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Building Reputation for Contract Renewal: Implications for Performance Dynamics and Contract Duration

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Abstract

Due to technological progress, recent performance is often more informative about future performance prospects than is older performance. We incorporate information decay in a career concern model in which performance depends on type and effort and contract renewal is based on the performance record.

In contrast with the career concern literature (e.g. Lewis, 1986; RJE), contractors work harder when the project approaches renewal date and when their reputation is better. Productive investment are crowded out by window-dressing effort in late contract periods, but it is boosted in early periods. More frequent contract renewals strengthen reputational effects and result in improved performance if the relative cost of investment is low, but otherwise long-term contracts induce more effort.

Our results are corroborated by some empirical studies showing that performance improves as the contract approaches renewal date.

Career concerns, contract renewal and dynamic incentives.

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1 Introduction

Casual observation and empirical evidence suggest that contract renewal can be a powerful incentive device to motivate agents: when the renewal decision depends upon past performance, the prospect of future rewards can induce the agent to exert noncontractible effort.

Examples abound. In the service industry, sellers may provide higher nonverifiable quality that increases their clients' satisfaction if this induces a repeated purchase. In the construction industry, contractors may contain cost more and ensure on time delivery if good performance affects the likelihood of being awarded a new contract. In private procurement, suppliers may adjust their productions to meet unexpected changes in product specifications if this helps to establish a long-term relationship with the buyer. In the academia, researchers may undertake duties that were not explicitly contracted upon if this helps them to obtain tenure.

Casual observation and empirical evidence also suggest that the incentive power of contract renewal is not necessarily constant throughout the contract life: often performance improves more as the contract approaches the renewal date. Using a panel of the 25 franchisees providing passenger services in the UK railway industry in the period 1997-2000, Affuso and Newbery (2002) found for example that nonverifiable investment by the contractors increased as the contract renewal date became nearer. In the water industry, Chong, Huet and Saussier (2006) found that operators reduced customer prices as expiry date approached. Their analysis was based on a database of 1102 French local public authorities in 2001.

In this paper we investigate the incentive power of contract renewal and its implications on performance dynamics and contract duration. Our first research objective is to provide a rationale for the above observations and thus explain when and why performance improves as the contract renewal date approaches.

In the economics literature, the incentive power of contract renewal has been rationalized in two ways. The first one is relational contracting. As first discussed by Kim (1998) in a repeated-game model with moral hazard, effort may be induced by the presence of an implicit agreement between the principal and the agent.[1] On the one hand, the agent exerts nonverifiable effort when the prospect of a long-term gain from contract renewal is greater than the one-shot saving on the cost of effort. On the other hand, the principal

[1]The idea in turn dates back to the repeat-purchase mechanism, first explored by Klein and Leffler (1981). In their model, the firm provides nonverifiable quality if the discounted stream of profit from quality provision is greater than the one-shot gain from underperformance.

renews the contract with a well-performing agent when the value of future cooperation is greater than the one-shot gain from reneging on the promised rent.

Relational contracting however does not explain why contract renewal should make the agent work harder as renewal date approaches. If the principal observes a deviation from the implicit agreement, she should retaliate and not renew the contract regardless of when the deviation was observed. The agent must then exert the same amount of effort in every and each period for the relational contract to be sustained. It is only if the principal suffered from bounded rationality and short memory that relational contracting could possibly explain why the agent works harder as renewal date approaches.

The second rationale for the incentive power of contract renewal hinges on career concerns (Holmstrom, 1982). In the standard career concern model the market uses a worker's current output to update its belief about the worker's ability and then bases future wages on these updated beliefs. The worker increases output by taking actions the market cannot observe, in an attempt to influence the market's belief. Reputation building then makes the agent works harder. Lewis (1986) applied this idea to a multiperiods principal agent model with adverse selection and moral hazard and found that effort is exerted to send favorable signals on project costs and induce the principal to continue financing the project.[2]

The career concerns literature however does also not explain why performance improves as contract approaches the renewal date. In fact, in Lewis the opposite occurs: the agent chooses higher effort in the earlier stages of the procurement contract as then the threat of project termination is stronger. This is in line with Holmstrom: when the agent's ability is constant, effort decreases over time because a shorter prospective career decreases the return to changing the market's belief. Dewatripont et al. (1999) extend Holmstrom to a multi-tasking framework and also find that effort falls over time.:

But an agent's ability is not a static concept. An agent who is high skilled in using the current technology may become a low performer when a new technology comes along. And as technology continuously evolves, recent performance is more informative about the agent's future performance prospects than old performance.

Starting from this observation, we incorporate information decay in a

[2]The incentive power of contract renewal is also discussed Laffont and Tirole (1993, Chapter 8). They proposed an adverse selection model with repeated auctions of incentive contracts, focussing on the bias towards the incumbent at renewal stage to improve incentives. In Dalen, Espen and Riis, (2006), quality is nonverifiable but competitors can be ranked according to their quality performance. Tournaments are then used at renewal in order to reward noncontractible quality.

model of career concerns and find that it results in contractual performance improving as the contract approaches the renewal date. We consider a three-period setting where performance (quality) is observable but nonverifiable and it depends on the agent's unobservable innate productivity and investment. There is information decay in the sense that the agent's productivity may change across periods due to exogenous factors. The principal cannot precommitt to a renewal policy or pricing policy. She uses performance information to update her belief about the agent's productivity and decide, at renewal date, whether to make a new offer to the agent. Contract renewal is rewarding for the agent because there is an asymmetry of information on cost at renewal stage.

Within this setting, we show that contract renewal acts as an implicit incentive mechanism to motivate the agent to invest in noncontractible quality. The agent invests to build a good reputation with the principal and increase his bargaining position at renewal stage (we call the positive effect of contract renewal on incentives, ‘ ’). The incentive power of contract renewal however changes over time. Due to information decay, good performance is more valuable the closer is the contract to renewal date, thus, the incentive to invest increases as the renewal date approaches (we call this ‘ ’). With linear investment cost, the agent never invests in the first period of the contract whilst he may invest in the second period. With convex cost, the agent may choose to make some investment in the first period, to reduce the overall cost of building reputation, but investment incentives remain stronger in the second period. A crowding out effect occurs as the second-period investment reduces the value of investing in the first period.

The strength of the incentive power of contract renewal depends on a variety of factors. With linear investment cost, a higher degree of information persistence increases the agent's incentive to invest. The effect is instead ambiguous when investment cost is convex. Higher discount factors or degree of asymmetric information on cost raise the expected rent and increase the agent's incentive to invest. (Weakly) higher renewal prices are also beneficial on investment as they act as commitment device to reward good performance. But while the implicit incentive provided by contract renewal increases welfare, underinvestment still occurs. Further, for given investment, instances arise where provision of the service is taken in house whilst delegation to the agent would be optimal.

The contract-renewal effect and the information-decay effect together generate a performance dynamics that is characterized by increasing (average) quality over time. A higher expected rent or a lower investment cost always sharpens the rise in performance. The degree of information persistence and

the discount factor instead affect performance dynamics in an ambiguous way, due to the crowding out effect.

The contract-renewal and the information-decay effects also arise when effort by the agent improves current performance but has no effect on innate productivity (we refer to such effort as "window dressing"). We show that the contractor chooses higher effort in later stages of the contract in order to send favorable signals to the principal regarding his productivity. He attempts to let the principal believe that recent improvements in performance will translate into better future prospects as they are due to an improvement in technology. The principal uses the contractor's past performance to update her belief about the contractors's future productivity but upon observing increasing performance, she is unable to disentangle between a contractor whose innate productivity has indeed increased from a contractor who is working harder. Thus, signal jamming arises.

We then identify one more effect: agents with better reputation have more incentives to exert window-dressing effort. We call this the ‘

’. Furthermore, effort crowds out productive investment in the second period of the contract, but it boosts it in first period. Thus in sectors where the potential for quality improvement is high we can expect both productive investment and window-dressing effort to take place; the former at the beginning of the contractual relationship whilst the latter towards the end.

A second question we address in the paper is how the duration of the contract affects the incentive power of contract renewal and what implications this generates on the choice of contract duration. We endogenize the frequency of renewal decisions in a repeated version of our basic model and compare two-period contracting with one-period contracting.

A limited number of papers have studied the pros and cons of long-term contracts. Longer contracts alleviate moral hazard problems by facilitating consumption smoothing (Lambert, 1983) and ease the hold up and the ratchet effects in the presence of specific investment (Laffont and Tirole, 1993). However, shorter contracts increase the flexibility to use new information as it comes along (Ellman, 2006) and increase participation. The positive effect of the frequency of interaction on reputational mechanisms was first noted by Shapiro (1983) for experience goods but it has been explored in depth only recently by Calzolari and Spagnolo (2007), who show that a more frequent re-auctionng of procurement contracts raises quality provision through relational contracting.

We contribute to this literature on optimal contract duration by focusing on career concerns and by showing how the optimal frequency depends on information decay. We show that an increase in the frequency of contract

renewal may either strengthen reputational effects and result in improved performance or yield the opposite, effect depending relative value of the cost of investment compared to the value of the contract for the agent. One-period contracting (resp. two period contracting) generates higher (average) quality when the cost of investment is low (resp. high).

Evidence on the determinants of contract duration shows that contracts are longer when relationship specific investment is important (Joskow, 1987) and shorter when flexibility becomes more relevant (Masten 1988). Bercovitz (1998) finds that franchise agreements tends to be of shorter duration in systems having the greatest potential for franchisee opportunism. Crocker and Masten's (1988) and Saussier (1999) find that contracts tend to be shorter in periods of higher uncertainty, which might be consistent with the benefit of shorter contracts in the presence of information decay.

The structure of the paper is as follows. In Section 2 we discuss the basic model where the agent can invest to enhance his innate productivity and the principal can neither commit to a renewal policy nor to a pricing policy. In Section 3 we extend the basic model to consider the possibility that the agent exerts effort to temporarily enhance quality, and we investigate the interaction between investment and effort over the contract life. We also relax the assumption of no commitment on pricing at renewal stage and investigate optimal pricing from a social perspective. In Section 4 we analyze the effect of the contract duration. To this purpose we extend the analysis to an infinitely repeated-game framework and compare the incentives to invest with two-period contracting and with one-period contracting. Section 5 concludes. All proofs missing from the text are in an Appendix.

2 Basic model

We start by investigating the incentive power of contract renewal and its implications on performance dynamics, when the agent has reputational concerns and building reputation enhances his bargaining power at renewal stage. To this purpose we consider a basic model, in which a principal (she) delegates to an agent (he) the provision of a good or service for 2 periods. Service quality is observable but nonverifiable and it depends in a deterministic way on the agent's current productivity, which can change over time due to technological factors. In each period the agent can make a nonverifiable investment to enhance his current productivity. When the contract expires, having observed the agent's performance in the past two periods, the principal chooses whether to make a new price offer to the agent or instead take provision in house. The principal cannot precommitt herself to a renewal

policy or to a price offer. Asymmetry of information between the principal and the agent on production cost at renewal stage ensures that the agent will enjoy a positive expected rent should he continue to provide the service.

After describing this basic model, in the subsequent two subsections we discuss the renewal decision of the principal and its implications on performance dynamics.

2.1 Framework

The agent's productivity (type) is uncertain and may change over time, due to technological progress and/or investment. More precisely, in each period the quality can take two values: high ( ) or low ( ); we will denote by Δ the quality differential ($\Delta = \quad - \quad$) and by $^{e}_{t}$ the expected quality in period . When the quality is initially , the agent can however upgrade it to by investing . For the sake of exposition, we will refer to the initial quality in period as the agent's "productivity" or "type", and will denote it by $_t$; we will use the term "quality" and the notation $_t$ to refer to the quality eventually provided in period . By assumption, $_t =$ if $_t =$; if instead $_t =$, then $_t =$ if the agent invests , and $_t =$ otherwise. We denote by $_t \in [0\ 1]$ the probability that the agent invests in period when the quality is low at the beginning of that period.

The initial productivity of the agent, $_1$, is randomly drawn and is equally likely to be or ; in the subsequent periods, the productivity follows a stationary first-order Markov process based on the agent's quality at the end of the previous period: $_t = \ _{t-1}$ with probability $\quad 1\ 2$. The parameter captures some information decay: a higher value for denotes slower changing technologies and therefore a higher probability that, absent any investment, the agent's quality in period remains the same as in period -1.

The agent's operating cost in period , $_t$, is also random and can take two values, $\underline{\ }$ and $\overline{\ }$, with respective probabilities and $1 -$. Under delegation, the agent's per period payoff is $_t = \ _t - \ _t - \ _t$ where $_t$ denotes the price paid to the agent in period . The realization of the cost is privately observed by the agent, and not by the principal. As we will see, this information asymmetry generates a rent for the agent, who therefore gains from convincing the principal to renew the contract.

Under delegation, the principal's per period payoff is $_t = \ _t - \ _t$. Inhouse production generates instead a per period payoff of $_t$, which is random

and uniformly distributed over the range $[\underline{\ }\ \overline{\ }]$.

At the beginning of period 3, having observed the qualities $_1$ and $_2$ provided in the first two periods as well as the realization of $_3$, the principal makes a take-it-or-leave-it offer (a price $_3$) to the agent for the future provision of the service.[3]

The timing of the game is as follows.

- Periods $= 1$ and $= 2$:
  - $_t$ is realized and observed by the agent;
  - If $_t =$, chooses whether to invest;
  - $_t$ is realized and observed by both parties.[4]
- Period 3:
  - is realized and observed by the principal;
  - The principal offers a price $_3$;[5]
  - $_3$ is realized and observed by the agent, who then accepts or rejects the principal's offer.

We will assume that the principal and the agent use the same discount factor when evaluating multiperiod payoffs.

2.2 Contract renewal

At the beginning of period 3, the principal observes the realized value of $_3$ and chooses the price $_3$. Since this is the last contracting period, the agent

[3] In this section, we focus on the renewal decision (including the price p_3) and its impact on the agent's behavior during the first periods; we will not need to discuss the determination of the prices p_1 and p_2. In section 4, we consider an infinite repetition of this basic framework and analyze the determination of prices in all periods.

[4] In this basic framework, the analysis does not depend on whether the principal observes the productivity θ_t and/or the investment decision; the quality q_t provides a "sufficient statistic" for period t. In later sections, in which the agent may temporary increase the perceived quality, the analysis does depend on whether the principal can detect or not such efforts.

[5] As usual, there is no loss of generality assuming that the principal always make an offer; offering a price p lower than $\underline{C}$, which will always be rejected, amounts to making no offer.

has no incentives to invest in case of low productivity; therefore, if delegating the provision to the agent, the principal expects a quality:

$$ {}^{e}_{3} = \quad [\ _3] = \begin{cases} \ + \ \Delta & \text{if} \quad _2 = \\ \ + (1 - \ ) \Delta & \text{if} \quad _2 = \end{cases} $$

Given that the agent's cost is either $\overline{\ }$ or $\underline{\ }$, the principal will offer $_3 = \overline{\ }$, $_3 = \underline{\ }$, or make an unacceptable offer ($_3$ $\underline{\ }$). The last option yields a payoff $_3$. If instead the principal offers $_3 = \underline{\ }$, then with probability , the agent observes $_3 = \underline{\ }$ and accepts the offer, whilst with probability $1 -$, the agent observes $_3 = \overline{\ }$ and rejects the offer, in which case the service is provided and the principal gets $_3$ in house. Thus, by offering $_3 = \underline{\ }$, the principal obtains an expected payoff equal to:

$$ [\ _3 \mid \ _3 = \underline{\ }] = \ ({}^{e}_{3} - \underline{\ }) + (1 - \ ) \ _3 $$

If instead the principal offers a high price, $_3 = \overline{\ }$, the agent always accept the offer and the principal thus obtains:

$$ [\ _3 \mid \ _3 = \overline{\ }] = {}^{e}_{3} - \overline{\ } $$

It can be checked that $[\ _3 \mid \ _3 = \overline{\ }] \quad [\ _3 \mid \ _3 = \underline{\ }]$ when

$$ _3 \quad \hat{\ } \equiv {}^{e}_{3} - \underline{\ } - \frac{\overline{\ } - \underline{\ }}{1 - } $$

This condition moreover implies $[\ _3 \mid \ _3 = \overline{\ }] \quad _3$. Therefore, when $_3$ $\hat{\ }$, the principal offers a high price ($_3 = \overline{\ }$), in which case the agent obtains a positive payoff $\overline{\ } - \underline{\ }$ with probability , and just covers his cost otherwise. If instead $_3$ $\hat{\ }$, then the principal either offers $_3 = \underline{\ }$, which may be accepted if $= \underline{\ }$, or an even lower price which is never accepted; in both cases, the agent obtains zero payoff. Therefore, the expected profit of the agent is equal to:

$$ [\Pi_3] = \Pr\left(\ _3 \leq \hat{\ } \right) \quad = \frac{\hat{\ } - \underline{\ }}{\overline{\ } - \underline{\ }} $$

where $\equiv (\overline{\ } - \underline{\ })$ denotes the agent's expected payoff from a high price $_3 = \overline{\ }$. Since the threshold $\hat{\ }$ increases with the expected quality ${}^{e}_{3}$, which is higher when the agent previously provided a good quality ($_2 =$ rather than $_2 =$), we have:

Proposition 1 ! "
#

Indeed, the better the previous performance of the agent, the greater the principal's expected payoff (net of the price) from delegation in period 3 and thus the higher the incentive of the principal to make a high-price offer. Since the agent earns a rent only when (his cost is low and) he receives such a high-price offer, a better past performance raises the expected rent of the agent.[6]

For the sake of exposition, we will normalize the distribution of as follows:

$ _ ≡ − _ − (‾ − _) (1 −) and ‾ ≡ − _ − (‾ − _) (1 −).

With this normalization, the probability of a high-price offer simply equals the prior in period 3; the agent's expected profit at the renewal stage is thus equal to:

$$[\Pi_3] = \begin{cases} & \text{if } {}_2 = \\ (1 - \ ) & \text{if } {}_2 = \end{cases}$$

2.3 Performance dynamics

We now analyze the agent's behavior during the first contract.

In period 2, the agent must decide whether to invest in case of low productivity (that is, if $_2 =$). Investing costs but upgrades the quality $_2$ from to , and thus increases the expected rent in period 3 from $(1 - \ )$ to . Therefore, the agent will invest if:

$$- \ + \qquad (1 - \ )$$

that is, if:

$$(2 \ - 1)$$

Note that the agent's decision does not depend on the observable history (i.e. $_1$ and $_2$), although it depends on the type $_2$.

When we also consider the agent's investment decision in period 1, we obtain:

Proposition 2 # # $ #
% & ' $_2 =$ ' ($^* \equiv (2 \ - 1)$ #
# % #) ( )
( )

[6] As already noted, the agent's rent derives here from asymmetric information about the operating cost. Absent such private information, at the renewal stage the principal would offer a cost-based contract extracting the whole surplus; this would nullify the potential role of the contract renewal as an incentive device.

Proof. Two cases can be distinguished.

Case 1: $\leq$ *. The agent will invest in period 2 whenever $_2 =$, and then obtain an expected payoff equal to $- +$ (when $=$ *, the agent is indifferent between investing or not, and either way obtains again $- +$). When instead $_2 =$, the agent obtains an expected payoff equal to . Therefore, investing in period 1 yields an expected payoff equal to

$$- + [( ) + (1 -) (- +)] = - (1 + (1 -)) + ^2$$

whereas in the absence of investment the agent's expected payoff is equal to

$$[(1 -) + (- +)] = - + ^2$$

The agent will thus choose again not to invest, since

$$(1 + (1 -))$$

Case 2: *. In this case, the agent will not invest in period 2 if $_2 =$. Therefore, investing in period 1 yields

$$- + [( ) + (1 -) (1 -)] = ^2 (1 - 2 (1 -)) -$$

whereas in the absence of investment the agent's expected payoff is equal to

$$[(1 -) + (1 -)] = ^2 2 (1 -)$$

The agent will thus choose again not to invest since $^2 (2 - 1)$ ■

Since quality is noncontractible, the principal cannot incentivize the agent to provide high quality via explicit contractual terms. However, the decision on contract renewal provides an implicit incentive (a " "). At the renewal stage, the principal indeed relies on the agent's past performance to update her belief about the agent's productivity and bases the terms of her contract offer on these updated beliefs. As a result, the expected rent of the agent depends on his contractual performance, which encourages the low-productivity agent to improve his performance by investing and upgrading his quality in the period before contract renewal, period 2.

There is however an " " which makes the incentive power of contract renewal become weaker as time moves away from renewal date. In this basic model, this effect removes any incentives to in the first period. Since quality in period 2 provides sufficient statistics for the agent's type in period 2, quality in period 1 has no informational value for the renewal

decision in period 3. Thus investing in period 1 could only be worthwhile if it reduced the overall cost for the agent of building a reputation in period 2. But this is never the case with linear cost: it is not worthwhile to invest in period 1 and incur with certainty just to reduce the probability of having to invest and incur in the following period.

The strength of this incentive mechanism depends on the persistence of the agent's productivity. Clearly, in case of full decay ($= 1$ 2), past performance tells nothing about future productivity; as a result, past performance will not affect the renewal decision, which in turn implies that the agent will never seek to upgrade the quality provided. As increases, past performance becomes more and more informative about the agent's future productivity, which in turn provides greater incentives for the agent to invest in quality if needed.

An implication of Proposition 2 is that the agent faces a greater incentive to deliver good quality as the renewal stage approaches, due to information decay (1) and discounting (1)

Corollary 1 ! * & + (

$$^{*} \equiv (2 \quad - 1)$$

Proof. When $=$ *, ${}^{e}_{1} =$ $[\ _1] =$ ${}^{e}_{2} =$ $[\ _2] = (\ + \ )$ 2; when instead *, then ${}^{e}_{1} = (\ + \ )$ 2 ${}^{e}_{2} =$. ■

The incentive power of contract renewal in multiperiod bilateral relationships was first analyzed by Lewis (1986). In his model the principal decides in each period whether to continue to finance the project or to terminate it. The threat of termination and the asymmetry of information on the project type between the principal and the agent may induce the agent to exert effort to limit costs. The benefit from the project accrues to the principal only when the project is completed; thus the threat of termination weakens as the project nears completion. This induces the agent to work less hard over time. In our model instead the benefit from the project accrues to the principal in each period whilst the renewal decision only occurs at the end of the contract. Together with the presence of information decay, absent in Lewis, this makes the agent work harder as the project nears completion.

, + It is useful to compare the condition under which a low-productivity agent invests in period 2 with the condition under which this investment is socially desirable. For the sake of exposition, we will assume here that even after observing a low quality in period 2 ($_2 =$), delegation to a low-cost agent is always more efficient than in-house provision,

that is:

$$+(1- \quad)\Delta - \underline{\ } \quad \overline{\ } \quad = \quad + \Delta - \underline{\ } - \frac{\overline{\ } \quad \underline{\ } \quad \overline{\ } \quad \underline{\ }}{1-} \iff \overline{\ } - \underline{\ } \quad (1- \quad) \quad \Delta$$

The following Corollary is then obtained.

Corollary 2 #) # S

$$^{S} \equiv \Delta + \quad (2 \quad -1)\frac{1+}{2}\Delta + \quad (2 \quad -1) \quad (\overline{\ } - \underline{\ })$$

\# - # # *

S $*$

Proof. Consider a low-productivity agent in period 2. If the agent does not invest, the principal will observe $_2 =$ and thus anticipate an expected quality $^{e}_{3} = \quad + (1- \quad)\Delta$ in period 3 . With probability $1-$ she will then offer a high price in period 3, which will always be accepted, whereas with probability she will offer a low price, which will be accepted only if the agent faces a low cost $\underline{\ }$; if the offer is rejected, the principal will resort to in-house provision and obtain in expected terms:

$$\frac{\int_{\hat{V}}^{\overline{V}}}{\int_{\hat{V}}^{\overline{V}}} = \frac{\hat{\ } + \overline{\ }}{2} = \quad - \underline{\ } - \frac{\overline{\ } \quad \underline{\ } \quad \overline{\ } \quad \underline{\ }}{1-} + \frac{2-}{2}\Delta$$

Total expected welfare is thus equal to (letting $\tilde{\ } = \overline{\ } + (1- \quad)\underline{\ }$ denote the expected operating cost in each period):

$$\begin{aligned} [\ _2(\ _2 = 0)] = & \quad - \tilde{\ } + \quad (\quad + (1- \quad)\Delta - \underline{\ }) \\ + & \ (1- \quad)(1- \quad)\left(\quad + (1- \quad)\Delta - \overline{\ }\right) \\ + & \ (1- \quad) \left(\quad - \underline{\ } - \frac{\overline{\ } \quad \underline{\ } \quad \overline{\ } \quad \underline{\ }}{1-} + \frac{2-}{2}\Delta \right) \end{aligned}$$

If instead the agent invests, the principal will observe $_2 =$ and thus anticipates an expected quality $^{e}_{3} = \quad + \quad \Delta$ in period 3. The probability that she will make a high price offer in period 3 will be , whereas with probability $1-$ she will offer a low price.

$$\begin{aligned} [\ _2(\ _2 = 1)] = & \quad + \Delta - \tilde{\ } - \quad + \quad (\quad + \quad \Delta - \underline{\ }) \\ + & \ (1- \quad) \ (\quad + \quad \Delta - \overline{\ }) \\ + & \ (1- \quad)(1- \quad)\left(\quad - \underline{\ } - \frac{\overline{\ } \quad \underline{\ } \quad \overline{\ } \quad \underline{\ }}{1-} + \frac{1+}{2}\Delta \right) \end{aligned}$$

Taking the difference between the two expressions above, we obtain ^{s} ■

When * ^{S}, underinvestment occurs: the agent does not invest even though doing so would be socially desirable. This is because when investing has no impact on the likelihood of a high price offer (i.e. $\hat{}$ or $\hat{\bar{}}$), then investing brings no benefit to the agent even though it enhances welfare, by increasing expected quality by $(2 \quad - 1)\,\underline{\Delta}$ whenever the agent ends up supplying the service (i.e. $\hat{}$ or $\hat{\bar{}}$ and $= \bar{}$). When instead investing induces the principal to switch from a low-price offer to a high-price offer, the value for the principal of the fall back low-price offer increases when the agent invests in quality, and, furthermore, offering a low price yields zero gain for the agent. It follows that it must be the case that:

$$
\begin{aligned}
\left({}_2 = 1 \quad = \bar{} \right) &= \quad \left({}_2 = 1 \quad = \bar{} \right) - \left({}_2 = 1 \quad = \bar{} \right) \\
\left({}_2 = 1 \quad = \_ \right) &= \quad \left({}_2 = 1 \quad = \_ \right) \quad \left({}_2 = 0 \quad = \_ \right)
\end{aligned}
$$

which implies:

$$
\left({}_2 = 1 \quad = \bar{} \right) \qquad \left({}_2 = 1 \quad = \bar{} \right) - \quad \left({}_2 = 0 \quad = \_ \right)
$$

Thus, the gain for the agent from investing in quality is lower than the welfare gain.

3 Extensions

In the basic model, the agent never invests in quality in the first period of the initial contract. This is because observed quality in period 2 suffices to perfectly infer the agent's type at the end of the contract, which is what matters for the renewal decision.

We now consider two extensions in which the agent may also find it desirable to maintain a good quality in the first period. First, we relax the assumption of linear investment cost and assume that cost is convex. As we shall see, investment may then be undertaken also in period 1, but it remains the case that incentives are stronger in period 2. Second, we introduce the possibility that the agent exerts effort (referred to as "window dressing") to improve his current performance but with no effect on innate productivity. We show that our main predictions continue to hold in this extended model though some new effects of contract renewal on performance dynamics arise.

In the basic model, the agent builds reputation to enhance his bargaining position at renewal stage, the rent for the agent stemming for the presence of asymmetry of information on cost. In this section, we further extend the

model to discuss the incentive value of the expected rent, considering the possibility that the principal commits to a pricing policy at renewal stage.

3.1 Variable investment

We assumed so far that, by investing , the agent could upgrade for sure his quality. We relax this assumption here and suppose instead that, when $_t =$, then the agent can upgrade his quality with any probability ≤ 1 by investing $( ) = \quad ^2 \; 2$.

Building on the above analysis, in period 2, a low-type agent ($_2 =$) will choose $_2$ so as to maximize:

$$\max_i - \; (\,_2) + [\,_2 \; + (1 - \,_2)\,(1 - \;)] \qquad = -\frac{}{2}\,(\,_2)^2 + \,_2\,(2 \; - 1) \qquad + (1 - \;) \qquad = \underline{\Pi}_2$$

That is, by investing $(\,_2)$ the agent increases the probability of earning the rent in period 3 by $_2\,(2 \; - 1)$; the agent will thus choose:

$$^*_2 = \min\left\{(2 \; - 1) \frac{}{} \; 1\right\}$$

By doing so, the agent obtains an expected payoff either equal to:

$$\underline{\Pi}_2 = \begin{cases} -\frac{}{2} + & \text{if} \quad \leq (2 \; - 1) \\ (1 - \;) \qquad + (2 \; - 1)^2 \, \frac{2 \quad 2}{2} & \text{if} \qquad (2 \; - 1) \end{cases}$$

If instead $_2 =$, then the agent's expected payoff is equal to $\overline{\Pi}_2 =$.

Given this, in period 1, a low-productivity agent ($_1 =$) will choose $_1$ so as to maximize:

$$\begin{aligned} & - \; (\,_1) + \; \left\{\,_1 \left[\; \overline{\Pi}_2 + (1 - \;)\,\underline{\Pi}_2\right] + (1 - \,_1)\left[(1 - \;)\,\overline{\Pi}_2 + \; \underline{\Pi}_2\right]\right\} \\ = \; & -\frac{}{2}\,(\,_1)^2 + \,_1 \; (2 \; - 1)\left(\overline{\Pi}_2 - \underline{\Pi}_2\right) + \left[(1 - \;)\,\overline{\Pi}_2 + \; \underline{\Pi}_2\right] \qquad (1) \end{aligned}$$

Proposition 3 # 1 2
2' $^*_2$' & + (#)
( )' ( )
( ). $^*_1$ $^*_2$' / '
& + (

Proof. If $\leq (2 \; - 1)$, expression (1) amounts to maximizing

$$-\frac{}{2}\,(\,_1)^2 + \,_1 \frac{}{2}\,(2 \; - 1)$$

which leads to

$$^{*}_{1} = \frac{}{2}(2 \quad - 1)$$

If instead $\quad (2 \quad - 1) \quad$, expression (1) amounts to maximizing

$$-\frac{}{2}(\ _1)^2 + \ _1 \ (2 \quad - 1)\left[(2 \quad - 1) \quad - (2 \quad - 1)^2 \frac{^{2} \ ^{2}}{2}\right]$$

which leads to

$$^{*}_{1} = \quad \left[1 - (2 \quad - 1)\frac{}{2}\right](2 \quad - 1)\frac{}{} = \left(1 - \frac{^{*}_{2}}{2}\right)(2 \quad - 1) \ ^{*}_{2} \quad ^{*}_{2}$$

In both cases, we have:

$$^{*}_{1} = \left(1 - \frac{^{*}_{2}}{2}\right)(2 \quad - 1) \ ^{*}_{2} \quad ^{*}_{2}$$

Taking the derivative of

$$\frac{^{*}_{1}}{^{*}_{2}} = \left(1 - (2 \quad - 1) \ \frac{}{}\right)(2 \quad - 1)$$

with respect to completes the proof. ■

In contrast with the linear case, with convex cost it is worth investing also in period 1. Note that the first two terms in expression (1) can be rewritten as:

$$-\frac{}{2}(\ ^{*}_{2})^2 + [\ _1 \ (2 \quad - 1) - \quad]\left[\ (2 \quad - 1) \quad - \left(\ ^{*}_{2}(2 \quad - 1) \quad - \frac{}{2}(\ ^{*}_{2})^2\right)\right]$$

This shows that the incentive to invest in period 1 derives from a combination of two effects First, an increase in or raises the net benefit for the agent from investing in period 1, thus $^{*}_{1}$ directly increases with or . Second, an increase in or increase $^{*}_{2}$, the investment in period 2, and this indirectly reduces the incentive to invest in period 1: a crowding out effect. When the ratio $^{*}_{1} \ ^{*}_{2}$ is considered, the direct effects of an increase in on $^{*}_{1}$ and $^{*}_{2}$ cancel out and only the indirect effect of $^{*}_{2}$ on $^{*}_{1}$ matter. As a result $^{*}_{1} \ ^{*}_{2}$ decreases in . The direct effects of and instead still matter as contract renewal is less important in period 1 than in period 2, because of information decay and discounting. This explains why the effect of and on $^{*}_{1} \ ^{*}_{2}$ is ambiguous.

Proposition 3 implies that the expected quality again increases as the contract approaches the renewal date: $_1^e \quad _2^e$, since:

$$\begin{aligned} {}_1^e &= \quad + \Pr(\ _1 = \quad) \\ &= \quad + \frac{1 + \ _1^*}{2}\Delta \qquad + \frac{1 + \ _2^*}{2}\Delta \end{aligned}$$

and:

$$\begin{aligned} {}_2^e &= \quad + \Pr(\ _2 = \quad) \\ &= \quad + \left\{ \frac{1 + \ _1^*}{2} [\quad + (1 - \quad)\ _2^*] + \frac{1 - \ _1^*}{2} [(1 - \quad) + \quad _2^*] \right\} \Delta \\ &= \quad + \left\{ \frac{1 + \ _2^*}{2} + \frac{\ _1^*}{2} (2 \quad - 1)(1 - \ _2^*) \right\} \Delta \qquad + \frac{1 + \ _2^*}{2}\Delta \end{aligned}$$

3.2 Window-dressing

We suppose here that, in each period, a low-type agent can exert an effort that, at cost , improves the quality (correctly or wrongly) perceived by the principal in that period. In contrast with the previous investment technology, this effort does not change the agent's underlying "type" : $_t$ coincides with $_{t-1}$ with probability , whatever the agent's effort. This effort, which is not observed by the principal, can thus be interpreted as pure window-dressing (however, the analysis applies unchanged to efforts that temporarily upgrade the true quality).

Let $_1 \in [0\ 1]$ denote the probability that a low-type agent exerts such effort in period 1 and by $_{q_1}$ the probability that a low-type agent exerts such effort in period 2, as a function of the quality $_1 \in \{\quad\}$ observed in period 1. Further, let $_t$ denote the probability that the principal assigns to facing a good type at the beginning of period , and $_t$ the corresponding probability at the end of that period. We let $_{q_1}$ and $_{q_1}$ denote respectively the prior and the posterior probability in period 3, for $_2 =$ and $_1 \in \{\quad\}$. Table 1 summarizes the principal's beliefs at time $= 1\ 2\ 3$.

[Insert Table 1 here]

At the beginning of period 3, the principal will again offer a high price $_3 = \bar{\ }$ only when she expects a sufficiently high quality $_3^e = \quad + \ _{q_1}\Delta$. As a result, the agent's expected profit becomes:

$$[\Pi_3] = \begin{cases} \quad _{q_1} & \text{if } \ _2 = \\ (1 - \quad) & \text{if } \ _2 = \end{cases}$$

Consider first the effort decision of a low-type agent in period 2 ($_2 =$). Exerting effort costs but yields $_2 =$, leading the principal to offer a high price with probability $_{q_1}$ in period 3; if instead the agent does not exert effort, $_2 =$ will be observed and the principal will offer a high price only with probability $(1 - \ )$ $_{q_1}$. The agent will thus exert effort if:

$$_{q_1} \quad - \quad \geq \quad (1 - \quad) \tag{2}$$

which holds when is sufficiently low.

Note that the agent's strategy in period 2 depends on how the principal's will interpret performance at $= 3$ ($_3$), which may in turn depend on the quality observed in period 1 ($_1$), but not on the unobservable type $_1$. Therefore, a low-productivity agent who exerted effort in period 1 ($_1 =$ $_1 =$) and an agent with high-productivity in period 1 ($_1 =$ $_1 =$) have the same incentive to exert effort in period 2.

Consider now the effort decision of a low-type agent in period 1 ($_1 =$). If the agent does not exert effort, with probability ($_L + (1 - \ ))$, $_2 =$ will be observed in period 2, in which case he will enjoy an expected rent of $_L$ from contract renewal. With complementary probability, $_2 =$ will be observed and the expected rent of the agent will be $(1 - \ )$. Thus, the agent's expected payoff from not exerting effort in period 1 is

$$[(\quad _L + (1 - \quad)) \quad _L + \quad (1 - \quad _L)\,(1 - \quad)] \quad ^2 \quad - \quad \frac{_L}{2} \tag{3}$$

Similarly, the expected payoff of the agent from exerting effort in period 1 is

$$[(\quad _H + (1 - \quad)) \quad _H + \quad (1 - \quad _H)\,(1 - \quad)] \quad ^2 \quad - \quad \frac{_H}{2} \quad - \tag{4}$$

Comparing (3) and (4), the agent will exert effort in period 1 if:

$$\frac{(\quad _H + (1 - \quad)) \quad _H - (\quad _L + (1 - \quad)) \quad _L - (1 - \quad) \ (\ _H - \ _L) \quad ^2}{1 + \quad (\ _H - \ _L) \quad ^2} \quad \geq \tag{5}$$

In order to highlight the role played by information decay, note first that the agent never exerts any effort in case of full information decay ($= \frac{1}{2}$), since reputation then does not matter ($_2 =$ $_3 = \frac{1}{2}$ whatever the past performance). When instead there is no information decay ($= 1$), exerting effort is only worthwhile if a good quality is observed in both periods: should instead performance be low in one period, the principal would then infer that the agent's type is low. Thus, exerting effort in one period makes sense only if the agent also exerts effort when needed in the other period. We then have:

Proposition 4 **0** $(\quad = 1)$'
! # ! **1** **)** '
2 #

Proof. $= 1$! **&%('** $_1 =$ $_L = 0$
$_1 =$ $_H$

$$\frac{1}{1 + \quad_1 \quad_H} \geq \tag{6}$$

$_1$ **&** **3(**

$$\frac{\quad_H}{1 + \quad_1 \quad_H} \frac{1}{1 + \quad_1 \quad_H} \geq \tag{7}$$

4 $\frac{\delta e_H}{1+e_1 e_H}$ 1 **&5(** ' $_1$ 0 **&6(**
' $_H = 1$ **0** **&5(**
' $_1 = 0$ $\frac{\delta e_H}{1+e_H}$ **&6(** $_H = 0$
! * $ % **0** #

$$^e_1 = \quad + \frac{1}{2}(1 + \quad_1)\Delta \tag{8}$$

$$^e_2 = \quad + \frac{1}{2}[\quad + (1 - \quad) \quad_H]\Delta + \frac{1}{2}[\quad_1(1 - \quad + \quad_H) + (1 - \quad_1)(1 - \quad + \quad_L)]\Delta \tag{9}$$

' '

$$^e_2 - \, ^e_1 = [\quad_H - \quad_1 - \quad(\quad_H - \quad_L)(1 - \quad_1)]\frac{\Delta}{2} \tag{10}$$

$= 1$ # $^e_2 = \, ^e_1 = 0$ $_1$ 0 $_H = 1$ $_1 = 0$
$_H = 0$ ■

The next proposition characterizes the agent's behavior when information decay is present.

Proposition 5 **&** **(** # !
' $_1$ 0 $_H = 1$
$_L$ 0 **1** ' # $_H$ $_L$ 0 $_1 = 0$ **&** **(** **1** #
! $_H \geq \quad_L$ * #
0 $_L$ 1 # **)**
( ) **&**)

Proof: see the Appendix.

Contract renewal provides here again an effective incentive device. The strength of this implicit incentive depends on the expected rent from renewal

and a number of other factors. First, there is again a information-decay effect: the incentive to exert effort increases as the renewal date approaches since, due to information decay, recent performance provides better information than past performance about the agent's underlying productivity. This information decay effect takes a less brutal form here, since the agent may exert effort in the first period as well as in the second one; yet, a necessary condition for effort to be exerted in period 1 is that it is exerted in period 2.[7] Second, a " " appears here: in the second period, the agent's incentive to exert effort increases with the reputation acquired in the first period: ${}_H \geq {}_L$. That is, the agent has more incentives to hide bad news when he is supposed to be good.

As in the basic model with only investment, and contrary to Lewis (1986), we find that performance improves as the contract approaches the expiry date. Moreover, with effort there is a new effect which again goes in the opposite direction as the one found in Lewis: incentives increase with existing reputation. This effect arises because on the one hand low quality in the current period perfectly reveals that the agent has low productivity; then by not investing, the agent loses all of his existing reputation. On the other hand, with effort, high quality in the current period is not sufficient statistics for true quality. Better past performance then (weakly) improves the perception of the principal as to the agent's true type, and the payoff from investing increases with existing reputation.

The contract renewal effect again has implications on the performance dynamics.

Corollary 3 ! * +
${}^e_2 \geq {}^e_1$

Proof. Consider the difference in expected quality given by expression (10),

$${}^e_2 - {}^e_1 = [\ {}_H - \ {}_1 - \ (\ {}_H - \ {}_L)(1 - \ {}_1)]\frac{\Delta}{2}$$

From Proposition 5, when ${}_1$ 0, ${}_H = 1$ and thus ${}^e_2 - {}^e_1 = (1 - \ {}_1)(1 - \ (1 - \ {}_L))\frac{\Delta}{2} \geq 0$; when ${}_1 = 0$ then ${}^e_2 - {}^e_1 = [(1 - \ )\ {}_H + \ {}_L]\frac{\Delta}{2} \geq 0$ with strict inequality for ${}_L$ or ${}_H$ 0 ■

[7] The role of information persistent is less clear-cut since both recent and past history matters. For example, when $q_1 = L$, an increase in ρ has a positive direct impact on the incentives to invest, taking as given the principal's belief at the beginning of period 2, but it tends to decrease this belief. In contrast, when $q_1 = H$, both the direct and the indirect effects enhance the incentives to invest.

3.3 Investment and window-dressing

Suppose now that both effort and investment are possible in each period and consider the agent's choice between investment and effort.

It is natural to assume that pure "window-dressing" is less costly than actual investment: . If follows that, in period 2, no investment ever takes place, since window-dressing is less costly and yet achieves the same result, namely, delivering a high quality in order to secure renewal with probability $_{q_1}$.

Consider now period 1 and suppose that $_1 = 0$. Then for given beliefs, the agent's expected payoff from not investing in period 1 is the same as the expected payoff from not exerting effort and it is given by expression (3).

The expected payoff of the agent from investing in period 1 is instead:

$$\left[(\; + (1 - \;) \; _H) \; _H + (1 - \;)^2 (1 - \; _H) \right] \; ^2 \quad - (1 - \;) \; \frac{H}{2} \; - \qquad (11)$$

where use has been made of the fact that investment changes the agent's current type.

Comparing (3) and (11), we obtain the net gain for the agent from investing in period 1 when there is no effort in period 1. Alternatively, the agent can choose not to invest and to exert effort. The net gain from exerting effort was analyzed in the previous section and it is given by the difference between expressions (3) and (4).

Proposition 6 $\rightarrow$ **1** '
**#** **#** **!**

Proof. Consider period 2. The agent invests if

$$_{q_1} \; - \; \geq \; (1 - \;)$$

whilst he exerts effort if condition (2) is satisfied. Since it follows that the agent has more incentives to exert effort than to invest. Consider now period 1, taking into account that in period 2 the agent may exert but not invest. The payoff of the agent at $= 1$ and $_1 = 0$ is given by

$$\left[(\; + (1 - \;) \; _H) \; _H + (1 - \; _H) (1 - \;)^2 \right] \; ^2 \quad - (1 - \;) \; _H \; - \qquad (12)$$

whilst the payoff of the agent in period 1 at $= 0$ and $_1 = 0$ is given by expression (3). Taking the difference between expressions (12) and (4), we obtain

$$(2 \; - 1) \; [(1 - \; _H) (\; _H - (1 - \;)) + \; _H \;] \quad - (\; - \;)$$

which is positive for $\to$ Thus, in period 1, the agent has more incentive to invest than to exert effort. ■

Contrary to the case with only productive investment, investment will take place also in period 1, provided that the cost of investment is not too high. Intuitively, both investment and effort are induced by the desire of the agent to build a reputation for being a good type and increase the expected rent from renewal. As effort reduces the informativeness of good performance observations in period 2, showing good performance becomes valuable also in period 1. Since investment has longer-term consequences but is costlier than effort, when the difference in cost is small, the agent will prefer to invest in early periods rather than to exert effort. In the second period, instead, since effort is cheaper than investment and they are observationally equivalent, the agent will improve performance only through effort.

Finally, consider how contractual performance changes over time.

Proposition 7 + #

Proof. If there is no investment in period 1, since there is never any investment in period 2 (Lemma 2), we are back to the case analyzed in the Section **??** where we showed that performance increases over time. Consider therefore the equilibrium where investment takes place in period 1. We now prove that it must be the case that $_H = 1$ (which implies $_L$ 0 from the proof of Proposition 5). First, suppose by contradiction that $_H$ 1. Then if $_H = \ _L = 0$ the payoff of the agent from investing in period 1, given $_1 = \ _H = \ _L = 0$ is (from (12))

$$\left[\ _H + (1 - \ )^2\right]\ ^2 \ - \qquad (13)$$

whilst the payoff of the agent from not investing in period 1, given $_1 = \ _H = \ _L = 0$ is (from 3)

$$\left[(1 - \ )\ _L + \ (1 - \ )\ ^2\ \right] \qquad (14)$$

Taking the difference between the above two expressions, after some simplifications, the agent invests in period 1 if

$$(2\ - 1)\,(\ _H - (1 - \ )\ _L)\ ^2\ \geq$$

Since $_H$ 1 implies $(2\ - 1)\ _H$ (from condition (2)) and $\geq$ we then have a contradiction.

Second, suppose that $_H$ 1 and $_H = \hat{}_H$ and $_L = \hat{}_L$ Then by the mixed strategy condition, the agent is indifferent between $_H = \hat{}_H$ and $_H = 0$ and $_L = \hat{}_L$ and $_L = 0$ Thus the payoffs of the agent in period 1 are still given by (13) and (14) above so the same reasoning as for the case of $_H = \ _L = 0$ applies here. ■

3.4 Limited commitment

So far we have assumed that the principal cannot commit to any renewal or pricing policy. As a result, while the renewal process does give the agent some incentives to invest, these are insufficient and may fail to induce the agent to invest whenever it is efficient to do so. Therefore, the principal would benefit from committing in advance to a renewal or pricing policy that enhances the agent's incentives to invest.

To see this, we now introduce some commitment ability for the principal, in that she can precommitt over the prices that she may propose in period 3. Ex post, the principal can opt for three types of offers: (i) an offer that the agent will accept whatever his cost, (ii) an offer that the agent will accept only if he faces a low cost, (iii) an offer that is never acceptable (equivalently the principal could choose not to make any offer), Clearly, among the admissible prices that would fall in each of the first two categories, the principal will chose the most favorable one. Therefore, without loss of generality, we can restrict attention to two relevant prices: a "high price" $\bar{\ } \geq \bar{\ }$ designed to be accepted by the agent whatever his cost, and a "low price" $\underline{\ } \in [\underline{\ }\ \bar{\ }]$ that the agent will accept only if he faces a low cost ($= \underline{\ }$).[8]

Committing to prices $\bar{\ }$ and $\underline{\ }$ higher than the corresponding costs, $\bar{\ }$ and $\underline{\ }$, generates two effects: keeping constant the probability of renewal, it increases the agent's expected rent; however, it also reduces the probability that the principal will wish to renew the contract. Let $\hat{\ }(\ _3^e)$ denote the threshold level for $\ _3$ below which the principal prefers to offer a high price and $\tilde{\ }(\ _3^e)$ the threshold above which she favors in-house provision. As long as they lie below $\bar{\ }$, they are defined by:

$$\hat{\ }(\ _3^e) \ : \quad \ _3^e - \bar{\ } = \ (\ _3^e - \underline{\ }) + (1 - \ )\ \hat{\ } \Leftrightarrow \hat{\ }(\ _3^e) = \ _3^e - \underline{\ } - \frac{\bar{\ } - \underline{\ }}{1 - \ } =$$

$$\tilde{\ }(\ _3^e) \ : \quad (\ _3^e - \underline{\ }) + (1 - \ )\ \tilde{\ } = \tilde{\ } \Leftrightarrow \tilde{\ }(\ _3^e) = \ _3^e - \underline{\ }$$

It is straightforward to check that the low-productivity agent still never invests in period 1 ($_1 = 0$), whilst he may invest in period 2 ($_2$ 0) when the expected rent from renewal is sufficient (without loss of generality, $\hat{\ }$ $\underline{\ }$ otherwise with no rent there would be no investment). In period 3, the

[8]The principal might also commit to pay P^0 if the contract is not renewed, a kind of severance pay. It is easy to check that the analysis remains similar, adjusting the prices $\overline{p}$ and $\underline{p}$ by the same amount. This therefore amounts to increase the expected rent at the renewal stage, which the principal can however retrieve ex ante by reducing accordingly the price for the first contract.

expected payoff of the principal writes as:

$$[\ \ _3] = \int_{\underline{V}}^{\hat{V}(q_3^e)} (\ {}_3^e - {}^-) \quad + \int_{\hat{V}(q_3^e)}^{\tilde{V}(q_3^e)} [\ (\ {}_3^e - \_) + (1 - \ \ )\ \ ] \quad + \int_{\tilde{V}(q_3^e)}^{\overline{V}}$$

whereas the expected payoff of the agent is given by:

$$[\Pi_3] = \int_{\underline{V}}^{\hat{V}(q_3^e)} \left({}^- - {}^{\tilde{}}\right) \quad + \int_{\hat{V}(q_3^e)}^{\tilde{V}(q_3^e)} \left(\_ - \_\right) \tag{15}$$

In period 2, investing in case of low productivity thus increases the expected quality $\ {}_3^e$ by $(2\ \ -1)\,\Delta$, from $\_{}_3 \equiv \ \ + (1 - \ \ )\,\Delta$ to ${}^-{}_3 \equiv \ \ + \ \ \Delta$; it therefore tends to increase the thresholds $\tilde{}\ (\ {}_3^e)$ and $\hat{}\ (\ {}_3^e)$ by the same amount and, as a result, enhances the payoff expected by the agent in period 3 by:

$$\Delta\Pi_3 = (2\ \ -1)\left[\left({}^- - {}^{\tilde{}}\right) - \quad \left(\_ - \_\right)\right]$$

where $\ \ \in [0\ \ 1]$ depends on the position of $\tilde{}\ (\_{}_3)$ with respect to $\overline{\ \ }$ and $\tilde{}\ ({}^-{}_3)$. It follows that, in order to enhance the agent's incentive to invest in quality, the principal should keep $\_$ as low as possible (i.e., $\_ = \_$) and instead increase ${}^-$ above $\overline{\ \ }$.

The principal can retrieve ex ante the agent's expected payoff from renewal through the price of the first contract; it will thus seek to maximize total expected welfare, which, assuming that investment is socially desirable, amounts to maximize

$$\begin{aligned}[\ \ _3] \ \ = \ \ & \left\{ \int_{\underline{V}}^{\hat{V}(\overline{q}_3)} \left({}^-{}_3 - {}^{\tilde{}}\right) \frac{}{\Delta} + \int_{\hat{V}(\overline{q}_3)}^{\tilde{V}(\overline{q}_3)} [\ \ ({}^-{}_3 - \_) + (1 - \ \ )\ \ ] \frac{}{\Delta} + \int_{\tilde{V}(\overline{q}_3)}^{\overline{V}} \frac{}{\Delta} \right\} \\ & + (1 - \ \ ) \left\{ \int_{\underline{V}}^{\hat{V}(\underline{q}_3)} \left(\_{}_3 - {}^{\tilde{}}\right) \frac{}{\Delta} + \int_{\hat{V}(\underline{q}_3)}^{\tilde{V}(\underline{q}_3)} \left[\ \left(\_{}_3 - \_\right) + (1 - \ \ )\ \ \right] \frac{}{\Delta} + \int_{\tilde{V}(\underline{q}_3)}^{\overline{V}} \frac{}{\Delta} \right\}\end{aligned}$$

subject to the constraint that $\Delta\Pi_3$ should be sufficient to induce a low productivity agent to invest in period 2, that is: $\ \ \le \ \ \Delta\Pi_3$. It follows that $\_ = \_$ not only maximizes the agent's incentive to invest, but moreover ensures that in-house provision is adopted only when it is efficient, that is, exactly when

$_3$ $\;_3^e - \_\_$.[9] It is thus optimal for the principal to adopt $\_ = \_\_$. As for $^-$, it may be necessary to increase it above $^{-}$ in order to provide sufficient incentives to invest. As this distorts the renewal decision, however, the principal will then choose the lowest level necessary to provide these incentives. We thus have:

Proposition 8 1

' $\_ = \_\_$. * $\hat{}^{s}$' $\hat{}^{s} \in (\; ^* \; ^s)$

' $-$) # $-$

2 # # *

Proof. Conditional on the investment being desirable, let $(2 \quad - 1)$ so that the agent does not invest. Then to induce investment the level of $^-$ needs to satisfy

$$(2 \quad -1) \quad (^- - \quad ^e) = \quad \Longleftrightarrow {}^- - {}^{-} = \hat{} \equiv \quad ^e + \frac{- \quad (2 \quad -1)}{(2 \quad -1)}$$

Let $\hat{}\;(\hat{})$ denote the level of such that the principal is indifferent between making a high-price offer or a low-price offer. We have

$$\begin{aligned} \hat{}\;(\hat{}) &= \quad + \quad \Delta - \_ - \frac{\hat{}}{1-} - \frac{- \; \_}{1-} = \\ &= \_ + \quad \Delta - \end{aligned}$$

where $\equiv \frac{\hat{\eta}}{1-\alpha}$ Expected welfare in period 2 is then

$$\begin{aligned} [\quad _2] = \quad & \left[(\quad + \quad \Delta - \quad ^e) \left(\quad - \frac{}{\Delta} \right) \right] + \int_{\hat{V}}^{\overline{V}} [\quad (\quad + \quad \Delta - \_) + (1 - \quad) \quad] \frac{}{\Delta} \\ & + (1 - \quad) \left[(\quad + (1 - \quad) \Delta - \quad ^e) \left(1 - \quad - \frac{}{\Delta} \right) \right] \\ & + \int_{\hat{V}(\underline{q},\eta)}^{\overline{V}(\overline{q},\eta)} [\quad (\quad + (1 - \quad) \Delta - \_) + (1 - \quad) \quad] \frac{}{\Delta} \end{aligned}$$

[9] Given $\tilde{q}_3$, consider an increase in $\underline{p}$, which thus decreases $\tilde{V} = \tilde{q}_3 - \underline{p}$, adjusting $\overline{p}$ so as to keep $\hat{V}$ constant; as long as $\tilde{V} \leq \overline{V}$ we have:

$$\frac{\partial E[W_3]}{\partial \underline{p}} = -\frac{\partial E[W_3]}{\partial \tilde{V}} = -\alpha \left(\tilde{q}_3 - \underline{C} - \tilde{V} \right) = -\alpha \left(\underline{p} - \underline{C} \right).$$

It is thus efficient to adopt $\underline{p} = \underline{C}$.

with

$$\frac{[\quad_2]}{} = -\frac{(\overline{\ } - \underline{\ }) + (1 - \quad)}{\Delta}$$

Thus, integrating over the change in welfare due to an increase in $\bar{\ }$ above $\bar{\ }$ is given by

$$\Delta_W = -\frac{(\overline{\ } - \underline{\ })}{\Delta} - \frac{(1 - \quad)^{\ 2}}{\Delta}$$

and as long as

$$^{*} \qquad \hat{\ }^{s} \equiv \ ^{s} - \Delta_W$$

where $\hat{\ }^{s} \in (\ ^{*} \quad ^{s})$ it is optimal to induce investment by raising $\bar{\ }$ above $\bar{\ }$

■

When private incentives are insufficient to induce the agent to invest, the principal may find it optimal to raise the agent's expected rent from contract renewal by raising $\bar{\ }$. Raising $\bar{\ }$ however also brings the cost of increasing the likelihood of in-house provision when delegation to the agent yields a higher welfare. With a higher $\bar{\ }$, the likelihood of a high price-offer to the agent decreases, whilst the likelihood of a low-price offer increases. This in turn increases the probability that the principal's offer is rejected by the agent and thus provision is taken in house. As the expected rent for the agent is costly to the principal but not to society as whole, delegation occurs to a suboptimal extent.

4 Contract duration

We have seen so far that contract renewal can act as an incentive device to induce the agent to provide noncontractible. quality. In this section we extend the analysis to consider how the incentive power of contract renewal changes with the length of the contract and how this affects the principal's choice of contract duration.

For this purpose we suppose that the principal-agent relationship presented in the basic framework is infinitely repeated. In each period, the principal can either delegate the provision of the good or service to the agent, in which case her payoff is of the form $_t - \ _t$, or keep the provision in-house, in which case she obtains $_t$, which is uniformly distributed over $[\underline{\ }\ \ \overline{\ }]$. In each period , with probability the agent's initial quality remains the same as in the previous period, and if it is low the agent can upgrade it by investing . For the sake of exposition, we assume that the principal observes the

agent's quality, $_t$, even when opting for in-house provision.[10] This simplifies the analysis by making the environment stationary. We will compare two settings, in which contracts last either one or two periods. For each setting, we characterize the (stationary) equilibrium levels of investment.

4.1 One period contracts (T=1)

Suppose that the contracts only last for one period. At each renewal date , the principal forms beliefs as to the agent's type based on the agent's past performance. We will focus on stationary Markov equilibria in which the principal's belief at the beginning of a period only depends on the agent's performance in the previous period. For simplicity, we let $= 1 \ 2$

Let $_t$ denote the probability that the principal assigns to the agent's quality being high in period ; this probability depends on the agent's type, which in turn is partly determined by the previous quality $_{t-1}$, and on the agent's investment in case of a low productivity (since the agent's incentive to invest only depends on the renewal stage, which in turn is driven by current performance; the agent's investment decision does not depend on past performance). For example, upon observing $_{t-1} =$, the principal anticipates that the quality will be high with probability:

$$_t = \bar{}_1 \equiv \quad + (1 - \quad) \tag{16}$$

If instead $_{t-1} =$, then the principal anticipates that quality will be high only with probability:

$$_t = \underline{}_1 \equiv 1 - \quad + \tag{17}$$

At renewal date , upon observing the realized value of $_t$, the principal has three options: she can either offer $_t = \bar{}$ or $_t = \underline{}$, or make no acceptable offer. Since her decision has no impact on subsequent performance and renewal stages (in particular, since in any event the principal keeps observing the agent's quality, the renewal decision does not affect investment), the principal chooses the price $_t$ so as to maximize her expected payoff for the current period. Offering a high price yields an expected payoff equal to:

$$_1\left(_t = \bar{} \right) = \quad + \ _t\Delta - \bar{}$$

whereas offering a low price yields:

$$_1(\ _t = \underline{}) = \frac{+ \ _t\Delta - \underline{}}{2} + \frac{_t}{2}$$

[10] This could for example be the case if the agent is involved in multiple relationships, to which the same productivity and investment patterns apply.

The principal then offers a high price when $_1\left(\ _t = \overline{\ }\right)$ $_1\left(\ _t = \underline{\ }\right)$ (which as before implies $_1\left(\ _t = \overline{\ }\right)$ $_t$), which under Assumption 1 happens with probability $_t$.

The agent obtains a rent $= \left(\overline{\ } - \underline{\ }\right)\ 2$ only when being offered a high price. Therefore, the agent's expected payoff is:

$$\Pi_1 = \begin{cases} \overline{\Pi}_1 = \overline{\ }_1 \quad - (1 - \quad) \quad + \quad \left(\overline{\ }_1 \overline{\Pi}_1 + (1 - \overline{\ }_1)\, \underline{\Pi}_1\right) & \text{if } \quad _{t-1} = \\ \underline{\Pi}_1 = \underline{\ }_{-1} \quad - \quad + \quad \left(\underline{\ }_{-1} \overline{\Pi}_1 + \left(1 - \underline{\ }_{-1}\right) \underline{\Pi}_1\right) & \text{if } \quad _{t-1} = \end{cases}$$

This determines the equilibrium payoffs for the agent, as a function of his investment decision; in particular, we have:

$$\begin{aligned} \overline{\Pi}_1 - \underline{\Pi}_1 &= \left(\overline{\ }_1 - \underline{\ }_{-1}\right) \quad + (2 \quad - 1) \quad + \quad \left(\overline{\ }_1 - \underline{\ }_{-1}\right)\left(\overline{\Pi}_1 - \underline{\Pi}_1\right) \\ &= \frac{(2 \quad - 1)\left((1 - \quad) \quad + \quad \right)}{1 - \quad (2 \quad - 1)(1 - \quad)} \end{aligned} \tag{18}$$

Given these continuation equilibrium payoffs, a low type agent's investment decision maximizes:

$$- \quad + \quad \left(\overline{\Pi}_1 - \underline{\Pi}_1\right)$$

This yields:

Proposition 9 **0** &! $= 1$(' *)
)) 1
& (1 $\frac{B}{c} \leq \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$' # # .
& (1 $\frac{B}{c}$ $\frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$' #)) $^*_1 \in (0\ 1)$' #
)

$$^*_1 = 1 - \frac{1 - \quad (2 \quad - 1)}{(2 \quad - 1)} -$$

*) # # #) ( )' ( ) ( )

Proof. **7** (18)' #

$$\frac{"\left(\overline{\Pi}_1 - \underline{\Pi}_1\right)}{"} = \frac{(2 \quad - 1)\left((1 - \quad (2 \quad - 1)) \quad - \quad \right)}{\left(1 - \quad (2 \quad - 1)(1 - \quad)\right)^2}$$

' $\geq 1 - \quad (2 \quad - 1)$' $\left(\overline{\Pi}_1 - \underline{\Pi}_1\right)$! $= 1$' * $(2 \quad - 1) \quad (\quad)$. # # 1 $1 - \quad (2 \quad - 1)$' $\left(\overline{\Pi}_1 - \underline{\Pi}_1\right)$ $(2 \quad - 1) \quad (\quad)$ $\frac{(2 \quad - 1)}{1 - \quad (2 \quad - 1)}$ $= 1$ $= 0$ **8** ' # #)) $

• 1 $\frac{c}{B} \geq \frac{\delta(2\rho-1)}{1-\delta(2\rho-1)}$' # # .

• 1 $\frac{c}{B}$ $\frac{\delta(2\rho-1)}{1-\delta(2\rho-1)}$' *) #
)) ${}^{*}_{1}$' *

$$\frac{(2\quad-1)\,((1-\quad)\quad+\quad)}{1-\quad(2\quad-1)\,(1-\quad)} =$$

$${}^{*}_{1} = 1 - \frac{1-\quad(2\quad-1)}{(2\quad-1)} -$$

■

By showing good performance, the agent raises the incentive of the principal to make a high-price offer at renewal stage, which in turn raises the agent's expected rent. But, if the agent were to invest in each period with probability 1, then the expected quality would be independent of past performance, which in turn would nullify the agent's incentive to invest.[11] For this reason no pure strategy equilibrium exists in which the agent invests in each period with probability 1.

In equilibrium, there is however a positive probability of investment if the relative benefit and the weight attached to the future are sufficiently important. In this equilibrium, the incentive effect of contract renewal is stronger when past performance provides a good indication about future performance, since this raises the principal's willingness to offer a high price upon observing good performance. The equilibrium value of investment thus decreases with information decay.

4.2 Two-period contracts (T=2)

Suppose now that each contract lasts for two periods: ! $= 2$. At renewal date , the principal looks again at past performance to form her expectation as to the quality that the agent will provide if the contract is renewed. As before, only quality $_{t-1}$ matters, however. This, in turn, implies that, as in the basic framework, the agent has no incentive to invest in the first execution period of a contract; he may however invest in the second period in order to increase the prospect of being offered a high price at the renewal stage.

[11] When $i = 1$, the expected rent $\overline{\Pi}_1 - \underline{\Pi}_1$ only comes from the reduction in the likelihood of having to invest in the future. But, as already noted, it is not worth investing for sure in a given period merely to reduce the future probability of investing.

Upon observing $_{t-1} =$, the principal assigns probability to $_t =$. Since there will be no investment in period , if the agent invests with probability in $+1$ the expected qualities will then be $+ \ \Delta$ in period and $+ \bar{}_2\Delta$ in period $+1$, where:

$$\bar{}_2 = 1 - 2 \ (1 - \ )(1 - \ )$$

Since the principal and the agent are similarly uncertain about the agent's future cost, the relevant prices for two-period contracts are a high price $\bar{} = \bar{} + \ ^e$ and a low price $\underline{} = \underline{} + \ ^e$. When offering $\bar{}$, the principal anticipates her expected payoff over the two periods to be equal to:

$$\bar{}_2(\ = \bar{}) = \ + \ \Delta - \bar{} + \ (\ + \bar{}_2\Delta - \ ^e) \tag{19}$$

A low price $\underline{}$ is accepted only with probability 1 2; the principal's expected payoff over the two periods is then equal to:

$$\bar{}_2(\ = \underline{}) = \frac{+ \ \Delta - \underline{} + \ (\ + \bar{}_2\Delta - \ ^e)}{2} + \frac{_t + \ \tilde{}}{2}$$

where $_t$ denotes the value of the in-house option in period and $^e \equiv (\underline{} + \bar{}) \ 2$ represents the expected value of the in-house option in period $+1$. It follows that the principal prefers to make a high-price offer if the realized value of the in-house option, $_t$, is such that:

$$_t \leq \underline{} + \ \Delta + \ (\ + \bar{}_2\Delta - \ ^e - \underline{} - \frac{\Delta}{2})$$

Therefore, when $_{t-1} =$ the principal offers a high price with probability:

$$\bar{} = \ + \ \bar{}_2 + \frac{\bar{} \ ^e \ \underline{} - \underline{} - \frac{\Delta}{2}}{\frac{\bar{}}{\bar{} - \underline{}}}$$

As in the case of one-period contract, the principal is more willing to offer a high price the more she expects the agent to provide high quality ($\bar{}_2$ high). The expected quality in turn depends on the agent's expected type and investment behavior. Compared with the case of one-period contracts, however, now the principal anticipates that the agent will not invest in the first period of the contract but only in the second one.

When instead $_{t-1} =$, the principal anticipates the expected quality to be $+ (1 - \ )\Delta$ in period and $+ \underline{}_2\Delta$ in period $+1$, where:

$$\underline{}_2 = 2 \ (1 - \ ) + (1 - 2 \ (1 - \ )) \ = \ + 2 \ (1 - \ )(1 - \ )$$

The probability of a high offer is therefore

$$\underline{\ } = 1 - \ + \ \underline{\ }_2 + \frac{- \ ^e - \ \underline{\ } - \frac{\Delta}{2}}{\underline{\ } \ - \ \underline{\ }}$$

Consider the agent's incentive to invest in second period of a contract. Let $\overline{\Pi}_2$ and $\overline{\Pi}_2$ denote the continuation payoffs of the agent in period given that $_{t-1} =$ and $_{t-1} =$ were observed, respectively. Following the same reasoning as for a one period contracts, we have

$$\overline{\Pi}_2 = \bar{\ } - 2 \ (1 - \ ) + \ ^2 \left[\bar{\ }_2 \overline{\Pi}_2 + (1 - \bar{\ }_2)\, \underline{\Pi}_2\right] \tag{20}$$

$$\underline{\Pi}_2 = \underline{\ } - [1 - 2 \ (1 - \ )] + \ ^2 \left[\underline{\ }_2 \overline{\Pi}_2 + (1 - \underline{\ }_2) \underline{\Pi}_2\right] \tag{21}$$

and thus:

$$\begin{aligned} \overline{\Pi}_2 - \underline{\Pi}_2 &= \left(\bar{\ } - \underline{\ }\right) + (2 \ - 1)^2 + \ ^2 \left(\bar{\ }_2 - \underline{\ }_2\right) \left(\overline{\Pi}_2 - \underline{\Pi}_2\right) \\ &= \frac{(2 \ - 1)\,(\ + \ (2 \ - 1)\,((1 - \ ) \ + \ ))}{1 - \ ^2 (2 \ - 1)^2 (1 - \ )} \end{aligned} \tag{22}$$

As for one period contracts, good performance in period $- 1$ bring three benefits to the agent at renewal stage: it increases the probability of receiving a high price offer by $\left(\bar{\ } - \underline{\ }\right)$, reduces the probability to have to invest in the second period of the next contract by $(2 \ - 1)^2$ and it raises by $\left(\bar{\ }_2 - \underline{\ }_2\right)$ the probability of enjoying $\overline{\Pi}_2$ rather than $\underline{\Pi}_2$ in the next renewal process. As in the case of one-period contracts, the agent never invests systematically with probability 1. He actually never invests in the first period of a contract; he may however invest with probability 1 in the second period of a contract, since the performance in that period affects the principal's belief for the following period and thus the likelihood of a high-price offer.

As before, in the period preceding a renewal stage, the agent will decide whether to invest so as to maximize:

$$- \ + \ \left(\overline{\Pi}_2 - \underline{\Pi}_2\right)$$

This leads to:

Proposition 10 0 (! $= 2$)' #
#
.

& (1 $\frac{B}{c} \leq \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$' # # .

& (1 $\frac{B}{c} \geq \frac{1-\delta^2(2\rho-1)^2}{\delta(2\rho-1)}$' #)) 1.

& (1 $\frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$ $\frac{B}{c}$ $\frac{1-\delta^2(2\rho-1)^2}{\delta(2\rho-1)}$' #))
$_2^* \in (0\ \ 1)$' #)

$$_2^* = \frac{1 + \quad (2 \quad - 1)}{(2 \quad - 1)} - \frac{1 - \quad ^2 (2 \quad - 1)^2}{^2 (2 \quad - 1)^2} -$$

*) # # #)
( )' ( ) ( )

Proof. 7 (22)' #

$$\frac{" \left(\overline{\Pi}_2 - \underline{\Pi}_2\right)}{"} = \frac{\left(1 - \quad ^2 (2 \quad - 1)^2 (1 - \quad)\right) \quad (2 \quad - 1)^2 (\quad - \quad)}{\left(1 - \quad ^2 (2 \quad - 1)^2 (1 - \quad)\right)^2} +$$

$$- \frac{^2 (2 \quad - 1)^3 (\quad + \quad (2 \quad - 1) ((1 - \quad) \quad + \quad))}{\left(1 - \quad ^2 (2 \quad - 1)^2 (1 - \quad)\right)^2}$$

$$= \frac{(2 \quad - 1)^2 (1 + \quad (2 \quad - 1))}{\left(1 - \quad ^2 (2 \quad - 1)^2 (1 - \quad)\right)^2} ((1 - \quad (2 \quad - 1)) \quad - \quad)$$

' $\frac{c}{B} \geq \frac{1}{1-\delta(2\rho-1)}$' $\left(\overline{\Pi}_1 - \underline{\Pi}_1\right)$! $= 1$'
*

$$\begin{aligned} (2 \quad - 1) (\quad + \quad (2 \quad - 1) \quad) &\leq (2 \quad - 1) ((1 - \quad (2 \quad - 1)) \quad + \quad (2 \quad - 1) \quad) \\ &= (2 \quad - 1) \end{aligned}$$

1 $\frac{c}{B}$ $\frac{1}{1-\delta(2\rho-1)}$' $\left(\overline{\Pi}_1 - \underline{\Pi}_1\right)$

$$(2 \quad - 1) (\quad + \quad (2 \quad - 1) \quad) \qquad \frac{(2 \quad - 1)}{1 - \quad (2 \quad - 1)}$$

$= 1$ $= 0$ 8

- 1 $\frac{c}{B} \geq \frac{\delta(2\rho-1)}{1-\delta(2\rho-1)}$' # # .
- 1 $\frac{c}{B}$ $\frac{\delta(2\rho-1)}{1-\delta^2(2\rho-1)^2}$' # #)) 1 .
- 7 ' $\frac{\delta(2\rho-1)}{1-\delta^2(2\rho-1)^2}$ $\frac{c}{B}$ $\frac{\delta(2\rho-1)}{1-\delta(2\rho-1)}$' *) #)) $_2^*$' *

$$\frac{(2 \quad - 1) (\quad + \quad (2 \quad - 1) ((1 - \quad) \quad + \quad))}{1 - \quad ^2 (2 \quad - 1)^2 (1 - \quad)} =$$

$$_2^* = \frac{1 + \quad (2 \quad - 1)}{(2 \quad - 1)} - \frac{1 - \quad ^2 (2 \quad - 1)^2}{^2 (2 \quad - 1)^2} -$$

■

As in the basic model, information decay makes the agent not to invest in the first execution period of the contract. Instead, when the relative benefit of investment is sufficiently high, the effect of past performance on renewal suffices to induce a low-productivity agent always to invest in the period preceding renewal. As with one-period contracts, investment incentives increase with the weight attached to the future and with the persistence of information, which enhances the effect of the investment on future expected quality.

4.3 Optimal contract duration

A natural question is whether incentives to invest are overall higher under two-period or one-period contracting. Note first that, in both regimes: (i) the agent never invests when $\frac{B}{c} \leq \frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$; and (ii) the agent invests with positive probability when $\frac{B}{c}$ $\frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$. Therefore, for the sake of exposition, we will focus here on the case where $\frac{B}{c}$ $\frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$.

Compared with one-period contracting, two-period contracting generates less investment in quality in the first period of the contracts, but more investment in their second period:

Proposition 11 **0** ' **#**

${}^{*}_{2} \geq {}^{*}_{1}$

Proof. **0** $\frac{B}{c}$ $\frac{1-\delta(2\rho-1)}{\delta(2\rho-1)}$ **#**

$$
\begin{aligned}
{}^{*}_{2} &= \max\left\{\frac{1+\ (2\ -1)}{(2\ -1)} - \frac{1-\ ^{2}(2\ -1)^{2}}{^{2}(2\ -1)^{2}} - 1\right\} \\
&= \max\left\{\frac{1+\ (2\ -1)}{(2\ -1)}\ {}^{*}_{1}\ 1\right\} \\
&\quad {}^{*}_{1}
\end{aligned}
$$

${}^{*}_{1}$ 1 $\frac{1+\delta(2\rho-1)}{\delta(2\rho-1)}$ 1 ■

To see why this is the case, let us compare the stakes in continuation

values, which under two-period contracting can be expressed as

$$
\begin{aligned}
\overline{\Pi}_2 - \underline{\Pi}_2 &= \left(^{-} - \_\right) \quad + \quad (2 \quad - 1)^2 \quad + {}^2\left(^{-}{}_2 - {}_{-2}\right)\left(\overline{\Pi}_2 - \underline{\Pi}_2\right) \\
&= (2 \quad - 1)(1 + \quad (2 \quad - 1)(1 - \quad)) \\
&\quad + \; (2 \quad - 1)^2 \\
&\quad + {}^2 (2 \quad - 1)^2 (1 - \quad) \left(\overline{\Pi}_2 - \underline{\Pi}_2\right)
\end{aligned}
$$

and under one-period contracting can be expressed as (decomposing it over two periods, for comparison purposes):

$$
\begin{aligned}
\overline{\Pi}_1 - \underline{\Pi}_1 &= \left(^{-}{}_1 - {}_{-1}\right) \quad + (2 \quad - 1) \; + \; \left(^{-}{}_1 - {}_{-1}\right)\left(\left(^{-}{}_1 - {}_{-1}\right) \quad + (2 \quad - 1) \quad\right) \\
&\quad + {}^2 \left(^{-}{}_1 - {}_{-1}\right)^2 \left(\overline{\Pi}_1 - \underline{\Pi}_1\right) \\
&= (2 \quad - 1)(1 - \quad)(1 + \quad (2 \quad - 1)(1 - \quad)) \\
&\quad + (2 \quad - 1)(1 + \quad (2 \quad - 1)(1 - \quad)) \\
&\quad + {}^2 (2 \quad - 1)^2 (1 - \quad)^2 \left(\overline{\Pi}_1 - \underline{\Pi}_1\right)
\end{aligned}
$$

These stakes involve three components. First, a good reputation has a greater impact on the probability of a high price offer under $! = 2$ than under $! = 1$, due to "crowding out" in the latter case: the principal anticipates that no investment will take place in the first period of the following contract when $! = 2$, whilst some investment will take place when $! = 1$. Observing high quality is therefore less valuable when $! = 1$ than when $! = 2$ which is reflected by an additional discount factor $(1 - \quad)$.

The second component refers to the saving in future investment cost; this effect is lower under $! = 2$ than under $! = 1$, since there is no investment in the following period under $! = 2$.

The last component refers to the impact of reputation on future contract negotiations, and it is again reduced under $! = 1$ by a discount factor $(1 - \quad)$ due to crowding out. When $\quad = 1$ there is no crowding out and no cost saving. Therefore the benefit from investing coincides in both contracting environments. This explains in particular why the threshold level for positive investment coincides in both regimes. In contracts, it is easily checked that the first effect (resulting from crowding out) already dominates the second effect (cost-saving) whenever the agent invests with positive probability,[12] which implies that the incentive to invest in each period when $! = 1$ is lower than the incentive to invest in the second period of a contract when $! = 2$

[12] The overall impact of the two effects in $\left(\overline{\Pi}_2 - \underline{\Pi}_2\right) - \left(\overline{\Pi}_1 - \underline{\Pi}_1\right)$ is equal to $(2\rho - 1)\left[(1 + \delta(2\rho - 1))B - c - i\delta(2\rho - 1)\right]$, which is positive for any $i \leq 1$ when $\frac{B}{c} > 1 - \delta(2\rho - 1)$ and investment levels are nonzero only when $\frac{B}{c} > \frac{1 - \delta(2\rho - 1)}{\delta(2\rho - 1)} > 1 - \delta(2\rho - 1)$.

This leads to:

Proposition 12 ! # ' # * #) ' # *

Proof: see the Appendix.

When the relative cost is large, the investment levels are small; two-period contracting, which provides a greater incentive to invest in the period before contract renewal, then tends to generate a greater average quality. When instead the relative cost is small, average quality is better under one-period contracting, which induces the agent to invest in all periods rather than every other period.

5 Conclusions

In a multi-period principal agent relationship with moral hazard, adverse selection and information decay, we have investigated the incentive of the agent to build a reputation for good performance, and the implications of reputational concerns on performance dynamics and contract duration. We have shown that the prospect of a renewal of the contract can help to induce the agent to invest in noncontractible quality, especially when the contract approaches the renewal date. This has provided us with a rationale as to why we often observe better contractual performance as the contract gets closer to the expiry date. We have also shown that asymmetric information at renewal date can facilitate noncontractible investment since it empowers contract renewal with incentive properties. This has important implications for the choice of contract duration: whether short or long term contracts are preferable was shown to depend on the relative extent of this rent compared to the cost of the investment.

Our results highlight the importance of granting some discretion to public authorities involved in the selection of contractors for the provision of public services. First, discretion gives the principal the possibility to use past performance to make inference as to the agent's productivity and thus to improve her own choice of whether to take provision in house or contract it out. Second, by making past performance relevant to future contract opportunities, discretion induces the agent to invest in nonverifiable dimensions. Granting discretion to public authorities is thus particularly important for all public services such as educational services, clinical services and nursing homes, which involve many noncontractible dimensions.

Throughout the paper we have restricted our attention to a one principal-one agent relationship. It would be interesting to extend the analysis to allow for the possibility that alternative providers are available at renewal stage. At first sight, given the limited commitment ability of the principal and the incentive properties of the expected rent, our results suggest that restricting participation may facilitate investment (as in Calzolari and Spagnolo, 2007). An in depth analysis of the effect of potential competition on the incentive power of contract renewal could constitute an interesting scope for future research.

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6 Appendix

Proof of Proposition 5. Consider period 2-strategy. From expression (2) and Table 1, when $_1 =$, the agent exerts effort if

$$\#_H(\ _H) \equiv (2\ -1)\ {}^H_2(\ _H)\qquad \geq \tag{23}$$

Let $\hat{}_H \equiv \frac{((2\rho-1)\delta B-\gamma)(e_1(1-\rho)+\rho)}{\gamma(e_1\rho+(1-\rho))}$ then

$$\begin{aligned} _H &= 0 \text{ if } \geq \#_H(\ _H = 0) = \ \geq (2\ -1)\quad ; &(24)\\ _H &= 1 \text{ if } \leq \#_H(\ _H = 1) = (2\ -1)\frac{+\ _1(1-\ )}{1+\ _1}\quad ; &(25)\\ _H &= \hat{}_H \text{ otherwise.} &(26)\end{aligned}$$

When instead $_1 =$, the agent exerts effort if (from expression 2 and Table 1)

$$\#_L(\ _L) \equiv (2\ -1)\ {}^L_2(\ _L)\qquad \geq \tag{27}$$

Let $\hat{}_L \equiv (1-\ )\frac{(2\rho-1)\delta B-\gamma}{\gamma\rho}$. We then obtain

$$\begin{aligned} _L &= 0 \text{ if } \geq \#_L(\ _L = 0) = \quad (2\ -1)\,; &(28)\\ _L &= 1 \text{ if } \leq \#_L(\ _L = 1) = (2\ -1)(1-\ ) &(29)\\ _L &= \hat{}_L \text{ otherwise.} &(30)\end{aligned}$$

Consider now **$'%** strategy in period 1. Substituting for $_H$ and $_L$ in expression (5) and simplifying, the agent invests if

$$\#_1(\ _1\ _H\ _L) \equiv \frac{_H\ ^2\ (2\ -1)^2}{(\ +\ _1(1-\ )+\ _1\ \ _H + (1-\ )\ _H)\,((1+\ \ (\ _H -\ _L)))} \geq \tag{31}$$

Now we proceed in steps.

(i) First, incentives increase with existing reputation, that is $_H \geq\ _L$. Suppose by contradiction that $_H\quad _L$ From (23), (27), we must then have ${}^L_2\quad {}^H_2$ that is (from Table 1)

$$\frac{1-}{1-\ +\ _L}\qquad \frac{2}{1-\ _2+\ _2\ _H} \tag{32}$$

Since $_2\quad 1-$ condition(32) can never holds for $_L\quad _H$ and we have a contradiction.

(ii) Second, from (24), (28), $_H =\ _L = 0$ if and only if $\geq\quad (2\ -1)$.

(iii) Third, in any equilibrium where $_1$ 0 we must have: $_H = 1$ $_L$ 0 To see this, note that

$$\frac{\#_2}{(2 \quad -1) \quad ^2} \equiv \frac{(\quad _H + (1- \quad)) \ ^H_2(\ _1) - (\quad _L + (1- \quad)) \ ^L_2}{1+ \quad (\ _H - \ _L)} \qquad ^H_2(\ _1)$$

It follows that, when condition (5) holds, and thus $_1$ 0, condition (23) is satisfied as strict inequality at $_1 =$, which implies $_H = 1$. From (ii), this implies that when $_1$ 0 then either $_L = 1$ or $_L = \hat{}_L$

In light of (i)-(iii) we can restrict attention to the following cases: 1. $_H = 1$, $_L = 1$ any $_1$; 2. $_H = 1$, $_L = \hat{}_L$ any $_1$; 3 $_H = \hat{}_H$ $\hat{}_L = \hat{}_L$ and $_1 = 0$

CASE 1. Let $_H = 1$ and $_L = 1$ Substituting for $_H = \ _L = 1$ in $\#_1(\ _1 \ \ _H \ \ _L)$ in (31) we get

$$\#_H(\ _1 \ 1 \ 1) = \frac{^2 \quad (2 \quad -1)^2}{(\ _1 + 1)}$$

Thus, a necessary condition for $\{ \ _1 \ \ _H \ \ _L\} = \{0 \ 1 \ 1\}$to be an equilibrium is

$$\#_H(0 \ 1 \ 1) = \ ^2 \quad (2 \quad -1)^2 \leq$$

whilst a NEC for $\{1 \ 1 \ 1\}$ is

$$\#_H(1 \ 1 \ 1) = \frac{^2 \quad (2 \quad -1)^2}{2} \geq$$

Let $\hat{}_1 = \frac{\delta^2 B(2\rho-1)^2}{2\gamma} - 1$ denote the level of $_1$ such that $\#(\hat{}_1 \ 1 \ 1) =$ Then, a NEC for $\{\hat{}_1 \ 1 \ 1\}$ is

$$\frac{^2 \quad (2 \quad -1)^2}{2} \qquad\qquad ^2 \quad (2 \quad -1)$$

CASE 2. Let $_H = 1$ and $_L = \hat{}_L$. Substituting for $_H = 1$ and $_L = \hat{}_L$ in $\#_1(\ _1 \ \ _H \ \ _L))$, we obtain

$$\#_H(\ _1 \ 1 \ \hat{}_L) = \frac{^2 \quad (2 \quad -1) \quad - \quad (1+ \quad)}{1+ \quad \frac{\gamma-(1-\rho)(2\rho-1)\delta B}{\gamma}}$$

Thus a NEC $\{0 \ 1 \ \hat{}_L\}$ is

$$\#_H(0 \ 1 \ \hat{}_L) = \frac{(2 \quad -1) \quad ^2}{1+}$$

whilst a NEC for $\{1 \;\; 1 \;\; \hat{}_L\}$ is

$$\#_1(1 \;\; 1 \;\; \hat{}_L) - \quad 0 \Longleftrightarrow \quad \frac{(2 \quad - 1) \quad ^2}{2\,(1 + \quad)}$$

Finally, $\{\hat{}_1 \;\; 1 \quad _L\}$ is an equilibrium if $\#_1(\hat{}_1 \;\; 1 \;\; \hat{}_L) = \quad$ at $\;_1 = \hat{}_1 \in (0 \;\; 1)$ which requires

$$\in \left(\frac{(2 \quad - 1) \quad ^2}{2\,(1 + \quad)} \quad \frac{(2 \quad - 1) \quad ^2}{(1 + \quad)} \right)$$

CASE 3. Let $\;_H = \hat{}_H \quad _L = \hat{}_L$ then we know from (iii) that $\;_1 = 0$ and no additional condition is required beyond those for $\;_H = \hat{}_H \quad _L = \hat{}_L$

So to summarize

$$(2 \quad - 1)\,(1 - \quad) \qquad \frac{(2\rho-1)^2\delta^2 B}{2} \qquad \{1 \;\; 1 \;\; 1\}$$

$$(2 \quad - 1)\,(1 - \quad) \qquad \in \left[\frac{(2\rho-1)^2\delta^2 B}{2} \quad (2 \quad - 1)^2 \quad ^2 \right] \qquad \{\hat{}_1 \;\; 1 \;\; 1\}$$

$$(2 \quad - 1)\,(1 - \quad) \qquad (2 \quad - 1)^2 \quad ^2 \qquad \{0 \;\; 1 \;\; 1\}$$

$$\in [(2 \quad - 1)\,(1 - \quad) \qquad \frac{(2\rho-1)^2\delta^2 B}{2(1+\delta)}] \qquad \{1 \;\; 1 \;\; \hat{}_L\}$$

$$\in \left[(2 \quad - 1)\,(1 - \quad) \qquad (2 \quad - 1)\,\frac{\rho+\hat{e}_1(1-\rho)}{1+\hat{e}_1} \right] \qquad \{\hat{}_1 \;\; 1 \;\; \hat{}_L\}$$

$$\in \left[\frac{(2\rho-1)\delta^2 B}{2(1+\delta)} \quad \frac{\rho(2\rho-1)\delta^2 B}{1+\delta} \right]$$

$$\in [(2 \quad - 1)\,(1 - \quad) \qquad (2 \quad - 1) \qquad] \qquad \frac{\rho(2\rho-1)\delta^2 B}{1+\delta} \qquad \{0 \;\; 1 \;\; \hat{}_L\}$$

$$\in [(2 \quad - 1) \qquad (2 \quad - 1) \quad] \qquad \{0 \;\; \hat{}_H \;\; \hat{}_L\}$$

Proof of Proposition 12. Under one-period contracting, this average quality is given by:

$$^*_1 = (1 - \quad)\,\frac{\overline{\&}_1 + \underline{\&}_1}{2}$$

where $\overline{\&}_1$ and $\&_1^L$, which respectively denote the total expected discounted quality when the current quality is either high ($_t = \quad$) or low ($_t = \quad$), are

characterized by:

$$\begin{aligned}
\overline{\&}_1 &= \quad + \left[(\quad + (1 - \quad) \;{}^{*}_{1}) \overline{\&}_1 + (1 - \quad)(1 - \;{}^{*}_{1}) \underline{\&}_1 \right] \\
\underline{\&}_1 &= \quad + \left[(1 - \quad + \quad {}^{*}_{1}) \overline{\&}_1 + \quad (1 - \;{}^{*}_{1}) \underline{\&}_1 \right]
\end{aligned}$$

It follows from these conditions that:

$$\frac{\overline{\&}_1 + \underline{\&}_1}{2} = \frac{1}{1 - \quad} \left(\frac{\quad + \quad}{2} + \frac{\quad}{2} \frac{{}^{*}_{1}\Delta}{1 - \quad (2 \quad - 1)(1 - \;{}^{*}_{1})} \right)$$

Therefore:

$$\begin{aligned}
{}^{*}_{1} &= (1 - \quad) \frac{\overline{\&}_1 + \underline{\&}_1}{2} \\
&= \frac{\quad + \quad}{2} + \frac{\quad}{2} \frac{{}^{*}_{1}\Delta}{1 - \quad (2 \quad - 1)(1 - \;{}^{*}_{1})}
\end{aligned}$$

Under two-period contracting, the average quality is given by:

$$\begin{aligned}
{}^{*}_{2} &= (1 - \quad) \left(\frac{\quad + \quad}{2} + \quad \left(\frac{1 + \;{}^{*}_{2}}{2} \overline{\&}_2 + \frac{1 - \;{}^{*}_{2}}{2} \underline{\&}_2 \right) \right) \\
&= (1 - \quad) \left(\frac{\quad + \quad}{2} + \quad \left(\frac{\overline{\&}_2 + \underline{\&}_2}{2} + \frac{{}^{*}_{2} \left(\overline{\&}_2 - \underline{\&}_2 \right)}{2} \right) \right)
\end{aligned}$$

where $\overline{\&}_2$ and $\underline{\&}_2$ now respectively denote the total expected discounted quality, evaluated in the second period of a contract, when the current quality is either high ($_t =$) or low ($_t =$); these expected values are characterized by:

$$\begin{aligned}
\overline{\&}_2 &= \quad + \quad \left\{ \quad + \quad \left[(\quad + (1 - \quad) \;{}^{*}_{2}) \overline{\&}_2 + (1 - \quad)(1 - \;{}^{*}_{2}) \underline{\&}_2 \right] \right\} \\
&\quad + (1 - \quad) \left\{ \quad + \quad \left[(1 - \quad + \quad {}^{*}_{2}) \overline{\&}_2 + \quad (1 - \;{}^{*}_{2}) \underline{\&}_2 \right] \right\} \\
\underline{\&}_2 &= \quad + (1 - \quad) \left\{ \quad + \quad \left[(\quad + (1 - \quad) \;{}^{*}_{2}) \overline{\&}_2 + (1 - \quad)(1 - \;{}^{*}_{2}) \underline{\&}_2 \right] \right\} \\
&\quad + \quad \left\{ \quad + \quad \left[(1 - \quad + \quad {}^{*}_{2}) \overline{\&}_2 + \quad (1 - \;{}^{*}_{2}) \underline{\&}_2 \right] \right\}
\end{aligned}$$

After some simplifications, it follows that:

$$\frac{\overline{\&}_2 + \underline{\&}_2}{2} = \frac{1}{1 - \quad} \frac{\quad + \quad}{2} + \frac{\quad^2}{1 - \quad^2} \frac{{}^{*}_{2} \left(\overline{\&}_2 - \underline{\&}_2 \right)}{2}$$

Therefore:

$$\begin{aligned}
{}_2^* &= (1- \ ) \left(\frac{+}{2} + \left(\frac{\overline{\&}_2 + \underline{\&}_2}{2} + \frac{{}_2^*\left(\overline{\&}_2 - \underline{\&}_2\right)}{2} \right) \right) \\
&= \frac{+}{2} + \frac{}{2} \frac{1+ \ (2 \ -1)}{1+} \frac{{}_2^*\Delta}{1- \ ^2 (2 \ -1)^2 (1- {}_2^*)}
\end{aligned}$$

It is convenient to introduce the notation $' \equiv \ (2 \ -1)$ and $(\equiv \ $. The investment levels are then:

$$\begin{aligned}
{}_1^* &= 1 - \frac{1-'}{'} (\\
{}_2^* &= \max \left\{ \frac{1+'}{'} \ {}_1^* \ 1 \right\} \\
&= \max \left\{ \frac{1+'}{'} \left(1 - \frac{1-'}{'} (\right) \ 1 \right\}
\end{aligned}$$

and the average expected qualities are:

$$\begin{aligned}
{}_1^* &= \frac{+}{2} + \frac{}{2} \frac{{}_1^*\Delta}{1-'(1-{}_1^*)} \\
&= \frac{+}{2} + \frac{\Delta}{2'} \frac{'-(1-')(}{1-(1-')(}
\end{aligned}$$

and

$$\begin{aligned}
{}_2^* &= \frac{+}{2} + \frac{}{2} \frac{1+'}{1+} \frac{{}_2^*\Delta}{1-'^2(1-{}_2^*)} \\
&= \frac{+}{2} + \frac{\Delta}{2'} \frac{1+'}{(1+ \ )'} \frac{'-(1-')(}{1-(1-')(} \text{ if } (\quad \frac{'}{1-'^2} \text{ (that is, } {}_2^* \quad 1) \\
&= \frac{+}{2} + \frac{\Delta}{2} \frac{1+'}{1+} \text{ if } (\leq \frac{'}{1-'^2} \text{ (that is, } {}_2^* = 1).
\end{aligned}$$

It follows that, as long as $(\quad \frac{\sigma}{1-\sigma^2}$ (that is, ${}_2^* \quad 1$), ${}_1^* \quad {}_2^*$ (it suffices to note that $\frac{1+\sigma}{(1+\delta)\sigma} \quad 1$). When instead $(\leq \frac{\sigma}{1-\sigma^2}$, ${}_2^* = 1$ and then:

$$\begin{aligned}
{}_1^* - {}_2^* &= \frac{\Delta}{2'} \left(\frac{'-(1-')(}{1-(1-')(} - '\frac{1+'}{1+} \right) \\
&= \frac{\Delta}{2'} \left(1 - \frac{1-'}{1-(1-')(} - '\frac{1+'}{1+} \right)
\end{aligned}$$

This quality differential thus decreases with **(**, it is positive for **(** $= 0$:

$$ {}^{*}_{1} - {}^{*}_{2}|_{\lambda=0} = \frac{\Delta}{2}\left(1 - \frac{1+\,'}{1+}\right) = \frac{{}^{2}\Delta}{2}\frac{2\quad -1}{1+} \quad 0 $$

whilst it is negative for **(** $= \frac{\sigma}{1-\sigma^2}$:

$$ {}^{*}_{1} - {}^{*}_{2}|_{\lambda=\frac{\sigma}{1-\sigma^2}} = \frac{\Delta\,(\,'\quad -1)}{2\,(1+\quad)} \quad 0 $$

It then follows that there exists a **(**$^{*} \in \left(0 \;\; \frac{\sigma}{1-\sigma^2}\right)$ such that ${}^{*}_{1} \geq {}^{*}_{2}$ for **(** $\leq$ **(*** and ${}^{*}_{1} \quad {}^{*}_{2}$ for **(** **(***