The Dynamic Use of Loyalty Rebates*

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Abstract

This paper studies the vertical relations between a manufacturer and a retailer over two periods in the presence of a competitive recycling sector. I show that contracts without intertemporal quantity targets lead to inefficient outcomes: the manufacturer distorts first-period output downwards, in order to reduce the retailer's payoff under the outside option (i.e. only selling the recycled good) in period 2. I then show that including second-period quantity targets in the first-period contract restores the efficient outcome, without foreclosing the recycling sector. Such contracts can be implemented through loyalty rebates, which are shown to be efficiency-enhancing and non-exclusionary.

JEL classification: L12, L14, L42.

*Work in progress!

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1 Introduction

Loyalty rebates encompass a wide range of business practices, such as pure quantity discounts (which may be all-units discounts or incremental-units discounts), bundled discounts, and market-share discounts. There is a growing scholarly literature on these business practices, suggesting that their effects are still far from being well understood. This paper studies the use of loyalty rebates in a dynamic setting where output decisions taken today will also affect payoffs in the future.

I consider a two-period model where a manufacturer of a recyclable good has to decide how much to produce of its primary good in each period, and at which price to sell its output to the monopoly retailer. A competitive recycling sector will recover and recondition part of the first-period output. In period 2, the monopoly retailer will then sell the recycled quantity along with the second-period quantity of the manufacturer's primary good.

The efficient outcome calls for the retailer to carry both the primary and the recycled good in period 2 ("common agency"). If period-t contracts can only specify period-t quantity and price, then, in the second period, the retailer can reject the manufacturers offer and sell the recycled good only. The retailer's payoff from this outside option will therefore determine its share in the second-period common-agency profits. An inefficiency can arise when these outside profits are increasing in the amount of recycled output available, i.e. increasing in the manufacturer's first-period primary good output. If contracts cannot include intertemporal quantity targets, then the manufacturer distorts first-period output downwards in order to reduce the retailer's payoff under the outside option in period 2. While sacrificing some first-period profits, the manufacturer can appropriate a larger share of second-period profits.

This finding recalls some of the early literature on the impact of recycling on monopoly power (Gaskins (1974), Swan (1980), Martin (1982)) which found a similar downward distortion in the monopolist's first-period output. However, in this literature, the rationale for this first-period output contraction is that in the second period, the recycling sector will steal some of the demand for the monopolist's primary good output. Thus, by reducing first-period output, the monopolist can soften competition from the recycling sector in the next period. In my setting, there is no "business-stealing" motivation: under common agency, the monopoly retailer will always maximize joint profits. However, the manufacturer faces a kind of "hold-up" problem: the more the manufacturer produces in period 1, the higher is the retailer's share in period 2 profits. Thus, the manufacturer does not have the right incentives to produce the industry-profit maximizing quantity in period 1.

Next, I show that including second-period quantity targets in the first-period contract restores the efficient outcome: the retailer commits ex ante to the level of the primary good that it will sell in period 2, effectively giving up its outside

option of selling the recycled good only. Of course, the manufacturer will have to compensate the retailer for the loss of its outside option. However, once this compensation is paid, the retailer will have no further claims on profits in period 2, so that nothing prevents the manufacturer from choosing the efficient first-period output level.

These efficient contracts can be implemented through various types of loyalty rebates, for which this paper gives an efficiency explanation. On various occasions, market-share contracts have been considered as disguised exclusive dealing contracts. Therefore, I also investigate how exclusive dealing contracts compare to the efficient loyalty rebates. I find that exclusive dealing contracts are less efficient than loyalty rebates, because they exclude the recycling sector, but that they may improve over the inefficient contracts without intertemporal quantity targets. However, if the manufacturer can choose between exclusive dealing and loyalty rebates, it would always choose loyalty rebates.

Interestingly, while the term "loyalty" is indicative of a long-term business relationship where seller and buyer interact repeatedly, most of the recent, game-theoretic literature on loyalty rebates has studied a static framework in which seller and buyer only interact once. Most of these papers take a contract theoretic approach, and show how the various types of loyalty rebates can be used to overcome some of the standard problems in vertical relations, like double marginalization (Kolay, Shaffer, and Ordover (2004)), asymmetric information about the state of demand (Majumdar and Shaffer (2007), risk-aversion (Chioveanu and Akgun (2007)), and asymmetric information about product quality (Mills (2004)).

There are very few papers which study the use of loyalty rebates in a dynamic setting. Marx and Shaffer (2004) show how market-share targets can be used for rent shifting when firms contract sequentially. In their paper, a ban on such contracts can be harmful, leading to exclusion that otherwise would not occur. Ordover and Shaffer (2006) analyze a two-period model with two sellers and one buyer where the buyer incurs switching costs: purchasing a unit from a seller in period 1 locks-in the buyer to purchasing a unit from the same seller in period 2. In this setting, a dominant firm can profitably exclude an equally efficient, but financially constrained rival, monopolizing the market when the efficient outcome would have the buyer purchase one unit from each seller in each period.

To my knowledge, the only other formal model of exclusionary rebates is Karlinger and Motta (2007). There, an incumbent and a more efficient entrant simultaneously make offers to a number of buyers who differ in size. Network externalities among these buyers imply that the entrant can profitably serve the market only if it reaches a certain minimum size. The incumbent can then use rebates to play a "divide-and-conquer" strategy, thereby breaking entry equilibria that would exist under uniform linear prices.

Apart from these two papers, the economic literature has found loyalty rebates to be efficiency-enhancing. Yet, their possible anti-competitive effects are

a major concern to antitrust authorities. So far, US authorities and courts have refrained from interfering with the use of loyalty rebates, primarily to avoid setting adverse precedents. Instead, the European Commission has consistently found such rebates to be anti-competitive when employed by a dominant firm. In fact, market-share discounts and discounts based on purchase growth are per-se illegal. The only admissible type of rebate used to be a standardized quantity discount. However, in their surprising *Michelin II* decision of 2003, the European Commission even found such standardized quantity discounts to be unlawful.

The *Michelin II* case concerned the market for replacement truck tires in France. There are two types of tires: new tires and secondary ("retreaded") tires (where "retreaded" means that the tread of a worn tire casing was renewed). The retreading technology is available both to manufacturers of new tires and to a competitive sector of middle-sized firms specialized in retreading. While retreaded tires are considered inferior substitutes to new ones, they account for half of the market for replacement tires. Michelin is the dominant manufacturer of new truck tires in France, and sells its output partly through its own retail network (Euromaster), and partly through a number of independent dealers.

Michelin's vertical relations with its dealers where characterized by a number of rebate and bonus programs for which retailers could qualify. In one of them (the "club des amis Michelin"), the "club" members (some 375 outlets covering about 20% of the market) would guarantee for a certain "Michelin temperature", i.e. for certain sales volumes and market shares. In return, Michelin would contribute to investment and training of the retailer's staff, and also provide financial contributions.

In its decision, the European Commission found Michelin's rebate schemes to be exclusionary, in the sense that Michelin tried to monopolize the retail network so as to deter entry by competing truck tire manufacturers (like Goodyear or Continental). In particular, the "club des amis" was found to amount to a (per-se illegal) loyalty rebate scheme, i.e. a discount based on market shares rather than absolute sales thresholds. But there are good reasons to doubt whether Michelin's intention was really to exclude. The fact that Michelin's own retail network, Euromaster, also carried competing brands is clearly at odds with the allegation of exclusion.

This paper proposes an alternative explanation for Michelin's loyalty rebates: that of maximizing profits on its new tires in the presence of competition from the retreaded tires sector. The paper proceeds as follows: Section 2 presents the basic two-period model and develops the main results regarding inefficient contracts, efficient loyalty rebates, and exclusive dealing contracts. In Section 3, I solve a linear example which illustrates the main findings for a specific class of demand functions. Section 4 concludes.

2 The Basic Two-Period Model

Consider a monopoly producer, denoted M, and a monopoly retailer, R, who both operate for two periods, $t = 1, 2$. In each period, M produces a certain output level of primary good A, denoted q_{At}, at constant marginal cost c_A, and sells this output to R, who then resells it to final consumers. After $t = 1$, consumers scrap M's first-period output. Now, this scrap material can be recycled and offered in $t = 2$ as secondary (or recycled) good B. There is a competitive recycling sector which retrieves M's first-period output and transforms it into good B at constant marginal cost c_B. For technological reasons, only a share $\sigma \in (0, 1)$ of q_{A1} can be recovered, so that the quantity of recycled good available in $t = 2$ is $q_B \leq \sigma q_{A1}$. (Since there is no supply of recycled good in $t = 1$, we omit the time subscript for good B.) The retailer can buy this supply of good B from the competitive recycling sector at price c_B. For simplicity, let's assume that R does not incur any other retailing costs (apart from the payments R has to make to M and to the recycled good sector).

Consumers consider goods A and B as imperfect substitutes, where the primary good is considered to be of superior quality relative to the recycled good. Consumers can observe the two quality levels before buying, but they differ in their willingness-to-pay (WTP) for quality: If the price difference between goods A and B is sufficiently large, some consumers prefer to buy A, and others prefer to buy B. Denote consumers' demand for good A in $t = 1$ by $D_A(p_{A1})$, and their demand for goods A and B in $t = 2$ by $D_A(p_{A2}, p_B)$ and $D_B(p_{A2}, p_B)$, where p_{At} is good A's retail price in period t, and p_B is B's retail price in $t = 2$. (All consumers face the same flat prices, i.e. we rule out price discrimination by R.)

Assumption 1: There is a range of prices $(p_{A2}, p_B) \in R^2$ at which consumers have strictly positive demand for both goods, $D_A(p_{A2}, p_B) > 0$ and $D_B(p_{A2}, p_B) > 0$.

Assumption 1 implies that in $t = 2$, the retailer can choose between (i) selling good A only, or (ii) selling good B only, or (iii) selling both goods (we will refer to this case as the "common agency" solution). Define industry profits (retail revenue minus variable cost of production) in period $t = 1, 2$ as

$$\Pi(q_{At}, q_B) \equiv (p_{At} - c_A) q_{At} + (1 - t)(p_B - c_B) q_B \tag{1}$$

(We rewrite the problem as one of choosing quantity levels, rather than prices, and assume that the profit function is continuously differentiable in both arguments.) The highest possible profits under common agency are denoted by

$$\Pi(q_{A2}^*, q_B^*) = \max_{q_{A2}, q_B} \Pi(q_{A2}, q_B) \text{ s.t. } q_B \leq \sigma q_{A1} \tag{2}$$

where the available scrap from $t = 1$ imposes an upper bound on the quantity of good B that can be sold in $t = 2$. Analogously, define the maximum profit if

either good A or good B is sold as:

$$\Pi_{At} \equiv \max_{q_{At}} \Pi\left(q_{At}, 0\right) \text{ for } t = 1, 2 \tag{3}$$

$$\Pi_B \equiv \max_{q_B} \Pi\left(0, q_B\right) \text{ s.t. } q_B \leq \sigma q_{A1} \tag{4}$$

We will assume that common agency is more profitable than selling only one of the two goods.

Assumption 2: $\Pi_{A2} < \Pi\left(q_{A2}^*, q_B^*\right)$ and $\Pi_B < \Pi\left(q_{A2}^*, q_B^*\right)$

Note that offering both goods allows R to indirectly discriminate among consumers: the high-WTP consumers will prefer to pay a high price to obtain the primary (high-quality) good, while low-WTP consumers prefer to pay a low price to buy the recycled (low-quality) good. Now, offering the low-quality good along with the high-quality good allows R to serve some (low WTP) consumers who would not have bought the good otherwise, at the cost of cannibalizing some demand for the high-quality good (medium-WTP consumers who would buy good A if there is no alternative, but who prefer B if they have the choice). Thus, Assumption 2 states that the first effect dominates the second one, so that offering good B along with good A increases profits.

2.1 Contracts Without Intertemporal Quantity Targets

M can make a take-it-or-leave-it offer to R at the beginning of each period. Suppose that a contract signed in period t stipulates (i) the quantity of good A to be traded in period t, q_{At}, and (ii) the payment to be made to M in period t, $T\left(q_{At}\right)$. Denote such a contract by $\{q_{At}, T\left(q_{At}\right)\}$. In particular, contracts signed in $t = 1$ cannot include provisions regarding period-2 quantities of either good A or B. I will argue that such contracts will not implement the efficient outcome.[1]

Let us first consider the retailer's problem in period 2. Under the common agency solution, the retailer chooses how much to sell of goods A and B:

$$\begin{aligned} &\max_{q_{A2}, q_B} \Pi\left(q_{A2}, q_B\right) \\ &\text{s.t. } q_B \leq \sigma q_{A1} \end{aligned} \tag{5}$$

We denote the solution to this problem as (q_{A2}^*, q_B^*), so that R will want to buy q_{A2}^* units of good A from M, and q_B^* units of good B from the competitive recycling sector. To determine the price at which M will sell these q_{A2}^* units, we have to distinguish two cases: (i) the constraint is binding, or (ii) the constraint is not binding. We will deal with each of these two cases .

Case 1: The constraint is slack, i.e. $q_B^* < \sigma q_{A1}$

Period 2 Contracting

If R rejects M's offer, R's outside option is to sell good B only. Thus, R's reservation payoff is Π_B as defined in expression (4).

[1] Note that I will use the term "efficiency" in the sense of "maximizing joint profits" (not in the sense of maximizing social welfare).

Assumption 3: The constraint $q_B \leq \sigma q_{A1}$ is always binding under the outside option, so that R will sell exactly σq_{A1} if it rejects M's offer.

(In the linear example presented later in this paper, this condition is always satisfied.) It follows that Π_B is a function of q_{A1}.

Then, total industry profits will be $\Pi\left(q_{A2}^*, q_B^*\right)$ as defined in expression (2), and M can claim at most $\Pi\left(q_{A2}^*, q_B^*\right) - \Pi_B\left(q_{A1}\right)$ out of these profits. Thus, M will propose the following contract to R:

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$$\begin{array}{c} q_{A2} = q_{A2}^* \\ T\left(q_{A2}\right) = \Pi\left(q_{A2}^*, q_B^*\right) + c_A q_{A2}^* - \Pi_B\left(q_{A1}\right) \end{array} \tag{6}$$

R will accept this contract, sell q_{A2}^* units of good A and q_B^* units of good B, and generate total profits of $\Pi\left(q_{A2}^*, q_B^*\right)$.

Period 1 Contracting

Without any recycled good available, R's outside option in $t = 1$ is zero. Given the optimal first-period output level, q_{A1}^*, M will therefore extract the full first-period revenue from R:

½ ¾

$$\begin{array}{c} q_{A1} = q_{A1}^* \\ T\left(q_{A1}\right) = \Pi\left(q_{A1}^*, 0\right) + c_A q_{A1}^* \end{array} \tag{7}$$

Now, what is the optimal first-period output level for M? Note that q_{A1} determines not only period-1 profits, $\Pi\left(q_{A1}, 0\right)$, but also period-2 profits, namely through the value of R's outside option, $\Pi_B\left(q_{A1}\right)$. Let M's discount factor on period-2 profits be $\delta \in (0, 1)$. Then, M will choose q_{A1} to maximize the present discounted value of period-1 and period-2 profits:

$$q_{A1}^* = \arg\max_{q_{A1}} \left\{\Pi\left(q_{A1}, 0\right) + \delta\left[\Pi\left(q_{A2}^*, q_B^*\right) - \Pi_B\left(q_{A1}\right)\right]\right\} \tag{8}$$

Note that as long as $q_B^* < \sigma q_{A1}$ (which holds by assumption in Case 1), the joint profit under common agency, $\Pi\left(q_{A2}^*, q_B^*\right)$, is not affected by the choice of q_{A1}. Thus, M's first-order condition reads:

$$\frac{\partial \Pi\left(q_{A1}, 0\right)}{\partial q_{A1}} = \delta \frac{\partial \Pi_B\left(q_{A1}\right)}{\partial q_{A1}} \tag{9}$$

Now, given that $q_B = \sigma q_{A1}$ whenever R exercises the outside option (i.e. R will be quantity-constrained when selling good B only), we have that

$$\frac{\partial \Pi_B\left(q_{A1}\right)}{\partial q_{A1}} > 0$$

Thus, the right-hand side of equation (9) is positive, which means that M will want to produce *less* than what would be optimal under "myopic" profit-maximization (i.e. if period-1 output levels were chosen without taking into account period-2 outcomes):

$$q_{A1}^* < \arg\max_{q_{A1}} \Pi\left(q_{A1}, 0\right)$$

Note that the early literature on the impact of recycling on monopoly (Swan (1980), Martin (1982)) found a similar downward distortion in the monopolist's first-period output. However, in this literature, the rationale for this first-period output contraction is that in the second period, the recycling sector would steal some of the demand for the monopolist's primary good output. Thus, by reducing first-period output, the monopolist can soften competition from the recycling sector in the next period.

In my setting, there is no "business-stealing" motivation: M does not compete neck-and-neck with the competitive recycling sector. Instead, the intermediation of a monopoly retailer ensures that the efficient quantities of both goods are sold, and that all the rents from both types of goods will be absorbed. However, producing more of good A in period 1 means that M indirectly improves R's outside option in period 2, and this in turn implies that the share M can claim of the period-2 profits decreases. Hence, M will want to reduce period-1 output as long as M's losses on period-1 profits are overcompensated by the (discounted) gains in M's share of period-2 profits.

Case 2: The constraint is binding, i.e. $q_B^* = \sigma q_{A1}$

Now, the common agency solution is different from Case 1 because the retailer is constrained in its choice of q_B. The solution to R's problem as stated in expression (5) is now $q_B^* = \sigma q_{A1}$ and some q_{A2}^*.[2]

Period 2 Contracting

Again, R's share in the joint profits will be determined by its outside option, which is to sell nothing of good A, and σq_{A1} units of good B. Of course, the constraint $q_B \leq \sigma q_{A1}$ will also be binding under the outside option, so that, as before, R's outside payoff Π_B as defined in expression (4) is a function of q_{A1}.[3]

Thus, total industry profits will be $\Pi\left(q_{A2}^*, \sigma q_{A1}\right)$, and M can claim at most $\Pi\left(q_{A2}^*, \sigma q_{A1}\right) - \Pi_B\left(q_{A1}\right)$ out of these profits. M's contract offer will therefore be analogous to the one of Case 1, just that the common-agency profits are now different:

$$\left\{ \begin{array}{l} q_{A2} = q_{A2}^* \\ T\left(q_{A2}\right) = \Pi\left(q_{A2}^*, \sigma q_{A1}\right) + c_A q_{A2}^* - \Pi_B\left(q_{A1}\right) \end{array} \right\} \tag{10}$$

Period 1 Contracting

Again, M can extract th06TD.020d52R45333.3(in)]TJTT141Tf2ned e2.3(43.8(d)-345.8(1).2(,)-873.4

where q_{A1}^* maximizes the present discounted value of period-1 and period-2 profits:

$$q_{A1}^* = \arg\max_{q_{A1}} \{\Pi(q_{A1}, 0) + \delta[\Pi(q_{A2}^*, \sigma q_{A1}) - \Pi_B(q_{A1})]\} \tag{12}$$

Unlike Case 1, though, M's choice of q_{A1} will now affect not only R's payoff in period 2, but also the common-agency profits. Every additional unit of good A that M produces in period 1 will relax the constraint on good B in period 2.

Thus, a forward looking producer will want to increase period-1 output above the first-period-profit maximizing level. M's first-order condition therefore reads:

$$\frac{\partial\Pi(q_{A1}, 0)}{\partial q_{A1}} + \delta\frac{\partial\Pi(q_{A2}^*, \sigma q_{A1})}{\partial q_{A1}} = \delta\frac{\partial\Pi_B(q_{A1})}{\partial q_{A1}} \tag{13}$$

As in Case 1, the right-hand side of equation (13) will be positive, because R is quantity-constrained in its outside option. At the solution, the sum of the two terms on the left-hand side is therefore positive as well . It follows that q_{A1}^* will fall short of the efficient first-period output level.

Again, we find that M's first-period output will be distorted downwards, although the trade-off that M faces is now somewhat different: In Case 1, M sacrifices some *first*-period profits to increase its share in the second-period common agency profit. In Case 2, M sacrifices some *second*-period profits in order to reduce R's share in these same profits.

In both cases, though, M faces a kind of "hold-up" problem: a high output level in period 1 (which is in both players' interest) enables R to take advantage of M in period 2 (by increasing R's claims to period-2 profits). Thus, M does not have the right incentives to produce the industry-profit maximizing quantity in period 1.

Proposition 1 *If the period-1 contract between M and R can only specify period-1 quantity and price, then contracting is inefficient: M will distort period-1 output downward to reduce R's share in period-2 profits, thereby reducing the present discounted value of joint profits.*

Proof: We argued above that in Case 1 (quantity constraint on good B is slack under common agency in $t = 2$), efficiency requires M to produce the output level that maximizes first-period profits,

$$q_{A1}^{eff} = \arg\max_{q_{A1}} \Pi(q_{A1}, 0)$$

which yields industry profits of Π_{A1} as defined in expression (3). However, M will instead produce

$$q_{A1}^* = \arg\max_{q_{A1}} \{\Pi(q_{A1}, 0) + \delta[\Pi(q_{A2}^*, q_B^*) - \Pi_B(q_{A1})]\}$$

which yields the same second-period industry profits as the efficient output level q_{A1}^{eff}, namely $\Pi(q_{A2}^*, q_B^*)$, but lower first period profits:

$$\Pi(q_{A1}^*, 0) < \Pi_{A1} \text{ because } q_{A1}^* < q_{A1}^{eff}$$

In Case 2 (quantity constraint on good B is binding under common agency in $t = 2$), the efficient first-period output is

$$q_{A1}^{eff} = \arg\max_{q_{A1}} \{\Pi(q_{A1}, 0) + \delta\Pi(q_{A2}^*, \sigma q_{A1})\}$$

whereas M will produce

$$q_{A1}^* = \arg\max_{q_{A1}} \{\Pi(q_{A1}, 0) + \delta[\Pi(q_{A2}^*, \sigma q_{A1}) - \Pi_B(q_{A1})]\}$$

which is again smaller than the efficient output level. If M produces q_{A1}^* instead of q_{A1}^{eff}, industry profits in the second period will be lower, $\Pi(q_{A2}^*, \sigma q_{A1}^*) < \Pi\left(q_{A2}^*, \sigma q_{A1}^{eff}\right)$, because the quantity constraint on good B in period 2 will be tighter if q_{A1}^* is produced instead of q_{A1}^{eff}. Period 1 profits are likely to be higher in this case, but the net effect on the present discounted value of joint profits is unambiguously negative:

$$\Pi(q_{A1}^*, 0) + \delta\Pi(q_{A2}^*, \sigma q_{A1}^*) < \Pi\left(q_{A1}^{eff}, 0\right) + \delta\Pi\left(q_{A2}^*, \sigma q_{A1}^{eff}\right)$$

Thus, both in Case 1 and in Case 2, first-period output will be distorted downwards, which will result in lower joint profits than the efficient output level would yield.□

2.2 Contracts When Intertemporal Quantity Targets Are Possible

I will now argue that allowing period-1 contracts to include targets regarding period-2 quantities will be sufficient to restore the efficient outcome. I will assume that the inclusion of such quantity targets requires the retailer's approval, which can only be obtained if these contracts give the same present discounted payoff to the retailer as the inefficient contracts analyzed in Section 2.1. Thus, the quantity targets will only serve the purpose of eliminating the inefficiency, but they will not allow M to appropriate any of the payoff that R enjoys under the inefficient contract.

Our benchmark contracts are denoted as follows: Period-1 contracts now take the form $\{q_{A1}, T(q_{A1}), q_{A2}, q_B\}$, while period-2 contracts reduce to $\{T(q_{A2})\}$. Thus, when R accepts such a period-1 contract, R also commits to the output levels to be sold in period 2, i.e. q_{A2} and q_B. (We assume that such a commitment is possible.)

Period 2 Contracting

With R being committed to selling exactly q_{A2} and q_B, R no longer has an outside option in $t = 2$, and so M will appropriate the full period-2 profits:

$$T(q_{A2}) = \Pi(q_{A2}, q_B) + c_A q_{A2} \tag{14}$$

leaving R with zero payoff in $t = 2$.

Period 1 Contracting

Let us start with the determination of $T(q_{A1})$. R anticipates that if it accepts the period-1 contract, it will be left without any payoff in period 2. Thus, for R to accept such a contract, M must compensate R for the loss of the outside option in $t = 2$. Therefore, M cannot extract the full period-1 revenue, but has to leave $\delta\Pi(0, \sigma q^*_{A1})$ to the retailer:

$$T(q_{A1}) = \Pi(q_{A1}, 0) + c_A q_{A1} - \delta\Pi(0, \sigma q^*_{A1}) \tag{15}$$

It is important to note that q^*_{A1} refers to the inefficient output levels of equations (8) and (12) respectively.

As for the period-2 quantity targets, it is straightforward that M will set them at the joint profit maximizing levels:

$$q_{A2} = q^*_{A2}, q_B = q^*_B$$

The crucial point is the choice of the period-1 output level. M knows that R will have no share in period-2 profits, and so the output decision in $t = 1$ will be unaffected by any period-2 payoff-sharing considerations.

M's maximization problem now reads

$$\max_{q_{A1}} \{T(q_{A1}) - c_A q_{A1} + \delta[T(q_{A2}) - c_A q_{A2}]\}$$

In Case 1, this is equivalent to

$$\max_{q_{A1}} \{\Pi(q_{A1}, 0) - \delta\Pi(0, \sigma q^*_{A1}) + \delta\Pi(q^*_{A2}, q^*_B)\} \tag{16}$$

With the last two terms being constant in q_{A1}, M will therefore choose the efficient first-period output level

$$q^{eff}_{A1} = \arg\max_{q_{A1}} \Pi(q_{A1}, 0)$$

In Case 2, M's maximization problem now reads

$$\max_{q_{A1}} \{\Pi(q_{A1}, 0) - \delta\Pi(0, \sigma q^*_{A1}) + \delta\Pi(q^*_{A2}, \sigma q_{A1})\} \tag{17}$$

which will again lead to the efficient output level

$$q^{eff}_{A1} = \arg\max_{q_{A1}} \{\Pi(q_{A1}, 0) + \delta\Pi(q^*_{A2}, \sigma q_{A1})\}$$

Note: Rather than writing two contracts, M and R might as well just contract once, namely in $t = 1$. Such a contract would then also include the payment to be made in $t = 2$: $\{q_{A1}, T(q_{A1}), q_{A2}, q_B, T(q_{A2})\}$. In this case, any pair $\{T(q_{A1}), T(q_{A2})\}$ such that

$$T(q_{A1}) + \delta T(q_{A2}) = \delta\Pi(0, \sigma q^*_{A1})$$

will be accepted by R, and will give rise to the same payoff to M as the benchmark contracts.

Proposition 2 *If period-1 contracts can include quantity targets for period 2, then efficiency is restored: M will produce the joint profit maximizing output level in period 1.*

Proof: follows from above

2.3 Discussion: Loyalty Rebates and Exclusive Dealing

Under the efficient contracts, R receives a positive payoff in period 1, while R's period-1 payoff is zero under the inefficient contracts. Thus, one could interpret R's period-1 payoff, $\delta\Pi\left(0, \sigma q_{A1}^{*}\right)$, as a "bonus" or "compensation" paid to R for accepting the quantity targets in $t = 2$. In this sense, the efficient contracts could take the form of **loyalty rebates**: the retailer is rewarded for carrying a minimum quantity of the dominant manufacturer's product, and for accepting an upper limit on the quantity R sells of the competing product.

Such a contract would be looked at with great skepticism by the European Commission: M would be suspected of trying to monopolize the industry, and the benchmark contracts defined above would even be per se illegal. But as was shown above, the purpose of these quantity targets is not to exclude the recycling sector. Exclusion is neither an intention nor an effect of these contracts. On the contrary, the efficient contracts induce M to expand period-1 output, thereby raising the sales of the recycling sector in period 2 above the level they would reach under the inefficient contracts.

Can we rephrase the efficient contracts $\{q_{A1}, T\left(q_{A1}\right), q_{A2}, q_B\}$ and $\{T\left(q_{A2}\right)\}$ analyzed above in a way that would be lawful under EU competition law? There are of course many ways in which the benchmark contracts could be implemented in practice. For instance, if the contract specifies the quantity target on good A, q_{A2}^{*}, the target on good B could be stated as a share rather than in levels: Instead of requiring $q_B = q_B^{*}$, the contract could require that R's share of sales of good B must not exceed

$$s_B = \frac{q_B^{*}}{q_{A2}^{*} + q_B^{*}}$$

or that R's share of sales of good A must not fall below

$$s_A = \frac{q_{A2}^{*}}{q_{A2}^{*} + q_B^{*}}$$

Given that market size in $t = 2$ is perfectly predictable in $t = 1$, each of these market-share targets is equivalent to a quantity target on good B. However, such contracts, usually called fidelity rebates (i.e. discounts based on the share of a buyer's needs sourced from a supplier) are per se illegal in the EU as well.

Alternatively, given that the contract specifies the first-period output level, $q_{A1} = q_{A1}^{eff}$, the two quantity targets relating to period 2 could be specified as percentages of the first-period output level:

$$g_A = \frac{q_{A2}^{*}}{q_{A1}^{eff}}, g_B = \frac{q_B^{*}}{q_{A1}^{eff}}$$

But again, this formulation would infringe European competition law: rebates must not be based on purchase growth, i.e. on R's past purchases. Until the Michelin II case, one type of contract was for sure lawful, namely "pure quantity discounts": applied to our setting, this means that period-1 contracts may only specify an *absolute* quantity target on good A (i.e. neither as a share of R's total sales in period 2, nor as a share of period-1 sales of good A), but must not make reference to good B. While we could implement our benchmark contract $\{q_{A1}, T(q_{A1}), q_{A2}, q_B\}$ even if we drop q_B from the list, it is not clear whether such a contract would still be accepted by the European Commission.

The reason why loyalty rebates are outlawed in the EU is that they are considered to be disguised exclusive dealing contracts. In the context of our model, it is straightforward to show that loyalty rebates and exclusive dealing contracts are not equivalent: since common agency in period 2 maximizes industry profits, M has no interest in excluding the recycling sector. Thus, if M could choose between loyalty rebates and exclusive dealing contracts, M would always prefer loyalty rebates. However, exclusive dealing contracts can be a second-best solution if loyalty rebates are not available.

Proposition 3 *(i) Exclusive dealing contracts are less efficient than loyalty rebates. (ii) Exclusive dealing contracts may improve over the inefficient contracts without intertemporal quantity targets.*

Proof:

(i) If R signs an exclusive dealing contract with M, then R can only sell good A in $t = 2$, but not good B. Thus, instead of making common-agency profits $\Pi(q_{A2}^*, q_B^*)$, the industry will make profits Π_{A2}, which, by Assumption 2, are lower than the common-agency profits. To make R sign such a contract, M has to compensate R for the loss of its outside option, $\delta\Pi(0, \sigma q_{A1}^*)$. Once R gave up the outside option, M can produce the efficient output in period 1, $q_{A1}^{eff} = \arg\max_q$

A similar argument can be made for Case 2. Suppose that the quantity constraint in $t = 2$ is very tight, i.e. let σ go to zero, so that hardly any of the first-period output can be recycled in period 2. Then, the common agency profits converge to the period-2 profits under exclusive dealing (where sales of good B are ruled out by contract) from above:

$$\lim_{\sigma \to 0} \Pi\left(q_{A2}^*\left(q_{A1}\right), q_B^*\left(q_{A1}\right)\right) = \Pi_{A2}$$

This is true for any level of period 1 output, in particular for $q_{A1} = \arg\max_{q_{A1}} \Pi\left(q_{A1}, 0\right)$, i.e. the output level that M will produce under exclusive dealing. Now, the actual output that M will produce under loyalty rebates is

$$q_{A1}^{eff} = \arg\max_{q_{A1}} \left\{\Pi\left(q_{A1}, 0\right) + \delta\Pi\left(q_{A2}^*, \sigma q_{A1}\right)\right\}$$

This output level generates higher discounted profits than any other q_{A1}, in particular $q_{A1} = \arg\max_{q_{A1}} \Pi\left(q_{A1}, 0\right)$. Therefore, the discounted profits under loyalty rebates must be higher than those under exclusive dealing: M produces a higher output than the one that maximizes first-period profits, because this relaxes the quantity constraint in the second period. Thus, the additional profits made in $t = 2$ overcompensate for the profits foregone in period 1.

(ii) Under the inefficient contracts, M's payoff is

- in Case 1:

$$\Pi\left(0, q_{A1}^*\right) + \delta\Pi\left(q_{A2}^*, q_B^*\right) - \delta\Pi\left(0, \sigma q_{A1}^*\right)$$

- in Case 2:

$$\Pi\left(0, q_{A1}^*\right) + \delta\Pi\left(q_{A2}^*\left(q_{A1}^*\right), q_B^*\left(q_{A1}^*\right)\right) - \delta\Pi\left(0, \sigma q_{A1}^*\right)$$

Consider Case 1: By Assumption 2, we have $\Pi_{A2} < \Pi\left(q_{A2}^*, q_B^*\right)$, so that the inefficient contract yields higher second-period profits than exclusive dealing. R's payoff is the same under both types of contracts: $\delta\Pi\left(0, \sigma q_{A1}^*\right)$. As for period-1 contracts, we have that $\Pi_{A1} > \Pi\left(0, q_{A1}^*\right)$, i.e. exclusive dealing yields higher first-period profits than our inefficient contracts. Therefore, inefficient contracts will be less profitable than exclusive dealing contracts whenever

$$\Pi\left(0, q_{A1}^*\right) + \delta\Pi\left(q_{A2}^*, q_B^*\right) < \Pi_{A1} + \delta\Pi_{A2}$$

i.e. whenever the additional profits from common agency in period 2 do not outweigh the profits lost in period 1 because of the inefficiently low first-period output.□

For the linear example analyzed in Section 3, I will show that the above inequality can go in both directions: depending on parameters, exclusive dealing contracts may be more or less efficient than the contracts without intertemporal quantity targets. (see Section 3.4)

3 A Linear Example

There are two goods indexed by $i = A, B$. Good A is the primary good, while good B is the recycled good. Each good is characterized by a quality index s_i, where good A is superior to good B, so that $s_A > s_B > 0$. Each consumer consumes one unit of good A, or one unit of good B, or zero units. A consumer has the following Mussa-Rosen utility function:

$$U = \begin{cases} \theta s_i - p_i & \text{if consumer buys good } i = A, B \\ 0 & \text{if consumer does not buy} \end{cases} \tag{18}$$

In each period, there is a mass 1 of consumers. Consumers differ with respect to the taste parameter $\theta \in [0, 1]$. Consumers with a high θ are more willing to pay to obtain the high quality good. Parameter θ is distributed in the population according to the cumulative distribution function $F(\theta)$. Let θ be uniformly distributed, so that $F(\theta) = \theta$.

If only good i is supplied at price p_i, then the demand for this good is equal to the mass of consumers with taste parameter θ such that $\theta s_i - p_i \geq 0$. In other words, all consumers with $\theta \geq \frac{p_i}{s_i}$ will buy, so that demand for good i is:

$$D_i(p_i) = 1 - F\left(\frac{p_i}{s_i}\right) = 1 - \frac{p_i}{s_i} \tag{19}$$

If instead both goods are offered, then consumers will buy good A rather than good B whenever

$$\theta s_A - p_A \geq \theta s_B - p_B$$

There is a consumer with $\theta = \tilde{\theta}$ who is indifferent between good A and B:

$$\tilde{\theta} = \frac{p_A - p_B}{s_A - s_B}$$

All consumers with $\theta \geq \tilde{\theta}$ will buy good A:

$$D_A(p_A, p_B) = 1 - F\left(\tilde{\theta}\right) = \frac{p_A - p_B}{s_A - s_B} \tag{20}$$

Those consumers with $\theta \in \left[\frac{p_B}{s_B}, \tilde{\theta}\right]$ will buy good B,

$$D_B(p_A, p_B) = F\left(\tilde{\theta}\right) - F\left(\frac{p_B}{s_B}\right) = \frac{p_A - p_B}{s_A - s_B} - \frac{p_B}{s_B} \tag{21}$$

while consumers with $\theta < \frac{p_B}{s_B}$ will not buy at all. For good B to have positive demand (i.e. for good B not to be dominated by good A), we must have $\tilde{\theta} > \theta_B$, which can be expressed as

$$\frac{s_A}{p_A} < \frac{s_B}{p_B}$$

3.1 Period 2

3.1.1 The Retailer's Outside Option

Let us take for granted that the retailer will always be quantity-constrained when exercising the outside option, no matter if $\sigma q_{A1} \geq q_B^*$ or not. (We will show below that this is indeed the case.) If R rejects M's offer, R will sell good B only. With σq_{A1} units available, R will charge the price that balances supply and demand:

$$D_B(p_B) = 1 - \frac{p_B}{s_B} = \sigma q_{A1}$$

Rearranging terms, we get

$$p_B = s_B(1 - \sigma q_{A1})$$

which yields profits

$$\Pi_B(0, \sigma q_{A1}) = [s_B(1 - \sigma q_{A1}) - c_B]\,\sigma q_{A1} \tag{22}$$

3.1.2 The Common-Agency Solution

If both goods are offered, then the industry profit maximization problem reads as follows:

$$\begin{aligned}&\max_{p_{A2}, p_B} \Pi(p_{A2}, p_B) = (p_{A2} - c_A)\,D_A(p_{A2}, p_B) + (p_B - c_B)\,D_B(p_{A2}, p_B)\\ &\text{s.t. } D_B(p_{A2}, p_B) \leq \sigma q_{A1}\end{aligned} \tag{23}$$

where $D_A(p_{A2}, p_B)$ and $D_B(p_{A2}, p_B)$ are defined in expressions (20) and (21), respectively, c_A is the constant marginal cost that M incurs to produce good A, while c_B is the wholesale price at which good B is provided by the competitive recycling sector.

To make sure that Assumptions 1 and 2 are satisfied, let us impose the following two regularity conditions:

$$s_A - s_B > c_A - c_B \tag{24}$$

$$\frac{s_B}{c_B} > \frac{s_A}{c_A} \tag{25}$$

Case 1: The quantity constraint on good B is slack
Then, the first-order conditions to our problem read

$$\begin{aligned}1 - \frac{2p_{A2} - p_B}{s_A - s_B} + \frac{c_A}{s_A - s_B} + \frac{p_B - c_B}{s_A - s_B} &= 0\\ \frac{p_{A2} - c_A}{s_A - s_B} + \frac{p_{A2} - 2p_B + c_B}{s_A - s_B} - \frac{2p_B - c_B}{s_B} &= 0\end{aligned}$$

Solving these FOCs for prices, we obtain

$$p_{A2}^* = \frac{1}{2}(s_A + c_A) \tag{26}$$
$$p_B^* = \frac{1}{2}(s_B + c_B) \tag{27}$$

which we can insert into the demand functions (20) and (21) to obtain the optimal quantities:

$$q_{A2}^* = \frac{1}{2}\frac{(s_A - s_B) - (c_A - c_B)}{s_A - s_B} \tag{28}$$
$$q_B^* = \frac{1}{2}\frac{c_A s_B - c_B s_A}{(s_A - s_B)\, s_B} \tag{29}$$

The regularity conditions (24) and (25) imposed above ensure that $q_{A2}^* > 0$ and $q_B^* > 0$. Moreover, they imply that the common-agency profits are higher than the profits from either selling good A only or good B only.

Figure 1 below illustrates this solution. The solid red line shows consumers' willingness-to-pay for good A as a function of $1 - \theta$, while the solid green line shows the corresponding function for good B. If both goods are offered, those consumers with the highest willingness-to-pay (those with $\theta \in \left[\tilde{\theta}, 1\right]$) will buy good A, while consumers with $\theta \in \left[\frac{p_B}{s_B}, \tilde{\theta}\right]$ will buy good B (these consumers derive a higher net surplus from consuming good B than from consuming good A). The red hatched area shows profits made on sales of good A, while the green hatched area represents profits made on sales of good B. The sum of these two areas gives the total profits under common agency.

Case 2: The quantity constraint on good B is binding

In this case, we have that

$$D_B(p_{A2}, p_B) = \frac{p_{A2} - p_B}{s_A - s_B} - \frac{p_B}{s_B} = \sigma q_{A1}$$

which we can rearrange to have

$$p_B = \frac{s_B}{s_A}\left[p_{A2} - \sigma q_{A1}(s_A - s_B)\right]$$

Thus, our maximization problem (23) reduces to choosing the optimal p_{A2}. After inserting for p_B and solving the first-order condition, we obtain

$$p_{A2}^* = \frac{1}{2}(s_A + c_A) \tag{30}$$
$$p_B^*(q_{A1}) = \frac{s_B}{s_A}\left[\frac{1}{2}(s_A + c_A) - \sigma q_{A1}(s_A - s_B)\right] > p_B^* \tag{31}$$

Interestingly, the optimal price for good A is the same no matter if the supply of good B is constrained or not. Only the price for good B will change:

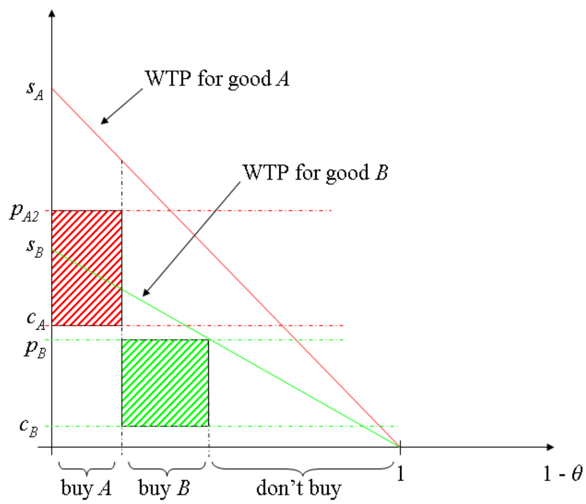

Figure 1: Willingness-to-pay for goods A and B, and common-agency profits

if the constraint is slack, good B is cheaper than if the constraint is binding. It follows that more of good A, and less of good B, is sold if the constraint is binding.

$$q^*_{A2}(q_{A1}) = \frac{1}{2s_A}(s_A - c_A - 2\sigma q_{A1} s_B) > q^*_{A2} \tag{32}$$

$$q^*_B(q_{A1}) = \sigma q_{A1} < q^*_B \tag{33}$$

3.2 Period 1

I will first characterize the efficient solution, and then show that if intertemporal quantity targets are not possible, M will distort period-1 output downwards.

3.2.1 The Efficient Solution

Recall that in $t = 1$, there is no supply of the recycled good, so the primary good is the only one to be sold.

Case 1: Constraint on Good B in $t = 2$ is not binding

Then, the industry profit maximization problem reads:

$$\max_{q_{A1}} \left\{ \Pi\left(q_{A1}, 0\right) + \delta \Pi\left(q_{A2}^{*}, q_{B}^{*}\right) \right\}$$

We see that period-2 profits do not depend on q_{A1} as long as $\sigma q_{A1} \geq q_B^*$, so that the problem can be rewritten as

$$\max_{q_{A1}} \Pi\left(q_{A1}\right) = \left(p_{A1}\left(q_{A1}\right) - c_A\right) q_{A1} = \left(s_A\left(1 - q_{A1}\right) - c_A\right) q_{A1}$$

where $p_{A1}\left(q_{A1}\right) = s_A\left(1 - q_{A1}\right)$ is the indirect demand function for good A in period 1.

Solving for the optimal quantity and price, we obtain

$$q_{A1}^{eff} = \frac{1}{2}\left(\frac{s_A - c_A}{s_A}\right), p_{A1}^{eff} = \frac{1}{2}\left(s_A + c_A\right) \tag{34}$$

Note that the efficient price in the first period, p_{A1}^{eff}, is the same as the optimal common-agency price for good A in the second period, p_{A2}^{*}, as defined in expressions (26) and (30). This is just a special feature of the simple linear specification analyzed here.

Case 2: Constraint on Good B in $t = 2$ is binding

In this case, the industry profit maximization problem becomes truly intertemporal:

$$\max_{q_{A1}} \left\{ \Pi\left(q_{A1}, 0\right) + \delta \Pi\left(q_{A2}^{*}\left(q_{A1}\right), q_{B}^{*}\left(q_{A1}\right)\right) \right\}$$

We can insert for these profit terms to have

$$\max_{q_{A1}} \left\{ \left(s_A\left(1 - q_{A1}\right) - c_A\right) q_{A1} + \delta\left[\left(p_{A2}^{*}\left(q_{A1}\right) - c_A\right) q_{A2}^{*}\left(q_{A1}\right) + \left(p_{B}^{*}\left(q_{A1}\right) - c_B\right) q_{B}^{*}\left(q_{A1}\right)\right] \right\}$$

where $p_{A2}^{*}\left(q_{A1}\right)$, $q_{A2}^{*}\left(q_{A1}\right)$, $p_{B}^{*}\left(q_{A1}\right)$, and $q_{B}^{*}\left(q_{A1}\right)$ are as defined in equations (30), (32), (31), and (33), respectively.

Solving for the efficient q_{A1}, we get

$$q_{A1}^{eff} = \frac{1}{2} \frac{s_A - c_A + \delta\sigma\frac{1}{s_A}\left(c_A s_B - s_A c_B\right)}{s_A + \delta\sigma^2 \frac{s_B}{s_A}\left(s_A - s_B\right)} \tag{35}$$

It is straightforward to show that q_{A1}^{eff} is larger than the "myopic" profit-maximizing quantity, $\frac{1}{2}\left(\frac{s_A - c_A}{s_A}\right)$. In other words, it is optimal to "overproduce" in the first period (thereby sacrificing some first-period profits) in order to relax the quantity constraint on the recycled good in the second period.

We can now show that R will always be quantity-constrained when exercising the outside option in $t = 2$.

Suppose R rejects M's offer and sells good B only. If R was not quantity-constrained, R would choose q_B to solve

$$\max_{q_B} \Pi(p_B) = (p_B(q_B) - c_B)\, q_B$$

where $p_B(q_B) = s_B(1 - q_B)$ is the indirect demand function for good B in period 2. The solution to this problem is

$$q_B = \frac{1}{2}\left(\frac{s_B - c_B}{s_B}\right)$$

Our first regularity condition, inequality (24), implies that this quantity is larger than the optimal q_B^* under common agency:

$$\frac{1}{2}\left(\frac{s_B - c_B}{s_B}\right) > q_B^* = \frac{1}{2}\frac{c_A s_B - c_B s_A}{(s_A - s_B)\, s_B}$$

In Case 2, where the quantity constraint on good B is binding even under common agency, $\sigma q_{A1} < q_B^*$, the quantity constraint on R's outside option is trivially binding: $\sigma q_{A1} < q_B^*$ and $q_B^* < \frac{1}{2}\left(\frac{s_B - c_B}{s_B}\right)$ implies that $\sigma q_{A1} < \frac{1}{2}\left(\frac{s_B - c_B}{s_B}\right)$.

It remains to show that R's quantity constraint will be binding even in Case 1, i.e. where $\sigma q_{A1} \geq q_B^*$. In this case, we have that

$$\sigma q_{A1} < q_{A1}^{eff} = \frac{1}{2}\left(\frac{s_A - c_A}{s_A}\right)$$

where q_{A1}^{eff} is the efficient period-1 output level in Case 1. The inequality follows from $\sigma < 1$ (only a fraction of period-1 output can be recycled) and $q_{A1} \leq q_{A1}^{eff}$ (if contracts are inefficient, period-1 output will fall short of the efficient level, otherwise the efficient quantity q_{A1}^{eff} is produced).

Now, we also have that

$$q_{A1}^{eff} = \frac{1}{2}\left(\frac{s_A - c_A}{s_A}\right) < \frac{1}{2}\left(\frac{s_B - c_B}{s_B}\right)$$

where the inequality is implied by regularity condition 2, as defined in (25). It follows that

$$\sigma q_{A1} < \frac{1}{2}\left(\frac{s_B - c_B}{s_B}\right)$$

i.e. even in Case 1, R will be quantity constrained when exercising the outside option.

3.2.2 Inefficient Output Choice

Suppose that period-1 contracts can only specify period-1 output levels. Then, the choice of q_{A1} determines not only period-1 profits on the primary good market, but also R's period-1 profits under the outside option.

Case 1: We argued in Section 2.1 that M's optimal first period output is given by

$$q_{A1}^{*} = \arg\max_{q_{A1}} \left\{ \Pi\left(q_{A1}, 0\right) + \delta\left[\Pi\left(q_{A2}^{*}, q_{B}^{*}\right) - \Pi_{B}\left(q_{A1}\right)\right] \right\}$$

Inserting for the relevant profit terms, M solves the following problem:

$$\max_{q_{A1}} \left\{ \left(s_{A}\left(1 - q_{A1}\right) - c_{A}\right) q_{A1} - \delta\left[s_{B}\left(1 - \sigma q_{A1}\right) - c_{B}\right] \sigma q_{A1} \right\}$$

where we drop $\Pi\left(q_{A2}^{*}, q_{B}^{*}\right)$ from the objective function because it is not a function of q_{A1}. The solution to the above problem is

$$q_{A1}^{*} = \frac{1}{2} \frac{s_{A} - c_{A} - \delta\sigma\left(s_{B} - c_{B}\right)}{s_{A} - \delta\sigma^{2} s_{B}} \tag{36}$$

and we clearly have that

$$q_{A1}^{*} < q_{A1}^{eff} = \frac{1}{2}\left(\frac{s_{A} - c_{A}}{s_{A}}\right)$$

i.e. M will distort first-period output downwards.

Case 2: The problem is analogous, just that now the common-agency profits also depend on q_{A1}. M has to find

$$q_{A1}^{*} = \arg\max_{q_{A1}} \left\{ \Pi\left(q_{A1}, 0\right) + \delta\left[\Pi\left(q_{A2}\left(q_{A1}\right), \sigma q_{A1}\right) - \Pi_{B}\left(q_{A1}\right)\right] \right\}$$

Again, we can insert for the relevant profit terms,

$$\begin{aligned}
\Pi\left(q_{A1}, 0\right) &= \left(s_{A}\left(1 - q_{A1}\right) - c_{A}\right) q_{A1} \\
\Pi\left(q_{A2}\left(q_{A1}\right), \sigma q_{A1}\right) &= \left(p_{A2}^{*} - c_{A}\right) q_{A2}^{*}\left(q_{A1}\right) + \left(p_{B}^{*}\left(q_{A1}\right) - c_{B}\right) q_{B}^{*}\left(q_{A1}\right) \\
\Pi_{B}\left(q_{A1}\right) &= \left[s_{B}\left(1 - \sigma q_{A1}\right) - c_{B}\right] \sigma q_{A1}
\end{aligned}$$

where p_{A2}^{*}, $q_{A2}^{*}\left(q_{A1}\right)$, $p_{B}^{*}\left(q_{A1}\right)$, and $q_{B}^{*}\left(q_{A1}\right)$ are defined in equations (30), (32), (31), and (33), respectively. The solution then reads

$$q_{A1}^{*} = \frac{1}{2} \frac{s_{A} - c_{A} - \delta\sigma s_{B}\left(\frac{s_{A} - c_{A}}{s_{A}}\right)}{s_{A} - \delta\sigma^{2} s_{B} \frac{s_{B}}{s_{A}}} \tag{37}$$

Again, we have that period-1 output falls short of the efficient quantity:

$$q_{A1}^{*} < q_{A1}^{eff} = \frac{1}{2} \frac{s_{A} - c_{A} + \delta\sigma \frac{1}{s_{A}}\left(c_{A} s_{B} - s_{A} c_{B}\right)}{s_{A} + \delta\sigma^{2} \frac{s_{B}}{s_{A}}\left(s_{A} - s_{B}\right)}$$

3.3 The Manufacturer's Offers

3.3.1 Inefficient Contracts

If period-1 contracts cannot specify period-2 quantity targets, then M will offer:

$$\{q_{A1} = q^*_{A1}, T(q_{A1}) = \Pi(q^*_{A1}, 0) + c_A q^*_{A1}\}$$

where q^*_{A1} is either as in expression (36) or as in expression (37).

Then, period-2 contracts specify both q_{A2} and $T(q_{A2})$, and M will make the following offers:

Case 1: $\{q_{A2} = q^*_{A2}, T(q_{A2}) = \Pi(q^*_{A2}, q^*_B) + c_A q^*_{A2} - \Pi(0, \sigma q^*_{A1})\}$

where q^*_{A2} and q^*_B are defined in equations (28) and (29), $\Pi(q^*_{A2}, q^*_B)$ is the common agency profit evaluated at (q^*_{A2}, q^*_B), and $\Pi_B(0, \sigma q_{A1})$ is the retailer's outside profit as defined in (22) evaluated at the inefficient first-period output level q^*_{A1} as in expression (36).

Case 2: $\{q_{A2} = q^*_{A2}(q^*_{A1}), T(q_{A2}) = \Pi(q^*_{A2}(q^*_{A1}), q^*_B(q^*_{A1})) + c_A q^*_{A2}(q^*_{A1}) - \Pi_B(0, \sigma q^*_{A1})\}$

which refers to the optimal quantities when the constraint on good B is binding, i.e. expressions (32) and (33), and the inefficient first-period output level q^*_{A1} as in expression (37). Note that the period-2 quantities of both good A and good B will be distorted because they depend on period-1 output, which falls short of the efficient output level.

3.3.2 Implementing the Efficient Solution

We showed in Section 2.2 that an efficient period-1 contract takes the form $\{q_{A1}, T(q_{A1}), q_{A2}, q_B\}$, where

$$q_{A1} = q^{eff}_{A1}, q_{A2} = q^*_{A2}, q_B = q^*_B$$

and

$$T(q_{A1}) = \Pi\left(q^{eff}_{A1}, 0\right) + c_A q^{eff}_{A1} - \delta\Pi(0, \sigma q^*_{A1})$$

where q^{eff}_{A1} refers to the efficient output level of either Case 1 (expressions (34) or Case 2 (35). In Case 1, q^*_{A2} and q^*_B are given by (28) and (29). In Case 2, expressions (32) and (33), evaluated at the efficient output level q^{eff}_{A1} in (35), apply.

Since M can appropriate the full period-2 profits, M has no incentive to distort first-period output, and will therefore produce the efficient output level q^{eff}_{A1}. The payment to R, $\delta\Pi(0, \sigma q^*_{A1})$, is determined by the inefficient output level q^*_{A1} as in expression (36) or (37).

In Period 2, M will offer the following contract:

Case 1: $\{q_{A2} = q^*_{A2}, T(q_{A2}) = \Pi(q^*_{A2}, q^*_B) + c_A q^*_{A2}\}$

Case 2: $\left\{q_{A2} = q^*_{A2}\left(q^{eff}_{A1}\right), T(q_{A2}) = \Pi\left(q^*_{A2}\left(q^{eff}_{A1}\right), q^*_B\left(q^{eff}_{A1}\right)\right) + c_A q^*_{A2}\left(q^{eff}_{A1}\right)\right\}$

where q^{eff}_{A1} is defined in (35).

3.4 Exclusive Dealing Contracts

We argued that exclusive dealing contracts will generate industry profits of

$$\Pi_{A1} + \delta\Pi_{A2} = (1+\delta)\frac{1}{4}\frac{1}{s_A}(s_A - c_A)^2$$

I will give two numerical examples: In Example 1, exclusive dealing contracts perform even worse than the inefficient contracts, while in Example 2, exclusive dealing contracts improve over the profits achieved under the inefficient contracts.

Example 1: $s_A = 4, s_B = 2, c_A = 1.5, c_B = 0.5, \delta = 0.8, \sigma = 0.5$

These parameters satisfy the regularity conditions (24) and (25). Under the exclusive dealing contract, industry profits are

$$(1+\delta)\frac{1}{4}\frac{1}{s_A}(s_A - c_A)^2 = 0.70313$$

Consider now the inefficient contracts without quantity targets. Under common agency in $t = 2$, the (unconstrained) optimal quantity of good B is

$$q_B^* = \frac{1}{2}\frac{c_A s_B - c_B s_A}{(s_A - s_B)\, s_B} = 0.125$$

If Case 1 applies, i.e. if $\sigma q_{A1} \geq q_B^*$, then M will produce the following (inefficient) output level in $t = 1$:

$$q_{A1}^* = \frac{1}{2}\frac{s_A - c_A - \delta\sigma(s_B - c_B)}{s_A - \delta\sigma^2 s_B} = 0.26389$$

Thus, we have that $\sigma q_{A1}^* = 0.13195$ is indeed sufficient to supply $q_B^* = 0.125$ in period 2. Then, industry profits over both periods are

$$\Pi(q_{A1}^*, 0) + \delta\Pi(q_{A2}^*, q_B^*) = 0.70617$$

We see that the inefficient contracts perform better than exclusive dealing contracts: $0.70313 < 0.70617$

However, both types of contracts are outperformed by the efficient loyalty rebate, which yields

$$\Pi\left(q_{A1}^{eff}, 0\right) + \delta\Pi(q_{A2}^*, q_B^*) = 0.71563$$

Example 2: $s_A = 7, s_B = 4, c_A = 1.5, c_B = 0.5, \delta = 0.8, \sigma = 0.5$

The values for s_A and s_B are now higher, while all the other values are unchanged. Again, these parameters satisfy the regularity conditions (24) and (25). Under the exclusive dealing contract, industry profits are now

$$(1+\delta)\frac{1}{4}\frac{1}{s_A}(s_A - c_A)^2 = 1.9446$$

Next, consider the inefficient contracts without quantity targets. Under common agency in $t = 2$, the (unconstrained) optimal quantity of good B is

$$q_B^* = \frac{1}{2}\frac{c_A s_B - c_B s_A}{(s_A - s_B)\, s_B} = 0.10417$$

If Case 1 applies, i.e. if $\sigma q_{A1} \geq q_B^*$, then M will produce the following (inefficient) output level in $t = 1$:

$$q_{A1}^* = \frac{1}{2}\frac{s_A - c_A - \delta\sigma (s_B - c_B)}{s_A - \delta\sigma^2 s_B} = 0.33065$$

Thus, we have that $\sigma q_{A1}^* = 0.16533$ is indeed sufficient to supply $q_B^* = 0.10417$ in period 2. Then, industry profits over both periods are

$$\Pi (q_{A1}^*, 0) + \delta\Pi (q_{A2}^*, q_B^*) = 1.9324$$

We see that now, exclusive dealing contracts generate higher profits than the inefficient contracts: $1.9446 > 1.9324$. Thus, if loyalty rebates are not available, but exclusive dealing contracts are, M might use the latter as a second-best solution to the contracting problem. But again, neither of these contracts performs as well as the efficient loyalty rebate, which yields

$$\Pi \left(q_{A1}^{eff}, 0 \right) + \delta\Pi (q_{A2}^*, q_B^*) = 1.9595$$

4 Conclusion

This paper considered the vertical relations between a manufacturer and a retailer in the presence of a competitive recycling sector. Efficiency requires that the retailer sell both the primary and the recycled good in the second period. I argued that contracts without intertemporal quantity targets lead to inefficient outcomes: the manufacturer distorts first-period output downwards, in order to reduce the retailer's payoff under the outside option. Thus, the manufacturer trades off some first-period profits against a larger share in the second period industry profits.

I then showed that including second-period quantity targets in the first-period contract restores the efficient outcome: If the retailer commits to selling both goods in the second period, thereby eff

The linear example developed in the last section shows that the inequality may go either way: depending on parameter values, exclusive dealing contracts may be more or less efficient than the inefficient contracts.

The contribution of this paper is to show how loyalty rebates can be used in a dynamic setting where first-period output choices affect future payoffs. The findings also shed light on the possible motivations for Michelin's use of market-share targets in its vertical contracts with tire retailers: rather than excluding rival manufacturers of tires, the objective may have been to maximize industry profits in the presence of a competitive tire-recycling sector.

There are several interesting issues that deserve further investigation: for instance, there may be other vertical restraints than the ones analyzed here (e.g. bundling) or combinations of several vertical restraints (e.g. bundling plus exclusive dealing) which can replicate the efficient outcome. Another avenue for future research is the structure of the scrap output market, i.e. the market on which first-period consumers sell their used goods to the recycling sector. The model analyzed in this paper is silent about the functioning of this market, which may have important implications for the contracting outcomes between manufacturer and retailer.

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