Market Effects of Patent Reform in the U.S. Semiconductor Industry

Jeff Thurk
University of Texas at Austin

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Abstract

The increase in US patenting activity behavior is a much publicized and researched topic. The US semiconductor industry is particularly interesting as evidence suggests firms use patents not to appropriate rents or protect R&D investments, but rather to strategically enable future R&D efforts. I study this industry in the context of early 1980's US legal reform which strengthened patent rights. Prior to the reform, the industry was highly concentrated and characterized by vertically-integrated firms. Post reform, however, entry of niche product firms led to a fragmented industry composed of both specialty design firms and traditional production firms. The concurrent increase in patent propensity has led other authors to hypothesize that this relationship is causal. As the early 1980's was a period of significant change for the industry (e.g., introduction of the PC), I seek to understand how much of the change in market structure is from patent reform.

To answer this question, I introduce a model of innovation and patenting in which firms choose R&D effort, licensing expense, and patent stock. Innovation requires both novel "in-sourced" innovation and the licensing of others' ideas. The level of patent protection determines to what degree a firm must license others' ideas. Firms gain value through both traditional production and licensing revenue, where the latter is a function of their patent stock. I modify the notion of an *Oblivious Equilibrium* introduced by Weintraub, Benkard, and Von Roy (2008) to approximate the Markov-Perfect Equilibrium with a dominant firm (Texas Instruments).

1 Introduction

The early to mid 1980s marked a period of important shifts in US patent policy. Perhaps the most significant event occurred in 1982 with the establishment of the Court of Appeals for the Federal Circuit (CAFC). The establishment of the CAFC was given national jurisdiction over patent claims, hence unifying and standardizing the legal treatment of patents.[1] The resulting case law has largely strengthened the rights of patent-owners.

The effects of patent reform on innovation and patenting intensity is less obvious. Studies indicate that patent reform did not affect firms' reliance on patents as a conduit to appropriating rents from innovation (see Cohen et al. [2000]). Despite these findings, patent applications increased dramatically during the 1980s and 1990s. The natural question is why do firms file for patents when they acknowledge that patents are a poor mechanism for appropriating rents?

This patent paradox exists in many industries, but perhaps no more notably than in the US semiconductor industry. The semiconductor industry is perhaps the best example of an industry with little reason to patent, yet does so intensely. Innovation within this industry is fast-paced, meaning that any invention today will likely be antiquated tomorrow. It is, however, an industry where innovation is cumulative and products sufficiently complex that any new product requires a pool of patentable ideas. Nonetheless, Hall and Ziedonis [2001] note that semiconductor R&D managers use patents to serve alternative, strategic purposes and that stronger levels of protection enable small firms to enter certain niche markets. Both of these trends are

Of course, the static model is too sample to actually match the data we see in the world. I model an environment similar to Ericson and Pakes [1995] where firms choose R&D effort, licensing expense, and patent stock. Innovation requires both novel "in-sourced" innovation and the licensing of others' ideas. The level of patent protection determines to what degree a firm must license others' ideas. Firms gain value through both traditional production and licensing revenue. The latter is a function of their patent stock. Strong protection fragments the industry by enabling niche product firms to use licensing revenue to overcome fixed costs of entry.

I modify the notion of an *Oblivious Equilibrium* (OE) introduced by Weintraub et al. [2008] to approximate the Markov-Perfect Equilibrium with a dominant firm (Texas Instruments). The following section outlines empirical and survey data. Sections 5 and 6 introduce the model and equilibrium concept. Sections 7 and 8 provide details on the calibration and estimation strategy, while section 9 provides the results. I conduct a counterfactual exercise in section 10, in which I simulate the model without patent reform. I provide concluding remarks in section 11.

2 Motivation

2.1 US Patent Reform

Congress' creation of the Court of Appeals for the Federal Circuit (CAFC) in hopes of unifying case law regarding patent claims. While unifying patent judgments does not necessarily imply stronger protection, the court's pro-patent judgments increased the value of patent rights. Prior to the creation of CAFC, district appeals courts invalidated patent rights approximately 60 percent of the time. The CAFC, on the other hand, was found to uphold the majority of patent claims, invalidating claims approximately 30 percent of the time.[2] It is important to note that the creation of the CAFC in 1982 should not be viewed as an instantaneous shift in US patent law, but rather the decisions made by this court through the early and mid-1980s substantively changed the definition and value of patent rights. One should view the following years as a transition towards a more pro-patent regime.

2.2 Changing Dynamics in the Semiconductor Industry

The US semiconductor industry is a large and influential component of the global economy, creating the essential building blocks of our computer age. Comparative advantage comes through innovation and innovation is a fast-paced, cumulative effort in which tomorrow's new product depends heavily on a broad set of today's.

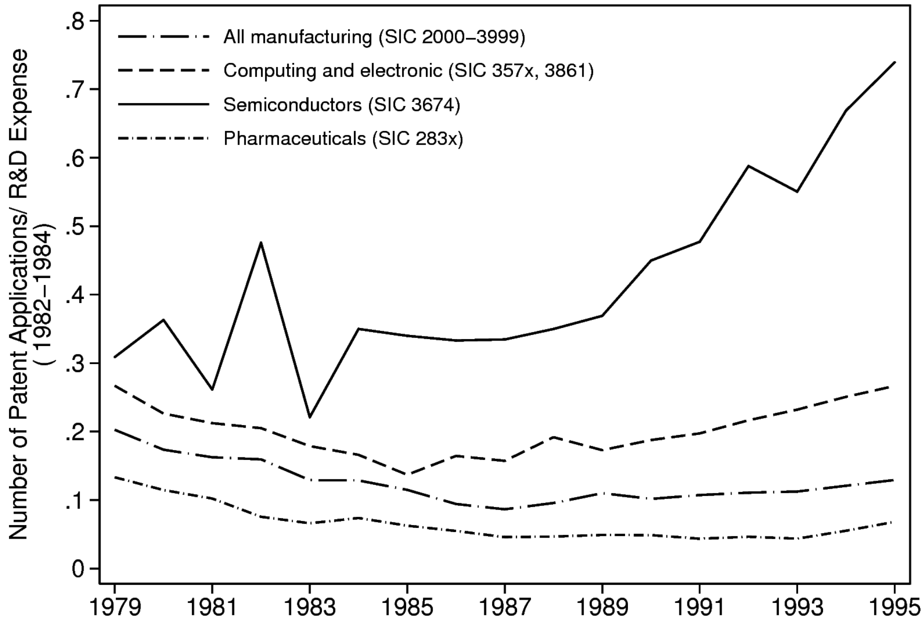
To analyze some fundamental trends in this market, I constructed a data set similar to Hall and Ziedonis [2001]. Since I'm interested in competition and innovation within the semiconductor industry, I isolated attention to publicly-traded firms whose principal business line is semiconductors and related devices (SIC3674). I used the NBER Patent Citation Database to compile a detailed collection of these firms' patenting efforts and matched the results with innovation-related financial information Compustat.[3] The result is a fairly comprehensive sample of semiconductor firms, their propensity to patent, their financial performance, their R&D efforts, and their size.

The data set enables me to compare firms' patent propensity across time. This analysis becomes increasingly interesting when comparing across industries.

[2] See Hunt [1999]

[3] Details regarding the data set are in the appendix.

Figure 1: Comparing Patent Propensity Across Industries



The above figure is similar to the one presented by Hall and Ziedonis [2001] and shows how semiconductor firms have become increasingly active in the patenting their inventions.[4] The increasing dashed line indicates that the number of patent applications per million of research and development expense has steadily increased since the early to mid 1980s. It is further instructional to note the sharp increase in patent applications around the 1982 creation of the CAFC. The increase may indicate firms decision to patent previous trade secrets, while the subsequent dip may represent a return to trend. This hypothesis would suggest that the more pronounced upward patenting trend occurring in the mid to late 80s is the result of CAFC-related pro-patent judgments.

Hall and Ziedonis [2001] also note that the composition of firms has changed substantially.

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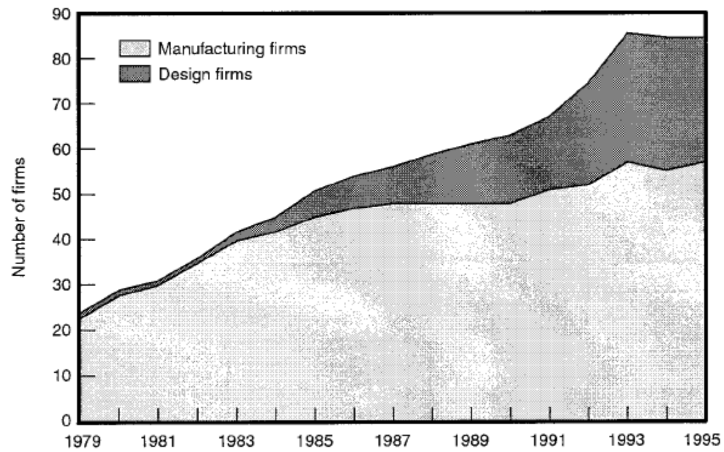
[4] There is one fundamental di erence. Whereas Hall and Ziedonis [2001] correlate patent grants with R&D expense, I compare the number of patent applications to a rm's R&D expense. Since I'm interested in rm's decisions whether or not to patent an innovation, using applications (and the year the application is made) seems more appropriate.

Figure 2: Firm Composition over Time



Source: Hall and Ziedonis (2001)

Early in the sample, firms were largely one-stop shops, integrating design through production and sales. This concentrated industry changes quite a bit through the sample, as the entrance and increasing prevalence of specialized design firms marked a significant shift in market structure. Whereas a traditional firm such as Intel can design, produce, and sell its product in-house, these design firms lacked any kind of production/ fabrication abilities. They instead focused on designing next generation technologies and selling/ licensing these technologies to the fabrication firms.

This market shift is also evident along other metrics.

Table 1: Descriptive Statistics

	1970-1982	1982-1995	Comments
Number of Firms (N)	40:9	73:0	
Patenting Firms (N)	14:0	28:1	
Average			
- Revenue ($MM)	323	366	
- Income ($MM)	15	25	
- Patents (N)	20:0	47:4	Patenting firms
- Patents/ R&D	0:38	0:50	Patenting firms
Market Size ($MM)	12; 913	27; 835	

Table 1 compares the increasing propensity to patent with other market statistics. Discussed above is the increasing propensity to patent, where patent applications per R&D $million increased from 0.38 to 0.50, as well as the increasing number of firms in the industry (41 to 73). While the share of patenting firms increases slightly (14/41 to 28/73) suggests a slight increase in patenting firms, one should not rely on this statistic to much since it may not capture patenting activities of new firms.[5] The most notable data statistic is the 225% increase in real market size across the sample. This increase reflects the increasing importance of semiconductor-related products such as the personal computer.

[5] Approximately 65% of the US rms holding a patent are accounted for in the NBER data set. The remaining 35% of patents are unmatched either because of di culty matching names or because the rm was not publicly-traded in 1989, the data of the matching rm name le.

Survey evidence is the final piece of the puzzle. Hall and Ziedonis [2001] interviews with semiconductor R&D managers indicates that these managers believe patents are a poor mechanism to appropriate rents from innovative efforts. These managers believe that one can leverage the comparative advantage of a particular idea by keeping it a secret, beating the competition to market, and by leveraging the firm's production advantages.

This reluctance to use patents in the traditional manner (i.e., gain monopoly rents) does not mean these managers view patents as inherently worthless. Instead, the authors show that managers are using their patent portfolios in more strategic ways. For big production firms, large patent portfolios can insulate them from the risk of becoming bogged down in infringement litigation down the line. The opposite is true for small firms. Small firms often have a novel niche idea but lack the production and/or industry knowledge to successfully commercialize the idea. Strong protection enables these firms to protect their ideas and enter the market leveraging their patent rights to gain licensing revenue.

3 Related Literature and Contribution

The literature is largely based on empirical, reduced-form studies. Boldrin and Levin [2008] ask whether increasing levels of protection encourage innovation and provide an extensive review of studies indicating the answer is no. In the semiconductor industry, early studies focused on the patenting motives (Tilton 1971, Taylor and Silbertson 1973, Levin 1982, von Hippel 1988), while later studies have focused on linking patenting behavior with R&D expense. Kortum and Lerner [1998] focus less on innovative effort and more on explaining patenting trends in national data. In particular, they test whether increasing US protection during the 1980s caused the increase in patenting. They find little evidence in the aggregate data, and instead conclude that the increase is due to an increase in R&D productivity amongst firms.

Hall and Ziedonis [2001] ask a similar question, but restrict their analysis to just the semiconductor industry. They reason that the aggregate data used in Kortum and Lerner [1998] hid industry-specific effects of the protection shift, and that across industries these effects canceled out. Using both reduced-form and survey evidence, they conclude that patent reform had two effects. First, it promoted fragmentation by enabling small niche firms to enter the industry. Second, it resulted in large firms becoming engaged in patent portfolio races in order to streamline future innovation.

Hunt [1996] studies spill-over effects in the US semiconductor industry. He finds evidence of a significant shift in competition during the late 1980s or early 1990s. Whereas reverse-engineering had previously enabled innovations to diffuse to competitors, the data indicates semiconductor firms moved to protect their innovations with patents. The consequence was a shift towards creating next-generation technologies based on competitors' licensed, rather than imitated, ideas.

This paper represents a contribution along many fronts. First, it addresses how much of the change in market structure can be attributed to the change in patent protection- a question the empirical literature cannot address. Second, the paper melds many of the ideas presented in the literature. I use the findings expressed inBoldrin and Levin [2008] and model patent protection without imposing any assumptions as to how protection may influence innovative effort. Instead, I use the findings of Hall and Ziedonis [2001] to explain the market shift. Finally, the paper represents a methodological contribution. I create a stationary equilibrium concept with dominant firms, an *Partial Oblivious Equilibrium*, which is a modification of the *Oblivious Equilibrium* developed by Weintraub et al. [2008]. I use this structure to extend the recent work of Yu [2008] and Qi [2008] to evaluate the transitional dynamics of government policy.

4 The Semiconductor Industry

TBD

5 Model

The model captures many of the trends and industry dynamics noted in the literature. Firms earn profit by producing and selling a product, but they may also earn licensing revenue. Firms choose how much R&D to conduct, how much to spend on licensing others' ideas, and whether to adjust the patent stock. High levels of patent protection require the firm to license competitors' technologies, hence increasing its licensing expense. Total market licensing revenue is endogenous in the model and is a function of the level of protection. A firm's licensing revenue is related to its share of patents in the industry. Increasing protection leads to more licensing expense and larger patent portfolios. It also leads to increased entry as small firms use licensing revenue, rather than profit, to overcome the fixed cost of entry.

5.1 Environment and Notation

The model is an extension of Ericson and Pakes [1995] in which firms make innovation, licensing, and patent decisions in discrete periods (t) over an infinite horizon, where $t \in \mathbb{N}$. Firms are heterogenous in their level of productivity ($x = 1, ..., \bar{x}$) and their number of patents ($k = 1, ..., \bar{k}$) Define $_t \in \mathcal{M}$ as the distribution firms where $_t(x, k)$ is the mass of firms with productivity level x and stock of patents k.

Incumbent firms enter each period with state $(x, k, \ )$ and earn profits and licensing revenue in spot markets. That is, the firm cannot influence either through its decisions. The expected profit for firm i $(x_{it}, \ _t)$ is an increasing function of its productivity (x_{it}) and a decreasing function[6] of distribution of firm productivity $(\ _t)$. Similarly, expected licensing revenue $(k_{it}, \ _t)$ is increasing in the stock of patents and decreasing in the distribution of patent stocks.[7]

Incumbent firms make three choices. First, I endogenize exit using the following mechanism. Each period incumbent firms observe an idiosyncratic "sell-off" value $\sim \Upsilon$. The value of is private information. An exiting firm is one whose is greater than the discounted value of remaining in the industry. The exiting firm enters the period, earns profit and licensing revenue, earns the sell-off value, and exits permanently.

Second, they choose how much to invest R&D through a combination of in "in-house" R&D (call this) and licensing of others' ideas (call this). Patent protection ( ) influences the substitutability of these inputs. While the outcome of R&D is stochastic, the firm can improve its chances of attaining a better productivity by increasing its R&D investment. After the outcome of R&D is known, incumbent firms to adjust their stock of patents.

Third, after observing the outcome of their R&D, firms decide the number of patent applications. The firm incurs patent adjustment cost $C(k_{t+1}, k_t)$ where k_{t+1} is the stock of patents in the next period. Since the data indicates the existence of a non-trivial application rejection rate, the actual fraction of granted patents is distributed $G(k_{t+1}, k_t)$. Note a one-period lag between application and grant - applications made today are granted by the USPTO next period.

The final component is firm entry. A firm choosing to enter the industry pays a one-time fixed cost f_e. Ex ante, all firms are identical and draw initial productivity from a distribution (F_e). An entering firm pays f_e

[6] By this I mean that as the average productivity increases, the rm's pro ts decrease.

[7] Or more speci cally, decreasing in the industry stock of patents.

during the current period and enters the following period, after observing its draw from F_e. Firms enter the industry if, and only if, the expected discounted value is greater than f_e.

Events occur in the following order:

1. Incumbent firms enter with state $(x_t; k_t; \ _t)$
2. Incumbent firms observe their sell-off values
3. Firms simultaneously set prices and earn profits
4. Exit and entry occur
5. Incumbent firms invest in R&D, pay licensing fees, and earn licensing revenue
6. Research outcomes observed (x_{t+1})
7. Firms decide how many patent applications to file
8. The number of accepted patents is realized (k_{t+1})
9. State space moves to $(x_{t+1}; k_{t+1}; \ _{t+1})$

5.2 Incumbent Firms

Define $\mathcal{S}$ as the set of all decision rules with element $\ \in \mathcal{S}$. Consider an individual firm i with decision rule $\ _i \in \mathcal{S}$ and define $\ _{-i} \in \mathcal{S}$ as the decision rule(s) of other firms. Translate the environment to a recursive structure, where $x_t = x; x_{t+1} = x'$. Given state $(x; k; \ )$, firm i solves the following recursive problem

$$
\begin{aligned}
V(x;k;\ ) &= \ (x;\ ) + \ (k;\ ) + \max\{\ (x;k;\ );\ \} && (1)\\
(x;k;\ ) &= \max_{\iota,\lambda} \left\{ \begin{array}{l} - \ \cdot c_{res} - \\ +E_{F(x'|x,I),H(\mu)}[v(x';k;\ )] \end{array} \right\} && (2)\\
v(x';k;\ ) &= \max_{k'}\{-C(k';k) + \ E_G[V(x';k';\ ')]\} && (3)\\
I &= f(\ ;\ ;\ ) \\
' &= H(\ ;\ _i;\ _{-i}) \\
c_{res} &= \text{unit cost of marginal investment} \\
C(k';k) &= \text{cost of adjusting patent stock}
\end{aligned}
$$

where $(\ _i;\ _{-i})$ is implied in $V(\cdot)$, $(\cdot)$, and $(\cdot)$. As detailed above, the firm earns and in spot markets. Second, it decides whether to stay in the market and earn expected discounted profits or sell its operations and earn . Third, it decides how much to invest in "in-house" R&D ( ) and how much to spend on licensing ( ). The composition depends on $f(\ ;\ ;\ )$ where $\ \in (0;1)$ captures the strength of protection (ie, protection increases as $\ \uparrow 1$). Finally, the firm decides how many patents to have in the next period (k').[8] Given the outcome of the firm decision rules in the industry, the distribution of firms evolves according to $H: \ (x;k) \to \ (x';k')$.

8 I implicitly assume that the rm may choose to increase its patent stock even if R&D is not successful. This assumption is founded on the idea that rms often patent ideas/ technologies which they fail to commercialize.

5.3 Entering Firms

Firms enter if, and only if, the expected discounted profits are greater than the cost.

$$\sum_{x'=1,\ldots,\overline{x}} [V(x';0;\ \ ')|F_e] \geq f_e \tag{4}$$
$$F_e = \text{ Initial prod. distribution}$$
$$f_e = \text{ fixed-cost of entry}$$

Free-entry implies (4) holds with equality. Define $e(\ \ )$ as the entry decision rule and assume there exist N potential entrants.

Proposition 1 *Given 1) the number of prospective entrants N is sufficiently large, 2) prospective entrants play mixed strategy, entering with positive probability; then firm entry follows a Poisson random variable. Moreover, there exists a cut-off mass of entering firms (e) such that equation (4) holds with equality.*

Proof See appendix. ■

5.4 Markov-Perfect Equilibrium

De nition- A *Markov-Perfect Equilibrium* (MPE) is a set of functions $\{V;\ \ ;W;\ \ ;\ \ ;\ \ = (\ \ ;\ \ ;\ \ ;h;e);H\}$ such that

1. satisfies equation (1) - firm continuation
2. ; satisfy equation (2) - investment
3. satisfies equation (3) - patenting
4. e satisfies equation (4) - firm entry
5. $H(\ \ ;\ \ )$ is given by

$$(x';k') = \sum_x \sum_k 1 \begin{Bmatrix} F(x'|x;I) = x' \\ G(\ \ (x';k)) = k' \end{Bmatrix} (x;k)$$

6. Firm strategies are optimal

$$\sup_{\sigma_i \in \mathcal{S}} V(x;k;\ \ |\ \ _i;\ \ _{-i}) = V(x;k;\ \ |\ \ )\ \ \forall x;k \in \mathbb{N};\forall\ \ \in \mathcal{M}$$

The final condition guarantees that there does not exist an optimal deviation; hence the decision rules are optimal.

5.5 Modifying the Equilibrium: An Oblivious Equilibrium

As noted by numerous authors, solving the problem above is computationally intensive (or impossible) for industries with a large number of firms (e.g., $N > 5$) or a large state space (e.g., 100 points). I remedy the *curse of dimensionality* by assuming firms are "boundedly rational," making decisions according to the long-run expected state. I approximate the MPE with the equilibrium concept developed by Weintraub et al. [2008].

De nition- An *Oblivious Equilibrium* (OE) is a MPE in which firm decisions are based on the long-run industry state (˜)

6 Accounting for a Dominant Firm

The OE concept requires that all firms are sufficiently small such that they can't single-handedly affect evolution of the firm distribution. This assumption is not relevant to the US semiconductor industry, where Texas Instruments maintains a significant market share. In this section, I modify the model and equilibrium to account for a dominant firm. A *Partial Oblivious Equilibrium* (POE) is a stationary equilibrium in which a small number of firms can affect aggregate state evolution but the majority cannot. Conceptually, the POE is a steady-state equilibrium in which the decision rules of these two types of firms are consistent.

6.1 Partial Oblivious Equilibrium

In the POE, I extend the equilibrium concept to allow for some firms sufficient sway so as to affect state evolution (following Weintraub et al. [2007], I call these dominant firms). Dominant firms make decisions taking into account how their actions influence expected state evolution. The remaining firms (which I call *fringe* firms) make decisions based on the expected evolution of the aggregate state and do not account for their own ability to influence state evolution. In other words, fringe firms follow similar strategies to that of an OE, though I do allow for them to account for dominant firm behavior.

Define $\mathcal{D} = \{i_1; i_2; ::: ; i_N\}$ as the set of dominant firms. The identity of the n firms does not change over time and should a dominant firm choose to leave the industry, it does so permanently and the set of dominant firms decreases. I further make a simplifying assumption that fringe firms cannot become dominant firms. Finally, all new firms become fringe firms. This specification captures the idea that, in the short run, the set of dominant firms is unlikely to change.[9]

6.2 Defining POE Strategies

As in the OE, all firms make expectations regarding the state of fringe firms. I account for deviations of dominant firms by introducing a finite set of statistics that depend on the state of these firms.[10]. Define $\mathcal{W}$ as the set of potential finite statistics with element $! \in \mathcal{W}$.[11]

$$\begin{aligned} V(x; k; !; \ ) &= \ (x; \ ) + \ (k; \ ) + \max\{ \ (x; k; !; \ ); \ \} && (5)\\ (x; k; !; \ ) &= \max_{\iota,\lambda} \left\{ \begin{array}{l} - \ \cdot c_{res} - \\ +E_{F(x'|x,I),H(\mu)}\left[v(x'; k; !; \ )\right] \end{array} \right\} && (6)\\ v(x'; k; !; \ ) &= \max_{k' \geq \delta k} \{-C(k'; k) + \ E_G[V(x'; k'; w'; \ ')]\} && (7)\\ I &= f(\ ; \ ; \ ) \\ ' &= H(\ ; \ _i; \ _{-i}) \\ w' &= D(w) \\ c_{res} &= \text{unit cost of marginal investment} \\ C(k'; k) &= \text{cost of adjusting patent stock} \end{aligned}$$

The key addition here is the addition of the dominant firm law of motion D. Fringe firms use D to forecast the expected state of the dominant firm in the next period.

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[9] This is of course true in the case of the semiconductor industry, where Texas Instruments has been the dominant rm throughout the sample.

[10] This strategy is similar to Krusell et al. [1998] and the OE with aggregate shocks detailed in Weintraub et al. [2008].

[11] In this model of one dominant rm, w will simply be the aggregate rm's state (x, k)

Prospective fringe firms observe industry state $(w; \ )$ and enter if, and only if, the expected discounted profits are greater than the cost.

$$\sum_{x'=1,\ldots,\overline{x}} [V(x'; 0; !; \ ')|F_e] \geq f_e \tag{8}$$
$$F_e = \text{ Initial prod. distribution}$$
$$f_e = \text{ fixed-cost of entry}$$

Free-entry implies (8) holds with equality. Define $e(w)$ as the entry decision rule and assume there exist N potential entrants.

Since dominant firms make decisions with the knowledge that their moves influence the aggregate state, I solve their problem using techniques from the Political-Economy literature[12]. For simplicity, I assume that dominant firms do not make entry or exit decisions.[13] Therefore, a dominant firm with state $(x; k)$ solves a one-shot deviation problem.

$$\tilde{V}(x; k; !; \ ) = (x; \ ) + (k; \ ) + \max_{\tau, \lambda} \left\{ \begin{array}{l} - \ \cdot c_{res} - \\ + E_{F(x'|x,I),H(\mu)}[v(x'; k; !; \ )] \end{array} \right\} \tag{9}$$

$$v(x'; k; !; \ ) = \max_{k'} \{ -C(k'; k) + \ E_G[V(x'; k'; w'; \ ')] \} \tag{10}$$

$$I = f(\tilde{\ }; \tilde{\ }; \ )$$
$$' = H(\ ; \ _i; \ _{-i})$$
$$w' = D(w; \tilde{\ }; \tilde{\ }; \tilde{h})$$
$$c_{res} = \text{ unit cost of marginal investment}$$
$$C(k'; k) = \text{ cost of adjusting patent stock}$$

where $V(\cdot)$ is the value function from solving the fringe firm OE. Note that the dominant firm's deviation $(\tilde{\ }; \tilde{\ }; \tilde{\ })$ influences the evolution of w.

I'm now ready to define the non-stationary decision rules. Define the pair $(\ ^F; \ ^D) \in \mathcal{S}_p$ as the decision rules for fringe and dominant firms, respectively. The fringe firm decision rule $\ ^F(x; k; w)$ is defined as

- $(x; k; !)$ - firm continuation
- $(x; k; !)$ - "in-house" R&D investment
- $(x; k; !)$ - R&D licensing expense
- $(x; k; !)$ - patenting
- $e(!)$ - firm entry

and $\ ^D(x; k; !)$ is defined as

- $\tilde{\ }(x; k; !)$ - "in-house" R&D investment
- $\tilde{\ }(x; k; !)$ - R&D licensing expense
- $\tilde{\ }(x; k; !)$ - patenting

[12] For example, see Krusell et al. [1997] and D'Erasmo et al. [2008].

[13] This assumption is conservative since my dominant rm, Texas Instruments, does not exit the industry, and no rm (fringe or prospective) enters and gains dominant share

where the dominant firm state (w) may be single-valued or a vector.

De nition- A *Partial Oblivious Equilibrium* (POE) is a set of functions $\{V; v; \tilde{V}; \tilde{v}; \ ; \ ; \ ^F; \ ^D; H; D\}$ such that

1. for fringe firms:
   (a) V satisfies an OE
   (b) satisfies equation (5) - firm continuation
   (c) $;$ satisfy equation (6) - investment
   (d) h satisfies equation (7) - patenting
   (e) e satisfies equation (8) - firm entry
   (f) Conditional on D, firm strategies are optimal

$$\sup_{\sigma_i \in \mathcal{S}} \tilde{V}_t(x; k; ! | \ _i; \ _{-i}) = \tilde{V}_t(x; k; ! | \ ) \ \forall x; k \in \mathbb{N}; \forall \ \in \mathcal{M}$$

2. for dominant firms:
   (a) $\tilde{} ; \tilde{}$ satisfy equation (9) - investment
   (b) $\tilde{h}$ satisfies equation (10) - patenting

3. $H(\ ; \ ^F; \ ^D)$ is given by

$$(x'; k') = \sum_x \sum_k 1 \left\{ \begin{array}{l} F(x'|x; I) = x' \\ G(\ (x'; k)) = k' \end{array} \right\} (x; k)$$

4. $D : \mathcal{W} \to \mathcal{W}$

7 Calibration

The model presented depends on specifications for distributions $\{F; G\}$, functions $\ ; \ ; I(\ ; \ ; \ ); C(k'; k)$, and parameters $\{\overline{x}; \underline{k}; \overline{k}; \ ; \ ; c_{res}; \ ; \ ; f_e\}$[14]. I'll close many of these by imposing the following structure.

7.1 Profit and Licensing Functions

There exists a representative consumer who values differentiated products (q_t).

$$U_t = \left[\sum_x \sum_k \ (x; k) q_t(x)^\alpha \right]^{\frac{1}{\alpha}} \tag{11}$$

where is the distribution of firms over the productivity and patent space, hence U_t represents Dixit-Stiglitz preferences. These preferences yield the following demand function for good q_t

$$q_t = \frac{M_t}{P_t} \left(\frac{p_t}{P_t} \right)^{-\epsilon} \tag{12}$$

[14] Note that I've excluded α (the heterogenous good coe cient) since the model implies a 1-1 mapping between α and ϵ. Namely, $\alpha = \frac{\epsilon - 1}{\epsilon}$

where is the elasticity of substitution between goods,[15] M_t is the market size (aggregate spending) in the industry, p_t is the price of good q_t, and P_t is the price index.

$$P_t = \left[\sum_x \sum_k \ (x; k) p_t(x)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{13}$$

Within the differentiated product industry, firms have the same non-increasing returns to scale production function

$$q_t = x_t l_t^{\varphi} \tag{14}$$

where $' \in (0; 1]$ and x_t is the firm-specific labor productivity factor where $x_t \in \mathbb{N} \bigcap [1; \overline{x}]$.

Each period, firms compete in spot profit markets so profit is not time dependent. Consequently, I'll suppress the time subscripts. For simplicity, assume we're in the case of the OE, so all firms are atoms. Firms choose labor, output, and pricing to maximize profits assuming these decisions have no effect on the aggregate price index. They solve

$$\max_l \{ P^{\frac{1}{1-\epsilon}} M^{\frac{1}{\epsilon}} x$$

Deriving the licensing revenue is a little bit different. First, the total licensing expense of firms (Y_L) is an endogenous object defined as

$$Y_L = \sum_x \sum_k s(x;k) \quad (x;k)$$

Secondly, I assume any existing patents are equally valuable to all firms and that firms are matched with ideas to license uniformly.[16]

This set-up implies a that licensing revenue is proportional to the firm's share of total industry patents

$$(k; \quad) \quad = \quad Y_L \cdot \frac{k}{\sum_x \sum_k \quad (x;k)k} \tag{16}$$

where,

$$\begin{aligned} Y_L &= \text{total licensing expense} \\ &= \sum_x \sum_k s(x;k) \quad (x;k) \end{aligned}$$

The intuition is straight-forward: as the firm gains a larger share of total industry patents, it receives greater share of the total licensing expense.

Extending these functions to include dominant firms requires solving for optimal dominant firm labor and patent decisions conditional of firm expected states.

7.2 Transition Probabilities and Other Functions

I now assume the following forms for transition functions $F; G; F_e; \Upsilon$

1. The former, F, maps firm productivity x to x' conditional on the level of R&D intensity (I). Since the state space for productivity is discrete, I follow the IO literature and assume that F takes the following form:

$$F(x'|x;I) = \begin{cases} \frac{(1-\delta)I}{1+I}; & x' = x+1 \\ \frac{1-\delta+\delta I}{1+I}; & x' = x \\ \frac{\delta}{1+I}; & x' = x-1 \end{cases}$$

$$I = \quad + (1 - \quad)$$

2. I assume the USPTO accepts patent applications at a constant rate p_{acc}. The stock of accepted patents

[16] Alternative mechanisms would be to create a matching function in which rms prefer to license ideas from rms either close to their level of productivity (the patented ideas may be more relevant/ implementable) or from rms close to the technological frontier (the patented ideas may be more \cutting-edge"). These mechanisms would not presumably a ect the license expense decision (λ) but would a ect the look of the licensing revenue function(η).

then follows a binomial distribution and the transition function for the stock of patents is

$$G(n; k'; p_{acc}) = \begin{cases} B(n; k'; p_{acc}); & k' \geq k-1 \\ 0; & \text{otherwise} \end{cases}$$

where,

$$\begin{aligned} n &= \text{number of patent applications} \\ &= h(x; k) - (k-1) \\ B(n; k'; p_{acc}) &= \text{binomial distribution} \\ &= \begin{pmatrix} n \\ k' \end{pmatrix} p_{acc}^{k'} (1 - p_{acc})^{n-k'} \end{aligned}$$

Note that I assume one patent expires each period.[17]

3. The time-invariant entry distribution (F_e) is assumed degenerate s.t.

$$F_e(x') = \begin{cases} 1; & x' = xe \\ 0; & x' \neq xe \end{cases}$$

$$xe \in [1; \overline{x}]$$

4. The distribution of idiosyncratic firm sell-off values (Υ) is assumed exponential with parameter (a).

5. The cost of adjusting the patent stock is simply

$$C(k'; k) = \begin{cases} c_{app} \cdot (k' - k - 1); & k' > k - 1 \\ 0; & \text{otherwise} \end{cases}$$

In total, these specifications introduce four new parameters: $\{\ ; p_{acc}; xe; c_{app}\}$.

7.3 Other Parameters

I parameterize many of the remaining parameters as follows:

Table 2: Calibrated Parameters

Variable	Value	Description
$\overline{x}$	3	Productivity upper bound
$\underline{k}$	0	Patent stock lower bound
$\overline{k}$	100	Patent stock upper bound
'	1.0	Labor returns to scale coefficient
p_{acc}	0.5	USPTO Application acceptance rate
xe	3	Entering firm productivity
c_{res}	$24	Cost of research (ie, wage)
c_{app}	$1,350	USPTO patent application fee
	0.945	Discount/ firm survival rate

*See appendix for methodology in setting these parameters

[17] One could also use patent statistics to try and estimate the number of applications that expire or become obsolete each period. I have assumed this structure for simplicity.

8 Computation

I solve for the remaining parameters $\{\ ;\ ;\ ; a; f_e\}$ using simulated method of moments. That is, I choose parameter values such that the model discussed in Section 6 matches moments observed in the data. Denote $\Theta \in \mathbb{R}^4$ as the set of model parameters and define $\hat{\Theta}$ as the set of parameters that generate model moments that most closely fit the data. That is, $\hat{\Theta}$ implies equilibrium decision rules that generate the following simulated moments:

$$\frac{1}{N_s} \sum_{s=1}^{N_s} M^m(\hat{\Theta})$$

where N_s is the total number of simulations.[18] These simulated moments "fit" the data weighted squared difference between data and model moments is sufficiently small.

$$\hat{\ } = \text{argmin}[M' \, \Omega^{-1} M \ \ ] \tag{17}$$

where

$$\begin{aligned}
\Theta_{5\times 1} &= \{\ ;\ ;\ ; a; f_e\} \\
M &= \begin{bmatrix} M_1^d - \frac{1}{N_s}\sum_{s=1}^{N_s} M_1^m(\hat{\Theta}) \\ \vdots \\ M_5^d - \frac{1}{N_s}\sum_{s=1}^{N_s} M_5^m(\hat{\Theta}) \end{bmatrix} \\
M^d &\equiv \text{data moment vector} \\
\Omega^{-1} &\equiv \text{weight matrix}
\end{aligned}$$

Since the model is just-identified, Ω^{-1} is the identity matrix and "sufficiently small" means close to zero. Details of the computational algorithm are in the appendix.[19]

8.1 Moments

Table 3 connects each of the unknown parameters with an identifying moment from the data.

[18] In practice, I use the equilibrium distribution of rms to calculate many, if not all, of the simulated moments. Since the distribution is an expectation, the two strategies are identical.

[19] Given parameter guess , computation of the equilibrium takes, on average, 18 minutes. I use a simulated annealing minimization algorithm to limit the risk of falling into local and not global minima. This does increase the computational burden of the minimization substantially. To remedy this, I adapted the minimization algorithm for parallel processing, thereby decreasing computational burden proportional to the number of processors used.

Table 3: Variables and Identifying Moments

Variable	Description	Moment	Data (1979-1989)
	Patent protection	Patents/ $R&D	0.37
	Elasticity of substitution	Avg. revenue ($MM)	313
	Stochastic depreciation rate	$N(< 10\%)$	4.7
a	Sell-off parameter	Mean exit rate	1.9
f_e	Fixed cost of entry	Number of entering firms	4.6

Now for a discussion on identification. Firms to spend more on licensing and invest more on developing patents as the patent protection factor ( ) increases. The elasticity of substitution ( ) drives substitutability between products and, hence the shape of the firm productivity distribution. Since revenues are correlated with productivity, average revenue and will be correlated. Similarly, pulls firm productivity down, so and $N(< 10\%)$, defined as the number of firms in the bottom 10-percentile of market share, will be positively correlated. The sell-off parameter (a) influences the probability a firm exits the industry. Finally, the correlation between f_e and the expected entry rate should be negative - as f_e falls, firms can cover the fixed cost of entry easier so more choose to enter.

8.2 Estimation Strategy

Recall that the objective of this paper is to ascertain how much of change in market structure was due to patent reform. I answer this question using a two-step approach. First, I assume the parameters $;a;$ and f_e are time-invariant. I solve (17) using data moments for the period 1979-1989, and recover these point estimates.[20]

The second stage is similar in concept, though the data moment(s) chosen are assumed to reflect a transition between steady-states. Specifically, I assume the industry was in steady-state prior to legal reform in the early 1980s and this reform caused the industry to transition to a new, to-be-determined steady-state. I take the parameters from stage one and re-estimate the patent protection factor to match the average growth in patent propensity after the legal reform.[21]

9 Results

The estimation results are based on solving (17). Results are preliminary and incomplete (ie, estimation has not converged yet). The following tables present those available.

Table 4: Estimation Results

	Stage 1		Stage 2	
Variable	Estimate	SE	Estimate	SE
	0.14	-	0.48	-
	3.2	-	-	-
	0.12	-	-	-
a	10.88	-	-	-
f_e	1,419.64	-	-	-

[20] Accordingly, market size (M) is set as the average market size for years 1979-1989

[21] I assume that the observed market size in 2008 is the new steady-state market size.

10 Counterfactuals

I now use the parameter estimates from Section 9 to estimate the contribution of patent reform to the changing market structure. The idea of this exercise is to use the model to ascertain what the patenting rates and market structure would have looked like if no patent reform would have taken place. Methodologically, I simulate the model again using the level of protection found using 1979-1989 data ($_{79-89}$) and the new steady-state market size.

TO BE COMPLETED

11 Conclusion

A wide variety of data and empirical studies indicate the US semiconductor industry experienced a dramatic shift during the 1980s. A once consolidated industry composed of vertically-integrated firms became a fragmented industry of both traditional production and specialized design firms. At the same time firm patent propensity increased sharply. Others have hypothesized that the legal strengthening of patent protection in the early to mid 1980s is responsible for both the shift in patenting and the change in market structure. In this paper I asked how much of the change in market structure is due to patent reform. The resulting structural model replicates key industry statistics and provides a foundation to examine the contribution of patent reform to changing market structure. Preliminary estimates indicate the reforms increased licensing and patenting behavior by 248%, though this result requires further testing for robustness. The paper also represents a methodological contribution, extending the OE concept of Weintraub, Benkard, and Von Roy (2008) to a stationary equilibrium with a dominant firm.

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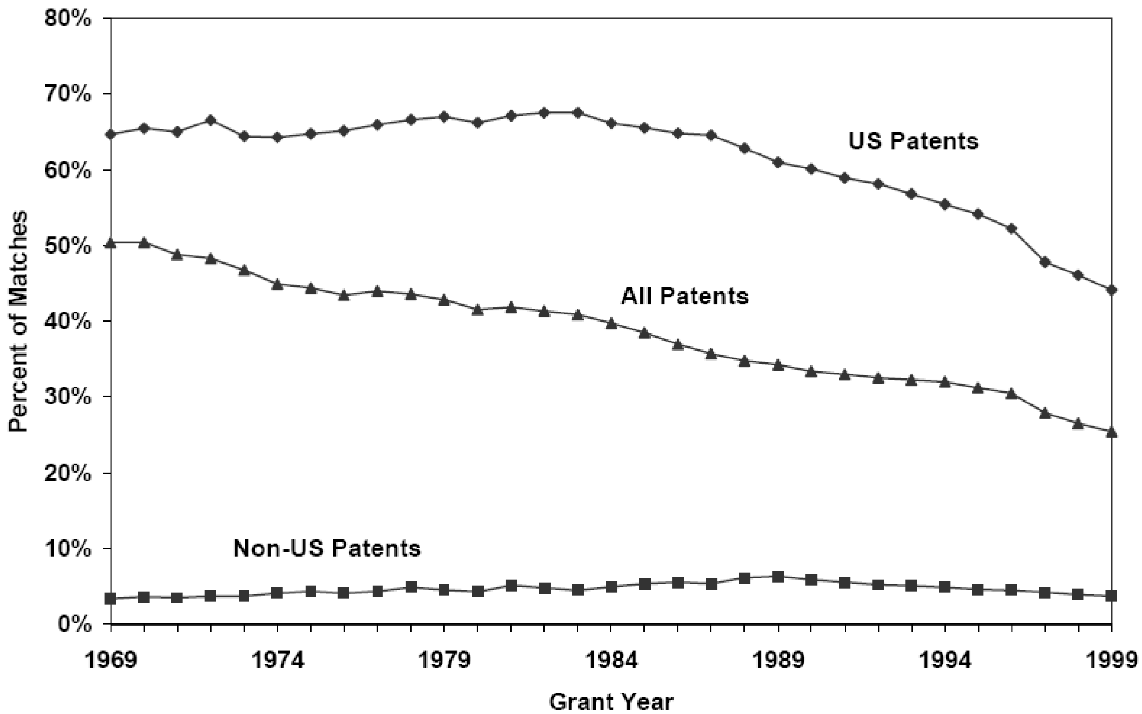
12 Appendix

12.1 Data Sources

I construct my data set by matching financial data of approximately 100 publicly-traded companies with patent grant information. The financial information is from Compustat, while the patent information is from the NBER *Patent Citation Data File.* Hall, Jaffe, and Trajtenberg matched patent grant data from the USPTO with Compustat CUSIP identification. The match was based on the assignee names in the USPTO data and the list of firm names in Compustat.

Figure 3: Compustat Match Rate



The decreasing match rate in the latter part of the sample is largely due to the fact that the authors' used Compustat firms from 1989. The CUSIP provides unique identification with Compustat financial information, so the final data set is a list of all patent applications from 1970-1995 and the financial information associated with the assignee. The match is made using the application year-CUSIP combination. The following table presents some summary descriptive statistics of the semiconductor industry.

Table 5: Industry Descriptive Statistics (1970-1995)

Category	Variable	Mean	Standard Deviation	Maximum	Minimum
Financial Information	Number of Firms (pr year)	70.7	29.2	142	38
	Revenue ($MM)	324.2	1,178.3	16,202	0.0
	Market Share (%)	1.5	5.2	52.4	0.0
	Income ($MM)	25.4	170.0	3,566.0	-409.0
	Research ($MM)	33.1	104.9	1,269.0	0.0
	-Firms Reporting R&D (%)	99	1	100	98
	Plant, Property, & Equipment ($MM)	220.4	774.4	11,792	0.0
	Employees (000)	3.2	10.9	89.9	0.0
Patent Information	Number of Patenting Firms	59.5	18.1	102	36
	-Share (%)	87	8	95	72
	Patents (N)	38	90	758	1
	Patents/ R&D	1.79	6.1	65.4	0.0

*All dollar values are real (1982-1984 base)

I used this, and other, data to calibrate relevant model parameters

1. The productivity upper bound $\overline{x}$:
   Simplifying assumption. The only important factor here is that $\overline{x}$ be sufficiently high to guarantee the distribution is stationary and not truncated.

2. The patent stock lower $\underline{k}$ and upper $\overline{k}$ bounds:
   Simplifying assumption. The lower bound makes sense since firms cannot carry negative patent stocks and I did not want to require firms to hold positive stocks. As with the productivity grid, I set $\overline{k}$ sufficiently high to guarantee the distribution is stationary and not truncated.

3. The labor returns to scale coefficient $'$:
   I set this parameter to one (constant returns to scale) in order to simplify some of the analysis. Empirical trade papers with similar production functions have used estimates of 0.7, derived from Cooper & Haltiwanger.

4. The USPTO acceptance rate p_{acc}:
   I set this figure using the congressional testimony of Jon Dudas, Under Secretary of Commerce for Intellectual Property and Director of the USPTO

   > Our patent examiners completed over 362,000 patent applications in 2007, the largest number ever, while maintaining for the second year in a row an examination compliance rate1 of 96.5 percent, the highest in a quarter of a century. The allowance rate for patents is currently 44%. This is in contrast to allowance rates in excess of 70% just eight years ago.

   In order to be conservative, I set this parameter to 50%.

5. The firm entry state xe:
   Simplifying assumption. Further revisions will revise this assumption based on firm entry data.

6. The labor wage cost of research c_{res}:
   This is set assuming the average researcher earns $50,000 per yer and works 2,080 hrs per year (40 hrs per week times 52 weeks per year). Of course, defining this variable in terms of hourly wage is not important to the results (it only impacts how we think of) but it is consistent with the parlance of our times where "wage" indicates hourly wage.

7. USPTO patent application fee c_{app}:
   Based on Helfgott [1993]. This figure includes the official filing fee and any agent fees but does not account for legal fees. Legal fees are increasing in the level of idea complexity. Conservative estimates range from $4,500 - 7,500$. Since semiconductor-related technologies are quite complex and specialized, it is likely that this range is on the low-side.

8. The time discount factor :
   This is based on the long-run rate of return of 7% on federal bonds.

12.2 Proofs

Proposition 1 *Given assumption (??), firm entry follows a Poisson random variable. Moreover, there exists a cut-off mass of entering firms ( ) such that* $\cdot E[V(a'; S')|G; \ ] = f_e$.

Proof I first show that entry follows a Poisson random variable. The proof follows Weintraub et al. [2008]. There are F potential entrants where F is assumed sufficiently large and each firm enters with the same probability p_F. Define $V_F(f)$ as the expected discounted profit flows for each entering firm if f firms enter simultaneously. By construction, $V_F(f)$ is decreasing in f. Free entry then implies the following equation must hold

$$\sum_{f=0}^{F-1} p_F^f (1 - p_F)^{F-f-1} V_F(f+1) - f_e = 0 \tag{18}$$

for which there exists a unique solution $p_F^* \in (0; 1)$ for each sufficiently large F, where assumption (??) guarantees an interior solution. Further, define $= \lim_{F \to \infty} F p_F^*$ and X_F as a binomial random variable with parameters $(F; p_F^*)$. Then $\lim_{F \to \infty} X_F = Z$ where Z is a Poisson random variable with parameter .

So long as the number of potential entrants is sufficiently large and each potential entrant plays the same mixed entry strategy, then entry can be modeled as a Poisson random variable with expected entry . ■

12.3 About Intellectual Property Rights

The paper largely deals with "utility" patents[22]. Utility patents largely grant the holder

> the right to exclude others from making, using, offering for sale, or selling the invention... or importing... products made by that process (USPTO)

In exchange, the inventor makes his/ her invention public information. The state hopes this revelation enables others to learn from the invention, enhancing their own inventiveness, and leading to more productive innovations for the country at large. This revelation is actually quite a commitment since any patent application must

> disclose the invention in a manner sufficiently clear and complete for the invention to be carried out by a person skilled in the art (WTO)

The cost of this trade-off is particularly stark in the U.S. where patent applications become public information 180 days after submission.[23] Consequently, the firm may reveal its idea before even taking advantage of the idea itself, thereby enabling competitors to piggy-back on its R&D efforts.

[22] For example other forms of intellectual property right protection are copyrights and trademarks.

[23] The combination of a sharp increase in number, breadth, and complexity of patent applications, has inundated the USPTO. In response, it uses the input of the outside world to help verify the validity of patent claims. Ironically, the patent o ce is using the strategy of open-source software development to aid in its e orts.

12.4 Computational Algorithm

In this section, I outline the computational algorithm used to solve for the OE and the POE. It is important to note that the equilibrium computed is a function of input parameters Θ, and that the remainder of the algorithm takes these as given.

The algorithm used is similar in methodology to Weintraub et al. [2008]. I begin with some introductory remarks and then proceed to outline the algorithm more systematically.

12.4.1 Useful Operators

Note that the distribution of firms $\in \mathcal{M}$ is a measure on $(\mathbb{R}^2; \mathcal{B})$ where B is a Borel -algebra. Let $\mathcal{P}(\mathbb{R}^2)$ be the space of probability measures on $(\mathbb{R}^2; \mathcal{B})$ and define the operator $\Phi : \mathcal{P}(\mathbb{R}^2) \to \mathcal{P}(\mathbb{R}^2)$ as

$$(\Phi \ )(x'; k') = \sum_x \sum_k 1_{\{F(x'|x,I)=x', h(x,k)=k'\}} \ (x; k) \tag{19}$$

Note that given stationary decision rules, one can use Φ to create a stationary distribution ^{SS}. Firms will use this steady-state distribution as a forecast of the long-run industry average.

I solve for equilibrium entry using a contraction mapping similar to Berry et al. [1995]. Define the following operator

$$\begin{aligned} \Psi\tilde{e} &= \tilde{e} + \log(\text{EVF}) - \log(f_e); \text{ where} \\ \tilde{e} &= \log(e) \\ \text{EVF} &= \sum_{x'=1,\ldots,\overline{x}} [\ V_e(x'; 0; \ ')|G] \end{aligned} \tag{20}$$

Note that the value function is dependent upon on the entry rate (V_e). The mechanics are fairly straightforward. High expected discounted profit leads to a new, higher guess of entry, thereby driving expected discounted profit down. The inverse is, of course, also true. A fixed point occurs where $\log(\text{EVF}) - \log(f_e) = 0$ which implies (4) holds with equality. While the model's mechanics make it difficult to prove that Ψ is a contraction, numerical experiments confirm that it works for a variety of demand structures.

12.4.2 The OE Algorithm

Given parameters (Θ) and parameterized functions $I; C(k'; k)$, I do the following to solve for the OE :

1. Guess mass of entrants (e)
2. Guess initial firm distribution (s^0)
3. Guess decision rules ($_0$; $_0$; $_0$; h_0)
4. Given s^0 and $_0$, solve for and
5. Use and to solve incumbent firm's problem, equations (1), (2), and (3). Call these decision rules ($_1$; $_1$; $_1$; h_1)
6. Solve for the invariant distribution of firm productivity and patents (s_1) using the following operator Φ. Define the stationary distribution as s_1
7. If $\|s_1 - s_0\|$small, then continue. Otherwise, set $(s_0; \ _0; \ _0; \ _0; h_0) = (s_1; \ _1; \ _1; \ _1; h_1)$ and return to (3). In order to mitigate the risk of cycling, update the decision rules smoothly (e.g., use a convex combination)

8. Check if (4) holds (free-entry). If not, use Ψ to revise the guess for e and return to (1)

12.4.3 The POE Algorithm

The POE algorithm involves finding decision rules (actually laws of motion) for the dominant firms and decision rules for fringe firms that are consistent. The solution is a steady-state in which dominant firms stay in the same position and the expected state of fringe firms is constant.

1. Define $D^n(w)$, $w = (x; k)$ as the dominant firm's decision rule for iteration n, where I use the identity function when n=1
2. Given $D^n(w)$, solve the OE for fringe firms.
3. Solve for the POE. From Step 2, we have a OE given a dominant firm decision rule. Now, we need to check that given this OE, the dominant firm will not want to deviate from its decision rule.
   (a) Start from the non-dominant model OE and use the data to identify the initial location of the dominant firm
   (b) Use the OE value function computed in Step 2 $V(\cdot)$ and construct a one-shot deviation value function for the dominant firm ($\tilde{V}(x; k; d = 1; w)$. Solve the dominant firm's problem and use the transition probabilities to solve for the expected value of the dominant firm's next state $(x'; k')$
   (c) Solve for next period's distribution ($'$)
   (d) Use the one-shot deviations to create a simulated data set $\{x_j; k_j\}_{j=1}^{J}$
   (e) Run OLS regressions (See section 12.4.4 for assumed OLS structure) on the simulated data set to create D^{n+1}
   (f) If $\| D^n - D^{n+1} \|_\infty$ is small then move to the next step. If not, let $n = n + 1$ and go back to (1).
4. We now have a solution for D. The final step is to find the steady-state by solving the following set of linear equations presented in (12.4.4)

12.4.4 Laws of Motion

I approximate the law of motion for the dominant firms, both on and off the equilibrium path, using using structure similar to Krusell et al. [1998]. Specifically, I assume that states evolve according to the following processes:

- Productivity

$$x' = a_0 + a_1 x + a_2 k$$

- Patent stock

$$k' = b_0 + b_1 x + b_2 k$$