Information Exchange and Competition in Communications Networks§

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Abstract

We develop a model of information exchange between calling parties. We characterize the equilibrium when two interconnected networks compete for such users by charging both for outgoing and incoming calls. We show that networks have reduced incentives to use off-net price discrimination to induce a connectivity breakdown when calls originated and received are complements in the information exchange. This breakdown disappears if operators are allowed to negotiate reciprocal access charges. We also study the relationship between sending and receiving retail charges as a function of the level of access charges. We identify circumstances where private negotiations over access charges induce first-best retail prices. In other words, endogenously chosen and unregulated access charges can internalize externalities between callers belonging to competing networks.

JEL Classification Codes: L41, L96.

Keywords: Interconnection, Access charges, Reception charges, Bill-and-keep, Information exchange.

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1. Introduction

People use electronic communications in order to exchange information. This exchange may take very different forms. Take, as an example, a situation with two individuals, A and B. A and B are academics trying to work on a joint paper but live in different places, thus they rely on telephone conversations and e-mail exchanges to conduct their work. One day, A and B are struggling to solve an equation. A finds the solution and calls B: B is obviously happy, but then does not need to call A back since the problem has been solved. Another day, A has an embryo of an idea and calls B. B needs some time to ponder over this rudimental idea and then calls A back to tell what she thinks about it: in this second case B is still happy of having received a call from A since it may lead to an improvement of their paper, but now the original call has initiated a chain reaction that has led B to return the call.

These simple examples illustrate a more general feature, namely that communications services are not consumed in isolation and involve interdependencies between callers. They also exemplify a phenomenon which seems particularly relevant for users, where an exchange of information may create the need for further exchanges of information, leading to a stimulation of calls. From a theoretical standpoint, two things are worth noticing. Firstly, receivers of an electronic communication enjoy a positive benefit (a *call externality*). Secondly, each individual has a demand for calls that depends not only on the price she has to pay, but also on the amount of calls received from the other party. Thus there is interdependency among incoming and outgoing messages and the demand functions depend on the prices charged to *both* parties. This paper studies a model of network competition when demand functions depend on the way information is exchanged between users.

Existing models of competition between interconnected networks typically assume that consumers derive utility only from making calls. This literature,[1] initiated by the seminal works of Armstrong (1998) and Laffont et al. (1998), asks if free negotiations over inter-carrier (wholesale) access charges can lead operators to choose (retail) fees that are detrimental to social welfare, for instance because they may be used to sustain collusive linear prices in the retail market. Another source of concern arises when access fees provide a commitment mechanism to limit competition over investment in infrastructure (Valletti and Cambini, 2005). If operators are of differing size they may not be able to reach a deal over the level of

[1] See the surveys of Armstrong (2002) and Vogelsang (2003).

reciprocal access charges (Carter and Wright, 2003). These works ignore the role played by call externalities which, as we argued above, are pervasive in electronic communications.

Given the normative intent of this literature, the absence of call externalities is potentially worrying since they have a strong impact on the (efficiency properties of the) competitive process. Recent literature has started to look into the problem of how to set retail charges in an environment with call externalities (Kim and Lim, 2001; DeGraba, 2003; Jeon et al., 2004; Berger, 2004 and 2005; Hermalin and Katz, 2005). Jeon et al. (2004; JLT hereafter) and Hermalin and Katz (2005; HK) are the papers closest to ours. They both determine competitive retail and reception charges in a setting where outgoing and incoming calls are perfectly separable. JLT show that, in the presence of a reception charge, the call price goes down as it reflects a pecuniary externality: when a network lowers its price, the subscribers to the *rival* network will have to pay more as they receive more calls. Hence an increase in the rival's reception charge makes it more desirable for a network to expand its output. JLT also show that, in the presence of network-based discrimination both on final prices and on receiving charges, "connectivity breakdowns" arise. In order to discourage subscribers from connecting to the rival, a network has an incentive to charge extremely high off-net call prices or off-net receiver prices. These results are worrying from a policy perspective and call for intervention (e.g., regulation of reception charges). HK also find that carriers have a tendency to reduce off-net traffic and that, under rather general conditions, access charges cannot be used to achieve full efficiency, even if set by a benevolent regulator.

All these works, however, ignore the interdependencies among users' calls and assume that the sets of calls/messages sent and received are independent of each other.[2] This basic assumption contrasts with empirical findings on telecommunications demand that show how incoming and outgoing calls are strongly interdependent. The starting point of our approach derives from this empirical evidence, started by the work of Larson *et al.* (1990) on point-to-point demand. This framework disaggregates traffic into two distinct elements: autonomous traffic – independent of the traffic level between the networks – and induced traffic, which is influenced by the traffic volume. Induced traffic strictly depends on how information between two customers is exchanged. Taylor (2004) reviews the empirical literature from the early 90's that applied point-to-point models to communications between city

[2] An exception is represented by Hermalin and Katz (2004) who analyze two-way communications. Their focus is on the possible strategic game between the communicating parties as to who will be the sender and who the receiver. The concern of Hermalin and Katz (2004) is on pricing under uncertainty about the parties' values of an exchange while they do not analyze multiple networks and the problem of interconnection.

pairs in Canada (inter-provincial traffic) and in the US (inter-LATA traffic), and between countries (Canada/US among others). This literature *rejects* separability between outgoing and incoming traffic, and finds strong and statistically relevant reverse-calling phenomena. Calls tend to generate calls in return.[3]

In this paper we take at face value these empirical findings that have been neglected so far by the theoretical literature. Call externalities have been either ignored or modeled in a simple fashion by assuming perfect separability between incoming and outgoing calls. We start by studying the process of information exchange which is the primitive that leads parties to enjoy utility from calls. Utility is not generated by calls *per se*, but rather calls are inputs to the production of information. Calls made and received could be substitutes (if the information received does not require the receiver to call back) or complements (if information becomes "productive" only by responding to queries initiated by one the parties). We embed the fundamental process of information exchange between two parties calling each other into the IO framework of network competition.

We propose a general formulation of the information exchange process between senders and receivers to study the role of inter-network access charges. Firstly, we show that, when access charges are taken as given, the risk of a "connectivity breakdown" is much diluted when induced traffic is positive (i.e., calls made and received are complements in the information exchange). In addition, we consider if mild forms of intervention might allow the industry to self-regulate. We show that the breakdown is completely eliminated if operators can choose reciprocal access charges (neither JLT nor HK address the case of access charges endogenously chosen by operators). Intuitively, although reciprocal deals give firms an instrument to tacitly collude, there is no need to destroy off-net traffic. On the contrary, operators prefer to choose low access charges so that off-net prices are also low: in this way the intensity of competition for the market is reduced as customers find a "big" network less appealing than a "small" one. While concerns for a breakdown are weakened, we still show how there exists some (limited) room for intervention to improve the outcome of private negotiations, for instance by using suitable reception charges.

Secondly, we describe the interaction between access charges and retail prices, both for outgoing and incoming calls, when all of them are determined endogenously. We show that operators set positive reception charges *only* when access charges are sufficiently low. Thus we provide an explanation for the common practice in telecommunications to charge

[3] "A call in one direction stimulates something like one-half to two-thirds of a call in return" (Taylor, p. 129).

only for outgoing calls and not for incoming calls when access (termination) charges are high enough, although receiver charges could also be set in principle.[4] We also show the result that, when induced traffic is positive, there exist conditions such that unregulated operators set reciprocal wholesale access charges that induce efficient retail pricing structures. This finding is quite striking and in stark contrast with both JLT and HK. It also clarifies the crucial role played by induced traffic: if incoming and outgoing calls are perfectly separable instead, we also find it impossible to achieve the first-best, as in the rest of the literature.

From the positive point of view our paper is about competitive pricing in the presence of externalities. Our analysis is also relevant, from the normative point of view, to assess the impact of recent regulatory interventions over inter-network charges. The discussion on the regulation of fixed-to-mobile termination charges in Europe, where CPP is in place, includes the adoption of charges based on Long Run Incremental Costs (LRIC), and the introduction of RPP. We establish a link between outgoing and incoming charges as a function of the access charge and we argue that RPP might emerge endogenously only if access charges are set sufficiently low. Thus we clarify some debate in the existing literature on the topic, where it is sometimes considered that CPP or RPP are mutually exclusive pricing policies (Crandall and Sidak, 2004; Littlechild, 2005). The determination of mobile-to-mobile termination is typically less intrusive and it includes reciprocal arrangements, and in some instances "bill-and-keep" deals. "Bill-and-keep" has been advocated as a way of sharing efficiently the value created by a call when both callers and receivers benefit from it (DeGraba, 2002). We give conditions under which this system is indeed efficient and we study if it can ever emerge from private negotiations.

The remainder of the paper is organized as follows. Section 2 introduces a model of information exchange between calling parties. Section 3 solves a model of network competition with uniform pricing. Section 4 analyzes the case with network-based price discrimination. Since price discrimination does not exhibit the result of profit neutrality with respect to access charges, we also ask what level would be chosen by negotiating operators as opposed to a social planner. In Sections 3 and 4 reception charges are fixed or set by a regulator, while Section 5 considers the case of market-determined reception charges. Section 6 concludes.

[4] The practice of charging customers both for originating and receiving calls is usually referred to as the Receiver Pays Principle (RPP) which is applied in the mobile markets of countries such as Canada, China, Hong Kong, and the U.S.. Positive reception charges make sense only if the receiver cares about being called, otherwise no one would ever answer a call. The practice of charging only for sending calls is known as the Calling Party Pays (CPP).

2. The information exchange process and call externalities

Imagine there are two users. One user subscribes to network i and the other user to network j and they communicate with each other. For the moment assume the networks are different, hence i can also denote the first user and j the second user. Each user is assumed to derive utility from the *amount of information created from originating and receiving calls* in a typical point-to-point connection. Information for user i, denoted I^{ij}, is produced according to the following production function: $I^{ij} = f^i(Q^{ij}, Q^{ji})$, where Q^{ij} is the quantity of outgoing calls user i originates to contact user j and Q^{ji} is the quantity of incoming calls that user i receives from user j. The production function $f(\cdot, \cdot)$ is one of the important primitives in our model and it can describe different cases when calls are substitutes or complements in the information creation process. $\partial I^{ij} / \partial Q^{ij} = f_1^i$ and $\partial I^{ij} / \partial Q^{ji} = f_2^i$ denote respectively how information changes at the margin with calls sent and received by i when communicating with j, and they are both assumed to be positive. We use superscripts ij to denote the exchange of information between i (the originator) and j (the receiver). This notation can also accommodate the case when both users are connected to the same network, in which case $j = i$ and $I^{ii} = f^i(Q^{ii}, Q^{ii})$.

Denote with p_i (respectively p_j) the price charged by network i (j) for calls originated by user i (j). Users have identical preferences over consumption of information and $U_i(I^{ij})$ denotes a standard (concave) utility function of user i when communicating with user j. We assume for the moment that users cannot be charged for receiving calls. Users i and j determine simultaneously the quantity of calls to consume by maximizing the following problems:

$$\text{User } i\text{: } \begin{cases} \max\limits_{Q^{ij}} U_i(I^{ij}) - p_i Q^{ij} \\ s.t. \quad I^{ij} = f^i(Q^{ij}, Q^{ji}) \end{cases} \qquad \text{User } j\text{: } \begin{cases} \max\limits_{Q^{ji}} U_j(I^{ji}) - p_j Q^{ji} \\ s.t. \quad I^{ji} = f^j(Q^{ji}, Q^{ij}) \end{cases}$$

The first-order conditions are respectively $U_i' f_1^i = p_i$ and $U_j' f_1^j = p_j$, which determine the equilibrium level of calls originated by user i (directed to j) and user j (directed to i): $Q^{ij} = Q^{ij}(p_i, p_j)$ and $Q^{ji} = Q^{ji}(p_j, p_i)$. Thus the quantity originated by a user is affected both by the price she pays and by the price other people pay.[5] Let $Q_1^{ij} = \partial Q^{ij}(p_i, p_j) / \partial p_i$ denote the "direct" price effect on quantities and $Q_2^{ij} = \partial Q^{ij}(p_i, p_j) / \partial p_j$ the "induced" price effect.

[5] We assume that both callers send a positive amount of calls. We discuss below the role played by this assumption.

Their sign is found by totally differentiating the first-order conditions. Denoting with $A^{ij} = U_i''\left(f_1^i\right)^2 + U_i' f_{11}^i < 0$ and $B^{ij} = U_i'' f_1^i f_2^i + U_i' f_{12}^i$, it results:

$$\begin{bmatrix} A^{ij} & B^{ij} \\ B^{ji} & A^{ji} \end{bmatrix} \begin{bmatrix} dQ^{ij} \\ dQ^{ji} \end{bmatrix} = \begin{bmatrix} dp_i \\ dp_j \end{bmatrix}$$

In a symmetric equilibrium with $p_i = p_j$, $A^{ij} = A^{ji} = A$, $B^{ij} = B^{ji} = B$ and:

$$\text{(1)} \qquad \begin{cases} Q_1^{ij} = A/(A^2 - B^2) \\ Q_2^{ij} = -B/(A^2 - B^2) \end{cases}$$

If both users are connected to the same network, i.e., calls are “on-net”, then $p_i = p_j$. For “on-net” calls the total effect of a price change is:

$$dQ^{ii} / dp_i = Q_1^{ii} + Q_2^{ii} = 1/(A + B).$$

In summary, a point-to-point exchange of information between two parties leads them to call each other, generating demand functions, $Q^{ij}(p_i, p_j)$ and $Q^{ji}(p_j, p_i)$. These demand functions are determined as a Nash equilibrium of the game played by the two users, where each user controls only one input of the information exchange. Demand functions react to the prices charged to both parties. The direct effect has a standard sign: $sign\left[\partial Q^{ij} / \partial p_i\right] = sign[A]$ $= sign\left[U''\left(f_1\right)^2 + U' f_{11}\right] < 0$. There is also an indirect effect that induces additional traffic: $sign\left[\partial Q^{ij} / \partial p_j\right] = sign[-B] = sign\left[-\left(U'' f_1 f_2 + U' f_{12}\right)\right]$. The sign of B depends on the properties of the information production function. It turns out to be zero only when utility is perfectly separable between the benefit derived from outgoing and incoming calls, as it is assumed by JLT and HK. As anticipated by the Introduction, separability does not seem to hold in the data. Calls tend to generate calls in return, thus sign[B] > 0. This would imply that incoming and outgoing calls are complements in the information exchange ($f_{12} > 0$).[6]

[6] The sign of the cross-price effects could also be thought in terms of the slope of the “best-response function” of each user, thus paralleling the cases of strategic complements and substitutes that are common to I.O. readers. In the problem of information exchange, however, users are not competing against each other in the usual sense.

2.1 A discussion of the information exchange

The exchange of messages is clearly a dynamic phenomenon. Our formulation should be seen as a reduced-form static representation of a more complex dynamic interaction between senders and receivers that we do not model explicitly. Thus one should not interpret the static Nash formulation described above as a game where the two end users must decide at a single, specific point in time how much they are going to call each other. To continue with our co-author example from the Introduction, we are not assuming that A, at the beginning of the collaboration with B, must decide how many messages he will send to B over the entire course of their collaboration, while B makes a simultaneous decision of the messages he will send to A.[7] However, it is still true that when A calls B because, at a given moment, he has information relevant to both, this would benefit B and might trigger further messages from B to A in a later period. It is this sequence of calls that the model of information exchange captures, not the one-way calling at each point in time. The cumulate amount of calls $Q^{ij}(p_i, p_j)$ from i to j is affected by the price paid by both parties, and is a simple way to capture, in a reduced-form, the interaction between users, which represents a significant departure from the existing literature that assumes independence between messages sent and received.

Having clarified the interpretation to the Nash representation, we also emphasize that the Nash solution described in Section 2 presupposes that there is an *interior* solution where both parties make some calls, i.e., both $Q^{ij} > 0$ and $Q^{ji} > 0$. It is easy to find light restrictions on functional forms that ensure that the Nash equilibrium game between users has an interior solution with a positive quantity of calls for both users and degenerate cases do not arise.[8]

These restrictions are reasonable to describe the information exchange. To see why, let us discuss what our approach rules out. We rule out corner solutions that would arise, for instance, if the amount of information sent and received were perfect substitutes, e.g., $I^{ij}= f(Q^{ij} + Q^{ji})$.[9] This functional form would be problematic for two reasons. Firstly, it is not clear why, any time the amounts of calls sent and received differ, the sender and the receiver must benefit in equal terms, as the functional form would imply. On the contrary, the amount of calls sent and/or received is expected to confer different benefits to the sender and to the re-

[7] To date, no one of the authors of this paper has had a privilege of working with such an organized co-author!

[8] These conditions are sufficient to ensure an interior solution: (i) $U_i(.)$ is differentiable and concave, (ii) $U_i(f^i(0,0)) = 0$, (iii) $U_i(f^i(0, Q_{ji}) > 0$ and there exists a value $\overline{Q}_{ij}$ such that $U_i(f^i(\overline{Q}_{ij}, Q_{ji}) - p_i\overline{Q}_{ij} > U_i(f^i(0, Q_{ji})$, (iv) $\lim_{Q_{ij}\to\infty} U_i(f^i(Q_{ij}, Q_{ji})) - p_i Q_{ij} < 0$, (v) $U(f^i(Q_{ij}, Q_{ji})) \neq U(f^j(Q_{ji}, Q_{ij}))$, when $Q_{ij} \neq Q_{ji}$.

[9] This functional form violates condition (v) of footnote 8.

ceiver. Secondly, this functional form that we rule out would imply that, when the two parties are symmetric, and p_i differs from p_j by a very small amount, only one party (the one that faces the cheaper price) would call, while the other party would never make a call, *for the entire duration of the information exchange*. This is not realistic in our view. Even in the extreme case where the parties are able to achieve a good level of co-ordination, so that the party that faces the cheaper price would make most calls, we can easily think of some point in time when the party that faces the more expensive prices would still have something to say and initiate the call, notwithstanding the price difference. This is precisely what our approach tries to capture and justifies its reliance on the manipulation of the first-order conditions of interior solutions.

Information exchange is a metaphor for representing the complex interaction between callers. It is easy to accommodate this framework also in the presence of reception charges. If network i (j) charges its customers a price r_i (r_j) per call for receiving calls, then the Nash equilibrium described in Section 2 that generates $Q^{ij}(p_i, p_j)$ and $Q^{ji}(p_j, p_i)$ is unchanged as long as customers accept every call, which indeed would happen if $U_i' f_2^i \geq r_i$ and $U_j' f_2^j \geq r_j$. We call this situation where reception charges do not "bind", either because they are zero or "low" enough, a situation of "sender sovereignty".

When reception charges do bind, the same analysis can be applied. In particular, if both reception charges are "high" enough compared to sending charges, then each customer effectively controls the amount of calls *received*. The first-order conditions in this case are respectively $U_i' f_2^i = r_i$ and $U_j' f_2^j = r_j$, which determine the volume of calls at the Nash equilibrium: $Q^{ij}(r_j, r_i)$ and $Q^{ji}(r_i, r_j)$. This situation of "receiver sovereignty" arises when sending charges do not bind: $U_i' f_1^i \geq p_i$ and $U_j' f_1^j \geq p_j$.

Finally, a last possibility arises when the binding prices (both for sending and receiving calls) are set by one network alone. This case is denoted as a situation of "network sovereignty". For instance, if network i is sovereign, then user i chooses both inputs in the information exchange. The first-order conditions in this case are $U_i' f_1^i = p_i$ and $U_i' f_2^i = r_i$, which determine the volume of calls: $Q^{ij}(p_i, r_i)$ and $Q^{ji}(r_i, p_i)$. Notice that this is a simple optimization problem conducted by user i, since user j is passive. This situation of "network i sovereignty" arises when user j accepts all calls received ($U_j' f_2^j \geq r_j$) and cannot send additional calls since they are truncated by i ($U_j' f_1^j \geq p_j$). The four situations of sender/receiver/network

i/network j sovereignty exhaust all possible cases for the determination of the volume of calls. Our analysis in Sections 3 and 4 concentrates on sender sovereignty since it is the more relevant case from an empirical standpoint. We do so by considering reception charges that are regulated and "low" enough, so that they do not bind. We do examine also the other situations when we analyze market-determined reception charges in Section 5.

3. The model of network competition

Demand for calls between a pair of users is generated according to the model described in the previous Section and summarized by the price effects of eq. (1). The model of competition follows Armstrong (1998), Laffont et al. (1998) and its extension by JLT. There are two networks, differentiated à la Hotelling. A unit mass of consumers is uniformly located on the segment [0,1] while the network operators are located at the two extremities. We denote by 1 (respectively 2) the firm located at the origin (respectively at the end) of the line. Customers pay a three-part tariff, $T_i(q) = F_i + p_i q_o + r_i q_r$, $i = 1, 2$, where the fixed fee F_i can be interpreted as a subscriber line charge, p_i as the unit price for originating q_o calls, r_i as the unit price that network i's consumer pays for receiving q_r calls.[10]

When a consumer located at l buys from firm i located at l_i, her utility is given by $y + v_0 - |l - l_i|/(2\sigma) + w_i$, where y is the income, v_0 is a fixed surplus component from subscribing (sufficiently large such that all customers always choose to subscribe to a network) and w_i is the indirect utility derived from making and receiving calls that we describe below. The parameter σ represents an index of substitutability between the networks.[11]

The consumer indifferent between the two networks determines the market share of the two firms. In particular firm i's share is α_i where:

(2) $$\alpha_i = \alpha(w_1, w_2) = 1/2 + \sigma(w_i - w_j)$$

where:

(3) $$w_i = \alpha_i v(p_i, p_i) + \alpha_j v(p_i, p_j) - r_i\left(\alpha_i Q^{ii}(p_i, p_i) + \alpha_j Q^{ji}(p_j, p_i)\right) - F_i$$

[10] The case with network-based price discrimination is considered in Section 4.
[11] HK concentrate most of their analysis on the case of undifferentiated networks ($\sigma \to \infty$) without fixed costs, which leads to possible tipping effects where all users subscribe to a single network.

(4) $$v(p_i, p_i) = U_i(f^i(Q^{ii}(p_i, p_i), Q^{ii}(p_i, p_i)) - p_i Q^{ii}(p_i, p_i)$$

(5) $$v(p_i, p_j) = U_i(f^i(Q^{ij}(p_i, p_j), Q^{ji}(p_j, p_i)) - p_i Q^{ij}(p_i, p_j)\,.$$

This system of equations describes a situation where a customer has many point-to-point information exchanges. A customer belonging to network *i* has α_i exchanges "on net" and α_j exchanges "off net". Each on-net exchange generates a quantity of calls Q^{ii} originated by the user connected to network *i* and destined to a user connected to the same network. This quantity depends only on p_i, and $v(p_i, p_i)$ is the corresponding (indirect) utility derived from each point-to-point exchange on net. Similarly, $v(p_i, p_j)$ is the (indirect) utility derived from each off-net exchange (this depends not only on p_i but also on p_j since the reverse traffic is originated by the rival network).

Both networks have full coverage. Serving a customer involves a fixed cost *f* of connection and billing. The marginal cost is *c* per call at the originating end and *t* at the terminating end. The total marginal cost is *c* + *t*. Networks pay each other a reciprocal two-way access charge, denoted by *a*, for terminating each others calls. Thus, network *i*'s profit is given by:

(6) $$\begin{aligned}\pi_i &= \alpha_i \left\{ \alpha_i \left(p_i - c - t + r_i \right) Q^{ii}(p_i, p_i) + \alpha_j (p_i - c - a) Q^{ij}(p_i, p_j) + F_i - f \right\} + \\ &+ \alpha_i \alpha_j (a - t + r_i) Q^{ji}(p_j, p_i)\end{aligned}$$

The first term in (6) is the profit deriving from originating calls terminated on the same network ($\alpha_i^2 \left(p_i - c - t + r_i \right) Q^{ii}$); the second term is profit from originating calls destined to the rival network ($\alpha_i \alpha_j (p_i - c - a) Q^{ij}$); the last term is the profit from termination and reception of incoming calls originated by rival customers ($\alpha_i \alpha_j (a - t + r_i) Q^{ji}$).

We consider the following timing of the game. First interconnection (*a*) and reception (*r*) terms are set in stage I. Then operators compete in two-part prices (*F* and *p*) in stage II. In particular, we assume that reception charges are always set "low" enough. In this way, we can concentrate on the case of "sender sovereignty" only. This analysis obviously includes the case where reception charges do not exist (*r* = 0) as is the case in many countries that adopt CPP. It also deals with the case of reception charges regulated in stage I by a benevolent social planner that makes sure that they do not "bind" in stage II.

3.1. Equilibrium with uniform pricing under "sender sovereignty"

In the Appendix we obtain the following expression for calling prices at equilibrium:

$$(7) \qquad p_i^* = c + t + \alpha_j (a - t)(\Phi - \Gamma) - \alpha_i r_j \Phi + (\alpha_i U_j' f_2^j \Phi - U_i' f_2^i \Theta)$$

where:

- $\Phi = \dfrac{\partial Q^{ij} / \partial p_i}{\partial Q_o^i / \partial p_i} = \dfrac{Q_1^{ij}}{\alpha_i (Q_1^{ii} + Q_2^{ii}) + \alpha_j Q_1^{ij}}$, where $Q_o^i = \alpha_i Q^{ii} + \alpha_j Q^{ij}$ denotes the total quantity of calls originated by network *i* and destined both on-net and off-net. Φ represents the ratio between the marginal change in outgoing off-net calls and the average marginal change in all outgoing calls (both off- and on-net, including the return effect on the latter), caused by an increase of price p_i; Φ is a summary of the "outgoing return phenomenon".
- $\Gamma = \dfrac{\partial Q^{ji} / \partial p_i}{\partial Q_o^i / \partial p_i}$ is the ratio between the marginal change in incoming off-net calls and the average marginal change in all outgoing calls, caused by an increase of price p_i. Γ is a summary of the "incoming return phenomenon".
- $\Theta = \dfrac{\partial Q_r^i / \partial p_i}{\partial Q_o^i / \partial p_i}$, with $Q_r^i = \alpha_i Q^{ii} + \alpha_j Q^{ji}$ denoting the total quantity of calls received by users connected to network *i*. Θ represents the ratio between the average marginal change in all incoming calls and the average marginal change in all outgoing calls, caused by an increase of price p_i. Θ is a summary of "incoming-to-outgoing" calls.

Eq. (7) can be interpreted as follows:

- $c + t + \alpha_j (a - t)(\Phi - \Gamma)$ represents the network's direct perceived marginal cost. The possible termination mark-up for off-net calls is multiplied by the difference between the off-net outgoing and incoming ratios, reflecting the two-way effect of call propagation;[12]

[12] Empirically, one would expect Φ ≥ Γ. Also notice that the sign of the termination mark-up does not matter if Φ = Γ, i.e., when outgoing and incoming calls are perfect complements.

- $-\alpha_i r_j \Phi$ is the pecuniary externality due to the presence of the reception charge r_j. This is multiplied only by the factor Φ, the outgoing return ratio from *i* to *j*, since this is the only direction that matters for the pecuniary externality imposed on *j*'s customers.
- $\alpha_i U'_j f_2^j \Phi - U'_i f_2^i \Theta$ represents the difference between the marginal utility of user *j* from receiving information through calls originated by users *i* (the marginal utility per point-to-point exchange is multiplied by the off-net outgoing ratio Φ and by *i*'s share) and the marginal utility of receiving calls by user *i* (user *i* gets information both on- and off-net, thus the marginal utility per exchange is multiplied by the incoming-to-outgoing ratio Θ). Prices thus depend on the difference between the positive externalities generated by the networks, as in a typical two-sided market setting (Rochet and Tirole, 2003 and 2005).

Eq. (7) generalizes previous results obtained in the literature. When the utility function is perfectly separable between outgoing and incoming calls, as it is in JLT, it results $Q_1^{ii} = Q_1^{ij}$ and $Q_2^{ij} = Q_2^{ii} = 0$, so that the incoming ratio is Γ = 0 and the outgoing ratio is Φ = 1, while the incoming-to-outgoing ratio simplifies to Θ = α_i. Then eq. (7) simplifies to $p_i^* = c + t + \alpha_j (a - t) - \alpha_i r_j + \alpha_i \left(U'_j f_2^j - U'_i f_2^i \right)$, that corresponds to (and partly amends) the result of JLT.[13] If users do not receive utility from being called (f_2 = 0) and are not charged for reception (r = 0), then (7) becomes simply $p_i^* = c + t + \alpha_j (a - t)$, as in Laffont et al. (1998). Under our more general specification we obtain the following:[14]

Proposition 1. *Under "sender sovereignty", the symmetric equilibrium with uniform multi-part pricing is characterized by the following conditions:*

a) the call price is given by:

$$p^* = c + t + \frac{2}{2+x}\left[\frac{a-t}{2}(1-x) - \frac{r}{2} - xU' f_2\right] \tag{8}$$

where x is a "propagation" factor given by: $x = \dfrac{\Gamma}{\Phi} = \dfrac{\partial Q^{ji} / \partial p_i}{\partial Q^{ij} / \partial p_i}$;

[13] Proposition 1 of JLT (p. 95) misses the last term in brackets, which is zero only in a symmetric equilibrium. This term represents the difference in users' marginal utility for receiving calls. It shows that if the marginal utility from receiving calls is higher for a rival user than for a own user, then, ceteris paribus, a network would *increase* its own price in order to reduce its outgoing calls and reduce the benefits of users of the rival network.
[14] All the proofs are relegated to the technical Appendix.

b) the fixed charge is given by: $F^{*} = f + 1/2\sigma - (p^{*} + r - c - t)Q(p^{*})$*;*

c) each network's profit is equal to $1/4\sigma$.

The result of profit neutrality (part c)) is common in symmetric models of network competition with multi-part uniform pricing. The more interesting feature of Proposition 1 lies in the expression for the call price given by eq. (8) where we have introduced a new element, the propagation factor *x*. This denotes the ratio between the change of off-net (return) calls and the direct change of on-net calls when the own price changes. In other words the propagation factor measures the relative impact of the reverse traffic phenomenon.

Using eq. (1) and the notation from Section 2, the expression for the propagation factor in a symmetric equilibrium is:

$$x = -B/A = -\left(U''f_1 f_2 + U'f_{12}\right)/[U''\left(f_1\right)^2 + U'f_{11}].$$

Since *A* is negative, the sign of *x* is the same as the sign of *B*. As we argued in the Introduction, the empirical evidence available to us suggests that incoming and outgoing calls are complements in the information production function, hence *x* should take positive values, although there is nothing in theory to prevent substitution effects, leading to negative values of *x*. It is also realistic to imagine that $x < 1$, while $x = 1$ corresponds to the case of perfect complements. This factor reduces to zero when the separability condition on utility holds, as in JLT, in which case eq. (8) reduces to $p^{*} = c + t + (a - t - r)/2$.

The impact of the propagation factor on the operators' pricing is two-fold. On the one hand, a higher *x* tends to decrease the price in order to induce the "call generates calls" phenomenon which is enjoyed by customers: this is the last term of the square bracket in eq. (8). On the other hand, call propagation also impacts on call prices via access imbalances and reception charges. The "perceived" off-net cost $(a - t)/2$ is pushed down the higher is the propagation factor *x* as every call off-net (for which $a - t$ is paid) induces a fraction of return calls (for which $a - t$ is received).

3.2 Welfare analysis

We now compare the equilibrium call charges with the social optimum. This is found by maximizing total welfare, i.e., the utility of the calling and the receiving parties minus the total cost, leading to $U'(f_1 + f_2) = c + t$. The optimal outcome reflects the externality and, under sender sovereignty (i.e., $U'f_2 \geq r$), could be implemented by setting the following call charge:

$$p = c + t - U'f_2 .$$

The optimal call charge should be set below the marginal cost, accounting for the positive externality from receiving calls. By equating eq. (8) to the optimal charge, one derives the following condition that would ensure efficiency:

(9) $$(a-t)(1-x) - r = -(2-x)U'f_2 .$$

Since there are two prices to be set (*a* and *r*), several cases can arise:

- Imagine there is no reception charge ($r = 0$), as is the case under a CPP system, then efficiency can be ensured only with below-cost access charges. In this case in fact from eq. (9) one gets $a = t - (2-x)U'f_2/(1-x) < t$. To understand this result, recall that efficiency dictates that the call price is set below cost: without a reception charge the only way to decrease the retail price is to cut the "perceived" marginal cost of a network, thus $a < t$. If the benefit from receiving calls is high, this might imply a "bill-and-keep" ($a = 0$).
- Imagine termination is set at the LRIC level, $a = t$, then efficiency is increased with a positive reception charge; however the first-best is not compatible with "sender sovereignty" if $x < 1$. In this case in fact from eq. (9) one gets $r = (2-x)U'f_2$. The level of the reception charge decreases with the level of the propagation factor but violates the constraint $U'f_2 \geq r$. Only the case of perfect complements ($x = 1$) makes LRIC regulation compatible with efficiency and sender sovereignty. This last result is more general: if $x = 1$ efficiency can always be ensured for *any* access charge if $U'f_2 = r$.

- A corollary of the previous case is that efficiency cannot be ensured under sender sovereignty if $a > t$ since from eq. (9) $r = (2-x)U'f_2 + (a-t)(1-x) > U'f_2$; thus no reception charge can induce to "call more" since the receivers would hang up.
- Imagine the reception charge is set at the network termination cost, $r = t - a$, then efficiency can be reached only if access is priced below marginal cost. In this case in fact from eq. (9) one gets $a - t = -U'f_2 < 0$. Notice that these conditions imply $U'f_2 = r$ and $a = t - r$, thus they just satisfy the constraint of "sender sovereignty". When the externality is high, this might imply a zero termination charge ($a = 0$) and a reception charge equal to the marginal cost of termination ($r = t$).

4. Network-based discrimination

Under uniform pricing operators are indifferent with respect to the level of the reciprocal access charge. This makes the model not very interesting to understand if the industry could "self-regulate" with a minimal degree of intervention, e.g., by simply imposing a reciprocity clause. To address this question we now consider the case of network-based discrimination, both on final prices and on receiving charges. In other words, customers are offered five-part tariffs of the form: $T_i(q) = F_i + p_i q_o + \hat{p}_i \hat{q}_o + r_i q_r + \hat{r}_i \hat{q}_r$, $i = 1, 2$, where $\hat{r}_i$ is the unit price for receiving calls originated on network $j \neq i$ and $\hat{p}_i$ is the unit price for off-net calls.

Firm i's share is still given by eq. (2) where now:

$$w_i = \alpha_i v(p_i, p_i) + \alpha_j v(\hat{p}_i, \hat{p}_j) - r_i \alpha_i Q^{ii}(p_i, p_i) - \hat{r}_i \alpha_j Q^{ji}(\hat{p}_j, \hat{p}_i) - F_i$$

is the net surplus for customers connected to network i. Notice that with the notation above we have again implicitly assumed that reception charges are "low" enough so that they do not bind in determining call volumes that are determined instead by senders alone. Network i's profit becomes:

$$\begin{aligned}\pi_i = {} & \alpha_i \left\{ \alpha_i (p_i - c - t) Q^{ii}(p_i, p_i) + \alpha_j (\hat{p}_i - c - t) Q^{ij}(\hat{p}_i, \hat{p}_j) + F_i - f \right\} + \\ & + \alpha_i \left\{ \alpha_i r_i Q^{ii}(p_i, p_i) + \alpha_j \hat{r}_i Q^{ji}(\hat{p}_j, \hat{p}_i) \right\} + \alpha_i \alpha_j (a - t) \left[Q^{ji}(\hat{p}_j, \hat{p}_i) - Q^{ij}(\hat{p}_i, \hat{p}_j) \right]\end{aligned} \tag{10}$$

The timing of the game is as before. First interconnection (a) and reception (r and $\hat{r}$) terms are set in stage I. Then operators compete in multi-part prices (F, p and $\hat{p}$) in stage II.

4.1. Equilibrium in stage II: price competition

Operators can internalize everything on-net, resulting in an efficient amount of on-net calls. On the contrary, there are uninternalized externalities off-net and thus inefficiency can arise with off-net communications. In the Appendix we prove the following:

Proposition 2. *When a symmetric equilibrium with sender sovereignty and termination-based discrimination exists, this equilibrium is characterized by:*

a) the on-net unit price is at the socially optimal value: $p^* = c + t - U'f_2$;

b) the off-net unit price is given by the following expression:

(11) $\hat{p}^* = c + a - (a-t)x - \hat{r} + U'f_2(1-x)$;

c) the fixed charge is given by the following expression:

(12) $F^* = f - (p^* - c - t)Q(p^*, p^*) + 1/2\sigma - \hat{r}Q(\hat{p}^*, \hat{p}^*) - \left(v(p^*, p^*) - v(\hat{p}^*, \hat{p}^*)\right)$;

d) a "connectivity breakdown" is less (more) likely to arise when the propagation factor x is positive (negative).

JLT find that when the utility function is separable and the utility from incoming calls is a fraction β of the utility from outgoing calls, there is always a "connectivity breakdown" if $\beta \geq 1$ via infinitely high off-net prices. To compare our results with JLT, denote with $\beta = f_2 / f_1$ so that $U'f_2 = \beta\hat{p}$. From eq. (11) the equilibrium off-net price is re-written as:

(11') $$\hat{p}^* = \frac{c + a - (a-t)x - \hat{r}}{1 - \beta(1-x)},$$

which takes finite values for $0 \leq \beta < \frac{1}{1-x}$. Even if β tends to 1 we do not have connectivity breakdown if the propagation factor is positive. In our framework, the breakdown appears (and is even more pronounced) when calls are substitutes (x < 0) in the production of information since it could occur also for values of β < 1, while the phenomenon of network breakdown is less likely when calls are complements (x > 0). Notice that when x = 1, i.e., when a

change of price induces the same change in outgoing and incoming volumes, then the off-net price simplifies to $c + t - \hat{r}$. The access charge a does not matter at all since for any outgoing call (for which a has to be paid) there will also be an incoming return call (for which a will be received).

A positive propagation factor helps ensure that the off-net exchange of information is not cut via a breakdown induced by high retail call charges when senders determine volumes.[15] A mirror result arises in the case of "receiver sovereignty" when calling prices do not bind. In this case, connectivity breakdown would occur in JLT if $\beta < 1$, via infinitely high off-net reception prices. In our model, we do not have a breakdown if the propagation factor is positive since reception charges take finite values for $\beta > 1 - x$.[16]

All in all, the magnitude of the connectivity problem is reduced compared to JLT if calls originated and received are complements in the information production function. To understand this, recall that the breakdown typically happens because one network (say the receiving) has the opposite interest to the other network (say the originating). However, once calls are complements to generate information, then the receiving network will need to allow termination of calls if it wants also its own originating calls to become "productive".

4.2 Stage I and the role of access charges

We now ask what level of access charges would be chosen in the first stage by negotiating firms as opposed to a social planner, to see if and how private and social interests diverge. In case of a discrepancy, in stage I a benevolent regulator may also set a reception charge.

The stage-II profit in a symmetric equilibrium is given by:

$$\pi^* = \tfrac{1}{4}\left(p^* + r - c - t\right)Q(p^*, p^*) + \tfrac{1}{4}\left(\hat{p}^* + \hat{r} - c - t\right)Q(\hat{p}^*, \hat{p}^*) + \tfrac{1}{2}\left(F^* - f\right),$$

where the expressions for the equilibrium off-net prices and fixed fees are given respectively by eq. (11') and (12): these are the only prices affected by access charges and by off-net reception charges. We imagine that in the first stage network operators are free to negotiate reciprocal access prices. We also suppose that a low-enough $\hat{r}$ is set exogenously and may be

[15] Since we have assumed so far that the volume of traffic is determined by callers, it must be $\beta\hat{p} \geq \hat{r}$, which from eq. (11') can be re-written as $\hat{r} \leq \beta\left[a(1-x) + c + tx\right]/(1+\beta x)$.

[16] A formal proof is available from the authors.

controlled only by the regulator. Given the symmetry, negotiations over access charges are equivalent to maximizing each operator's profit. The reciprocal access charge is set such that

$$(13)\qquad \frac{\partial \pi^*}{\partial a} = \frac{1}{4}\left[\frac{\partial Q(\hat{p}^*, \hat{p}^*)}{\partial a}\left(\hat{p}^* - c - t + \hat{r}\right) + Q(\hat{p}^*, \hat{p}^*)\frac{\partial \hat{p}^*}{\partial a}\right] + \frac{1}{2}\frac{\partial F^*}{\partial a} \leq 0 .$$

The effect of a on the volume of off-net calls is:

$$(14)\qquad \frac{\partial Q(\hat{p}^*, \hat{p}^*)}{\partial a} = \left[Q_1(\hat{p}^*, \hat{p}^*) + Q_2(\hat{p}^*, \hat{p}^*)\right]\frac{\partial \hat{p}^*}{\partial a} = (1+x)Q_1(\hat{p}^*, \hat{p}^*)\frac{\partial \hat{p}^*}{\partial a} .$$

The effect of a on the off-net price $\hat{p}^* = \dfrac{c + a - (a-t)x - \hat{r}}{1 - \beta(1-x)}$ is quite intricate in general. We cannot suppose that the ratio β of marginal utilities does not change with a. To simplify the problem, we assume from now that the propagation effect does not substantially change with a change in prices. In other words, if calls "propagate" at certain vector of prices – for instance every call sent induces 0.5 calls in return – a change of prices would not change the information exchange and calls would still "propagate" in the same manner with x = 50%. This assumption is restrictive but is arguably realistic for small price changes.[17] Under this simplifying assumption, we obtain the following:

Proposition 3. *If an interior solution exists, network operators jointly agree on the following reciprocal access charge:*

$$(15)\qquad a^* = t + \frac{1}{(1-x)(1+2\beta)}\left[-\beta(3-x)(c+t) + \hat{r}(2+\beta+\beta x) + \frac{Q(\hat{p}^*, \hat{p}^*)}{Q_1(\hat{p}^*, \hat{p}^*)}\frac{1-\beta(1-x)}{1+x}\right].$$

In order to understand eq. (15), consider the standard case with no reception charge ($\hat{r} = 0$) and no benefit from receiving calls (β = 0). Then eq. (15) simplifies to $a^* = t + Q/Q_1 < t$. This is the finding of Gans and King (2001): firms "collude" by setting be-

[17] This assumption is strictly true with utilities functions such as $U^i = (Q^{ij} + \rho Q^{ji})^{\alpha}$ with α < 1 and 0 < ρ < 1. In this case the propagation factor is constant and equal to x = -ρ < 0. This particular specification also exhibits β = constant. An example with a constant and positive propagation factor is $U^i = Q^{ij}(1 + \rho Q^{ji}) - {Q^{ij}}^2/2$, with x = ρ > 0. In this case the ratio β is not constant.

low-cost access charges, in the limit by adopting a "bill-and-keep" system. This is because the off-net calling charge becomes cheaper than the on-net charge, and customers then prefer to belong to the "smaller" network so that they make many cheap off-net calls. Operators are less aggressive, since it pays to be small. In equilibrium, this results in higher profits. This mechanism carries forward to our more general framework. The first term in the square bracket of eq. (15) is negative when customers care about receiving calls: part of this benefit is internalized by the operators, and they tend to push off-net calling prices down via lower access charges. The second term in the bracket reflects the impact of the reception charge. As operators try to avoid competition by pushing off-net prices down, and as the reception charge already achieves this via the pecuniary externality, then there is no need to push *a* too low when this effect is ensured by the reception charge. The last term is the effect of Gans and King, corrected by the call externality and by the propagation factor.

Eq. (15) is only a candidate equilibrium charge since, as we discuss in the Appendix, restrictions are needed to ensure that profit is concave in *a*. Also, this solution holds so long as the reception charge does not bind, i.e., $\hat{r} \leq \beta\hat{p}$. When it exists and is positive, an interesting implication is that connectivity breakdown would *not* happen even when β is very high: when *a* is endogenized and takes the value of eq. (15), then $\hat{p}^*$ from (11') becomes:

(16) $$\hat{p}^* = \frac{c+t+\hat{r}}{1+2\beta} + \frac{Q}{Q_1(1+2\beta)(1+x)},$$

that does not go to infinity for *any* value of the propagation factor and of the utility from receiving calls. This contrasts with Proposition 2, where we found that, under some parameter configuration, a connectivity breakdown would happen. The difference is that now operators can adjust the access charge as well: as the utility from receiving calls increases or as the propagation factor goes up, the access charge is pushed down so that the off-net price always takes finite values. In fact, it is easy to see that, if the reception charge is zero, then from eq. (16) the off-net price is strictly lower than the on-net price: $\hat{p}^* < p^* = (c+t)/(1+\beta)$.

This result has two immediate implications. Firstly, connectivity breakdown concerns are totally eliminated if operators are allowed to negotiate over access charges. Secondly, in the absence of reception charges, negotiations would not necessarily deliver efficient outcomes, since they would result in off-net prices that are "too" low. The latter point leads to

our next policy question. How does the privately chosen access charge compare to the socially optimal one? The access charge that maximizes social welfare is the one that makes the privately optimal off-net price in eq. (11') equal to the socially efficient price:

$$\hat{p}^* = p^* \Rightarrow \frac{c+a-(a-t)x-\hat{r}}{1-\beta(1-x)} = \frac{c+t}{1+\beta}.$$

Thus, the socially optimal access charge is:

$$(17) \qquad a^W = t - \frac{(c+t)\beta(2-x)}{(1-x)(1+\beta)} + \frac{\hat{r}}{1-x}.$$

Note that $a^W = t$, the so-called LRIC rule, is typically not socially optimal in the absence of reception charges, as we already discussed in Section 3.2 under uniform pricing. If $\hat{r} = 0$, then $a^W < t$ is socially efficient and so is a "bill-and-keep" system when β is sufficiently high. The same arguments apply here. Evaluating the difference between eq. (15) and eq. (17), we get:

$$(18) \qquad a^* - a^W = \frac{1-\beta(1-x)}{(1+\beta)(1+2\beta)(1-x)}\left[-\beta(c+t)+\hat{r}(1+\beta)+\frac{Q}{Q_1}\frac{1+\beta}{1+x}\right].$$

Eq. (18) is informative about the difference between privately- and socially-optimal access charges. In the special case when $\beta = 1/(1-x)$ there is no wedge between the two charges: the industry would self-regulate with no need of intervention. However, this result does not hold in general. In the absence of reception charges, negotiations over reciprocal access charges would set them too low from a social point of view, as both the first and the third term of the RHS of (18) are negative. Still, this may not be a problem if the socially-optimal charge from eq. (17) is already equal to zero: operators could not go below this level, thus they would agree on an efficient "bill-and-keep" system without having to mandate it.

If, on the other hand, both a^* and a^W take positive values, then it possible for the regulator to use a regulated reception charge to achieve efficiency. This level of the reception charge is determined by equating the LHS of (18) to zero, finding:

$$(19) \qquad \hat{r}^W = -\frac{Q(\hat{p}^*, \hat{p}^*)}{Q_1(\hat{p}^*, \hat{p}^*)}\frac{1}{1+x} + \frac{\beta}{1+\beta}(c+t)\,.$$

Our results on welfare are summarized in the following proposition:

Proposition 4. *In stage I of the game:*

a) unregulated negotiations over reciprocal access can eliminate the "connectivity breakdown" problem for off-net calling charges;

b) an efficient "bill-and-keep" system can be chosen by negotiating operators;

c) when negotiations lead to a positive access charge, the regulator can use positive reception charges to get closer to efficient choices, although he never achieves the first-best.

In summary, we have found two encouraging results, and a mildly encouraging one. We have shown that free negotiations over the reciprocal access charge would ensure that connectivity breakdown problems disappear. This is the first encouraging result. The intuition can be obtained once again by recalling that operators are better off by trying not to compete too hard against each other: only by allowing *cheap* off-net calls they can ensure that this is achieved. Their mutual interest goes *opposite* to a breakdown that is equivalent to infinitely high off-net charges. We have also seen that, under some circumstances, both a regulator and private firms would choose a "bill-and-keep" system, in which case efficient outcomes are reached with minimal intervention (only a reciprocity clause is needed). This is the second encouraging result. When negotiations are not efficient, it is still possible for a regulator to improve efficiency with an appropriate reception charge, although a lot of information on costs and demand is required on the regulator's side to achieve this outcome: this is our mildly encouraging result. In fact, it is impossible even for a perfectly informed regulator to achieve the first best via the regulation of reception charges: they should be set so high that they would violate sender sovereignty.

5. Market-determined reception charges

In the previous section we discussed the role that can be played by reception charges when set by a benevolent regulator. If they were instead negotiated by operators in stage I, it is clear that operators would have an interest divergent from the regulator's. The intuition that

we gave for negotiated access charges would carry forward to negotiated reception charges as well. Operators would try to obtain cheap off-net calls to reduce the intensity of competition. When off-net charges are already the lowest possible (i.e., “bill-and-keep” or $a = 0$), then the off-net price from (11’) is $\hat{p}^{*} = [c + tx - \hat{r}]/[1 - \beta(1 - x)]$: operators could start introducing *positive* reception charges as this allows them to push off-net retail prices even lower, via the pecuniary externality. This typically results in reception charges that are “too high” from the social point of view. Conversely, if the freely negotiated access charge is positive (but “too low” from a social point of view), then eq. (16) holds and the off-net prices increases with the reception charge. The regulator and the negotiating operators have conflicting interests: the former would like to introduce a positive reception charge as the overall resultint 57 0 TDpe uty the atnte(hav exactldy theop poa)]TJ1935807 0 TD0.0005 Tc011815 Tw[iteo interets and thyr wo

zero mean, and strictly positive density $g(\cdot)$ over the support. ε_i is i.i.d. for each pair of customers. Consider an off-net information exchange between i and j:

- If $\varepsilon_i \geq \hat{\varepsilon}_i = \hat{r}_i - U_i' f_2^i$ and $\varepsilon_j \geq \hat{\varepsilon}_j = \hat{r}_j - U_j' f_2^j$, then there is a "sender sovereignty" regime that generates calls $Q^{ij} = Q^{ij}(\hat{p}_i, \hat{p}_j)$ and $Q^{ji} = Q^{ji}(\hat{p}_j, \hat{p}_i)$;
- If $\varepsilon_i < \hat{\varepsilon}_i$ and $\varepsilon_j \geq \hat{\varepsilon}_j$, then there is a "network i sovereignty" regime that generates calls $Q^{ij} = Q^{ij}(\hat{p}_i, \hat{r}_i - \varepsilon_i)$ and $Q^{ji} = Q^{ji}(\hat{r}_i - \varepsilon_i, \hat{p}_i)$;
- If $\varepsilon_i \geq \hat{\varepsilon}_i$ and $\varepsilon_j < \hat{\varepsilon}_j$, then there is a "network j sovereignty" regime that generates calls $Q^{ij} = Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{p}_j)$ and $Q^{ji} = Q^{ji}(\hat{p}_j, \hat{r}_j - \varepsilon_j)$;
- If $\varepsilon_i < \hat{\varepsilon}_i$ and $\varepsilon_j < \hat{\varepsilon}_j$, then there is a "receiver sovereignty" regime that generates calls $Q^{ij} = Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i)$ and $Q^{ji} = Q^{ji}(\hat{r}_i - \varepsilon_i, \hat{r}_j - \varepsilon_j)$.

The volume of calls between i and j can be written as:

$$
\begin{aligned}
D^{ij} &= D^{ij}\left(\hat{p}_i, \hat{p}_j, \hat{r}_i, \hat{r}_j\right) = Q^{ij}(\hat{p}_i, \hat{p}_j)(1 - G(\hat{\varepsilon}_i))(1 - G(\hat{\varepsilon}_j)) + \\
&(1 - G(\hat{\varepsilon}_j))\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} Q^{ij}(\hat{p}_i, \hat{r}_i - \varepsilon_i) g(\varepsilon_i) d\varepsilon_i + (1 - G(\hat{\varepsilon}_i))\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{p}_j) g(\varepsilon_j) d\varepsilon_j \\
&+ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i) g(\varepsilon_i) g(\varepsilon_j) d\varepsilon_i d\varepsilon_j.
\end{aligned}
$$

The first term of the RHS is the demand under sender sovereignty, the second term is the demand when network i is sovereign and customer j accepts incoming calls, the third term is the demand when network j is sovereign and customer i accepts the calls, finally the fourth term is the demand under receiver sovereignty. The indirect utility for a customer belonging to network i from off-net exchanges with network j is $\alpha_j\left(\hat{v}^{ij} - \hat{r}_i D^{ji}\right)$ where:

$$
\begin{aligned}
\hat{v}^{ij} &= \hat{v}\left(\hat{p}_i, \hat{p}_j, \hat{r}_i, \hat{r}_j\right) = U_i(f^i(Q^{ij}(\hat{p}_i, \hat{p}_j), Q^{ji}(\hat{p}_j, \hat{p}_i)))(1 - G(\hat{\varepsilon}_i))(1 - G(\hat{\varepsilon}_j)) \\
&+ (1 - G(\hat{\varepsilon}_j))\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} U_i(f^i(Q^{ij}(\hat{p}_i, \hat{r}_i - \varepsilon_i), Q^{ji}(\hat{r}_i - \varepsilon_i, \hat{p}_i))) g(\varepsilon_i) d\varepsilon_i \\
&+ (1 - G(\hat{\varepsilon}_i))\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} U_i(f^i(Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{p}_j), Q^{ji}(\hat{p}_j, \hat{r}_j - \varepsilon_j))) g(\varepsilon_j) d\varepsilon_j \\
&+ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} U_i(f^i(Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i), Q^{ji}(\hat{r}_i - \varepsilon_i, \hat{r}_j - \varepsilon_j))) g(\varepsilon_i) g(\varepsilon_j) d\varepsilon_i d\varepsilon_j - \hat{p}_i D^{ij}.
\end{aligned}
$$

which is used to determine market shares as in eq. (2). The profit of network i is:

$$\text{(20)} \qquad \pi_i = \alpha_i \left[\alpha_i \left(p_i + r_i - c - t \right) D^{ii} + \alpha_j \left(\hat{p}_i - c - a \right) D^{ij} + F_i - f + \alpha_j (\hat{r}_i + a - t) D^{ji} \right].$$

We now illustrate the equilibrium in stage II, when operators compete in five-part prices. As before, on-net prices are efficient as everything is internalized and their expressions are not reported. The following proposition characterizes off-net prices as noise converges to zero:

Proposition 5. *As the noise vanishes,*

- *the only symmetric candidate equilibrium without connectivity breakdown when senders are sovereign most of the time is given by:*

$$\textit{(21)} \qquad \begin{cases} \hat{p}^* = \dfrac{c + a + (a-t)\left[1 - x(1+x)\right]}{1 - \beta + \beta x(1+x)} \\ \hat{r}^* = t - a + x^2 \dfrac{a - t + \beta(c+a)}{1 - \beta + \beta x(1+x)} \end{cases} \quad \textit{iff} \quad \frac{t - \beta c}{1+\beta} < a \le t + x^2 \frac{\beta(c+t)}{(1-x)(1+x-\beta)} \quad \textit{and}$$

$$\beta < \frac{1}{1 - x - x^2};$$

$$\textit{(22)} \qquad \begin{cases} \hat{p}^* = \dfrac{c + a - (a-t)x}{1 - \beta(1-x)} \\ \hat{r}^* = 0 \end{cases} \quad \textit{iff} \ a > t + \frac{x^2 \beta(c+t)}{(1-x)(1+x-\beta)} \ \textit{and} \ \beta < \frac{1}{1-x}.$$

- *The only symmetric candidate equilibrium without connectivity breakdown when receivers are sovereign most of the time is given by:*

$$\textit{(23)} \qquad \begin{cases} \hat{p}^* = a + c + x^2 \dfrac{a - t + \beta(c+a)}{\beta - 1 + x(1+x)} \\ \hat{r}^* = \dfrac{\beta\left[-2a - c + t + (a+c)x(1+x)\right]}{\beta - 1 + x(1+x)} \end{cases} \quad \textit{iff} \quad -c + \frac{x^2(c+t)}{\beta - 1 + \beta x} \le a < \frac{t - \beta c}{1+\beta} \quad \textit{and}$$

$$\beta > 1 - x - x^2;$$

$$\textit{(24)} \qquad \begin{cases} \hat{p}^* = 0 \\ \hat{r}^* = \dfrac{\beta\left[t - a(1-x) + cx\right]}{\beta - 1 + x} \end{cases} \ \textit{iff} \ a \le -c + \frac{x^2(c+t)}{\beta - 1 + \beta x} \ \textit{and} \ \beta > 1 - x.$$

- *There is a symmetric candidate equilibrium without connectivity breakdown and efficient prices only in the presence of a positive propagation factor:*

$$\text{(25)} \quad \begin{cases} \hat{p}^* = p^* = \dfrac{t+c}{1+\beta} \\ \hat{r}^* = \beta\hat{p}^* = \dfrac{\beta(t+c)}{1+\beta} \end{cases} \textit{iff } a = \frac{t-\beta c}{1+\beta} \textit{ and } 1-x < \beta < 1/(1-x).$$

We discuss eq. (21) and (22) as they relate to the case of sender sovereignty most of the time, which is the case people are familiar with in practical terms, as the number and length of calls is predominantly set by the caller and not the receiver for price reasons. The range of validity of the two equations is also plotted in Figure 1, where we have put the marginal utility from receiving calls on the horizontal axis and the level of the access charge on the vertical axis. Connectivity breakdown occurs in the bottom left corner (the downward-sloping curve describes the limiting condition for having sender sovereignty most of the time: $a > (t-\beta c)/(1+\beta)$; in this region reception charges are set so high that a connectivity breakdown arises) and to the right of the diagram (the vertical line is $1/(1 - x - x^2)$; in this region sending charges are set very high). In the other regions there is a candidate for a symmetric equilibrium with senders determining volumes most of the time.

Eq. (21) says that when termination charges are set above cost ($a > t$ = LRIC), then it is possible that competing operators should set *negative* reception fees. By paying consumers to receive calls, operators directly promote increased termination revenue. Indeed we have found several practical examples of such behavior in mobile telephony. However, negative reception charges may raise several problems. Firstly, telecom operators might lack the ability to set such charges because of the technical complexity associated to keeping track of receiving calls – this difficulty was present in the past. Secondly, negative prices may give rise to opportunistic behavior. For instance, some time ago in Italy mobile operators did set negative reception fees for a while only to find that people were calling their mobile phones from office lines so as to obtain these rebates. The scheme had to be withdrawn.[18] Thus in Proposition 5 and Figure 1 we also impose a non-negativity constraint on reception charges. This generates the upward-sloping curve in Figure 1. In the region above this curve, reception charges are zero and off-net call charges are given by eq. (22).

[18] http://www.cellularitalia.com/contrattigsm.html

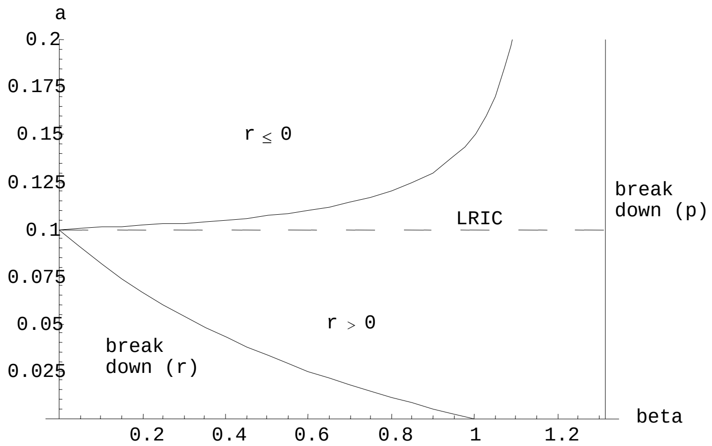

Figure 1 – Market-determined reception charges under "sender sovereignty"

[Parameter values: x = 0.2, t = 0.1, c = 0.1]

If, on the other hand, negative reception charges do not represent a problem and could be implemented, then eq. (21) is also valid in the region above the upward sloping curve. From eq. (21), it can also be found that the sum of outgoing and incoming off-net charges is:

$$\hat{p}^* + \hat{r}^* = c + t + \frac{(1-x)\left[a(1+\beta) - t + \beta c\right]}{1 - \beta + \beta x(1+x)} > c + t\,.$$

Thus, in the domain of validity of eq. (21) ($a > (t - \beta c)/(1+\beta)$) there is always an inefficiency result: too few calls are sent and received as the benefits of the receiver are not accounted for and the access charge cannot be set low enough to revert this tendency. The inefficiency disappears only when $a = (t - \beta c)/(1+\beta)$, in which case eq. (25) is valid. This is the limiting value of the access charge that indicates a shift between receiver and sender sovereignty. Efficiency emerges as firms are moving along the line $\beta\hat{p} = \hat{r}$ and thus internalize everything when setting their off-net call price.

These results shed light on the debate of CPP versus RPP: the ability of charging customers for receiving calls does not imply that reception charges are actually used by operators. This question can be answered only if the level of the access charge is known, otherwise different regimes can occur as illustrated by Figure 1. Our findings can explain the empirical observation that consumers are often *not* charged for receiving calls in countries with high termination charges, even when there is nothing in principle to prevent operators from introducing such charges. This is in line, for instance, with the European experience of calls to

mobile phones. On the contrary, if termination charges are set at or below cost then operators should start introducing reception charges if they have the ability to do so. This is in line with the North-American experience of mobile (cellular) telephony. Marcus (2004) discusses how the call termination system in the U.S. has a strong tendency toward symmetry in the rates charged for reciprocal compensation. In particular he shows that:

- ILEC-CLEC and ILEC-mobile reciprocal compensation rates are generally symmetric, and set at a rate that reflects the marginal cost of the ILEC.[19]
- ILEC-ILEC, CLEC-CLEC, CLEC-mobile, and mobile-mobile reciprocal compensation rates are determined through voluntary negotiations, and in many cases are set to zero ("bill-and-keep"), in particular for ILEC-ILEC and mobile-mobile interconnection.

The mobile sector is a particularly good candidate to test our positive findings since it is indeed a case of network-based competition where roughly symmetric mobile firms have to terminate calls on each other's network. The rate for reciprocal compensation is established through unregulated commercial negotiations. These agreements are generally on a "bill-and-keep" basis. Mobile operators also charge their customers for receiving calls (RPP): this is expected from our analysis in the presence of below-cost termination rates.[20]

5.1 A first-best result when the propagation factor is positive

The North-American evidence supports our findings in the last stage of the game, as it links reception charges to below-cost access (termination) charges. It also points to another empirical fact, namely that operators seem to agree on below-cost reciprocal termination charges in an environment where both outgoing and incoming charges are market-determined. This fact is related to our last step in the technical analysis of this paper. We now ask what level of reciprocal termination rates would be selected by unregulated negotiations in stage I.

Proposition 6. *When the propagation factor is positive, the following reciprocal access charge is a candidate to emerge from unregulated commercial negotiations:* $a^* = (t - \beta c)/(1+\beta)$*. Under this charge, if* $1 - x < \beta < 1/(1-x)$*, there is no connectivity*

[19] ILECs are the incumbent fixed-line operators, while CLECs are their fixed-line competitors. This is in contrast with European countries where asymmetries in termination rates exist between fixed-line operators (who are subject to regulation) and mobile operators (who historically have not been subject to regulation).
[20] FCC, *In the Matter of developing a Unified Intercarrier Compensation Regime*, CC Docket 01-92, §95.

breakdown and market-determined retail prices are efficient. If the propagation factor is zero or negative, there is always a connectivity breakdown.

This last result is quite striking. First of all, it points quite clearly to the importance of the propagation factor. In the absence of the propagation factor ($x = 0$) there would never be a possibility of reaching unregulated agreements over access charges for any value of β. Operators would be using extremely high charges either for sending or for receiving calls, according to the value the utility from receiving calls, to induce a break-down. Although they do not conduct such analysis, this would be the outcome in JLT of endogenizing reciprocal access charges. This would also occur if calls are substitutes in the information exchange ($x < 0$). On the contrary, if calls are complements in the information exchange – which we recall is the empirically-supported case – this breakdown does not arise in a suitable range of values of β. In the limit, if x tends to one, the range extends to all possible values of β.

The second aspect emerging from Proposition 6 is that, if negotiations do not induce a breakdown, they are efficient. From a normative point of view, this remarkable result implies that the industry can indeed "self-regulate" under a mild form of intervention such as reciprocity. Intuitively, operators recognize that, by choosing suitable access charges they can affect the intensity of retail competition. In our earlier analysis in Section 4, with regulated or zero reception charges, the access charge could just affect off-net call prices. As we showed in Proposition 3, operators would then try to be cheap off-net, in order not to compete too vigorously for customers. In the presence of market-determined reception charges a very low access charge is more problematic: if it is set too low ($a < a^*$), it would increase the reception charge and induce a break-down. In this case competition for customers would be very intense since all off-net exchanges would be cut down and customers would prefer to belong to the bigger network. Operators then face two options when selecting the reciprocal value for a. Either they ensure that off-net communications do take place, or they induce a break-down.

Proposition 6 finds that it is in the joint interest of firms to avoid a breakdown, not because allowing off-net traffic generates profits per se, but because competition for customers is less intense. In fact, both with and without breakdown off-net profits are zero (in the former case because there is no traffic, in the latter case because prices are efficient, so the sum of off-net prices is just equal to marginal costs). As also on-net profits are zero (on-net prices are efficient with and without a breakdown), profits are only raised via the fixed fee, which is lower in the presence of a breakdown due to more intense competition for customers.

This intuition, while being helpful to understand why the trade-off between allowing and not allowing off-net exchanges is resolved in favor of permitting them, does not precisely tell why efficiency is also reached. This last aspect can be understood starting from an equilibrium with sender sovereignty most of the time, i.e., reception charges do not bind most of the time. Then we have an intuition similar to Proposition 3 in this restricted range: operators are better off by reducing a. In fact, they reduce a precisely to the point $a = a^*$ where it does not violate sender sovereignty. They do not go below it otherwise they would trigger a breakdown. As found by Proposition 5, at this point prices are given by eq. (25), where off-net charges achieve efficiency.[21]

Notice that, in the range described by Proposition 6 when the first-best is attained, the equilibrium multi-part tariffs take a simple expression: $\hat{p}^* = p^* = (t+c)/(1+\beta)$, $\hat{r}^* = \beta p^*$, $F^* = f + 1/(2\sigma)$ and firms do not engage in network-based price discrimination.[22] However, this result has been reached by granting firms the possibility to engage in on-net/off-net price discrimination. Given that the first-best is attained, it does not seem relevant in this context to propose a policy that forbids operators to practice price discrimination.

6. Conclusions

This paper has developed a model of information exchange between calling parties and used it to analyze competition between network operators. We have found that a relevant role is played by the "propagation factor", which measures how return traffic is induced by sending calls. When inter-network (access) charges are taken as given, we have shown that the "connectivity breakdown" problem highlighted by JLT and HK is enhanced when calls made and received are substitutes in the information production function, and reduced when they are complements. When access charges are endogenized and chosen by negotiating operators, we have also shown that the breakdown problem can be totally eliminated.

The empirical literature has found that "calls generate calls", i.e., the propagation factor is positive. As mentioned by Taylor (2004), this earlier literature has used aggregate data

[21] Notice the value that the optimal $a^* = (t - \beta c)/(1+\beta)$ might take in practice. If the cost of termination and origination are similar, and if the value from receiving calls is sufficiently high, then a^* can be approximated by a "bill-and-keep", even before taking into account transaction and metering costs.

[22] HK find a related result that, when firms charge the same prices on-net and off-net, they also set the fixed subscription fee at zero. Recall that, in their model, $f = 0$ and networks are identical (the degree of substitutability is infinity).

and might have captured both the propagation factor and network externalities. It is an important question for further empirical research to study the information exchange process and correctly identify the propagation factor using micro-data. At a theoretical level, it would be very interesting to consider a more strategic dynamic game between callers, to see under what conditions this could generate the reduced-form representation summarized by the information exchange framework.

Until better analysis is available, however, the evidence available to us so far points to a positive propagation factor, thus our results suggest that connectivity breakdowns should not be seen as a major source of concern under a mild form of regulation (reciprocity). Indeed, we have shown how, under some circumstances, the industry can self-regulate and achieve first-best allocations via negotiated access charges that internalize externalities.

Our results also imply that the debate over the merits of CPP versus RPP must be assessed against the level set for the access charge. We have shown that, in the presence of below-cost access charges, operators may introduce positive reception charges. When the benefit from receiving calls is sufficiently high, below-cost access charges (in the limit a "bill-and-keep" system) have good properties and may be selected by operators themselves. Given that optimal regulation is difficult and costly to achieve, our simplest policy message is to allow operators to negotiate over reciprocal access charges, while granting them full pricing flexibility.

Our findings can be applied when operators are symmetric and calling patterns balanced. If traffic were significantly imbalanced, voluntary negotiations of symmetric rates would be more difficult to achieve. Off-net price discrimination could also be used for anti-competitive purposes. We believe that further work is needed in this area to see if "light" regulation (e.g., reciprocity deals or non-discrimination requirements) could work also in asymmetric settings.

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Appendix

Derivation of eq. (7).

Following JLT we study the optimization program of network i assuming that α_i is given. Let $\tilde{F}_i = F_i + r_i\left[\alpha_i Q^{ii}(p_i, p_i) + \alpha_j Q^{ji}(p_j, p_i)\right]$ be a generalized fixed fee incorporating also the cost for receiving calls. Given eq. (2) we can write $w_i - w_j = (\alpha_i - 1/2)/\sigma$ and, using eq. (3), we obtain:

$$\tilde{F}_i = \tilde{F}_j + (1/2 - \alpha_i)/\sigma + \alpha_i v(p_i, p_i) + \alpha_j v(p_i, p_j) - \alpha_j v(p_j, p_j) - \alpha_i v(p_j, p_i).$$

Network i's profit in eq. (6) can be rearranged in the following way:

$$\begin{aligned}\pi_i = {} & \alpha_i \left\{(p_i - c - t)\left[\alpha_i Q^{ii}(p_i, p_i) + \alpha_j Q^{ij}(p_i, p_j)\right] + \tilde{F}_i - f\right\} + \\ & + \alpha_i \alpha_j (a - t)\left[Q^{ji}(p_j, p_i) - Q^{ij}(p_i, p_j)\right]\end{aligned}$$

The FOC with respect to p_i is given by:

$$\begin{aligned}\frac{\partial \pi_i}{\partial p_i} = {} & \alpha_i \left[\alpha_i Q^{ii}(p_i, p_i) + \alpha_j Q^{ij}(p_i, p_j) + (p_i - c - t)\left(\alpha_i \frac{\partial Q^{ii}}{\partial p_i} + \alpha_j \frac{\partial Q^{ij}}{\partial p_i}\right)\right] + \\ & + \alpha_i \frac{\partial \tilde{F}_i}{\partial p_i} + \alpha_i \alpha_j (a - t)\left(\frac{\partial Q^{ji}}{\partial p_i} - \frac{\partial Q^{ij}}{\partial p_i}\right)\end{aligned} \tag{A1}$$

where

$$\begin{aligned}\frac{\partial \tilde{F}_i}{\partial p_i} &= \frac{\partial \tilde{F}_j}{\partial p_i} + \alpha_i \frac{\partial v(p_i, p_i)}{\partial p_i} + \alpha_j \frac{\partial v(p_i, p_j)}{\partial p_i} - \alpha_i \frac{\partial v(p_j, p_i)}{\partial p_i} \\ \frac{\partial \tilde{F}_j}{\partial p_i} &= r_j \alpha_i \frac{\partial Q^{ij}}{\partial p_i}.\end{aligned}$$

For calls originated by network i and terminated on network j, the indirect utility is given by eq. (5). Thus it results:

$$\frac{\partial v(p_i, p_j)}{\partial p_i} = -Q^{ij}(p_i, p_i) + U_i' f_2^i \frac{\partial Q^{ji}}{\partial p_i}.$$

The effect of a price change on the indirect utility of customers connected to network i is given by the sum of a “standard” loss in utility on each originated call ($-Q^{ij}$) and a term that reflects the change in utility due to way the recipient j reacts to the information exchange, thus leading j to change the number of calls sent to customer i ($U'f_2 \partial Q^{ji} / \partial p_i$).

For calls originated by network j and terminated on network i, where $v(p_j, p_i) = U_j(f^j(Q^{ji}(p_j, p_i), Q^{ij}(p_i, p_j)) - p_j Q^{ji}(p_j, p_i)$, it results:

$$\frac{\partial v(p_j, p_i)}{\partial p_i} = U'_j f_2^j \frac{\partial Q^{ij}}{\partial p_i}.$$

In this case, the marginal change in indirect utility depends only on the impact that network *i*'s price has on the calls received by users *j*.

For calls originated and terminated on the same network, where the indirect utility is given by eq. (4), it results:

$$\frac{\partial v(p_i, p_i)}{\partial p_i} = -Q^{ii}(p_i, p_i) + U'_i f_2^i \frac{dQ^{ii}}{dp_i}.$$

Note that in the last term the effect of p_i influences both call origination and call reception, $dQ^{ii}/dp_i = Q_1^{ii} + Q_2^{ii}$.

After substituting the previous expressions in eq. (A1), eq. (7) results. QED

Proof of Proposition 1.

In a symmetric equilibrium $p_i^* = p_j^* = p^*$, $r_i = r_j = r$, $U_i = U_j = U$, α = ½. We also have $Q_1^{ij} = Q_1^{ii}$ and $Q_2^{ij} = Q_2^{ii}$. Thus it results $\Phi|_{sym} = \frac{2}{2+x}, \Gamma|_{sym} = \frac{2x}{2+x}, \Theta|_{sym} = \frac{2x+1}{2+x}$, where x is a *propagation* factor defined as $x = \Gamma/\Phi = Q_2^{ji}/Q_1^{ij}$. Substitution into eq. (7) gives eq. (8).

The level of the fixed fee is determined by studying the derivative w.r.t. α_i:

$$\pi_i = \alpha_i \left\{(p_i - c - t)\left[\alpha_i Q^{ii} + \alpha_j Q^{ij}\right] + \tilde{F}_j - f + (1/2 - \alpha_i)/\sigma\right\} +$$
$$\alpha_i \left\{\alpha_i v(p_i, p_i) + \alpha_j v(p_i, p_j) - \alpha_j v(p_j, p_j) - \alpha_i v(p_j, p_i)\right\} + \alpha_i \alpha_j (a - t)\left[Q^{ji} - Q^{ij}\right]$$

Since in $p_i = p_i^*$ it results $\frac{d\pi_i}{d\alpha_i} = \frac{\partial \pi_i}{\partial \alpha_i} + \frac{\partial \pi_i}{\partial p_i^*}\frac{\partial p_i^*}{\partial \alpha_i} = \frac{\partial \pi_i}{\partial \alpha_i}$, we have:

$$\frac{\partial \pi_i}{\partial \alpha_i} = \left\{(p_i - c - t)\left[\alpha_i Q^{ii} + \alpha_j Q^{ij}\right] + \tilde{F}_j - f + (1/2 - \alpha_i)/\sigma\right\} +$$
$$+ \left\{\alpha_i v(p_i, p_i) + \alpha_j v(p_i, p_j) - \alpha_j v(p_j, p_j) - \alpha_i v(p_j, p_i)\right\} + (1 - 2\alpha_i)(a - t)\left[Q^{ji} - Q^{ij}\right] +$$
$$+ \alpha_i \left\{(p_i - c - t)\left[Q^{ii} - Q^{ij}\right] - r_j (Q^{jj} - Q^{ij}) - 1/\sigma + v(p_i, p_i) - v(p_i, p_j) + v(p_j, p_j) - v(p_j, p_i)\right\} = 0.$$

In a symmetric equilibrium the above expression reduces to:

$$\left.\frac{\partial \pi_i}{\partial \alpha_i}\right|_{sym} = (p^* - c - t)Q(p^*)/2 + \tilde{F}^* - f - 1/2\sigma = 0$$

where $\widetilde{F}^* = F^* + rQ(p^*)$ and $Q(p^*) = Q^{ii}(p^*, p^*) + Q^{ij}(p^*, p^*)$, giving part b). Finally, substituting F* and p* in eq. (6) gives part c): $\pi^* = 1/4\sigma$.[23] QED

Proof of Proposition 2.
We follow the same procedure as in Section 3 with α_i given. Result a) is standard, as for on-net prices a network maximizes joint surplus with all customers, hence reaching the socially optimal value. To get result b) consider only the profit for off-net calls (both outgoing and incoming). After manipulations of eq. (10) we get:

$$\text{(A2)}\quad \begin{aligned} \hat{\pi}_i(\hat{p}_i;\alpha_i,\hat{r}_j) = \alpha_i\left\{\alpha_j\left[v(\hat{p}_i,\hat{p}_j) + (\hat{p}_i - c - a)Q^{ij}(\hat{p}_i,\hat{p}_j)\right] + \alpha_i\hat{r}_j Q^{ij}(\hat{p}_i,\hat{p}_j)\right\} + \\ + \alpha_i\left\{\alpha_j(a-t)Q^{ji}(\hat{p}_j,\hat{p}_i) - \alpha_i v(\hat{p}_j,\hat{p}_i)\right\}. \end{aligned}$$

The FOC is:

$$\begin{aligned} \frac{\partial\hat{\pi}_i}{\partial\hat{p}_i} = \alpha_i\left\{\alpha_j\left[-Q^{ij}(\hat{p}_i,\hat{p}_j) + U_i' f_2^i Q_2^{ji} + Q^{ij}(\hat{p}_i,\hat{p}_j) + (\hat{p}_i - c - a)Q_1^{ij}\right] + \alpha_i\hat{r}_j Q_1^{ij}\right\} + \\ + \alpha_i\left[\alpha_j(a-t)Q_2^{ji} - \alpha_i U_j' f_2^j Q_1^{ij}\right] \end{aligned}$$

that can be rewritten as:

$$\frac{\partial\hat{\pi}_i}{\partial\hat{p}_i} = \alpha_i Q_1^{ij}\left\{\alpha_j\left[\hat{p}_i - \left(c + a - xU_i' f_2^i - x(a-t)\right)\right] + \alpha_i\left(\hat{r}_j - U_j' f_2^j\right)\right\}$$

where $x = Q_2^{ji}/Q_1^{ij}$ is the propagation effect. Solving $\frac{\partial\hat{\pi}_i}{\partial\hat{p}_i} = 0$ we get:

$$\hat{p}_i^* = c + a - (a-t)x - U_i' f_2^i x - \tfrac{\alpha_i}{\alpha_j}\left(\hat{r}_j - U_j' f_2^j\right)$$

that, if a symmetric equilibrium exists, reduces to eq. (11). In order to show under what conditions this price is indeed a maximum, we have to determine the SOC. Denote with $\beta^i = f_2^i / f_1^i$ such that $U_i' f_2^i = \beta^i \hat{p}_i$. In the same way, define $\beta^j = f_2^j / f_1^j$. Since $\partial\left(U_i' f_2^i\right)/\partial\hat{p}_i = \beta^i + \hat{p}_i\left(\partial\beta^i/\partial\hat{p}_i\right)$ and $\partial\left(U_j' f_2^j\right)/\partial\hat{p}_i = \hat{p}_j\left(\partial\ \ ^j/\partial\hat{p}_i\right)$

$$\left.\frac{\partial^2 \hat{\pi}_i}{\partial \hat{p}_i^2}\right|_{sym} = \frac{1}{4} Q_1^{ij} \left\{ 1 + x\left(\beta + \hat{p}\frac{\partial \beta^i}{\partial \hat{p}_i} + x'(\beta\hat{p} + a - t) \right) - \hat{p}\frac{\partial \beta^j}{\partial \hat{p}_i} \right\}.$$

Consider first the case of JLT with independent outgoing and incoming call, i.e., x = 0. The SOC is $\left.\frac{\partial^2 \hat{\pi}_i}{\partial \hat{p}_i^2}\right|_{\substack{sym \\ x=0}} = \frac{1}{4} Q_1^{ij} < 0$. It is easy to find sufficient conditions that ensure a negative SOC more in general. Recalling the definition of the propagation factor, we can write:

$$\frac{\partial \beta^i}{\partial \hat{p}_i} = \frac{\left(\partial f_2^i / \partial \hat{p}_i\right) f_1^i - \left(\partial f_1^i / \partial \hat{p}_i\right) f_2^i}{\left(f_1^i\right)^2} = Q_1^{ij}\, \frac{f_{12}^i - \beta^i f_{11}^i + x(f_{22}^i - \beta^i f_{12}^i)}{f_1^i}$$

$$\frac{\partial \beta^j}{\partial \hat{p}_i} = Q_1^{ij}\, \frac{f_{22}^j - \beta^j f_{12}^j + x(f_{12}^j - \beta^j f_{11}^j)}{f_1^j}.$$

In a symmetric equilibrium the SOC becomes:

$$\left.\frac{\partial^2 \hat{\pi}_i}{\partial \hat{p}_i^2}\right|_{sym} = Q_1^{ij} \left\{ 1 + x\left[\beta + x'(\beta\hat{p} + a - t)\right] - \hat{p} Q_1^{ij} (f_{22} - \beta f_{12})(1 - x^2)/ f_1 \right\} / 4 .$$

Suppose that calls are perfect complements, i.e., x = 1. A sufficient condition to have $\left.\frac{\partial^2 \hat{\pi}_i}{\partial \hat{p}_i^2}\right|_{\substack{sym \\ x=1}} < 0$ is that $x' \approx 0$, i.e., the propagation factor is not affected by a change in prices.

Since we have assumed sender sovereignty, it must be that the reception charge is low enough, i.e., $U' f_2 \geq \hat{r}$ in a symmetric equilibrium. Finally, to determine the fixed fee of the multi-part tariff, we maximize network i's profit from eq. (10) w.r.t. to α_i, after substituting:

$$\begin{aligned} F_i = F_j &+ (1/2 - \alpha_i)/\sigma + \alpha_i v(p_i, p_i) + \alpha_j v(\hat{p}_i, \hat{p}_j) - r_i \alpha_i Q^{ii}(p_i, p_i) - \hat{r}_i \alpha_j Q^{ji}(\hat{p}_j, \hat{p}_i) \\ &- \alpha_j v(p_j, p_j) - \alpha_i v(\hat{p}_j, \hat{p}_i) + r_j \alpha_j Q^{jj}(p_j, p_j) + \hat{r}_j \alpha_i Q^{ij}(\hat{p}_i, \hat{p}_j), \end{aligned}$$

$$\begin{aligned} \frac{\partial \pi_i}{\partial \alpha_i} &= \left\{ \alpha_i (p_i - c - t) Q^{ii} + \alpha_j (\hat{p}_i - c - t) Q^{ij} + F_j + (1/2 - \alpha_i)/\sigma + \alpha_i v(p_i, p_i) \right. \\ &\left. + \alpha_j v(\hat{p}_i, \hat{p}_j) - \alpha_j v(p_j, p_j) - \alpha_i v(\hat{p}_j, \hat{p}_i) + r_j \alpha_j Q^{jj} + \hat{r}_j \alpha_i Q^{ij} - f \right\} \\ &+ \alpha_i \left\{ (p_i - c - t) Q^{ii} - (\hat{p}_i - c - t) Q^{ij} - 1/\sigma + v(p_i, p_i) - v(\hat{p}_i, \hat{p}_j) + \right. \\ &\left. + v(p_j, p_j) - v(\hat{p}_j, \hat{p}_i) - r_j Q^{jj} + \hat{r}_j Q^{ij} \right\} + (1 - 2\alpha_i)(a - t)\left(Q^{ji} - Q^{ij}\right) = 0. \end{aligned} \tag{A3}$$

In a symmetric equilibrium (A3) is: $\left.\frac{\partial \pi_i}{\partial \alpha_i}\right|_{sym} = \left(p^* - c - t\right) Q(p^*, p^*) + F^* - f + \hat{r} Q(\hat{p}^*, \hat{p}^*) - 1/2\sigma + \left(v(p^*, p^*) - v(\hat{p}^*, \hat{p}^*)\right) = 0$, which gives eq. (12). QED

Proof of Proposition 3.
The off-net price is given by eq. (11'), and under the assumption $x' \approx 0$, we have:

(A6) $$\frac{\partial \hat{p}^*}{\partial a} = \frac{(1-x)[1-\beta(1-x)] + (1-x)\frac{\partial\beta}{\partial a}[c+a-(a-t)x-\hat{r}]}{(1-\beta(1-x))^2} = \frac{1-x}{1-\beta(1-x)}\left(1+\hat{p}^*\frac{\partial\beta}{\partial a}\right),$$

where $\frac{\partial\beta}{\partial a} = \frac{\partial\beta}{\partial\hat{p}}\frac{\partial\hat{p}^*}{\partial a}$. Notice how $\partial p^*/\partial a$ is positive under rather general conditions (it suffices that $\partial\beta/\partial a$ is positive or not "too" negative). From eq. (12) the effect on F* is $\frac{\partial F^*}{\partial a} = \frac{\partial v(\hat{p}^*,\hat{p}^*)}{\partial a} - \hat{r}\frac{\partial Q(\hat{p}^*,\hat{p}^*)}{\partial a}$, where $\frac{\partial v(\hat{p}^*,\hat{p}^*)}{\partial a} = U'f_1\frac{\partial Q}{\partial a} + U'f_2\frac{\partial Q}{\partial a} - Q\frac{\partial\hat{p}^*}{\partial a} - \hat{p}^*\frac{\partial Q}{\partial a} = \left[-Q + \beta\hat{p}^* Q_1(1+x)\right]\frac{\partial\hat{p}^*}{\partial a}$. Therefore, it results:

(A7) $$\frac{\partial F^*}{\partial a} = \left[-Q(\hat{p}^*,\hat{p}^*) + (\beta\hat{p}^* - \hat{r})Q_1(\hat{p}^*,\hat{p}^*)(1+x)\right]\frac{\partial\hat{p}^*}{\partial a}.$$

Substituting eq. (14), (A6) and (A7) in eq. (13) we obtain:

(A8) $$\frac{\partial\pi^*}{\partial a} = \frac{1}{4}\frac{\partial\hat{p}^*}{\partial a}\left\{-Q(\hat{p}^*,\hat{p}^*) + Q_1(\hat{p}^*,\hat{p}^*)(1+x)\left[\hat{p}^*(1+2\beta) - c - t - \hat{r}\right]\right\} \le 0$$

which gives eq. (15) at an interior solution when $x \neq 1$. In order to determine if the optimal access charge is a local maximum, we consider the sign of the SOC. Denoting by $\Psi = -Q + Q_1(1+x)\left[\hat{p}^*(1+2\beta) - c - t - \hat{r}\right]$, the SOC evaluated in a = a* reduces to $\left.\frac{\partial^2\pi^*}{\partial a^2}\right|_{a=a*} = \frac{1}{4}\frac{\partial\hat{p}^*}{\partial a}\frac{\partial\Psi}{\partial a}$. Since $\partial p^*/\partial a$ is positive, we need to study the sign of $\partial\Psi/\partial a$:

$$\frac{\partial\Psi}{\partial a} = (1+x)\left\{\frac{\partial\hat{p}^*}{\partial a}(Q_{11}+Q_{12})\left[\hat{p}^*(1+2\beta) - c - t - \hat{r}\right] + Q_1\left[2\beta\frac{\partial\hat{p}^*}{\partial a} + \hat{p}^*(1+2\frac{\partial\beta}{\partial a})\right]\right\},$$

$$\left.\frac{\partial\Psi}{\partial a}\right|_{a=a^*} = (1+x)\left\{\frac{\partial\hat{p}^*}{\partial a}(Q_{11}+Q_{12})\frac{Q}{Q_1} + Q_1\left[2\beta\frac{\partial\hat{p}^*}{\partial a} + \hat{p}^*(1+2\frac{\partial\beta}{\partial a})\right]\right\}.$$

The sign of the last expression is not obvious a priori. If demand functions are linear, and $\partial\beta/\partial a$ is positive or not "too" negative, then it results $\left.\partial\Psi/\partial a\right|_{a=a^*} < 0$ and the access charge in (15) is a local maximum. However, this is not a general property. QED

Proof of Proposition 4.
Parts a) and b) are already discussed in the main text. To complete part b) imagine a "bill-and-keep" system is in place, a = 0. From (17) the regulator can induce efficiency by choosing a reception charge equal to:

$$\hat{r}^W = \frac{\beta(c+t)(2-x)}{1+\beta} - t(1-x),$$

which satisfies the constraint $\hat{r} \le \frac{\beta}{1+\beta}(c+t)$ if $(\beta c - t)(1-x)/(1+\beta) \le 0$. In other words the first-best can be achieved if either the propagation factor is 1, or if $x < 1$ and $\beta c < t$. In particular, if β is sufficiently low, reception charges may not be needed at all ($\beta \le \frac{t(1-x)}{t+c(2-x)} \Rightarrow \hat{r} = 0$). Finally, part c) points to a potential problem that the regulator faces when trying to introduce a regulated termination charge in order to drive a^* towards efficiency. Such charge, if it exists, is given by eq. (19) when both a^* and a^W are positive. This charge has also to satisfy "sender sovereignty", which is written as $\hat{r} \le \frac{\beta}{1+\beta}(c+t)$ when the efficient retail price is induced. It is straightforward to see that this inequality is always violated by (19), unless the propagation factor is infinitely high. Thus the regulator can at most set the highest possible reception charge that does not violate the constraint. QED

Proof of Proposition 5.
We follow again the same procedure used earlier keeping market shares constant. From (A2), we denote with

$$\text{(A9)} \quad \psi(.) = \alpha_j \left(v^{ij}(.) + (\hat{p}_i - c - a)Q^{ij}(.)\right) + \alpha_i \hat{r}_j Q^{ij}(.) + \alpha_j (a-t)Q^{ji}(.) - \alpha_i v^{ji}(.)$$

the off-net per customer profit, where (.) depends on the different pricing regimes (sender, receiver, or network sovereignty). We consider the maximization of the profit given by eq. (20) as the noise vanishes while keeping a large support to avoid price indeterminacy.[24]

Network *i*'s off net profit can be written as follows:

$$\text{(A10)} \quad \begin{aligned} \hat{\pi}_i = \alpha_i \Big\{ & (1 - G(\hat{\varepsilon}_i))(1 - G(\hat{\varepsilon}_j))\psi(\hat{p}_i, \hat{p}_j) + (1 - G(\hat{\varepsilon}_j))\int_{\hat{\varepsilon}_i}^{\bar{\varepsilon}} \varepsilon_i Q^{ji}(\hat{p}_i, \hat{p}_j) g(\varepsilon_i) d\varepsilon_i - \\ & (1 - G(\hat{\varepsilon}_i))\int_{\hat{\varepsilon}_j}^{\bar{\varepsilon}} \varepsilon_j Q^{ij}(\hat{p}_i, \hat{p}_j) g(\varepsilon_j) d\varepsilon_j + (1 - G(\hat{\varepsilon}_j))\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \psi(\hat{p}_i, \hat{r}_i - \varepsilon_i) g(\varepsilon_i) d\varepsilon_i + \\ & (1 - G(\hat{\varepsilon}_j))\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \varepsilon_i Q^{ji}(\hat{r}_i - \varepsilon_i, \hat{p}_i) g(\varepsilon_i) d\varepsilon_i - G(\hat{\varepsilon}_i)\int_{\hat{\varepsilon}_j}^{\bar{\varepsilon}} \varepsilon_j Q^{ij}(\hat{p}_i, \hat{r}_i - \varepsilon_i) g(\varepsilon_j) d\varepsilon_j + \\ & (1 - G(\hat{\varepsilon}_i))\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} \psi(\hat{r}_j - \varepsilon_j, \hat{p}_j) g(\varepsilon_j) d\varepsilon_j + G(\hat{\varepsilon}_j)\int_{\hat{\varepsilon}_i}^{\bar{\varepsilon}} \varepsilon_i Q^{ji}(\hat{p}_j, \hat{r}_j - \varepsilon_j) g(\varepsilon_i) d\varepsilon_i - \\ & (1 - G(\hat{\varepsilon}_i))\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} \varepsilon_j Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{p}_j) g(\varepsilon_j) d\varepsilon_j + \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} \psi(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i) g(\varepsilon_i) g(\varepsilon_j) d\varepsilon_i d\varepsilon_j + \\ & \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} \varepsilon_i Q^{ji}(\hat{r}_i - \varepsilon_i, \hat{r}_j - \varepsilon_j) g(\varepsilon_i) g(\varepsilon_j) d\varepsilon_i d\varepsilon_j - \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} \varepsilon_j Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i) g(\varepsilon_i) g(\varepsilon_j) d\varepsilon_i d\varepsilon_j \Big\} \end{aligned}$$

Consider first the case of sender sovereignty most of the time. The threshold levels of the noise are given by $\hat{\varepsilon}_i = \hat{r}_i - U_i' f_2^i(\hat{p}_i, \hat{p}_j)$ and $\hat{\varepsilon}_j = \hat{r}_j - U_j' f_2^j(\hat{p}_j, \hat{p}_i)$. Since we deal with the case of sender sovereignty most of the time, we solve the maximization problem for $\hat{\varepsilon}_i, \hat{\varepsilon}_j < 0$. Maximizing (A10) w.r.t. $\hat{p}_i$ and $\hat{r}_i$ and evaluating the corresponding FOCs in a symmetric equilibrium, after tedious calculations and simplifications, we obtain:

[24] To get the results we impose a regularity condition. Given a continuous function $F(\cdot)$, a regular sequence of distributions $G_n(\varepsilon)$ of the random variable ε with zero mean satisfies: $\lim_{n\to\infty} G_n(F(\varepsilon) | \varepsilon \ge \varepsilon_0) = F(\varepsilon = \varepsilon_0)$ for $\varepsilon_0 > 0$, $\lim_{n\to\infty} G_n(F(\varepsilon) | \varepsilon \le \varepsilon_0) = F(\varepsilon = 0)$ for $\varepsilon_0 > 0$ (see Feller, 1971).

$$
\begin{aligned}
&\frac{\partial \hat{\pi}_i}{\partial \hat{p}_i} = \frac{1}{2}\Bigg\{ \left(1 - G(\hat{\varepsilon}_i)\right)^2 \frac{\partial \psi(\hat{p}_i, \hat{p}_j)}{\partial \hat{p}_i} + \left(1 - G(\hat{\varepsilon}_i)\right) \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \frac{\partial \psi(\hat{p}_i, \hat{r}_i - \varepsilon_i)}{\partial \hat{p}_i} g(\varepsilon_i) d\varepsilon_i + \\
&\left(1 - G(\hat{\varepsilon}_i)\right) \left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \varepsilon_i Q_2^{ji}(\hat{r}_i - \varepsilon_i, \hat{p}_i) g(\varepsilon_i) d\varepsilon_i + \int_{\hat{\varepsilon}_i}^{\bar{\varepsilon}} \varepsilon_i Q_2^{ji}(\hat{p}_i, \hat{p}_j) g(\varepsilon_i) d\varepsilon_i \right] - \\
\text{(A11)} \quad &\int_{\hat{\varepsilon}_i}^{\bar{\varepsilon}} \varepsilon_i \left[\left(1 - G(\hat{\varepsilon}_i)\right) Q_1^{ij}(\hat{p}_i, \hat{p}_j) + G(\hat{\varepsilon}_i) Q_1^{ij}(\hat{p}_i, \hat{r}_i - \varepsilon_i) \right] g(\varepsilon_i) d\varepsilon_i + \\
&\frac{\partial \hat{\varepsilon}_j}{\partial \hat{p}_i} g(\hat{\varepsilon}_i) \left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \hat{\varepsilon}_i \left(Q^{ij}(\hat{p}_i, \hat{p}_j) - Q^{ij}(\hat{p}_i, \hat{r}_i - \varepsilon_i) \right) g(\varepsilon_i) d\varepsilon_i + \int_{\hat{\varepsilon}_i}^{\bar{\varepsilon}} \varepsilon_i \left(Q^{ji}(\hat{p}_j, \hat{r}_j - \varepsilon_j) - Q^{ji}(\hat{p}_j, \hat{p}_i) \right) g(\varepsilon_i) d\varepsilon_i \right] + \\
&\frac{\partial \hat{\varepsilon}_i}{\partial \hat{p}_i} g(\hat{\varepsilon}_i) \left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \hat{\varepsilon}_i \left(Q^{ji}(\hat{p}_j, \hat{r}_j - \varepsilon_j) - Q^{ji}(\hat{p}_j, \hat{p}_i) \right) g(\varepsilon_i) d\varepsilon_i + \int_{\hat{\varepsilon}_i}^{\bar{\varepsilon}} \varepsilon_i \left(Q^{ij}(\hat{p}_i, \hat{p}_j) - Q^{ij}(\hat{p}_i, \hat{r}_i - \varepsilon_i) \right) g(\varepsilon_i) d\varepsilon_i \right] \Bigg\}.
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{\partial \hat{\pi}_i}{\partial \hat{r}_i} = \frac{1}{2}\Bigg\{ \left(1 - G(\hat{\varepsilon}_i)\right) \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \frac{\partial \psi(\hat{p}_i, \hat{r}_i - \varepsilon_i)}{\partial \hat{r}_i} g(\varepsilon_i) d\varepsilon_i + \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} \frac{\partial \psi(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i)}{\partial \hat{r}_i} g(\varepsilon_i) g(\varepsilon_j) d\varepsilon_i d\varepsilon_j + \\
&\left(1 - G(\hat{\varepsilon}_i)\right) \left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \varepsilon_i Q_1^{ji}(\hat{r}_i - \varepsilon_i, \hat{p}_i) g(\varepsilon_i) d\varepsilon_i \right] - G(\hat{\varepsilon}_i) \left[\int_{\hat{\varepsilon}_i}^{\bar{\varepsilon}} \varepsilon_i Q_2^{ij}(\hat{p}_i, \hat{r}_i - \varepsilon_i) g(\varepsilon_i) d\varepsilon_i \right] - \\
\text{(A12)} \quad &G(\hat{\varepsilon}_i) \frac{\partial \hat{\varepsilon}_i}{\partial \hat{r}_i} \hat{\varepsilon}_i g(\hat{\varepsilon}_i) Q^{ji}(\hat{p}_j, \hat{p}_i) + \frac{\partial \hat{\varepsilon}_i}{\partial \hat{r}_i} g(\hat{\varepsilon}_i) \int_{\hat{\varepsilon}_i}^{\bar{\varepsilon}} \varepsilon_i \left(Q^{ij}(\hat{p}_i, \hat{p}_i) - Q^{ij}(\hat{p}_i, \hat{r}_i - \varepsilon_i) \right) g(\varepsilon_i) d\varepsilon_i + \\
&\frac{\partial \hat{\varepsilon}_i}{\partial \hat{r}_i} g(\hat{\varepsilon}_i) \left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \hat{\varepsilon}_i Q^{ji}(\hat{p}_j, \hat{r}_j - \varepsilon_i) g(\varepsilon_i) d\varepsilon_i \right] + \\
&\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} \varepsilon_i Q_1^{ji}(\hat{r}_i - \varepsilon_i, \hat{r}_j - \varepsilon_j) g(\varepsilon_i) g(\varepsilon_j) d\varepsilon_i d\varepsilon_j - \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_i} \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_j} \varepsilon_j Q_2^{ij}(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i) g(\varepsilon_i) g(\varepsilon_j) d\varepsilon_i d\varepsilon_j \Bigg\}
\end{aligned}
$$

As the noise vanishes, from the regularity condition of the distribution function when the threshold value $\hat{\varepsilon}$ is negative, it follows for a continuous function $F(.)$ that:

$$
\int_{\underline{\varepsilon}}^{\hat{\varepsilon}} F(\varepsilon) dg(\varepsilon) = F(\hat{\varepsilon}) G(\hat{\varepsilon}), \quad \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} F(\varepsilon) dg(\varepsilon) = F(0)(1 - G(\hat{\varepsilon})).
$$

Using this property, in a symmetric equilibrium, (A11) and (A12) simplify to:

$$
\begin{aligned}
&\frac{\partial \hat{\pi}_i}{\partial \hat{p}_i} = \frac{1}{2}\left(1 - G(\hat{\varepsilon})\right) \left\{ \left(1 - G(\hat{\varepsilon})\right) \frac{\partial \psi(\hat{p}_i, \hat{p}_j)}{\partial \hat{p}_i} + G(\hat{\varepsilon}) \left. \frac{\partial \psi(\hat{p}_i, \hat{r}_i - \varepsilon_i)}{\partial \hat{p}_i} \right|_{\varepsilon_i = \hat{\varepsilon}} + G(\hat{\varepsilon}) \hat{\varepsilon} Q_2^{ji} \right\} \\
&\frac{\partial \hat{\pi}_i}{\partial \hat{r}_i} = \frac{1}{2} G(\hat{\varepsilon}) \Bigg\{ \left(1 - G(\hat{\varepsilon})\right) \left. \frac{\partial \psi(\hat{p}_i, \hat{r}_i - \varepsilon_i)}{\partial \hat{r}_i} \right|_{\varepsilon_i = \hat{\varepsilon}} + G(\hat{\varepsilon}) \left. \frac{\partial \psi(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i)}{\partial \hat{r}_i} \right|_{\varepsilon_i = \varepsilon_j = \hat{\varepsilon}} + \\
&\left(1 - G(\hat{\varepsilon})\right) \hat{\varepsilon} Q_1^{ji} + G(\hat{\varepsilon}) \hat{\varepsilon} \left[Q_1^{ji} - Q_2^{ij} \right] \Bigg\}.
\end{aligned}
$$

Since when $\hat{\varepsilon} < 0$ it results that $G(.) \rightarrow 0$, we can neglect the terms that go to zero faster and thus can write:

$$
\text{(A13)} \quad \frac{\partial \hat{\pi}_i}{\partial \hat{p}_i} \cong \frac{1}{2} \frac{\partial \psi(\hat{p}_i, \hat{p}_j)}{\partial \hat{p}_i}
$$

$$
\text{(A14)} \quad \frac{\partial \quad _i}{\partial r_i} \cong - \left\{ \left. \frac{\partial \quad p_i \;\; r_i - \quad _i}{\partial r_i} \right|_{i =} + \quad Q^{ji} \right\}
$$

The analysis of the RHS of eq. (A13) is identical to the analysis conducted in Proposition 2, thus the expression of the symmetric FOC w.r.t. $\hat{p}$ at equilibrium is:

$$\text{(A15)} \quad \frac{\partial \hat{\pi}_i}{\partial \hat{p}_i} = \frac{1}{4} Q_1^{ij} \left\{ \hat{p}\left[1 - \beta(1-x)\right] - \left(c + a - x(a-t)\right) - \hat{r} \right\}.$$

The analysis of the term $\psi(.)$ in eq. (A14) when network i is sovereign needs some more intermediate steps. As the traffic is only determined by i, the first-order conditions are $U_i' f_1^i = \hat{p}_i$ and $U_i' f_2^i = \hat{r}_i - \varepsilon_i$, which determine the optimal level of calls

$$\frac{\partial \hat{\pi}_i}{\partial \hat{r}_i} \cong \frac{1}{4} Q_1^{ji} \left\{ U'f_2 + a - t + \left(\hat{p} - c - a \right) x + \hat{r} - U'f_2 - \left[(U'f_1 - \hat{p}) + x(U'f_2 - \hat{r}) \right] \right\}.$$

Since we started by considering the case where senders are sovereign most of the time, i.e., $\hat{\varepsilon} < 0 \Leftrightarrow U'f_2 = \beta\hat{p} > \hat{r}$ for the receiver and $U'f_1 = \hat{p}$ for the sender, we finally get:

$$\text{(A17)} \quad \frac{\partial \hat{\pi}_i}{\partial \hat{r}_i} = \frac{1}{4} \left\{ Q_1^{ji} \left[\hat{r} + a - t + \left(\hat{p} - c - a \right) x - x(\beta\hat{p} - \hat{r}) \right] \right\}.$$

The analysis when there is receiver sovereignty most of the time follows the same procedure. The threshold levels of the noise are given by $\hat{\varepsilon}_i = \hat{r}_i - U'_i f_2^i(\hat{r}_j, \hat{r}_i) > 0$ and $\hat{\varepsilon}_j = \hat{r}_j - U'_j f_2^j(\hat{r}_i, \hat{r}_j) > 0$. Maximizing (A10) w.r.t. $\hat{p}_i$ and $\hat{r}_i$, and using the regularity condition we obtain after simplifications:

$$\frac{\partial \hat{\pi}_i}{\partial \hat{p}_i} = \frac{1}{2} \left(1 - G(\hat{\varepsilon}) \right) \left\{ \left(1 - G(\hat{\varepsilon}) \right) \frac{\partial \psi(\hat{p}_i, \hat{p}_j)}{\partial \hat{p}_i} + G(\hat{\varepsilon}) \left. \frac{\partial \psi(\hat{p}_i, \hat{r}_i - \varepsilon_i)}{\partial \hat{p}_i} \right|_{\varepsilon_i = \hat{\varepsilon}} + \left(1 - G(\hat{\varepsilon}) \right) \hat{\varepsilon} Q_2^{ji}(\hat{p}_i, \hat{p}_j) - G(\hat{\varepsilon}) \hat{\varepsilon} Q_1^{ij} \right\}$$

$$\frac{\partial \hat{\pi}_i}{\partial \hat{r}_i} = \frac{1}{2} G(\hat{\varepsilon}) \left\{ \left(1 - G(\hat{\varepsilon}) \right) \left. \frac{\partial \psi(\hat{p}_i, \hat{r}_i - \varepsilon_i)}{\partial \hat{r}_i} \right|_{\varepsilon_i = \hat{\varepsilon}} + G(\hat{\varepsilon}) \left. \frac{\partial \psi(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i)}{\partial \hat{r}_i} \right|_{\varepsilon_i = \varepsilon_j = \hat{\varepsilon}} - \left(1 - G(\hat{\varepsilon}) \right) \hat{\varepsilon} Q_2^{ij} \right\}.$$

Since when $\hat{\varepsilon} > 0$ it results that $G(.) \rightarrow 1$, we can further approximate:

$$\text{(A18)} \quad \frac{\partial \hat{\pi}_i}{\partial \hat{p}_i} \cong \frac{1}{2} \left\{ \left. \frac{\partial \psi(\hat{p}_i, \hat{r}_i - \varepsilon_i)}{\partial \hat{p}_i} \right|_{\varepsilon_i = \hat{\varepsilon}} - \hat{\varepsilon} Q_1^{ij} \right\}$$

$$\text{(A19)} \quad \frac{\partial \hat{\pi}_i}{\partial \hat{r}_i} \cong \frac{1}{2} \left. \frac{\partial \psi(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i)}{\partial \hat{r}_i} \right|_{\varepsilon_i = \varepsilon_j = \hat{\varepsilon}}$$

The analysis of the term $\psi(.)$ in (A18) is the mirror of the analysis of the term $\psi(.)$ in (A14) as now the regime is one of "network j" sovereignty. The FOC w.r.t. $\hat{p}_i$ of (A9) is the following at a symmetric equilibrium:

$$\frac{\partial \psi(\hat{p}_i, \hat{r}_i - \varepsilon_i)}{\partial \hat{p}_i} = \frac{1}{2} \left[-Q^{ij} + U'_i f_2^i \frac{\partial Q^{ji}}{\partial \hat{p}_i} + Q^{ji} + \left(\hat{p}_i - c - a \right) \frac{\partial Q^{ij}}{\partial \hat{p}_i} + \hat{r}_j \frac{\partial Q^{ij}}{\partial \hat{p}_i} \right] +$$

$$\text{(A20)} \quad \frac{1}{2} \left[(a - t) \frac{\partial Q^{ji}}{\partial \hat{p}_i} - \frac{\partial Q^{ji}}{\partial \hat{p}_i} (U'_j f_1^j - \hat{p}_j) - U'_j f_2^j \frac{\partial Q^{ij}}{\partial \hat{p}_i} \right] =$$

$$\frac{1}{2} \frac{\partial Q^{ij}}{\partial \hat{p}_i} \left\{ \left[\hat{p}_i - \left(c + a - x\hat{r}_i - x(a - t) \right) \right] - \left[x(U'_j f_1^j - \hat{p}_j) + (U'_j f_2^j - \hat{r}_j) \right] \right\}$$

where $x = \frac{\partial Q^{ji} / \partial \hat{p}_i}{\partial Q^{ij} / \partial \hat{p}_i} = \frac{-B^{ij}}{C^{ij}}$ is the outgoing propagation effect related to the calls originated by network *i*. After replacing (A20) in (A18) as the noise vanishes, when $\varepsilon_i = \hat{\varepsilon} = \hat{r} - U'f_2$, it results:

$$\frac{\partial \hat{\pi}_i}{\partial \hat{r}_i} \cong \frac{1}{4} Q_1^{ij} \left\{ \left[\hat{p}_i - \left(c + a - x\hat{r} - x(a-t) \right) \right] - x(U'f_1 - \hat{p}_j) \right\}.$$

Since we started by considering the case where receivers are sovereign most of the time, i.e., $\hat{\varepsilon} > 0$ and $U'f_1 = \hat{r} / \beta$ for the receiver, we finally get:

$$\text{(A21)} \quad \frac{\partial \hat{\pi}_i}{\partial \hat{p}_i} = \frac{1}{4} Q_1^{ij} \left\{ \left[\hat{p} - \left(c + a - x\hat{r} - x(a-t) \right) \right] - x(\hat{r}/\beta - \hat{p}) \right\}.$$

The analysis of the term $\psi(.)$ in (A19) corresponds to the case of "receiver sovereignty". The indirect utilities from the information exchange between *i* and *j* are:

$$v^{ij} = v(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i) = U_i(f^i(Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i), Q^{ji}(\hat{r}_i - \varepsilon_i, \hat{r}_j - \varepsilon_j)) - \hat{p}_i Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i)$$
$$v^{ji} = v(\hat{r}_i - \varepsilon_i, \hat{r}_j - \varepsilon_j) = U_j(f^j(Q^{ji}(\hat{r}_i - \varepsilon_i, \hat{r}_j - \varepsilon_j), Q^{ij}(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i)) - \hat{p}_j Q^{ji}(\hat{r}_i - \varepsilon_i, \hat{r}_j - \varepsilon_j).$$

The FOC w.r.t. $\hat{r}_i$ of (A9) is:

$$\frac{\partial \psi(\hat{r}_j - \varepsilon_j, \hat{r}_i - \varepsilon_i)}{\partial \hat{r}_i} = \left[\frac{\partial Q^{ij}}{\partial \hat{r}_i}(U_i' f_1^i - \hat{p}_i) + \hat{r}_i \frac{\partial Q^{ji}}{\partial \hat{r}_i} + (\hat{p}_i - c - a) \frac{\partial Q^{ij}}{\partial \hat{r}_i} \right] + \alpha_i \hat{r}_j \frac{\partial Q^{ij}}{\partial \hat{r}_i} +$$

$$\text{(A22)} \quad \alpha_j (a-t) \frac{\partial Q^{ji}}{\partial \hat{r}_i} - \alpha_i \left(\frac{\partial Q^{ji}}{\partial \hat{r}_i}(U_j' f_1^j - \hat{p}_j) + \hat{r}_j \frac{\partial Q^{ij}}{\partial \hat{r}_i} \right) =$$

$$= Q_1^{ji} \left\{ \alpha_j x \left[U_i' f_1^i - (c+a) \right] + \alpha_j \hat{r}_i + \alpha_j (a-t) - \alpha_i \left(U_j' f_1^j - \hat{p}_j \right) \right\}$$

where $\partial Q^{ij} / \partial \hat{r}_i = Q_2^{ij}, \partial Q^{ji} / \partial \hat{r}_i = Q_1^{ji}$ and $x = Q_2^{ij} / Q_1^{ji}$ is the propagation effect when the receiver is sovereign.[25] Denoting $\beta = f_2 / f_1$, and recalling that $\hat{r} = U'f_2$ when receivers are sovereign as the noise vanishes, after replacing (A22) in (A19), the FOC at the symmetric equilibrium is:

$$\text{(A23)} \quad \frac{\partial \hat{\pi}_i}{\partial \hat{r}_i} = \frac{1}{4} Q_1^{ji} \left\{ \hat{r} \left[1 - (1-x)/\beta \right] + (a-t) - x(c+a) + \hat{p} \right\}.$$

Putting all the results together (eqs. (A15), (A17), (A21), (A23)), these are the final expressions of the FOCs at a symmetric equilibrium as the noise vanishes:

[25] The study of the sign of the SOC is the same as in the proof of Proposition 2. Sufficient conditions for having $\left. \frac{\partial^2 \hat{\pi}_i}{\partial \hat{r}_i^2} \right|_{\substack{sym \\ x=0}} < 0$ are that either *x* is small, or that it is close to 1 and *x'* ≈ 0.

$$4\frac{\partial\hat{\pi}}{\partial\hat{p}}=\begin{cases}Q_1^{ij}\{\hat{p}[1-\beta(1-x)]-(c+a-x(a-t))-\hat{r}\} \text{ if } \beta\hat{p}\geq\hat{r}\\ Q_1^{ij}\{\hat{p}-(c+a-x(\hat{r}+a-t))-x(\hat{r}/\beta-\hat{p})\} \text{ if } \beta\hat{p}\leq\hat{r}\end{cases}$$

$$4\frac{\partial\hat{\pi}}{\partial\hat{r}}=\begin{cases}Q_1^{ij}\left[\hat{r}+a-t+(\hat{p}-c-a)x-x(\beta\hat{p}-\hat{r})\right] \text{ if } \beta\hat{p}\geq\hat{r}\\ Q_1^{ij}\{\hat{r}[1-(1-x)/\beta]+(a-t)-x(c+a)+\hat{p}\} \text{ if } \beta\hat{p}\leq\hat{r}\end{cases}$$

Notice that these FOCs boil down to those of JLT (p. 109-110) when $x = 0$. Consider first the case where $\beta\hat{p}>\hat{r}$ and senders determine most volume in equilibrium. The relevant FOCs give eq. (21) as the interior solution. This solution is valid if $\beta < 1/(1 - x - x^2)$ and compatible with the regime of sender sovereignty if $\beta\hat{p}^*>\hat{r}^*\Rightarrow a>(t-\beta c)/(1+\beta)$. If the reception charge $\hat{r}^*$ must take positive values: $\hat{r}^*\geq 0\Rightarrow a\leq t+x^2\dfrac{\beta(c+t)}{(1-x)(1+x-\beta)}$. If a is above this threshold, then the reception charge $\hat{r}^*$ is zero and $\hat{p}^*$ is obtained directly from eq. (11'), resulting in eq. (22). In this case the range of validity is $\beta < 1/(1 - x)$. The case where $\beta\hat{p}<\hat{r}$ and receivers determine most volume in equilibrium can be analyzed in a similar manner, giving eq. (23) and (24). Finally, consider the case where $\beta\hat{p}=\hat{r}$. After substituting this into the FOCs, we have the following candidate pair of prices when $a=(t-\beta c)/(1+\beta)$: $\hat{p}^*=p^*=(c+t)/(1+\beta)$, $\hat{r}^*=\beta p^*=\beta(c+t)/(1+\beta)$. These prices are efficient. However, when $a=(t-\beta c)/(1+\beta)$, imagine to set the reception charge at the efficient level and to increase the call price above it: $\hat{p}^*=p^*+\delta$, $\delta > 0$, and $\hat{r}^*=\beta p^*$. In this range $\beta\hat{p}^*>\hat{r}^*$ and the FOC w.r.t. $\hat{p}$ simplifies to:

$$4\frac{\partial\hat{\pi}}{\partial\hat{p}}=Q_1^{ij}\delta[1-\beta(1-x)].$$

Similarly, imagine $\hat{p}^*=p^*$ and $\hat{r}^*=\beta p^*+\delta$; then the FOC w.r.t. $\hat{r}$ is:

$$4\frac{\partial\hat{\pi}}{\partial\hat{r}}=Q_1^{ij}\delta(\beta+x-1)/\beta.$$

It follows that efficient prices can *never* be reached when the utility function is separable since, with $x = 0$, there is always an incentive to deviate and induce a break-down by setting either arbitrarily high call prices when $\beta > 1$ or arbitrarily high reception charges when $\beta < 1$. However, in the presence of a positive propagation factor there is a range $1-x<\beta<1/(1-x)$ such that the deviation is not profitable, thus prices are given by eq. (25) and are efficient. QED

Proof of Proposition 6.
We only consider the case of senders sovereignty most of the time. We also suppose that there is no problem associated with implementing negative reception charges. This means that retail prices are given by eq. (21) over the entire range $a>(t-\beta c)/(1+\beta)$, by eq. (25)

when $a=(t-\beta c)/(1+\beta)$, and induce an off-net breakdown when $a<(t-\beta c)/(1+\beta)$. As in Section 4.2, in a symmetric equilibrium the profit is given by:

$$\pi^* = \tfrac{1}{4}\left(p^* + r^* - c - t\right)Q(p^*, p^*) + \tfrac{1}{4}\left(\hat{p}^* + \hat{r}^* - c - t\right)Q(\hat{p}^*, \hat{p}^*) + \tfrac{1}{2}\left(F^* - f\right).$$

Since the sender is sovereign most of the time, the effect of a is as in the proof of Proposition 3, $\dfrac{\partial Q(\hat{p}^*, \hat{p}^*)}{\partial a} = (1+x)Q_1(\hat{p}^*, \hat{p}^*)\dfrac{\partial \hat{p}^*}{\partial a}$, $\dfrac{\partial v(\hat{p}^*, \hat{p}^*)}{\partial a} = \left[-Q + \beta\hat{p}^* Q_1(1+x)\right]\dfrac{\partial \hat{p}^*}{\partial a}$ while $\dfrac{\partial F^*}{\partial a} = -Q(\hat{p}^*, \hat{p}^*)\left(\dfrac{\partial(\hat{p}^* + \hat{r}^*)}{\partial a}\right) + \left(\beta\hat{p}* - \hat{r}^*\right)Q_1(1+x)\dfrac{\partial \hat{p}^*}{\partial a}$.

The FOC w.r.t. the reciprocal access charge is given by the following condition:

$$\frac{\partial \pi^*}{\partial a} = \frac{1}{4}\left[\frac{\partial Q(\hat{p}^*, \hat{p}^*)}{\partial a}\left(\hat{p}^* - c - t + \hat{r}^*\right) + Q(\hat{p}^*, \hat{p}^*)\frac{\partial(\hat{p}^* + \hat{r}^*)}{\partial a}\right] + \frac{1}{2}\frac{\partial F^*}{\partial a}.$$

Under the assumption that x does not change for a small variation in prices, we have:

(A23)
$$\begin{aligned}\frac{\partial \pi^*}{\partial a} &= \frac{1}{4}\left\{-Q(\hat{p}^*, \hat{p}^*)\frac{\partial(\hat{p}^* + \hat{r}^*)}{\partial a} + Q_1(\hat{p}^*, \hat{p}^*)(1+x)\frac{\partial \hat{p}^*}{\partial a}\left[\hat{p}^*(1+2\beta) - c - t - \hat{r}^*\right]\right\}\\ &= \frac{1}{4}\left\{-Q(\hat{p}^*, \hat{p}^*)\frac{\partial(\hat{p}^* + \hat{r}^*)}{\partial a} + Q_1(\hat{p}^*, \hat{p}^*)\frac{(1-x)(3+2x)}{1-\beta+\beta x(1+x)}\frac{\partial \hat{p}^*}{\partial a}\left[a(1+\beta) - t + \beta c\right]\right\}\end{aligned}$$

with:

- $\dfrac{\partial(\hat{p}^* + \hat{r}^*)}{\partial a} = \dfrac{1-x}{1-\beta+\beta x(1+x)}\left((1+\beta) + \dfrac{\partial \beta}{\partial a}\left[a + c + (\hat{p}^* + \hat{r}^* - c - t)\dfrac{1-x(1+x)}{1-x}\right]\right)$,
- $\dfrac{\partial \hat{p}^*}{\partial a} = \dfrac{1}{1-\beta+\beta x(1+x)}\left(2 - x(1+x) + \dfrac{\partial \beta}{\partial a}\hat{p}^*\left(1 - x(1+x)\right)\right)$,

where $\frac{\partial \beta}{\partial a} = \frac{\partial \beta}{\partial \hat{p}}\frac{\partial \hat{p}^*}{\partial a}$. Note that $\frac{\partial \hat{p}^*}{\partial a}$ is always positive when $\frac{\partial \beta}{\partial a}$ is small enough.

If (A23) is negative everywhere in the whole domain $a > \dfrac{t-\beta c}{1+\beta}$, when an interior equilibrium in retail prices exists, then the lowest threshold of a is chosen. At $a^* = \dfrac{t-\beta c}{1+\beta}$, eq. (25) is valid and $\left.\dfrac{\partial \pi^*}{\partial a}\right|_{a=a^*} = 0$. A sufficient condition for having (A23) negative everywhere is that both $\frac{\partial \hat{p}^*}{\partial a} > 0$ and $\frac{\partial(\hat{p}^* + \hat{r}^*)}{\partial a} > 0$. This is ensured by the following condition: $\frac{\partial \beta}{\partial a} \cong 0$,

i.e., the access charge does not influence much the marginal utility in receiving calls. This condition is by no means necessary.[26]

In order to analyse if a^* is a global maximum, we have to compare the profit level reached when $a = a^*$ with the profit level that can be reached by setting $a < a^*$ and inducing a connectivity breakdown. The comparison is simple since we can note that:

- on-net retail prices are always efficient in both cases, thus they do not generate differences;
- off-net calls also do not generate differences: in case of connectivity breakdown there are no off-net calls, thus profits are zero, while when $a = a^*$ off-net calls are efficient but also generate zero profits;
- the only differences arise from the fixed fee F^* given by eq. (12).

In both cases profits are raised only through the fixed fee. In case of connectivity breakdown the fixed fee is:

$$\begin{aligned} F^* &= f - \left(p^* - c - t\right)Q(p^*, p^*) + 1/2\sigma - \hat{r}Q(\hat{p}^*, \hat{p}^*) - \left(v(p^*, p^*) - v(\hat{p}^*, \hat{p}^*)\right) \\ &= f - \left(p^* - c - t\right)Q(p^*, p^*) + 1/2\sigma - v(p^*, p^*). \end{aligned}$$

In case $a = a^*$ the fixed fee is:

$$\begin{aligned} F^* &= f - \left(p^* - c - t\right)Q(p^*, p^*) + 1/2\sigma - \hat{r}Q(\hat{p}^*, \hat{p}^*) - \left(v(p^*, p^*) - v(\hat{p}^*, \hat{p}^*)\right) \\ &= f - \left(p^* - c - t\right)Q(p^*, p^*) + 1/2\sigma - r^*Q(p^*, p^*). \end{aligned}$$

Thus it is better to set $a = a^*$ than to induce a breakdown if $-r^*Q(p^*, p^*) > -v(p^*, p^*) \Rightarrow U(p^*, p^*) - (p^* + r^*)Q(p^*, p^*) > 0 \Rightarrow U(p^*, p^*) - (c+t)Q(p^*, p^*) > 0$ that is always verified. QED

[26] As an example, take the same specification as in footnote 17 with a constant and positive propagation factor $x < 1$, $U^i = Q^{ij}(1 + xQ^{ji}) - Q^{ij^2}/2$. This generates the following demand functions when senders are sovereign most of the time: $Q^{ij} = [1 - \hat{p}_i + x(1 - \hat{p}_j)]/(1 - x^2)$. At a symmetric equilibrium the impact of the price on the ratio of marginal utilities is different from zero: $\partial\beta / \partial\hat{p} = -x/[\hat{p}^{*2}(1 - x)]$. However, after substitutions one also gets: $\partial\hat{p}^* / \partial a = (1 - x)^2(2 + x)/(1 - x^2 - x^3)$ and $\partial(\hat{p}^* + \hat{r}^*)/\partial a = (1 - x)(1 - 2x)/(1 - x^2 - x^3)$. Both derivatives are strictly positive for $0 < x < ½$, thus (A23) is negative. Outside this range of x, it is possible to show that (A23) is still negative everywhere by direct substitution of equilibrium values and by taking into account restrictions on parameters that ensure that equilibrium quantities are positive.