then face a binding capacity constraint in the output market, producing each an output of k^*, while the large firm n will produce $r((n-1)k^*) \in (k^*, K-(n-1)k^*)$. Hence, by any measure of firm size s_i (capacity, book or market value, sales, output), $s_1 = ... = s_{n-1} < s_n$.

It remains to show that $\chi_1 = ... = \chi_{n-1} > \chi_n$. For $i \in \{1, ..., n-1\}$, we have

$$\chi_i = \frac{P((n-1)k^* + r((n-1)k^*))k^* - c(k^*)}{p^* k^*},$$

and so $\chi_1 = ... = \chi_{n-1}$. For the large firm n, Tobin's Q is

$$\chi_n = \frac{P((n-1)k^* + r((n-1)k^*))r((n-1)k^*) - c(r((n-1)k^*))}{p^*\left[K-(n-1)k^*\right]}.$$

We thus have $\chi_1 > \chi_n$ if and only if

$$P(Q^*) - \frac{c(k^*)}{k^*} > P(Q^*)\frac{r((n-1)k^*)}{K-(n-1)k^*} - \frac{c(r((n-1)k^*))}{K-(n-1)k^*},$$

where $Q^* = (n-1)k^* + r((n-1)k^*)$ is industry output. To see that this inequality does indeed hold, note that

$$\begin{aligned} P(Q^*) - \frac{c(k^*)}{k^*} &> P(Q^*) - \frac{c(r((n-1)k^*))}{r((n-1)k^*)} \\ &> P(Q^*)\frac{r((n-1)k^*)}{K-(n-1)k^*} - \frac{c(r((n-1)k^*))}{K-(n-1)k^*}, \end{aligned}$$

where the first inequality follows from $r((n-1)k^*) > k^*$ and the stricty convexity of c, and the second inequality from $r((n-1)k^*) < K-(n-1)k^*$. Hence, $\chi_1 > \chi_n$. ■

This prediction is consistent with the empirical evidence provided in Eeckhout and Jovanovic (2002). Using Compustat data, they show that (i) the market-to-book ratio (Tobin's Q) is lower for firms with larger book value, and

(ii) the market-to-book ratio is decreasing in firm sales.

5.3 Output and Market Structure Over the Business Cycle

In this subsection, we consider the comparative statics of our model with respect to the level of demand. We show that if K is just below the threshold level of capacity $\widehat{K}$, then a small slump in demand will be reinforced by a large contraction of output. But the change in output is asymmetric: all but one firm downsize, relinquishing their capacity to one large firm, which will then exhibit a low Tobin's Q. If on the other hand, K is just above $\widehat{K}$, then a small increase in demand will induce a large expansion of output. But, again, the expansion of output is asymmetric: the small (high-Q) firms will grow at the expense of the large (low-Q) firm. This result has potentially important implications for the economic effects of business cycles as it shows that an "efficient" allocation of capacity will result in a magnification of the business cycle.

Let $P(Q;\theta)$ denote inverse demand if the state of demand is given by $\theta \geq 0$. Conditional on θ, we make the same assumptions on the shape of inverse demand. Further, we assume that an increase θ will be associated with (i) an increase in demand, $\partial P(Q;\theta)/\partial\theta > 0$ for all $Q > 0$, $\lim_{\theta\to 0} P(Q;\theta)Q/n < c'(Q/n)$ for $Q > 0$, and $\lim_{\theta\to\infty} P(Q;\theta) + Q\partial P(Q;\theta)/\partial Q > c'(Q/n)$ for $Q > 0$; and (ii) less price-elastic demand, $\partial^2 P(Q;\theta)/\partial Q\partial\theta \geq 0$ for all $Q > 0$. These assumptions subsume the special case where an increase in the level of demand means a replication of the population of consumers, leaving consumers' tastes and incomes unchanged, and so inverse demand can be written as $P(Q;\theta) \equiv \widetilde{P}(Q/\theta)$ and satisfies $\widetilde{P}'(\cdot) < 0$.

Proposition 7 *There exists a threshold demand level $\widehat{\theta}(K)$ such that $\widehat{K}(\theta) < K$ if and only if $\theta < \widehat{\theta}(K)$ and $\widehat{K}(\theta) > K$ if and only if $\theta > \widehat{\theta}(K)$. That is, if demand is low, $\theta < \widehat{\theta}(K)$, the efficient auction induces the asymmetric capacity allocation $(k^*(\theta), ..., k^*(\theta), K - k^*(\theta))$, while if demand is high, the efficient auction induces the symmetric capacity allocation $(K/n, ..., K/n)$.*

Proof. Let

$$\varphi(K;\theta) \equiv P(K;\theta)K - nc(K/n) - \{P(Q^*;\theta)Q^* - (n-1)c(k^*) - c(r((n-1)k^*;\theta))\}, \tag{13}$$

and

$$\psi(q;\theta) \equiv P((n-1)k^* + q;\theta) + r\frac{\partial P((n-1)k^* + q;\theta)}{\partial Q} - c^{\prime}(q), \tag{14}$$

where $r((n-1)k^*;\theta)$ is defined by the first-order condition $\psi(r((n-1)k^*;\theta);\theta) = 0$, $Q^* \equiv (n-1)k^* + r((n-1)k^*;\theta)$, and k^* (which depends on θ) maximizes the expression in curly brackets in equation (13). As we have shown before, the threshold capacity level $\widehat{K}$ is uniquely defined by $\varphi(\widehat{K};\theta) = 0$.

We first show that $d\widehat{K}/d\theta > 0$. Since $\partial\varphi(\widehat{K};\theta)/\partial K < 0$, it follows from the implicit function theorem that $d\widehat{K}/d\theta > 0$ if and only if $\partial\varphi(\widehat{K};\theta)/\partial\theta > 0$. Applying the envelope theorem (as k^* maximizes the expression in curly brackets above), we obtain

$$\begin{aligned}\frac{\partial\varphi(\widehat{K};\theta)}{\partial\theta} &= \widehat{K}\frac{\partial P(\widehat{K};\theta)}{\partial\theta} - Q^*\frac{\partial P(Q^*;\theta)}{\partial\theta} \\ &\quad - \left[P(Q^*;\theta) + Q^*\frac{\partial P(Q^*;\theta)}{\partial Q} - c^{\prime}(r((n-1)k^*;\theta))\right]\frac{\partial r((n-1)k^*;\theta)}{\partial\theta}.\end{aligned}$$

From the first-order condition (14), it follows that the expression in brackets is negative. Since $r((n-1)k^*;\theta)$ is the large firm's best response, we have $\partial\psi(r((n-1)k^*;\theta);\theta)/\partial q < 0$, and so (from the implicit function theorem), $\partial r((n-1)k^*;\theta)/\partial\theta > 0$ if and only if $\partial\psi(r((n-1)k^*;\theta);\theta)/\partial\theta > 0$. Indeed,

$$\frac{\partial\psi(r((n-1)k^*;\theta);\theta)}{\partial\theta} = \frac{\partial P(Q^*;\theta)}{\partial\theta} + r((n-1)k^*;\theta)\frac{\partial^2 P((n-1)k^* + q;\theta)}{\partial Q\partial\theta} < 0.$$

Hence, $\partial r((n-1)k^*;\theta)/\partial\theta$. We now claim that $\widehat{K}\partial P(\widehat{K};\theta)/\partial\theta < Q^*\partial P(Q^*;\theta)/\partial\theta$. To see this, recall that $\widehat{K} < Q^*$. From our assumption on the cross-partial derivative of inverse demand, it then follows $0 < \partial P(\widehat{K};\theta)/\partial\theta < \partial P(Q^*;\theta)/\partial\theta$. Hence, $\partial\varphi(\widehat{K};\theta)/\partial\theta > 0$, and so $d\widehat{K}/d\theta > 0$.

We now show that $\widehat{K} \to 0$ as $\theta \to 0$. Our assumptions on inverse demand imply that for any fixed $K > 0$, $\lim_{\theta\to 0}\varphi(K;\theta) < 0$. The assertion then follows from the observation that $\varphi(K;\theta)$ is strictly concave in K. Next, we show that $\widehat{K} \to \infty$ as $\theta \to \infty$. To see this, note that $\varphi(K;\theta)$ is maximized at $K = Q^C$,

the perfectly collusive cartel output, which is implicitly defined by

$$P(Q^C;\theta) + Q^C \frac{\partial P(Q^C;\theta)}{\partial Q} - c'(Q^C/n) = 0.$$

Observe that $Q^C \to \infty$ as $\theta \to \infty$. Otherwise, if Q^C were bounded from above, the l.h.s. of the above equation would become strictly positive for θ sufficiently large; a contradiction. Since $\widehat{K} > Q^C$, the assertion is indeed correct.

Summing up, we have shown that $\widehat{K}$ is strictly increasing with θ, $\widehat{K} \to 0$ as $\theta \to 0$, and $\widehat{K} \to \infty$ as $\theta \to \infty$. Hence, there exists a unique $\widehat{\theta}$ such that $K > \widehat{K}$ if and only if $\theta < \widehat{\theta}$ and $K < \widehat{K}$ if and only if $\theta < \widehat{\theta}$. ■

An immediate implication of the proposition is that in a demand slump ($\theta < \widehat{\theta}$) the industry structure is more asymmetric than during a boom ($\theta > \widehat{\theta}$).

6 Differentiated Bertrand Competition

In this section we show that our main results extend to the case where the downstream industry is modeled by differentiated Bertrand competition, and firms are strategic complements instead of substitutes. The purpose of analyzing this extension is to demonstrate that our results are not due to some particular property of the Cournot model.

Assume that there are two firms that simultaneously set prices, denoted by p_i $(i = 1, 2)$. The demands for their goods are $q_1 = Q(p_1, p_2)$ and $q_2 = Q(p_2, p_1)$, respectively, where Q is decreasing in its first and increasing in its second argument. The firms have capacity constraints k_i $(i = 1, 2)$, which are determined in the first stage of the game.

We model differentiated Bertrand competition subject to capacity constraints as Maggi (1996). If firm i faces a demand $q_i \leq k_i$ then its cost is $c(q_i)$; if $q_i > k_i$ then its cost is $c(q_i) + \theta(q_i - k_i)$, where c is a strictly increasing function and θ is a large positive number. Verbally, this means that the firms (constrained or not) always serve the entire demand they face; however, producing beyond their respective capacity constraints carries a drastic monetary penalty. This assumption allows us to ignore the issue of *rationing* when demand exceeds capacity.[8] Therefore, we can focus on the main qualitative difference between

[8] Rationing naturally does not arise in the Cournot model with capacity constraints. Using

Bertrand and Cournot models: strategic complements vs. strategic substitutes.

We assume that for all capacity allocations (k_1, k_2) with $k_1 + k_2 = K$, there exist prices (p_1, p_2) such that $Q(p_1, p_2) = k_1$ and $Q(p_2, p_1) = k_2$. In order to ensure that the price vector that gives rise to demands that equal the capacities is *uniqe*, we assume that for all (p_1, p_2), $Q_1(p_1, p_2) + Q_2(p_1, p_2) < 0$, where Q_i denotes $\partial Q/\partial p_i$ for $i = 1, 2$. As a result of this assumption, firm 1's "iso-demand curve," $p_2(p_1)$, defined implicitly by $Q(p_1, p_2(p_1)) \equiv k_1$, has a slope greater than one: $p_2' = -Q_1/Q_2 > 1$. Therefore, the iso-demand curves may only intersect once, hence the point (p_1, p_2) where $Q(p_1, p_2) = k_1$ and $Q(p_2, p_1) = k_2$ is unique.

Firm 1's profit function when its capacity constraint is slack is $\pi(p_1, p_2) = p_1 Q(p_1, p_2) - c(Q(p_1, p_2))$. Assume that π is strictly concave in p_1, and define firm 1's unconstrained reaction function as $r(p_2) = \arg\max_{p_1} \pi(p_1, p_2)$. By symmetry, r is the unconstrained reaction function of firm 2 as well. Assume that r is differentiable with $r' \in (0, 1)$, which implies that there exists a unique equilibrium without capacity constraints, where both firms set $p^B = r(p^B)$. These assumptions could be expressed in terms of the true fundamentals (the functions Q and c), but, in the interest of brevity, we keep them in this form.[9] Denote the per-firm equilibrium output in the unconstrained differentiated Bertrand model by $q^B = Q(p^B, p^B)$.

If $Q(r(p_2), p_2) > k_1$, that is, firm 1's best response to firm 2's price yields a demand for firm 1's good that exceeds its capacity, then by the concavity of $\pi(p_1, p_2)$, the optimal (constrained) response for firm 1 is to set $p_1 > r(p_2)$ such that $Q(p_1, p_2) = k_1$. By symmetry, the same is true for firm 2: In case its unconstrained best response is not feasible, $Q(r(p_1), p_1) > k_2$, then its constrained best response to p_1 is $p_2 > r(p_1)$ such that $Q(p_2, p_1) = k_2$.

Our first result is that for all initial capacity allocations, there is an equilibrium in the ensuing differentiated Bertrand model subjected to capacity constraints.

Lemma 2 *For all k_1, k_2 with $0 < k_1 \leq k_2$ and $k_1 + k_2 = K$, there exists an equilibrium in the capacity-constrained Bertrand game.*

Proof. If $k_1 \geq q^B$ then both firms are capable of producing the unconstrained

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this model, we essentially assume it away in the Bertrand model.

[9] The reader may consult chapter 6.2 of Vives (2000) for details.

Bertrand equilibrium output. It is immediate that both firms setting p^B forms an equilibrium.[10]

In the rest of the proof assume $k_1 < q^B$. Find p_1^0 such that $Q(p_1^0, r(p_1^0)) = k_1$. Note that $p_1^0 > p^B$ because $Q(p^B, r(p^B)) = q^B > k_1$ and $Q(p_1, r(p_1))$ is decreasing in p_1.[11] We distinguish two cases depending on whether or not k_2 exceeds $Q(r(p_1^0), p_1^0)$.

Case 1: $Q(r(p_1^0), p_1^0) \leq k_2$. We claim that $(p_1^0, r(p_1^0))$ is an equilibrium.

Firm 2 is best responding to firm 1's price without violating its capacity constraint, therefore it has no profitable deviation.

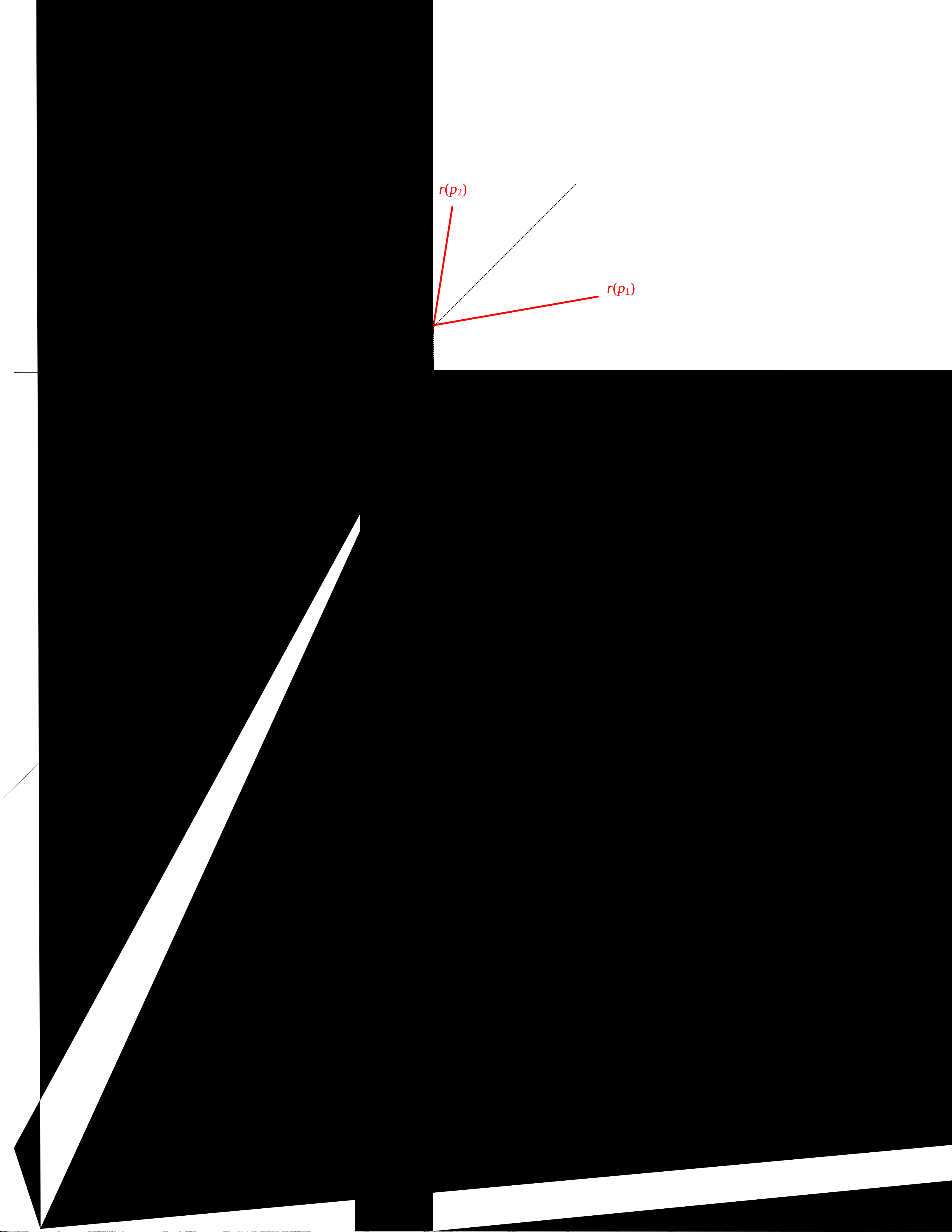
Firm 1's unconstrained best response to $r(p_1^0)$ would be $r(r(p_1^0))$. Since $p_1^0 > p^B$ and $r' \in (0,1)$, we have $p_1^0 > r(p_1^0) > p^B$, which then implies (by the same argument) that $r(p_1^0) > r(r(p_1^0)) > p^B$. But then $Q(r(r(p_1^0)), r(p_1^0)) > k_1$, that is, firm 1's best response to $r(p_1^0)$ violates its capacity constraint, because $Q(p_1^0, r(p_1^0)) = k_1$, $p_1^0 > r(r(p_1^0))$, and Q is decreasing in its first argument. Therefore firm 1's constrained best response to $r(p_1^0)$ is p_1^0, the price for which the capacity constraint holds as an equality.

Case 2: $Q(r(p_1^0), p_1^0) > k_2$. In this case, find (p_1^*, p_2^*) such that $Q(p_1^*, p_2^*) = k_1$ and $Q(p_2^*, p_1^*) = k_2$. We claim that (p_1^*, p_2^*) is an equilibrium.

First note that $p_1^0 < p_1^*$ and $p_2^* < p_1^*$. The first inequality holds because $Q(p_1^0, r(p_1^0)) = Q(p_1^*, p_2^*) = k_1$, $Q(r(p_1^0), p_1^0) > Q(p_2^*, p_1^*) = k_2$, and $Q_1 + Q_2 < 0$. Intuitively (graphically), we move along firm 1's iso-demand curve starting from $(p_1^0, r(p_1^0))$ in the direction where firm 2's demand decreases, so $p_1^* > p_1^0$ and $p_2^* > r(p_1^0)$. The second inequality follows because $k_1 < k_2$, and the firms are symmetric.

[10] The same prices form an equilibrium when the firms do not have capacity constraints. The only action that is not available to a firm without capacity constraint that is available to it with capacity constraint is decreasing its price so much that the capacity constraint becomes binding. However, such a move clearly cannot be profitable. Therefore there is no profitable deviation from equilibrium for either firm as long as their capacities exceed the equilibrium output without capacity constraints.

[11] This is so because $dQ(p_1, r(p_1))/dp_1 = Q_1 + Q_2 r' < Q_1 + Q_2 < 0$.



$r(p_2)$
$r(p_1)$

rewrite the joint profit as

$$P_1(k_1, k_2)k_1 - c(k_1) + P_2(k_1, k_2)k_2 - c(k_2). \tag{15}$$

We will assume that this expression is maximized in k_1 and $k_2 \equiv K - k_1$ at $k_1 = k_2 = K/2$. While (15) is symmetric in k_1 and k_2, this amounts to an additional (though mild) assumption. The assumption is made in the spirit of the original (Cournot) model, where the firms' joint profit maximizing quantity choice is symmetric as well.

Let K^C denote the joint production of a "cartel," that is, the value of K that maximizes $[P_1(K/2, K/2) + P_2(K/2, K/2)]\, K/2 - 2c(K/2)$. By definition (and the assumption in the previous paragraph), if the total capacity is K^C, then the optimal capacity allocation is $k_1 = k_2 = K^C/2$.

The symmetric allocation can only be optimal for K not exceeding the joint production in the unconstrained Bertrand equilibrium, $2q^B$. We now argue that even at $K = 2q^B$ it is strictly better for the firms to allocate the total capacity asymmetrically, so that the smaller firm (denoted by firm 1) becomes capacity constrained while the other firm becomes unconstrained in the ensuing equilibrium.

Suppose towards contradiction that each firm has capacity q^B and plays the unconstrained equilibrium by setting price p^B. Recall that $q^B = Q(p^B, r(p^B))$. Now reduce k_1 and increase k_2 by the same infinitesimal amount, $dk = [Q_1(p^B, p^B) + Q_2(p^B, p^B)r'(p^B)]dp$. Notice that by construction, firm 1 remains exactly capacity constrained if it increases its price by dp and firm 2 increases it by $r'(p^B)dp$. On the other hand, the same change in prices makes firm 2 unconstrained because the total demand decreases (as both prices go up) while the total capacity remains the same. Therefore, the resulting prices, $p^B + dp$ and $r(p^B + dp)$, form an equilibrium where firm 1 is constrained and firm 2 is unconstrained. We just need to show that the joint profit is higher in the new equilibrium. The change in the joint profit can be written as

$$\left. \frac{d\left[\pi(p_1, r(p_1)) + \pi(r(p_1), p_1)\right]}{dp_1} \right|_{p_1 = p^B} = \left[\pi_1(p^B, p^B) + \pi_2(p^B, p^B)\right] \left[1 + r'(p^B)\right],$$

where π_j denotes the derivative with respect to the jth argument. But this expression is positive because $\pi_1(p^B, p^B) = 0$ by the equilibrium condition, while $\pi_2(p^B, p^B) > 0$ and $r' > 0$.

We have so far established that for a total capacity level of $K = K^C$ the optimal capacity allocation is symmetric, while for $K = 2q^B$, the optimal allocation is asymmetric. By continuity (i.e., since the problem of optimal capacity allocation is continuous in K), there must exist an intermediate value of K, call it $\hat{K}$, where the optimal capacity allocation changes from symmetric to asymmetric. At such K, the joint profit of the firms is the same from splitting $\hat{K}$ equally and allocating it optimally in an asymmetric fashion. If the production technology exhibits strictly decreasing returns (i.e., c is strictly convex) then there is a discrete drop in the social surplus as K increases past $\hat{K}$. This is the exact same phenomenon that we found in the Cournot model. We summarize our findings regarding the differentiated Bertrand model in the following proposition.

Proposition 8 *In the differentiated Bertrand model, for some (low) values of K the efficient capacity allocation is symmetric, while for some other (high) values of K it is asymmetric. There exists a threshold value $\hat{K} \in (K^C, 2q^B)$ where the efficient capacity allocation changes from symmetric to asymmetric. Around $\hat{K}$, a small increase in the total available capacity reduces the social surplus.*

7 Conclusions, Future Research

It may be interesting and worthwhile to extend the model to allow for *substitutability* between the input obtained in the auction and other inputs. (In other words, we will treat the upstream good as "capital" rather than "capacity" as we do currently.) The intuition gained from our analysis suggests that our main results should generalize to that setup. Namely, we would expect that the resulting input allocation is asymmetric and suffers from the same types of inefficiencies that we identified in our model.

Another direction to future research is to explicitly incorporate upstream sellers (producers of the input or capacity used by the downstream firms) into the model. Our current model could easily be adapted to allow for a capacity

supply curve (i.e., the supply could be made more elastic). However, by explicitly considering upstream sellers it would be possible to compare different contracting modes between the sellers and buyers of the upstream good (the input or capacity). This extension would make it possible to formally compare centralized and decentralized input trading mechanisms.

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