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Endogenous Spatial Differentiation with Vertical Contracting

Frago Kourandi and Nikolaos Vettas

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Abstract

We set-up a linear city model with duopoly upstream and downstream. Consumers have a transportation cost when buying from a retailer, and retailers have a transportation cost when buying from a wholesaler. We characterize the equilibria in a five-stage game where location and pricing decisions (wholesale and retail) by all four firms are endogenous. The usual demand and price competition effects are modified and a strategic effect emerges, since the retailers' marginal costs become endogenous. Firms tend to locate farther away from the market center relative to the vertically integration case. When the wholesalers choose locations before the retailers, each wholesaler locates closer to the market center relative to the retail locations, and relative to when the wholesalers cannot move first. Each wholesaler does this to strengthen the strategic position of its retailer by credibly pulling him towards the market center. As a result, the intensity of competition is higher and industry profit is lower when upstream locations are chosen before downstream. Variations of the model and welfare analysis are provided.

JEL Classification: L13; R32

Keywords: Spatial differentiation; Locations; Vertical contracting; Linear city; Strategic commitment

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1 Introduction

A large number of studies has examined important aspects of how oligopolistic firms choose their locations (or horizontal differentiation) and then compete in prices. Maximal, minimal or intermediate differentiation may emerge in the market depending on the balance between a direct demand effect that pulls firms towards the centre of the market and a strategic price effect that pushes them away from one another. This literature typically focuses on the direct market relation between the oligopolists and the final consumers. In contrast, the nature of vertical relations are important in the great majority of the oligopolistic markets, that is, how upstream firms (wholesalers) trade with downstream firms (retailers) is interrelated with how downstream firms trade with final consumers. In this paper, we study endogenously the horizontal product differentiation (or locations) and pricing decisions in vertically linked duopolies, or, more simply put, we endogenize the locations and pricing decisions of wholesalers and retailers on a Hotelling line.

We study a two-tier horizontally differentiated duopoly model. We assume exclusive vertical contracting with each downstream firm able to turn one unit it gets from its upstream firm into one unit in the final market. Upstream and downstream firms are differentiated with respect to their horizontal locations (as usual, these can be interpreted either geographically or in the space of product characteristics). We study two versions of the model, first when any location on the real line can be chosen and, second, when location can only be chosen within the interval within which consumers are uniformly distributed. Choices are made sequentially. Allowing upstream firms and downstream firms to locate anywhere on the real line allows a clearer and richer characterization of the equilibrium location incentives.

The main questions that can be answered in a framework that combines vertical and product differentiation elements refer to the implications of vertical pricing rules for the location choices and the role that upstream and/or downstream differentiation plays for vertical trade and final prices. The strategic incentives are quite rich, as each firm's choice affects the subsequent choices of both firms and of the final consumers. For instance, upstream locations affect downstream locations, then wholesale prices and then the final prices. In addition, the quantity that each upstream firm will sell is equal to the quantity that the corresponding retailer will sell to the final consumers and this quantity depends (directly or indirectly) on all the choices made by the firms.

In models of horizontal differentiation without upstream firms, there are two opposite forces, one pushing firms close to each other, the "demand" effect, and one to the opposite direction, the "price competition" effect. In our model there is a third force affecting the wholesale prices and the transportation cost that the downstream firms pay when moving to the centre of the line. The interaction of these forces give the equilibrium result. We obtain a number of results. In our basic model (with locations anywhere on the real line) we find that there is a unique subgame perfect equilibrium. In this, all firms locate outside the unit interval but wholesalers always locate closer to the city center than retailers. Relative to the vertical integration benchmark, all locations are farther

away from the center (and therefore the social cost is also higher, including both the intermediate and the final transportation cost). Double marginalization emerges, with all firms making positive profits and final prices higher than under vertical integration. Double marginalization also emerges when locations are restricted within the unit interval, however in that case maximum differentiation is obtained, with each chain at an endpoint.

Important insights are obtained when we modify the order of location choices. When wholesalers chose their locations first, they choose to locate closer to the center relative to when location choices are simultaneous or when retailers move first. They do this to credibly pull their retailers towards the center and offer them a stronger strategic position in the final market. In equilibrium, this strategic behavior leads to more intense competition and lower profits when wholesalers choose their locations first. The industry would, thus, prefer retailers to choose locations first (or at the same time as the wholesalers).

The remainder of the paper is as follows. Related literature is briefly discussed in Section 2. Section 3 sets up the basic model. Section 4 derives the equilibrium and studies its properties. Extensions of the basic model are examined in Section 5. Section 6 modifies the basic model with firms allowed to locate only in the unit interval. An additional equilibrium for fixed upstream locations is presented in Section 7 where downstream firms may reverse their locations on the line. Section 8 concludes.

2 Related literature

Our paper contributes to two dinstict literatures, on product differentiation and on vertical contracting. Each of these contains a number of influential papers and is too vast to survey here.[1] We only refer to work that is more closely related to the specific setting of our model. The strand of the product differentiation literature we are building on, starts with the classic linear-city model of Hotelling (1929) modified by D' Aspremont *et al.* (1979) in a way the strategic price incentives can also be more easily characterized.[2] Nevertheless, only very few papers examine the vertical chain interactions in a horizontal differentiation framework. This creates a gap in the literature since in reality most market structures have an important vertical element with upstream firms supplying the downstream firms. Marginal production costs of the downstream firms are therefore

[1] For a review and key results on product differentiation see e.g. Anderson *et al.* (1992), Gabszewicz and Thisse (1992) and for vertical relations Rey and Tirole (2007) and Rey and Verge (2008).

[2] Many variants of the linear city model has been studied. Among other, Anderson and Neven (1991) solve the location-pricing game when oligopolists compete in quantities. Ziss (1993) examines the D' Aspremont *et al* model with heterogeneous production technologies. If marginal cost difference is sufficient small, a price and location equilibrium exists in which both firms enter and maximum differentiation emerges. Anderson and Engers (1994) study a price-taking equilibrium in the spatial setting. In Vettas (2003) and Vettas and Christou (2005), firms are horizontally as well as vertically differentiated. Tabuchi and Thisse (1995) and Lambertini (1997) allow firms to locate along the entire real line, while consumers are concentrated around the market center.

endogenously determined in the vertical chain. Thus, location choices by the upstream and/or downstream firms are affected by the transactions in a vertical environment.

Gupta *et al.* (1994) assume that an upstream monopolist sets the wholesale price based on its observation of the locations chosen by the downstream firms and that the downstream firms can price discriminate. Beladi *et al.* (2008) study an upstream monopoly and a downstream duopoly where two-part tariffs are signed and the downstream firms cannot produce all varieties demanded. Aiura and Sato (2008) examine a model with an upstream monopolist at the center of the city and supplying two retailers. Retailers do not pay wholesale prices but only a transportation cost and choose their locations and final prices. Here, we consider in contrast a model with two upstream and two downstream firms that pay linear wholesale prices. A location-price equilibrium is also analyzed by Brekke and Straume (2004) where each downstream firm has its own supplier, upstream firms bargain about the input prices with the downstream firms, but upstream firms are not product differentiated. Allain (2002) and Laussel (2006) examine the situation where two upstream firms are exogenously brand differentiated and two downstream firms are exogenously spatially differentiated, thus, consumers face four different products (and are distributed on a rectangle). In our paper, in contrast, consumers care only about spatial differentiation and we further endogenize the location choices in both levels assuming input prices are set by the wholesalers.

Previous work[3] by Matsushima (2004) has also studied the two upstream and two downstream firm structure on the line and finds equilibrium locations that depend on the transportation cost parameters. Our model differs in that upstream firms do not restrict wholesale prices to be equal to the rival firm's transportation cost, the upstream firms supply their own downstream firms and firms can locate outside the unit interval. Further, we assume that it is the downstream firms that pay the transportation costs when supplied by the upstream firms. We also present a sequential location choice by the upstream firms first and the downstream firms after, as opposed to simultaneous symmetric location choices by Matsushima (2004). Matsushima (2009) also studies the incentives for vertical mergers in a location model and Matsumura and Matsushima (2009) examines the mixed strategy equilibria under large cost differentials.

3 The basic model

We set up a model where upstream and downstream duopolies locate on a line as follows. Consumers are uniformly distributed on a [0,1] interval and have unit demands. There are two upstream firms, A and B, and two downstream firms, X and Y, each choosing unique a location on the entire real line. There is exclusive dealing and the downstream firms have a simple fixed-proportions technology: firm X turns each unit it purchases from firm A into one unit that it can sell to the

[3] Dobson and Waterson (1996) study the exclusivity in an exogenously non-horizontal differentiated successive duopoly in upstream and downstream level with consumers facing four varieties. Inderst and Shaffer (2007) analyse the impact of retail mergers on product variety in a non-Hotelling type differentiated model.

final consumers; likewise, for firms Y and B.

The locations of firms A and B are denoted by a and $1-b$, respectively. The locations of firms X and Y are denoted by x and $1-y$, respectively. Thus, for the A and X chain, a and x measure how much to the right of endpoint 0 each firm is located, while for the B and Y chain, b and y measure how much to the left of endpoint 1 each firm is located. Without loss of generality, we consider $1-a-b>0$, so that A is located to the left of B. In the main body of the paper we will also focus on the case where $1-x-y>0$ so that X is also to the left of Y; in Section 7 we also discuss the possibility that the downstream locations switch to the "wrong" side of the line relative to their upstream suppliers.

A transportation cost has to be paid both in the wholesale and retail market. So that our results are easily comparable to the literature, we follow the often used assumption that this cost is quadratic in distance. A consumer located at point z pays transportation cost $t(x-z)^2$ when purchasing a product from firm X and $t(1-y-z)^2$ when purchasing a product from firm Y. In turn, firm X pays transportation cost $\tau(x-a)^2$ when purchasing a unit from firm A and firm Y pays transportation cost $\tau(y-b)^2$ when purchasing a unit from firm B. These costs may be real transportation costs (for example depending on the weight or the volume of the product) or may be product characteristic transformation costs necessary to convert one unit of the upstream firm's input to one unit of final good. We consider linear pricing at the wholesale and retail level. A final consumer who purchases a unit from downstream firm X or Y pays the price set by this firm (p_X or p_Y) plus the transportation cost between the chosen firm's location and his own location. We also assume that the basic reservation value of each consumer is high enough so that each consumer purchases one unit of the product. A downstream firm who purchases a unit from upstream firm A or B pays the price set by this firm (w_A or w_B) plus the transportation cost between the two trading firms. Apart from locations, the products are homogeneous and production costs are assumed zero for simplicity. Note that final consumers care about the upstream locations and prices indirectly, that is, only to the extent they affect the downstream locations and prices.

Each of the four firms seeks to maximize its own profit and each consumer his own net surplus. We assume there are no information asymmetries and proceed to analyze a sequential game where all locations and prices are endogenous. We view locations as longer-run (and more difficult to change) variables than prices. Therefore, the main model we analyze is a five stage game as follows:

(1) Upstream firms A and B simultaneously choose their locations, a, b.

(2) Downstream firms X and Y simultaneously choose their locations, x, y.

(3) Upstream firms simultaneously set linear wholesale prices, w_A, w_B: firm A charges w_A to firm X and firm B charges w_B to firm Y.

(4) Having observed both wholesale prices, downstream firms simultaneously set their (final) product prices, p_X, p_Y.

(5) Having observed the firms' locations and final prices, each consumer purchases one unit of the product from one of the downstream firms X and Y.

We proceed backwards, solving for the subgame perfect Nash equilibria of the game.

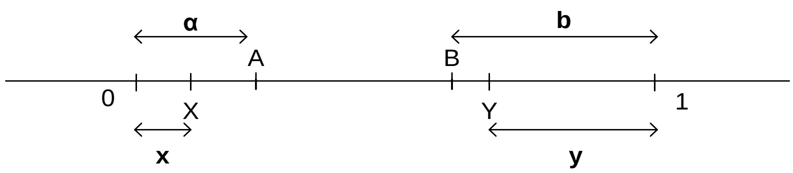


Figure 1: Upstream and downstream locations (example)

In subsequent Sections of the paper and following the analysis of our main case, we also consider alternative timing assumptions about the location choices, when downstream locations can be chosen before the upstream locations and also when all locations (upstream and downstream) are chosen simultaneously. We also examine other extensions of the analysis: when firm locations cannot be on the entire real line but are restricted within the unit interval; and when downstream locations can be at the opposite side on the line relative to their upstream suppliers. But first we proceed to the analysis of our main case.

4 Equilibrium analysis

To obtain a subgame perfect equilibrium, we proceed backwards. First, we consider trade in the downstream market to find equilibrium retail prices and consumer choices given the wholesale prices and the locations of all firms. Second, we derive the equilibrium wholesale prices given all firms' locations and anticipating equilibrium behavior in the retail market. Third, we derive the downstream firms' equilibrium locations given the upstream locations and anticipating pricing, wholesale and then retail. Finally, we derive upstream equilibrium locations anticipating equilibrium in all the subsequent stages. Once we derive the equilibrium, we study its properties. As should be expected when dealing with such a five-stage game, the analysis is tedious at times and some formal details are in the Appendix, or are presented in some brevity to facilitate the continuity of the intuition and the arguments.

4.1 Consumers choice and retail prices

This part of analysis corresponds to duopoly competition in the final (retail) market with fixed locations and potentially different unit costs.[4] For some locations and costs, both retailers sell in the market, while for locations and costs that significantly favor one of the retailers, we obtain a

[4] Ziss (1993) studies the linear city model with different unit costs. The analysis in this part parallels his.

corner solution where the rival makes no sales. The costs faced by the retailers are equal to the wholesale price plus the transportation cost they have to pay per unit.

Given the firms' locations and the wholesale prices, we calculate the demand functions for the downstream firms X and Y. Let z be the demand of firm X and $1-z$ the demand of firm Y. The firms' profit functions are

$$\begin{aligned}\Pi_X &= (p_X - f_X)z,\\ \Pi_Y &= (p_Y - f_Y)(1-z),\end{aligned}$$

where f_X (respectively f_Y) is the *aggregate* marginal cost, that is, the wholesale price *plus* the transportation cost of firm X (respectively Y) with $f_X \equiv w_A + \tau(x-a)^2$ and $f_Y \equiv w_B + \tau(y-b)^2$. When both firms sell positive quantities, demand for each firm is characterized by the presence of an indifferent consumer located at z, as follows:

$$p_X + t(x-z)^2 = p_Y + t(1-y-z)^2. \tag{1}$$

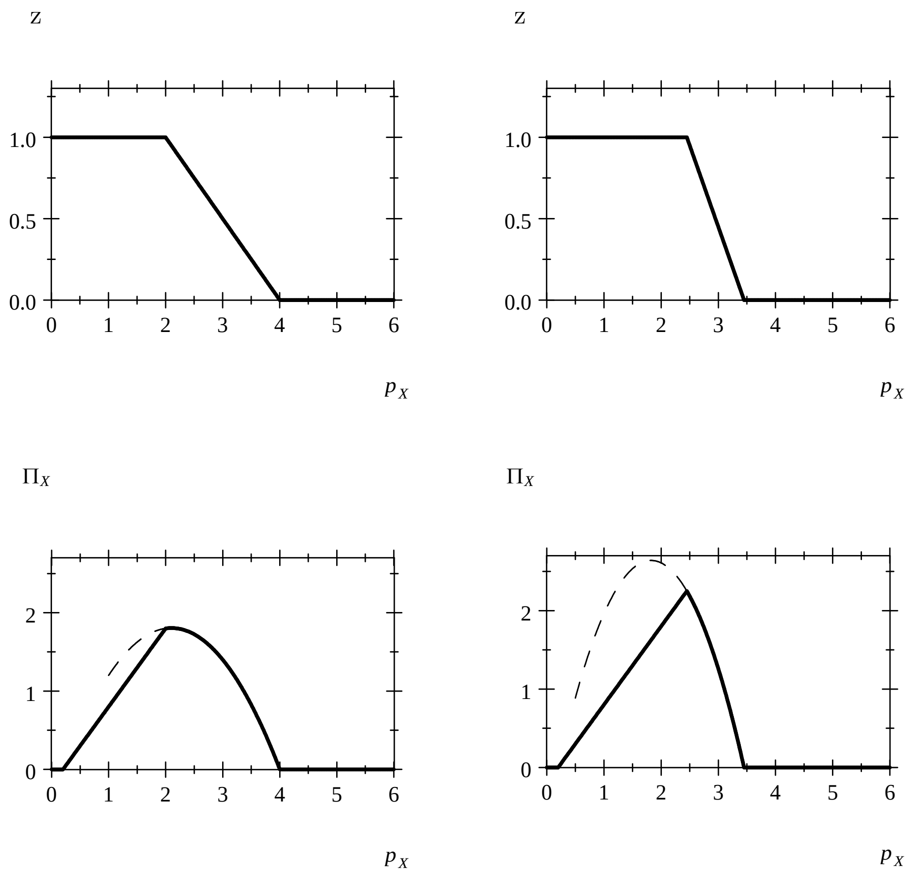
Of course, the location of the indifferent consumer depends on the downstream firms' locations, product prices and the transportation cost parameter t, that is, $z = z(p_X, p_Y, x, y, t)$; to simplify the exposition we suppress the arguments of this function and obtain

$$z = \frac{1+x-y}{2} + \frac{p_Y - p_X}{2t(1-x-y)}. \tag{2}$$

Further, taking into account the possibility that *all* consumers may choose to purchase from one, demand for firm X will be equal to

$$z = \begin{cases} 1 & if \quad \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} \geq 1 \\ \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} & if \quad 0 < \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} < 1 \\ 0 & if \quad \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} \leq 0. \end{cases} \tag{3}$$

We note that the demand and profit functions for the downstream firms are continuous in both firms' prices. For intermediate values of the prices, the market is shared, $z \in (0,1)$, and as firm X reduces p_X, the indifferent consumer moves to the right on the line (thus, z increases). When p_X decreases below the threshold stated in expression (3), firm X captures all the demand ($z=1$) and its profits is simply equal to its profit margin $p_X - f_X$. Likewise, for high values of p_X, firm X makes no sales and its profit is driven to zero. Figure 2 presents two examples of the demand and profit functions for different values of the parameters.



$x = 0,\ y = 0,\ t = 1,\ f_X = 0.2,\ p_Y = 3$ $\qquad$ $x = 0.2,\ y = 0.3,\ t = 1,\ f_X = 0.2,\ p_Y = 3$

Figure 2: Downstream demand and profits

Given the profit functions for the downstream firms, we now proceed to the characterization of equilibrium retail prices. The analysis is standard but special care should be taken for the characterization of the corner cases:

Lemma 1 *The equilibrium price and profit for firm X are:*

$$p_X = \begin{cases} f_Y - t(1-x-y)(1+y-x) & if \quad f_X \leq \overline{f_X} \\ \frac{1}{3}\left(t(1-x-y)(3+x-y) + f_Y + 2f_X\right) & if \quad \overline{f_X} < f_X < \widehat{f_X} \\ f_X & if \quad f_X \geq \widehat{f_X} \end{cases} \tag{4}$$

and

$$\Pi_X = \begin{cases} f_Y - t(1-x-y)(1+y-x) - f_X & if \quad f_X \leq \overline{f_X} \\ \frac{(t(1-x-y)(3+x-y)+f_Y-f_X)^2}{18t(1-x-y)} & if \quad \overline{f_X} < f_X < \widehat{f_X} \\ 0 & if \quad f_X \geq \widehat{f_X}, \end{cases} \tag{5}$$

where

$$\begin{aligned} \widehat{f_X} &\equiv f_Y + t(1-x-y)(3-y+x), \\ \overline{f_X} &\equiv f_Y - t(1-x-y)(3-x+y). \end{aligned}$$

Prices and profits for firm Y can be also derived in a symmetric way.

When firm X has low enough *aggregate* marginal cost (lower than the critical value $\overline{f_X}$) it is the only one that sells in the market, while Y has zero demand. The opposite is true when X has a high *aggregate* marginal cost (higher than $\widehat{f_X}$). When the difference in the marginal costs of the two firms is not too large, $\overline{f_X} < f_X < \widehat{f_X}$, then both downstream firms make sales in the final goods' market.

Two properties of the downstream market equilibrium should be noted. The first derivative of the final prices and the profits with respect to the transportation cost parameter t of the consumers is positive: the more differentiated the final goods are, the higher are the prices set by the retailers and the profits they obtain. Also, we observe that $\frac{dp_i}{dfi} = \frac{2}{3} < 1$. Thus, following an increase in his aggregate marginal cost, a retailer passes part of this to the final consumers but absorbs the rest.

4.2 Wholesale prices

In this stage, the two upstream firms compete by setting their wholesale prices. Each upstream firm exclusively supplies its retailer, firm A supplies firm X and firm B supplies firm Y. Even though each upstream firm cannot supply the rival retailer, competition takes place: the wholesale prices shape competition in the subsequent stage where retail prices and final demand is determined. If the wholesale price charged to retailer X by the upstream firm A is high, retailer X has a relative cost disadvantage compared with the rival retailer and, thus, the demand obtained by firm X (and, in turn, by firm A) is low.

Since a downstream firm turns one unit of the good it purchases in the wholesale market into one unit it sells in the retail market, the demand function of firms A and B are $D_A = z$ and $D_B = 1 - z$ and their profit functions are $\Pi_A = w_A z$ and $\Pi_B = w_B(1 - z)$. Assuming equilibrium will follow when setting retail prices, as characterized in the previous Subsection, the upstream profit functions can be expressed as:

$$
\begin{aligned}
\Pi_A &= w_A \\
&\quad \text{if } w_A \leq w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\
&= w_A\left(\tfrac{2t(1-x-y)(y-x)+w_B-w_A+\tau\left((y-b)^2-(x-a)^2\right)}{6t(1-x-y)} + \tfrac{1}{2}(1+x-y)\right) \\
&\quad \text{if } \left\{\begin{array}{l} w_A > w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\ w_A < w_B + \tau\left((y-b)^2 - (x-a)^2\right) - t\,(x-y+3)\,(x+y-1) \end{array}\right\} \\
&= 0 \\
&\quad \text{if } w_A \geq w_B + \tau\left((y-b)^2 - (x-a)^2\right) - t\,(x-y+3)\,(x+y-1)
\end{aligned}
\tag{6}
$$

and

$$
\begin{aligned}
\Pi_B &= 0 \\
&\quad \text{if } w_A \leq w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\
&= w_B\left(1 - \left(\tfrac{2t(1-x-y)(y-x)+w_B-w_A+\tau\left((y-b)^2-(x-a)^2\right)}{6t(1-x-y)} + \tfrac{1}{2}(1+x-y)\right)\right) \\
&\quad \text{if} \left\{ \begin{array}{l} w_A > w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\ w_A < w_B + \tau\left((y-b)^2 - (x-a)^2\right) - t\,(x-y+3)\,(x+y-1) \end{array} \right\} \qquad (7) \\
&= w_B \\
&\quad \text{if } w_A \geq w_B + \tau\left((y-b)^2 - (x-a)^2\right) - t\,(x-y+3)\,(x+y-1)\,.
\end{aligned}
$$

Note that these profits are functions of the wholesale prices and all four locations. When the *aggregate* marginal cost $w_A + \tau(a-x)^2$ faced by firm X is low enough compared to its rival, firm A captures the whole demand via its retailer X ($z = 1$). When the *aggregate* marginal cost faced by firm X is high enough, its demand (and by implication also the demand of firm A) becomes zero ($z = 0$). For intermediate values of $w_A + \tau(a-x)^2$ we obtain $z \in (0,1)$, both retailers make sales in the final market, and therefore, both wholesalers make sales in the upstream market. We should stress the role of the upstream and downstream locations in our model since they crucially affect the marginal costs of the retailers. For some locations, even if an upstream firm charges a zero wholesale price to its retailer, it cannot obtain positive demand and the *aggregate* marginal cost of one retailer is always lower than the rival is.[5] This happens when the distance of one retailer from its supplier is very large compared to the rival's distance from its own supplier. Clearly, these cases will tend to emerge when the transportation cost parameter τ is high.

Maximization of the profit functions (6) and (7) with respect to the wholesale prices (note that the profit functions are quasi-concave) implies the reaction functions of the upstream firms:

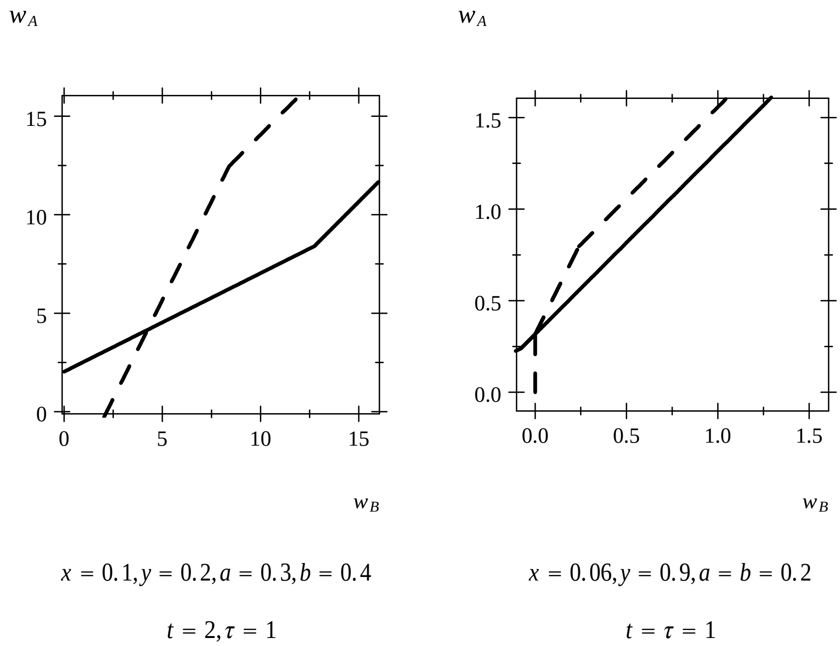
$$
\begin{aligned}
w_A = R_A(w_B) &= w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\
&\quad \text{if } w_B \geq \tau\left((x-a)^2 - (y-b)^2\right) + t\,(y-x+9)\,(1-x-y) \\
&= \tfrac{1}{2}w_B + \tfrac{t(x-y+3)(1-x-y)-\tau\left((x-a)^2-(y-b)^2\right)}{2} \\
&\quad \text{if} \left\{ \begin{array}{l} w_B > \tau\left((x-a)^2 - (y-b)^2\right) - t\,(x-y+3)\,(1-x-y) \\ w_B < \tau\left((x-a)^2 - (y-b)^2\right) + t\,(y-x+9)\,(1-x-y) \end{array} \right\} \\
&= 0 \\
&\quad \text{if } w_B \leq \tau\left((x-a)^2 - (y-b)^2\right) - t\,(x-y+3)\,(1-x-y)
\end{aligned}
$$

[5] To illustrate, take for example $t = \tau = 1, a = b = 0.2, x = 0.06, y = 0.9$. Then even if $w_B = 0$ firm A takes the whole demand.

and

$$
\begin{aligned}
w_B = R_B(w_A) \quad &= \quad 0 \\
&\quad\ \text{if } w_A \leq \tau\left((y-b)^2-(x-a)^2\right)-t\,(y-x+3)\,(1-x-y) \\
&= \quad \tfrac{1}{2}w_A + \frac{t(y-x+3)(1-x-y)-\tau\left((y-b)^2-(x-a)^2\right)}{2} \\
&\quad\ \text{if } \left\{ \begin{array}{l} w_A > \tau\left((y-b)^2-(x-a)^2\right)-t\,(y-x+3)\,(1-x-y) \\ w_A < \tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y) \end{array} \right\} \\
&= \quad w_A - \tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+3)\,(x+y-1) \\
&\quad\ \text{if } w_A \geq \tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)\,.
\end{aligned}
$$

In Figure 3 (I) we present these reaction functions for some parameters values t, τ and locations a, b, x, y for which both firms sell. In Figure 3 (II) we show a case where firm B faces a strong cost disadvantage as it is located far away from its supplier, and this implies a high aggregate marginal cost $w_B + \tau(y-b)^2$ for its retailer. Even if the wholesale price it sets is reduced to zero, firm B cannot obtain positive demand. In equilibrium, firm A serves the whole market via its retailer X. We also note that the wholesale prices are strategic complements, an increase in w_B leads to an increase to w_A and vice versa.



(I) (II)

Figure 3: $R_A(w_B)$: solid line, $R_B(w_A)$: dash line

From the reaction functions we obtain:

Lemma 2 *The equilibrium wholesale prices (as functions of the upstream and downstream locations) are:*

$$
\begin{aligned}
w_A &= \tau\left((y-b)^2-(x-a)^2\right)+t\,(y-x+3)\,(x+y-1) \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0 \\
&= \tfrac{\tau\left((y-b)^2-(x-a)^2\right)+t(x-y+9)(1-x-y)}{3} \\
&\quad \text{if } \left\{ \begin{array}{c} \tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)>0 \end{array} \right\} \qquad (8) \\
&= 0 \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\,(x-y+9)\,(1-x-y)\geq 0
\end{aligned}
$$

and

$$
\begin{aligned}
w_B &= 0 \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0 \\
&= \tfrac{\tau\left((x-a)^2-(y-b)^2\right)+t(y-x+9)(1-x-y)}{3} \\
&\quad \text{if } \left\{ \begin{array}{c} \tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)>0 \end{array} \right\} \qquad (9) \\
&= \tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+3)\,(x+y-1) \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\,(x-y+9)\,(1-x-y)\geq 0.
\end{aligned}
$$

In equilibrium, when the upstream locations are asymmetric enough relative to the downstream locations the disadvantaged upstream firm sets a zero price and there are no sales for itself (and its retailer) while the rival upstream firm sets the highest price that allows it to capture the whole demand. Otherwise, the market is shared (upstream and downstream). In either case, by substituting the equilibrium wholesale prices (8) and (9) into expressions (6) and (7), we obtain the equilibrium upstream profits in this stage.

4.3 Downstream locations

Taking as given the locations of the upstream firms and assuming that the retail and wholesale prices will be subsequently chosen in equilibrium, the downstream firms simultaneously choose their locations to maximize their profits,

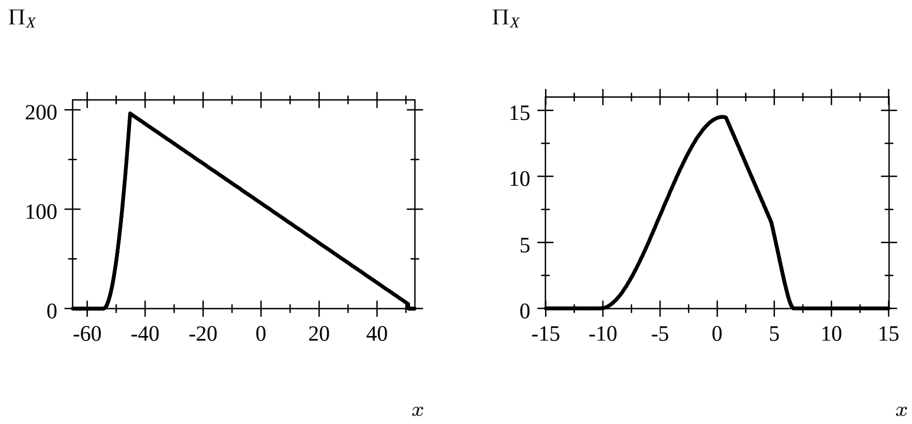
$$
\begin{aligned}
\Pi_X &= 2t(1-x-y) \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\le 0 \\
&= \frac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t(x-y+9)(1-x-y)\right)^2}{162t(1-x-y)} \\
&\quad \text{if } \left\{\begin{array}{l} \tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)>0 \end{array}\right\} \\
&= 0 \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\,(x-y+9)\,(1-x-y)\ge 0
\end{aligned} \tag{10}
$$

and

$$
\begin{aligned}
\Pi_Y &= 0 \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\le 0 \\
&= \frac{\left(\tau\left((x-a)^2-(y-b)^2\right)+t(y-x+9)(1-x-y)\right)^2}{162t(1-x-y)} \\
&\quad \text{if } \left\{\begin{array}{l} \tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)>0 \end{array}\right\} \\
&= 2t(1-x-y) \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\,(x-y+9)\,(1-x-y)\ge 0.
\end{aligned} \tag{11}
$$

In our main model firms are allowed to locate anywhere on the real line, that is, also outside the unit interval where consumers live. Like in the previous steps of analysis, special care should be taken about the corner cases. Is it possible that in equilibrium one retailer captures the whole demand and the other gets nothing? In other words, is it possible that for certain wholesale locations, a retailer cannot avoid obtaining zero demand no matter what location it would choose? If so, for what values of the upstream locations this may occur? We find that, if the upstream locations are asymmetric enough, we may obtain a corner solution in the continuation of the game, that is, one retailer has no way to avoid making zero sales. Upstream firms are located asymmetrically enough and this leads a retailer to take the whole demand. Take for example firm B that supplies firm Y to be located far away from the unit interval and firm A that supplies firm X to be located in the centre of the unit interval. Then, the transportation cost that retailer Y pays when supplied by firm B is high compared to the rival's transportation cost. Based on its cost advantage, firm X maximizes its profits by choosing a location x that allows it to capture the whole market. The alternative, choosing a location far away from the unit interval would lead to zero demand or on any event to demand lower than unity and firm X's profits would not be maximized. Figure 4 (I) presents the profit function of firm X when it captures the whole demand.

In contrast, in Figure 4 (II) firm X maximizes its profit when it shares the market with firm Y, as its cost advantage is not high enough to allow it take all the demand.



$a = 0.5, b = -100, y = -52, t = \tau = 1$ $\qquad$ $a = 0.5, b = -10, y = -7, t = \tau = 1$

(I) $\qquad$ (II)

Figure 4: Downstream profits

We have to determine exactly when a corner solution will emerge. From the profit functions above we calculate that firm X serves the whole demand if $\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0$. When firm B is located far away from the unit interval, its retailer Y has to decide where to locate in order not to pay a high transportation cost but also get some positive demand. If Y locates at its supplier's location, it minimizes its transportation cost but it is located far away from the consumers. In contrast, when it locates in or near the unit interval it is close to the final consumers but faces a high transportation cost. In equilibrium, Y chooses a location y that maximizes the expression $\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)$, that is, it minimizes the area where its demand is zero. By direct calculations it follows that $y=\frac{b\tau-4t}{t+\tau}$. Given this location y, firm X maximizes its own profits. This argument holds symmetrically when firm A is located far away from the unit interval and firm X has a cost disadvantage. Of course, when neither firm has a high cost disadvantage in equilibrium the firms share the demand. From this analysis we obtain:[6]

Lemma 3 *The equilibrium downstream locations (as functions of the upstream locations) are:*

[6] The second order conditions of the downstream firms are satisfied at the equilibrium locations. Profit margins are positive at the equilibrium point x,y if $a-b+45>\frac{9(-9t+16\tau(1-a-b))}{4\tau(1-a-b)}$ and $a-b-45<\frac{9(9t-16\tau(1-a-b))}{4\tau(1-a-b)}$. The second order conditions for firms X and Y are satisfied for $a-b-45<\frac{243t^2+32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$ and $a-b+45>\frac{-243t^2-32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$ respectively. This holds as $\frac{9(9t+16\tau(a+b-1))}{4\tau(1-a-b)}<\frac{243t^2+32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$ and $\frac{9(-9t+16\tau(1-a-b))}{4\tau(1-a-b)}>\frac{-243t^2-32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$.

$$
\begin{aligned}
(x,y) &= \left(\frac{\left(5t+a\tau-\sqrt{t\tau(a-b-9)(1-a-b)}\right)}{t+\tau}, \frac{b\tau-4t}{t+\tau}\right) \\
&\quad \text{if } 81t+4\tau(a-b-9)(a+b-1)\leq 0 \\
&= \left(\frac{\left(16a\tau^2(1-a-b)+4t\tau\left(17a+6b-a^2+b^2-7\right)-63t^2\right)}{4(t+\tau)(4\tau(1-a-b)+9t)}, \frac{\left(16b\tau^2(1-a-b)+4t\tau\left(17b+6a-b^2+a^2-7\right)-63t^2\right)}{4(t+\tau)(4\tau(1-a-b)+9t)}\right) \\
&\quad \text{if } \left\{\begin{array}{l} 81t-4\tau(a-b+9)(a+b-1)>0 \\ 81t+4\tau(a-b-9)(a+b-1)>0 \end{array}\right\} \\
&= \left(\frac{a\tau-4t}{t+\tau}, \frac{\left(5t+b\tau-\sqrt{t\tau(a-b+9)(a+b-1)}\right)}{t+\tau}\right) \\
&\quad \text{if } 81t-4\tau(a-b+9)(a+b-1)\leq 0.
\end{aligned} \tag{12}
$$

The equilibrium downstream profits in this stage can be obtained by substituting the equilibrium downstream locations (12) into the profits (10) and (11).

4.4 Upstream locations

In this stage, upstream firms choose their locations to maximize their profits,

$$
\begin{aligned}
\Pi_A &= \frac{6t\left(\tau(1-a-b)+\sqrt{t\tau(a-b-9)(1-a-b)}\right)}{t+\tau} \\
&\quad \text{if } 81t+4\tau(a-b-9)(a+b-1)\leq 0 \\
&= \frac{t(9t+2\tau(1-a-b))(81t+4\tau(a-b+9)(1-a-b))^2}{108(t+\tau)(9t+4\tau(1-a-b))^2} \\
&\quad \text{if } \left\{\begin{array}{l} 81t-4\tau(a-b+9)(a+b-1)>0 \\ 81t+4\tau(a-b-9)(a+b-1)>0 \end{array}\right\} \\
&= 0 \\
&\quad \text{if } 81t-4\tau(a-b+9)(a+b-1)\leq 0
\end{aligned}
$$

and

$$\begin{aligned}
\Pi_B &= 0 \\
&\quad \text{if } 81t + 4\tau\left(a-b-9\right)\left(a+b-1\right) \leq 0 \\
&= \tfrac{t(9t+2\tau(1-a-b))(81t+4\tau(b-a+9)(1-a-b))^2}{108(t+\tau)(9t+4\tau(1-a-b))^2} \\
&\quad \text{if } \left\{ \begin{array}{c} 81t - 4\tau\left(a-b+9\right)\left(a+b-1\right) > 0 \\ 81t + 4\tau\left(a-b-9\right)\left(a+b-1\right) > 0 \end{array} \right\} \\
&= \tfrac{6t\left(\tau(1-a-b)+\sqrt{t\tau(b-a-9)(1-a-b)}\right)}{t+\tau} \\
&\quad \text{if } 81t - 4\tau\left(a-b+9\right)\left(a+b-1\right) \leq 0.
\end{aligned}$$

An upstream firm will not choose a location that will make its own retailer face so high aggregate marginal cost that would make it obtain zero demand in the subsequent stage. Obviously, this is because such an outcome would also lead to zero demand and profits for this upstream firm. Therefore in equilibrium, both upstream firms avoid receiving zero demand and they share the market. The relevant profit functions reduce to:

$$\begin{aligned}
\Pi_A &= \frac{t\left(9t+2\tau\left(1-a-b\right)\right)\left(81t+4\tau\left(a-b+9\right)\left(1-a-b\right)\right)^2}{108\left(t+\tau\right)\left(9t+4\tau\left(1-a-b\right)\right)^2}, \\
\Pi_B &= \frac{t\left(9t+2\tau\left(1-a-b\right)\right)\left(81t+4\tau\left(b-a+9\right)\left(1-a-b\right)\right)^2}{108\left(t+\tau\right)\left(9t+4\tau\left(1-a-b\right)\right)^2}.
\end{aligned}$$

From the first order conditions and the fact that the profit margins are positive we obtain[7] the equilibrium upstream locations, $a = \frac{9t-5\tau-9\sqrt{\tau^2+t^2}}{8\tau}$, $b = \frac{9t-5\tau-9\sqrt{\tau^2+t^2}}{8\tau}$. Notice that $a < 0$ and $b < 0$, thus, upstream firms are located outside the unit interval for all values of the parameters t and τ. Combining this result with Lemmas 1-3 we obtain:

Proposition 1 *The unique subgame perfect equilibrium in the five stage game is:*

[7] The second order conditions are satisfied in equilibrium.

$$
\begin{aligned}
a^* &= b^* = \frac{9t - 5\tau - 9\sqrt{\tau^2 + t^2}}{8\tau}, \\
x^* &= y^* = \frac{-\left(5(t+\tau) + 9\sqrt{\tau^2 + t^2}\right)}{8(t+\tau)}, \\
w_A^* &= w_B^* = \frac{27t\left(t + \tau + \sqrt{\tau^2 + t^2}\right)}{4(t+\tau)}, \\
p_X^* &= p_Y^* = \frac{9t\left(9t^3 + 41t^2\tau + 73t\tau^2 + 32\tau^3 + \sqrt{\tau^2 + t^2}\,(-9t + 32\tau)(t+\tau)\right)}{32\tau(t+\tau)^2}, \\
z^* &= 0.5, \\
\Pi_A^* &= \Pi_B^* = \frac{27t\left(t + \tau + \sqrt{\tau^2 + t^2}\right)}{8(t+\tau)}, \\
\Pi_X^* &= \Pi_Y^* = \frac{9t\left(t + \tau + \sqrt{\tau^2 + t^2}\right)}{8(t+\tau)}.
\end{aligned}
$$

We note that, in equilibrium, the market is shared equally, all four firms obtain positive profits and all locations are outside the unit interval. Further, upstream firms locate closer to each other (and to the market center) relative to the downstream firms ($a > x$). We discuss the equilibrium properties in detail in the next Subsection.

4.5 Equilibrium properties

Here we discuss the equilibrium we have obtained just above, and compare its properties to the vertical integration case and to the social optimum. We start by noting that:

Remark 1 *Profits for all firms are strictly positive for all parameters. As the downstream transportation cost parameter t increases, all four prices and profit levels increase in equilibrium.*

We obtain this result by directly differentiating the relevant expressions in Proposition 1: wholesale and retail prices increase ($\frac{dw_A}{dt} > 0$, $\frac{dp_X}{dt} > 0$) and this also implies higher profit levels ($\frac{d\Pi_A}{dt} > 0$, $\frac{d\Pi_X}{dt} > 0$), with symmetric relations holding for firms B and Y. As should be expected, when final consumers view the products as less differentiated, competition becomes less intense.

Remark 2 *As the upstream transportation cost parameter τ increases, the wholesale prices and all four profit levels increase when $\tau > t$, and decrease otherwise. No general conclusion can be derived for the retail prices.*

By directly differentiating the relevant expressions in Proposition 1 we obtain:

$$
\begin{aligned}
\frac{dw_A}{d\tau} &= \frac{27t^2(\tau - t)}{4\sqrt{\tau^2 + t^2}\,(t+\tau)^2}, \\
\frac{dp_X}{d\tau} &= \frac{9t^2\left((t+\tau)\left(41\tau^3 - 32t\tau^2 + 18t^2\tau + 9t^3\right) - 9\sqrt{\tau^2 + t^2}\left(\tau\left(t\tau + \tau^2 + 3t^2\right) + t^3\right)\right)}{32\tau^2\sqrt{\tau^2 + t^2}\,(t+\tau)^3}, \\
\frac{d\Pi_A}{d\tau} &= \frac{27t^2(\tau - t)}{8\sqrt{\tau^2 + t^2}\,(t+\tau)^2}, \quad \frac{d\Pi_X}{d\tau} = \frac{9t^2(\tau - t)}{8\sqrt{\tau^2 + t^2}\,(t+\tau)^2}.
\end{aligned}
$$

To illustrate, take for example $\tau = 2t$. Then $\frac{dw_A}{d\tau} = 0.3354$, $\frac{dp_X}{d\tau} = 0.4107$, $\frac{d\Pi_A}{d\tau} = 0.1677$, $\frac{d\Pi_X}{d\tau} =$

0.05 59. When upstream transportation cost parameter τ is double the downstream transportation cost parameter t, an increase in the cost parameter τ, can be partly passed to the final consumers and the upstream firms absorb the rest. The upstream cost parameter matters more than the downstream.

Now we contrast our results to the vertical integration case.

Remark 3 *Under vertical integration, each vertically integrated pair of firms (X with A and Y with B) would locate at* $-\frac{1}{4}$ *from each endpoint of the unit interval and equilibrium prices will be equal to* $\frac{3t}{2}$.

The above result follows from an application of the D' Aspremont *et al* (1979) model when there is only one stage of competition and firms are allowed to locate anywhere on the real line (see e.g. Tabuchi and Thisse (1995) and Lambertini (1997)).

Remark 4 *In our model, final prices are higher than under vertical integration, and "double marginalization" obtains in equilibrium.*

This result did not have to necessarily hold since in our model we have duopoly competition in each stage. We find that each upstream firm raises its price to a level that makes its retailer charge a higher final price than the one would see in equilibrium under vertical integration. Only part of the negative effect to its retailer of an increased wholesale price is taken into account by the wholesaler, while the remainder of this effect emerges as an externality.

Now we turn to the locations.

Remark 5 *Compared to the vertical integration case, both upstream and downstream locations are farther away from the unit interval:* $x^* = y^* < a^* = b^* < -\frac{1}{4}$.

We will discuss in detail the forces that shape the equilibrium locations, but it is also useful at this point to briefly turn to the welfare properties of the equilibrium. Final consumers face unit demand, thus, the social cost (SC) simly equals the transportation cost paid by the retailers plus the transportation cost paid by the final consumers:

$$SC = \int_0^z t(k-x)^2 dk + \int_z^1 t(1-y-k)^2 dk + \tau(x-a)^2 z + \tau(y-b)^2 (1-z)\,. \tag{13}$$

By substituting the locations from Proposition 1, we calculate that in equilibrium total social cost is:

$$\begin{aligned} SC &= \int_0^{\frac{1}{2}} t(k-x)^2 dk + \int_{\frac{1}{2}}^1 t(1-y-k)^2 dk + \frac{\left(\tau(x-a)^2 + \tau(y-b)^2\right)}{2} \\ &= \frac{18t\left(\sqrt{\tau^2+t^2}\,(-9t+7\tau) + 7\tau\,(t+\tau) + 9t^2\right)}{64\tau\,(t+\tau)} + \frac{t}{12} > 0. \end{aligned}$$

A social planner in contrast, would minimize the total transportation cost (13). Standard calculations imply:

Remark 6 *The social optimum locations are* $a = b = x = y = \frac{1}{4}$.

Firm X should be located at the same point as its supplier A, and firm Y as its supplier B, so as to eliminate the transportation costs of the retailers. Also, they are both located inside the unit interval to minimize the transportation cost paid by the consumers. The optimum social cost is $SC = \frac{t}{48}$. We find that:

Proposition 2 *We have* $x^* = y^* < a^* = b^* < -\frac{1}{4} < \frac{1}{4}$.

Thus, in the equilibrium of our model, firms locate outside the unit interval in contrast to the socially optimum locations which are within the unit interval. The total social cost is strictly higher than the optimal and also strictly higher than the cost under vertical integration.

Now let us discuss the equilibrium locations in more detail. In models of horizontal differentiation with duopolists selling directly to final consumers, there are two opposite forces, one pushing firms close to each other to directly obtain higher demand and one in the opposite direction to reduce the intensity of price competition. In our model, these forces are modified and a third force also emerges. Given the locations of the other three firms, if the downstream firm moves closer to the centre of the unit interval this affects its aggregate marginal cost, the wholesale price plus the transportation cost. This effect may be working to bring firms either closer to each other or apart, depending on the locations of the upstream firms. The marginal production cost of the retailers is location dependent.

In a standard linear city duopoly model as in D'Aspremont *et al (1979)* the marginal production cost is exogenous. Denoting this cost by c, profits are $\Pi_X = (p_X - c)\, z$ for firm X. By the envelope theorem we obtain:

$$\frac{d\Pi_X}{dx} = (p_X - c)\left(\frac{dz}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{dx}\right),$$

evaluated at the equilibrium prices. The direct demand effect that pushes firms close to each other corresponds to $\frac{dz}{dx}$ while the indirect price competition effect that pushes firms to the opposite direction corresponds to $\frac{dz}{dp_Y}\frac{dp_Y}{dx}$. In our model the marginal production cost for the retailers is endogenous and equal to the wholesale price plus the transportation cost, $f_X = w_A + \tau(a-x)^2$. The profit function of firm X is $\Pi_X = (p_X - f_X)\, z$ and by the envelope theorem we now obtain:

$$\frac{d\Pi_X}{dx} = (p_X - f_X)\left(\frac{dz}{dx} + \left(\frac{dz}{dp_Y}\frac{dp_Y}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_X}\frac{df_X}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_Y}\frac{df_Y}{dx}\right)\right) - \frac{df_X}{dx}z.$$

A change in the location x affects the aggregate marginal costs of both retailers and the final prices directly and indirectly via the change in the marginal costs. The direct demand effect again corresponds to $\frac{dz}{dx}$ but the indirect price competition effect now corresponds to the expression $\frac{dz}{dp_Y}\frac{dp_Y}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_X}\frac{df_X}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_Y}\frac{df_Y}{dx}$ as final prices are affected by the endogenous marginal production costs of the retailers. A change in x, leads to a change in the wholesale prices and the retailers' transportation cost, therefore, final prices change too. Finally, profits are also affected directly by the change in the marginal cost itself, $\frac{df_X}{dx}$.

Remark 7 *In addition to the demand effect and the price competition effect, the aggregate marginal cost effect plays a role in shaping the retailer location incentives. These three forces co-determine the equilibrium locations.*

As the profit margin $(p_X - f_X)$ and the demand z are positive, the following derivatives determine the effect of x on the profit Π_X :

$$\frac{dz}{dx} = \frac{\tau\left((y-b)^2 - (x-a)^2\right) + t\,(9-y+x)\,(1-x-y)}{18t(1-x-y)^2} - \frac{x}{1-x-y},$$

$$\frac{dz}{dp_Y}\frac{dp_Y}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_X}\frac{df_X}{dx} + \frac{dz}{dp_Y}\frac{dp_Y}{df_Y}\frac{df_Y}{dx} = \frac{4\,(t\,(x-5) - \tau\,(a-x))}{9t(1-x-y)},$$

$$\text{and } \frac{df_X}{dx} = -\frac{2}{3}\,(t\,(x+4) + 2\tau\,(a-x))\,.$$

The price competition effect $\frac{4(t(x-5)-\tau(a-x))}{9t(1-x-y)}$ is negative if firm A is located to the right of firm X ($a > x$) and $x < 5$. Given a, b, y, if retailer X moves to right (that is, if x increases) this may either increase or decrease the aggregate marginal cost that it pays depending on the location of its supplier A: $\frac{df_X}{dx} = \frac{d\left(w_A + \tau(a-x)^2\right)}{dx} = -\frac{2}{3}\,(t\,(x+4) + 2\tau\,(a-x))$ where w_A is given by (8). If firm A is located to the right of firm X ($a > x$) with $x > 0$ then $\frac{df_X}{dx} < 0$ and as x increases the marginal cost of firm X decreases, which is a positive effect. So, by increasing x, the marginal cost is reduced, demand is increased (demand effect) and prices fall (price competition effect). But if firm A is located to the left of firm X ($a < x$), the sign of $\frac{df_X}{dx}$ depends on the values of the cost parameters. If $\frac{df_X}{dx} > 0$, this means that as x increases aggregate marginal cost increases too, which may lead to an increase in the retail price and a reduction in the demand. Therefore, as firm X moves to the centre, it is ambiguous if the demand effect is positive and the price competition effect negative. In equilibrium, the upstream firms are located closer to the unit interval compared to the downstream firms and all four firms are outside the unit interval ($x < a < 0, y < b < 0$). The forces pushing firms (X relative to Y and A relative to B) close to each other are dominated by the forces pushing firms to the opposite direction and this total effect is stronger for the downstream firms.

5 Modified order of location choices

We present now some extensions of the basic model which allows us to obtain additional insights into the problem, especially about the location incentives. We start, in this Section, by considering what happens when the wholesalers cannot choose their locations first. As we show, the equilibrium locations are affected in a systematic way.

5.1 Simultaneous location choices

Suppose now that the four firms make their location choices simultaneously in the first stage, while the pricing stages of the game remains the same. Now, in the second stage the wholesale prices are

determined, followed by the final prices, whereas final consumers decide which retailer to buy from. By inserting the wholesale and final prices we have obtained in our proceeding analysis into the profit functions of the upstream and downstream firms we obtain the profit functions depending on the four location choices. We observe that $\Pi_A = 3\Pi_X$ and $\Pi_B = 3\Pi_Y$ for all location values and parameters t, τ:

$$\begin{aligned}
\Pi_X &= \frac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)\right)^2}{162t(1-x-y)}, \\
\Pi_Y &= \frac{\left(\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\right)^2}{162t(1-x-y)}, \\
\Pi_A &= \frac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)\right)^2}{54t(1-x-y)}, \\
\Pi_B &= \frac{\left(\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\right)^2}{54t(1-x-y)}.
\end{aligned}$$

The four firms simultaneously seek to maximize their profits with respect to their locations. From the first order conditions of the upstream firms (and the fact that the profit margins are positive), we find that each wholesaler chooses to have the same location as the corresponding retailer:

$$a = x \text{ and } b = y.$$

Further, from the first order conditions of the retailers we obtain:[8] $a = x = b = y = -1.75$. Thus, we have:

Proposition 3 *In the simultaneous locations choice model, the equilibrium outcome is:*

$$\begin{aligned}
a^* &= b^* = x^* = y^* = -1.75, \\
w_A^* &= w_B^* = 13.5t,\ p_X^* = p_Y^* = 18t, \\
z^* &= 0.5, \\
\Pi_A^* &= \Pi_B^* = 6.75t,\ \Pi_X^* = \Pi_Y^* = 2.25t.
\end{aligned}$$

Like in our basic model, all firms are located outside the unit interval, now at -1.75. We observe that the equilibrium locations are independent of the parameter τ, as downstream firms are located at the same point of their suppliers, which means that they pay zero transportation costs in equilibrium. Equilibrium locations are also independent of the parameter t; this is a similar result to the standard linear city model where firms are located in equilibrium at -0.25. Compared to that model which serves as the vertical integration benchmark, firms in our model are located farther away ($-1.75 < -0.25$). The introduction of the wholesalers in the problem, which means endogenous production costs for the retailers, pushes the locations farther away. The forces that

[8] The second order conditions are satisfied.

push the firms closer to each other are dominated by the forces that push them to the opposite direction and the latter are stronger at a model with endogenous production costs.[9]

5.2 Downstream locations first

We could also consider our sequential locations model but with reversed location choices. Downstream firms now choose their locations first and then upstream location choices follow. This timing may be especially relevant if the retailers have a stronger position in the market than the wholesalers. The remaining game is the same. From the analysis of our simultaneous locations case just above, we know that in the second stage upstream firms locate on their retailers' locations ($a = x, b = y$). Given these upstream locations the equilibrium downstream locations in the first stage are $x = y = -1.75$. Therefore, the sequential locations outcome where downstream firms determine their locations first coincides with the simultaneous locations outcome (Proposition 3).

5.3 Comparison

It is important to compare the equilibrium locations in our simultaneous locations game (Proposition 3) with the sequential locations game (Proposition 1). In the previous Section, we found that $x < a < 0$ and $y < b < 0$, which means that all firms are located outside the unit interval with the upstream firms to be closer to [0,1]. We can further calculate that for all parameter values $-1.75 < x < a < 0$ and $-1.75 < y < b < 0$. Thus, we find that in the simultaneous locations game firms move farther away from the city center than when upstream locations are chosen first. In addition, profits for both upstream and downstream firms and wholesale and final prices are *higher* compared to the sequential location choice game. Why then do not upstream firms locate at -1.75 in our basic model, to obtain higher profits? Each upstream firm has a unilateral incentive to deviate from -1.75 and move closer to the unit interval given that its rival is located at -1.75 and that downstream locations will follow equation (12). Each wholesaler does this in an effort to offer a strategic commitment incentive to its own retailer, to help it move credibly closer to the market center and in this way strengthen its position in the final market. However both retailers become more competitive in this way and all firms obtain lower equilibrium profits. We can say that they are trapped in a Prisoners' Dilemma in locations when wholesalers choose locations first. Both upstream firms jointly prefer being located at -1.75 (which also leads to $x = y = -1.75$) however there is a unilateral incentive for each to move closer to the unit interval to increase their own demand and enjoy higher profits. For example, if firm B were located at -1.75 then firm A

[9] Brekke and Straume (2004) allow wholesale prices to be determined through bargaining between the upstream and downstream firms but they do not study the location choices of the upsream firms. In contrast, we assume that wholesale prices are set by the upstream firms and that their locations are endogenous. When the bargaining power of the upstream firms equals one, and there is no product differentiation at the upstream level ($\tau = 0$) the two models give the same results, firms locate at -1.75.

would like to deviate from -1.75 and move closer to the unit interval (a increases) and push firm X also closer to the unit interval (x increases too). Demand would then increase for firms X and A, as well as profits.

Remark 8 *In the sequential locations game, wholesalers locate first closer to the center in an effort to also push their retailers closer to the center, but they end up with lower equilibrium profits than in the simultaneous locations game.*

Thus, we conclude that firms would prefer to set their locations simultaneously compared to the sequential game where upstream firms move first. When upstream firms set their locations first, they can affect the downstream locations and this leads to a stronger competition between them.

When the location choices are reversed compared to our basic model and downstream firms choose first locations, the equilibrium locations coincide with the simultaneous equilibrium locations. All four firms prefer the downstream firms to locate first to make the upstream firms locate at -1.75 and not to move to a location closer to the unit interval. If retailers are located at -1.75, the suppliers have no other choice but to locate on their retailers' locations to maximize their profits. Overall, higher profits can simply be obtained either with simultaneous location choices or with the downstream firms to locate first compared to our basic model.

6 Locations in the unit interval

In this Section we modify our basic model so that firms are restricted to locate within the unit interval [0,1], an assumption often made in the literature. Our analysis of the game is as before, however now a, b, x and y cannot take negative values. Thus, we have to solve for the restricted location choices in the first and second stage of the game with the other stages remaining the same.

In the second stage the downstream firms set their locations and their profit functions are given by expressions (10) and (11). Can one of the two retailers in equilibrium serve the whole market? Firm X takes the whole demand if $\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0$. Given that all firms are located in the unit interval, this expression can be negative only when $\tau(y-b)^2$ is higher than $\tau(x-a)^2+t\,(y-x+9)\,(1-x-y)$. Thus, firm Y can avoid receiving zero demand by setting $y=b$, that is, locate at the same point as its supplier. Firm Y has some local monopoly power and always serves some consumers located close to that firm. Likewise, firm X avoids receiving zero demand by setting $x=a$. Therefore, in equilibrium no retailer can serve the whole demand, each retailer can assure at least some sales. The profit functions, thus, reduce to:

$$\Pi_X = \frac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t\,(x-y+9)\,(1-x-y)\right)^2}{162t(1-x-y)}, \tag{14}$$

$$\text{and } \Pi_Y = \frac{\left(\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\right)^2}{162t(1-x-y)}.$$

From the first order conditions we obtain (note that the profit margin is positive[10]):

$$x = \frac{\left(16a\tau^2(1-a-b)+4t\tau\left(17a+6b-a^2+b^2-7\right)-63t^2\right)}{4(t+\tau)(4\tau(1-a-b)+9t)},$$
$$\text{and } y = \frac{\left(16b\tau^2(1-a-b)+4t\tau\left(17b+6a-b^2+a^2-7\right)-63t^2\right)}{4(t+\tau)(4\tau(1-a-b)+9t)}.$$

However, the pair of locations (x, y) is positive if the numerators of x and y are positive (also note that $x, y < 1$). Otherwise, due to the concavity of the profit functions the equilibrium locations become zero. We get four cases, depending on the the values of x and y:

$$\begin{aligned} x &= \tfrac{\left(16a\tau^2(1-a-b)+4t\tau\left(17a+6b-a^2+b^2-7\right)-63t^2\right)}{4(t+\tau)(4\tau(1-a-b)+9t)} \\ &\quad \text{if } 16a\tau^2(1-a-b)+4t\tau\left(17a+6b-a^2+b^2-7\right)-63t^2>0 \\ &= 0 \\ &\quad \text{if } 16a\tau^2(1-a-b)+4t\tau\left(17a+6b-a^2+b^2-7\right)-63t^2\leq 0 \end{aligned} \tag{15}$$

and

$$\begin{aligned} y &= \tfrac{\left(16b\tau^2(1-a-b)+4t\tau\left(17b+6a-b^2+a^2-7\right)-63t^2\right)}{4(t+\tau)(4\tau(1-a-b)+9t)} \\ &\quad \text{if } 16b\tau^2(1-a-b)+4t\tau\left(17b+6a-b^2+a^2-7\right)-63t^2>0 \\ &= 0 \\ &\quad \text{if } 16b\tau^2(1-a-b)+4t\tau\left(17b+6a-b^2+a^2-7\right)-63t^2\leq 0. \end{aligned} \tag{16}$$

In the first stage, upstream firms choose their locations within the unit interval. The equilibrium locations are $(a, b) = (0, 0)$ and $(x, y) = (0, 0)$. The detailed proof is provided in Appendix A. The following results summarize the equilibrium outcome, the social cost in equilibrium and the equilibrium locations under vertical integration.[11]

Proposition 4 *The equilibrium outcome with locations restricted in the unit interval is:*

$$\begin{aligned} a^* &= b^* = x^* = y^* = 0, \\ w_A^* &= w_B^* = 3t,\ p_X^* = p_Y^* = 4t, \\ z^* &= 0.5, \\ \Pi_A^* &= \Pi_B^* = \frac{3t}{2},\ \Pi_X^* = \Pi_Y^* = \frac{t}{2}. \end{aligned}$$

Proposition 5 *The social cost in equilibrium with locations within the unit interval is equal to $\frac{t}{12}$. As the transportation cost parameter t increases, the social cost increases too.*

[10] The profit margin of firm X is positive when $\tau\left((y-b)^2-(x-a)^2\right)+t(x-y+9)(1-x-y)>0$ and the profit margin of firm Y is positive when $\tau\left((x-a)^2-(y-b)^2\right)+t(y-x+9)(1-x-y)>0$. Also, the second order conditions are satisfied in equilibrium.

[11] In equilibrium, the inequality $\overline{f_X} < f_X < \widehat{f_X}$ is valid and corresponds to $-3t < 0 < 3t$.

In equilibrium, the social cost is simply equal to the transportation cost of the consumers since this is a unit demand model (prices do not affect welfare) and the downstream firms are located at the same point as their suppliers:

$$SC = \int_0^{\frac{1}{2}} t(z-x)^2 dz + \int_{\frac{1}{2}}^1 t(1-y-z)^2 dz = \int_0^{\frac{1}{2}} tz^2 dz + \int_{\frac{1}{2}}^1 t(1-z)^2 dz = \frac{t}{12}.$$

Proposition 6 *Under vertical integration (firm X with its supplier A and firm Y with its supplier B) with final prices equal to t, the two vertically integrated pairs of firms locate at the two endpoints.*

As the transportation cost parameter t of the consumers increases, products become more differentiated and the profits of the upstream and downstream firms increase. In equilibrium, the retail price p equals the wholesale price w plus the transportation cost t, that is, the marginal cost of the downstream firm plus the transportation cost of the consumers, $p = w + t$ (see D'Aspremont *et al* (1979)). With production cost c zero (as in our model) the final prices under vertical integration would simply equal the transportation cost parameter t. But in our model under vertical separation, the final prices are higher than the transportation cost t due to the existence of the intermediaries, that is, pricing first at the wholesale level and then at the retail level. As in our basic model, a type of double marginalization emerges with both upstream and downstream firms having positive profit margins and final prices higher than under vertical integration.

We observe that the cost parameter τ that affects the transportation cost paid by the downstream firms when supplied by the upstream firms does not affect the equilibrium prices and profits as retailers are located at the same point as their suppliers. The wholesale and product prices in the restricted locations model are lower than in our basic model where firms locate anywhere on the real line. Again, there are three forces that affect the location choices. The demand effect, the price competition effect and the aggregate marginal cost effect. The forces that push firms farther away from each other dominate the forces that push them towards each other and this lead to maximum differentiation. This result was expected since in the unrestricted locations model we have obtained that all four firms locate outside the unit interval (and the profit functions are quasi-concave).

We should also note that if we modify the restricted locations model to have all four firms simultaneously choose their locations, we obtain that again upstream firms locate at the same point as their retailers and that maximum differentiation occurs ($a = x = 0$ and $b = y = 0$) as opposed to the unrestricted locations model where all firms locate at -1.75. So, in the simultaneous locations choice model with locations within [0,1] we obtain the same equilibrium outcome compared to the sequential location choices with restricted locations.[12]

[12] Matsushima (2004) solves the simultaneous restricted locations choice model so that upstream firms price discriminate among downstream firms. He finds that for some parameters values downstream firms are located closer to the center of the unit interval compared to the upstream firms in an effort to reduce the wholesale prices they pay. In our model, there is no such incentive since each retailer has its own supplier and no price discrimination takes place.

7 Retailers locating at the opposite side from the wholesalers

In our analysis we have assumed that upstream firm A locates to the left of B. This is simply a matter of labelling and without loss of generality. We have also assumed that each downstream firm locates at the same side of the line as the corresponding upstream (X with A and Y with B). Here, we investigate the possibility that, while A is to the left of B, X locates to the right of Y. We find that for some upstream locations and parameter values it could also be an equilibrium in the second stage of the game that the retailers locate on the line at the opposite side of the corresponding wholesaler (and we also show that profit is lower in this arrangement). We study this behavior in this Section. Still, we do not find an equilibrium of the entire game when all locations (upstream and downstream) are endogenous and are chosen so that wholesalers and retailers are chosen to the opposite side of one another.

We proceed with the analysis. Let us fix the upstream locations (with $1 - a - b > 0$). Thus far we have assumed that $1 - x - y > 0$. In Appendix B, we calculate the profit functions of the downstream firms when firm X is located at the same point of firm Y (with $1 - x - y = 0$) or to the right of firm Y (with $1 - x - y < 0$). The overall profit function of firm X is:

$$\Pi_X = \begin{cases} \Pi_X^L & if \quad 1 - x - y > 0 \\ 0 & if \quad 1 - x - y = 0 \\ \Pi_X^R & if \quad 1 - x - y < 0, \end{cases}$$

where Π_X^L is the profit when firm X is to the left (L) of firm Y and Π_X^R is the profit when firm X is to the right (R) of firm Y with:

$$\begin{aligned} \Pi_X^L &= 2t(1-x-y) \\ &\quad \text{if } \tau\left((x-a)^2 - (y-b)^2\right) + t\,(y-x+9)\,(1-x-y) \leq 0 \\ &= \frac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t(x-y+9)(1-x-y)\right)^2}{162t(1-x-y)} \\ &\quad \text{if } \left\{ \begin{array}{l} \tau\left((y-b)^2 - (x-a)^2\right) + t\,(x-y+9)\,(1-x-y) > 0 \\ \tau\left((x-a)^2 - (y-b)^2\right) + t\,(y-x+9)\,(1-x-y) > 0 \end{array} \right\} \\ &= 0 \\ &\quad \text{if } \tau\left((x-a)^2 - (y-b)^2\right) - t\,(x-y+9)\,(1-x-y) \geq 0 \end{aligned} \tag{17}$$

and

$$
\begin{aligned}
\Pi_X^R &= 2t\,(x+y-1) \\
& \quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+9)\,(x+y-1)\le 0 \\
\\
&= \tfrac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t(y-x+9)(x+y-1)\right)^2}{162t(x+y-1)} \\
& \quad \text{if } \left\{ \begin{array}{l} \tau\left((y-b)^2-(x-a)^2\right)+t\,(y-x+9)\,(x+y-1)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+9)\,(x+y-1)>0 \end{array} \right\} \qquad (18) \\
\\
&= 0 \\
& \quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\,(y-x+9)\,(x+y-1)\ge 0.
\end{aligned}
$$

We can write the profit function of firm Y in an analogous way.

In this Section we prove that for symmetric upstream locations ($a=b$) and sufficiently high transportation cost parameter t there is a second equilibrium when the downstream locations are set. In the second stage of the game, firm X given the location of firm Y (and the locations of the upstream firms and the cost parameters t,τ) chooses either to locate to the left or to the right of firm Y. We have already characterized in Section (4.3) that an equilibrium where firm X locates to the left of firm Y (see the complete proof in Appendix C). Now, we will prove that, for some parameter values, there is another equilibrium where firm X locates to the right of firm Y.

If there is an equilibrium where firm X is located to the right of firm Y, call it $(\widehat{x},\widehat{y})$, the optimal location choices of the downstream firms will be given by the maximization of the downstream firms' profits Π_X^R (equation (18)) and Π_Y^R respectively. From the first order conditions (and the fact that the profit margin of the downstream firms is positive) we find that:

$$
\begin{aligned}
\widehat{x} &= \frac{16a\tau^2\,(a+b-1)+4t\tau\left(19a+12b+a^2-b^2-11\right)+99t^2}{4\,(t+\tau)\,(4\tau\,(a+b-1)+9t)}, \qquad (19) \\
\widehat{y} &= \frac{16b\tau^2\,(a+b-1)+4t\tau\left(19b+12a+b^2-a^2-11\right)+99t^2}{4\,(t+\tau)\,(4\tau\,(a+b-1)+9t)}
\end{aligned}
$$

if

$$
\begin{aligned}
9t+2\tau\,(a+b-1) &> 0, \\
\frac{(81t+4\tau\,(a-b-9)\,(1-a-b))}{9t+4\tau\,(a+b-1)} &> 0, \frac{(81t+4\tau\,(a-b+9)\,(a+b-1))}{(9t+4\tau\,(a+b-1))}>0
\end{aligned}
$$

with

$$
\begin{aligned}
\Pi_X^R(\widehat{x},\widehat{y}) &= \frac{t\,(9t+2\tau\,(a+b-1))\,((81t+4\tau\,(a-b-9)\,(1-a-b)))^2}{324\,(t+\tau)\,(9t+4\tau\,(a+b-1))^2}, \\
\Pi_Y^R(\widehat{x},\widehat{y}) &= \frac{t\,(9t+2\tau\,(a+b-1))\,((81t+4\tau\,(a-b+9)\,(a+b-1)))^2}{324\,(t+\tau)\,(9t+4\tau\,(a+b-1))^2}.
\end{aligned}
$$

The first constraint is necessary to have positive equilibrium profits and the other two to have positive profit margins (to assure that the calculated quantities are not negative). Thus, if such an equilibrium exists it will be $(\widehat{x},\widehat{y})$. To better understand the problem see the following figure.

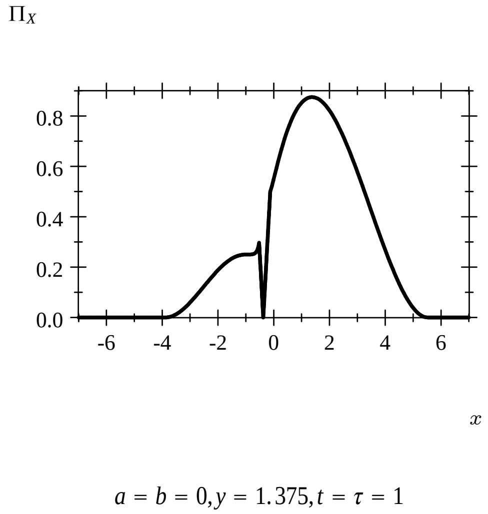


Figure 5: Downstream profits with firm X located either to the left or right of firm Y

Given that the upstream firms are located to the two opposite endpoints and the transportation cost parameters equal unity, we obtain that $\widehat{y} = 1.375$. In Figure 5 we plot the profit function of firm X allowing X to be located either to the left or to the right of firm Y. When firm X is located to the left of firm Y and far enough from the unit interval and its own supplier then it gets zero demand and profits. There is an interval where firm X is located to the left of firm Y and both firms sell to the final consumers. However, when X is located close to Y, either to the left or to the right, X captures the whole demand. Firm X has a cost advantage compared to firm Y and reduces its retail price to capture the whole demand. This cost advantage is greater when firm X is located to the right of firm Y since firm X is closer to its supplier, firm A. Thus, the profits of firm X when it captures the whole demand are higher to the side that is closer to its supplier. There is again an interval where the two retailers share the market, but now firm X is to the right of firm Y. Finally, if firm X is located to the right of firm Y and away from the unit interval, it obtains zero demand since it has a high cost disadvantage and is located away from [0,1] where consumers live.

We prove that for symmetric upstream locations and some parameter values t, τ, firm X (given $y = \widehat{y} = \frac{11t+4a\tau}{4(t+\tau)}$) obtains higher profits when it locates to the right of firm Y and shares the market with Y compared to the profits obtained when i) it locates to the left of Y and shares the market ii) it locates to the left of Y and captures the whole demand iii) it locates to the right of Y and captures the whole demand. The total maximum of firm X's profits is to the right of firm Y when both firms sell to the final consumers. Figure 5 presents an example. Employing symmetry, we prove that, for some parameter values and given $\widehat{x}$, the best response of firm Y is $\widehat{y}$.

Proposition 7 *For fixed and symmetric upstream locations and $t \geq \frac{4\tau(1-2a)}{9}$, the pair of locations $\widehat{x} = \widehat{y} = \frac{11t+4a\tau}{4(t+\tau)}$ constitutes an equilibrium at the second stage of the game.*

If the transportation cost parameter of the consumers t is high enough (compared to τ), it can also be an equilibrium in the second stage of the game that retailers locate at the opposite side of the corresponding wholesaler. In the extreme case where t is positive and τ is zero, retailers pay zero transportation costs when supplied by their wholesalers. It is equivalent to locating at the optimum points on the real line either to the same or the opposite side of their wholesalers. In Figure 5 we assume $a = b = 0$ and $\tau = 1$, therefore we need $t \geq \frac{4}{9}$, which is satisfied since $t = 1$.

Therefore, there are two equilibria in the second stage of the game for fixed upstream locations, $a = b$, and for $t \geq \frac{4\tau(1-2a)}{9}$. Retailers either locate to the same side (see (x^*, y^*) in Section (4.3), equation (12) for $a = b$) or to the opposite side of their wholesalers $(\widehat{x}, \widehat{y})$:

$$x^* = y^* = \frac{-7t + 4a\tau}{4(t+\tau)},\ \Pi_X^L(x^*, y^*) = \Pi_Y^L(x^*, y^*) = \frac{t(9t + 2\tau(1-2a))}{4(t+\tau)}$$

and

$$\widehat{x} = \widehat{y} = \frac{11t + 4a\tau}{4(t+\tau)},\ \Pi_X^R(\widehat{x}, \widehat{y}) = \Pi_Y^R(\widehat{x}, \widehat{y}) = \frac{t(9t - 2\tau(1-2a))}{4(t+\tau)}.$$

We compare the profits of the two equilibria and find that equilibrium (x^*, y^*) pareto dominates $(\widehat{x}, \widehat{y})$ since $\Pi_X^L(x^*, y^*) > \Pi_X^R(\widehat{x}, \widehat{y})$.[13] Downstream equilibrium profits are higher when retailers are located closer to their suppliers.

However, if we fix the upstream locations to the locations obtained at Proposition (1) ($a^* = b^* = \frac{9t-5\tau-9\sqrt{\tau^2+t^2}}{8\tau}$) we obtain the additional equilibrium $\widehat{x} = \widehat{y} = \frac{31t-5\tau-9\sqrt{\tau^2+t^2}}{8(t+\tau)}$ in the second stage of the game when $t \geq \frac{4}{3}\tau$. Retailer either locate at (x^*, y^*) or at $(\widehat{x}, \widehat{y})$ in the second stage of the game for these fixed upstream locations. He have proved numerically that the pair (a^*, b^*) is not an equilibrium pair of locations in the first stage of the game, when retailers, in the subsequent stage, locate at $(\widehat{x}, \widehat{y})$. Wholesalers have an incentive to deviate from (a^*, b^*).

8 Conclusion

Our paper contributes to the horizontal differentiation literature and the vertical contracting literature. We have studied a linear city model with duopoly upstream and downstream. Wholesalers and then retailers choose their locations and then their prices, before consumers make their choices. We derive a unique subgame perfect equilibrium of this five stage game and examine its properties. We find that wholesalers choose to become less differentiated (that is, locate closer to the unit interval) than the retailers and that differentiation is greater compared to the vertical integration benchmark (which in turn is greater than in the social optimum). We also find positive profit margins both upstream and downstream and that final prices are higher than under vertical integration ("double marginalization"). Thus, vertical separation in our duopoly implies both a higher social cost (of transportation) and higher final prices for the consumers.

Considering a number of extensions offers additional insights into the problem. When firms' locations are restricted to the unit interval, maximum differentiation is obtained, with final prices

[13] For symmetric upstream locations, we obtain $a = b < \frac{1}{2}$.

that are always higher than under vertical integration. We also modify the order of location choices. When locations are chosen simultaneously by all four firms or the downstream locations are chosen before the upstream, the upstream and the downstream firms in each pair choose the same location. This location is also farther away from the city center relative to the case when upstream locations are chosen first: in that case the wholesalers locate closer to the city center with the goal to also pull their retailers closer to the center, so that they can strategically strengthen their retailer's position in the downstream market. Still, since this happens for both vertical chains, the end result is a more competitive market (than when all locations are chosen simultaneously) and profits are lower.

The framework studied in this paper allows a number of additional extensions and modifications. While the results presented here take a relatively simple and clear form, the equilibrium calculations for this model or its variations are quite involved, as one should expect from a five stage game, especially when care has to be taken for possible corner solutions. Still, a number of extensions appear promising. In companion work we consider the role of price discrimination and non exclusive vertical relations. Modifications of the product differentiation structure, pricing and contracting and the timing of the game may offer additional important insights, and so would models where consumers may directly care also about the upstream choices (and not only indirectly, as in our model). Of course, empirical work that studies the interplay of product differentiation and vertical contracting will also be very important and hopefully our theoretical study of the topic also contributes in this direction.

Appendix A

Proof of Proposition 4. We have to show that, given $b = 0$, the best response of firm A is $a = 0$ and vice versa. The game is symmetric so it is sufficient to prove only the first part. Given $b = 0$, we obtain $y = 0$ for every a and t, τ since $4t\tau\left(6a + a^2 - 7\right) - 63t^2 \leq 0$. After we substitute equations (15) and (16) into the profit function of firm A, we compare profits when x is positive with when x is zero. Location x is positive when:

$$\begin{aligned} 16a\tau^2(1 - a - b) + 4t\tau\left(17a + 6b - a^2 + b^2 - 7\right) - 63t^2 &> 0 \\ \text{or } -4\tau(t + 4\tau)a^2 + 4\tau(17t + 4\tau)a - 7t(9t + 4\tau) &> 0 \\ \text{or} \qquad\qquad\qquad\qquad\qquad a &\in (a_1, a_2) \end{aligned}$$

with

$$\begin{aligned} a_1 &\equiv \frac{17t\tau + 4\tau^2 - \sqrt{9t^2\tau^2 + 16\tau^4 + 24t\tau^3 - 63t^3\tau}}{2(t + 4\tau)\tau} > 0, \\ a_2 &\equiv \frac{17t\tau + 4\tau^2 + \sqrt{9t^2\tau^2 + 16\tau^4 + 24t\tau^3 - 63t^3\tau}}{2(t + 4\tau)\tau} > 0. \end{aligned}$$

Thus, $\Pi_A(b = 0) = \begin{cases} \Pi_A(x > 0, y = 0) & if \quad a \in (a_1, a_2) \\ \Pi_A(x = 0, y = 0) & \qquad otherwise \end{cases}$
with $\Pi_A(x > 0, y = 0) =$

$$\frac{t\left(16\tau^2(t+16\tau)a^4 - 128\tau^2(7t-12\tau)a^3 + 24\tau\left(-572t\tau - 256\tau^2 + 27t^2\right)a^2 + 64\tau(59t+26\tau)(9t+4\tau)a - (319t+144\tau)(9t+4\tau)^2\right)^2}{3456(t+\tau)(9t+4\tau(1-a))^3\left(4\tau(t+4\tau)a^2 - 4\tau(21t+8\tau)a + (11t+4\tau)(9t+4\tau)\right)}$$

and $\Pi_A(x = 0, y = 0) = \frac{\left(-9t + a^2\tau\right)^2}{54t}$.

From the first order conditions of $\Pi_A(x = 0, y = 0)$ we find that the maximum is at $a = 0$. The first derivative of $\Pi_A(x > 0, y = 0)$ with respect to a is negative, therefore its maximum is at the minimum value of a. We also find that $\lim\limits_{a \to a_1^+} \Pi_A(x > 0, y = 0) = \Pi_A(x = 0, y = 0)$ at $a = a_1$, implying that firm A prefers to locate at $a = 0$. All these concern parameters values where $9t^2\tau^2 + 16\tau^4 + 24t\tau^3 - 63t^3\tau \geq 0$ is satisfied. The expression $9t^2\tau^2 + 16\tau^4 + 24t\tau^3 - 63t^3\tau$ is increasing in τ and becomes zero when $\tau = 1.12272t$.[14] Thus, $9t^2\tau^2 + 16\tau^4 + 24t\tau^3 - 63t^3\tau$ is non-negative when $\tau \geq 1.12272t$. Now, we have to prove that the best response of firm A is $a = 0$ for $\tau < 1.12272t$. Given $b = 0$, x is zero if $16a\tau^2(1 - a) + 4t\tau\left(17a - a^2 - 7\right) - 63t^2 \leq 0$. The expression $16a\tau^2(1 - a) + 4t\tau\left(17a - a^2 - 7\right) - 63t^2$ is decreasing in t when $t > \frac{\tau}{1.12272}$ and is negative at $t = \frac{\tau}{1.12272}$, implying that this expression is negative for all t greater than $\frac{\tau}{1.12272}$ and therefore $x = 0$. For $x = 0$ firm A prefers to locate at the endpoint $a = 0$.

The equilibrium $(a, b) = (0, 0)$ is the unique symmetric equilibrium. We prove that there is no symmetric equilibrium such that $a = b > 0$, since each firm has a unilateral incentive to deviate to

[14] This expression is zero at $\tau = \frac{\left(t - 4\sqrt[3]{\sqrt{\frac{63}{16}t^6} + \frac{127}{64}t^3}\right)^2}{16\sqrt[3]{\sqrt{\frac{63}{16}t^6} + \frac{127}{64}t^3}} \simeq 1.122\,72t$

a zero location. If we substitute $a = b > 0$ in equations (15) and (16) we obtain:

$$x = \begin{cases} \frac{4b\tau - 7t}{4(t+\tau)} & if \quad 4b\tau - 7t > 0 \\ 0 & if \quad 4b\tau - 7t \leq 0, \end{cases}$$

$$\text{and } y = \begin{cases} \frac{4b\tau - 7t}{4(t+\tau)} & if \quad 4b\tau - 7t > 0 \\ 0 & if \quad 4b\tau - 7t \leq 0. \end{cases}$$

For $b > \frac{7t}{4\tau}$ downstream locations are positive, otherwise they are zero. Also, for symmetric locations we have that $a = b < \frac{1}{2}$. Thus, for $\frac{7t}{4\tau} < \frac{1}{2}$ or $\tau > 3.5t$ the profit function of A becomes:

$$\Pi_A(a = b > 0; \tau > 3.5t) = \begin{cases} \frac{3}{2}t & if \quad a = b \leq \frac{7t}{4\tau} \\ \frac{3t(9t+2\tau-4b\tau)}{4(t+\tau)} & if \quad \frac{7t}{4\tau} < a = b < 0.5. \end{cases}$$

For $\tau \leq 3.5t$ downstream locations are zero for every upstream location and the profit function of A is given by:

$$\Pi_A(a = b > 0; \tau \leq 3.5t) = \left\{ \frac{3}{2}t \quad if \quad a = b < 0.5. \right.$$

Now, we prove that firm A, given $b > 0$, has an incentive to deviate from $a > 0$ and locate at $a = 0$. If we substitute $a = 0, b > 0$ into equations (15) and (16), we find that $x = 0$ for every b and $y > 0$ for $b \in (b_1, b_2)$ where $b_1 \equiv \frac{17t\tau+4\tau^2-\sqrt{9t^2\tau^2+16\tau^4+24t\tau^3-63t^3\tau}}{2\tau(t+4\tau)}$ and $b_2 \equiv \frac{17t\tau+4\tau^2+\sqrt{9t^2\tau^2+16\tau^4+24t\tau^3-63t^3\tau}}{2\tau(t+4\tau)}$. Again, $9t^2\tau^2 + 16\tau^4 + 24t\tau^3 - 63t^3\tau \geq 0$ is satisfied when $\tau \geq 1.12272t$, but as $b_1 > \frac{1}{2}$ when $\tau < 3.3925t$ and $b_2 > \frac{1}{2}$ for every τ, we have that $y > 0$ for $b \in (b_1, 0.5)$ when $\tau > 3.3925t$, otherwise (for $b \in (0, b_1]$) we have that $y = 0$. For $1.12272t \leq \tau \leq 3.3925t$ downstream location y cannot be positive and for $\tau < 1.12272t$ we can show (see above for x) that $y = 0$. Thus, the profit function of firm A becomes:

$$\Pi_A(a = 0, b > 0; \tau > 3.3925t) =$$

$$\begin{cases} \frac{t\left(38313t^3+360(1-b)(127-9b)t^2\tau+16\left(-126b+b^2+1121\right)(1-b)^2t\tau^2+256(9-b)(1-b)^3\tau^3\right)^2}{3456(t+\tau)\left(99t^2+4(1-b)(20-b)t\tau+16(1-b)^2\tau^2\right)(9t+4\tau(1-b))^3} & if \quad b \in (b_1, 0.5) \\ \frac{\left(9t+b^2\tau\right)^2}{54t} & if \quad b \in (0, b_1], \end{cases}$$

$$\Pi_A(a = 0, b > 0; \tau \leq 3.3925t) = \frac{\left(9t + b^2\tau\right)^2}{54t}.$$

The profits of firm A are higher when it deviates from a symmetric location within the unit interval and locates at the endpoint. We examine three areas depending on the parameter values t, τ. For $\tau \leq 3.3925t$ following direct calculations we find that:

$\Pi_A(a = 0, b > 0) = \frac{\left(9t+b^2\tau\right)^2}{54t} > \Pi_A(a = b > 0) = \frac{3}{2}t.$

For $3.3925t < \tau \leq 3.5t$, the profits of firm A when $a = 0$ depend on the rival's firm location b. When $b \in (0, b_1]$, $\Pi_A(a = 0, b \in (0, b_1]) = \frac{\left(9t+b^2\tau\right)^2}{54t} > \Pi_A(a = b \in (0, b_1]) = \frac{3}{2}t$. When $b \in (b_1, 0.5)$ the difference in the profits

$$\Delta\Pi = \frac{t\left(38313t^3+360(1-b)(127-9b)t^2\tau+16\left(-126b+b^2+1121\right)(1-b)^2t\tau^2+256(9-b)(1-b)^3\tau^3\right)^2}{3456(t+\tau)\left(99t^2+4(1-b)(20-b)t\tau+16(1-b)^2\tau^2\right)(9t+4\tau(1-b))^3} - \frac{3}{2}t$$

is decreasing in b ($\frac{d\Delta\Pi}{db} < 0$, the profits $\Pi_A(a = 0, b > 0)$ decrease when b increases) and is positive at the maximum value of b $\left(\lim\limits_{b\to 0.5^-} \Delta\Pi > 0\right)$ which implies that $\Delta\Pi$ is positive for all $b \in (b_1, 0.5)$.

For $\tau > 3.5t$, the profit function of firm A depends on the rival's firm location b when firm A locates either within the unit interval or at the endpoint. As $b_1 < \frac{7t}{4\tau}$, we examine the following cases. When $b \in (0, b_1]$, we have again that $\frac{\left(9t+b^2\tau\right)^2}{54t} > \frac{3}{2}t$. When $b \in (b_1, \frac{7t}{4\tau}]$, the difference in profits

$$\Delta\Pi = \frac{t\left(38\,313t^3+360(1-b)(127-9b)t^2\tau+16\left(-126b+b^2+1121\right)(1-b)^2t\tau^2+256(9-b)(1-b)^3\tau^3\right)^2}{3456(t+\tau)\left(99t^2+4(1-b)(20-b)t\tau+16(1-b)^2\tau^2\right)(9t+4\tau(1-b))^3} - \frac{3}{2}t$$

is decreasing in b and is positive at the maximum value of b ($b = \frac{7t}{4\tau}$), therefore, the profits are higher when firm A locates at the endpoint. Finally, when $b \in (\frac{7t}{4\tau}, 0.5)$ the difference in the profits

$$\Delta\Pi = \frac{t\left(38\,313t^3+360(1-b)(127-9b)t^2\tau+16\left(-126b+b^2+1121\right)(1-b)^2t\tau^2+256(9-b)(1-b)^3\tau^3\right)^2}{3456(t+\tau)\left(99t^2+4(1-b)(20-b)t\tau+16(1-b)^2\tau^2\right)(9t+4\tau(1-b))^3} - \frac{3t(9t+2\tau-4b\tau)}{4(t+\tau)}$$

is increasing in b ($\frac{d\Delta\Pi}{db} > 0$, profits $\Pi_A(a = 0, b > 0)$ and $\Pi_A(a = b > 0)$ decrease as b increases, but the decrease in the latter is higher since a is closer to b) and is positive for the minimum value of b $\left(\lim\limits_{b\to \frac{7t}{4\tau}^+} \Delta\Pi > 0\right)$, thus, firm A has an incentive to deviate and locate at $a = 0$. ■

Appendix B

Overall profit function of firm X. In Section (4.3) we provide the profit functions of the downstream firms when $1 - x - y > 0$, that is, firm X is located to the left of firm Y. Now, we also study the case where $1 - x - y = 0$ and $1 - x - y < 0$ where firm X is located at the same point or to the right of firm Y. When $1 - x - y < 0$ we cannot simply replace in the profit functions (10) and (11), x with $1 - y$ and y with $1 - x$ since there is exclusive dealing between firm X and its supplier A and between firm Y and its supplier B. If firm X is located to the right of firm Y, it is still supplied by firm A which is located to the left of firm B. Thus, when firm X is located to the right of firm Y is not completely symmetric to when firm Y is located to the right of firm X, since they have different suppliers.

Assume $1 - x - y = 0$, so that retailers are located at the same point on the line. There is no product differentiation downstream and consumers buy one unit of the product from the cheapest retailer. The demand of firm X, z, becomes:

$$z = D_X = D_A = \begin{cases} 1 & if \quad p_X < p_Y \\ \frac{1}{2} & if \quad p_X = p_Y \\ 0 & if \quad p_X > p_Y. \end{cases}$$

Downstream competition is very intense (Bertrand competition) and pushes final prices to the marginal cost. Firm X faces aggregate marginal cost $f_X = w_A + \tau(x - a)^2$ and firm Y $f_Y = w_B + \tau(y - b)^2$. Thus, final price is set equal to $\max\{f_X, f_Y\}$ and the firm with the lowest aggregate marginal cost captures the whole demand and enjoys positive profits. The profit function of firm

X becomes:

$$\Pi_X = \begin{cases} w_B - w_A + \tau\left((y-b)^2 - (x-a)^2\right) & if \quad w_A + \tau(x-a)^2 < w_B + \tau(y-b)^2 \\ 0 & if \quad w_A + \tau(x-a)^2 \geq w_B + \tau(y-b)^2. \end{cases}$$

Since firm X is supplied by firm A, the profit function of firm A is:

$$\Pi_A = \begin{cases} w_A & if \quad w_A < w_B + \tau\left((y-b)^2 - (x-a)^2\right) \\ 0 & if \quad w_A \geq w_B + \tau\left((y-b)^2 - (x-a)^2\right). \end{cases}$$

Downstream competition is transferred to the upstream level and the wholesale prices are reduced to the difference in the transportation costs of the two retailers (minus $\varepsilon \simeq 0$). If firm B charges a wholesale price higher than that level, firm A can undercut w_B and capture the whole demand via its retailer X. Thus:

$$\begin{aligned} w_A &= \begin{cases} \tau\left((1-x-b)^2 - (x-a)^2\right) & if \quad (1-x-b)^2 - (x-a)^2 > 0 \\ 0 & otherwise, \end{cases} \\ w_B &= \begin{cases} \tau\left((x-a)^2 - (1-x-b)^2\right) & if \quad (x-a)^2 - (1-x-b)^2 > 0 \\ 0 & otherwise. \end{cases} \end{aligned}$$

The profits of X reduce to zero, $\Pi_X = 0$, as $\tau\left((1-x-b)^2 - (x-a)^2\right) - \left(\tau\left((1-x-b)^2 - (x-a)^2\right)\right) = 0$. In an analogous manner $\Pi_Y = 0$.

Assume $1-x-y < 0$, so that firm X is located to the right of firm Y. The indifferent consumer is given by:

$$p_Y + t(z - (1-y))^2 = p_X + t(x-z)^2$$

and the demand of firm Y (z) that is located on the left of firm X is:

$$z = D_Y = D_B = \begin{cases} 1 & if \quad \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} \geq 1 \\ \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} & if \quad 0 < \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} < 1 \\ 0 & if \quad \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} \leq 0. \end{cases}$$

The profit functions of firms X and Y are:

$$\begin{aligned} \Pi_X^R &= (p_X - f_X)(1-z), \\ \text{and } \Pi_Y^R &= (p_Y - f_Y)\, z. \end{aligned}$$

The superscript R refers to the case where firm X is to the right of firm Y. From the first order conditions, we obtain the equilibrium final price for firm X (likewise, for firm Y):

$$p_X^R = \begin{cases} f_Y - t(x+y-1)(1+x-y) & if \quad f_X \leq f_Y - t(3-y+x)(x+y-1) \\ \frac{1}{3}\left(t(x+y-1)(3+y-x) + f_Y + 2f_X\right) & if \quad \left\{ \begin{array}{c} f_Y - t(3-y+x)(x+y-1) < f_X \\ f_X < f_Y + t(3-x+y)(x+y-1) \end{array} \right\} \\ f_X & if \quad f_X \geq f_Y + t(3-x+y)(x+y-1) \end{cases}$$

with the respective profits

$$\Pi_X^R = \begin{cases} f_Y - t\,(x-y+1)\,(x+y-1) - f_X & if \quad f_X \le f_Y - t\,(3-y+x)\,(x+y-1) \\ \frac{(t(x+y-1)(3+y-x)+f_Y-f_X)^2}{18t(x+y-1)} & if \quad \left\{ \begin{array}{l} f_Y - t\,(3-y+x)\,(x+y-1) < f_X \\ f_X < f_Y + t\,(3-x+y)\,(x+y-1) \end{array} \right\} \\ 0 & if \quad f_X \ge f_Y + t\,(3-x+y)\,(x+y-1)\,. \end{cases}$$

In the third stage, upstream firms seek to maximize their profits $\Pi_A^R = w_A(1-z)$ and $\Pi_B^R = w_B z$ with respect to their wholesale prices. Assuming equilibrium in the subsequent stage, the profit function of firm A becomes:

$$\begin{aligned} \Pi_A^R &= 0 \\ & \quad \text{if } w_A \ge w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\ &= w_A\left(1 - \left(\frac{2t(y-x)(x+y-1)+w_A-w_B+\tau\left((x-a)^2-(y-b)^2\right)}{6t(x+y-1)} + \frac{1}{2}(1+x-y)\right)\right) \\ & \quad \text{if } \left\{ \begin{array}{l} w_A < w_B + \tau\left((y-b)^2 - (x-a)^2\right) + t\,(y-x+3)\,(x+y-1) \\ w_A > w_B + \tau\left((y-b)^2 - (x-a)^2\right) - t\,(x-y+3)\,(x+y-1) \end{array} \right\} \\ &= w_A \\ & \quad \text{if } w_A \le w_B + \tau\left((y-b)^2 - (x-a)^2\right) - t\,(x-y+3)\,(x+y-1)\,. \end{aligned}$$

When the aggregate marginal cost faced by firm X, $w_A + \tau(x-a)^2$, is high enough, the demand of firm X reduces to zero, thus, firm A gets zero demand too. For intermediate prices, w_A, the market is shared with firm B and for very low prices firm A captures the whole demand. As in section (4.2), we calculate the equilibrium wholesale prices for firm A (analogously for firm B):

$$\begin{aligned} w_A^R &= \tau\left((y-b)^2 - (x-a)^2\right) - t\,(x-y+3)\,(x+y-1) \\ & \quad \text{if } \tau\left((x-a)^2 - (y-b)^2\right) + t\,(x-y+9)\,(x+y-1) \le 0 \\ &= \frac{\tau\left((y-b)^2-(x-a)^2\right)+t(y-x+9)(x+y-1)}{3} \\ & \quad \text{if } \left\{ \begin{array}{l} \tau\left((y-b)^2 - (x-a)^2\right) + t\,(y-x+9)\,(x+y-1) > 0 \\ \tau\left((x-a)^2 - (y-b)^2\right) + t\,(x-y+9)\,(x+y-1) > 0 \end{array} \right\} \\ &= 0 \\ & \quad \text{if } \tau\left((x-a)^2 - (y-b)^2\right) - t\,(y-x+9)\,(x+y-1) \ge 0. \end{aligned}$$

The corresponding profits for firm X are:

$$\begin{aligned}
\Pi_X^R &= 2t\,(x+y-1) \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+9)\,(x+y-1)\leq 0 \\
&= \frac{\left(\tau\left((y-b)^2-(x-a)^2\right)+t(y-x+9)(x+y-1)\right)^2}{162t(x+y-1)} \\
&\quad \text{if } \left\{\begin{array}{l} \tau\left((y-b)^2-(x-a)^2\right)+t\,(y-x+9)\,(x+y-1)>0 \\ \tau\left((x-a)^2-(y-b)^2\right)+t\,(x-y+9)\,(x+y-1)>0 \end{array}\right\} \\
&= 0 \\
&\quad \text{if } \tau\left((x-a)^2-(y-b)^2\right)-t\,(y-x+9)\,(x+y-1)\geq 0.
\end{aligned}$$

Section (4.3) provides the profits of firm X when located to the left of firm Y (Π_X^L) and Appendix B presents the profits of firm X when X is located at the same point or to the right of firm Y (Π_X^R). Thus,

$$\Pi_X = \begin{cases} \Pi_X^L & if \quad 1-x-y>0 \\ 0 & if \quad 1-x-y=0 \\ \Pi_X^R & if \quad 1-x-y<0. \end{cases}$$

■

Appendix C

Proof of the downstream equilibrium locations. In Section (4.3) we prove that there is an equilibrium in the second stage of the game, when firm X is located to the left of firm Y, say (x^*, y^*) with:

$$\begin{aligned}
x^* &= \frac{\left(16a\tau^2\,(1-a-b)+4t\tau\left(17a+6b-a^2+b^2-7\right)-63t^2\right)}{4\,(t+\tau)\,(4\tau\,(1-a-b)+9t)}, \\
y^* &= \frac{\left(16b\tau^2\,(1-a-b)+4t\tau\left(17b+6a-b^2+a^2-7\right)-63t^2\right)}{4\,(t+\tau)\,(4\tau\,(1-a-b)+9t)}.
\end{aligned}$$

However, we do not compare the profits of firm X when located either to the left or the right of firm Y. We assume that firm X is located to the left. Now we complete the proof and show that given $y = y^*$, firm X prefers to locate at the left of firm Y at point x^*, compared to the right of firm Y. For simplicity, we provide the proof for symmetric upstream locations ($a = b$) since this will be the equilibrium outcome.

Initially we prove that given $y = y^* = \frac{-7t+4a\tau}{4(t+\tau)}$, firm X cannot capture the whole demand either located to the left or to the right of firm Y. Firm X would serve the whole market located to the left of firm Y if:

$$\tau\left((x-a)^2-(y-b)^2\right)+t\,(y-x+9)\,(1-x-y)\leq 0$$

$$
\begin{aligned}
\text{or } x &\in (x_1, x_2) \text{ with} \\
x_1 &\equiv \frac{20t + 4a\tau - 3\sqrt{t\,(9t - 16\tau + 32a\tau)}}{4\,(t+\tau)}, \\
x_2 &\equiv \frac{20t + 4a\tau + 3\sqrt{t\,(9t - 16\tau + 32a\tau)}}{4\,(t+\tau)}.
\end{aligned}
$$

Since $1-y^* < x_1 < x_2$, we have that inequality $\tau\left((x-a)^2 - (y-b)^2\right) + t\,(y - x + 9)\,(1 - x - y) \leq 0$ is not satisfied. Firm X would serve the whole market located to the right of firm Y if:

$$
\tau\left((x-a)^2 - (y-b)^2\right) + t\,(x - y + 9)\,(x + y - 1) \leq 0
$$

$$
\begin{aligned}
\text{or } x &\in (x_4, x_3) \text{ with} \\
x_3 &\equiv \frac{-16t + 4a\tau + 3\sqrt{t\,(81t + 16\tau - 32a\tau)}}{4\,(t+\tau)}, \\
x_4 &\equiv \frac{-16t + 4a\tau - 3\sqrt{t\,(81t + 16\tau - 32a\tau)}}{4\,(t+\tau)}.
\end{aligned}
$$

Since $1 - y^L > x_3 > x_4$, we have that inequality $\tau\left((x-a)^2 - (y-b)^2\right) + t\,(x - y + 9)\,(x + y - 1) \leq 0$ is not satisfied.

Thus, we have to prove that firm X prefers to share the market with firm Y and locate to the left of Y. Firm X maximizes its profits to the left at x^* with:

$$
\begin{aligned}
\Pi_X^L(x^*, y^*) &= \frac{t\,(9t - 2\tau\,(2a-1))}{4\,(t+\tau)}, \\
x^* &= \frac{-7t + 4a\tau}{4\,(t+\tau)}
\end{aligned}
$$

and to the right at x^+ with

$$
\begin{aligned}
\Pi_X^R(x^+, y^*) &= \frac{(9t - 2\tau\,(2a-1))\,(9t - 16\tau + 32a\tau)^2}{8748t\,(t+\tau)}, \\
x^+ &= \frac{51t + 16\tau - 20a\tau}{12\,(t+\tau)}
\end{aligned}
$$

for

$$
9t + 16\tau\,(2a-1) > 0.
$$

The constraint $9t + 16\tau\,(2a-1) > 0$ is necessary so that both firms sell, when X is located to the right of Y. If a is low enough, that is, upstream firms are located away from the unit interval, firm X can never sell when located to the right of firm Y since its supplier A is far away. We compare the profits $\Pi_X^L(x^*, y^*)$ and $\Pi_X^R(x^+, y^*)$ and obtain $\Pi_X^L(x^*, y^*) > \Pi_X^R(x^+, y^*)$. ■

Appendix D

Proof of the second equilibrium in the second stage of the game for fixed and symmetric upstream locations. For symmetric upstream locations ($a = b$) and given $y = \widehat{y} = \frac{11t+4a\tau}{4(t+\tau)}$, the profits of firm X when it locates to the right of firm Y and shares the market are

maximized at $\widehat{x}$ where:

$$\begin{aligned}
\Pi_X^R(\widehat{x},\widehat{y}) &= \frac{t\left(9t+2\tau\left(2a-1\right)\right)}{4\left(t+\tau\right)},\\
\widehat{x} &= \frac{11t+4a\tau}{4\left(t+\tau\right)}.
\end{aligned}$$

Firm X maximizes its profits when it locates to the left of firm Y and shares the market at x^- with:

$$\begin{aligned}
&\Pi_X^L(x^-,\widehat{y})\\
&= \frac{\left(4\left(-9t+4\tau\left(2a-1\right)\right)\sqrt{\left(9t-8\tau+16a\tau\right)^2}+4\left(-81t^2+288t\tau\left(2a-1\right)+32\tau^2\left(2a-1\right)^2\right)\right)^2}{279\,936t\left(t+\tau\right)\left(9t+4\tau\left(1-2a\right)+\sqrt{\left(9t-8\tau+16a\tau\right)^2}\right)},\\
x^- &= -\frac{\left(15t+2\tau\left(a-2\right)+\frac{1}{2}\sqrt{\left(9t-8\tau+16a\tau\right)^2}\right)}{6\left(t+\tau\right)}.
\end{aligned}$$

When firm X locates to the left of firm Y and captures the whole demand, its profits are maximized at x_m^L with:

$$\begin{aligned}
\Pi_X^L(x_m^L,\widehat{y}) &= 2t(1-x-y) = \frac{t\left(4\tau\left(1-2a\right)-27t+3\sqrt{t\left(81t+16\tau\left(2a-1\right)\right)}\right)}{2\left(t+\tau\right)},\\
x_m^L &= \frac{20t+4a\tau-3\sqrt{t\left(81t+16\tau\left(2a-1\right)\right)}}{4\left(t+\tau\right)}.
\end{aligned}$$

Finally, firm X captures the whole demand located to the right of firm Y and maximizes its profits at x_m^R where:

$$\begin{aligned}
\Pi_X^R(x_m^R,\widehat{y}) &= \frac{t\left(-9t-4\tau+8a\tau+3\sqrt{t\left(9t-16\tau\left(2a-1\right)\right)}\right)}{2\left(t+\tau\right)},\\
x_m^R &= \frac{-16t+4a\tau+3\sqrt{t\left(9t+16\tau\left(1-2a\right)\right)}}{4\left(t+\tau\right)}.
\end{aligned}$$

The $(\widehat{x},\widehat{y})$ pair of locations is an equilibrium in the second stage of the game when:

$$\begin{aligned}
\Pi_X^R(\widehat{x},\widehat{y}) &\geq \Pi_X^L(x^-,\widehat{y}),\\
\Pi_X^R(\widehat{x},\widehat{y}) &\geq \Pi_X^L(x_m^L,\widehat{y}),\\
\text{and } \Pi_X^R(\widehat{x},\widehat{y}) &\geq \Pi_X^R(x_m^R,\widehat{y})
\end{aligned}$$

are satisfied. By direct calculations we find that when $\Pi_X^R(\widehat{x},\widehat{y}) \geq \Pi_X^L(x^-,\widehat{y})$ then $\Pi_X^R(\widehat{x},\widehat{y}) \geq \Pi_X^L(x_m^L,\widehat{y})$ and $\Pi_X^R(\widehat{x},\widehat{y}) \geq \Pi_X^R(x_m^R,\widehat{y})$. Inequality $\Pi_X^R(\widehat{x},\widehat{y}) \geq \Pi_X^L(x^-,\widehat{y})$ is satisfied when:

$$t \geq \frac{4\tau\left(1-2a\right)}{9}.$$

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