Non-Random Consumer Search

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Abstract

Consumer search is rarely random. In the supermarket, certain products stand out more than others; on the internet, some sponsored links are placed higher up the page than others. Firms believe that 'prominence' matters - and pay large sums to secure it. But why should prominence matter so much, when the cost of visiting an extra supermarket aisle, or clicking an additional web link, is so small? This paper provides a simple answer to this 'puzzle'. I demonstrate that products in prominent locations face a more elastic demand curve. Prominent products are therefore cheaper. Consumers understand this, and are therefore less likely to search non-prominent products. This disadvantages the latter, reducing their profitability. Prominence therefore has a large effect on price and profits, explaining why firms pay so much for it.

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1 Introduction

> "The most expensive real estate in New York City isn't on Park or Fifth avenues. It's at your grocery store"[1]

Models of consumer search often take for granted the assumption that search is random. In reality certain products are always more prominent than others. In a supermarket, items that are placed in end-of-aisle displays immediately jump out. The same is true of firms which appear high up in Google's list of sponsored ads. Shelf space and sponsored ads are sold - like real estate - to the highest bidder. Manufacturers are willing to pay large sums of money to secure a prominent location for their product, believing that this will greatly increase sales. However this seems to present a puzzle. When the cost of walking down an extra aisle or clicking on an additional link is so small, why should prominence influence consumer behaviour so much? In this paper I demonstrate that prominent products face a more elastic demand curve. Consequently consumers rationally expect prominent products to be cheaper, and therefore tend to buy them much more frequently. This is true even if the products are ex ante symmetric. The model also provides insights into, for example, the way supermarkets design their store layout.

Slotting fees were first introduced in the 1980s, and now earn US grocery stores an estimated $9 billion a year. The actual term 'slotting fee' covers a broad range of payments made by manufacturers to retailers, including payments for premium shelf space. Even well-established brand names such as Heinz pay large sums of money to secure a prominent position for their products.[2] Slotting fees have also spread beyond the grocery sector. For example a publisher can pay Barnes & Noble $10,000 a month and in return get its book onto a front-of-store promotional table.[3]

The online equivalent of a slotting fee is the charge made by websites for displaying sponsored ads. These advertisements earn Google, Yahoo! and Microsoft an estimated $10 billion in revenues each year.[4] Each time somebody clicks on a sponsored link, the search engine receives a click-through

[1] Quoted by Ralph Nader in 'Shelf Space Wars: Slotting Fees Hike Prices', Charleston Gazette (West Virginia) November 14, 1994

[2] For example, "Heinz hopes to save... $145m [out of a presumably much larger amount] from reducing the deals offered to retailers in return for premium shelf space." ('Heinz cuts 2,700 jobs to fend off billionaire', The Guardian, June 2, 2006)

[3] 'The Book Business; Cash Up Front', The New York Times, June 5, 2005

[4] Athey and Ellison [2]

price from the company that owns the link. The click-through price is determined by auction and, other things equal, companies who bid more appear higher up in search lists. On travel websites, companies that refuse to pay sufficient slotting fees may still be listed, but put at a disadvantage. For example Expedia threatened that if United Airlines did not pay commissions, consumers would have to make extra clicks to view United's prices.[5] United gave in almost immediately, and paid.

What is particularly interesting is that (at least from casual observation) prominent products tend also to be cheaper. An item that is moved from its regular place in a supermarket to an end-of-aisle display tends to temporarily decrease in price. A bookshop's 'book of the week' is often placed in a prominent location and sold at a discounted price, or as part of a special offer. Anecdotal evidence suggests that many consumers do click on sponsored web links even though they can often be avoided - suggesting that consumers do expect these firms to offer good value for money.[6]

The theoretical literature, however, does not give a clear prediction as to whether or not prominent products should be cheaper. The three papers most relevant to our discussion here are Arbatskaya [1], Wilson [12], and Armstrong et al [3]. In Arbatskaya and Wilson, more prominent firms charge higher prices, whilst the opposite is true in Armstrong et al. Perry and Wigderson [9] present a model of search without recall (for example travelling along a motorway looking for petrol stations), in which the relationship between price and prominence is non-monotonic. Chen and He [5] construct a model of search and advertising on the internet. Firms differ in how likely they are to match a consumer's preference, but amongst the firms that do match, all are equally good. Consumers do not know their valuation until finding a suitable firm. In equilibrium all firms charge the same price - equal to the standard monopoly price. This is due to Diamond's [7] Paradox. Athey and Ellison [2] also consider a model of internet search, but focus on auction design. They fix the prices of all firms exogenously and therefore

[5] 'The Sum Of All Fares; How Online Booking Sites Influence You', The Washington Post July 28, 2002

[6] "...over 80 percent of Polish Internet users who recognise sponsored links from other search results, declare that they consciously click on such links... Over half of those surveyed said that making use of this form of advertising is helpful. The research also proved that over 33 percent of web users who clicked on a sponsored link remember the website and return to it later." ('Over 80 Percent Web Users Click on Sponsored Links', Polish News Bulletin, March 15, 2006)

totally abstract away from the issues considered in our model.[7]

In Arbatskaya [1] and Wilson [12], firms sell a homogeneous product and consumers have heterogeneous search costs. In Arbatskaya's model search must take place in a pre-specified order (for example in an oriental bazaar). In Wilson's duopoly model, there exists a set of parameters such that one firm unilaterally obfuscates and makes it costlier for consumers to find it (for example locating in a backstreet). This obfuscation softens competition, thereby raising prices and profits. In both these papers, a firm that is more prominent tends to charge a higher (expected) price because it has monopoly power over consumers who have a high search cost. Crucially though, as the (distribution of) search costs tends towards zero, this monopoly position is lost, and prominence has no effect on price.

Armstrong et al [3] is different both because firms sell heterogeneous products, and because consumers have the same positive search cost. Prior to searching a firm, consumers have no idea of their valuation for the firm's product. The authors distinguish between fresh demand (which is price-sensitive) and return demand (which is not price-sensitive). Fresh demand comes from consumers who have not yet searched the whole market; return demand comes from consumers who have searched every firm and then return to buy their best available product. More prominent firms have proportionally more fresh demand and therefore face a more elastic demand curve, which causes them to charge lower prices. Importantly, however, as the search cost tends towards zero, prominence has no effect on price. Intuitively this is because when the search cost is so low, consumers find it worthwhile to search every firm before buying.

In this paper I present two different but closely related models. In the first model, there are many different products and none of them are in direct competition with one another. However some of these products end up in prominent positions, and others do not. The model therefore applies to a mulitproduct monopolist which has a single store and must arrange its products between different aisles. The model also applies to a marketplace in which different stalls sell different products, but some stalls have a better location than others. In the second model, there is a single product, with many different producers each selling a single differentiated version of it.

[7] Specifically relating to slotting fees, there are papers which look at whether these promote efficient use of shelf space; optimal sharing of risk between manufacturers and retailers; exclusion of smaller retailers; greater choice for consumers. For a detailed overview, see Bloom et al [4] and FTC [8].

There is no need for any firm to be more prominent than any other, but a platform can exogenously choose to make one firm prominent if it so wishes. The model therefore applies best to the case where consumers want to search on Google for a particular product.

In both of the models that I present, more prominent products have a lower price. Unlike Armstrong *et al* [3], this is despite the fact that I assume the cost of searching an additional supermarket aisle/market stall/sponsored link to be *almost* costless. As in Armstrong *et al*, I assume that consumers have heterogeneous valuations. Instead the key driver of my results is the fact that consumers know their valuations for products *before* they start search. Intuitively, as a consumer searches further down a list of sponsored links, he signals that he is looking for something in particular. Having exerted costly effort (however tiny) to find what he wants, he will not be deterred from buying if he finds the price slightly more expensive than he originally anticipated. Firms that are listed lower down therefore have less elastic demand, because consumers reveal more match information when visiting them. Consumers foresee this, expect higher prices further down the list, and are therefore less likely to search that far. Only consumers with especially strong valuations for the non-prominent firms search on - merely encouraging the non-prominent firms to charge even more. The same is true in a supermarket - when a consumer visits a relatively obscure aisle, he does so because he has a strong valuation for the products placed there. The supermarket has every incentive to charge a high price and exploit this. By contrast, when a consumer walks past an end-of-aisle display, he is probably passing on his way to buy something else. The consumer reveals little information about match value, so the firm sets a lower price.

2 Search in a (Super)Market

2.1 Assumptions

There are T firms, each of which produces $n \geq 2$ differentiated goods at zero marginal cost. The nT products are neither substitutes nor complements, and consumers demand at most one unit of each. A consumer's valuation for good j at firm t, or simply (j, t), is denoted by $v_{j,t}$. Valuations for each good are independently drawn from the interval $[a, b]$ using a common distribution function $F(v_{j,t})$ (with corresponding density $f(v_{j,t})$). $F(v_{j,t})$ is strictly increasing, atomless, twice continuously differentiable, and satisfies:

- (A1) At least one of the following holds

  (i). $p[1 - F(p)]$ is strictly concave in p

  (ii). $f(p)$ is logconcave in p

The firms must be searched sequentially and in a speci…ed order (think for example of a marketplace with a single entrance and T stalls spaced out one after the other). It costs s to visit the first firm, a further s to visit the second, and so forth. A consumer must visit a store in order to buy any of its products, and no firm before it can be missed out. I will compare outcomes under two different scenarios:

1. **Known Layout** Consumers know precisely which products are stocked by which firm

2. **Unknown Layout** Consumers have no prior idea about where any of the nT products are located

In both scenarios I assume that consumers know their valuation for each good before visiting any firms. Crucially, however, consumers only learn the price of a good after searching the firm that sells it.[8] For simplicity I restrict attention to equilibria in which a firm charges the same price on all its n products. Let p_t be the actual price charged by the t^{th} firm. Consumers form a (point) expectation p_t^E of the price charged by the t^{th} firm. Consumers are rational, risk-neutral, and use their price expectations to optimally decide how many firms to search. Firms set their prices to maximise profits, given

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consumers' expectations and subsequent search behaviour. In equilibrium consumer expectations must be correct.

When $s = 0$ consumers learn all prices, and each firm charges the standard monopoly price $p^m = \arg\max p\left[1 - F(p)\right]$ for each good. For reasons that will be clearer later on, I assume $p^m > a$.

2.2 Solving for an equilibrium

As an extra piece of notation, $\Pr\left(t; s, \mathbf{v}, \mathbf{p^E}, L\right)$ is the probability that a consumer visits store t. This probability depends upon the search cost s, whether or not the store layout L is known, as well as the consumer's vector of valuations $\mathbf{v}$ and the vector of expected prices $\mathbf{p^E}$. We can then write the demand for good 1 at store t as

$$D_{1,t} = \int_{p_t}^{b} f\left(v_{1,t}\right) \Pr\left(t; s, \mathbf{v}, \mathbf{p^E}, L\right) dv_1$$

A consumer buys the good if (a) he values it more than the actual price and (b) he turns up to the firm selling the good. Part (a) explains why we only integrate over $v_{1,t} \geq p_t$, whilst part (b) explains why the probability term $\Pr\left(t; s, \mathbf{v}, \mathbf{p^E}, L\right)$ must also be added. We can derive a necessary equilibrium condition by differentiating $p_t D_{1,t}$ with respect to p_t and then imposing $p_t = p_t^E$ (rational expectations).

Lemma 1 An equilibrium price for good 1 at …rm t must satisfy

$$\int_{p_t^E}^{b} f\left(v_{1,t}\right) \Pr\left(t; s, \mathbf{v}, \mathbf{p^E}, L\right) dv_{1,t} - p_t^E f\left(p_t^E\right) \Pr\left(t; s, v_{1,t} = p_t^E, \mathbf{v}, \mathbf{p^E}, L\right) = 0 \tag{1}$$

Given the vector of price expectations $\mathbf{p^E}$, the actual price $p_{1,t}$ must maximise the firm's profit. This implies that small changes in actual price around the expected level should have zero effect on profit. To understand (1), consider a small increase in the actual price p_t above the expected level p_t^E. The firm gains revenue on existing consumers, who have mass equal to demand (the first term). It loses revenue on consumers who stop buying the good following the price rise (the second term). Any consumer who stops buying (a) has a marginal valuation for the good and (b) visits the store given his price expectations. Part (a) explains the term $f\left(p_t^E\right)$ whilst part (b) explains the term $\Pr\left(t; s, v_{1,t} = p_t^E, \mathbf{v}, \mathbf{p^E}, L\right)$.

Remark 2 In any equilibrium where some consumers visit …rm t, $p_t^E \geq p^m$

Proof: $\Pr\left(t; s, \mathbf{v}, \mathbf{p^E}, L\right) \geq \Pr\left(t; s, v_{1,t} = p_t^E, \mathbf{v}, \mathbf{p^E}, L\right)$ so the lefthand side of (1) weakly exceeds $\Pr\left(t; s, v_{1,t} = p_t^E, \mathbf{v}, \mathbf{p^E}, L\right)\left[1 - F\left(p_t^E\right) - p_t^E f\left(p_t^E\right)\right]$, which by strict quasiconcavity of $p\left[1 - F(p)\right]$ is strictly positive for any $p_t^E < p^m$. ■

I now make the following two assumptions:

- (A2) $s \to 0$
- (A3) n is 'large'

Studying equilibrium behaviour as $s \to 0$ renders the model tractable and allows us to fully characterise the solution. The intuition gained from looking at this special case is persuasive, and suggests that similar qualitative results could be expected for general levels of s. The assumption that n is large guarantees that an equilibrium always exists, and will be explained (and made more precise) shortly.

2.3 Prominence with a known layout

Lemma 3 If consumers know the layout, equilibrium prices solve

$$\frac{1 - F\left(p_T^E\right)}{p_T^E f\left(p_T^E\right)} - \left[1 - F\left(p_T^E\right)^{n-1}\right] = 0 \qquad (2)$$

$$\frac{1 - F\left(p_t^E\right)}{p_t^E f\left(p_t^E\right)} - \left[1 - F\left(p_t^E\right)^{n-1} \prod_{z=t+1}^{T} F\left(p_z^E\right)^n\right] = 0 \text{ for } t = 1, ..., T-1 \qquad (3)$$

The above equations are (1) rewritten but accounting for $s \to 0$. Because the cost of searching a firm is trivially small, any consumer with $v_{1,t} > p_t^E$ is prepared to search (at least as far as) firm t. Consequently demand for good 1 at firm t (in the limit) is simply $1 - F\left(p_t^E\right)$. Following Rhodes [10], demand comprises two types of consumer[9]:

[9] This terminology parallels that already used in the search literature. A 'shopper' usually has $s = 0$ and visits a store regardless of his valuation (and therefore reveals no match information). In Diamond's [7] Paradox any consumer visiting a firm reveals himself to have a valuation strictly above the expected price, which the firm then exploits.

- **Shoppers** for $(1,t)$ are consumers whose decision to search firm t is made independently of their valuation for product $(1,t)$
- **Diamond consumers** for $(1,t)$ only search firm t because of the positive surplus they expect to earn on product $(1,t)$

A shopper reveals no information about his valuation for good $(1,t)$ when visiting firm t, so the firm would like to charge the consumer p^m. By contrast a Diamond consumer reveals that his valuation for good $(1,t)$ strictly exceeds the expected price, so if the firm charged slightly more than p_t^E such a consumer would continue to buy. Consequently small increases in p_t above p_t^E only affect the behaviour of shoppers. Since $s \to 0$, a shopper for $(1,t)$ must expect to earn strictly positive surplus on at least one other good sold by firm t or at some later firm. This happens with probability $1 - F\left(p_t^E\right)^{n-1} \prod_{z=t+1}^{T} F\left(p_z^E\right)^n$.

There always exists a Diamond equilibrium in which consumers expect each good at each store to cost b. No consumer ever visits any of the firms, who then find it a (weak) best response to charge b.[10] Similarly, there also exist equilibria in which consumers expect some subset of firms $t \geq \tau$ (where τ is an arbitrary integer between 1 and T) to charge b, in which case consumers never search beyond firm $\tau - 1$. Firms from τ onwards charge b and make zero profit, so if there are any fixed production costs these 'inactive' firms prefer not to exist. It is therefore sensible to look for other 'non-degenerate' equilibria, in which each firm makes positive profit.

To find these non-degenerate equilibria, the game should be solved backwards. Since $s \to 0$, a consumer's decision about whether or not to search the last firm is independent of his valuations for (and the expected prices of) any of the products sold by the $T-1$ preceding firms. The last firm's optimisation problem is therefore identical to that of a single isolated firm that sells n differentiated products. In Rhodes [10] I demonstrate that such a problem has non-degenerate equilibria, provided n is large enough. Intuitively a consumer visits a store only if he has a high valuation for at least one of the goods on sale. This gives the firm an incentive to raise price above the expected level and exploit those with high valuations. If $n = 1$ this incentive to raise price is so strong that the market breaks down completely (Diamond Paradox). But when n is large enough, turning up does not reveal too much

[10] Notice that $p_t^E = b \forall t$ solves equations (2) and (3).

about valuation for any single product, meaning that the firm has less reason to exploit consumers. This enables a non-degenerate equilibrium to exist.

Assumption A2 guarantees that n is large and therefore that a non-degenerate equilibrium price p_T^E exists. This implies that some consumers optimally search through to the final store. One can then deduce that in equilibrium, each of the preceding $T-1$ stores must also charge a non-degenerate price.[11] One slight complication is that equations (2) and (3) may admit multiple (non-degenerate) sets of solutions. Nevertheless there always exists one equilibrium in which each expected price is (weakly) lower than it is in any other equilibrium. Given that prices strictly exceed p^m and profit per good limits (as $s \to 0$) to $p\left[1-F\left(p\right)\right]$, both consumers and producers prefer prices to be lower. Consequently there exists a Pareto dominant equilibrium.

Proposition 4 In the Pareto dominant equilibrium

$$p_T^E > p_{T-1}^E > ... > p_1^E > p^m$$

In the Pareto dominant equilibrium, firms that are more prominent charge a strictly lower price for each product that they sell. This is because more prominent firms attract a higher number of shoppers and consequently face a more elastic demand curve. In order to reach the less-prominent firms and buy their products, consumers must pass by (and hence learn the prices charged by) the more-prominent firms. A high percentage of consumers who are marginal for good $(1,t)$ visit firm t - either because they want to buy other products at firm t, or because they want to search on to a later firm. Visiting a prominent firm therefore reveals little information about valuation for the products on sale at the store, giving the firm less incentive to exploit high-valuation consumers by increasing price. By contrast, visiting a later firm reveals more information - a consumer who searches that far is most likely doing so because he values highly the products sold by that retailer. This gives later firms an incentive to exploit consumers by charging prices above what they expected. This drives up equilibrium prices at the later retailers. The fact that all prices are strictly above p^m is a simple consequence of the

[11] For example, it certainly cannot be an equilibrium for any preceding firm to charge b. If consumers expected this, they would only be visiting the store to get to another. Everybody would be shoppers, so the store would optimally charge p^m, invalidating the original expectation.

fact that, even at the most prominent firm, not all marginal consumers will visit, so if consumers expected $p_1^E = p^m$ the firm would raise actual price a little and earn more revenue on Diamond consumers.

2.4 Prominence with an unknown layout

Analogous to the case of known layout, there always exist Diamond-type equilibria in which (some) firms are never visited, but (more sensible) non-degenerate equilibria also exist. In general the optimal search rule is complex. If a consumer has done no search, he has no idea which firms sell which products. In deciding whether to search firm 1, the consumer must consider all firm-product combinations (i.e. every possible way of equally distributing the nT goods to the T firms). He must then decide whether or not it is optimal to pay s and search firm 1, accounting for direct surpluses that might be earned on products sold by firm 1, and also accounting for possible payoffs that might be earned from future searches. Once firm 1 has been searched, the consumer learns what products it sells, and can then infer the set of products sold by firms 2 through to T. In deciding whether or not to search firm 2, the consumer must again consider all possible firm-product combinations, and again take into account both direct and indirect surpluses that might be earned. This process is repeated after each one-step search, and clearly becomes unwieldy for general s. Fortunately when $s \to 0$ the optimal search rule takes the following simple form:

Remark 5 Suppose the consumer has searched F …rms (for $F = 0, 1, ..., T - 1$). He should search the $F + 1^{th}$ …rm provided there is at least one product (a). that he has yet to …nd and (b). which he values at strictly more than the minimum price he expects to …nd at stores $F + 1$ through to T

To better understand this, suppose that the minimum expected price set by firms $F + 1$ through to T is set by firm F'. Picking any particular good at random, firm F' stocks it with probability $\frac{1}{T-F}$. It costs $(F' - F)\, s$ to search as far as firm F', so provided at least one valuation weakly exceeds $p_{F'}^E + (T - F)(F' - F)\, s$, the consumer should search $F + 1$ to learn what that firm stocks, and then redecide whether he wants to search further. As $s \to 0$, so does $(T - F)(F' - F)\, s$. Therefore a sufficient condition to search firm $F + 1$ is that the consumer has a valuation strictly exceeding $p_{F'}^E$. This is also necessary, because if all valuations were below $p_{F'}^E$ it would definitely not pay to search any further.

With this optimal search rule in mind, it is simple to rule out equilibria in which a more prominent firm charges strictly more than another firm which is less prominent. To understand why, suppose to the contrary that $p_1^E > p_2^E$ is an equilibrium. A consumer who is marginal for good $(1,1)$ (i.e. has $v_{1,1} = p_1^E$) searches firm 1 because he is uninformed about store layout, and knows that with some probability actually firm 2 stocks that good. But then using (1), firm 1's first order condition is $1 - F\left(p_1^E\right) - p_1^E f\left(p_1^E\right) = 0$, which is the same for a standard monopolist. The solution is therefore p^m and not the putative p_1^E (since $p_1^E > p_2^E \geq p^m$ from Remark 2). Intuitively the expectation that store 2 is cheaper causes all consumers who are marginal for firm 1's products to visit firm 1, which causes that firm to want to charge strictly less than what was expected, thus breaking the 'equilibrium'. Having ruled out equilibria in which more prominent firms charge strictly higher prices, it is then simple to prove the following:

Proposition 6 There always exist non-degenerate equilibria. In any such equilibrium

$$p_T^E > p_{T-1}^E > ... > p_1^E > p^m$$

Furthermore these prices satisfy the equations

$$\frac{1 - F\left(p_T^E\right)}{p_T^E f\left(p_T^E\right)} - \left[1 - F\left(p_T^E\right)^{n-1}\right] = 0 \quad (4)$$

$$\frac{1 - F\left(p_t^E\right)}{p_t^E f\left(p_t^E\right)} - \left[1 - F\left(p_t^E\right)^{(T-t+1)n-1}\right] = 0 \text{ for } t = 1, ..., T-1 \quad (5)$$

The equilibrium price of the last firm p_T^E solves (4), which is the same as (2). Therefore the last firm faces the same optimisation problem, regardless of whether or not consumers know the layout. By following the search rule in Remark 5, anybody who values at least one of firm T's products at strictly more than p_T^E visits the store. Once stores 1 through to $T-1$ have been searched, the consumer perfectly infers what products firm T offers. Therefore given p_T^E, the same set of consumer-types visits firm T regardless of whether layout was known or not. Rhodes [10] then demonstrates that since n is large, (4) has non-degenerate solutions.

More prominent firms charge a strictly lower equilibrium price, but prices are always above the standard monopoly price p^m. We already demonstrated that in any equilibrium, prices must increase at least weakly as firms become

less prominent. Given this fact and the optimal search rule in Remark 5, a consumer who is marginal for good 1 at firm t searches firm t provided that he values one (or more) of the other $(T-t+1)\,n-1$ goods sold by firms $t, t+1, ..., T$ at more than p_t^E.[12] More prominent firms (those with low t) attract more consumers because there is greater uncertainty about what these firms sell, and later firms can only be accessed by first searching the prominent stores. Consequently turning up to a store that is relatively prominent reveals little information about a consumer's valuation for any of the products sold there. Alternatively, more prominent stores attract more shoppers and fewer Diamond consumers, because there are more reasons to visit prominent firms even if you have a marginal valuation for one product. Consequently a firm that is more prominent has less incentive to exploit Diamond consumers by raising prices, and instead has more incentive to cut prices to attract custom from shoppers. Alternatively, one can view more prominent stores as facing a more locally elastic demand curve. More marginal consumers are attracted to prominent firms by the possibility that other goods - which they value highly - are stocked there. This makes demand more sensitive at the margin to small price changes, which encourages the more prominent firms to cut price.

When store layout is unknown, there may be multiple sets of prices which solve equations (4) and (5) and satisfy $p_T^E > p_{T-1}^E > ... > p_1^E > p^m$. However unlike the case where layout is known, all equilibria have prominence. If there are multiple equilibria, there is always a Pareto dominant equilibrium. If we compare the Pareto dominant equilibria across the two cases, we get the following result:

Proposition 7 If we compare the Pareto dominant equilibria when layout is known and unknown, we …nd that
(1). p_T^E is the same under both cases
(2). p_t^E is strictly lower when layout is unknown, for all $t < T$

When layout is unknown consumers have a greater incentive to search an extra store, just in case a product that they like happens to be offered there. Consumers have less incentives to search when layout is known, because they know which products are sold by non-prominent retailers at high prices. These high expected prices at non-prominent firms deter some

[12] This explains why both demand and thickness of demand in (1) can be simplified down to give the expressions in (4) and (5).

consumers from starting search at all, which reduces the number of marginal consumers searching even prominent stores. Thus demand is more elastic when layout is unknown, which causes firms to charge lower prices. This also explains why p_T^E is unaffected by whether layout is known or unknown - since by the time a consumer has searched the other $T-1$ firms, he knows perfectly what is sold at the last store.

2.5 Discussion

The model's results apply particularly to marketplaces and supermarkets, and to an extent also shopping malls. When applying to a marketplace, there is a single entrance with T stalls located one after the other. As consumers pass by one stall they observe all the products sold there and the prices charged. In the supermarket interpretation, one firm owns nT products but, because of space considerations, must arrange them into T rows. Since all the products are independent in valuation and use, equilibrium prices are the same as in Lemma 3 and Proposition 6, despite the fact that one firm sets all prices at the same time.[13] As consumers travel through the store to find products, they notice also the prices of other products. The mall interpretation is somewhat more problematic because the models assumes that consumers must pay s to travel between market stalls/supermarket aisles, but once at a stall/aisle observe prices for free. This is less plausible in a mall, where consumers may have to pay an additional s to enter individual stores.

Remark 8 More prominent products are cheaper, and earn more pro...t

If a product costs p and the cost of search $s \to 0$, then the profit earned on that good is simply the profit earned by a standard monopolist when there is no search cost, i.e. $p[1-F(p)]$. In our model prices strictly exceed p^m, and $p[1-F(p)]$ is strictly quasiconcave. Consequently more prominent products earn strictly greater profit, because they are priced more cheaply than less prominent products. In a marketplace, stallholders should be willing to pay more to secure a prime location near any entrance. In a supermarket, manufacturers should pay slotting fees to ensure their products are placed in prominent positions, such as at the entrance or at the end of

[13] Although technically one must assume that consumer expectations are 'passive'. See the later model for more details on this.

aisles. Their products are then sold for less than usual, but contrary to the standard intuition (which is that discounted products earn less profit) they make higher profit.

The model can also explain (and give a different interpretation to) many supermarket practices. Using Proposition 7, equilibrium prices are lower (and profits per good higher) when store layout is unknown. Therefore supermarkets should periodically redesign stores, and frequently change the set of products that are promoted in end-of-aisle displays. In contrast to the standard view (which says that supermarkets move products around and put expensive items in prominent places to tempt consumers to buy things they don't want), the model says that supermarket redesigns help keep prices lower.

Supermarkets should place popular and frequently-bought items towards the back of stores. To see this, extend the earlier model by assuming there is a $T+1^{th}$ store, and that its products have $p^m = a$. Rhodes [10] demonstrates that in a single-firm model with $p^m = a$ and $s \to 0$, there exists an equilibrium in which every good is priced at p^m. (If consumers expect prices of a, almost surely they find it worthwhile to turn up, so everybody turns up, in which case it is as if $s = 0$, and so the firm charges $p^m = a$). In the current model, these products can be viewed as goods which everybody is willing to buy in equilibrium, perhaps because valuations are high. So in the current model with multiple firms, if these products are placed at the back and expected to cost a, almost surely everyone is prepared to search in order to find them. In which case everybody also walks past all the other stores, allowing them to also charge the standard monopoly price for their products - so maximum profit is extracted from consumers by everybody.

When product layout is fixed (and known), and the supermarket is considering which of the T sets of n products to advertised, we get the following result:

Lemma 9 Consider the Pareto dominant equilibrium. Then the supermarket should advertise the prices of the goods which are ***least*** prominent

Advertising the price of a product commits the firm to charging it. Consumers can be sure what the price is before they actually observe the product. The supermarket should optimally advertise low prices for products that are far back (and scattered around) its store, because this encourages consumers to search more in the store. Store traffic is increased, and more marginal

consumers are present for all the other products that are on sale. This makes demand more elastic, encouraging the firm to cut prices. Consumers recognise this, revise downwards their expectations, increasing the flow of marginal consumers still further, and causing the firm to further cut prices. This increases equilibrium profits. Notice that when some prices are advertised in this way, it will not be true that prices get strictly more expensive as one ventures deeper into a store, but it will be true that the prices of unadvertised goods display the prominence effect. The relationship between advertising and prominence runs counter to Armstrong et al [3], in which the authors argue that advertising may cause products to be more prominent. Here we argue the opposite - that advertised products should be made non-prominent, because this encourages consumers to search more.

I end this section by briefly discussing how sensitive the results are to the assumptions of the model. Although the set-up is one-shot, Rhodes [10] analyses a single-firm model, and demonstrates that pricing is similar in both a static model and a dynamic model in which consumers can learn prices over time. The assumption that $s \to 0$ makes the model tractable, but is unlikely to affect the result that more prominent firms are cheaper. Even for larger s, more prominent firms are still visited more compared to other firms, and hence less information is revealed when visiting a prominent firm. Consequently prominent firms still face a weaker sample selection problem, and hence less incentive to charge high prices. What probably would be affected is the ranking of prices under known and unknown layout. When $s \to 0$, it is always worth searching the next firm on the off chance that it may sell something the consumer wants. However the chances that the next firm stocks a particular product is small, and so when s is large, such 'speculative' search is less attractive. Only people with many strong valuations would search, which would create a stronger sample selection effect and higher prices, as compared to known layout.

3 Sponsored Ads

3.1 Assumptions

There are $n \geq 3$ firms each costlessly producing a single differentiated product. Consumers have unit demand and wish to buy at most one product. Each consumer cares only about two randomly selected goods, which I label i and j.[14] Letting v_i and v_j denote the valuations attached to these two products, the difference $d = v_i - v_j$ is assumed to be distributed on an interval $[-D, D]$ according to a symmetric, logconcave and continuously differentiable density function $h(d)$.[15] v_i and v_j are assumed to be large enough such that in equilibrium consumers always buy a product.

Consumers are perfectly informed about their valuations, but are uninformed about prices. Furthermore, I assume that consumers do not know which ...rm sells which product. There exists a website which has a list of n links - one for each firm. After clicking on a firm's link, a consumer learns what product the firm sells and what price it charges. Clicking a link and accessing this information costs a strictly positive amount s, where I set $s \to 0$. Consumers have perfect recall and can buy (at no extra cost) from any firm that they have previously searched. The website can arrange the links in one of two ways.

1. **Without prominence** Each firm has an equal probability $(\frac{1}{n})$ of being in any position in the list

2. **With prominence** One firm is always first in the list and must be searched first. All other firms have an equal probability $(\frac{1}{n-1})$ of being in any of the other $n-1$ positions in the list

The 'with prominence' case is meant to correspond to the sponsored links which one often finds when visiting websites. Firms are not able to condition their price on how many links a consumer has previously clicked on. Consumers themselves are risk-neutral, understand how the list is made, and search optimally through the list given their expectations of prices. These

[14] Without loss of generality, the valuation attached to any other good (apart from i and j) can be set to $-\infty$. The valuation structure can then be viewed as a generalisation of the Spokes model (Chen and Riordan [6]).

[15] For example d could be uniformly or normally distributed Or v_i and v_j could be *iid* uniform.

expectations are passive - meaning that if one firm charges a price different from that expected, the consumer maintains his prior beliefs about the prices charged by other firms in other (as yet unclicked) links.

As a benchmark consider the case where $s = 0$, and look for a symmetric equilibrium in which each firm charges p^*. Consumers search through the entire list, and become fully informed about prices and matches. Suppose one firm charges price p. With probability $\frac{2}{n}$ a consumer likes the firm's product and is a potential customer. Potential customers buy from the firm if it gives a higher payoff than their other favourite good, i.e. if $d \geq p - p^*$. Therefore the firm has demand $\frac{2}{n}[1 - H(p - p^*)]$. Solving the first order condition, we find that $p^* = \frac{1}{2h(0)}$.[16] Intuitively, each firm gets on average half the consumers who are interested in it, and small changes in price around the equilibrium value attract consumers with $d = 0$ who are otherwise indifferent about which of their two favourite firms to buy from.

3.2 Optimal search strategy

Now return to the case $s \to 0$. By way of notation, split the list of firms into two groups, A and B. When there is no prominence, all firms are in group B and group A is empty. When there is prominence, the prominent firm is in group A and the other $n-1$ firms are listed in group B. I look for symmetric equilibria in which each firm in group B is expected to charge the same price p_B^E. By convention, suppose that within the ordered list of n firms, the first of the two firms that the consumer likes is called i and the second one in the list that the consumer likes is called j.

Lemma 10 A consumer's optimal search rule is
(i). Click on links (in the specified order) until firm i is found
Suppose that firm i is found at link k
(ii). Then if $d \geq p_i - p_j^E - s\frac{n+1-k}{2}$ stop searching and buy i
Otherwise search on (in the specified order) until firm j is found
Then buy i if $v_i - v_j \geq p_i - p_j$, otherwise buy j

A consumer should always click on links until finding the first of his two favoured firms (part (i)), because by assumption $s \to 0$ and valuations are

[16] Since $h(d)$ is logconcave, so is demand. This implies that profit is logconcave and hence quasiconcave. This means that the first order condition is both necessary and sufficient to characterise the optimum.

high relative to (expected) prices. Part (ii) of the search rule is more interesting. Notice that $s\frac{n+1-k}{2}$ is the same as $\sum_{z=1}^{n-k} s\frac{z}{n-k}$ - which equals the expected search costs a consumer would incur if he clicked on every remaining link and only stopped when he found firm j. Therefore the decision about whether or not to click just the next link, is fully equivalent to deciding whether or not to click on every remaining link until …rm j is found. The intuition is as follows. Suppose links 1 through to k have been clicked, firm i has been found but firm j has not. Suppose further that the consumer, behaving optimally, is just indifferent about whether or not to click on link $k+1$. If link $k+1$ is clicked, firm j is found immediately with probability $\frac{1}{n-k}$ (in which case no further search occurs), and with complementary probability firm j lies in one of the remaining $n-k-1$ links. In the latter case, once link $k+1$ has been 'eliminated', there are fewer links to search, so search is more attractive than it was before. Hence if the consumer was just indifferent about clicking on link $k+1$, once that link has been clicked, he strictly prefers to click on all other links until he finds firm j.[17]

3.3 Pricing when there is no prominence

All firms are in group B and their position within the list is randomly determined after each firm has simultaneously chosen a price for its product.

Lemma 11 There exists a symmetric equilibrium in which each …rm charges $p_B^* = \frac{1}{h(0)}$

Because consumers know v_i and v_j before searching, in equilibrium they never decide to return to (and buy from) a firm that they sampled previously. The introduction of a small search cost causes the equilibrium price to exactly double in magnitude compared to the situation when $s=0$. However unlike in the earlier (super)market model, this increase in price actually benefits the firms, because all consumers continue to buy a product. To understand why price doubles, recall the distinction between shoppers and Diamond consumers, defined here as:

- A **shopper** for the k^{th} firm is a consumer (1) who likes k's product and (2) whose other favoured product is listed below k

[17] This differs from Weitzman's [11] optimal search rule, because here the payoff distribution for any given link depends upon the number (and contents) of links that have already been clicked.

- A **Diamond consumer** for the k^{th} firm is someone (1) who likes k's product but (2) whose other favoured product is listed above k

If a firm's product is a priori unattractive to a consumer (which happens with probability $\frac{n-2}{2}$), then such a consumer is irrelevant to the firm's pricing decision. Other consumers, who are potentially interested in buying the firm's product, then divide into shoppers and Diamond consumers.

Shoppers reveal no match information when clicking on a firm's link. This is because they have yet to find their other favoured product. Since $s \to 0$ and price expectations are symmetric, if a firm changes its price slightly, it affects the behaviour of those shoppers with $d \approx 0$. Shoppers are therefore as responsive to small price changes as are consumers in the case when $s = 0$.

Diamond consumers, on the other hand, are totally unresponsive to price changes. Roughly half choose not to search beyond the first favoured product they find. They do not learn (so cannot respond to) the price of their other favoured product. The half who do search beyond their first favoured product do so precisely because it is a strictly less good offer than the one they expect to receive further down the list. They value the second product strictly more than the first, and so will still buy even if its price is increased slightly.

A firm drawn at the top of the list gets only shoppers, and a firm at the bottom of the list gets only Diamond consumers. Since prices are set before firms learn where they are in the list, on average a firm expects half of potential customers to be shoppers, and the other half to be Diamond consumers. Demand is therefore only half as responsive to price changes compared to when $s = 0$, so price is twice as high.[18]

3.4 Pricing when there is prominence

I now consider the case in which one firm is made prominent, and must be searched first.

Lemma 12 There is a unique equilibrium, and prices satisfy

$$p_A^E = \frac{1 - H\left(p_A^E - p_B^E\right)}{h\left(p_A^E - p_B^E\right)} \text{ and } p_B^E = \frac{1}{h(0)} + \frac{1 - H\left(p_B^E - p_A^E\right)}{\frac{n-2}{2} h(0)}$$

[18] Requiring firms to set price before learning their list position seems crucial, but probably is not. Firms are symmetric and therefore consumers may click links randomly, in which case there is no point conditioning price on list position.

In equilibrium consumers follow the search rule in Lemma 10 and never return to a firm that they previously visited. The prominent firm's equilibrium price is uniquely defined by $p_A^E = \frac{1-H\left(p_A^E - p_B^E\right)}{h\left(p_A^E - p_B^E\right)}$. This is the same price that the firm would set in a duopoly problem with $s = 0$ when its rival charges p_B^E. Intuitively, in the current problem every consumer observes the prominent firm's price (just as they would do if in fact $s = 0$). Furthermore, consumers (are correct to) expect that non-prominent firms charge p_B^E. Since the cost of accessing these rivals is arbitrarily small, the prominent firm is effectively playing a standard game with $s = 0$ in which all firms' prices are observed, and in which its rivals choose price p_B^E.

The price charged by non-prominent firms is uniquely defined by $p_B^E = \frac{1}{h(0)} + \frac{1-H\left(p_B^E - p_A^E\right)}{\frac{n-2}{2}h(0)}$. If the non-prominent firms competed only with each other, they would charge $\frac{1}{h(0)}$ - the same price derived in Lemma 11 when nobody is prominent. However they also compete with the prominent firm. With probability $1 - H\left(p_B^E - p_A^E\right)$ a consumer (1) likes the prominent firm's product but (2) searches beyond it; such consumers strictly prefer one of the products available at the non-prominent firms, and are therefore Diamond consumers. Small increases in price above p_B^E therefore lose none of these consumers, whose mass is proportional to $1 - H\left(p_B^E - p_A^E\right)$.

Given the above discussion, it is apparent that non-prominent firms charge strictly more than $\frac{1}{h(0)}$. Making one firm prominent therefore causes the $n-1$ non-prominent firms to strictly increase their prices. The firms that are made non-prominent know that they can only be drawn in positions $2, 3, ..., n$ when a consumer searches the market. The firms therefore face a stronger sample selection problem, because a consumer reveals more information when visiting them; fewer visitors are shoppers, and more are Diamond consumers. What happens to the price charged by the firm that is made prominent, is actually ambiguous. One the one hand, demand is composed only of shoppers and therefore becomes more elastic. But on the other hand, the prominent firm's competitors charge more, and competition is in strategic complements. When $h(d)$ is 'sufficiently close to being uniform', the prominent firm's price falls when it is made prominent. This is true because $p_A^E = \frac{1}{h\left(p_A^E - p_B^E\right)} - \frac{H\left(p_A^E - p_B^E\right)}{h\left(p_A^E - p_B^E\right)}$, $H\left(p_A^E - p_B^E\right) > 0$ and for a uniform distribution $h\left(p_A^E - p_B^E\right) = h(0)$.

Although it is difficult to make general statements about whether the prominent firm's price increases or decreases, we do get a very strong result on the difference in prominent and non-prominent prices:

Proposition 13 The prominent …rm charges a strictly lower price than the non-prominent …rms

The prominent firm faces a more elastic demand curve, because everybody searches it and therefore reveals no match information for the firm to exploit. Non-prominent firms are more likely to be searched second rather than first by consumers who like their product; they therefore face a demand curve which is less elastic. A greater proportion of people who click on their links are Diamond consumers, and there is less opportunity for them to inform consumers about the price that they charge.

What is particularly noteworthy about these results is that they hold even though $s \to 0$. A small change in the way links are presented, and a small cost of clicking on additional links, has a large impact on firms' pricing decisions and profits.

Remark 14 The prominent …rm earns strictly more pro…t than the non-prominent …rms

The prominent firm could always choose to mimic the non-prominent firms and price as they do, but instead can do better by cutting price and taking advantage of his more elastic demand curve. Since the prominent firm makes more profit, companies are willing to bid more to secure the top spot in sponsored ads - much like we observe in reality.

3.5 Discussion

One important feature of the model concerns the way in which options are presented. To illustrate, consider the market for loans. Suppose that each bank sells a single loan product, and that loans are differentiated by both their term and the amount that can be borrowed. Our model supposes that the consumer visits Google, types in "loans", and is then presented with a randomly generated list of banks. But, if the consumer knows that he values most highly a £5,000 loan repayable over two years, why not just type that into Google? The reason is as follows. If Google returned just a single link to the firm offering that particular loan product, then everybody who visits

the firm is a Diamond consumer. The firm understands that anybody who visits its site values its product highly and will not be deterred from buying if the actual price is slightly higher than was expected. As in the Diamond [7] Paradox, the market breaks down. Therefore our model suggests that even if consumers do visit a search engine with very specific queries, a generic randomly drawn results list should still be returned - otherwise over time firms will respond by raising prices, until the market ceases to exist.

Another important feature of the model is the assumption that, when one firm is made prominent, all consumers search it first. The website could of course force consumers to search prominent firms first, perhaps by requiring additional searches/clicks to find information on non-prominent firms. However as Armstrong et al [3] point out, the fact that prominent firms can be expected to set lower prices means that it is entirely reasonable for consumers to search them first anyway. If consumers understand that some firms (have paid more money and) expect to be searched first, they expect these firms to be cheaper and therefore do indeed search them first.

I end this Section with a short example, which illustrates many of the points which have been discussed

Example Suppose that $d \sim U[-D, D]$

When there is no prominence, all firms charge $2D$

With prominence, $p_A^E = \frac{D}{n-1}\left[\frac{3n}{2} - 1\right] < 2D < \frac{D}{n-1}[2n-1] = p_B^E$. So when one firm is made prominent, it becomes strictly cheaper and all other firms become strictly more expensive. The difference in prices $p_B^E - p_A^E = \frac{D}{2}\frac{n}{n-1}$ is significant, and especially for low n, the first firm receives a large proportion of total clicks. It can also be shown that if the platform can extract firm profits, it prefers to make one firm prominent, even though this harms overall welfare (and therefore also harms consumer welfare).

A Proofs

Proof of Lemma 3: Take (1), let $s \to 0$, and start with demand. If $p_t^E < b$ then $p_t^E + ts < b$ and consumers with $v_{1,t} \geq p_t^E + ts$ definitely search for the firm and buy. So rewrite $\int_{p_t^E}^{b} f(v_{1,t}) \Pr\left(t; s, \mathbf{v}, \mathbf{p^E}, L\right) dv_{1,t}$ as $1 - F\left(p_t^E + ts\right) + \int_{p_t^E}^{p_t^E+ts} f(v_{1,t}) \Pr\left(t; s, \mathbf{v}, \mathbf{p^E}, L\right) dv_{1,t}$; since $s \to 0$ this limits to $1 - F\left(p_t^E\right)$. Now consider thickness of demand. If $v_{1,t} = p_t^E$, don't visit store t if $v_{j,t} \leq p_t^E \forall j \geq 2$ and $v_{j,z} \leq p_z^E \forall j, \forall z > t$; hence $\Pr\left(z; \cdot, v_{1,z} = p_z^E\right) < 1 - F\left(p_t^E\right)^{n-1} \prod_{z=t+1}^{T} F\left(p_z^E\right)^n$. If $v_{1,t} = p_t^E$, definitely visit t if $v_{j,t} \geq p_t^E + ts$ some $j \geq 2$ or $v_{j,z} \geq p_z^E + tz$ some j and some $z > t$; hence $\Pr\left(z; \cdot, v_{1,z} = p_z^E\right) \geq 1 - F\left(p_t^E + ts\right)^{n-1} \prod_{z=t+1}^{T} F\left(p_z^E + sz\right)^n$. So as $s \to 0$, $\Pr\left(z; \cdot, v_{1,z} = p_z^E\right) \to 1 - F\left(p_t^E\right)^{n-1} \prod_{z=t+1}^{T} F\left(p_z^E\right)^n$.

We still need to check that the first order condition in (1) is sufficient to characterise the equilibrium. Following Rhodes [10] the problem is concave in p_t for $p_t \leq p_t^E + ts$ provided that the standard monopoly problem $\max p\left[1 - F(p)\right]$ is concave for $p \leq p_t^E + ts$. This will be true as $s \to 0$ provided that n is large enough. So $p_t = p_t^E$ is strictly preferred to all other prices up to $p_t^E + ts$. For $p_t \geq p_t^E + ts$ profit is equal to $p_t\left[1 - F(p_t)\right]$, which is quasiconcave and decreasing in p_t. ■

Proof of Proposition 4: Consider firm t's equilibrium condition (3). This is strictly positive when $p_t^E = p^m$ because $p_z^E \geq p^m$ by Remark 2. Using Rhodes [10] and the fact that n is large, there exist non-degenerate equilibrium prices for firm T's problem. Pick the lowest. Firm $T-1$'s equilibrium condition is strictly negative if $p_{T-1}^E = \min p_T^E$ (where with abuse of notation, $\min p_T^E$ is the lowest equilibrium solution to firm T's problem) and continuous, so $\min p_{T-1}^E < \min p_T^E$. Choose $\min p_{T-1}^E$ and work backwards in an analogous fashion. Hence when one picks the lowest equilibrium prices, $p_T^E > p_{T-1}^E > ... > p_1^E > p^m$. It remains to show that such prices Pareto dominate all other equilibria. If there is a unique set of equilibrium prices we are done. If not, pick any vector of equilibrium prices. Given $p_2^E, ..., p_n^E$, consider firm 1's equilibrium price. If there are multiple equilibria and the lowest is not being picked, picking it raises aggregate welfare. Pick it. Given $p_3^E, ..., p_n^E$, consider firm 2's equilibrium price. If there are multiple equilibria and the lowest is not being picked, picking it raises welfare on firm 2's goods. It also means that at the old p_1^E, firm 1's equilibrium condition is negative, so there exists a new lowest p_1^E which is lower than the previous - so aggregate

welfare has definitely increased. Proceeding like this for all firms, aggregate welfare must strictly increase for all equilibrium price vectors except the one where we already chose the lowest possible prices. ■

Proof of Proposition 6: Take (1), let $s \to 0$, assume $p_t^E < b$ and start with demand. If $\frac{1}{T}\left(v_{1,t} - p_t^E\right) \geq ts$ a consumer definitely searches to firm t and buys, so rewrite $\int_{p_t^E}^{b} f\left(v_{1,t}\right) \Pr\left(t; s, \mathbf{v}, \mathbf{p^E}, L\right) dv_{1,t}$ as $1 - F\left(p_t^E + Tts\right) + \int_{p_t^E}^{p_t^E+Tts} f\left(v_{1,t}\right) \Pr\left(t; s, \mathbf{v}, \mathbf{p^E}, L\right) dv_{1,t}$; since $s \to 0$ this limits to $1 - F\left(p_t^E\right)$. Consider thickness of demand at firm t. Note from the text that in any equilibrium $p_t^E \leq p_{t+1}^E$. If $v_{1,t} = p_t^E$, no consumer visits firm t if they value all the other products sold by firm t onwards at less than p_t^E; hence $\Pr\left(t; \cdot, v_{1,t} = p_t^E\right) < 1 - F\left(p_t^E\right)^{(T-t+1)n-1}$. If $v_{1,t} = p_t^E$, a consumer definitely visits firm t if they value any of the other products sold by firm t onwards at more than $p_t^E + Tts$[19]; hence $\Pr\left(t; \cdot, v_{1,t} = p_t^E\right) \geq 1 - F\left(p_t^E + Tts\right)^{(T-t+1)n-1}$. So as $s \to 0$, $\Pr\left(t; \cdot, v_{1,t} = p_t^E\right) \to 1 - F\left(p_t^E\right)^{(T-t+1)n-1}$. This explains (4) and (5).

Find a solution for p_T^E to (4). In (5) when $t = T-1$, the lefthand side is continuous, strictly positive when $p_{T-1}^E = p^m$ and strictly negative when $p_{T-1}^E = p_T^E$. Therefore there exist equilibrium values for p_{T-1}^E that lie strictly between p^m and p_T^E. Proceeding in the same manner for all prices yields the inequality $p_T^E > p_{T-1}^E > ... > p_1^E > p^m$.

These equilibrium conditions can be shown to be both necessary and sufficient, using a similar technique as for the case of known layout. ■

Proof of Lemma 9: Calculate total profits earned when 'firm' t's prices are advertised (at their optimal level p_t^*). Now suppose we advertise firm $t+1$'s prices instead, but at level p_t^*. Prices (and hence profits) of firms $t+2, ..., T$ are the same. Since $s \to 0$, firm $t+1$ earns the same profit that firm t previously earnt. But firm t now earns more than firm $t+1$ did, because in equation (3) thickness is larger so the lowest equilibrium price is smaller. This then implies that thickness of demand is also greater for firms $1, ..., t-1$ so all these firms now earn strictly greater profit. This is based on the assumption that firm $t+1$ uses the advertised prices p_t^* that were optimal when firm t was doing the advertising. In fact we can increase profits even more by reoptimising and choosing different prices for firm $t+1$. Since this arguments holds for all t, firm T's prices should be advertised. ■

[19] At the beginning, firm t stocks one of these products with probability $\frac{1}{T}$, and it costs ts to search that far.

Proof of Lemma 10: Consider the second part of (ii). Suppose $\{1,...,y\}$ have been searched, i has been found, and that once $y+1$ is clicked the consumer optimally clicks on all other links until finding j. Not clicking $y+1$ gives payoff $v_i - p_i$; clicking $y+1$ gives payoff $v_j - p_j^E - \sum_{z=1}^{n-y} s\frac{z}{n-y}$. The latter increases in y since $\sum_{z=1}^{n-y} s\frac{z}{n-y} = s\frac{n+1-y}{2}$ decreases in y. Therefore if i is at link k and $d < p_i - p_j^E - s\frac{n+1-k}{2}$ each subsequent link should be clicked until j is located. For the first part of (ii), suppose $d \in \left[p_i - p_j^E - s\frac{n+2-y}{2}, p_i - p_j^E - s\frac{n+1-y}{2}\right)$ and $y \in \{k+1,...,n-1\}$. Once $\{1,...,y\}$ are searched, the consumer clicks all links until j is found (since $d < p_i - p_j^E - s\frac{n+1-y}{2}$). Hence if only $\{1,...,y-1\}$ are searched, the consumer will not click on y (since $d \geq p_i - p_j^E - s\frac{n+2-y}{2}$). Clicking on $y-1$ therefore gives $\frac{1}{n-y+2}\left(v_j - p_j^E\right) + \frac{n-y}{n-y+2}\left(v_i - p_i\right) - s$, and not clicking gives $v_i - p_i$. Since $d > p_i - p_j^E - s\left(n+2-y\right)$, not clicking is preferred. Similar logic implies no previous links are clicked on either; a similar procedure is used if $d \geq p_i - p_j^E - s$. Therefore if i is at link k and $d \geq p_i - p_j^E - s\frac{n+1-k}{2}$ search is stopped immediately. ■

Proof of Lemma 11: See the proof of Lemma **12**. If $p_A^E = p_B^E - D$ then non-prominent firms get no demand from the first firm, so it is as if there is no first firm and the size of the market (which has no effect on marginal decisions) has been changed. ■

Proof of Lemma 12 and Proposition 13: From Steps A and B (see below), $p_B^E = \frac{1}{h(0)} + \frac{1-H\left(p_B^E - p_A^E\right)}{\frac{n-2}{2}h(0)} \geq \frac{1}{h(0)}$ and $1 - H\left(p_A^E - p_B^E\right) - p_A^E h\left(p_A^E - p_B^E\right) \leq 0$. In a corner solution, $p_A^E = p_B^E - D$ and $p_A^E = p_B^E - D$, so $p_A^E = \frac{1}{h(0)} - D$. To satisfy A's equilibrium condition, we need $1 - p_A^E h\left(-D\right) \leq 0$, or $\frac{1}{h(0)} - D \geq \frac{1}{h(D)}$. But by logconcavity $h\left(D\right) < h\left(0\right)$ so $\frac{1}{h(D)} > \frac{1}{h(0)} > \frac{1}{h(0)} - D$, yielding a contradiction. So any solution must be interior.

Therefore any equilibrium satisfies $p_A^E = \frac{1-H\left(p_A^E - p_B^E\right)}{h\left(p_A^E - p_B^E\right)}$ and $p_B^E = \frac{1}{h(0)} + \frac{1-H\left(p_B^E - p_A^E\right)}{\frac{n-2}{2}h(0)}$. Writing $y = p_A^E - p_B^E$, we have $y = \frac{1-H(y)}{h(y)} - \frac{1}{h(0)} - \frac{H(y)}{\frac{n-2}{2}h(0)}$ (*). The lefthand side strictly increases in y whilst the righthand side strictly decreases in y so there is at most one solution. When $y = -D$, the righthand side is $\frac{1}{h(y)} - \frac{1}{h(0)} > 0$ so the lefthand side is bigger. When $y = 0$ the righthand side is $\frac{-1}{2h(0)} - \frac{1}{(n-2)h(0)} < 0$ so the lefthand side is smaller. By continuity there exists a unique solution $y \in (-D, 0)$. This uniquely pins down both p_A^E and p_B^E. ■

Step A $1-H\left(p_A^E-p_B^E-s\frac{n}{2}\right)-p_A h\left(p_A^E-p_B^E-s\frac{n}{2}\right)\leq 0$

Proof: a consumer who likes the prominent firm's product buys it (without further search) if $v_i-p_A\geq v_j-p_B^E-s\frac{n}{2}$, so the prominent firm's profit is proportional to $p_A\left[1-H\left(p_A-p_B^E-s\frac{n}{2}\right)\right]$. This is logconcave (and so quasiconcave) in p_A, and the first order condition gives $1-H\left(p_A^E-p_B^E-s\frac{n}{2}\right)-p_A h\left(p_A^E-p_B^E-s\frac{n}{2}\right)\leq 0$. ■

Step B $p_B^E=\frac{1}{h(0)}+\frac{1-H\left(p_B^E-p_A^E\right)}{\frac{n-2}{2}h(0)}$

Proof: A non-prominent firm has any list position $k\in\{2,...,n\}$ with probability $\frac{1}{n-1}$, and with probability $\frac{2}{n}$ a consumer likes their product. Suppose it sets price p_B.

With probability $\frac{n-k}{n-1}$ a potential customer has not found their other favourite product, and buys (without further search) if $v_i-p_B\geq v_j-p_B^E-\left(\frac{n+1-k}{2}\right)s$. This makes a contribution to demand which is equal to $\frac{2}{n(n-1)}\sum_{k=2}^{n}\frac{n-k}{n-1}\left[1-H\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)\right]$.

With probability $\frac{1}{n-1}$ the consumer's other favourite product is supplied by the prominent firm. The consumer searches for his non-prominent favourite only if (in equilibrium) $v_i-p_A^E<v_j-p_B^E-\frac{n}{2}s$, and buys from the non-prominent favourite only if $v_i-p_A^E<v_j-p_B$ (because it is costless to return to a previously sampled firm). This contribution to the non-prominent firm's demand is then $\frac{2}{n(n-1)^2}\sum_{k=2}^{n}\min\left\{H\left(p_A^E-p_B^E-\frac{n}{2}s\right),H\left(p_A^E-p_B\right)\right\}$ or simply $\frac{2}{n(n-1)}\min\left\{1-H\left(p_B^E-p_A^E+\frac{n}{2}s\right),1-H\left(p_B-p_A^E\right)\right\}$.

With probability $\frac{1}{n-1}$ the consumer's other favourite lies in position $j\in\{2,...,k-1\}$, 6(i)21793f

or $\frac{2}{n(n-1)}\left[\frac{n-2}{2}+1-H\left(p_B^E-p_A^E+\frac{n}{2}s\right)\right]$. $\left.\frac{\partial D}{\partial p_B}\right|_{p_B=p_B^E}=-\frac{2}{n(n-1)}\sum_{k=2}^{n}\frac{n-k}{n-1}h\left(\left(\frac{n+1-k}{2}\right)s\right)$. So letting $s\to 0$,

$$p_B^E=\frac{1}{h(0)}+\frac{1-H\left(p_B^E-p_A^E\right)}{\frac{n-2}{2}h(0)}$$

It is now necessary to show that setting $p_B=p_B^E$ is globally optimal.

Consider the demand $\tilde{D}=\frac{2}{n(n-1)}\sum_{k=2}^{n}\frac{n-k}{n-1}\left[1-H\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)\right]+\frac{2}{n(n-1)}\left[1-H\left(p_B^E-p_A^E+\frac{n}{2}s\right)\right]+\frac{2}{n(n-1)}\sum_{j=2}^{n}\frac{n-j}{n-1}\left[1-H\left(\left(\frac{n+1-j}{2}\right)s\right)\right]$. If $p_B\leq p_B^E+s$ then $D=\tilde{D}$; otherwise $\tilde{D}>D$. The second derivative of $p_B\tilde{D}$ with respect to p_B is a weighted sum of terms of the form $-2h\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)-p_Bh'\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)$. These terms are certainly negative when $p_B\leq p_B^E+s$ since then $p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\leq 0$, and $h'(x)\geq 0$ for any $x\leq 0$ by logconcavity. Hence $p_B\tilde{D}$ is concave in this region.[20] The terms are also negative for $p_B\in\left(p_B^E+s,p_B^E+\frac{n}{2}s\right)$. Even if $p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s>0$, we know that $h'(0)=0$ so by continuity for any $\delta>0$, there exists $\epsilon>0$ such that for all $s<\epsilon$, $h'\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)<\delta$. Therefore by choosing s to be small enough, $p_B\tilde{D}$ is still concave in p_B. p_B^E is preferred to any other price below p_B^E+s when demand is D', so the same is true when demand is in fact D.

To prove that p_B^E is also preferred to any $p_B\geq p_B^E+\frac{n}{2}s$, it is sufficient to prove that the first derivative of p_BD is negative in this region. To prove that, it is sufficient to show that (1). $1-H\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)-p_Bh\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)$, (2). $1-H\left(p_B-p_A^E\right)-p_Bh\left(p_B-p_A^E\right)$ and (3). $1-H\left(p_B-p_B^E\right)-p_Bh\left(p_B-p_B^E\right)$ are all negative. Notice that (1). is the first derivative of $p_B\left[1-H\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)\right]$, which is quasiconcave in p_B. Setting the first derivative to zero, we find $p_B=\frac{1-H\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)}{h\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)}$ where the lefthand side increases in p_B and the righthand side decreases in p_B. If we set $p_B=p_B^E$, we find a contradiction because $p_B^E>\frac{1}{2h(0)}\approx\frac{1-H\left(-\left(\frac{n+1-k}{2}\right)s\right)}{h\left(-\left(\frac{n+1-k}{2}\right)s\right)}$.

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[20] One additional step is the following. Demand is really a sum of demand segments. If p_B is low enough, $p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s<-D$, the firm gets all demand on the k^{th} segment, and this demand segment is unresponsive to price changes. The derivative with respect to price of this demand segment is 1 (i.e. demand). When $p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s=-D$, this demand segment suddenly becomes responsive to price, and the derivative with respect to price of this demand segment is $1-h\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)$. I.e. the profit function is more positive at low prices, and hence is concave - although it is kinked at several points, because there are several demand segments.

So the optimal p_B would be below p_B^E, implying that the first derivative of $p_B\left[1-H\left(p_B-p_B^E-\left(\frac{n+1-k}{2}\right)s\right)\right]$ would be (weakly) negative whenever $p_B \geq p_B^E+\frac{n}{2}s$. A similar approach can be used for (3). Also for (2). which is the first derivative of $p_B\left[1-H\left(p_B-p_A^E\right)\right]$ which is also quasiconcave in p_B. Setting the first derivative equal to 0 gives $p_B = \frac{1-H\left(p_B-p_A^E\right)}{h\left(p_B-p_A^E\right)}$, where again the lefthand side increases in p_B but the righthand side decreases. Trying $p_B = p_A^E$ again gives a contradiction because $p_A^E > \frac{1}{2h(0)}$, so the p_B that solves $p_B = \frac{1-H\left(p_B-p_A^E\right)}{h\left(p_B-p_A^E\right)}$ is below $p_A^E < p_B^E$. Therefore (2). is certainly (weakly) negative when $p_B \geq p_B^E+\frac{n}{2}s$. ■

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