

# Participation Screen for Collusion in Auctions

Janne Tukiainen\*

University of Helsinki, RUESG and HECER

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## Abstract

I propose a new statistical method of testing for collusion. This test utilizes data on entry decisions. I compare this method with an existing test with Monte Carlo analysis and show that my method is robust to unobserved heterogeneity unlike the existing test with otherwise similar properties. I apply both methods to procurement auctions that contract snow removal in schools of Helsinki. Two of the bidders seem to participate in a contract allocation scheme.

Keywords: Auctions; Collusion; Discrete choice models; Entry; Simultaneous equations estimation

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# 1 Introduction

I propose one new statistical method and use one existing method to test for collusion in a territorial allocation setting. Both of these tests can be applied to any institutional setting where independent and mutually exclusive markets can be defined. In general, these tests can be applied in any static discrete games with binary outcomes. However, I discuss them in the light of my application, which is a procurement auction. Due to their simultaneous nature, auctions are particularly well suited for the static estimation methods that are applied here. Moreover, in auctions it is easy to argue the independence of the markets under scrutiny. These tests are based on the participation decision of the bidders instead of the bid levels. Collusion is defined here as an explicit arrangement among a group of bidders that limits competition between the participants. Collusion can take many forms in auctions. Typical methods include different contract allocation mechanisms, like territorial allocation or job rotation, and submitting phony bids. The previous literature on detecting collusion in auctions has studied only phony bidding scenarios. The tests that I use here can also be applied in phony bidding scenarios.

I test whether the participation of one bidder affects the participation decision of other bidders. In the competitive setting the identity of competitors should not affect the participation decision, given that the auctioned contracts are identical and the bidders are symmetric. For strategic reasons bidders would like to avoid each other, but if they are symmetric, bidder C has no reason to avoid bidder A more than bidder B. Porter and Zona (1999) (denoted PZ) propose a test based on the correlation of the residuals of single equation participation choice models. Negative correlation between two bidders' residuals implies territorial allocation and positive correlation phony bidding. PZ use it to detect phony bidding. I propose a test that is robust to unobserved heterogeneity, unlike the PZ test. This is based on solving the simultaneous equations model of participation. I use estimation techniques proposed by Tamer (2003).

The central difficulty in detecting collusion is that similar market outcomes can be a result of either collusive or competitive behavior. Territorial allocation can be a result of either an explicit agreement or due to cost advantages that firms have in different areas. Due to transaction costs for example, firms could decide to bid only on those markets that are near the location of their operations. With different locations, territorial allocation emerges as a competitive result. We get suspicious if the territories overlap, but firms still systematically avoid bidding for the same contracts. Unfortunately, this can be again a result of competitive behavior if the contracts are heterogeneous. Some firms may have costs advantages

in some types of contracts. Therefore with heterogeneous contracts and asymmetric bidders, participation patterns of any kind may emerge in the competitive setting. However, if we control for bidder and contract heterogeneity, then the identity of other participants should not affect the participation decision of any bidder in the competitive setting. This makes testing for collusion possible. I apply the methods to school yard snow removal auctions in the City of Helsinki held in the autumns of the years 2003-2005. In Figure 1. I present the spatial participation pattern in these auctions in the year 2003. It marks on the city map the schools that each bidder has participated in and the location of bidders' and city's garages. The map shows that two bidders (A and K) seem to avoid each other. Moreover they systematically avoid each other in an overlapping geographic area, the city center. This suggests collusive behavior in this market. This I put to test.

I make two contributions to the literature on collusion in auctions. First, I propose a new test to detect collusion. I will show with Monte Carlo analysis that it is robust to unobserved heterogeneity unlike the existing method by PZ. Second, the empirical application is important in itself because it is the first empirical study of a territorial allocation scheme. The minor contribution of this paper is the policy implications of the empirical application.

This study is related to two different fields of empirical industrial organization. The first is the literature on the detection of collusion. The second is the entry literature, as it is possible to think of this problem as an entry game with a single auction as an analog of a single market. Harrington (2005) provides a recent survey on detecting cartels. Also Levenstein and Suslov (2006) have a survey on cartel studies but they do not address auctions nor the detection of cartels. Berry and Tamer (2007) provide a survey on empirical analysis of entry models. The existing studies on the detection of collusion in auctions (e.g. Bajari and Ye (2003), Baldwin, Marshall and Richard (1997), Banerji and Meenakshi (2004), Porter (1983), Porter and Zona (1993,1999), Price (2008)) have only applications to phony bidding scenarios.

In Section 2, I present both the PZ test and my own test. I conduct Monte Carlo analysis to examine the finite sample properties of these tests in Section 3. In Section 4, I present the market of the application and analyze its characteristics with respect to collusion. Then I present the data and descriptive statistics in Section 5 and the results in Section 6. Finally, Section 7 concludes.

## 2 Testing

In this section, I present both the existing PZ test and propose a new test. I discuss how the new test nests the existing test and I also discuss their relative strengths and weaknesses. While I present both tests here only in a two-bidder case, they extend to a  $n$ -bidder case by conducting pairwise analysis for all the possible pairs of bidders. The PZ test could also be extended to the  $n$ -bidder case by using multivariate probit analysis.

### 2.1 The model

Assume that there are two competing firms, denoted 1 and 2, that do not know ex ante to which markets the other firms are going to bid. Assuming that the value of the outside option is zero and the payoffs are linear in observed factors  $x$ , a standard simultaneous single market entry game can be presented with the following payoffs:

$$\begin{array}{cc} y_2 = 0 & y_2 = 1 \\ y_1 = 0 & 0, 0 \quad 0, x_2\beta_2 - u_2 \\ y_1 = 1 & x_1\beta_1 - u_1, 0 \quad x_1\beta_1 + \delta_1 - u_1, x_2\beta_2 + \delta_2 - u_2 \end{array}$$

This game maps directly into a following model:

$$\begin{aligned} y_1^* &= x_1\beta_1 + y_2\delta_1 + u_1, \\ (1) \quad y_2^* &= x_2\beta_2 + y_1\delta_2 + u_2, \\ y_i &= 1 \text{ if } y_i^* \geq 0, \text{ otherwise } y_i = 0, \quad i = 1, 2. \end{aligned}$$

Now  $y_i^*$  denotes the latent continuous variable that determines the participation decision. In an auction setting,  $y_i^*$  is the expected profit of bidder  $i$  from submitting a bid. Bidder  $i$  submits a bid to an auction if  $y_i^* > 0$ .  $x$  includes all the observable variables that affect the bidder's costs and its probability of winning the auction. These include the contract and bidder characteristics. We observe  $y_i = 1$  if the bidder  $i$  submitted a bid and  $y_i = 0$  if it did not. This model nests both the PZ test and the new test. Next I present both the tests. I show that given some assumptions, the  $\delta_i$ 's are a measure of collusion.

## 2.2 The PZ test

PZ propose several tests to detect collusion in auctions. They utilize both the participation decisions and the bid levels to test whether some bidders submitted phony bids. PZ use legal evidence to create a control group made up of non-defendant firms that bid on Ohio school milk contracts. They compare the behavior of this control group with the behavior of defendant firms. I present and use here one of their many tests that can be used, similar to the new test that I propose, to detect territorial allocation and can be applied outside an auction setting. They test for the statistical independence in the probability of bidding using a standard pairwise procedure. PZ state: "Under the null hypothesis of independent action based on public information and the maintained specifications of our probit submission model, knowledge of whether one particular firm bids should not help predict whether another firm has also bid. In the case of complementary bidding, if one cartel member bids, then other ring members also bid. In this case the unexplained portion of the competitive bidding equation is positively correlated across cartel firms. In the case of territorial allocation, if a particular cartel member bids, then other cartel members will tend to not bid. Then the unexplained portion of the competitive bidding equation is negatively correlated across cartel firms." They propose to use the Spearman correlation coefficients computed using pairs of weighted residuals based on the control group probit models. PZ use the control group estimates also for the cartel group to address the problem of endogeneity that arises because the participation decisions of cartel firms are affected by collusion. This biases the estimates of the effect of observables on their participation. Assuming that all the bidders are identical, the control group estimates can be used as unbiased estimates also for the treatment group. There is a trade-off between the endogeneity problem and the need to make the assumption that all the firms react identically to changes in the explanatory variables when deciding whether to use the control group or not, when such is available. In situations where bidders have different production technologies or differ in some other important respects, it could be better not use a control group at all.

To get the PZ test from the system of equations (1), define  $\epsilon_1 = y_2\delta_1 + u_1$  and  $\epsilon_2 = y_1\delta_2 + u_2$  and assume a bivariate normal distribution of these new error terms  $\epsilon_i$ . The PZ test is a test of correlation between the error terms  $\epsilon_i$ . This they carry out by estimating the two equations of the system (2) separately by univariate probit and then calculating a Spearman correlation between the error terms of these two probit equations.

$$\begin{aligned}
y_1^* &= x_1\beta_1 + \epsilon_1, \\
(2) \quad y_2^* &= x_2\beta_2 + \epsilon_2, \\
y_i &= 1 \text{ if } y_i^* \geq 0, \text{ otherwise } y_i = 0, \quad i = 1, 2. \\
(\epsilon_1, \epsilon_2) &\sim IIDN \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_\epsilon \\ \rho_\epsilon & 1 \end{bmatrix} \right)
\end{aligned}$$

If the Spearman correlation is negative and statistically significant, we can conclude that we are missing some variable from the estimation that affects the bidders differently and significantly. If we have no other missing variables, this is the competitor's decision to bid. For example, firm 1 bids on those contracts that are allocated to it in the collusion scheme and firm 2 avoids those contracts as agreed. The benefit of this test is that it does not require many observations and that it is identifiable even when the firms under scrutiny never bid to the same contracts. It is also computationally very fast, easy to implement and has better convergence properties than simultaneous equation methods. The test hypothesis in PZ case is:

$$\begin{aligned}
\text{No collusion, } H_0: \text{Corr}(\epsilon_1, \epsilon_2) &= 0, \\
\text{Collusion, } H_1: \text{Corr}(\epsilon_1, \epsilon_2) &\neq 0.
\end{aligned}$$

PZ detect positive correlation and thus conclude phony bidding. Since the PZ test is a test of endogeneity in a bivariate probit model, there are numerous other ways to test for endogeneity in this model. This includes the standard trio of likelihood ratio, Lagrange multiplier and Wald tests. The first test of endogeneity in a bivariate probit model was introduced by Kiefer (1982) to the statistics literature. Monfardini and Radice (2007) survey and compare these tests with a Monte Carlo analysis in a recursive probit framework. The PZ test hypothesis can also be written as

$$\begin{aligned}
\text{No collusion, } H_0: \rho_\epsilon &= 0, \\
\text{Collusion, } H_1: \rho_\epsilon &\neq 0.
\end{aligned}$$

### 2.2.1 Identification in the PZ test

Although not explicitly stated in their study, PZ need to make the following two key assumptions:

**Identification assumption 1:**  $x_1$  and  $x_2$  capture entirely the competitive effect.

Firms would prefer being the only bidder to competing against other firms. If this is not controlled for in the estimation, it creates negative correlation in the residuals that would make us point out innocent firms as guilty of territorial allocation or make it harder to detect phony bidding. For this reason PZ include the observed competitors' characteristics in  $x$ . They seem to implicitly assume that this captures all the strategic reasons for the bidders to avoid each other in a competitive setting. With these assumptions,  $\rho_\epsilon$  is a measure of collusion since it captures the effect of the  $\delta_i$ 's. It means that knowledge of whether one particular firm bids should not help to predict whether another firm has also bid when firms are not colluding.

**Identification assumption 2:** We have an iid sample  $\{(y_{1i}, y_{2i}), x_{1i}, x_{2i}\}$  such that

$0 < \Pr[y_1, y_2 | (x_1, x_2)] < 1$  for all  $(y, x_1, x_2) \in Y \times R^{d_1} \times R^{d_2}$  where  $x = (x_1, x_2) \in R^d$  and  $Y = \{(0, 0), (1, 1), (0, 1), (1, 0)\}$ .

This standard assumption is also required for both the tests.

**Identification assumption 3:**  $(\epsilon_1, \epsilon_2) \sim IIDN \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_\epsilon \\ \rho_\epsilon & 1 \end{bmatrix} \right)$ , where  $\epsilon_i = y_j \delta_i + u_i$ ,  $i = 1, 2$ .

This does not seem like an innocuous assumption because the  $y$ 's are discrete variables and yet the error terms are assumed to follow a smooth continuous distribution.

**Identification assumption 4:**  $Cov(u_1, u_2) = \rho_u = 0$ .

Given this assumption,  $\rho_\epsilon = \delta_1 \delta_2 Cov(y_1, y_2)$ , and then a test for the significance of  $\rho_\epsilon$  can be used as a test for collusion, since it is then essentially a test for the joint significance of the  $\delta_i$ 's in the system (1). This assumption is the main weakness of the PZ test. The test is not robust to such unobserved heterogeneity that is observed by both the firms and unobserved by the econometrician. Any missing variable that is correlated with the participation decision and affects both bidders (directly or indirectly and in the same or in a different direction) will enter the residuals and thus corrupt the test. Next, I propose a new test that is robust to such missing variables.

## 2.3 The new test

The main contribution of this paper is the following. I propose to test for collusion by estimating the simultaneous equation system (1) fully and basing the collusion test on whether the  $\delta_i$ 's differ significantly

from zero. Tamer (2003) provides an estimation method to this model. The main benefit of this test is that it is robust to unobserved heterogeneity. In the case of territorial allocation, both  $\delta_i$ 's are negative and in the case of phony bidding they are positive. The test hypothesis is now:

No collusion,  $H_0$ :  $\delta_i = 0$  for all  $i = 1, 2$

Collusion,  $H_1$ :  $\delta_i \neq 0$  for some  $i = 1, 2$ .

The main difference between the new test and the PZ test is that I estimate the  $\delta_i$ 's separately from  $\rho_u$  whereas PZ estimate their joint effect. It is also possible to assume any known distribution for the error terms  $u_i$ . More specifically, my test of collusion is a test of whether the  $\delta_i$ 's are non-zero (and of the same sign). I need not make the problematic identification assumptions 3 and 4, but I have to maintain the identification assumptions 1 and 2. The significance of the  $\delta_i$ 's can be calculated with separate t-tests and it should be possible also to use for example the Wald test to test for joint significance.

### 2.3.1 Identification in the new test

The identification rests on the following four assumptions. Assumptions 2-4 are the identification assumptions 1-3 in Tamer (2003, p. 153). The assumption 1 is needed for me to be able to interpret the model, in particular the  $\delta_i$ 's, as a collusive model instead of a competitive model as Tamer (2003) does.

**Identification assumption 1:** as above.

There are two reasons for the firms to avoid each other. The first is the strategic reason that is in play when the firms are competing. The second is the possible collusive agreement. When the firms are colluding, they communicate their entry decision. Thus firm  $i$  knows the exact value of  $y_j$ . However, when the firms are competing and there are private shocks, they can only build expectations on the other firm's participation decision. This allows for separating the competitive and collusive effects of the competitor's participation in the estimation. Identification of this model is based on the assumption that when the firms compete, the shocks are private and therefore  $\Pr(y_j = 1|x_j)\mu_i$  captures the strategic effects more accurately than using the actual participation decision. Firm  $i$  forms an expectation on  $\Pr(y_j = 1|x_j)$  based on its competitor's observed characteristics. Therefore we get  $\Pr(y_j = 1|x_j)\mu_i = d_j\mu_i$ , where  $d_j$  denotes those characteristics of bidder  $j$  that are observed by bidder  $i$ ,  $i \neq j$ . Then the strategic element is



controlled for by simply including the characteristics of the analyzed competitor in  $x$ . The characteristics of other competitors are thought of as contract characteristics and are therefore in the  $x$  vector as well. Now  $y_j\delta_i$  consists only of the collusive effect. Because colluding firms do not compete against each other, we have  $\delta_i < 0$  (territorial allocation) or  $\delta_i > 0$  (phony bidding) and  $\mu_i = 0$  when the firms collude. When the firms are competing,  $\delta_i = 0$  and  $\mu_i < 0$ . If the firms collude only in some auctions and compete in others, both the effects could be negative but are still correctly identified since the collusive effect is estimated based on the actual participation and the competitive effect on the expected participation. Instead of  $d_j$ , one could include more refined estimates of  $\Pr(y_j = 1|x_j)$  in the equation. One could utilize for example the two stage approach proposed by Bajari et al. (2007a). Because an improvement in this dimension is not the objective of this paper, I maintain this linear identification assumption that PZ make for the sake of simplicity.

For us to be able to control for the strategic behavior in the way explained above, firms must get private shocks to the profitability of entry when competing. With incomplete information, the bidders form beliefs about other bidders' participation decisions. Given these beliefs, they have a unique response in a two-bidder game, given their own shock. These beliefs can be controlled for by including the competitors' characteristics in the  $x$ . With complete information  $u_1$  and  $u_2$  are common knowledge to the firms but unobserved by the econometrician. With incomplete information  $u_i$  is private information and thus observed only by firm  $i$ . This assumption makes a difference to the econometric analysis. There is currently no method for determining which information structure the data generating process follows. Fewer bidders and more experience in the market make complete information more plausible. The incomplete information assumption that these tests need to make seems to be natural in most auction settings when firms are competing. Gibbons (1992) for example, uses auctions as an example of incomplete information games in his influential textbook on game theory. This assumption was made for example in the entry analysis conducted by Seim (2006). However, when firms are colluding it seems plausible that they communicate these private shocks to each other, thus making the environment that of complete information.

**Identification assumption 2:** as above.

**Identification assumption 3\*:** Let  $U = (u_1, u_2)$  be a random vector independent of  $x$  with a known joint conditional distribution  $F_u$  that is absolutely continuous with mean 0 and unknown covariance matrix  $\Omega$ .

Note that since there are no restrictions on  $\Omega$ , this assumption implies that the correlation of the error terms is allowed. Therefore, the  $y$ 's are allowed to be correlated with the error terms. Only the independence of  $x$  from  $u$ 's is needed. Moreover, Tamer (2003, page 154) discusses that even the assumptions on the independence of  $x$ 's from  $u$ 's and that the distribution  $F_u$  is known can be relaxed.

When I assume that  $F_u$  is bivariate normal, that is  $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \sim IIDN \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_u \\ \rho_u & 1 \end{bmatrix} \right)$ , model (1) nests also the PZ testing approach, because when  $\delta_i = 0$ ,  $u_i = \epsilon_i$ . This shock structure allows for common shocks, i.e. missing variables that are observed by both the firms but not by the econometrician. These enter into the common component of the error term  $\rho_u$ . When estimating all the parameters simultaneously,  $\rho_u$  captures the effects of all the missing variables that affect the entry decisions of both bidders (directly or indirectly through the other bidder and in the same or in a different direction). Therefore the  $\delta_i$ 's are estimated correctly even in the presence of unobserved heterogeneity. For example there could be differences in production technologies that make it more costly for bidder 1 to provide the service to certain schools, but the characteristics of these particular schools do not hinder bidder 2. For example firm 1 has larger vehicles and thus trouble getting them through the gates of certain schools. This is unobserved by the econometrician but observed by bidder 2. Then bidder 1 would usually avoid those schools and therefore bidder 2 would bid more often to those schools. This will make  $\rho_u$  negative. The PZ test would be corrupted but my test would not. Variables that are observed by one bidder and unobserved by the other bidders and the econometrician enter in the private part of the error term. Both the old and the new test are robust to such missing variables. Therefore the new test is robust all such missing variables that would corrupt the PZ test.

**Identification assumption 4\*:**  $\delta_1 \times \delta_2 > 0$ .

Note that this assumption allows both the territorial allocation case (both  $\delta$ 's negative) and the phony bidding case (both  $\delta$ 's positive).

**Identification assumption 5:** There is one *unique* continuous regressor in either the  $x_1$  or  $x_2$ .

Tamer (2003) makes this identification assumption in his Theorem 1 (page 153). This means that an exclusion restriction is required. Typically the firm characteristics provide these. These assumptions joint with his comments on Theorem 1 on page 154 lead to the following conclusions: Assuming  $x$ 's have a full rank, the parameter vector  $(\beta_1(\text{including constant}), \beta_2(\text{including constant}), \delta_1, \delta_2)$  is identified. Moreover,

if  $x$ 's have rich enough support, all the parameters in the covariance matrix  $\Omega$  should be point identified. Tamer (2003) argues that since this is a threshold crossing model, it is possible to normalize the variances in  $\Omega$  to one. Then the identification of  $\Omega$  boils down to identification of  $\rho_u$ .

Because my test relaxes two of the identification assumptions needed for the PZ test, it can be thought of as a nested structure. However, the PZ test does not require an exclusion restriction. Therefore my test nests the PZ test only in some data sets. Since Tamer (2003) shows that the separate effects of the  $\beta$ 's and the  $\delta$ 's (and in some cases the  $\rho_u$ ) are identified from this simultaneous equations model (1), there is no endogeneity problem in this testing approach even though by construction the  $y$ 's are correlated with the  $u$ 's whenever  $\rho_u \neq 0$ .

### 2.3.2 Estimation in the new test

Model (1) is exactly the one discussed in Tamer (2003). He assumes that the error terms are common knowledge for the firms but unobserved by the econometrician. This assumption allows him to interpret the actual participation decision as a relevant measure of the competitive effect. He also assumes that the firms play only pure strategies. According to Tamer (2003), this game always has multiple equilibria for large enough supports of the error terms. This needs to be taken into account in the estimation. There are some recent studies on structural analysis of entry games in auctions. They have different approaches to addressing the problem of multiplicity of equilibria. For Bajari and Hortacsu (2003) multiple equilibria is not a concern as they only consider the number of bids, not the identity of the entrant. Athey et al. (2004) abstract away from the multiple equilibria problem by arguing that "as is often the case with entry models, there may be many equilibria, as a result, our results compare sets of equilibria across auction methods". Krasnokutskaya and Seim (2006) verify the uniqueness of the equilibrium entry probabilities numerically. Li and Zheng (2006) take the fully structural approach to estimate a model that allows for endogenous entry, an uncertain number of actual bidders, unobserved heterogeneity and mixed strategy entry equilibrium under the independent private values paradigm. Their model requires observations where  $n \geq 2$ . Most of the auctions in my application data have only 1 bidder. Also Li (2005) allows for mixed strategies. Both of these papers assume symmetric bidders. Bajari et al. (2007b) use simulations to calculate all the equilibria. One common econometric goal of all these structural auction papers is to estimate the distribution of bidder's private costs and the distribution of entry costs. They do not

provide any methods to test for collusion, which is the aim of this study. I follow the existing literature on detecting collusion in auctions and use reduced form methods. It is much easier to answer my questions in a reduced form. This is an unrealistic but hopefully innocuous assumption.

Various methods to conduct this estimation have been proposed in the literature. For example, Greene (1998) states that this kind of model can be estimated with a bivariate probit model without having to pay any heed to the simultaneity problem. It seems that Greene (1998) is implicitly assuming a unique equilibrium, as he states: "in the bivariate probit model, unlike in the linear simultaneous equations model, if the two dependent variables are jointly determined, we just put each other on the right-hand side of the other equation and proceed as if there were no simultaneity problem". I do not use this method in the application because according to a Monte Carlo analysis, it always overestimated the negative effect of  $\delta_i$ 's, thus making the empirical size 100% in every model in Table 1. I used STATA command "biprobit" to estimate a seemingly unrelated simultaneous probit equation. It also had severe convergence problems.

In territorial allocation setting it is natural to assume that  $\delta_i$ 's are both negative. Then, according to Tamer (2003), it is easy to see that  $Pr[(0,0|x)] + Pr[(0,1|x)] + Pr[(1,0|x)] + Pr[(1,1|x)] > 1$ . This is an example of an incoherent model. Tamer (2003) argues that although the system (1) is an example of an incoherent model, with some restrictions on the parameters it becomes a coherent model<sup>1</sup>. It remains an incomplete model but Tamer (2003) then shows that this incompleteness will not present any problems for the identification. Typically, econometricians have imposed a coherency condition  $\delta_1 * \delta_2 = 0$ . This condition changes the model into a recursive one and thus eliminates the simultaneity. Bresnahan and Reiss (1990 and 1991, denoted BR) and Berry (1992) transform the model into one that predicts the joint outcome [(0,1) or (1,0)]. This provides consistent point estimates for the parameters of interest but involves loss of information. Bjorn and Vuong (1985) and Kooreman (1994) assume that unique outcome is chosen with known probability in the region of incompleteness. According to Tamer (2003) this may lead to inconsistent estimates. Toivanen and Waterson (2005) eliminate the possibility of multiple equilibria by assuming that the entry game proceeds as Stackelberg competition. Seim (2006) uses simulations to show that her model has a unique equilibrium.

BR provides one estimation method that is useful in my case. Instead of using an ordered probit as

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<sup>1</sup>A structural model is complete and coherent if for any value of regressors there exists a unique value for the responses. Tamer (2003) uses coherency to refer to an existence of a solution to the model and calling the model complete if the solution is unique. An analog in games is the existence of Nash equilibria and whether it is unique.

BR do, I use the simultaneous equation formulation of their idea provided by Tamer (2003). Tamer (2003) also proposes a new and more efficient estimator. It is however computationally more challenging and conducting Monte Carlo analysis using it would take a lot of time. Moreover, it uses multidimensional kernel smoothing that requires more data points than I have available in the application. For these two reasons, that estimation method is not used here.

The maximum likelihood estimator presented by Tamer (2003) that uses the BR idea, is defined by a following log-likelihood

$$(3) \quad L_{ML}(b) = \sum_{t=1}^T \left[ \begin{aligned} & y_{t1}y_{t2} \log(P_1(x_t, b)) + (1 - y_{t1})(1 - y_{t2}) \log(P_2(x_t, b)) \\ & + ((1 - y_{t1})y_{t2} + y_{t1}(1 - y_{t2})) \log(1 - P_1(x_t, b) - P_2(x_t, b)) \end{aligned} \right],$$

where  $P_1(x_t, b) = \Pr[(y_{t1} = 1, y_{t2} = 1)|x] = \Pr(u_{t1} \geq -x_{t1}\beta_1 - \delta_1; u_{t2} \geq -x_{t2}\beta_2 - \delta_2)$  and  $P_2(x_t, b) = \Pr[(y_{t1} = 0, y_{t2} = 0)|x] = \Pr(u_{t1} < -x_{t1}\beta_1; u_{t2} < -x_{t2}\beta_2)$

There are  $t = 1, \dots, T$  auctions in the data.  $y_{t1}$  gains value one if bidder 1 submitted a bid in auction  $t$ , otherwise it is zero. Assuming that  $u_1$  are distributed bivariate normal,  $P$ 's are known functions and (3) can be maximized with standard numerical optimization methods.

The benefit of this test is that it is robust with respect to missing variables, but that comes with a cost. This test requires more observations than the PZ test. It also requires that there are some auctions in the data where both bidders have submitted a bid. The need to rely on numerical optimization methods means that it is also harder to implement, may have convergence problems and is computationally time consuming.

### 3 Monte Carlo analysis

I conduct Monte Carlo analysis to compare the finite sample properties of the new test that I propose (called BR) with the existing PZ test. This is done by comparing the empirical power and size of these different tests. The Monte Carlo model is chosen to reflect the characteristics of the actual application. Variable  $x$  can be thought of as a contract characteristic, like contract size and variables  $z$ 's as the bidder characteristics, like distance. Following the application, I discretize the  $z$  variables. I assume that there are three different firms in the markets, of which the collusion test is conducted only for the firms 1 and

2. The BR estimation is based on the model (4) and is estimated with the equation (3). The PZ model is estimated with single equation probits omitting  $y_j\delta_i$  from the model (4).

$$\begin{aligned}
y_1^* &= \beta_{10} + x\beta_{11} + z_1\beta_{12} + \min(z_2, z_3)\beta_{13} + y_2\delta_1 + u_1 \\
(4) \quad y_2^* &= \beta_{20} + x\beta_{21} + z_2\beta_{22} + \min(z_1, z_3)\beta_{23} + y_1\delta_2 + u_2 \\
(u_1, u_2) &\sim IIDN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right).
\end{aligned}$$

I use eight different specifications in the data generating process. They are summarized in Table 1. To address the multiplicity of equilibria, the data generation is conducted in a following way. First I generate  $x$  from a uniform (0,1) distribution. The three  $z$ 's are discrete variables that gain values 0,1,2,3,4,5 or 6 each with equal probability. Then I calculate the following cell probabilities for  $y$ 's. Let  $\beta_{10} + x\beta_{11} + z_1\beta_{12} + \min(z_2, z_3)\beta_{13} = x_1\beta_1$  and  $\beta_{20} + x\beta_{21} + z_2\beta_{22} + \min(z_1, z_3)\beta_{23} = x_2\beta_2$ . Then, I assign the multiple region (see Tamer (2003)) an equal change of being a bid by either bidder. This is done by reducing  $[(P_{00} + P_{01} + P_{10} + P_{11}) - 1]/2$  from both  $P_{01}$  and  $P_{10}$ . Then I use these new cell probabilities to randomly assign simultaneously values for the pair  $(y_1, y_2)$ .

$$\begin{array}{ll}
y_2 = 0 & y_2 = 1 \\
y_1 = 0 \quad P_{00} = BVN(-x_1\beta_1, -x_2\beta_2, \rho) & P_{01} = BVN(-x_1\beta_1 - \delta_1, x_2\beta_2, \rho) \\
y_1 = 1 \quad P_{10} = BVN(x_1\beta_1, -x_2\beta_2 - \delta_2, \rho) & P_{11} = BVN(x_1\beta_1 + \delta_1, x_2\beta_2 + \delta_2, \rho)
\end{array}$$

Table 1. Different model specifications used in the Monte Carlo analysis

Model	$\beta_{10}, \beta_{20}$	$\beta_{11}, \beta_{21}$	$\beta_{12}, \beta_{22}$	$\beta_{13}, \beta_{23}$	$\delta_1, \delta_2$	$\rho$
1.	0	0.3	-0.3	0.2	0	0
2.	0	0.3	-0.3	0	-0.3	0
3.	0	0.3	-0.3	0	-0.45	0
4.	0	0.3	-0.3	0	-0.6	0
5.	0	0.3	-0.3	0.2	0	-0.5
6.	0	0.3	-0.3	0	-0.45	-0.5
7.	0	0.3	-0.3	0.2	0	0.5
8.	0	0.3	-0.3	0	-0.45	0.5

I have run numerous different Monte Carlo analyses over the course of the last two years for these models. An example of these is presented in Table 2. The PZ results were obtained using a STATA "simulate" routine. For models with no unobserved heterogeneity (1.-4.), the PZ test has much better

power. It detects guilty firms more often. For models with unobserved heterogeneity (5.-8.), which is generated with  $\rho$ , PZ fails utterly as expected. Either having huge amount of type I errors (model 5.) when there is such unobserved heterogeneity that makes the firms avoid each other (negative  $\rho$ ), or very low power (model 8.) when unobserved heterogeneity makes the firms bid to the same contracts (positive  $\rho$ ). The BR results were obtained using the "optim" command in R. BR estimation does roughly as well with unobserved heterogeneity as without it. The only concern with BR is the low power results. This concern is alleviated by the fact that the BR method produces on average accurate estimates for all the parameters. Because  $\rho$  is estimated accurately on average, BR can be used jointly with PZ as a way to evaluate whether any variables are missing. Therefore, according to this Monte Carlo analysis, in cases with unobserved heterogeneity, the BR method works much better than PZ and in the case with no unobserved heterogeneity is complementary to PZ. However, the results in the Table 2. are misleading because of the following two reasons. First, the results for the BR test were obtained by giving the Nelder-Mead algorithm the correct starting value vector from Table 1. The results are worse when a vector of zeroes is used as starting values. Second, in Table 2, I used the default number of iterations for the Nelder-Mead in R (500). Typically, convergence takes a few thousand iterations. If allowed to converge, the results worsen. Other standard algorithms do even worse than Nelder-Mead. The main conclusion therefore is that, in small and moderate samples, maximizing (3) is a difficult optimization problem to solve. First, depending on the algorithm used, it converges very slowly or not at all. Second, it has numerous local maxima since starting the optimization routine from different starting values leads to different results. Third, it is difficult to identify the  $\delta_i$ 's and the  $\rho$  separately if the equation (3) is used as such. However, it is possible to solve all these problems.

Table 2. Power and Size results for the Monte Carlo comparison of the tests with 9,75 % significance level.

Model	Obs	PZ, power	PZ, size	BR, power	BR, size	BR, $\rho$
1.	250		4.5 %		7.3 %	0.04 ; 6 %
2.	250	49.9 %		24.6 %		0.05 ; 7 %
3.	250	74.9 %		30.3 %		0.06 ; 6 %
4.	250	88.6 %		36.9 %		0.04 ; 6 %
5.	250		97.8 %		8.6 %	-0.46 ; 29 %
6.	250	98.7 %		26.9 %		-0.53 ; 35 %
7.	250		0.1 %		4.5 %	0.52 ; 37 %
8.	250	8.1 %		30.7 %		0.55 ; 38 %

In the last column titled "BR,  $\rho$ ", the first number is the mean of the estimated  $\rho$ 's and the second number is the share of these estimates that are significantly different from zero. In both the tests the actual size is 9,75 % since the BR test is based on either of the  $\delta_i$ 's being significant at 5 % level. The PZ test is adjusted accordingly. I do not report the  $\rho$  for the PZ test, because it is a different  $\rho$  than in the BR test and thus comparing the two is not relevant. 1000 repetitions are used in this table.

The problem of identification is solved by imposing the identification assumption 4 ( $\delta_1 \times \delta_2 > 0$ ) on the likelihood function. This nonlinear constraint is imposed easily by using a penalty function. I use a distance based penalty function ( $constant \times \delta_1 \times \delta_2$ ) because that is well suited to be used with evolutionary algorithm methods. The problem of multiple local maxima is solved by using the evolutionary algorithm methods, in particular the "rgenoud" package in R (Mebane and Sekhon 2009). Since this algorithm chooses the starting population randomly, the researcher does not have to be concerned with the problem of what starting values to choose. It is enough to choose a large enough starting population. When you let the algorithm run long enough, it is probable that it finds the slope of the global maximum. Unfortunately, evolutionary algorithms are notoriously slow at hill climbing. Therefore I use a Nelder-Mead optimization routine on the best parameter vector after 10 generations. Nelder-Mead seems to work better than the BFGS which is an option in the rgeound package in terms of power, size and convergence. It is very slow in this particular problem but always converges given time. This approach is very time consuming. Running Monte Carlo with 250 observations takes about 1 hour to converge on 3 Ghz processor and the computer time is linear in observations. Even when ran on many processors simultaneously, it takes quite a while. Therefore, the Monte Carlo analysis is still under construct. The results obtained thus far are presented in Table 3. To be able to compare the results with



Table 2, I use the 9,75 % significance level. In addition to separate, both one-sided (first number) and two-sided (second number), t-tests, I use a Wald tests to test for the joint significance of the  $\delta_i$ 's (third number).

Table 3. Power and Size results for the Monte Carlo analysis of the BR test applying rgenoud with 9,75 % significance level.

Model	Obs	BR, power	BR, size	BR, $\rho$	rep
1.	250		15 %; 8 %; 6 %	0.09(0.60)	100
2.	250	26 %; 0%; 0 %		0.10(0.60)	100
3.	250	0 %; 0 %; 0%		-0.10(0.57)	100
4.	250	43 %; 0 %; 7 %		-0.05(0.57)	100
5.	250		27 %; 16 %; 14 %	-0.24(0.48)	159
6.	250	57%; 36 %; 31 %		-0.56(0.39)	154
7.	250		7 %; 20 %; 7 %	0.42(0.48)	157
<b>7.</b>	<b>1000</b>		<b>6 %; 12 %; 10 %</b>	<b>0.45(0.27)</b>	<b>139</b>
8.	250	0 %; 0 %; 0%		0.42(0.63)	155
<b>8.</b>	<b>1000</b>	<b>56 %; 45% ; 41 %</b>		<b>0.49(0.31)</b>	<b>141</b>

The preliminary results in Table 3 show that the new test seems to work very well if there are 1000 observations in the data. Unfortunately, with only 250 observations the proposed optimization method does not work. A closer look on the Monte Carlo results reveals that the problems in the simulations with only 250 observations arise due to the corner solutions with respect to  $\rho$  ( $\rho$  either 1 or -1). These corner solutions are not present with 1000 observations. Removing the outlying observations in terms of rho improves the performance of these tests with 250 observations to acceptable levels. I am currently looking for yet another improvement on the optimization approach that would allow it to be used with data as small as 250 observations. One possibility is to use only the evolutionary algorithm and let it run for a longer period. Unfortunately, looking into that with Monte Carlo analysis is probably not feasible. A second option is to run an evolutionary algorithm for a shorter period but many times for the same set of observations and then use the means of the estimated parameter vectors as the final estimate. Again this is costly in terms of computer time. I am currently trying to get some evidence on how these approaches would work in practice. A third possibility is to use the proposed algorithm but discard the result if  $\rho$  is either 1 or -1.

## 4 School yard snow removal market

I will apply both the PZ test and the new test to detect possible collusion in the school yard snow removal markets in the City of Helsinki. This particular market is interesting because it allows the analysis of territorial allocation. This is the first empirical application that tries to detect that form of collusion. This is surprising because territorial allocation is fairly typical form of collusion. For example of all the reported Finnish cartels from a period 1959 - 1990, when cartels were legal, 7.2 % of the cartels were of this form. On the other hand, the cases where bidders use the territorial allocation scheme with overlapping territories may be limited. In this Section, I explain the rules of the auction in question and give a general description of the market. Harrington (2005) states that "it has been shown that cartel formation is more likely with fewer firms, more homogenous products and more stable demand". In this Section, I also show that these are all true for the market under scrutiny.

In Figure 1, I present the spatial participation pattern. It marks on the city map the schools that each bidder has submitted bids on in year the 2003. Also the location of the bidders' and city's garages is marked on the map (bold and larger letters). Most firms seem to participate more actively near their garages than further away. The map shows the bidders A and K seem to avoid each other. Moreover they systematically avoid each other in an overlapping geographic area, as we can see at the lower left corner. Of the other bidders, bidder R submits bids to all but two contracts and three small bidders, T, S and P, only a few bids. This map raises suspicions about collusive territorial allocation. Maps for years 2004 and 2005 are in Appendix 1. It is interesting to note that in the year 2005 the participation pattern no longer implies collusive behavior.

Figure 1. Bidder participation in school yard snow removal auctions in Helsinki 2003.



Small capital letters present the location of schools and which bidders (A,K,P,R,S,T) have bid to a given school. "-" means that there were no bids. The approximate location of bidders' and city's (C) garages are marked with larger and bold capital letters.

Starting from the autumn 2003, the City of Helsinki has auctioned the snow removal services for school yards. All the contracts are auctioned simultaneously. The bidders submit single sealed bids for four different type of services for each school. First service type consists of snow ploughing and sanding.

Second service is the transportation of snow from the school to the snow dump. Third service is the transportation of sand from the school to the snow dump and the fourth is washing the yard. The last two services are needed only once every spring. Different services can be allocated to different firms within the same school. The lowest bid wins the given contract and the winner is paid their unit bid times the respective size of the contract. After the auction, all the bids are public knowledge. Thus all bidders detect deviators from collusive agreements easily. The bids are in unit costs. In the first service for example, winner's bid is in euro per square meter. That times the school yard size in square meters is the payment per ploughing. Snow has to be ploughed every time there is 5 cm of snow on the ground. Typically, bidders submitted bids to all of the services, but there are exceptions. For example one firm participated only in the snow transportation service and they bid for all the schools. I consider only the bidding to the first service type because it is the most important in monetary terms. For the purpose of this study, the chosen service type does not matter. The amount of schools contracted differs from year to year. In the year 2003, there were 153 schools, in 2004 37 schools and in 2005 65 schools. This number varied according to how much of the services the City wanted to provide itself. I restrict the discussion in this Section mainly to the year 2003 because that is the year that I suspect that the collusion took place.

It states in the invitation to tender that "the buyer reserves the rights to transfer some of the contracts to be serviced by the city itself". This means that city announces that it has set a secret reserve price for the contract. The secret reserve price means that the city does not accept bids that are too high. In this case too high means a bid higher than the costs that the City would incur by providing the service itself. It seems that this secret reserve price is binding for many firms in most auctions. In 2003, a total of six bidders participated in the snow ploughing and sanding services. Of 153 contracts there were 2 with zero bidders, 85 with one bidder, 60 with two bidders, 5 with three bidders and 1 with four bidders. If the secret reserve price was not binding, we would expect all the potential bidders that are not capacity constrained to submit a bid in all the auctions. Also entry costs could limit the participation. However, there is little reason to suspect that bid preparation includes large costs to the firms in these markets because the bidding process is very simple and they have previous experience from providing the service under contract. An industry expert explained that it would take him about two minutes to calculate a bid in this sort of market because costs are very well known. The actual number of submitted bids can still change due to capacity constraints and different number of potential bidders in different areas of the city. PZ observe a similar distribution of actual bidders on their data set of school milk bidding. They suggest

that small number of actual bidders indicates that "there may not be significant firm-specific information in the markets. If bidders knew their costs as well as the costs of the other potential suppliers, then under a set of standard assumptions either one or two bids would be observed. The low cost supplier would submit a bid just below the cost of the next-lowest-cost supplier, and the next-lowest supplier would be indifferent between bidding at its own cost and not bidding." In contrast to the markets analyzed in PZ, there may be more uncertainty about the costs of other bidders evident in this market. The bidders use somewhat different equipment, they have different main activities and possibly efficiency differences. It is also implausible that the asymmetries among bidders would be so large that it is common knowledge which will be the cost ordering of the bidders in all the auctions. I think that the explanation of a binding secret reserve price possibly jointly with territorial allocation is more plausible. Also capacity constraints could limit the bidding of especially the small bidders.

Snow removal is typically a secondary activity for the firms. The main activities of three larger participants are construction, paving, delivery services and landscaping. Three smaller firms do real estate maintenance as their main activity. The common feature for all these firms is that they use the snow removal equipment for these main activities outside the winter period. Flambard and Perrigne (2006) argue in their study of snow removal contracts in Montreal, that because snow removal is a secondary activity to supplement income, capacity constraints do not seem to be a major issue in their auctions. For the Helsinki auctions, this is probably true for the larger companies. On the other hand, the smaller companies are typically one man firms with very limited amount of equipment. Three smaller firms only submit from three to six bids to schools located near their office. Another reason to suspect that the large firms are not constrained by capacity is the fact that they have subcontracting deals with each other. Thus they have access to additional capacity beyond their own. These firms also participate in other snow removal auctions that the City holds. The secondary nature of the activity also acts as an entry barrier. No seller can enter just the snow removal activity alone. The required equipment is too expensive in relation to the industry's part-year nature for it to be profitable. On the other hand there are numerous construction firms in the area that already have the necessary equipment.

Flambard and Perrigne (2006) investigate the potential asymmetry among bidders. They find empirical evidence of asymmetry resulting from firm location, because in the urbanized part of the city the storage costs are prohibitive. Their assessments of most of the market conditions hold also for the snow removal contracts here. The only difference being that in they study streets and I study schools. They argue that because of the equipment size and weather conditions, firms located far from the snow

removal location will have to rent storage space for their equipment. This additional cost can induce some asymmetry among firms. They further argue that this asymmetry may prevent the least efficient firms from participating to the auction as their bids will not be competitive.

Markets can be described by the nature of demand, the nature of the production process and the nature of competitive interaction among bidders. Demand for snow removal services is very inelastic, because the weather is not affected by the price. Neither do the conditions stated in the invitation to the tender about when the service should be provided depend on prices. This property makes collusion more profitable because the increase in prices due to collusion does not reduce demand. The product is homogenous. There can be very little quality differences in snow removal. It is either removed or not. On the other hand, the existence of the secret reserve price makes the demand elastic. If cartel bids too high, the contract may not be awarded to anyone. Thus reservation prices reduce the incentives to collude. The production processes can be different due to differences in snow removal equipment.

Besides the fairly inelastic demand, there are other characteristics in this market that may facilitate collusion. First, firms compete only on prices, which simplifies the cartel operations. Thus the cartel needs only to coordinate the participation or the level of bids. Second is that publicly announcing all the bids and the bidder identities make it easier for the cartel to detect deviation. Markets are easily defined, allowing the assignment of territories. The set of participating firms is small and there are entry barriers making it possible to submit higher cartel bids. Subcontracting is typical in this market. This provides an easy way to distribute the cartel rents and also facilitates direct communication and a pretext for the meetings of the cartel. The representative of the buyer (City service center PALMIA) thinks it is plausible that some of the firms could be colluding. However, there is no legal outside evidence. On the other hand, the simultaneous nature of these auctions makes it more difficult to sustain collusion. Bidders can punish from deviation only in the next year auction. However if the bidders meet in the other markets that they are active on, for example construction, they can possibly punish there. Also subcontracting deals allow a way to punish deviators.

As can be seen from the participation maps (Figures 1-3) the behavior of bidder A changes over time. In the year 2005 it bids to seven same schools as bidder K whereas in the year 2003 they never bid to the same school. K generally bids to the same schools in 2005 as in 2003. Therefore, with respect to equipment and location, it would probably have been possible for A to compete with K also in the year 2003, because I am not aware of any technology or location changes for A. This is further evidence for collusive behavior in 2003.

Job rotation is a similar phenomenon to territorial allocation. In a sequential auction setting job rotation can exist either as a result of collusion or as a result of an efficient outcome of a competitive bidding process when capacity constraints or decreasing returns to scale matter (Hendricks and Porter 1989). This makes the detection of collusion more difficult in sequential setting. In contrast to sequential auctions where the winners of previous auctions are observed, in simultaneous auctions the bidders do not observe how much capacity is already committed when making the decisions of participating in a given auction. Thus there is no backlog. In a simultaneous setting, capacity constraints or decreasing returns to scale affect only the total number of auctions that seller participates in. If there is enough uncertainty about other bidders' costs, competitive bidding should not result in the case where certain bidders systematically avoid each other. Assuming that bidders don't know to which homogenous auctions the competitors are going to bid, we can think that firms randomly submit bids to contracts up to their capacity. Then it is highly unlikely that some firms manage to systematically avoid each other when there are many contracts. Also in sequential auctions, the bidders may signal their preferences to other bidders more easily than in simultaneous auctions. Territorial allocation can be a result of competitive behavior when there are large observable cost differences among bidders. Still if these differences are controlled for we should not observe that identity in itself matters in a competitive setting.

The important players are probably not capacity constrained. If they were, they would have more incentives to avoid bidding to the same contracts. In a simultaneous game it is not possible to exactly know where the others are going to bid. Therefore we should observe that bidders anyway sometime bid to the same contracts if there is no way of communicating to each other what actions firms are going to take. Explicit communication is explicit collusion. If the game is played repeatedly, bidders can perhaps infer each others' future actions from past decisions. If incumbency for example explains a lot of participation decision, the collusion could as well be tacit. Existence of capacity constraints does not make this testing approach invalid but it can change the interpretation on the type of possible collusion. Unfortunately the information on contracts in 2002 was not available and thus the effect of incumbency cannot be checked.

## 5 Data and modeling choices

There are 258 auctions in the data with 335 bids submitted. 28 auctions did not receive any bids, of which 19 were held in 2004. Nine bidders participated in these auctions. Six in 2003, three in 2004 and six in 2005. Three firms exited the market after 2003 and three new entered in 2005. The participation decisions of bidders are described in Table 4 along with the bid levels. It shows the number of bids submitted, the number of contracts won, the number of contracts won conditional on facing any competition and bid level information for each bidder. It also shows to which city areas a given bidder submitted bids and in which years the bidder submitted any bids. Only three players submitted bids every year. By looking at the map we notice that only A and K avoid each other in the same city area. Therefore I conduct the tests only for the bidders A and K. Moreover, bidder R submitted too many bids in the year it participated and bidders T, S, H and O too few bids to be of any use in analysis of discrete choice models. There is too little variation in their decisions to use the tests for them.

Table 4. Descriptive statistics for the bidders in years 2003-2005. Participation and bids.

	<b>A</b>	<b>K</b>	<b>R</b>	<b>T</b>	<b>S</b>	<b>P</b>	<b>H</b>	<b>J</b>	<b>O</b>
<b># of bids</b>	42	98	151	3	3	16	1	19	2
<b># wins</b>	33	89	97	1	1	4	1	2	2
<b># of wins com</b>	16	66	12	1	1	1	0	0	2
<b>mean bid</b>	0,097	0,071	0,089	0,100	0,090	0,107	0,050	0,103	0,097
<b>sd bid</b>	0,005	0,020	0,005	0,017	0,052	0,021	NA	0,022	0,018
<b>min bid</b>	0,082	0,040	0,085	0,080	0,060	0,068	0,050	0,071	0,084
<b>max bid</b>	0,110	0,156	0,980	0,110	0,150	0,135	0,050	0,140	0,110
<b>South(centre)</b>	Yes	Yes	Yes	No	No	No	Yes	No	No
<b>Northwest</b>	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No
<b>North</b>	No	Yes	Yes	No	No	No	No	Yes	Yes
<b>East</b>	No	Yes	Yes	No	Yes	No	No	No	No
<b>Year 03</b>	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No
<b>Year 04</b>	Yes	Yes	No	No	No	Yes	No	No	No
<b>Year 05</b>	Yes	Yes	No	No	No	Yes	Yes	Yes	Yes

The data includes information on contract characteristics and bidder characteristics. Contract characteristics include the school yard size and some measures of its shape or "tightness" that is intended to capture how difficult it is to plough the yard. These are the number of walls or fences that surround the yard, the number of permanent obstacles like trees or small buildings in the yard and dummy for whether



the yard includes tight spaces. The shape variables are obtained by looking at the 1:1250 -maps of the school areas. This allows detail to the point of a single tree. Contract characteristics also include the distance to the schools from the city's garages to account for possible changes in the secret reservation price. The information on the bidders is limited. Bidders did not agree to be interviewed. They only answered a few short questions. The most important variable that we are interested in is the location of bidder garages. We could use that to calculate the distances from garages to the contracted schools. I did not receive the accurate addresses of the garages but the firms gave their location information on postal code level. I could assume that the garage is located in the middle of the given postal code area. This creates measurement errors to distance variable. Another and perhaps even more important reason to use something else than just the distance to capture the cost shifters of the firms is that according to an industry expert another important factor is where the bidders' main activities are located at the time. Bidders prefer schools near their construction sites for example. This is not observable. Thus we need to construct proxy variable for firms' cost shifters. I also use four area dummies in the analysis to capture the bidders' strengths in larger area around the school. These area dummies also capture the possible changes in the number of potential bidders.

To proxy all firm specific cost shifters, I construct a variable called "ofsix" - to how many of the six nearest schools of a given school, a given bidder submitted a bid in a given year. This variable captures not only the distance, but also the overall costs of the bidder in the close proximity of a given school. There is a possible endogeneity problem with this proxy. Bidder might have or might not have bid to some of the near schools due to collusion instead of cost conditions. To capture the competitive effect, I calculate the maximum of the "ofsix"-variable among all the given bidder's competitors for each school. I use this variable instead of including the "ofsix" variables of all the competitors, because then the "ofsix" variable works then as the exclusion restriction.

Another source of possible endogeneity is that unlike PZ, I do not have a control group. PZ use parameter estimates gained from control group probit estimations in testing. This is because the participation of collusive firms is affected by collusion thus biasing the estimated effect of observables on their participation. Assuming symmetry the control group estimates can be used as unbiased parameter estimates also for the test group. I do not have any outside evidence nor enough bidders to form a control group. I need to assume that this possible bias in the parameters of the control variables does not bias the estimates of the test statistics. However, I do not need to assume bidder symmetry like PZ.

Figure 2 in Appendix 1 shows the scatter plots of bids in relation to school yard size for each bidder

separately. Bidder A participated in smaller auctions than other bidders. The reason for this could be that they operate only in the center of the city where yards are typically smaller. It can also be because they specialize in smaller yards due to their different equipment. We note that unit bids are decreasing in yard size, implying economies of scale. These seem to be decreasing. Thus I include yard size and its square in the econometric analysis.

## 6 Results

The results of both the tests are presented in Table 5. In the PZ test, I have estimated probit models separately for the bidders A and K. I estimate the model using data for all the three years 2003-2005. It is not possible to conduct the BR test separately for the different years. The activity variable "ofsix" is significant at all standard levels for both the bidders and has the expected positive sign. Bidder A seems to bid close to the city garages, more in the south region of the city and more to difficult yards. This is in line with the fact that they advertise having equipment best suited for difficult yards. Surprisingly, the competition variable, i.e. the maximum of competitors' activity, has a positive and significant sign, which means that bidder A bids more to auctions where tough competition is expected. Bidder K seems to get some returns to scale from yard size. The residuals of these two probit models are negatively correlated. This correlation is significant at 1 % level. This implies that collusion occurred at least during some of the three years in the data. If the years 2003 and 2005 are analyzed separately with the PZ test, there is even stronger evidence of collusion in 2003 and no evidence in 2005. Collusion is also supported by the fact that competitors' characteristics are not important for bidder K and for bidder A the effect of competition is opposite to what it should be under competition. Thus the results of the PZ test suggest collusive behavior. This is assuming that I have not overlooked any important explanatory variable.

The results from estimating the simultaneous equation model (1) with only the Nelder-Mead algorithm are presented also in Table 5. Unfortunately, I have not yet come up with an optimization approach that works well for data with only 258 observations. Using the evolutionary algorithm jointly with the Nelder-Mead led exactly to such a corner solution that seemed to invalidate the Monte Carlo analyses for 250 observations. Calculation took days but in the end it converged to parameter values where  $\rho = 1$  and  $\delta_i$ 's both negative and all of them significant. The Hessian matrix was well behaving and the log likelihood gained value -141. However, based on the Monte Carlo analysis, I trust that result less than

the one I report in Table 5 which has a lower value for the likelihood function. I report the results that were obtained using only the Nelder-Mead algorithm with a large number of different, more or less arbitrary, starting values. Despite not allowing it to converge (typically that results to poorly behaving Hessian), the reported results below have some desired properties. First, with the starting values chosen, the likelihood function reached a higher value at the iteration limit than for any other set of starting values (not counting the one obtained with *rgenoud*). Second, the Hessian matrix allowed for computing standard errors for all the variables. It is clear from the problems in the Monte Carlo analyses and the problems in the numerical optimization in this application (very slow convergence if any, results are not robust to different starting values nor to different algorithms, Hessian is only rarely well behaving, corner solution for the evolutionary method) these results should be treated with utmost caution until I come up with a better optimization approach or more data. However, in many cases the tests seemed to suggest collusion.

The activity variable "ofsix" is significant for both the bidders and has the expected positive sign. Competitors' characteristics are not important. A also bids more to schools with difficult yards. K is significantly more active in 2005. A bids more to south area and seems to bid close to the city garages. The test statistic is negative for both firms and is also significant for  $\delta_K$ . K avoid schools where A actually bids. Therefore, bidders A and K seem to collude according to both the tests. Collusion is also implied by the fact that the strategic elements do not seem to be important for the bidders, because they do not avoid bidding to contracts that their competitors are likely to bid to.

The behavior of firms A and K seems to be more consistent with a collusive than a competitive model. In terms of Harrington (2005), this is a screening result, which means that I have identified this market as suspect to collusion. This can also be thought of as the verification of the cartel because the method by construct identifies the exact model of collusion (territorial allocation). This is not however sufficient for the prosecution of the colluding firms. Screening is useful in fairly quickly analyzing the market to detect those where more attention should be put to find legal evidence.

Table 5. The estimation results

	<b>PZ</b>		<b>BR</b>	
	<b>Bidder A</b>	<b>Bidder K</b>	<b>Bidder A</b>	<b>Bidder K</b>
<b>constant</b>	- *	- ****	-	-
<b>city dist</b>	- ****	+	- ***	-
<b>yard size</b>	-	+ *	-	-
<b>yard size sq</b>	+	-	+	+
<b>walls</b>	+ **	+	+	-
<b>obstacles</b>	+	-	+ **	-
<b>shape</b>	+ **	+	+	+
<b>t03</b>	ref group	ref group	ref group	ref group
<b>t04</b>	+	+	-	+
<b>t05</b>	+	+	+	+ **
<b>OfsixA</b>	+ ****	NA	+ *	NA
<b>Max Ofsix -A</b>	+ **	NA	-	NA
<b>OfsixK</b>	NA	+ ****	NA	+ ***
<b>Max Ofsix -K</b>	NA	-	NA	-
<b>Area S</b>	+ ****	-	+ **	-
<b>Area NW</b>				
<b>Area N</b>	ref group	ref group	ref group	ref group
<b>Area E</b>				
<b>delta</b>	NA	NA	-0.27	<b>-0.76*</b>
<b>Rho</b>	<b>-0.165***</b>		-0.098 (s.e. 0.65)	
<b>Log lik</b>	-24.8	-144.2	-148.9	

A unit of observation is school.  $n = 258$ . "city dist" is the distance from the nearest City garage. "yard size" is the yard size of the school and "yard size sq" its square. "walls" is the number of walls surrounding the yard, "obstacles" the number of obstacles in the yard and "shape" a dummy for yards including tight areas. "t03 - t05" are the year dummies and "Area X" the area dummies. "Ofsixi" is the activity of bidder i. It tells to how many of the six nearest schools of a given school the bidder i has submitted a bid on. "Max Ofsix -i" is the maximum of the Ofsix variable among i's competitors. "delta" denotes the test variable  $\delta$ . "\*" means 10 % significance level, "\*\*\*" means 5 % significance level, "\*\*\*\*" means 1 % significance level and "\*\*\*\*\*" means 0,1 % significance level for two-sided tests (one-sided for delta).

## 7 Conclusions

I have proposed a new test to detect collusion that is based on participation decisions. The test is conducted by estimating two simultaneous discrete choice equations with methods proposed by Tamer (2003). This test can be applied to all environments where independent and mutually exclusive markets can be defined. Auctions are only one potential application environment. The test is best suited for detecting territorial allocation schemes. It can also be used to detect phony bidding but there are also other tests for that purpose. Preliminary Monte Carlo analysis shows that this test has the desired properties. Namely that it is robust to missing variables unlike the existing test with otherwise similar properties by PZ. The PZ test is better in a sense that it requires less from the data, both in terms of quantity and the nature of observations. The PZ test also has better convergence properties and better power. The new test complements it as it can be used to check whether the model has important missing variables that would invalidate the existing test approach.

I apply both these tests to school yard snow removal auctions in the City of Helsinki and find some evidence of collusion. Both the tests suggest collusion. Moreover the analyzed firms do not behave strategically as we would expect in the competitive setting. Two bidders seem to participate in a contract allocation scheme. The collusive regime seems to last only the year 2003. However due to the possible endogeneity problems and problems in the numerical optimization, these results should be treated with caution. Still, this analysis should validate closer legal study to support the prosecution of these two companies.

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## Appendix 1. Participation patterns

Figure 2. Scatter plots of bids and school yard size for each bidder separately for snow ploughing and sanding contract (bid 1 + bid 2) in year 2003.

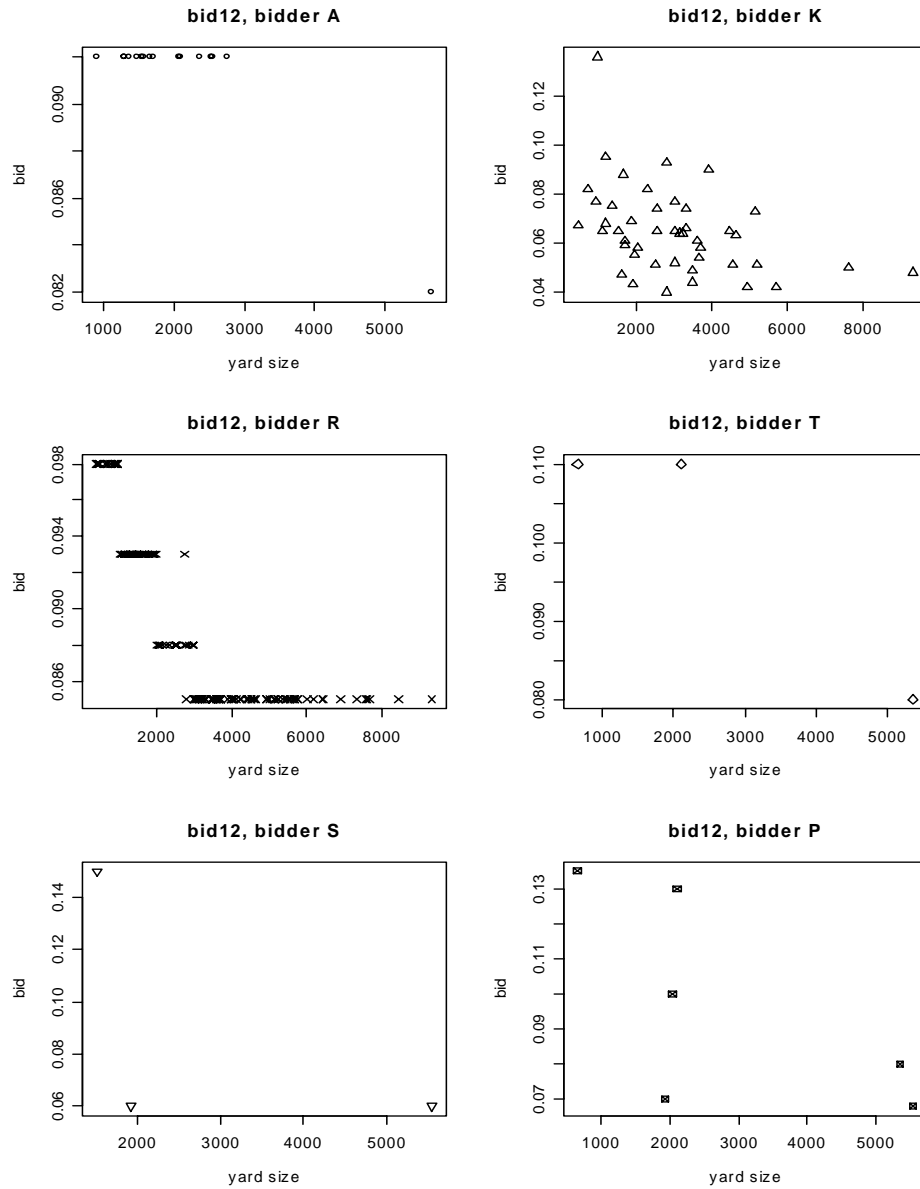




Figure 3. Bidder participation in school yard snow removal auctions in Helsinki 2004.



Small capital letters present the location of schools and which bidders (A, K, P) have bid to a given school. - means that there were no bids. The approximate location of bidders' and city's (C) garages are marked with larger and bold capital letters.

Figure 4. Bidder participation in school yard snow removal auctions in Helsinki 2005.



Small capital letters present the location of schools and which bidders (A, H, J, K, O, P) have bid to a given school. - means that there were no bids. The approximate location of bidders' and city's (C) garages are marked with larger and bold capital letters.