Recovering the Sunk Costs of R&D: The Moulds Industry Case*

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Abstract

R&D firms coexist with non-R&D firms, even in narrowly defined industries. In this paper I estimate the magnitude of the R&D sunk costs which rationalize firm behaviour observed in the data. On the methodological side, I develop an estimable model with dynamic competition where firms can decide to invest in physical capital and R&D. By assuming that firms' individual states are private information, the industry state is summarized by the aggregate (payoff relevant) state. This has two advantages for estimation purposes: (i) it avoids the 'curse of dimensionality', typical in dynamic industry models and; (ii) it deals with unobserved firms in the data, a problem neglected in the literature, but severe if one wants to estimate from the equilibrium conditions.

I apply the model to the Portuguese Moulds Industry and estimate the average sunk costs of R&D to be 2.6 million euros (1.7 times the average firm sales). Finally, I evaluate the impact of a reduction in the sunk costs of R&D on equilibrium market structure, productivity and capital stock. The results corroborate the idea that sunk costs of R&D have implications for policies which target at promoting R&D. Policy makers should be concerned with reducing the large sunk costs of R&D and promote R&D start-ups.

Keywords: Incomplete Information, Investment, Markov Equilibrium, Moulds Industry, R&D, Structural Estimation, Sunk Costs

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1 Introduction

Even in narrowly defined industries R&D firms coexist with non-R&D firms. Since most existing theories focus in the continuous rather than the discrete decision, they predict that in general, either all or no firms perform R&D (e.g. Cohen and Klepper, 1996; Klette and Kortum, 2004; Vives, 2004). In this paper I explore the discrete decision to become an R&D firm with two main contributions. First, I develop a model which deals with the 'curse of dimensionality', typical of dynamic industry models. This is achieved by using an aggregate (payoff relevant) state to represent the state of the industry. This way, instead of keeping track of all competitors' state, each firm just tracks individual state and the aggregate state, considerably reducing the size of the state space. The second contribution is the quantification of sunk costs of R&D. I estimate the sunk costs of R&D at the level which is consistent with the actual R&D decisions observed in the data. This is done using a methodology recently developed by Bajari, Benkard and Levin (2007) to estimate dynamic industry models.

The Portuguese Moulds industry has been very successful and is recognized worldwide for its quality standards, technology and competitive prices. A report for the US international trade commission (USITC, 2002) emphasizes the fast delivery, technology, quality and competitive price as the main strengths of the Portuguese Moulds Industry. There has been also a considerable effort in moving upstream in the value chain by also supplying design and propotyping services. Some firms have also developed new materials for the moulds. This creates value for the clients since it allows them to reduce the costs of production. Eventough this upstream move and technology shift requires considerable investment in Research and Development, more than 60% of firms in the industry do not perform R&D and these firms are considerably less productive. I estimate the size of the costs required to rationalize this wedge.

Firms have a similar evolution within the industry. First, they are typically founded by an ex-employee(s) (managers of engineers) who left another firm in the industry to start his (their) own business and the size of the start-up is normally very small (less than 10 workers). If the firm is sucessfull and able to secure some client base, they grow by investment and increment in producing capaticty. Later in their life, they might decide to increase their supply of services to design and propotypying and also develop new products and materials which is achieved by performing R&D.

There is a considerable cost of becoming a pro-active firm who besides producing moulds is also able to supply their clients with moulds conception and design skills, mould testing and development and development of new materials, all at a competitive price. A successful innovative firm should be able to produce not only the mould itself but also all the pre and post production services required by their clients. The costs can range from training and hiring of new employees, investment in new machinery and even the establishment of

links with universities and public research agencies. These costs strongly support the idea of sunkness since they can not be recovered, particularly in this industry, and they are also easy to reconcile with the fact that R&D firms are bigger than their non-R&D counterparts.

Sunk costs have for a long time been regarded as one potential source of inefficiency in the economy. The earlier literature puts most of the emphasis in the failure of the contestability theory in the presence of sunk entry costs, which results in market failures because the industry will not be competitive and firms can maintain some degree of market power (Baumol and Willig, 1981; Stiglitz, 1987). The issue is of great importance for policy makers and regulators since their existence results in a market failure which induces the need for policy intervention.

Sunk costs of R&D, in particular, have been widely studied in the industrial organization literature, especially following the work by Sutton (1991, 1998). The main purpose of this research was to explore the relationship between R&D and market structure. Especially, firms can use R&D as a strategic tool to increase barriers to entry and maintain a dominant position even for large market size. One constraint raised by Schmalensee (1992) is that, in cases R&D does not have a 'forever lasting' effect and therefore does not create a 'forever lasting' advantage/barrier, it is not clear how the incumbent can maintain a dominant position. However, the study of more complex dynamics for the outcome of R&D requires a fully dynamic model that goes beyond the two period approach and this type of framework was at the time in an early development stage. Dixit (1988) acknowledges this in his work by referring that

> *"Perhaps the most important aspect ignored here is the possibility of partial progress (state variables) in the R&D race. That has so far proved intractable at any reasonably general level, but remains an important problem for future research".* Dixit (1988: 326)

The objective in this paper is quantifying the magnitude of R&D sunk costs and, the implications for industry R&D and innovation. I will estimate the sunk costs of R&D in a fully dynamic setting and I find these to be of significant magnitude (1.7 times the yearly average sales in the industry). To achieve this I develop a dynamic framework for productivity and physical capital accumulation which incorporates a constant elasticity demand specification. In this area, several dynamic industry equilibrium models have been developed (Jovanovic, 1982; Hopenhayn, 1992; Ericson and Pakes, 1995; Klette and Kortum, 2004). Since it allows for optimal R&D and investment choices, I use the framework studied in Ericson and Pakes (1995), but I summarize the industry state by the aggregate (payoff relevant) state.

The paper will progress in two parts. In the first part I develop a model which can be applied to the type of financial databases available and avoids the 'curse of dimensionality'. In the second part of the paper I estimate the model for a panel of firms in the Portuguese Moulds industry and recover the sunk costs of R&D. The framework is the following: firms

can enter and exit the market, invest in physical capital and decide to engage in R&D by paying a setup sunk cost. There are both linear and quadratic costs with total irreversibility for the physical capital investment. Productivity follows a first ordef Markov process which depends on whether the firm is an R&D performer or not. To become an R&D performer, firms have to pay a setup sunk cost. Finally, they compete in the market where demand is modeled by a representative consumer Constant Elasticity of Substitution framework.

Most firm level datasets[1] contain information on financial variables (balance sheet, profits and losses, number of workers) for a subset of the total population of firms in the industry. This creates problems on estimating game theoretic type of models, because in these cases we need information on the whole population of firms, otherwise we need to control for unobserved players. Most studies in empirical Industrial Organization have then focused on oligopolies or regulated industries where there is information for all players in the market, but this leaves aside a large part of industries which are relevant and interesting cases.

Furthermore, for the question I am interested in, oligopolist markets are less attractive since in most cases all firms are large and perform R&D. However, it will be the coexistence in equilibrium of both R&D and non-R&D firms that allows the identification of sunk costs. A second problem arising in the estimation is the 'curse of dimensionality' which occurs when the state space grows exponentially, either by increasing the number of firms or the number of states per firm.

To deal with the problems mentioned above, I introduce the assumption of incomplete information. By doing this the industry state is summarized by the (payoff relevant) aggregate state. The equilibrium definition is then very intuitive. The evolution of the aggregate industry state is consistent with optimal behavior and given the beliefs about the evolution of the aggregate state agents behave optimally (single agent decision conditional on beliefs). The assumption addresses the two problems both avoiding the 'curse of dimensionality' by reducing the dimensionality of the state space and dealing with unobserved firms in the data since it only requires that the aggregate industry state is observed.[2]

The problem can then be represented as a single agent dynamic model where beliefs about the evolution of the industry must be consistent with actual play. I have also developed an

[1] Examples of these are Standard & Poor's COMPUSTAT Database for US firms, Bureau Van Dijk's FAME (UK) and AMADEUS (Europe) database or Thomson Financial's DATASTREAM database (UK). Only census data would contain observations for all firms present in the industry and even in this case smaller firms are sampled.

[2] To better understand the "curse of dimensionality' problem, consider a model with several state variables per firm and/or large numbers of firms. Equilibria and policy rules are then impossible to compute since the size of the problem grows exponentially. For example, call $\mathbf{s}$ the industry state (i.e. if we define s_{it} the state vector of firm i at time t, then the industry state at time t is $\mathbf{s}_t \quad s_{1t}, ...s_{Nt}$), finding the industry state transition, $Q \; \mathbf{s}_{t+1}|\mathbf{s}_t$, for an industry with 50 firms and 2 binary state variables would mean calculating a $^{50} \times \; ^{50}$ transition matrix. If one assumes the typical anonymity and symmetry (Pakes and McGuire, 2001) the problem will be greatly reduced but still intractable ($^{4} \times \; ^{4}$). The 'curse of dimensionality' is not only a computational problem but will also arise in the estimation. As we will see ahead, if one tries to estimate a flexible policy function on the whole industry state like proposed by Bajari, Benkard and Levin (2007), since this industry state is very large, it will require a large quantity of data (not available on most firm level dataset).

algorithm, which is represented in figure 1, to solve the model which resembles a nested fixed point where the inside loop solves the dynamic programing problem and the outside loop solves for equilibrium beliefs. I can use this algorithm to recalculate the model for different structural parameters and perform policy simulations. This would not be possible in the Full Information case for the Moulds Industry where the average number of firms is above 500.

In related research Weintraub, Benkard and Van Roy (2007) propose the use of a different equilibrium concept, the approximated "Oblivious Equilibrium". In this type of equilibrium firms disregard the current state of the industry and base their decisions solely upon the long run industry state. As the number of firms in the industry grows, this converges to the MPNE provided the industry state distribution satisfies a 'light tail' condition. This result resembles Hopenhayn (1992) and when the number of firms grows large, with no aggregate shocks, the equilibrium is deterministic.

The firm's information can work as a link between Weintraub, Benkard and Van Roy (2007) and Ericson and Pakes (1995). To see this recall that in Ericson and Pakes, individual states are public information (perfect information) whereas in Weintraub et al. it is as if only the long run industry distribution is common knowledge (minimum information). In the setting I will develop below, individual states are privately observed but the aggregate industry state is publicly observed (partial information).

Introducing incomplete information has some potential drawbacks by implicitly imposing more structure on the type of strategic interactions since firms now react to the 'average' competitor (i.e. firm A's reaction to competitors B and C is identical either they are similar or very different). In some cases, like oligopolist industries, this comes at a cost. However, for industries where competition is well summarized by the aggregate state variables, this restriction is minor. This is the case in industries where there is a large number of players, there are no market leaders and products are differentiated. Examples of these type of industries are for example Industrial Machinery Manufacturing or Metalworking Machinery Manufacturing (moulds, dies, machine tools). What all these industries share in common is the fact that each firm sells specialized products, prices are contract specific and information is kept secret from competitors.

The earlier dynamic models only accounted for the effects of entry and exit and did not allowed for investment or R&D (Jovanovic, 1982; Hopenhayn, 1992). Ericson and Pakes (1995) develop an attractive framework where players interact strategically using Markov strategies which generates a Markov Perfect Nash Equilibrium (MPE) as defined by Maskin and Tirole (1988, 2001).

However, the MPNE brings two complications. One was the possibility of non-existence of equilibrium which Doraszelski and Satterthwaite (2005) addressed with the introduction of privately observed i.i.d. shocks. The second, is the 'curse of dimensionality' and the com-

putational burden. Recent algorithms (e.g. Pakes and McGuire (2001)) are very successfull in avoiding this second problem and solve the model for cases up to 10-15 firms, by using techniques similar to the artificial intelligence literature. However, they still cannot solve problems where there is a larger number of firms in the market.

Other theoretical models exist that study the R&D decision in an industry framework. Vives (2004) for example does this in a static setting, but since it does not incorporate any heterogeneity, it cannot explain the coexistence of R&D and non-R&D firms. Klette and Kortum (2004) use an interesting dynamic framework with the advantage of providing an analytical solution. However, the model cannot be extended to account for R&D sunk costs and it does not allow for aggregate uncertainty making it unattractive for the question I want to address.

There has been an increased interest in the literature on estimating dynamic industry model with some successful applications to oligopolies (Benkard, 2004; Ryan, 2005; Schmidt Dengler, 2006). Several alternative techniques have been developed and this is currently an area under research (Aguirregabiria and Mira, 2007; Bajari, Benkard and Levin, 2007; Pakes, Ostrovsky and Berry, forthcoming; Pesendorfer and Schmidt-Dengler, forthcoming).

I use a method similar to Hotz et al. (1994) as developed by Bajari, Benkard and Levin (2007) because it allows for both continuous and discrete actions. The estimation is done in two steps. In the first step I recover the static parameters (production function, demand elasticity, policy function and transition functions). By assumption, estimated policies are profit maximizing for the actual equilibrium observed in the data. I can then estimate continuation values by simulating industry paths far enough in the future using the estimated policies and transitions. Using non-optimal policies by slightly perturbing the estimated policy functions I can simulate alternative (non-profit maximizing) continuation values. With these optimal and non-optimal continuation values and exploring the property that the value function is linear in the dynamic parameters, I can recover the parameters by imposing the equilibrium condition that optimal continuation values must be larger then non-optimal continuation values, without needing to recalculate continuation values for each set of parameters.

The minimum distance estimator explores the optimality condition by searching for the parameters that minimize the cases where the continuation values for the non-optimal policies are larger than the continuation values for the estimated policies. These are the parameters which are consistent with actual behavior being optimal.

One alternative I have not explored here is the possibility of using a nested fixed point estimator as suggested by Rust (1987). The reason why I can do this is because conditional on equilibrium beliefs for the evolution of the industry state, agents solve a simple dynamic programing problem. The equilibrium beliefs can be directly recovered from the data and

parameters estimated using a single agent approach.[3]

To confirm the validity of the used assumptions, I perform some specification tests. The main objective of introducing incomplete information into the model is to solve the 'curse of dimensionality' problem by summarizing the industry state distribution into the aggregate industry state. This allows the restriction to Markovian strategies on own state and aggregate (payoff relevant) industry state to work. However, problems occur if the (equilibrium) aggregate industry state does not follow a first order Markov process because Markovian strategies are no longer optimal and previous lags of the aggregate industry state are relevant, potentially making optimal strategies history dependent. This can be checked by testing the significance of previous lags (t-2 and above). An alternative I have also explored, is to test the significance of further moments of the industry state distribution in predicting the evolution of the aggregate state. If previous lags of the aggregate state and/or further moments of the industry state distribution are not significant in predicting the aggregate industry state, the model is well specified.

The data I use has been collected by the Bank of Portugal ("Central de Balancos") yearly for the period 1994-2003. This industry competes in the international market (90% of total production is exported, mainly supplied to the auto industry) and it has recently faced increased competition from Asian countries. The strategy adopted by most players has been to reinforce strong links with clients, to develop new materials (product innovation) and minimize waste (process innovation). Given the state of the industry, most firms would be expected to perform R&D since according to the experts it is the only way to survive the strong competition. The sector has developed partnerships with Universities to achieve this and has been quite successful internationally. However, a significant proportion of firms (56% in my sample for the year 2003) still reports no R&D. Some under reporting could be happening because the accounting rules to qualify as R&D expenditures are quite restrictive. However, under reporting cannot explain the large number of firms not reporting R&D. I will argue that it is due to high sunk costs of starting R&D and I will estimate these from the data. Since the industry is populated by many small firms and the products and prices are contract specific, the industry fits very well in the models' assumptions. Finally I evaluate the impact of a 25% reduction in the sunk costs of R&D. This results in an 11% increase in average productivity and 18% increase in average capital stock.

The rest of the paper is organized as follows. In section 2 I outline the model, section 3 provides details of the application, section 4 describes the estimation, section 5 gives a brief introduction to the moulds industry, section 6 summarizes the data, section 7 contains the results and section 8 the policy experiments, in section 9 I provide some possible extensions and future research and finally section 10 concludes.

[3] The main problem with such approach is its computational cost even for the single agent case when there are several individual state variables.

2 The aggregate state dynamic model

2.1 States and actions

This section describes the elements of the general model. Time is discrete and every period, $t = 1, 2, ..., \infty$, there are N firms in the market (N_t incumbents and $N_t^* = N - N_t$ potential entrants) and a typical firm is denoted by $i \in \{1, ..., N\}$

States Agents are endowed with a state (discrete or continuous) $s_{it} \in \mathfrak{s}_i$ and a vector of payoff shocks $\varphi_{it} \in \mathfrak{J}$. Both the state and the payoff are privately observed by the players. The econometrician observes the states, s_{it}, but not the payoff shocks, φ_{it}.

Assumption 2.1 *(a) Individual states and actions are private information and;*
(b) $p(\mathbf{s}_t|S_t, ..., S_0) = p(\mathbf{s}_t|S_t)$

The industry state is $\mathbf{s}_t = (s_{1t}, ..., s_{Nt}) \in \mathfrak{s}_i^N$. The vector of payoff shocks is drawn i.i.d. and assumed to depend on the actions of the players. This satisfies Rust's (1994) conditional independence assumption and allows the value function to be written as a function of the state variables which keeps the number of payoff relevant state variables small.

Actions Incumbents choose $l = l^c + l^d$ actions that can be continuous $a_{it}^c \in \mathfrak{A}^c \subset \mathbb{R}^{l^c}$ or discrete (exit, R&D start-up) $a_{it}^d \in \{0,1\}^{l^d}$ and $a_{it} = \{a_{it}^c, a_{it}^d\} \in \mathfrak{A} \subset \mathbb{R}^{l^c} \times \{0,1\}^{l^d}$. Throughout the analysis I will restrict discrete actions to be binary for a matter of simplicity and I also use one continuous variable (investment) and one discrete variable (entry/exit). For example, if a_{it}^d represents 'status' and firms choose to exit the industry they set $a_{it}^d = 0$ and receive a 'scrap' value, φ_i^{scrap}. Potential (short lived) entrants may choose to pay a privately observed entry cost (φ_i^{entry}) and enter the industry.

Assumption 2.1 implies that the only common information is the aggregate state. Moreover it says that all we can learn about the state of the industry $\mathbf{s}_t$ is contained in S_t and history $(S_{t-1}, ..., S_0)$ brings no more additional information.

State transition

Assumption 2.2 *(No Spillover) Conditional on current state and actions, own state evolves according to the transition function*

$$p(s_{it+1}|s_{it}, a_{it})$$

Per period payoff Time is discrete and firms receive per period returns which depend on the state of the industry, current actions and shocks ($\pi(a_{it}, \mathbf{s}_t, \varphi_{it})$).

Assumption 2.3 *(a) There exists a function ($S : \mathfrak{s}^N \to \mathfrak{S} \in \mathbb{R}$) that maps the vector of firm's individual states ($\mathbf{s}_t$) into an aggregate state ($S(s_{1t}, s_{2t}, ..., s_{Nt})$) and this state is observed with noise ($S_t = S(s_{1t}, s_{2t}, ..., s_{Nt}) + \varepsilon_t$, where ε_t is i.i.d. distributed $\Phi(\varepsilon_t)$ and bounded support).*

(b) Per period returns can be written as

$$\pi(a_{it}, \mathbf{s}_t, \varphi_{it}) = \pi(a_{it}, s_{it}, S_t, \varphi_{it})$$

Under this assumption, S_t is the payoff relevant variable observed by all agents. The random shock, ε_t, guarantees that there is no perfectly informative state S_t from which we can exactly recover $(s_{1t}, ..., s_{Nt})$. The intuition for this error term is the following, imagine s_{it} is marginal cost which affects pricing in the stage game so that the price is a function of the state $p(s_{it}, S_t)$. If players make pricing mistakes, the actual price they set is $p(s_{it}, S_t)+\varepsilon^i_t$, where ε^i_t is i.i.d, the aggregate state is then $S_t = \sum_{i=1}^{N} p(s_{it}, S_t) + \varepsilon^i_t = \sum_{i=1}^{N} p_{it} + \varepsilon_t$, since ε^i_t is i.i.d. we can rewrite $\varepsilon_t = \sum_{i=1}^{N} \varepsilon^i_t$. Note that the payoff relevant shocks (φ_{it}) have no impact on the stage game pricing. One type of demand which meets this assumption is:

Example 1 *With CES demand where the state is individual price (p_i) and demand elasticity is σ: $S_t = \sum_{i=1}^N p_{it} + \varepsilon_t$.*

The timing is the following:

1. States ($\mathbf{s}_t$) and shocks (φ_{it}) are formed,
2. Actions ($\mathbf{a}_t = (a_{1t}, .., a_{Nt})$ are taken simultaneously (given the observed state),
3. Firms compete in the market and receive period returns ($\pi(.)$).
4. Exitors leave the market and collect the exit fee and entrants pay the entry fees. All costs are incurred
5. Both stochastic and deterministic outcomes of actions are realized. New state is formed ($\mathbf{s}_{t+1}$).

2.2 Strategies

For each state firms can take actions in some space $a_{it} \in \mathfrak{A}$. Restricting to Symmetric Markovian Pure Strategies,[4] these strategies, σ map the set of states into the action space $\sigma : \mathfrak{s} \times \mathfrak{S} \times \mathfrak{J} \to \mathfrak{A}$ $(\sigma_{it}(s_{it}, S_t, \varphi_{it}) = (\sigma^c_{it}(s_{it}, S_t, \varphi_{it}), \sigma^d_{it}(s_{it}, S_t, \varphi_{it})))$ where the action space is

[4] Anonymity as defined in Ericson and Pakes is imposed by assuming that firms do not observe each others state.

$$\mathfrak{A}(s_{it}, S_t, \varphi_{it}) = \begin{cases} \{0,1\} \times [0, \bar{a}^c] & \text{if } s_{it} \neq s^e \\ \{0,1\} & s_{it} = s^e \end{cases}$$

Using symmetry we can drop the i subscript and imposing stationarity we can drop the t subscript: $\sigma_{it}(s_{it}, S_t, \varphi_{it}) = \sigma(s_{it}, S_t, \varphi_{it})$.

Proposition 2 *Under assumptions 2.1 to 2.3 the industry aggregate state conditional distribution takes the form $q(S'|S)$.*

Proof. See appendix. ■

So while the industry state is a vector $\mathbf{s}_t = (s_{1t}, s_{2t}, ..., s_{Nt})$, S_t is a scalar variable that maps individual firm's states into an aggregate industry state $S_t = g(s_{1t}, s_{2t}, ..., s_{Nt}) + \varepsilon_t$. Since this result allows me to focus on optimal strategies that just depend on individual and the aggregate state, I test the validity of this result in the empirical section.

When some actions and states are not observed, the firm has to condition its strategies on the expected actions and state of the competitors. When nothing is observed about the competitors, the firm will have the same expectation about the state and actions for all competitors. To understand the implications of the incomplete information assumption, recall that in the Ericson and Pakes framework with the symmetry and anonymity assumption firms keep track of the industry state distribution and not the whole industry state vector as it would be the case with no anonymity. In the incomplete information case what matters is just the first moment of this same distribution so this imposes slightly stronger conditions than the usual symmetry and anonymity. The structure implicitly imposed upon the strategic interactions is that the firm will have the same expectations for all different competitors and will react symmetrically to all of them (i.e. firm A's reaction to competitors B and C is identical either they are similar or very different, what matters is the average competitor). Notice that implicitly, knowledge about the own state is considered to have no impact on the evolution of the aggregate state conditional on knowing the current state, i.e., $q(S_{t+1}|s_{it}, S_t) = q(S_{t+1}|S_t)$.

Corollary 3 *Under assumptions 2.1 to 2.3 and when $S_t = \sum_{i=1}^{N} h(s_{it}) + \varepsilon_{it}$, as N becomes large $q(S'|S)$ is approximately normally distributed with conditional mean $\mu_{S'|S} = (1 - \rho_S)\mu_S + \rho_S S$ and standard deviation $\sigma_{S'|S} = \sigma_S(1-\rho^2)^{1/2}$. Where $\mu_S, \sigma_S^2, \rho_s$ are respectively the unconditional mean, variance and autocorrelation for the S_t process.*

Proof. By the Central Limit Theorem. ■

Corollary 4 *Three moments of the aggregate state distribution, $(\mu_S, \sigma_S, \rho_S)$ fully characterize $q(S'|S)$.*

Proof. Follows from Corollary 3. ■

Value function Given Proposition 2 and Assumption 2.2, we can write the ex-ante value function defined as the discounted sum of future payoffs before player specific shocks are observed and actions taken, as

$$\begin{aligned} & V(s_{it}, S_t; p(a_{it}|s_{it}, S_t), q) \\ = & \int_{a_{it}^c} \sum_{a_{it}^d} \left[\begin{array}{c} \int_{\varphi_{it}} \pi(a_{it}, s_{it}, S_t, \varphi_{it}) p(\varphi_{it}|a_{it}) \\ +\rho \int_{s_{it}+1, S_t} V_i(s_{it+1}, S_{t+1}) p(ds_{it+1}|s_{it}, a_{it}) q(dS_{t+1}|S_t) \end{array} \right] p(a_{it}^c|s_{it}, S_t) p(a_{it}^d|s_{it}, S_t) da_{it}^c d\varphi_{it} \end{aligned}$$

This continuation value depends on the beliefs about the transition of the aggregate state. These beliefs depend on the equilibrium strategies played by all players. Notice that since firm i does not observe $s_{jt}, \forall j \neq i$, it can only form an expectation on its rivals actions conditional on the information available S_t, $p(s_j|S) = \int_{s_j} \bar{\sigma}(s_j, S) g(s_j|S) ds_j$ where $g(s_j|S)$ is the probability density function of firm j's state conditional on S and $\bar{\sigma}(s_j, S) = \int_{\varphi_j} \sigma(s_j, S, \varphi_j) p(\varphi_j|a_j) d\varphi_j$. The assumption has a similar effect to mixed strategies or privately observed information in Doraszelsi and Saterthwaite (2005) which smooths out the continuation value and guarantees existence of equilibria.[5]

2.3 Equilibrium

The equilibrium concept is Markov Perfect Bayesian Equilibrium in the sense of Maskin and Tirole (1988, 2001). Since I restrict to Markovian pure strategies that the firm can take actions $a_{it} \in A(s_{it}, S_t, \varphi_{it})$ the problem can be represented as:

$$V(s_{it}, S_t, \varphi_{it}; q) = \sup_{a \in \mathfrak{A}(s,S,\varphi)} h(s, S, \varphi, a, V; q)$$

where

$$\begin{aligned} & h(s, S, \varphi, a, V; q) \\ = & \left\{ \pi(s_{it}, S_t, \varphi_{it}, a_{it}) + \rho E\{V(s_{it+1}, S_{t+1}, \varphi_{it+1})|s_{it}, S_t, a_{it}; q\} \right. \end{aligned}$$

Definition 5 *A collection of strategies and beliefs $(\sigma, q())$ constitute a Markov perfect equilibrium if:*

(i) Firms' strategies $(\sigma_{it} = \sigma^*(s_{it}, S_t, \varphi_{it}; q))$ conditional on beliefs about industry evolution (q) maximize the value function

$$V(s_{it}, S_t, \varphi_{it}; q) = h(s, S, \varphi, \sigma(s, S; q), V_i; q) \ \forall \sigma \in \mathfrak{t}, s \in \mathfrak{s}, S \in \mathfrak{S}, \varphi \in \mathfrak{J}$$

[5] Note that this was a problem with the original Ericson and Pakes framework which Doraszelski and Satterthwaite have shown to cause the model not to have an equilibrium in pure strategies in some cases.

where $E\left[V(s_{i,t+1}, S_{t+1}, \varphi_{it+1})|s_{it}, S_t, \varphi_{it}\right] = \int_{s\in\mathfrak{s}}\int_{S\in\mathfrak{S}} V(s_{i,t+1}, S_{t+1})\tilde{q}(ds_{it+1}, dS_{t+1}, \varphi_{it+1}|s_{it}, S_t, \varphi_{it})$ *and*

$\tilde{q}(s_{it+1}, S_{t+1}, \varphi_{it+1}|s_{it}, S_t, , \varphi_{it}) = q(S_{t+1}|S_t)p(s_{it+1}|s_{it}, \bar{\sigma}_{it}(.|q))p(\varphi_{it+1}|s_{it+1})$

(ii) all players use Markovian strategies $\sigma(s_{it}, S_t, \varphi_{it})$

(iii) The transition matrix $(q^*(S_{t+1}|S_t; \bar{\sigma}^*(s_{it}, S_t|q)))$ *resulting from using optimal strategies* (σ^*_{it}) *defined above is consistent with beliefs* $q(S_{t+1}|S_t)$

The solution to the dynamic programming problem conditional on q provides optimal strategies $\sigma^*(.|Q)$ and a solution exists, under Blackwell's regularity conditions. These strategies will then characterize the industry conditional distribution $q(S_{t+1}|S_t;\sigma^*)$ and the equilibrium is the fixed point to a mapping from the beliefs used to obtain the strategies into this industry state transition

$$\Upsilon(q)(S_{t+1}|S_t) = q^*(S_{t+1}|S_t; \sigma^*(.|q))$$

where firm's follow optimal strategies $\sigma^*(.)$. An equilibrium exists when there is a fixed point to the mapping $\Upsilon(q) : \mathfrak{Q} \to \mathfrak{Q}$

Theorem 6 *An equilibrium* q^* *exists.*

Proof. See appendix. ■

2.3.1 Uniqueness

The problem of multiple equilibrium is recurrent in this type of games and has been widely discussed in the literature. One of the main problems is the significant difficulties that arise in estimating the model when one cannot fully characterize the whole set of possible equilibria.

> *"However, discrete games with incomplete information have a very different equilibrium structure than games with complete information. For example, in a static coordination game Bajari, Hong, Krainer and Nekipelov (2006) show that the number of equilibria decreases as the number of players in the game increase. In fact, the equilibrium is typically unique when there are more than four players. In a complete information game, by comparison, the average number of Nash equilibrium will increase as players are added to the game (see McKelvey and McLennan (1996)). Thus, the assumption of incomplete information appears to refine the equilibrium set."* Bajari, Hong and Ryan (2007: 11)

Given the structure of the game developed above, I can compute the set of equilibrium. Using corollary 4 the equilibrium is defined by a triple $(\mu_S, \sigma_S\,, \rho_S)$. Given this triple I can solve the model for any starting vector $(\mu^0_S, \sigma^0_S\, \rho^0_S)$ and compute the resulting equilibrium.

Figure 2 represents the configuration for any starting value of (μ_S)[6] and corroborates the findings by Bajari et al (2006) supporting the idea of uniqueness of equilibrium for this model. Whereas in general uniqueness is difficult to prove, with this framework it can be checked by looking at all possible equilibrium configurations $(\mu_S, \sigma_S, \rho_S) \in \mathfrak{M} \times \mathfrak{D} \times \mathfrak{R}$ and $\mathfrak{M} \times \mathfrak{D} \times \mathfrak{R}$ is a compact set.

2.4 Discussion

The model above brings some of the ideas developed in Doraszelski and Satterthwaite (2005) to address the (in)existence of equilibrium in Ericson and Pakes (1995). Instead of introducing i.i.d. stochastic shocks for the discrete decisions, I introduce incomplete information in an extreme form where no firm knows their competitors' individual state. Firms attach probabilities to the outcomes which smooths the continuation values and eliminates the discreteness that caused non-existence problems in the Ericson and Pakes framework.

Reducing the industry state into the payoff relevant aggregate state by introducing incomplete information avoids the 'curse of dimensionality'. As noted before, this imposes more structure on the type of strategic interactions by making strategic reactions identical to all competitors. In a sense this condition imposes slightly stronger restrictions than the usual anonymity and symmetry assumptions which are also fundamental to reduce the dimension of the state space. Symmetry and anonymity are a restriction that allows the state space to be characterized more compactly as a set of "counting measures" (i.e. the industry state distribution).

In a different area of research, Krusell and Smith (1998) use a similar idea whereby the evolution of the aggregate variables in the economy is well approximated by some summary statistics even in the presence of substantial heterogeneity in the population.

Empirical applications avoid the calculation of the equilibrium but they require estimating $\Pr(\mathbf{s}'|\mathbf{s})$ from the data (Pakes, Ostrovski and Berry, forthcoming) or estimating the policy functions $\sigma(\mathbf{s}, \varphi)$ (Bajari, Benkard and Levin, 2007). However, if the industry state is large, since it does not solve the 'curse of dimensionality', it will require a very large amount of data to flexibly estimate either $\Pr(\mathbf{s}'|\mathbf{s})$ or $\sigma(\mathbf{s}, \varphi)$. Estimating very flexible policies can lead to serious bias in the second stage estimates which arise because the first stage parameters enter nonlinearly in the second stage. Therefore any error in the first stage can be greatly magnified into the second stage (Aguirregabira and Mira, 2007). In an empirical application to the Portland Cement Industry, Ryan (2005) used the sum of competitors capacities as the state variable rather than the individual capacities of competitors. While doing this for tractability reasons, it is in fact imposing that the players strategies are of the form $\sigma(s, S, \varphi)$ instead of $\sigma(\mathbf{s}, \varphi)$.

[6] σ, ρ are held constant only for simplicity in order to provide a visual representation.

Assumptions 2.3 and 2.1 might be seen as restrictive in some settings.[7] The first is satisfied by most reduced form profit functions whenever S is payoff relevant. The algorithm is therefore flexible enough to allow different demand structures provided the aggregate state is the payoff relevant variable.

The second assumption is more restrictive as it imposes that firms do not observe each other's states (and actions) and also that history of the aggregate state is irrelevant conditional on the current state. For example, imagine the state variable is price, this means that firms observe industry aggregate prices (e.g. published by some entity or magazine) but they do not observe firms individual prices because this would involve incurring in costly market research. In several industries firms try to keep their prices secret. In fact, when R&D is important, we would probably expect to see industrial secrecy being used as a strategic tool.

In industries where there are market leaders, Assumption 2.1 will possibly not hold. However, the model can be extended in these cases by enlarging the state space to include the state of the market leaders. Instead of one there are two problems to solve, one for the leader and one for all other firms and the state space becomes (s_{it}, S_t, s_{Lt}) where s_{Lt} is the state of the leader.

Once $q(S_{t+1}|S_t)$ is known the problem can be represented as a standard dynamic programming problem which can be estimated with available techniques for single agent models (Rust (1987), Hotz and Miller (1993), Aguirregabiria and Mira (2002)). Alternatively, we can apply the estimators developed for dynamic games.

3 Recovering the Sunk Costs

To estimate the sunk costs of R&D, I use a model where firms sell differentiated products facing a CES demand. They can invest both in physical capital and decide to engage in R&D for which they have to pay a sunk cost. This sunk cost can have several sources from building an R&D lab to the costs involved in internally changing the firm's organization or even credit constraints. Finally potential entrants can enter and incumbents can exit.

3.1 State and action space

The state space is represented by four variables: Physical capital, productivity, R&D status and operating status (enter/exit)

$$s_{it} = (K_{it}, \omega_{it}, R_{it}, \chi_{it})$$

where $K_{it} \in \mathfrak{K}, \omega_{it} \in \Omega, R_{it} \in \{0, 1\}, \chi_{it} \in \{0, 1\}$ where $\chi = 1$ means the firm is active

[7] Assumption 3 ('no spillover') is standard in the literature and it allows us to write down the transition for the individual state conditional on the firms' actions independently of the other firms' action/states.

and $R = 1$ means the firm has built the R&D lab.

After entering the industry, firms can invest in physical capital, pay a sunk cost and engage in R&D (this is done only once and R&D can be done forever) and finally decide on exit from the industry.

$$a_{it} = (a^c_{it}, a^d_{it}) = (I_{it}, R_{it+1}, \chi_{it+1})$$

where $I_{it} \in \Im$

This generates a law of motion for the state variables that depends on actions

$$s_{i,t+1} = s(s_{it}, a_{it})$$

As will be explained below, this law of motion will be stochastic for productivity and deterministic for all other state variables.

3.2 Parametrization

Per period returns are a primitive of the model which must be specified $\pi(s_{it}, S_t, a_{it}, \varphi_{it})$. I first define the demand and production functions and then, assuming Bertrand pricing, I solve for the reduced form period returns. The period return function satisfies Rust's (1994) conditional independence and additive separability assumptions

$$\pi(s_{it}, S_t, a_{it}, \varphi_{it}) = \tilde{\pi}(s_{it}, S_t, a_{it}) + \varphi_{it}(a_{it})$$

3.2.1 Demand

Using the Dixit-Stiglitz monopolistic competition framework,[8] there are N_t available goods, each supplied by a different firm so there are N_t firms in the market. Consumers choose quantities of each good Q_i to consume and pay P_i with the following preferences

$$U\left(\left(\sum_i Q_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, Z\right)$$

With $U(.)$ differentiable and quasi-concave and Z represents aggregate industry shifter. The aggregate price index is

$$\tilde{P} = \left(\sum_{i=1}^{N} P_i^{-(\sigma-1)}\right)^{-\frac{1}{\sigma-1}} \tag{1}$$

and demand becomes (see Appendix A.3):

[8] As explained before the model may work with other demand structures.

$$Q_i = \tilde{Y}\tilde{P}^{\sigma-1}P_i^{-\sigma} \tag{2}$$

Where $\left(\frac{\tilde{Y}}{\tilde{P}}\right) = \frac{\sum_{i=1}^{N_t} P_i Q_i}{\tilde{P}}$ is total industry deflated revenues. If the goods were perfect substitutes (σ is infinite), then there can be no variations in adjusted prices across firms, $P_i = \tilde{P}$ and $\frac{\tilde{Y}_i}{\tilde{P}} = Q_i$ for all firms.

3.2.2 Production function

The production technology uses both capital (K) and labor (L) with a given productivity factor (ω) according to a Cobb Douglas

$$Q_i = \omega_i L_i^{\alpha} K_i^{\beta} \tag{3}$$

It is easy to show[9] that maximizing out for labor, $\tilde{\pi} = P(Q)Q - wL$ becomes,

$$\tilde{\pi}(\omega_i, K_i, S; \sigma, \beta) = \frac{1}{\gamma}\left(\frac{\sigma-1}{\sigma}\right)\tilde{Y}\frac{\left(\omega_i K_i^{\beta}\right)^{\gamma}}{\sum_j \left[\omega_j K_j^{\beta}\right]^{\gamma}} \tag{4}$$

where $\gamma \in [0, (1-\alpha)^{-1}]$, $S_t = \tilde{Y}_t / \sum_j \left[\omega_j K_j^{\beta}\right]^{\gamma}$ is the state of the industry and (σ, β) are the elasticity of substitution and capital coefficient, respectively.

Notice that since in the short run, productivity and physical capital are fixed, the only way to adjust production is through labour which is assumed to be perfectly flexible.

Productivity The assumption os that R&D generates stochastic innovations that affect the profits. The source of the effect on profits can be via revenues (product innovation, i.e. either by developing a new product or improving the quality of an existing product R&D outcome changes the revenues and therefore the profits of the firm) or via costs (process innovation, i.e. by changing a current process of production, or improving the use of resources the R&D outcome will affect the costs of the company and therefore its prices). In general, product and process innovation are difficult to disentangle from each other unless one has firm level price data. Since in my data I do not have price data I consider them to be indistinguishable in the model and restrict the analysis to the effect on productivity, ω. The model can however be extended to allow for quality in the demand specification (see Melitz, 2000). This distinction would be important to model other type of phenomena like dynamic pricing, where the effects of product and process innovation would be qualitatively different.

This 'internal' source of uncertainty distinguishes R&D investment from other firm's decisions like capital investment, labor hiring, entry and exit which have deterministic outcomes and where the only source of uncertainty is 'external' to the company (e.g. the environment,

[9] See Appendix A.4.

competition, demand). This distinction is important since the stochastic R&D outcome will determine (together with entry and exit) the stochastic nature of the equilibrium (Markov).

3.2.3 Cost function

Investment cost Investment cost takes the traditional quadratic adjustment form (Hayashi 1982). I do not introduce non-linear adjustment costs (Cooper and Haltiwanger 1995 and 2000) because in the data there are not many observations with zero investment. I however introduce investment irreversibility by restricting it to be positive, $I_t \in \mathbb{R}_+$ so that there is no disinvestment. Bond and Van Reenen (2003) provide a good survey on investment models. The investment costs take the following form

$$C^K(I_t) = \mu_1 {I_t}^2 + (\mu_2 + \mu_3 \varphi_{it}^{inv}) I_t \tag{5}$$

R&D technology To start up R&D, firms have to pay a sunk cost of $(\varpi_0 + \varpi_1 \varphi_{it}^{RD})$ (e.g. to build the R&D lab). From there onwards the assumption is that the level of R&D is set at an exogenous so I will not deal with the continuous R&D choice. The reason why I do this is to avoid the estimation for the productivity evolution dependent on the R&D level chosen. This would require some type of parametrization and could introduce more error in the second stage estimates. Given the evidence from the empirical literature, the assumption is not restrictive since R&D intensity is typically highly autocorrelated which supports the idea that firms set the R&D at an exogenous optimal level, independent from the current state.

Notice that we have to introduce a binary state variable to track if the R&D sunk cost has been paid or not $R \in \{0, 1\}$. The productivity evolves stochastically depending on whether the R&D sunk cost has been paid or not, i.e.

$$p(\omega_{i,t+1} | \omega_{it}, R_{it})$$

Entry cost Potential entrants are short lived and cannot delay entry. Upon entry, firms must pay a (privately observed) sunk entry fee of $\nu_3 + \nu_4 \varphi_{it}^{entry}$ to get a draw of ω with distribution $p(\omega' | \chi = 0)$ next period. The capital stock level upon entry is fixed $K = \underline{K}$ and $R = 0$, i.e., firms enter the market with a capital stock of $\underline{K}$ and no R&D. Active firms take a value $\chi = 1$ and inactive firms $\chi = 0$.

Exit value Every period the firm has the option of exiting the industry and collect a scrap exit value of $\nu_1 + \nu_2 \varphi_{it}^{scrap}$.

Payoff shocks The vector of payoff shocks $\varphi = (\varphi^{inv}, \varphi^{RD}, \varphi^{entry}, \varphi^{scrap})$ are i.i.d. standard normal.

3.2.4 State transition

Productivity is stochastic and follows a Markov process.

$$\omega_{it+1} = E(\omega_{it+1}|\omega_{it}, R) + \nu_{it}$$

The capital stock depreciates at rate δ and investment add to the stock:

$$K_{i,t+1} = (1-\delta)K_{it} + I_{it}$$

If a firm decides to start R&D, the sunk cost is paid only once and does not need to be paid ever again while the firm stays in the industry:

$$R_{i,t+1} = \begin{cases} 1 & \text{if } R_{it} = 1 \text{ or } R_{i,t+1} = 1 \\ 0 & \text{otherwise} \end{cases}$$

If a firm exits it sets $\chi_{i,t+1} = 0$ and if it enters it sets $\chi_{i,t+1} = 1$

$$\chi_{i,t+1} = \begin{cases} 1 & \begin{array}{c} \text{if } \chi_{it} = 0 \text{ and firm } i \text{ enters OR,} \\ \chi_{it} = 1 \text{ and firm } i \text{ stays} \end{array} \\ 0 & \begin{array}{c} \text{if } \chi_{it} = 0 \text{ and firm } i \text{ does not enter OR,} \\ \chi_{it} = 1 \text{ and firm } i \text{ exits} \end{array} \end{cases}$$

3.2.5 Period Returns

Using the above specification I can write the per period return function

$$\pi(\omega_{it}, K_{it}, R_{it}, \chi_{it}, R_{it+1}, \chi_{it+1}, I_{it}, S_t) =$$
$$= \left\{ \begin{array}{c} \chi_{it} \left(\begin{array}{c} \frac{1}{\gamma}\left(\frac{\sigma-1}{\sigma}\right)\tilde{Y}_t \frac{\left(\omega_{it}K_{it}^{\beta}\right)^{\gamma}}{\sum_j \left[\omega_{jt}K_{jt}^{\beta}\right]^{\gamma}} - \mu_1 I_{it}^2 - \mu_2 I_{it} - \varphi_{it}^{inv} I_{it} \\ -(\varpi_0 + \varpi_1 \varphi_{it}^{RD})(R_{it+1} - R_{it})R_{it+1} + (1-\chi_{it+1})(\nu_1 + \nu_2 \varphi_{it}^{scrap}) \end{array} \right) \\ -(1-\chi_{it})\chi_{it+1}(\nu_3 + \nu_4 \varphi_{it}^{entry}) \end{array} \right\}$$

Where $\varphi_{it} = \{\varphi_{it}^{entry}, \varphi_{it}^{scrap}, \varphi_{it}^{inv}, \varphi_{it}^{RD}\}$ is a collection of i.i.d. standard normal payoff shocks. Using the demand specified above (2) there are two 'external' variables that affect company's revenues. One is market size ($\tilde{Y}$) and the other is competitors' adjusted price index ($\tilde{P}$). Since individual prices are determined by productivity and physical capital ($P_i^* = P(\omega_i, K_i, \tilde{P}, \tilde{Y})$, see appendix), the price index is a mapping from individual firms' productivity and capital stock onto a pricing function so that we get the final result for the aggregate state variable

$$S_t = \tilde{Y}_t / \sum_j \left[\omega_{jt}K_{jt}^{\beta}\right]^{\gamma} \tag{6}$$

It is important to recall that as explained before, firms adjust production to maximize short run profits through the only flexible input, labor.

3.3 Value Function

The value function for the firm is

$$V(s_{it}, S_t, \varphi_{it}; q) = \sup_{a \in \mathfrak{A}} h(s_{it}, S_s, \varphi_{it}, a_{it}, V_{it}; q)$$

where

$$\begin{aligned} & h(s_{it}, S_t, \varphi_{it}, a_{it}, V_{it}; q) \\ = \ & \tilde{\pi}(s_{it}, S_t, a_{it}) + \varphi_{it}(a_{it}) + \rho E\{V(s_{it+1}, S_{t+1})|s_{it}, S_t, a_{it}; q\} \end{aligned}$$

s_{it} and a_{it} have been defined above and the expectation $E[.|s_{it}, S_t, a_{it}; q]$ is taken over $p(\omega'|\chi = 0)q(S'|S)$ if $\chi = 0$ and $p(\omega'|\omega, R)q(S'|S)$ if $\chi = 1$. So the firms decide on next period capital investment (K'), R&D start-up (R') and next period operating status, i.e. entry and exit (χ').

Firms optimally choose their entry, exit, R&D and investment given the knowledge about the evolution of the industry $q(S'|S)$.

There are two different value functions depending on the firm being an incumbent ($\chi_{it} = 1$) or a potential entrant ($\chi_{it} = 0$). For incumbents, the value function is the sum of current returns and the expected continuation value which depends on current individual state (s_{it}), current industry state (S_t) and actions taken (a_{it}). For the potential entrant the value function is either zero if it chooses to remain outside ($\chi_{it+1} = 0$) or the sum of the entry cost with the continuation value which depends on the aggregate industry state (S_t) and the entry state distribution ($p(s_{it+1}|\chi_{it} = 0)$).

4 The Estimation

There are currently several proposed alternatives to estimate dynamic industry models in the recent surge of estimation techniques which extend the work of Hotz and Miller (1993) for single agent models (see Pesendorfer and Schmidt-Dengler, forthcoming; Aguirregabiria and Mira, 2007; Bajari, Benkard and Levin, 2007; and Pakes, Ostrovsky and Berry, 2007). I will follow the technique developed by Bajari, Benkard and Levin (2007) since this allows for both discrete and continuous choices and is easily applicable to the model outlined above. This framework has been applied by Ryan (2006) to study the impact of environmental regulation changes on capacity investment for the cement industry in the US. The industry state is the

sum of competitors' capacities rather than the individual capacities of competitors and this resembles the model I am about to estimate.

The estimation proceeds in two steps. In the first step I recover the production function parameters (α, β), the demand elasticity (σ), unobserved productivity (ω_{it}) as well as the state transition $(\Pr(\omega'|\omega, R)$ and $q(S'|S))$ and equilibrium policy function $(\sigma(.))$. In the second step, I impose the equilibrium conditions to estimate the linear and quadratic investment costs parameters, R&D sunk costs and exit costs $(\mu_1, \mu_2, \varpi_0, \nu_1)$.

By simulating actions and states from a starting configuration and collecting these paths through time, I can calculate the present-value for a given path and a given set of parameters. Slightly perturbing the policy functions allows me to generate alternative paths and different present-values for a given parameter vector. The observed policy functions were generated by profit-maximizing firms who chose the actions with the highest expected discounted value. This means that at the true parameters, the discounted value generated by the observed actions should be greater than those generated by any other set of actions.

My main interest is recovering the R&D sunk costs, ϖ_0. Getting a good estimate of sunk costs of R&D is important because these will determine R&D performance and consequently innovation and productivity which are topics of extreme importance for policy makers. Second, these will have an effect on market structure and competition as explained by Sutton (1998).

For most industries, the R&D/Sales ratio is not very high (2%-5%). This is puzzling if we recall that only a fraction of the firms actually perform R&D. The reason must then be that either the returns to R&D are too low or that there are very high sunk costs involved that prevent firms from engaging in R&D (credit constraints could also be a cause and they could be modeled in a similar fashion). With all dynamic cost parameters recovered, I can then do some policy analysis to study changes in the amount of R&D and industry structure when one changes the sunk costs of R&D.

One hotly debated (and unsolved) issue is the link between competition and R&D performance. Aghion et al. (1999) provide a theoretical explanation and some empirical evidence arguing that there is an inverted U-shape relationship between these two, whereby innovation is higher for mid levels of competition but lower for either very competitive or little competitive industries. However, since both market structure and R&D performance are jointly determined in equilibrium, it is not easy to disentangle these effects without a dynamic model that addresses the market structure endogeneity issue.

4.1 First step

In the first step I recover the static parameters (production function, demand, policies and transitions). This then allows me to compute the per period returns, simulate actions for a given state using the estimated policies and update the states using the transitions which

will be the hearth of the second step.

4.1.1 Productivity

Productivity is not directly observed but there are methods[10] to estimate it as the residual from a production function estimation (Olley and Pakes, 1995; Levinshon and Petrin, 2003; De Loecker, 2007). To be coherent with the theoretical model I use a methodology similar to De Loecker (2007) which allows me to recover both the production function parameters and the demand elasticity when one uses deflated sales instead of quantities. The main problem with De Loecker (2007) is that it is inconsistent with the model I have outlined. The reason for the inconsistency arises from the fact that input demand function depend on the industry state, more precisely on the aggregate industry state. This means that the elasticity of demand cannot be recovered in the first step since the input demand is also a function of the aggregate sales and I can only recover it only in the second step together with the capital coefficient. To see this notice that sales are $P.Q$ so taking the logs and using (2) and (3) from above:

$$y_{it} = p_{it} + q_{it} = \frac{1}{\sigma}\tilde{y}_t + \frac{\sigma-1}{\sigma}\tilde{p}_t + \frac{\sigma-1}{\sigma}\left(\omega_{it} + \alpha_k k_{it} + \alpha_l l_{it}\right)$$

or

$$y_{it} - \tilde{p}_t = \frac{1}{\sigma}(\tilde{y}_t - \tilde{p}_t) + \frac{\sigma-1}{\sigma}\left(\omega_{it} + \alpha_k k_{it} + \alpha_l l_{it}\right)$$

The first problem is then that the unobserved productivity term ω_{it} is possibly correlated with the inputs. Levinshon and Petrin (2003) propose the use of materials to control for the unobservable. To see this recall that input demand is a function of individual states and the aggregate state

$$m_{it} = m(\omega_{it}, k_{it}, \tilde{y}_t)$$

Assuming that m_{it} is monotonically increasing in ω_{it} this can be inverted

$$\omega_{it} = \omega(k_{it}, \tilde{y}_t, m_{it})$$

and the unobservable is now a function of observables. Imposing that productivity is governed by a first order Markov process we get

$$\omega_{it} = E[\omega_{it}|\omega_{it-1}] + \eta_{it}$$

[10] Ackerberg et al. (forthcoming) provide a survey on the literature for estimating production functions.

First stage From above we can rewrite the production function as (using deflated sales as variables $y_{it}^p = y_{it} - \tilde{p}_t$)

$$\begin{aligned} y_{it}^p &= \frac{1}{\sigma}\tilde{y}_t^p + \frac{\sigma-1}{\sigma}\left(\alpha_k k_{it} + \alpha_l l_{it}\right) + \frac{\sigma-1}{\sigma}\omega_{it} + \varepsilon_{it} \\ &= \frac{\sigma-1}{\sigma}\alpha_l l_{it} + \phi(k_{it}, \tilde{y}_t^p, m_{it}) + \varepsilon_{it} \end{aligned}$$

where

$$\phi(k_{it}, \tilde{y}_t^p, m_{it}) = \frac{1}{\sigma}\tilde{y}_t^p + \frac{\sigma-1}{\sigma}\alpha_k k_{it} + \frac{\sigma-1}{\sigma}\omega(k_{it}, \tilde{y}^p, m_{it})$$

And we can estimate this non-parametrically using an nth-order polynomial. This provides estimates of $\widehat{\frac{\sigma-1}{\sigma}\alpha_l}$ and $\hat{\phi}$.

Second stage For the second stage I use the estimated values to construct

$$\hat{\phi}_{it} = \hat{y}_{it} - \widehat{\frac{\sigma-1}{\sigma}\alpha}_l l_{it}$$

with this we can give an estimate of $\frac{\sigma-1}{\sigma}\omega_{it}$ for a given $\widetilde{\frac{\sigma-1}{\sigma}\alpha_k}$ and $\widetilde{\frac{1}{\sigma}}$

$$\widehat{\frac{\sigma-1}{\sigma}}\omega_{it} = \hat{\phi}_{it} - \widetilde{\frac{1}{\sigma}}\tilde{y}_t + \widetilde{\frac{\sigma-1}{\sigma}}\alpha_k k_{it}$$

Using this we can approximate non-parametrically $E[\omega_{it}|\omega_{it-1}]$ with an nth-order polynomial

$$\begin{aligned} y_{it} - \widehat{\frac{\sigma-1}{\sigma}}\alpha_l l_{it} &= \frac{1}{\sigma}\tilde{y}_t + \frac{\sigma-1}{\sigma}\alpha_k k_{it} + E[\omega_{it}|\omega_{it-1}] + \eta_{it} + \varepsilon_{it} \\ &= \frac{1}{\sigma}\tilde{y}_t + \frac{\sigma-1}{\sigma}\alpha_k k_{it} + \\ &\quad + \begin{bmatrix} \gamma_0 + \gamma_1\left(\hat{\phi}_{it-1} - \frac{1}{\sigma}\tilde{y}_{t-1} - \frac{\sigma-1}{\sigma}\alpha_k k_{it-1}\right) \\ +... + \gamma_n\left(\hat{\phi}_{it-1} - \frac{1}{\sigma}\tilde{y}_{t-1} - \frac{\sigma-1}{\sigma}\alpha_k k_{it-1}\right)^n \end{bmatrix} \\ &\quad + \eta_{it} + \varepsilon_{it} \end{aligned} \tag{7}$$

Using non-linear least squares allows us to finally recover an estimate for $\frac{1}{\sigma}$ and $\frac{\sigma-1}{\sigma}\alpha_k$.

Potential problems of the second stage For the second stage estimation to work, the error term of equation (7), $\eta_{it} + \varepsilon_{it}$, must be uncorrelated with k_{it} and $\tilde{y}_t$. While this might be a reasonable assumption for k_{it} due to the timing of investment that makes k_{it} independent from 'news' in period t, the same is not necessarily true for $\tilde{y}_t$ if in the productivity shock η_{it} there is an aggregate time component η_t. One potential instrument

is the use of lagged $\tilde{y}_{t-1}$.

4.2 Policies and transitions

4.2.1 Policies

With all state variables observed $(\omega, K, \tilde{Y}, R)$, the policy functions can be easily estimated. To recover the equilibrium policies I use a polynomial expansion on the state variables $\sigma(s, S)$. For the investment function I use OLS while for entry and R&D I use a probit specification. I have tried different degrees for the polynomials and there is a clear preference over polynomials with smaller degrees because they produce policy functions with less noise. Since errors in the policy functions enter nonlinearly in the second step, this can significantly bias the estimates in small samples.

4.2.2 The transition function

Aggregate state From Corollary 4 the observed aggregate state should have a conditional normal distribution with mean $\mu_{S'|S} = (1-\rho_S)\mu_S + \rho_S S$ and variance $\sigma_{S'|S} = \sigma_S(1-\rho_S^2)^{1/2}$. Where $(\mu_S, \sigma_S, \rho_S)$ are respectively the unconditional mean, variance and autocorrelation for S and are easily estimable from the data.

Productivity Since the model does not impose any parametric restrictions on the transition for individual productivity, I estimate it separately for R&D and non-R&D firms using a polynomial on lagged productivity $(g^{RD}(\omega_{i,t-1}), g^{NRD}(\omega_{i,t-1}))$. The main assumption is that

$$\omega_{i,t+1} = E(\omega_{i,t+1}|\omega_{it}, R_{it}) + \varepsilon^R_{it+1} = \alpha^R_0 + \alpha^R_1\omega_{it} + \alpha^R_3\omega^2_{it} + \alpha^R_3\omega^3_{it} + \varepsilon^r_{it+1}$$

which is estimated separately for R&D firms and non-R&D firms

4.3 Second Step (minimum distance estimator)

Assuming the policy and transition functions are consistently estimated, starting from a state configuration (s_0, S_0), I can draw vectors of payoff shocks $\varphi = (\varphi^{inv}, \varphi^{RD}, \varphi^{entry}, \varphi^{scrap})$, simulate actions (a_0) by reading off the policy functions and update states (s_1, S_1) by reading off the transition functions. Doing this for long enough periods (each path has been simulated for $\bar{T}$ periods), I compute a sequence of actions and states $\{a_t(s_0, S_0, \varphi_0), s_t(s_0, S_0), S_t(s_0, S_0)\}_{t=1}^{\bar{T}}$ from a starting configuration (I have used n_s different starting configuration combinations for (s_0, S_0)). With this sequence of actions and states, I can compute the discounted stream of profits for a given parameter vector θ and a given first step estimate $\tilde{\alpha}$, $\sum_{t=0}^{\bar{T}} \rho^t \pi(a_t, s_t, S_t, \varphi_t; \tilde{\alpha}, \theta)$ which gives me an estimate of the expected value from a

starting configuration $EV(s_0, S_0; \tilde{\alpha}, \theta) = \sum_{t=0}^{\bar{T}} \rho^t \pi(a_t, s_t, S_t, \varphi_t; \tilde{\alpha}, \theta)$.[11] For each starting configuration I simulate n_J different path to get an average estimate $EV(s_0, S_0; \tilde{\alpha}, \theta) = \frac{1}{n_J} \sum_{j=1}^{n_J} \sum_{t=0}^{\bar{T}} \rho^t \pi(a_t^j, s_t^j, S_t^j, \varphi_t^j; \tilde{\alpha}, \theta)$.

In order for a strategy, σ, to be an equilibrium then it must be that for all $\sigma' \neq \sigma$

$$V(s, S; \sigma, Q(S'|S); \theta) \geq V(s, S; \sigma', Q(S'|S); \theta)$$

So the set of dynamic parameters θ, must rationalize the strategy profile σ. I just consider the case where θ is point identified whereas Bajari et al. (2007) also develop the method for (bounds) set identification on θ.

Given the linearity of the value function on the dynamic parameters I can write

$$V(s, S; \sigma, Q(S'|S); \theta) = W(s, S; \sigma, Q(S'|S)) * \theta$$

where $W(s_t, S_t; \sigma, Q(S'|S)) = E_{\sigma|s_t, S_t} \sum_{s=t}^{\infty} \rho^s w_t$ and $\theta = [1, \mu_1, \mu_2, \varpi_0, \nu_1]$, $w_t = \big[\pi(s_s, S_s; \sigma, Q), I_s, I_s^2, 1(R_{s+1}$

.

I construct alternative investment, R&D and exit policies (σ') by drawing a mean-zero normal error and adding it to the estimated first stage policies. With these non-optimal policies I construct alternative expected value following the same procedure as before to get $W(s_0, S_0; \sigma', Q(.))$ (I calculate these values for n_σ alternative policies).

I then compute the differences between the optimal and non-optimal value functions for several (X_k) policies and states ($X_k, k = 1, ...n_I$), where $n_I = n_\sigma * n_s$

$$\hat{g}(x; \theta, \tilde{\alpha}) = \left[\hat{W}(s, S; \hat{\sigma}, \hat{Q}(S'|S)) - \hat{W}(s, S; \hat{\sigma}', \hat{Q}(S'|S))\right] * \theta$$

Since the estimated policies should be optimal, then the expected value when using σ should be bigger than using alternative σ'. The empirical minimum difference estimator then minimizes the square of the violations ($g(x, \theta, \tilde{\alpha}) < 0$)

$$\hat{J}(\theta; \tilde{\alpha}) = \frac{1}{n_I} \sum_{k=1}^{n_I} \left(\min\{\hat{g}(X_k; \theta, \tilde{\alpha}), 0\}\right)^2$$

and

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \frac{1}{n_I} \sum_{k=1}^{n_I} \left(\min\{g(X_k; \theta, \tilde{\alpha}), 0\}\right)^2$$

Notice that I set the time of each path $\bar{T} = 100$, the number of starting configurations $n_s = 100$, the number of simulations for each configuration $n_J = 150$ and the number of alternative policies $n_\sigma = 200$, so that I get the number of differences $n_I = 20,000$

[11] I set the discount factor at ρ . which is in line with other studies.

4.3.1 Standard errors

Since the estimated parameters in the first step are used in the second step, the standard errors of the parameters are determined by the first stage standard errors. The easiest way to estimate them is to use subsampling or the bootstrap.

4.3.2 Optimization

When the objective functions lacks smoothness (e.g. problems with discontinuous, non-differentiable, or stochastic objective functions) using derivative based methods might produce inaccurate solutions. For this reason, to minimize the empirical minimum distance ($\hat{J}$) I use a derivative free optimization method (Nelder-Mead) which circumvents this problem. Non-smoothness might occur with finite n_I, because of the min operator in the objective function, $\hat{J}$, which takes only the negative values of $g(.)$ and this creates discontinuities even if $g()$ is continuous in θ.

4.4 Identification

Identification of the static parameters follows the identification strategy used in De Loecker (2007) with the main difference that the demand elasticity cannot be recovered in the first stage since it enters the input demand function (in order to be consistent with the model above). Therefore, as explained above, both the capital coefficient and demand elasticity are recovered in the second stage.

The sunk costs of R&D are identified from the observed behavior of the firms. Under the assumption that observed actions are profit maximizing, the sunk costs of R&D are identified through the comparison between observed (optimal) behavior and alternative (non-optimal) behavior. The sunk costs are such that other policies are sub-optimal. Similarly, investment costs and exit values are estimated from the observation of optimal behavior and comparing with non-optimal behavior. So the identification of the dynamic parameters is achieved by comparing actual with alternative actions. Note that if policies are estimated with error, the parameters might be wrongly estimated. Because of this I have chosen polynomials of lower degree (1st and 2nd) to approximate the policy functions.

A second potential problem is that identification of the policy functions only works provided there are no unobservable state variables. This is actually a potential concern and a reason why one might consider the use of a fixed effects specification in the first step.

5 The moulds industry

The Portuguese moulds industry is a case study of success and ability to compete in a global environment. The industry exports 90% of its production and supplies 60% of its

production to the very competitive car manufacturing industry accounting for more than

clients due to the boom of the plastics and packaging sectors. This put pressure for the introduction of new technologies (e.g. CAD, CAM, Complex process, In-mold Assembling) and an increasingly importance of innovation and R&D. For example, Exhibit 3 shows a computer operated machine for building moulds which is completely different from the techniques used in the 1970's and 1980's. These state of the art Industrial Machinery allows flexibility at a low cost besides a close collaboration with the client in the phase pre mould construction. The design team can work closely with the clients' engineers and produce a 3D virtual version of the mould which is then just programed into the machine to start production.

Figure 5 presents firm size distribution (number of workers) and it is clear that the sector is mainly populated by small and medium firms. In 2004, Portugal was the 9th biggest world exporter and 3rd European exporter (Figure 6). The industry invests in R&D and has established strong links with Universities. Most of the R&D is targeted at the development and use of new materials (product innovation), and minimization of waste and worked hours (process innovation).

The Wikipedia website provides a quote about a Portuguese moulds manufacturer (SIMOLDES) that illustrates the importance of the industry:

> *Simoldes is a Portuguese mould maker company headquartered in Oliveira de Azeméis.*
>
> *Considered to be Europe's largest mould maker, Simoldes Group Mould Division is the world leader in plastic injection moulds for the automotive industry.* *(*http://en.wikipedia.org/wiki/Simoldes)

However, a puzzling fact about the Portuguese moulds (and most industries in general) is that 56% of the firms in my sample in 2003 claim to do no R&D. With increasing competition from low wage countries, the low end specialization does not seem an optimal strategy so why do we still observe firms performing no R&D? The potential reason I will explore is the existence of R&D sunk costs and if so, is this the reason why most R&D is done by bigger firms. To answer these questions I estimate a fully structural dynamic model where firms decide on physical capital investment and R&D.

Since each mould is (quasi) unique, prices depend on the mould specification and are typically contract specific and agreed between the producer and the client. Therefore, individual prices are not observed and even if they were observable they would not be difficult to comparable due to the nature of the product. Most firms establish strong cooperative relations with their clients in order to improve the quality of the product. Firms tend to specialize in a particular type of mould and therefore potential clients approach firms with the expertise in their product. For this reason the industry fits very well within the monopolistic competition framework. This is appropriate since firms sell a differentiated product

and along this product they have some degree of market power. Also the assumption that firms react to aggregate movements in the industry and not to any particular competitor is not stringent since the market is quite fragmented. The incomplete information is not violated since firms do not directly observe their competitors prices or productivity. Because of all these facts, the industry fits very well in the framework developed above.

I have observations for both big and small firms but I do not observe all firms in the market since the data is collected by sampling. These type of datasets are common and as explained before the complete information model would have problems because of the non-observed firms in the market. However, for the incomplete information case, I just need to observe aggregate variables which are available from the National Statistics Office (INE). Another important advantage of the Portuguese Moulds Industry is the fact that I observe R&D behavior and this is what will allow me to recover the R&D sunk costs.

6 The data

The data is part of a database compiled yearly by the Portuguese Central Bank ("Central de Balanços"). I have extracted the observations for the period between 1994-2003 for the five-digit NACE (rev 1.1) industry, 29563. This database collects, financial information (balance sheet and P&L) together with other variables like number of workers, occupation of workers (5 levels), total exports, R&D, founding year and current operational status (e.g. operating, bankrupt, etc). I have also collected industry aggregate variables for sales, number of firms, employment and value added from the Portuguese National Statistics Office (INE, 2007) and industry price data from IAPMEI (2006).

6.1 Descriptive statistics

The dataset has 231 firms over the period 1994-2003 and 1,290 observations. There are 265 observations with positive R&D that corresponds to 59 firms. I observe 49 cases of R&D start-ups after 1994 (defined as a firm not reporting R&D ever before in the sample). On average, an R&D firm reports positive R&D for 2.5 year periods (tables 2 and 3).

Due to the short nature of the panel, there are few observations on entry and exit. A further complication arises due to the way data has been collected. Since answering the questionnaire is not compulsory, some firms might not be reported in the dataset but still be active in the industry. This complicates the identification of exitors and entrants since the firms might enter the dataset but could have been operating in the market before first appearing in the dataset. I address these problem with two variables that help to identify entry and exit. For entry, firms report their founding year so I match the founding year with the year the firm first appeared in the sample and if it is within a 2 year window I consider it to be a new entrant (this is reported in Table 2 under the column entry in the

industry). Regarding exit, the central bank collects a variable that reports the "status" of the firm. The problem with this variable is that some firms that might have closed down are still reported as "active", so I can only capture a fraction of the total exits. Using this methodology I identify a total of 48 entries and 7 exits from the panel.

In tables 4 and 5 I present some summary statistics for the main variables. The average firm in my sample sells 1.5 million Euros and employs 32 workers with an average labor productivity of 20,452 euros. Over the period 1994-2003, real sales have grown at an average 9.9% and labor productivity at 6%.

After a decline until 1998, the total number of firms in the industry has grown up to a maximum of 738 in 2003, employing 8,766 employees so, the industry is populated by small and medium firms and there are no market leaders.

R&D performers are bigger and older and their labour productivity is on average 20% higher. Average sales growth is higher for non-R&D firms and they also invest more but the also the variance is larger.

7 Results

7.1 First stage

7.1.1 Production Function

Table 6 reports the results for the production function estimates using the methodology defined above. Because of problems that could arise in the first stage, and bias the estimates of α_L due to potential unobserved state variables, I have also tried using a fixed effects specification with no overall impact on the results.

The estimated labor and capital coefficients imply decreasing returns to scale while the estimated demand elasticity implies a price-cost margin of 13%. These values are at a reasonable level and within the range of parameters found in the literature for other industries. To test the method I also report the results using a fixed effects and a first differences specification. All these are prone to the input endogeneity problem but Table 6 shows that the first difference results are very similar to the two step procedure. This evidence seem to corroborate some of the findings by Bond and Soderbom (2005) according to which, in the presence of adjustment costs for the inputs and autocorrelation in productivity consistent estimation of production functions parameters becomes possible by quasi-first differencing and using lagged levels of inputs as instruments.

In order for the firms to be willing to pay a sunk cost for R&D, it must be that its because they expect to have a bigger productivity. To check if the productivity distribution for R&D firms stochastically dominates the distribution of productivity for the non-R&D firms I plot in figure 7 the two distributions. As we can see, there is evidence that R&D

firms have better productivity draws and so the productivity estimates are according to the expected.

7.1.2 Transition function

Aggregate state For the aggregate state, I calculate the mean, variance and autocorrelation and use corollary 4 to specify the aggregate state transition and these are:

$$\mu_S = 16.43$$
$$\sigma_S = 0.39$$
$$\rho_S = 0.97$$

In table 7 I test the normality results against a non-parametric approximation using a polynomial expansion with very similar results.

Specification test Since the model might be misspecified and assumptions violated I test the implications by testing if the aggregate follows a first order Markov process. The problem arises that if the aggregate state is not a first order Markov process, then Markovian strategies would not be optimal. If assumption 2.1 is violated, then the use of the lagged values of the aggregate variable would be insufficient and potentially all history could matter. The data does not reject the null hypothesis that the aggregate state is a first order Markovian process and this is good news since it supports the implications of my assumptions.

This is an important specification test of the model since the idea that the industry state can be summarized by the aggregate state is a crucial result to resolve the 'curse of dimensionality' problem.

I also directly perform a test of the following implication:

$$p(S_{t+1}|g(\mathbf{s}_t), S_t) = p(S_{t+1}|S_t)$$

I use both standard deviation and skewness as further measures of changes in industry distribution. Beside sales distribution I also add capital stock and productivity distribution. The results are summarized in Table 7b. Due to the lack of sufficient observations, I test one measure in each column. The overall result is that none of the measures are statistically significant once we control current aggregate state. This strongly suggests that the model is well specified and that the current aggregate industry state is a sufficient statistic.

Productivity For the individual productivity, I estimate a third order polynomial for ω separately for R&D and non R&D firms and results are shown in table 8.

7.1.3 Investment, R&D and Exit policies

The final part of the first step involves the estimation of the investment and R&D policy functions. These will be at the heart of the second stage where it is imposed that they represent optimal behavior. I have used different polynomial approximations (1st, 2nd and 3rd) and opted for a 2nd order polynomial. The reason for doing so is because higher order polynomials can create more noise in the estimates and this is magnified in the second step as these variables enter non-linearly in the maximum distance estimator (Aguirregabiria and Mira, 207). The results are presented in table 9. The R&D policy function was estimated using a probit model whereas the investment policy function was estimated using OLS.

For the exit policies due to data limitations, I have adopted a probit model only on productivity and aggregate sales.

7.2 Second Stage

In the second stage I use the minimum distance estimator outlined above to recover the linear and quadratic investment cost, R&D sunk cost and exit value, reported in Table 10. Standard errors were estimated using the bootstrap. I have introduced per period R&D expenditures for firms who decide to do R&D at 1% of their sales level. This is a fixed cost component for any firm who choose to do R&D and has to be paid every period to keep the "R&D lab" operating.

The values are estimated at the right expected signs. Specially, investment has positive quadratic adjustment costs. The exit value are positive and estimated at around 534,000 euros which is slightly higher then the average capital stock of exitors (420,684 EUR). Finally for the parameter we are interested in, the R&D sunk costs are estimated at 2.6 million euros which is 1.7 times the average sales in the industry and 87% the average sales of an R&D firm.

The magnitude of the sunk costs is high so I want to test the robustness of these results. In order for the estimates to be biased upwards we would need either the continuation values for no-R&D to be too low or the continuation values for R&D to be too high. Since the functions which are estimated differently for R&D and non-R&D firms are the investment and R&D policies and the productivity dynamics, the only way the estimated would be biased is when these policies are imprecisely estimated. Since the estimated sunk costs are very similar using the first or the second order polynomial approximation, so there is no evidence that results have been biased by policy function estimation error.

Finally, I present the estimated value function for the 1st order polynomial case in figure 8 for the average $\bar{S}$. We can see that this is quite well behaved and increasing both in K and ω as expected.

8 Counterfactual Experiments

In this section I perform a policy experiment where the sunk cost of R&D is decreased in 25% to access the impact of this shock on industry R&D, productivity and investment. To perform these experiments I now need to solve for the equilibrium of the model. Particularly I have to solve for equilibrium industry evolution, $q(S'|S)$. This requires defining entry costs and specify the productivity distribution for entrants. I specify the actual mean and variance for the productivity of entrants in my dataset and use an entry value which is consistent with the number of firms existent in the industry. After setting these I use the algorithm in figure 1 to calculate the equilibrium for the model using the estimated structural parameters. Notice that these experiments could not be performed without the incomplete information assumption that allows the use of the aggregate state instead of the full industry state. To solve the model in the complete information case with 200 firms in the market would be computationally impossible, but it is feasible and relativelly fast in the "aggregate state" case.

Results are presented in table 11. The first thing to notice is the decrease of the number of firms in the market. This happens because with lower sunk costs of R&D, each firm is now on average bigger and so the entry condition is met with less firms in the market. Secondly there is an increase in the percentage of firms performing R&D, which doubles. This increase in the number of R&D firms translates into an increase in average productivity of 11% and in average capital stock of 18%.

9 Conclusion

In this paper I have estimated the sunk costs of R&D for the Portuguese Moulds Industry and developed a model which is computationally tractable and empirically implementable with the typical firm level datasets. The model both avoids the 'curse of dimensionality' and the existence of unobserved firms in the data.

The idea I explored was to summarize the industry state into the payoff relevant aggregate state by introducing incomplete information in the model. As explained, this implicitly imposes more structure in terms of strategic interactions, specifically the firms react symmetrically to all its competitors. This is not restrictive for the moulds manufacturing industry because each firm specializes in a particular product, they do not observe what their competitors offer, firms produce almost per piece and prices are contract specific. This means that demand can be well approximated with a constant elasticity framework.

Finally I apply this setup to recover the sunk cost of R&D for the Portuguese moulds industry. I have estimated these to be around 2.6 million euros (or 1.7 times the average yearly firm sales level). The magnitude of the sunk costs suggest that choice of subsidies cannot disregard the discreteness of the R&D decision. Particularly, R&D subsidies targeted

at R&D start-ups will be more effective.

I have not explored two ways of making use of the simplification introduced by the model. First, since given the beliefs about the aggregate state evolution, the problem can be represented as a single agent one, I can apply the Nested Fixed Point Algorithm as developed in Rust (1987). Second, the existence of serially correlated unobserved variables might bias the first stage estimates. This bias can be magnified to the second stage because of the nonlinear relationship between the first stage and the second stage parameters. Aguirregabiria and Mira (2007) propose a method to deal with this which makes bigger use of the equilibrium conditions. I have not explored the fact that since my model avoids the curse of dimensionality, I can recalculate the equilibrium for a given parameter set and use the equilibrium conditions in a similar way. I think a future line of research is to make use of these alternatives to increase the efficiency of the estimator.

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A Appendix

A.1 Proof of proposition 2

Proof. Using Assumptions 2.3 to 2.2, S_t is the payoff relevant variable and $p(\mathbf{s}_t|S_{t,}...S_0) = p(\mathbf{s}_t|S_t)$ the aggregate (industry) state transition is

$$\begin{aligned}
p(S_{t+1}|S_t, S_{t-1}, ...) &= \int_{\mathbf{s}_{t+1}:S_{t+1}=S(\mathbf{s}_{t+1})+\varepsilon_{t+1}} p(\mathbf{s}_{t+1}|S_t, ..., S_0)d\mathbf{s}_{t+1} \\
&= \int_{\mathbf{s}_{t+1}:S_{t+1}=S(\mathbf{s}_{t+1})+\varepsilon_t} \int_{\mathbf{s}_t} p(\mathbf{s}_{t+1}|\mathbf{s}_t)p(d\mathbf{s}_t|S_t, ...S_0)d\mathbf{s}_{t+1} \\
&= \int_{\mathbf{s}_{t+1}:S_{t+1}=S(\mathbf{s}_{t+1})+\varepsilon_t} \int_{\mathbf{s}_t} p(\mathbf{s}_{t+1}|\mathbf{s}_t)p(d\mathbf{s}_t|S_t)d\mathbf{s}_{t+1} \\
&= \int_{\mathbf{s}_{t+1}:S_{t+1}=S(\mathbf{s}_{t+1})+\varepsilon_{t+1}} p(\mathbf{s}_{t+1}|S_t)d\mathbf{s}_{t+1} \\
&= q(S_{t+1}|S_t)
\end{aligned}$$

■

A.2 Sketch proof of theorem 6

Some preliminary lemmas:

Lemma 7 *$s_{it}|S_t$ is independently and identically distributed across firms with density function $g(s_{it}|S_t; q)$.*

Proof. By the independence assumption (no spillovers). ■

Lemma 8 *The distribution $g(s_{it}|S_t)$ is continuous with positive densities and bounded support.*

Proof. $S_t = S(s_{1t}, ..., s_{Nt}) + \varepsilon_t$ with ε_t distributed i.i.d. $\Phi(\varepsilon_t)$ and bounded support. Then S_t is never perfectly informative and therefore $g(s_{it}|S_t) > 0\ \forall s_{it}, S_t$. ■

Rewriting the state transition

$$\begin{aligned} q(S_{t+1}|S_t) &= \int_{\mathbf{s}_{t+1}:S_{t+1}=S(\mathbf{s}_{t+1})+\varepsilon_{t+1}} p(d\mathbf{s}_{t+1}|S_t;q)d\Phi(\varepsilon_{t+1}) \quad (8)\\ &= \int_{(s'_1,..s'_N):S'=S(s'_1,..s'_N)+\varepsilon'} p(ds'_1|S;q)...p(ds'_N|S;q)d\Phi(\varepsilon') \end{aligned}$$

$$\begin{aligned} p(s_{it+1}|S_t;q) &= \int_{s_{it}} p(s_{i,t+1}|s_{it}, a(s_{it},S_t), \chi(s_{it},S_t))g(ds_{it}|S_t;q) \quad (9)\\ &= \int_{s_{it}} \sum_{\iota_i\in\{0,1\}} p(s_{i,t+1}|s_{it}, a_{it}, \iota_i)\xi(S)^{\iota_i}(1-\xi(S)^{(1-\iota_i)})g(ds_{it}|S_t;q) \end{aligned}$$

Lemma 9 *$a(s_{it}, S_t)$ is continuous in q.*

Proof. Standard dynamic programming argument. ■

Lemma 10 *$\xi(S_t)$ is continuous in q.*

Proof. Notice that $\xi(S) = \int_s \chi(s,S)g(ds|S)$.

$$\chi(s,S;q) = \begin{cases} 1 & \text{if } c \le \bar{c}(s,S;q) \\ 0 & \text{otherwise} \end{cases}$$

Where we can define

$$\begin{aligned} \bar{c}(s,S;q)/\rho &= \{E[V(s',S')|a,\chi=1] - E[V(s',S')|a,\chi=0]|s,S\} \\ &= \begin{bmatrix} \int_S\int_s V(s',S')p(s'|s,a,\chi=1)q(S'|S)dsdS \\ -\int_S\int_s V(s',S')p(s'|s,a(s,S),\chi=0)q(S'|S)dsdS \end{bmatrix} \\ &= \int_S \begin{bmatrix} \int_s V(s',S')p(s'|s,a(s,S),\chi=1)ds \\ -\int_s V(s',S')p(s'|s,a(s,S),\chi=0)ds \end{bmatrix} q(S'|S)dS \end{aligned}$$

and since $\bar{c}(s,S;q)$ is continuous in q (because $V(s,S)$ is continuous in q and $a(s,S)$ is also continuous in q), also will $\xi(S)$. ■

Conjecture 11 *$g(s_{it}|S_t)$ is continuous in q.*

Since $\bar{c}$ is continuous in q as shown above, this means that for a tiny change in q (close to zero), there is only a small fraction of firms affected by this as $\bar{c}$ also changes only slightly due to continuity (just remind that $a(s_{it}, S_t)$ is also continuous in q). This means that the

steady state distribution of states will not have any discrete jump and should therefore be continuous in q.

Proof of Theorem 6. From lemmas 7-10 and conjecture 11, $q(S'|S) \in \mathfrak{Q}$ is a continuous self map on a non-empty compact and convex set $\mathfrak{Q} \in \mathcal{BC}[\underline{S}, \bar{S}]$ to which Schauder's Fixed Point Theorem can be applied. This proves the result. ■

A.3 Demand

Assuming individuals have the following demand

$$U\left(\left(\sum_i Q_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, Z\right)$$

With $U(.)$ differentiable and quasi-concave and Z represents aggregate industry shifters. Setting up the Lagrangian for $i = 1, ..., N$ $\left(\left(\sum_i Q_i^{\frac{\sigma-1}{\sigma}}\right) = \tilde{Q}\right)$

$$U\left(\left(\sum_i Q_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, Z\right) - \lambda\left(\sum P_i Q_i - \tilde{Y}\right)$$

Take the First Order Conditions

$$U_1^{\frac{1}{\sigma-1}}\left(\sum_i Q_i^{\frac{\sigma-1}{\sigma}}\right) Q_i^{\frac{\sigma-1}{\sigma}} Q_i^{-1} = P_i \lambda \tag{10}$$

Rearranging

$$\left(\lambda^{-1} U_1^{\frac{1}{\sigma-1}}\left(\sum_i Q_i^{\frac{\sigma-1}{\sigma}}\right)\right)^{\sigma} P_i^{-\sigma} = Q_i$$

$\left(\lambda^{-1} U_1 \tilde{Q}^{\frac{1}{\sigma-1}}\right) Q_i^{-1/\sigma} = P_i$

Using the budget constraint $\tilde{Y} = \sum P_i Q_i$ and (10) from above

$$\tilde{Y} = \lambda^{-1} U_1^{\frac{1}{\sigma-1}}\left(\sum_i Q_i^{\frac{\sigma-1}{\sigma}}\right) \sum Q_i^{\frac{\sigma-1}{\sigma}} \tag{11}$$

Using (11) from above and replacing for Q_i

$$\tilde{Y} = \left(\lambda^{-1} U_1^{\frac{1}{\sigma-1}}\left(\sum_i (Q_i)^{\frac{\sigma-1}{\sigma}}\right)\right)^{\sigma} \sum P_i^{-(\sigma-1)}$$

Finally replacing back in the FOC and rearranging, demand is

$$Q_i = \frac{\tilde{Y}}{\sum P_i^{-(\sigma-1)}} P_i^{-\sigma}$$

A.4 Derivation of the reduced form profit function

Since ω_i and K_i are fixed factors, the only adjustable factor is labor: $\pi = P(Q(L_i))Q(L_i) - wL_i$ where w is the wage rate. The first order conditions are

$$\frac{\sigma-1}{\sigma}\alpha P\left[Q(L_i)\right]\frac{Q(L_i)}{L_i} = w \tag{12}$$

Rewriting we get

$$L_i^* = \left[\left[\frac{(\sigma-1)\alpha}{\sigma w}\right]^{\sigma}\tilde{Y}(\tilde{P}\omega_i K_i^{\beta})^{(\sigma-1)}\right]^{1/[\sigma-\alpha(\sigma-1)]} \tag{13}$$

Replacing back in the production function (3)

$$Q_i^* = \omega_i L_i^{\alpha} K_i^{\beta} = \left[\left(\omega_i K_i^{\beta}\right)\left(\frac{(\sigma-1)\alpha}{\sigma w}\left(\tilde{P}^{(\sigma-1)}\tilde{Y}\right)^{1/\sigma}\right)^{\alpha}\right]^{\sigma/[\sigma-\alpha(\sigma-1)]} \tag{14}$$

Prices can be written from the Demand Function (2)

$$P_i^* = \left[\omega_i K_i^{\beta}\left(\frac{(\sigma-1)\alpha}{\sigma w}\right)^{\alpha}\left(\left(\frac{\tilde{Y}}{\tilde{P}}\right)^{1/\sigma}\tilde{P}\right)^{-\sigma(1-\alpha)}\right]^{-1/[\sigma-\alpha(\sigma-1)]} \tag{15}$$

Finally sales are

$$P_i Q_i = \left[\left(\frac{(\sigma-1)\alpha}{\sigma w}\right)^{\alpha(\sigma-1)}\tilde{Y}\left(\tilde{P}\omega_i K_i^{\beta}\right)^{(\sigma-1)}\right]^{1/[\sigma-\alpha(\sigma-1)]} \tag{16}$$

The quality adjusted price index is

$$\tilde{P} = \left(\sum P_i^{-(\sigma-1)}\right)^{-\frac{1}{\sigma-1}} \tag{17}$$

From (15) above we can express this as

$$(P_i^*)^{-1} = \left[\omega_i K_i^{\beta}\left(\frac{(\sigma-1)\alpha}{\sigma w}\right)^{\alpha}\left(\tilde{Y}\tilde{P}^{(\sigma-1)}\right)^{-(1-\alpha)}\right]^{1/[\sigma-\alpha(\sigma-1)]} \tag{18}$$

So that the quality adjusted price index is

$$\tilde{P} = \left[\left(\frac{(\sigma-1)\alpha}{\sigma w}\right)^{\alpha}\tilde{Y}^{-(1-\alpha)}\right]^{-1}\left(\sum\left[\omega_i K_i^{\beta}\right]^{(\sigma-1)/[\sigma-\alpha(\sigma-1)]}\right)^{-(\sigma-\alpha(\sigma-1))/(\sigma-1)} \tag{19}$$

Using this in the equation for profit

$$VA(\omega_i, K_i, N; \theta) = P(Q_i)Q_i - wL_i =$$

$$= \left(\frac{\sigma-\alpha(\sigma-1)}{\sigma}\right)\left[\left(\frac{(\sigma-1)\alpha}{\sigma w}\right)^{\alpha(\sigma-1)}\tilde{Y}\left(\tilde{P}\omega_i K_i^{\beta}\right)^{(\sigma-1)}\right]^{1/[\sigma-\alpha(\sigma-1)]}$$

$Y_i = \frac{\left(\omega_i K_i^{\beta}\right)^{\gamma}}{\sum\left[\omega_i K_i^{\beta}\right]^{\gamma}}\tilde{Y}$
Writing $\gamma = (\sigma-1)/(\sigma-\alpha(\sigma-1))$

$$VA(\omega_i, K_i, N; \theta) = \frac{1}{\gamma}\left(\frac{\sigma-1}{\sigma}\right)\left(w\frac{\sigma}{(\sigma-1)\alpha}\right)^{-\alpha\gamma}\tilde{Y}^{\gamma/(\sigma-1)}\left(\omega_i K_i^{\beta}\tilde{P}\right)^{\gamma} \tag{20}$$

Using the expression for $\tilde{P}$, (19) we finally get the one period returns

$$VA(\omega_i, K_i, N; \theta) = \frac{1}{\gamma}\left(\frac{\sigma-1}{\sigma}\right)\tilde{Y}\frac{\left(\omega_i K_i^{\beta}\right)^{\gamma}}{\sum\left[\omega_i K_i^{\beta}\right]^{\gamma}} \tag{21}$$

A.5 Data and sample construction

I have collected data for the aggregate variables from the Portuguese National Statistics Office (INE), together with data on industry price deflators (from IAPMEI, 2006). I have merged these aggregate variables with the sample for the 5 digit NACE code industry 29563 (Moulds Industry). The capital stock was calculated using the perpetual inventory method formula. Value added was constructed as total sales subtracted from materials and services. Both aggregate and individual sales and value added were deflated with the industry price deflator.

In 11 observations the number of workers reported was zero which occurs in the year the firms enter or exit the industry. Since the owner of the firm is never reported as a worker I add one to all firms with zero reported workers. The results are robust to dropping these observations.

I identified 9 holes in the sample (firms that interrupt reporting for 1 or more consecutive years). In these cases either the earlier or later periods are dropped, minimizing the total number of observations lost.

Entry and exit are difficult to identify since it is not compulsory for firms to report to the central bank. However, the dataset has information on the founding year and current firm "status" (i.e. active, bankrupt, merged, etc). Using this information I identify 48 actual entries and 7 exits.

I have winsorized at 1% (0.5% on each tail) the variables for ln(K), I, ln(Materials), ln(Value Added), value added growth, sales growth.

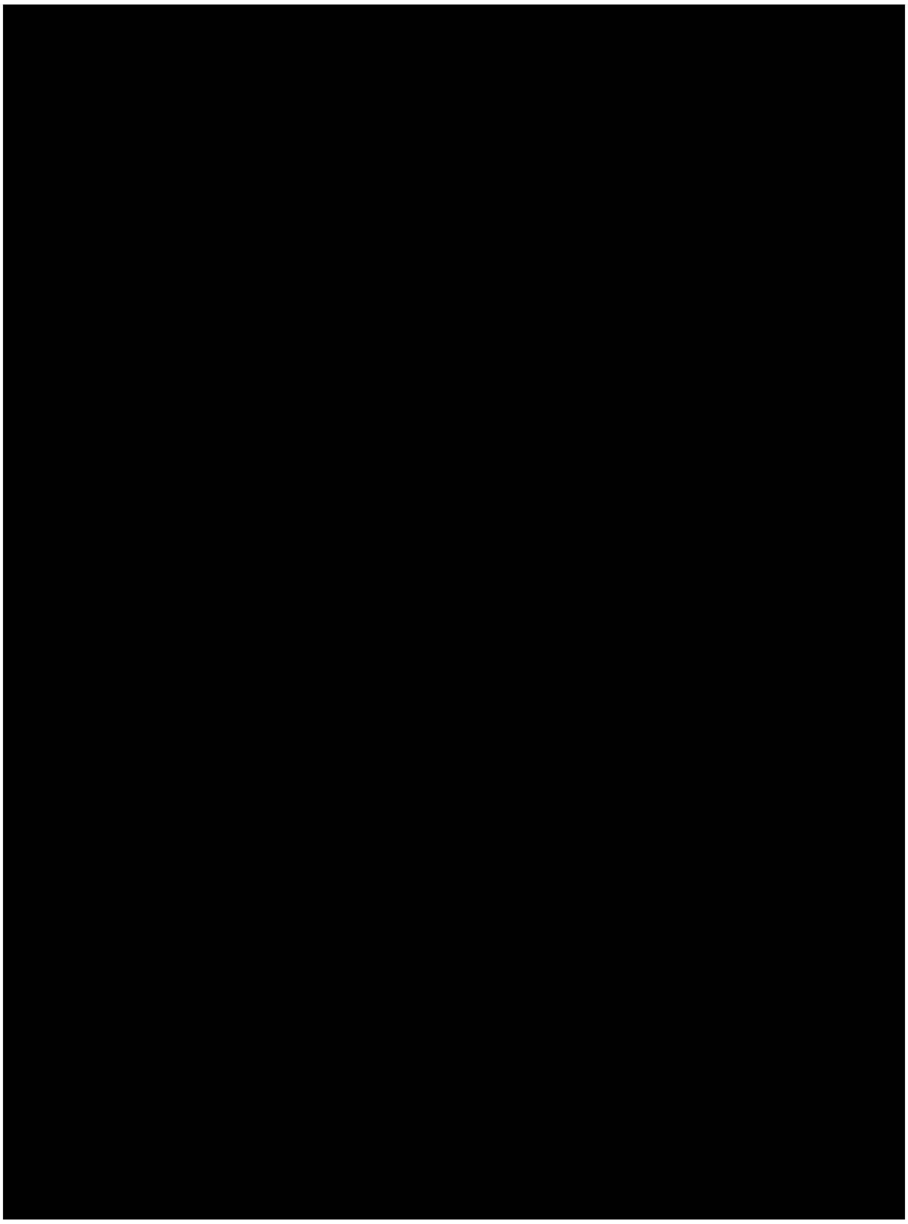

Exhibit 1: Plastics' Mould (1950's): Toy's head

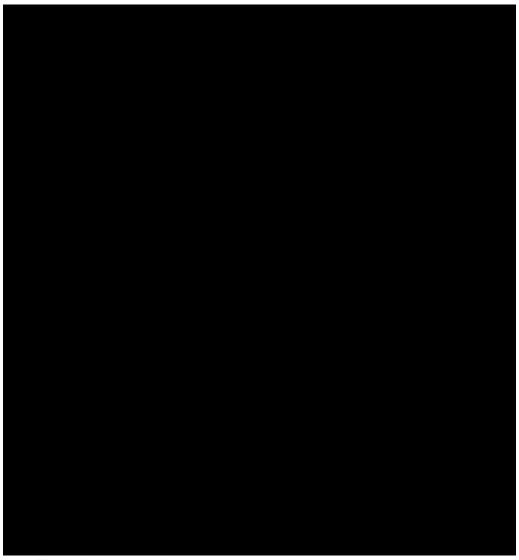

Exhibit 2: Metals' mould (1950's): spoon

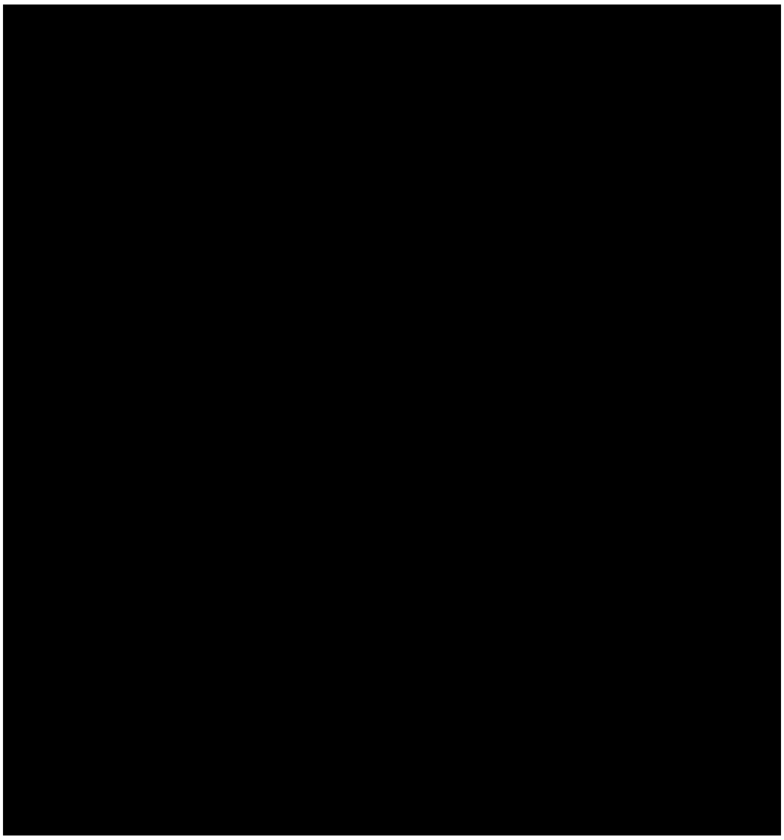

Exhibit 3: CNC (Computer Numerical Control) Machining (2006)

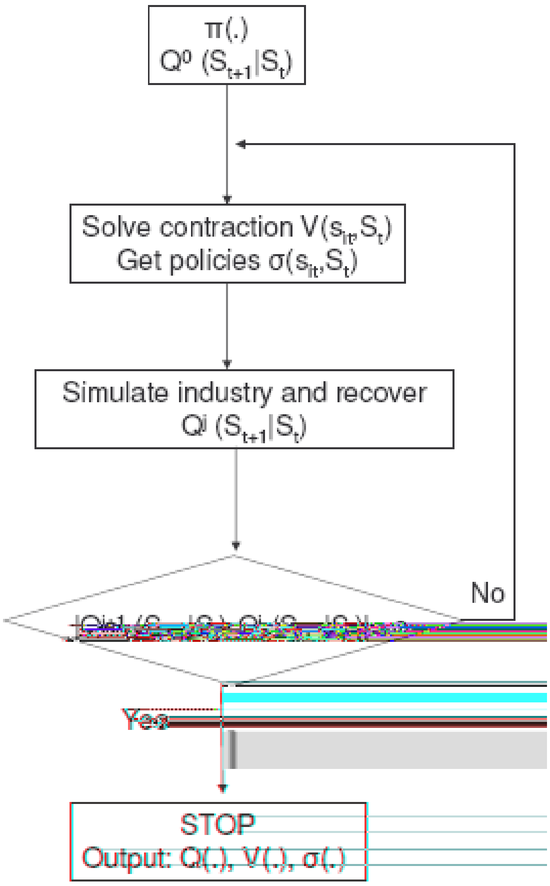

Figure 1: Algorithm

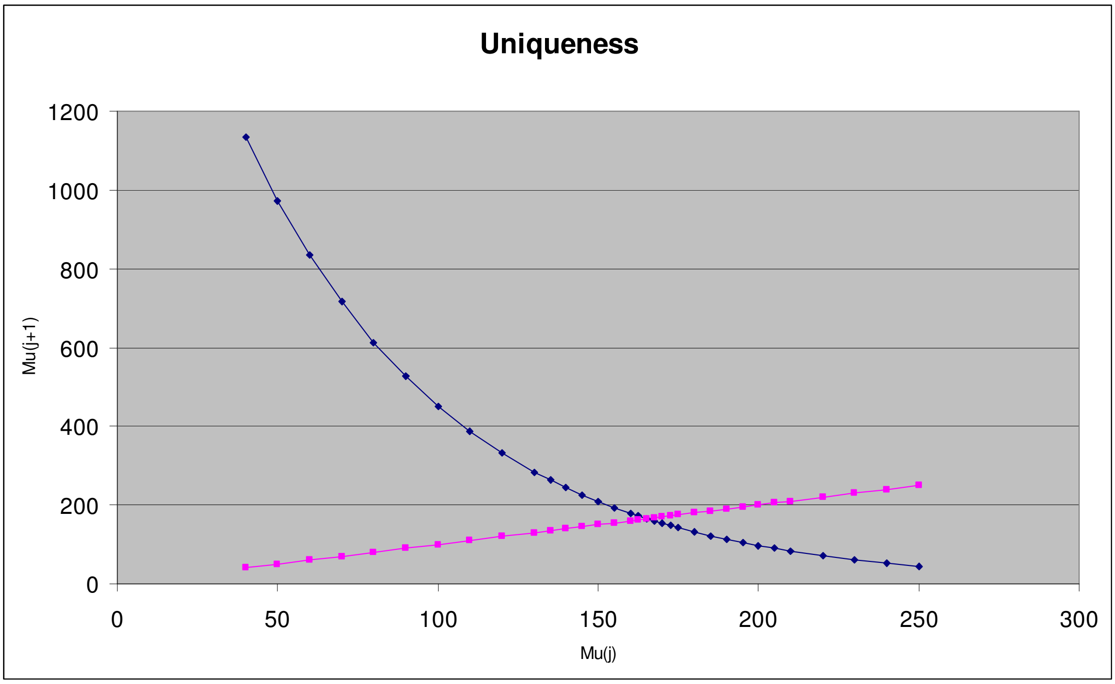

Figure 2: Uniqueness

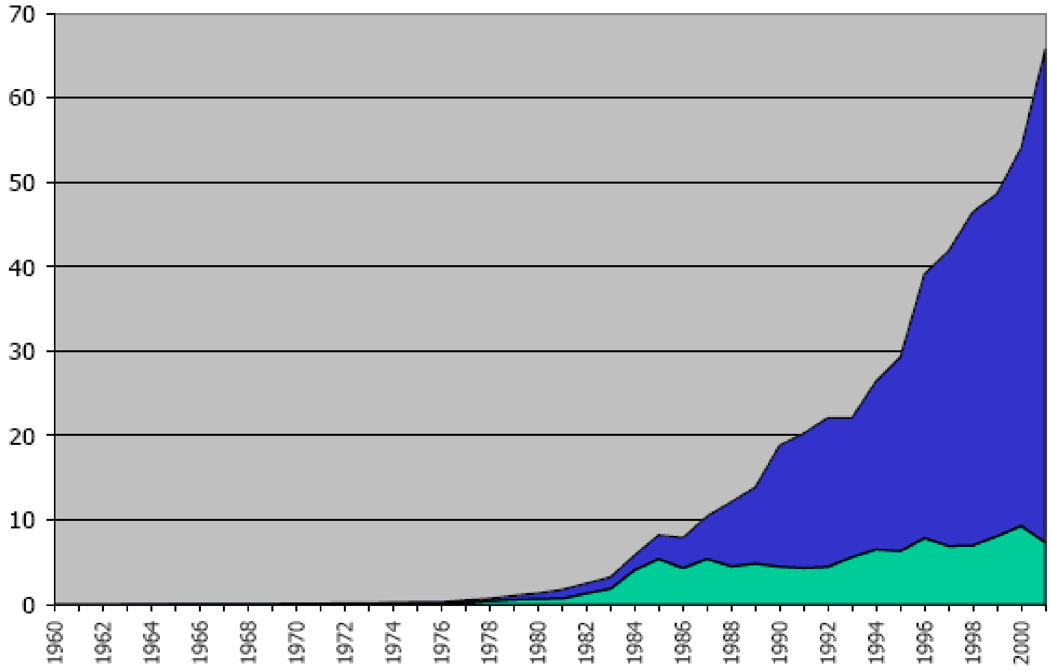

Figure 3: Total exports (blue) and US exports (green) 1960-2001 (millions of euros)

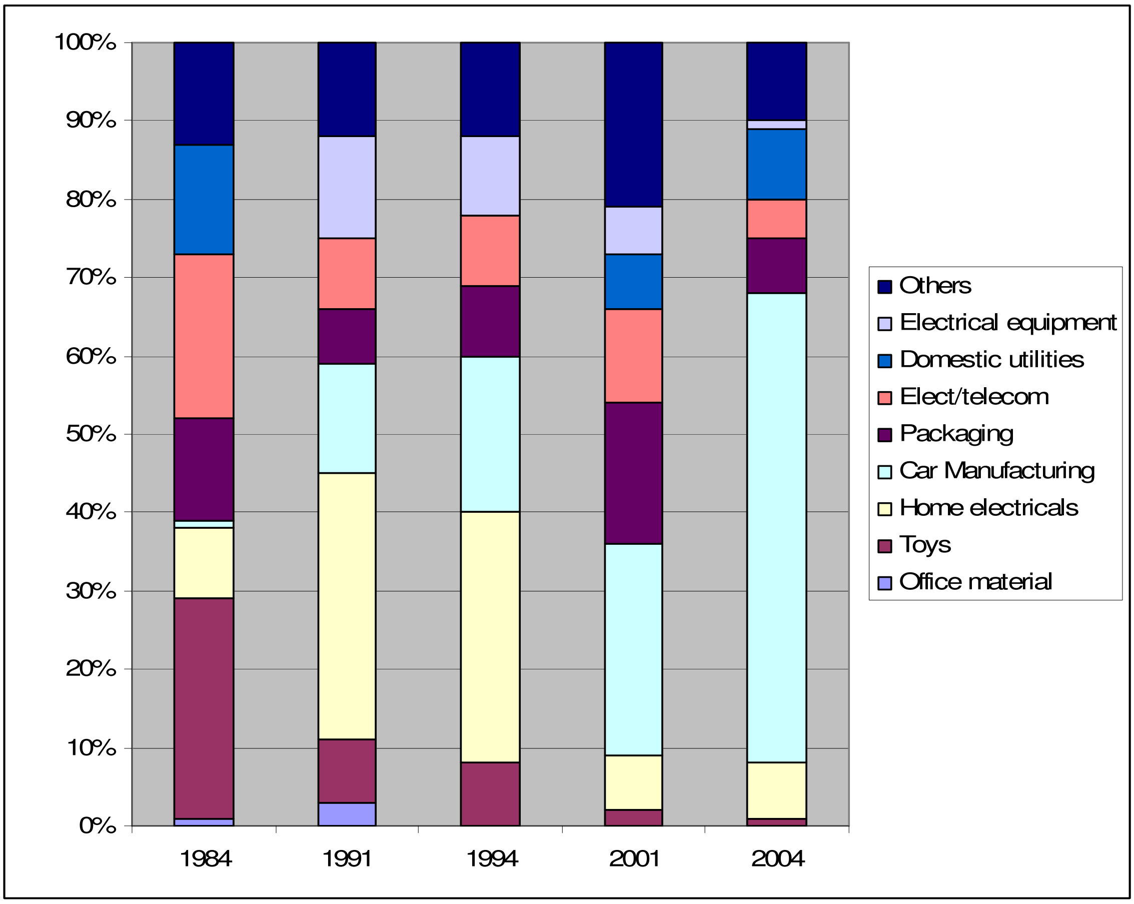

Figure 4: Export composition (% share of total exports), by client/product type

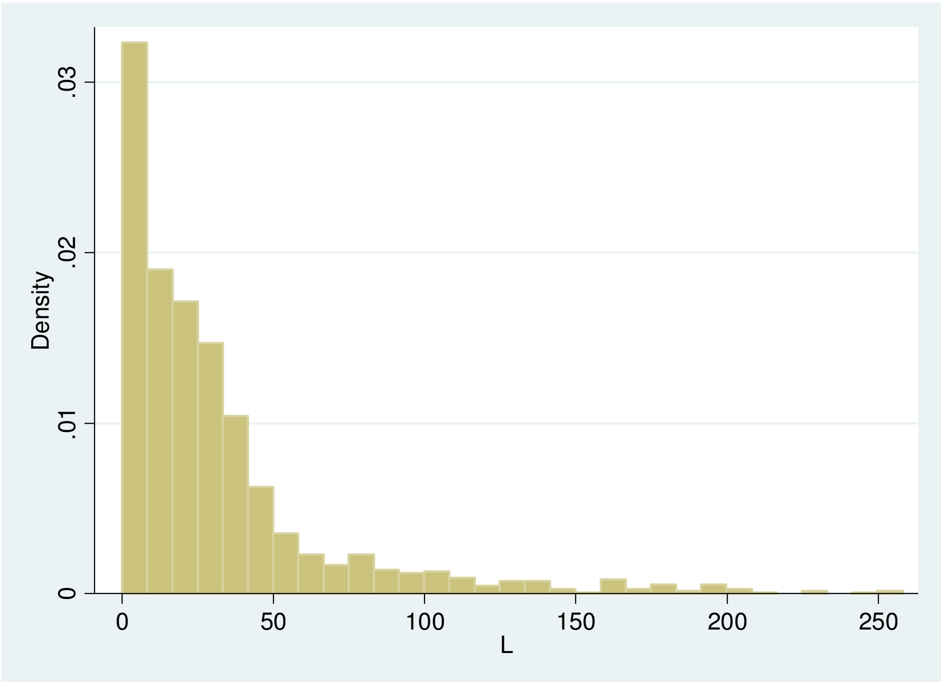

Figure 5: Distribution of firm size (number of workers per firm)

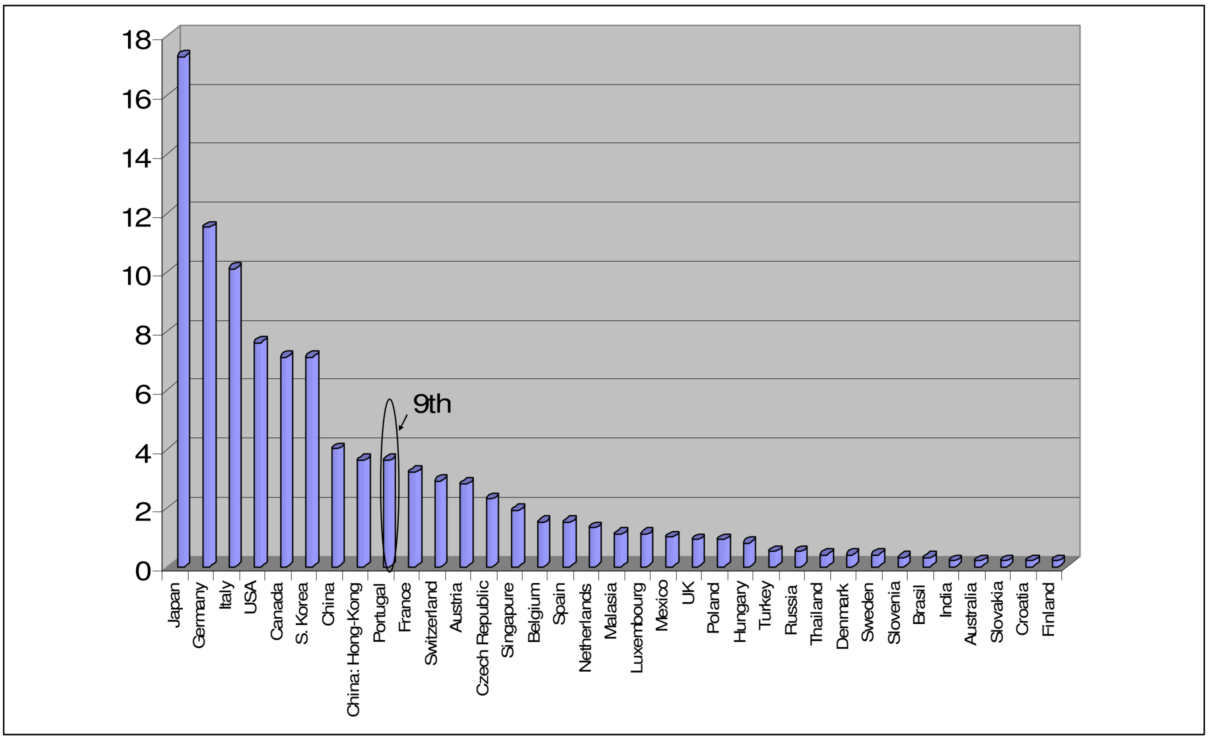

Figure 6: World moulds exports in 2004 (%)

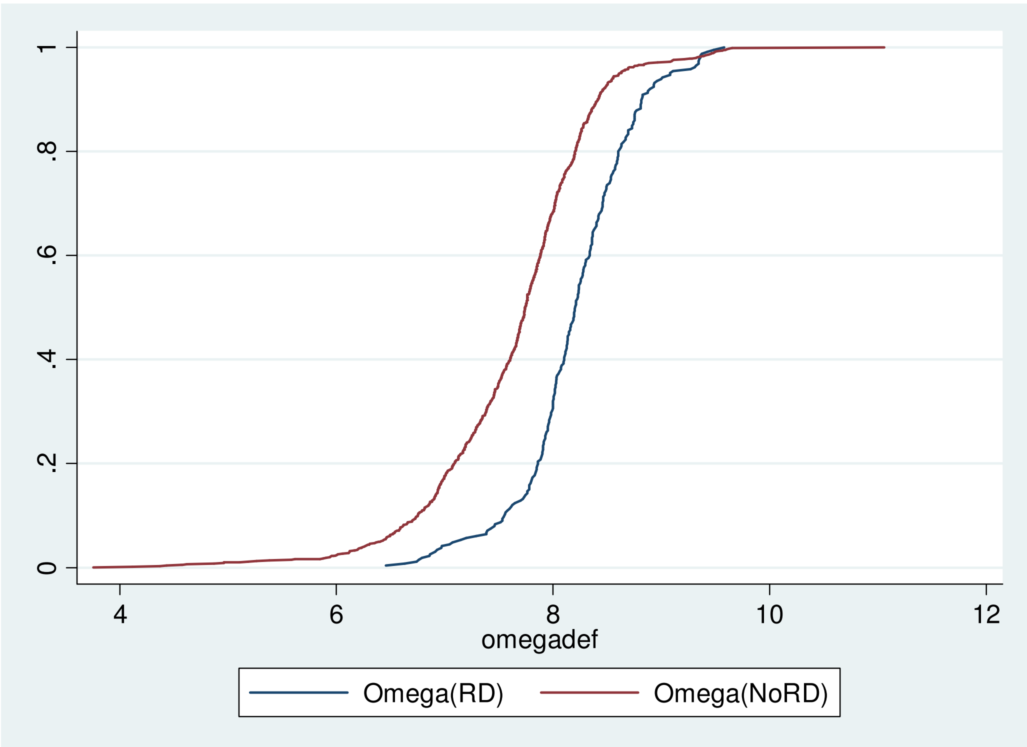

Figure 7: Productivity distribution for R&D and non-R&D firms

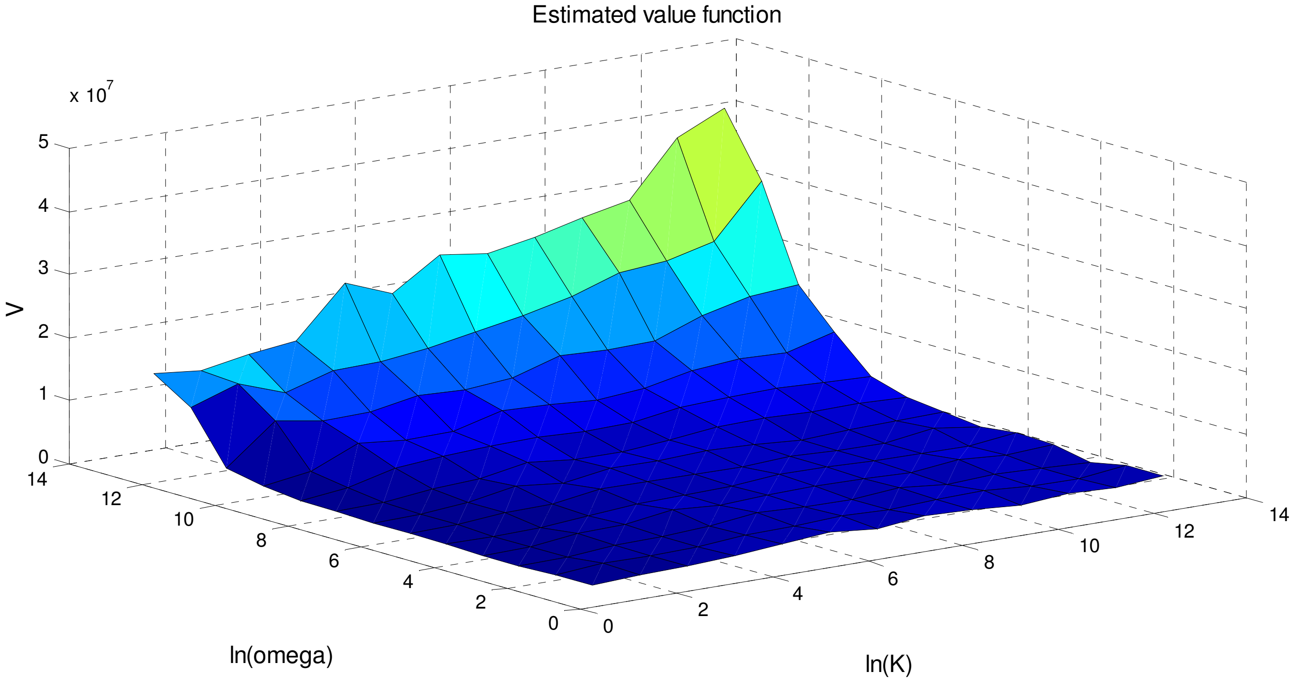

Figure 8: Estimated value function for the 1st order polynomial parameters

Year	Number of firms	Number of non R&D firms	Number of R&D firms	R&D start-ups	Number of entries in the Industry	Number of entries in the dataset	Exits
1994	144	134	10	-	2	3	0
1995	157	137	20	10	12	14	2
1996	165	141	24	4	8	14	0
1997	170	145	25	2	11	20	2
1998	164	135	29	7	9	33	0
1999	136	108	28	3	2	46	1
2000	92	68	24	7	2	8	0
2001	88	56	32	9	1	5	0
2002	88	53	35	4	1	2	0
2003	86	48	38	3	0	0	2
Total	1290	1025	265	49	48	145	7

Table 2: Firms, entry, exit and R&D behavior per year

Consecutive years of positive R&D	Number of firms
1	26
2	12
3	6
4	6
5	2
6	4
7	2
8	1

Table 3: Distribution for the number of periods with positive R&D

Variable	Mean	p50	Std. Dev.	Min	Max
Sales (EUR)	1,570,166	697,310	2,866,806	3,292	34,700,000
Value Added (EUR)	792,605	389,231	1,437,950	494	15,200,000
Employment	32	20	39	1	258
Capital Stock (EUR)	1,092,534	406,775	2,169,156	135	23,900,000
Labor Productivity (EUR)	20,452	19,279	9,150	386	73,313
Sales growth	9.9%	8.1%	32.9%	-171.9%	207.5%
Value added growth	10.1%	8.6%	35.8%	-169.1%	226.0%
Labor productivity growth	6.0%	5.7%	34.5%	-188.5%	176.2%

Table 4: Summary statistics

Year	Number of firms	Employment	Sales (euros)	Value added (euros)	Price Index	Price Variation	Sales Growth	Value added growth
1994	644	5,133	171,300,000	152,600,000	96.73	-	-	-
1995	570	5,796	193,400,000	172,300,000	100.00	3%	13%	13%
1996	452	7,316	244,200,000	217,500,000	101.80	2%	26%	26%
1997	477	7,821	292,700,000	246,200,000	101.87	0%	20%	13%
1998	461	7,740	322,400,000	258,800,000	97.50	-4%	10%	5%
1999	549	8,429	362,200,000	277,300,000	99.91	2%	12%	7%
2000	604	8,879	411,800,000	299,300,000	104.90	5%	14%	8%
2001	612	8,919	421,000,000	368,800,000	105.90	1%	2%	23%
2002	722	9,312	378,000,000	359,200,000	98.87	-7%	-10%	-3%
2003	738	8,766	402,800,000	358,600,000	90.51	-8%	7%	0%

Table 5: Aggregate variables, per year

(i) First stage (GLS)			(ii) Second Stage (NLLS)			(iii) Fixed Effects estimates			(iv) First difference estimates		
	Coef.	*s.e.*		Coef.	*s.e.*		Coef.	*s.e.*		Coef.	*s.e.*
ln(L)	0.61	*0.03*	ln(K)	0.18	*0.02*	ln(K)	0.76	*0.04*	D.ln(K)	0.53	*0.05*
			ln(S)	0.29	*0.10*	ln(L)	0.24	*0.02*	D.ln(L)	0.20	*0.03*
			$\gamma 0$	0.13	*0.69*	ln(S)	0.35	*0.03*	D.ln(S)	0.33	*0.14*
			$\gamma 1$	0.59	*1.07*	Constant	-1.43	*0.51*	Constant	0.04	*0.03*
			$\gamma 2$	0.32	*0.11*						
			$\gamma 3$	0.02	*0.05*						
R squared	92%			75%			91%			21%	
Observations	1,269			1,038			1,269			1,038	
Firms	231			227			231			224	
Labor coefficient	0.86						1.18			0.79	
Capital coefficient	0.26						0.37			0.30	
PC Margin	29%						35%			33%	

Notes: columns (i) and (ii) report the results for the first and second stage of the production function estimates. Columns (iii) and (iv) report the production function results using a fixed effects and a first differences specification, respectively.

Table 6: Production function estimates, dependent variable ln(Value Added)

Dependent Variable: Aggregate State ln(S)	(i) Coef.	(i) s.e.	(ii) Coef.	(ii) s.e.	(iii) Coef.	(iii) s.e.	(iv) Coef.	(iv) s.e.
ln[S(t-1)]	0.95	*0.08*	1.25	*0.34*	-2.33	*7.34*	570.85	*598.06*
ln[S(t-1)]^2			-0.35	*0.32*				
ln[S(t-1)]^3					0.10	*0.23*	-34.94	*36.56*
ln[S(t-2)]							0.71	*0.74*
Constant	1.01	*1.23*	1.76	*1.50*	27.66	*59.81*	-3096.74	*3260.35*
Observations	11		10		11		11	
Adjusted R Squared	0.95		0.93		0.95		0.95	
Mean ln(S)	16.43							
St. Dev ln(S)	0.39							
Autocorrelation ln(S)	0.97							

Notes: Column (i) specifies a linear first order markov process and column (ii) a second order Markov process. Column (iii) and (iv) present results for a second and third order polynomial approximation for the first order Markov process.

Table 7: Tests on the aggregate state variable

Dependent Variable: ln[S(t+1)]	(i) Coef.	(i) s.e.	(ii) Coef.	(ii) s.e.	(iii) Coef.	(iii) s.e.	(iv) Coef.	(iv) s.e.	(v) Coef.	(v) s.e.	(vi) Coef.	(vi) s.e.	(vii) Coef.	(vii) s.e.	(viii) Coef.	(viii) s.e.
ln[S]	0.95	*0.08*	0.90	*0.09*	0.84	*0.09*	0.79	*0.10*	0.68	*0.18*	0.88	*0.07*	0.83	*0.08*	0.69	*0.20*
std(y)	-	-	-0.62	*0.59*	-	-	-	-	-	-	-	-	-	-	-1.77	*0.87*
skew(y)	-	-	-	-	-0.02	*0.18*	-	-	-	-	-	-	-	-	-0.68	*0.28*
std(k)	-	-	-	-	-	-	0.49	*0.47*	-	-	-	-	-	-	-0.25	*0.83*
skew(k)	-	-	-	-	-	-	-	-	-0.15	*0.15*	-	-	-	-	0.07	*0.13*
std(ω)	-	-	-	-	-	-	-	-	-	-	-0.58	*0.28*	-	-	0.07	*0.26*
skew(ω)	-	-	-	-	-	-	-	-	-	-	-	-	0.04	*0.05*	-0.12	*0.10*
const	1.01	*1.23*	2.64	*1.27*	2.65	*1.51*	2.81	*1.29*	5.14	*2.85*	2.50	*1.06*	2.83	*1.36*	7.46	*3.49*
Obs.	11		9		9		9		9		9		9		9	
R2	94%		94%		92%		94%		94%		96%		93%		99%	

Notes: Column (i) specifies a linear first order markov process and columns (ii)-(viii) test the significance of further moments (standard deviation and skewness) of the distribution of sales (y), capital stock (k) and TFP (ω).

Table 7b: Further tests on the aggregate state variable

Dependent variable: ω(t)

	(i)		(ii)	
	Non-R&D firms		R&D firms	
	Coef.	*s.e.*	Coef.	*s.e.*
ln[ω(t-1)]	0.72	*0.03*	0.75	*0.09*
$\ln[\omega(t-1)]^2$	0.16	*0.01*	0.07	*0.10*
$\ln[\omega(t-1)]^3$	-0.03	*0.01*	-0.01	*0.03*
constant	0.14	*0.03*	0.24	*0.05*
R-squared	67%		80%	
Obs.	784		254	
Firms	198		59	

Note: Results for the productivity transition using a 3rd order polynomial expansion.

Table 8: Transition function for productivity, OLS results

	(ii)		(iii)		(i)		(iv)	
Dep. Var.	Investment				R&D start-up		Exit Probit	
	R&D firms		Non R&D firms					
	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*	Coef.	*s.e.*
ln(S(t-1))	-0.36	*0.27*	-0.16	*0.20*	0.26	*0.25*	-0.02	*0.53*
ln(K(t-1))	-2.08	*1.13*	1.20	*0.37*	0.16	*0.06*		
$\ln(K(t-1))^2$	0.10	*0.04*	-0.02	*0.02*	-	-	-	-
ln(ω(t-1))	1.17	*0.37*	0.25	*0.10*	0.07	*0.14*	-0.17	*0.20*
$\ln(\omega(t-1))^2$	-0.25	*0.13*	0.17	*0.06*	-	-	-	-
Constant	26.67	*8.90*	1.90	*3.98*	-7.90	*4.05*	-2.07	*8.66*
R Squared	53%		30%		4%		1%	
Observations	206		832		1038	832	1038	
Firms	51		213		264	213	224	

Notes: Columns (i) and (ii) presents the results for the invesment OLS results for the non-R&D and R&D firms on a second order approximation on the state variables (w,K,S). Column (iii) presents the results for the R&D start-up probit regression on a first order approximation on the state variables (w,K,S). Finally column (iv) presents the results for the exit probit regression on a first order approximation on the state variables (w,S)

Table 9: Estimated policy function

	μ_1	μ_2	ω_0	ν_1
Coefs	-0.46	5.77	2,598,000	-534,000
s.e.	*1.61*	*7.17*	*1,020,524*	*1,020,162*

Table 10: Investment cost, R&D sunk cost and exit value

	ω_0=2,598,000	ω_0=1,948,500	% change
Market size	*4,228,255*	*6,514,233*	*43%*
Number of firms	*267*	*227*	*-15%*
% of R&D firms	*16%*	*33%*	*16%*
Average Productivity	*2.92*	*3.24*	*11%*
Average Capital Stock	*77,637*	*92,865*	*18%*
Entry Rate	*4%*	*5%*	*1%*
Exit Rate	*4%*	*5%*	*1%*

Table 11: Counterfactual results for a 25% reduction in sunk costs of R&D (ω_0)