*

Ming Gao[†]

Abstract

1 Introduction

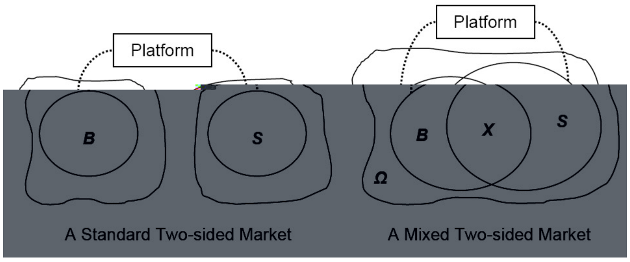
mixed

standard

*

†

mgao.phd2005@london.edu


Platform
B
S
Platform
B
X
S
Ω
A Standard Two-sided Market
A Mixed Two-sided Market

degree

of mixedness

seller-buyers

Bundling with Network Effects

---

ceteris paribus

2 Modeling Set-Up

$[0, 1]$

2.1 The platform

Separate sales

$\mathbf{P}_s \equiv (a_s^S, a_s^B, A^S, A^B) \in \mathbb{R}^{+4}$

A^S A^B a_s^S

a_s^B

Bundling

$\mathbf{P}_m \equiv (a_m^S, a_m^B, A) \in \mathbb{R}^{+3}$ A

a_m^S a_m^B

A

a_m^B a_m^S

F^S F^B

c F

c $\max(F^S, F^B) \leq$

$F \leq F^S + F^B$

A^S A^B A

2.2 The agents

$[0, 1]$ Ω $\omega \in \Omega$
v^S_ω v^B_ω

v^S v^B

Assumption 1 *v^S and v^B are drawn independently from respective cumulative distributions $G^S(\cdot)$ and $G^B(\cdot)$, with respective density functions $g^S(\cdot)$ and $g^B(\cdot)$.*

membership decision

Y

trading decision
seller **buyer**
S
B
seller-buyer **pure seller** **pure buyer**
PS PB
X

$$S = PS \cup X; \quad B = PB \cup X;$$
$$X = S \cap B; \quad Y = S \cup B = PS \cup PB \cup X.$$

Ex post $X \neq \varnothing$

The number of sellers $N^S \equiv \Pr[\omega \in S]$
The number of buyers $N^B \equiv \Pr[\omega \in B]$
The number of seller-buyers $N^X \equiv \Pr[\omega \in X]$
The number of users $N \equiv \Pr[\omega \in Y] = N^S + N^B - N^X$

---

v^S_ω v^B_ω
ω
v^S_ω v^B_ω
v^S v^B
B^S B^B B^S B^B

2.3 The trade

N^S

N^B $N^S \cdot N^B$

a^S a^B

v^S

$$u^S \equiv (v^S - a^S)N^B$$

v^B

$$u^B \equiv (v^B - a^B)N^S$$

$u^S + u^B - A^S - A^B$ $u^S + u^B - A$

2.4 Agents' decisions

- *Any agent's outside option (i.e. not joining the platform) gives zero surplus.*
- *Whenever an agent is indifferent between joining and not joining, she doesn't join.*

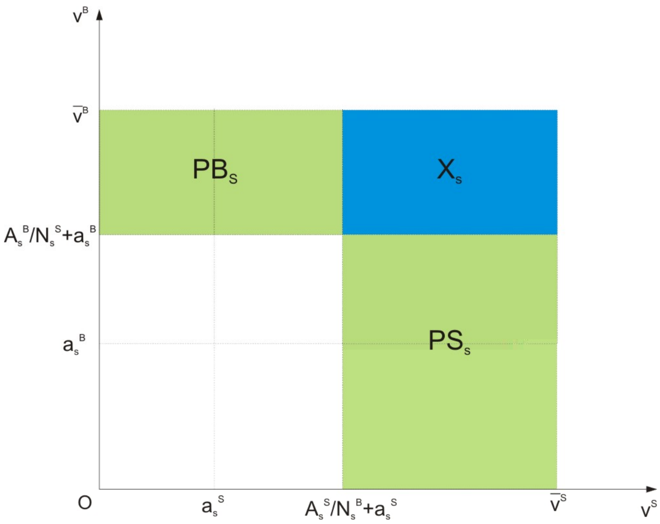
3 Separate Sales

$$\mathbf{P}_s = (a_s^S, a_s^B, A^S, A^B)$$

$\mathbf{P}_s$

$$N_s^S = \Pr[u^S - A^S > 0] = 1 - G^S(\frac{A^S}{N_s^B} + a_s^S)$$
$$N_s^B = \Pr[u^B - A^B > 0] = 1 - G^B(\frac{A^B}{N_s^S} + a_s^B)$$

N_s^S N_s^B $\mathbf{P}_s$ $\mathbf{P}_s$



$\mathbf{P}_s$

$$\Pi_s(\mathbf{P}_s) \equiv \underbrace{(A^S - F^S)N_s^S} + \underbrace{(A^B - F^B)N_s^B} + \underbrace{(a_s^S + a_s^B - c)N_s^S N_s^B}$$

Lemma 1 *(Redundancy of membership fees) For any* $\mathbf{P}_s = (a_s^S, a_s^B, A^S, A^B) \in \mathbb{R}^{+4}$*, there exists a degenerate price vector* $\mathbf{P}_{s0} = (a_s^{S\prime}, a_s^{B\prime}, 0, 0) \in \mathbb{R}^{+4}$*, such that* $\mathbf{P}_{s0}$ *exactly replicates the demands and profit under* $\mathbf{P}_s$*.*

Proof. $\mathbf{P}_s = (a_s^S, a_s^B, A^S, A^B) \in \mathbb{R}^{+4}$ N_s^S N_s^B

---

s

N_s^S N_s^B

v^S $[0, \bar{v}^S]$ v^B $[0, \bar{v}^B]$

$\mathbf{P}_s$

$$a_s^{S\prime} \equiv a_s^S + \frac{A^S}{N_s^B}$$
$$a_s^{B\prime} \equiv a_s^B + \frac{A^B}{N_s^S}$$

$\mathbf{P}_{s0} = (a_s^{S\prime}, a_s^{B\prime}, 0, 0)$ $\mathbb{R}^{+4}$ N_s^S N_s^B $\mathbf{P}_{s0}$ $\Pi_s(\mathbf{P}_s) = \Pi_s(\mathbf{P}_{s0})$ $\blacksquare$

$\mathbf{P}_s$

$\mathbf{P}_{s0}$

$$\mathbf{P}_{s0}^* \equiv (a_s^{S*}, a_s^{B*}, 0, 0)$$
$$\mathbf{P}_{s0}^* \in \arg\max_{\mathbf{P}_s} \Pi_s(\mathbf{P}_s)$$

4 Bundling

$$\mathbf{P}_m = (a_m^S, a_m^B, A)$$

A

4.1 Bundling with Full Agent Rationality

$\mathbf{P}_m$

u^S u^B $u^S + u^B$ $\mathbf{P}_m$ $\mathbf{P}_m$ $\max(u^S, u^B, u^S + u^B)$ A $\mathbf{P}_m$

$$Y_{m1} \equiv \{\omega \mid \max(u^S, u^B, u^S + u^B) > A\}$$

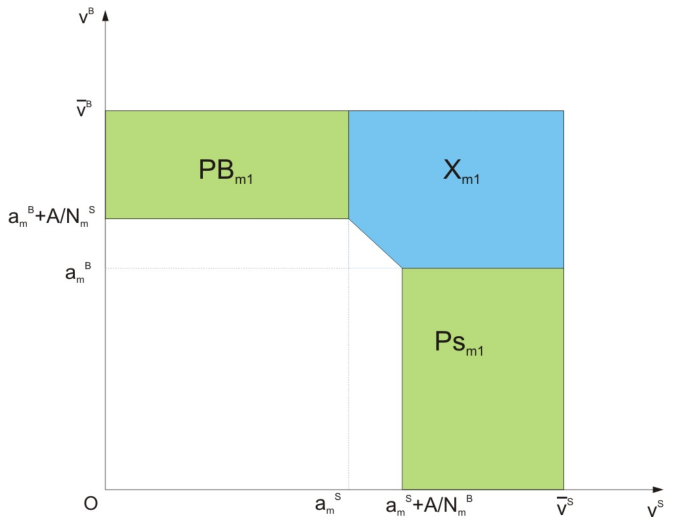
$m1$

$\mathbf{P}_m$

$$\begin{aligned} X_{m1} &\equiv \{\omega \in Y_{m1} \mid \max(u^S, u^B, u^S + u^B) = u^S + u^B\} \\ &= \{\omega \mid u^S > 0, u^B > 0 \qquad u^S + u^B > A\} \end{aligned}$$

$G^S(\cdot)$ $G^B(\cdot)$ a^S_m a^B_m A

$\mathbf{P}_m$



$\mathbf{P}_m = (a^S_m, a^B_m, A)$

$$\begin{aligned} N^S_{m1} &= n^S_{m1}(a^S_m, a^B_m, A) \\ N^B_{m1} &= n^B_{m1}(a^S_m, a^B_m, A) \\ N_{m1} &= n_{m1}(a^S_m, a^B_m, A) \end{aligned}$$

a^S_m a^B_m A

$\mathbf{P}_m$

$$\Pi_{m1}(\mathbf{P}_m) = \underbrace{(A - F)N_{m1}} + \underbrace{(a^S_m + a^B_m - c)N^S_{m1}N^B_{m1}}$$

N^S_{m1} N^B_{m1}

$\mathbf{P}_m$

N^S_{m1} N^B_{m1}

$N^S_{m1}, N^B_{m1} \leq 1$

Lemma 2 *At any variable fees* $(a^S, a^B) \in \mathbb{R}^{+2}$*, the degenerate separate-sales strategy* $\mathbf{P}_{s0} = (a^S, a^B, 0, 0)$ *and the degenerate bundling strategy* $\mathbf{P}_{m0} = (a^S, a^B, 0)$ *produce exactly the same demand in each market segment.*

Proof. ■

a^S a^B $A^S = A^B = A > 0$

$A^S = A^B = \frac{1}{2}A > 0$

α $v_\alpha^S \in (a^S, \frac{A^S}{N^B} + a^S)$ $v_\alpha^B > \frac{A}{N^S} + a^B$
$0 < u_\alpha^S < A^S$ $u_\alpha^B > A > A^B$
$u_\alpha^S + u_\alpha^B > A$

The Bundling Effect

$\Pi_s(\mathbf{P}_{s0}^*)$ $\mathbf{P}_{s0}^* = (a_s^{S*}, a_s^{B*}, 0, 0)$

$\mathbf{P}'_{m0} \equiv (a_s^{S*}, a_s^{B*}, 0)$

$\Pi_{m1}(\mathbf{P}'_{m0})$

N^S N^B

$\mathbf{P'}_{m0}$

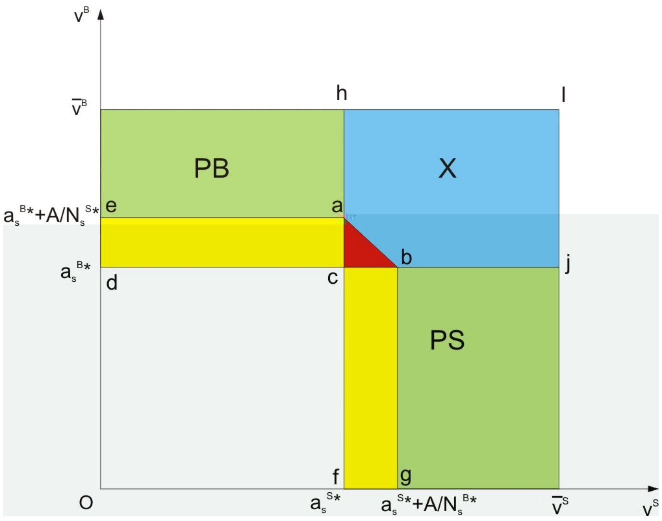
$\Pi_{m1}(\mathbf{P'}_{m0})$

Proposition 1 The Bundling Effect *If separate sales price* $\mathbf{P}^*_{s0} = (a_s^{S*}, a_s^{B*}, 0, 0)$ *is optimal, i.e.* $\mathbf{P}^*_{s0} \in \arg\max_{\mathbf{P}_s} \Pi_s(\mathbf{P}_s)$ *then at bundling price* $\mathbf{P'}_{m0} = (a_s^{S*}, a_s^{B*}, 0)$, *we have* $\frac{\partial \Pi_{m1}}{\partial A}(\mathbf{P'}_{m0}) > 0$. *That is, at* $\mathbf{P'}_{m0}$ *raising the bundled membership fee from zero strictly increases bundling profit.*

Proof. ■

after

$\mathbf{P'}_{m0}$ A



$\mathbf{P'}_{m0} = (a_s^{S*}, a_s^{B*}, A = 0)$

a_s^{S*} a_s^{B*} $jcf\bar{v}^S$

A $bcfg$

$jbg\bar{v}^S$ A $acde$

abc A

N^S N^B

N^S N^B

$N^S \cdot N^B$

F

A

A

A

a_s^{S*} a_s^{B*}

$\frac{1-N_s^{B*}}{N_s^{B*}}$ $\frac{1-N_s^{S*}}{N_s^{S*}}$

a_s^{S*} a_s^{B*}

F^S

F^B $F \geq \max[F^S, F^B]$

A

abc A^2

N_s^{X*}

A

Comparison with McAfee, McMillan and Whinston (1989)

pure *mixed*

F^S F^B F

bundling

pure

increase

decrease

A N^X

A

The Degree of Mixedness

during

$\Pi_{m1}(\mathbf{P}'_{m0})$

$\Pi_s(\mathbf{P}^*_{s0})$

$F - F^S \geq 0$

$F - F^B \geq 0$

$F^S + F^B - F \geq 0$

Proposition 2 *Suppose* $\min(F^S, F^B) > 0$*, then bundling strictly dominates separate sales if*

$$\frac{N_s^{X*}}{N_s^*} \geq \frac{F - \min(F^S, F^B)}{\min(F^S, F^B)}$$

where $\frac{N_s^{X*}}{N_s^*}$ *is the proportion of seller-buyers in all users at the optimal separate sales prices* $\mathbf{P}^*_{s0}$*, which measures the degree of mixedness, and* $\frac{F-\min(F^S,F^B)}{\min(F^S,F^B)}$ *is the percentage increase in the lower-cost group's per-user fixed cost due to bundling.*

Proof. ■

a_s^S a_s^B

a_m^S a_m^B A

(a_s^{S*}, a_s^{B*})

A

4.2 Bundling with Bounded Agent Rationality

$$\mathbf{P}_m = (a_m^S, a_m^B, A)$$

A Motivating Example of Bounded Rationality

ex ante

Assumption M2 *Given bundling price $\mathbf{P}_m$, each agent makes her membership decision based only on her more urgent need - the higher of u^S and u^B. The less urgent need only affects her trading decision after becoming a member.*

$\mathbf{P}_m$

$$Y_{m2} \equiv \{\omega \mid \max(u^S, u^B) > A\}$$

ex post $\mathbf{P}_m$

$$\begin{aligned} X_{m2} &\equiv \{\omega \in Y_{m2} \mid \min(u^S, u^B) > 0\} \\ &= \{\omega \mid \max(u^S, u^B) > A \qquad \min(u^S, u^B) > 0\} \end{aligned}$$

$m2$

$\max(u^S, u^B) > A$

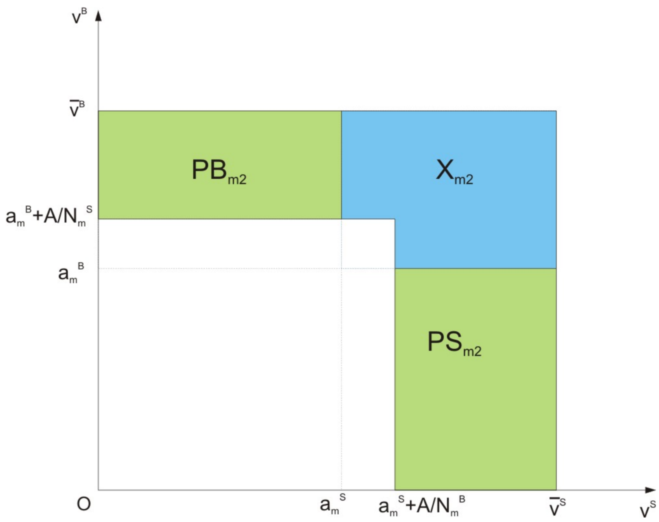
$\min(u^S, u^B) > 0$

X_{m2}

$\mathbf{P}_m$

a^S a^B A

$\mathbf{P}_m$



$\mathbf{P}_m = (a^S, a^B, A)$

$$N^S_{m2} = n^S_{m2}(a^S, a^B, A)$$
$$N^B_{m2} = n^B_{m2}(a^S, a^B, A)$$
$$N_{m2} = n_{m2}(a^S, a^B, A)$$

---

a^S a^B A

N^S_{m2} N^B_{m2} $\mathbf{P}_m$

$\mathbf{P}_m$

$$\Pi_{m2}(\mathbf{P}_m) = (A - F)N_{m2} + (a^S + a^B - c)N_{m2}^S N_{m2}^B$$

The Bundling Effect under Bounded Agent Rationality

Proposition 3 *Propositions 1 and 2 also hold with bounded agent rationality*

Proof. ■

X_{m1} X_{m2}

4.3 Bundling with Activation Cost

activation cost

Assumption AC *The platform requires that each user need to activate each kind of service individually after becoming a member. An activation procedure will involve an arbitrarily small but positive cost $\gamma > 0$ for the agent. The platform does not incur the fixed cost of the relevant service until an agent activates it.*

Under both separate sales and bundling, if an agent only activates the buying service (resp. selling service), the platform only incurs F^B (resp. F^S) for her; if an agent activates both services, the platform incurs F for her. $F \in [\max(F^S, F^B), F^S + F^B]$

$F \geq \max(F^S, F^B)$

F^S F^B

2γ F $F^S + F^B$

γ F^B F

γ F^S F

2γ F

F^S F^B F

Proposition 4 *When there is activation cost $\gamma > 0$, bundling strictly dominates separate sales whenever $X^* \neq \varnothing$.*

Proof. ■

5 Conclusion

Bundling with

Network Effects

References

Rand Journal of Economics

Review of Network Economics

Rand Journal of Economics

Rand Journal of Economics

Rand Journal of Economics

Rand Journal of Economics

Bell Journal of Economics

Quarterly Journal of Economics

Journal of the European Economic Association

Rand Journal of Economics

6 Appendix - Proofs

Lemma 2

Demand under separate-sales $\mathbf{P}_s = (a^S, a^B, A^S, A^B)$

$$N_s^S = 1 - G^S(\frac{A^S}{N_s^B} + a^S)$$
$$N_s^B = 1 - G^B(\frac{A^B}{N_s^S} + a^B)$$
$$N_s^X = N_s^S \cdot N_s^B$$
$$N_s = N_s^S + N_s^B - N_s^X$$

$\mathbf{P}_m = (a^S, a^B, A)$ N_{m1}^S N_{m1}^B

$$N_{m1}^S = G^B(a^B)[1 - G^S(\frac{A}{N_{m1}^B} + a^S)] + N_{m1}^X$$
$$N_{m1}^B = G^S(a^S)[1 - G^B(\frac{A}{N_{m1}^S} + a^B)] + N_{m1}^X$$

$$N_{m1}^X = [1 - G^S(a^S)][1 - G^B(a^B)] - \int_0^A [G^B(\frac{A-z}{N_{m1}^S} + a^B) - G^B(a^B)]dG^S(\frac{z}{N_{m1}^B} + a^S)$$

$$N_{m1} = N_{m1}^S + N_{m1}^B - N_{m1}^X = G^B(a^B)[1 - G^S(\frac{A}{N_{m1}^B} + a^S)] + G^S(a^S)[1 - G^B(\frac{A}{N_{m1}^S} + a^B)] + N_{m1}^X$$

$\mathbf{P}_{s0} = (a^S, a^B, 0, 0)$ $\mathbf{P}_{m0} = (a^S, a^B, 0)$

$$N_s^X = N_{m1}^X = [1 - G^S(a^S)][1 - G^B(a^B)]$$
$$N_s^S = N_{m1}^S = 1 - G^S(a^S)$$
$$N_s^B = N_{m1}^B = 1 - G^B(a^B)$$
$$N_s = N_{m1} = 1 - G^S(a^S)G^B(a^B). \blacksquare$$

Proposition 1

Step 1:

$\mathbf{P}_{s0} = (a_s^S, a_s^B, 0, 0) \in$

$\mathbb{R}^{+4}$

$$N_s^S = 1 - G^S(a_s^S)$$
$$N_s^B = 1 - G^B(a_s^B)$$
$$\Pi_s(\mathbf{P}_{s0}) = (a_s^S + a_s^B - c)N_s^S N_s^B - F^S N_s^S - F^B N_s^B$$
$$\mathbf{P}_{s0}^* = (a_s^{S*}, a_s^{B*}, 0, 0)$$
$$N_s^{S*} = 1 - G^S(a_s^{S*})$$
$$N_s^{B*} = 1 - G^B(a_s^{B*})$$
$$\frac{\partial N_s^S}{\partial a_s^S}(\mathbf{P}_{s0}^*) = -g^S(a_s^{S*})(< 0)$$
$$\frac{\partial N_s^B}{\partial a_s^B}(\mathbf{P}_{s0}^*) = -g^B(a_s^{B*})(< 0)$$

a_s^{S*} a_s^{B*}

$\frac{\partial \Pi_s}{\partial a_s^S}(\mathbf{P}_{s0}^*) = N_s^{S*}N_s^{B*} + [(a_s^{S*} + a_s^{B*} - c)N_s^{B*} - F^S]\cdot\frac{\partial N_s^S}{\partial a_s^S}(\mathbf{P}_{s0}^*) = 0$
$\frac{\partial \Pi_s}{\partial a_s^B}(\mathbf{P}_{s0}^*) = N_s^{S*}N_s^{B*} + [(a_s^{S*} + a_s^{B*} - c)N_s^{S*} - F^B]\cdot\frac{\partial N_s^B}{\partial a_s^B}(\mathbf{P}_{s0}^*) = 0 \Rightarrow$

$$(a_s^{S*} + a_s^{B*} - c)N_s^{B*} - F^S = \frac{N_s^{S*}N_s^{B*}}{g^S(a_s^{S*})}(> 0)$$

$$(a_s^{S*} + a_s^{B*} - c)N_s^{S*} - F^B = \frac{N_s^{S*}N_s^{B*}}{g^B(a_s^{B*})}(> 0)$$

Step 2:
Demand under bundling $\mathbf{P}_m = (a^S, a^B, A)$

$N_{m1}^X = [1 - G^S(a^S)][1 - G^B(a^B)] - \int_0^A [G^B(\frac{A-z}{N_{m1}^S} + a^B) - G^B(a^B)]dG^S(\frac{z}{N_{m1}^B} + a^S)$
$N_{m1}^{PS} = G^B(a^B)[1 - G^S(\frac{A}{N_{m1}^B} + a^S)]$
$N_{m1}^{PB} = G^S(a^S)[1 - G^B(\frac{A}{N_{m1}^S} + a^B)]$
$N_{m1}^S = N_{m1}^{PS} + N_{m1}^X$
$N_{m1}^B = N_{m1}^{PB} + N_{m1}^X$
$N_{m1} = N_{m1}^{PS} + N_{m1}^{PB} + N_{m1}^X$

A

$\frac{\partial N_{m1}^X}{\partial A}(\mathbf{P}_m) = -\int_0^A \{g^B(\frac{A-z}{N_{m1}^S} + a^B)\cdot[\frac{1}{N_{m1}^S} - \frac{A}{N_{m1}^{S2}}\cdot\frac{\partial N_{m1}^S}{\partial A}(\mathbf{P}_m)]\cdot g^S(\frac{z}{N_{m1}^B} + a^S)$
$-[G^B(\frac{A-z}{N_{m1}^S} + a^B) - G^B(a^B)]\cdot\frac{1}{N_{m1}^{B2}}\cdot[\frac{g^{S\prime}(\frac{z}{N_{m1}^B} + a^S)}{N_{m1}^B} + g^S(\frac{z}{N_{m1}^B} + a^S)]\cdot\frac{\partial N_{m1}^B}{\partial A}(\mathbf{P}_m)\}dz$
$\frac{\partial N_{m1}^S}{\partial A}(\mathbf{P}_m) = -G^B(a^B)\cdot g^S(\frac{A}{N_{m1}^B} + a^S)\cdot\frac{N_{m1}^B - A\cdot\frac{\partial N_{m1}^B}{\partial A}(\mathbf{P}_m)}{N_{m1}^{B2}} + \frac{\partial N_{m1}^X}{\partial A}(\mathbf{P}_m)$
$\frac{\partial N_{m1}^B}{\partial A}(\mathbf{P'}_{m0}) = -G^S(a^S)\cdot g^B(\frac{A}{N_{m1}^S} + a^B)\cdot\frac{N_{m1}^S - A\cdot\frac{\partial N_{m1}^S}{\partial A}(\mathbf{P}_m)}{N_{m1}^{S2}} + \frac{\partial N_{m1}^X}{\partial A}(\mathbf{P}_m)$

$A = 0$ $\frac{\partial N_{m1}^X}{\partial A} = 0$

$\mathbf{P'}_{m0} = (a_s^{S*}, a_s^{B*}, 0)$

$N_{m1}^S = N_s^{S*}$
$N_{m1}^B = N_s^{B*}$
$N_{m1}^X = N_s^{S*}\cdot N_s^{B*}$
$\frac{\partial N_{m1}^X}{\partial A}(\mathbf{P'}_{m0}) = 0$ A N_{m1}^X $\mathbf{P'}_{m0}$
$\frac{\partial N_{m1}^S}{\partial A}(\mathbf{P'}_{m0}) = \frac{\partial N_{m1}^{PS}}{\partial A}(\mathbf{P'}_{m0}) = -\frac{G^B(a_s^{B*})g^S(a_s^{S*})}{N_s^{B*}}(< 0)$
$\frac{\partial N_{m1}^B}{\partial A}(\mathbf{P'}_{m0}) = \frac{\partial N_{m1}^{PB}}{\partial A}(\mathbf{P'}_{m0}) = -\frac{G^S(a_s^{S*})g^B(a_s^{B*})}{N_s^{S*}}(< 0)$
$\Pi_{m1}(\mathbf{P}_m) = (A - F)N_{m1} + (a_m^S + a_m^B - c)N_{m1}^S N_{m1}^B$

$$\begin{aligned}\frac{\partial \Pi_m}{\partial A}(\mathbf{P'}_{m0}) &= N_{m1} + \frac{\partial N_{m1}^B}{\partial A}(\mathbf{P'}_{m0})\cdot[(a_s^{S*} + a_s^{B*} - c)N_s^{S*} - F] \\ &\quad + \frac{\partial N_{m1}^S}{\partial A}(\mathbf{P'}_{m0})\cdot[(a_s^{S*} + a_s^{B*} - c)N_s^{B*} - F]\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \Pi_m}{\partial A}(\mathbf{P'}_{m0}) &= N_s^{S*} \cdot N_s^{B*} + \frac{G^S(a_s^{S*})g^B(a_s^{B*})}{N_s^{S*}} \cdot (F - F^B) + \frac{G^B(a_s^{B*})g^S(a_s^{S*})}{N_s^{B*}} \cdot (F - F^S) \\
&= N_s^{S*} \cdot N_s^{B*} + [-\frac{\partial N_{m1}^{PB}}{\partial A}(\mathbf{P'}_{m0}) \cdot (F - F^B) - \frac{\partial N_{m1}^{PS}}{\partial A}(\mathbf{P'}_{m0}) \cdot (F - F^S)]
\end{aligned}
$$

$\frac{\partial N_{m1}^{PS}}{\partial A}(\mathbf{P'}_{m0}) < 0$ $\frac{\partial N_{m1}^{PB}}{\partial A}(\mathbf{P'}_{m0}) < 0$ $F \geq \max(F^S, F^B)$ $\frac{\partial \Pi_{m1}}{\partial A}(\mathbf{P'}_{m0}) > 0$ $N_s^{S*} \cdot N_s^{B*} \neq 0$.■

Proposition 2

$\mathbf{P}_{s0}^* = (a_s^{S*}, a_s^{B*}, 0, 0)$

$$\Pi_s(\mathbf{P}_{s0}^*) = (a_s^{S*} + a_s^{B*} - c)N_s^{S*}N_s^{B*} - F^S N_s^{S*} - F^B N_s^{B*}$$

$N_s^{S*} = 1 - G^S(a_s^{S*})$
$N_s^{B*} = 1 - G^B(a_s^{B*})$

$\mathbf{P'}_{m0} = (a_s^{S*}, a_s^{B*}, 0)$

$$\Pi_{m1}(\mathbf{P'}_{m0}) = (a_s^{S*} + a_s^{B*} - c)N_s^{S*}N_s^{B*} - F(N_s^{S*} + N_s^{B*} - N_s^{X*})$$

$$\Pi_{m1}(\mathbf{P'}_{m0}) - \Pi_s(\mathbf{P}_{s0}^*) = F N_s^{X*} - (F - F^S)N_s^{S*} - (F - F^B)N_s^{B*}$$

$\min(F^S, F^B) > 0$ $0 < F^S \leq F^B \leq F$
$\frac{N_s^{X*}}{N_s^*} \geq \frac{F - \min(F^S, F^B)}{\min(F^S, F^B)} = \frac{F - F^S}{F^S} \Rightarrow F^S N_s^{X*} \geq (F - F^S)N_s^* \Rightarrow (F^S + F - F)N_s^* \geq (F - F^S)N_s^*$
$\Rightarrow F N_s^* \geq (F - F^S)(N_s^* + N_{s0}^X) = (F - F^S)(N_s^{S*} + N_s^{B*}) \geq (F - F^S)N_s^{S*} + (F - F^B)N_s^{B*}$
$\Rightarrow \Pi_{m1}(\mathbf{P'}_{m0}) - \Pi_s(\mathbf{P}_{s0}^*) = F N_s^{X*} - (F - F^S)N_s^{S*} - (F - F^B)N_s^{B*} \geq 0.$
$\Pi_{m1}(\mathbf{P'}_{m0}) - \Pi_s(\mathbf{P}_{s0}^*) > 0$
$\Pi_{m1}(\mathbf{P'}_{m0}) - \Pi_s(\mathbf{P}_{s0}^*) = 0$
A ■

Proposition 3

N_{m2}^S N_{m2}^B $\mathbf{P}_m = (a^S, a^B, A)$

$N_{m2}^S = 1 - G^S(a^S) - G^B(\frac{A}{N_{m2}^S} + a^B)[G^S(\frac{A}{N_{m2}^B} + a^S) - G^S(a^S)]$
$N_{m2}^B = 1 - G^B(a^B) - G^S(\frac{A}{N_{m2}^B} + a^S)[G^B(\frac{A}{N_{m2}^S} + a^B) - G^B(a^B)]$

$$N^X_{m2} = [1 - G^S(\tfrac{A}{N^B_{m2}} + a^S)][1 - G^B(a^B)] + [1 - G^B(\tfrac{A}{N^S_{m2}} + a^B)][1 - G^S(a^S)]$$
$$- [1 - G^S(\tfrac{A}{N^B_{m2}} + a^S)][1 - G^B(\tfrac{A}{N^S_{m2}} + a^B)]$$
$$N_{m2} = N^S_{m2} + N^B_{m2} - N^X_{m2} = 1 - G^S(\tfrac{A}{N^B_{m2}} + a^S)G^B(\tfrac{A}{N^S_{m2}} + a^B)$$

$$\mathbf{P'}_{m0} = (a^{S*}_s, a^{B*}_s, 0)$$

$$N^S_{m2} = 1 - G^S(a^{S*}_s)$$
$$N^B_{m2} = 1 - G^B(a^{B*}_s)$$
$$N^X_{m2} = [1 - G^S(a^{S*}_s)][1 - G^B(a^{B*}_s)]$$
$$N_{m2} = 1 - G^S(a^{S*}_s)G^B(a^{B*}_s)$$

$\mathbf{P}^*_{s0}$ $\mathbf{P'}_{m0}$ ■

Proposition 4

$$\mathbf{P}_s = (a^S_s, a^B_s, A^S, A^B)$$
$$\Pi^{AC}_s(\mathbf{P}_s) = (a^S_s + a^B_s - c)N^S_s N^B_s + (A^S - F^S)N^{PS}_s + (A^B - F^B)N^{PB}_s + (A^S + A^B - F)N^X_s$$
$$\mathbf{P}_m = (a^S, a^B, A)$$
$$\Pi^{AC}_m(\mathbf{P}_m) = (a^S_m + a^B_m - c)N^S_m N^B_m + (A - F^S)N^{PS}_m + (A - F^B)N^{PB}_m + (A - F)N^X_m$$
$$\mathbf{P}^*_{s0} = (a^{S*}_s, a^{B*}_s, 0, 0) \qquad \mathbf{P'}_{m0} = (a^{S*}_s, a^{B*}_s, 0)$$
$$\Pi^{AC}_s(\mathbf{P}^*_{s0}) = \Pi^{AC}_m(\mathbf{P'}_{m0}) = (a^{S*}_s + a^{B*}_s - c)N^{S*}_s N^{B*}_s - F^S N^{PS*}_s - F^B N^{PB*}_s - F N^{X*}_s$$

$\mathbf{P}^*_{s0}$ $\mathbf{P'}_{m0}$

a^{S*}_s a^{B*}_s

$$\frac{\Pi^{AC}_s}{\partial a^S_s}(\mathbf{P}^*_{s0}) = N^{S*}_s N^{B*}_s + (a^{S*}_s + a^{B*}_s - c) \cdot N^{B*}_s \cdot \frac{\partial N^S_s}{\partial a^S_s}(\mathbf{P}^*_{s0})$$
$$- F^S \cdot \frac{\partial N^{PS}_s}{\partial a^S_s}(\mathbf{P}^*_{s0}) - F^B \cdot \frac{\partial N^{PB}_s}{\partial a^S_s}(\mathbf{P}^*_{s0}) - F \cdot \frac{\partial N^X_s}{\partial a^S_s}(\mathbf{P}^*_{s0}) = 0$$

$$\frac{\partial N^{PS}_s}{\partial a^S_s}(\mathbf{P}^*_{s0}) = (1 - N^{B*}_s)\frac{\partial N^S_s}{\partial a^S_s}(\mathbf{P}^*_{s0})$$
$$\frac{\partial N^{PB}_s}{\partial a^S_s}(\mathbf{P}^*_{s0}) = -N^{B*}_s \frac{\partial N^S_s}{\partial a^S_s}(\mathbf{P}^*_{s0})$$
$$\frac{\partial N^X_s}{\partial a^S_s}(\mathbf{P}^*_{s0}) = N^{B*}_s \frac{\partial N^S_s}{\partial a^S_s}(\mathbf{P}^*_{s0})$$

$$(a^{S*}_s + a^{B*}_s - c) \cdot N^{B*}_s = \frac{N^{S*}_s N^{B*}_s}{g^S(a^{S*}_s)} + F^S(1 - N^{B*}_s) + (F - F^B)N^{B*}_s$$

$\frac{\Pi_s^{AC}}{\partial a_s^B}(\mathbf{P}_{s0}^*) = 0 \Rightarrow$

$$(a_s^{S*} + a_s^{B*} - c) \cdot N_s^{S*} = \frac{N_s^{S*} N_s^{B*}}{g^B(a_s^{B*})} + F^B(1 - N_s^{S*}) + (F - F^S)N_s^{S*}$$

$\frac{\Pi_m^{AC}}{\partial A}(\mathbf{P'}_{m0}) :$

$$\begin{aligned}
\frac{\Pi_m^{AC}}{\partial A}(\mathbf{P'}_{m0}) &= N_m + \frac{\partial N_m^B}{\partial A}(\mathbf{P'}_{m0}) \cdot [(a_s^{S*} + a_s^{B*} - c)N_s^{S*} - F^B] \\
&\quad + \frac{\partial N_m^S}{\partial A}(\mathbf{P'}_{m0}) \cdot [(a_s^{S*} + a_s^{B*} - c)N_s^{B*} - F^S]
\end{aligned}$$

$$\begin{aligned}
\frac{\Pi_m^{AC}}{\partial A}(\mathbf{P'}_{m0}) &= N_s^{S*} N_s^{B*} + (F^S + F^B - F) \cdot [G^S(a_s^{S*}) \overline{g}^B(a_s^{B*}) + G^B(a_s^{B*}) \overline{g}^S(a_s^{S*})] \\
&= N_s^{S*} N_s^{B*} + (F^S + F^B - F)[-N_s^{S*} \frac{\partial N_m^{PB}}{\partial A}(\mathbf{P'}_{m0}) - N_s^{B*} \frac{\partial N_m^{PS}}{\partial A}(\mathbf{P'}_{m0})]
\end{aligned}$$

$\frac{\partial N_m^{PS}}{\partial A}(\mathbf{P'}_{m0}) < 0$ $\frac{\partial N_m^{PB}}{\partial A}(\mathbf{P'}_{m0}) < 0$ $F \leq F^S + F^B$ $N_s^{S*} N_s^{B*} \neq 0$
$X^* \neq \varnothing$ $\frac{\partial \Pi_m}{\partial A}(\mathbf{P'}_{m0}) > 0.\blacksquare$