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## Information-Sharing Between Competition Authorities: The Case of a Multinational Merger

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# Information-Sharing Between Competition Authorities: the Case of a Multinational Merger

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## Abstract

The increasing number of antitrust cases that affect more than one country calls for more active cooperation between competition authorities. I analyze the impact of exchange of confidential information between two authorities deciding on a multinational merger. The authorities, who differ in their leniency towards the merger, want to permit it if it enhances expected welfare in their country. The firm can secretly manipulate the precision with which it transmits the information about the merger's welfare implications.

Under no information exchange, the firm's strategy consists in choosing an extreme level of precision (very low or very high) depending on the average merger welfare implications. Under information exchange, the firm's behavior depends on the level of cooperation in the decision making between the countries. If the authorities' exert their veto power, the firm will use a lower level of precision more often making the more lenient authority strictly worse off. Further cooperation in the decision making modifies the firm's payoff structure which induces it to make a greater use of intermediate and higher levels of precision.

Other situations where the model can be applied abound in industrial organization and political economy. For instance, a politician trying to gain support for a policy in front of two audiences with different opinions about the policy.

*JEL Classification:* D82, D83, K21, L4.

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# 1 Introduction

“Most officials believe that the issue of confidentiality is the chief limitation of enforcement cooperation agreements and hence it is submitted that the majority of effort should be concentrated on overcoming this particular obstruction to effective cooperation between antitrust agencies.”

Marsden and Whelan (2005), p.24.

With globalisation, competition has an increasingly significant international dimension. A clear example is the existence of international cartels, such as the vitamin cartel which took place between January 1990 and February 1999. Another

example is the use of a common number in the telephone area code 16(e)9(r)21(a)10(t)8(i)7(o)pl71(i)6(c)9(y)-081(i)6(s)8(s71(u)12et)8searis38(c81(u)12(s)7st)8(e)9(d)-081(s)8(u)11(c)36(h)-081(a)theOm(p)-1fianlyoucaims38(e)9(f69(u)11(l)276[(i)6(n)11(s38(t)8(r)12ur)11(m)17(e)9(n)38(t91(s)]TJ/ment91(s)-442AsJapanedget b the fistuotationve oe of themain limitationonment91(s)2667(i)6(s)-685(t)8(h)11(e)2646(i)6(m)72(p)-16oass71il71bdrindmidenforhissueoexchan121(g)10(e-051(o)10(p)-08[(c)9oa)10(n)11(n)176(d)11(e)9(n)38(t89(i)6(a)11(f)-081(i)6. tions.-39481e80(n-20)8(2)10(0)11(0)102601(a)10(n)11(c)9(e)9an-20MicroeieseAM)reues7(tc)9(e)9an-209(( typeo agreementsis theebetween the. and the.

European Commission as support for a complaint against Intel. AMD believed that many of the issues in the U.S. case were similar to the questions under investigation by the E.C. This request was done under 28 U.S.C. § 1782, which allows a district court to order the production of documents "for use in a proceeding in a foreign or international tribunal" upon request by "any interested person". This request was first denied by the district court and then reversed by the Ninth Circuit Court of Appeals.<sup>4</sup>

A minority of competition policy agreements expressly provide for the exchange of confidential information. Within this group we find the bilateral agreement between the US and Australia<sup>5</sup>, the trilateral agreement between Iceland, Norway and Denmark<sup>6</sup> and, more recently, the agreement between competition authorities of the European Member States<sup>7</sup>.

In our analysis we ignore a very important cost attached to the exchange of confidential information: the danger of leakage of commercially sensitive information to third parties. This can be clearly an issue when cooperation involves competition agencies in countries where the law for protecting confidential information is weak or where the credibility of the agency is low. We also ignore many benefits. For instance, the exchange of information facilitates the discussion about the case and minimises the possibility of missing an issue that needs an enforcement action. In the merger cases, it also eliminates conflicting decisions and remedies.<sup>8</sup> Our focus is only on the impact on the incentives of the firm to provide precise information when undertaking a multinational merger.

In our model, the welfare implications of the merger correspond to an underlying real-valued state variable. In order to have the merger assessed, the multinational firm has to provide information about this variable (i.e. the merger's type) to the

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<sup>4</sup>In the Supreme court opinion two justices concurred the judgment, one dissented and one took no part. Intel Corp. v. Advanced Micro Devices, Inc., No. 02-572, available at: [www.supremecourtus.gov/opinions/03pdf/02-572.pdf](http://www.supremecourtus.gov/opinions/03pdf/02-572.pdf)

<sup>5</sup>See the Agreement between the Government of the United States of America and the Government of Australia on Mutual Antitrust Enforcement Assistance, available at [www.apeccp.org.tw/doc/USA/Cooperation/usaus7.htm](http://www.apeccp.org.tw/doc/USA/Cooperation/usaus7.htm)

<sup>6</sup>[www.globalcompetitionforum.org/regions/europe/Denmark/Agreement1.pdf](http://www.globalcompetitionforum.org/regions/europe/Denmark/Agreement1.pdf)

<sup>7</sup>See the E.C. Regulation 1/2003 available at <http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2003:001:0001:0025:EN:PDF>.

<sup>8</sup>A successful example of a merger involving cooperation between agencies that illustrates these points was the Holnam/Lafarge case. A waiver granted by the parties allowed the U.S. and Canadian agencies to improve the effective coordination, which ended up in a more informed decision-making (see Valentine (2000)).

competition authority. Neither the firm nor the authority have private information<sup>9</sup> about this variable and the only mechanism available to transmit it is through a noisy signal<sup>10</sup>. The authority only observes a random realisation of the signal. The signal is unbiased, that is, the firm cannot misrepresent the merger's welfare implications. However, the firm can choose to (secretly) manipulate the precision with which he transmits this information to the authority at no cost. We model this by allowing the firm to choose the variance of the signal. The choice of the noise is unobservable because the authority does not actually know how much information the firm has. Moreover, a larger quantity of information does not imply more precision as some of it may be irrelevant.

The policy decision (to clear the merger or not) depends only on the realisation of the signal. The authority is restricted<sup>11</sup> to adopt a cut-off rule whereby if the realisation of the signal is above some threshold she clears the merger and she blocks it otherwise.

First, we determine the optimal choice of variance when the firm deals with a single authority. We find that the optimal variance chosen by the firm does not depend on how *good* or *bad* the average merger is (i.e. how far above or below the average merger type is with respect to the policy threshold) but rather on whether it is good or bad (i.e. above or below the threshold). In particular, a firm with an average bad merger will choose the risky strategy of high variance to have more chances to be thought "good". By contrast, a firm with an average good merger will choose low variance to increase the likelihood of obtaining a high realisation of the signal. Furthermore, this strategy does not change depending on whether the authority can commit to a particular threshold ex-ante. We also establish the optimal response (i.e. policy threshold) of the authority to the firm's behaviour. If the authority can commit, this equilibrium threshold, when the average merger is welfare detrimental (enhancing), is stricter (more lenient) than the full information threshold. Moreover, when there is uncertainty about the merger's undesirability,

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<sup>9</sup>We also discuss at the end of the paper, how the results change if we allow the firm to have private information about this variable.

<sup>10</sup>The justification for such a signal relies on the fact that the production technology (i.e. the way in which the firm compiles and transmits the information about the merger) may be noisy. Another justification for the existence of noisy signals is the imperfect perception of the receiver. For instance, Kolstad et al. (1990) analyze the optimal use of ex ante safety regulation and ex post tort liability. In their model, there is uncertainty about how a court will interpret whether or not the injurer met the standard of due care.

<sup>11</sup>The monotone likelihood ratio property holds, which is a sufficient condition for a cut-off rule to be optimal.

the authority's ability to commit makes her set a more lenient threshold in order to induce the firm to provide a more precise information.

Then, we analyse the case of an international merger, where the multinational wants to undertake a merger in two countries at the same time. We consider the case where one authority is more lenient than that of the other country (i.e. their policy thresholds differ<sup>12</sup>), either because their tastes for mergers differ or because the same underlying state variable has different welfare implications in the two countries. We consider three different rules that deal with the situation when the competition authorities disagree (that is, when the realisation of the signal lies between their thresholds). We explore the impact of the information-sharing regime on the incentives of the firm to provide precise information. Information-sharing means that the authorities receive the same realisation of the signal sent by the multinational, so the multinational is left to make a unique choice of variance for both countries.

When the authorities share information, we find that the agreement has no impact when the average merger is either good or bad for both countries at the same time. However, the agreement does have an impact on the firm's behaviour when the average merger is good for one country but bad for the other. This impact will depend on the particular rule that governs the disagreement.

If the authorities can exert their veto power (i.e. one country can always unilaterally block the merger), then information-sharing it is always a bad idea as it will induce the firm to send a very imprecise signal to both authorities (making the more lenient country strictly worse off). However, if cooperation goes beyond the information-sharing stage, and there is some informal bargaining/persuasion process (not explicitly modeled in this paper) taking place between the competition authorities, then there is some scope for information-sharing to be beneficial as long as the more lenient country is not always the same one.

We model the outcome of the bargaining process as the merger being approved with some probability in case of disagreement. First, we take this probability as being exogenous which may be interpreted as the authorities' relative bargaining power. In particular, if the authorities have equal bargaining power, the choice of the variance may be non-monotonic in the expected merger. When the average merger is relatively bad for the less lenient country, the firm will choose either low variance or high variance. In particular, if the asymmetry between authorities is very large, the firm will first choose high variance in order to maximise the chances to be

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<sup>12</sup>We take these thresholds as given and assume that the authorities can commit to them.

above the second threshold and then will switch to low variance in order to maximise the chances of being cleared at least by the more lenient country. As the average merger gets less welfare detrimental for the stricter country, it becomes safer to play a riskier strategy by increasing the variance to an intermediate level. Furthermore, this variance decreases as the average merger approaches to the policy threshold of the stricter country. This is because the chances of having the merger cleared are high anyway and, by reducing the variance, the probability of having it blocked by both authorities is decreased.

We also consider the case, where the probability of clearing the merger depends on the particular realisation of the report. This intends to capture the idea that the less lenient country would be less reluctant to clear the merger if the realisation falls near its threshold as compared to when it falls very far from it. In particular, we consider the case where this probability function is increasing and linear in the signal realisation. From the point of view of the firm, it is as if there were a unique but uncertain threshold. Because of the linearity, the firm considers the expected threshold and behaves as in the national merger case with respect to this threshold.

To sum up, further cooperation modifies the firm's payoff structure in a way the firm makes a greater use of intermediate and lower levels of noise (as compared to the veto power case) and therefore, it can be potentially good if lenient countries are not always the same ones.

## 1.1 Related literature

This is a signal-jamming model (see Holmstrom (1982, 1999) and Meyer and Vickers (1997)) where the firm jams the signal not by manipulating its mean, but instead by changing its variance, and hence its information content.

The choice of variability as a strategic variable, has been considered in a large variety of setups.<sup>13</sup> The general conclusion of all these papers is that the players that are at disadvantage tend to optimally choose more risky strategies than those who are in a favourable position. However, (as far as we are aware) a distinctive feature of our model is that the noise (i.e. the distribution of the signal) is not

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<sup>13</sup>For instance, Cabral (2003) analyses a model of R&D races where there are two contestants who are allowed to choose the variance of their stochastic R&D technology. Tsetlin, Gaba and Winkler (2004) consider the choice of variability of the performance distribution in a multi-round contest. Krakel, Nieken and Przemeck (2008) undertake a similar analysis in the auction context. In the compensation literature, Gaba and Kalra (1999) introduce the level of dispersion of the probability distribution of sales as a choice variable besides the level of effort.

observed by the receiver (in their setup the buyer). To get an intuition of why observability matters, consider the paper by Johnson and Myatt (2006). The authors consider how much information a monopolist would want to provide to his potential customers. In particular, they show how the seller's supply of information affects the shape of the distribution of buyers' expected valuations and hence generates rotations of the demand curve. They show that these rotations generate a convexity in the monopolist's profits which explains the optimality of the monopolist's extreme choices in Lewis and Sappington (1994). In their model, the informativeness of the information transmitted by the seller is observed by the receiver. To see why this makes a difference, consider the situation where the product is bad on average, that is, in the absence of any information the buyer will not buy it (at a given price). To convince the buyer to buy, the seller has to generate a high realisation of the signal and wants the buyer to attach a high weight to this signal in her updating. If the noise is observable, adding noise to the signal will make the buyer attach a lower weight to the signal realization so the optimal strategy is actually to reduce the noise as much as possible. However, if the noise is not observable, the buyer will not adjust the prior and signal weights to account for the noise and, therefore, the optimal strategy of the seller is to increase the noise of the signal.

The paper proceeds as follows. Section 2 introduces the benchmark model. Section 3 first presents the results for the commitment case. It then presents the no-commitment case results. Then, Section 4 introduces and analyses the multinational merger setup. Finally, Section 6 concludes.

## 2 The model

We consider a multinational firm (M) proposing a merger that must be cleared under the competition law of the country. The welfare consequences of the merger can be summarised in the real-valued state variable  $\theta$ , with support on  $[-\infty, +\infty]$ . Therefore, the mergers above 0 are welfare enhancing while the ones below are welfare detrimental. Furthermore, the higher the  $\theta$ , the more desirable it is for the country to have the merger cleared by the competition regulator (R). If R were to observe the true  $\theta$  (full information framework), she would like to clear the merger with probability one whenever  $\theta \geq 0$  and block it otherwise. The multinational always has a net benefit of 1 from the merger (otherwise he would not have proposed it in the first place) and, therefore, he would like to have the merger cleared for any  $\theta$ . As



in the career concerns literature, the firm does not have private information about its type  $\theta$ . For instance, the firm may be giving information about the merger's market effect or the merger's effect on third parties, for which the firm may not have more information than the competition authority.<sup>14</sup> The parameter  $\theta$  is distributed according to a Normal distribution,  $f(\theta)$ , with mean  $\mu$  and variance  $\eta^2$  and this distribution is common knowledge.

The actions available to the players are the following. M sends a message containing the information about his type  $\theta$  to R but he does so through a noisy signal. In particular, the signal has the following form:

$$S = \theta + \varepsilon$$

where  $\varepsilon$  is a random variable distributed according to a Normal distribution with mean zero and variance  $V$ . M cannot choose to send a message to R different from his type (i.e. lying is not possible and therefore, the signal should be on average equal to the true  $\theta$ ) but he can choose,  $V$ , which reflects the lack of precision with which the message is sent. In particular, the higher the  $V$ , the less informative is the realisation of signal about the true type of M. For simplicity, we assume that M can only choose within this interval of variances:  $V \in [V_L, V_H]$ , where  $0 < V_L < V_H$ . Therefore, the distribution of the signal  $S$ ,  $g(s)$ , is Normal with mean  $\mu$  and variance  $\eta^2 + V_i$ , where  $i$  is the subscript for the particular variance in  $[V_L, V_H]$  and  $G(s)$  is the cumulative distribution.

R observes the realisation of the signal,  $s$ . Given  $s$ , R updates her beliefs about M and chooses the appropriate probability of clearance  $p(s)$ . We consider the case where  $p(s)$  is a simple cut-off rule where  $\hat{s}$  will be optimally determined by R:<sup>15</sup>

$$p(s) = \begin{cases} 0 & \text{if } s < \hat{s} \\ 1 & \text{if } s \geq \hat{s} \end{cases}$$

The timing of the game depends on whether R has the ability to commit to a particular threshold before the information is transmitted by M. We will deal with each case in turn.

The timing of the game *under commitment* is as follows. First, R commits to

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<sup>14</sup>We discuss in Section 5 the consequences of relaxing this assumption by allowing the firm to have private information about its type.

<sup>15</sup>The monotone likelihood ratio property (MLRP) is a sufficient condition for a cut-off rule to be optimal, because when this property holds a higher realisation of  $s$  is more likely to come from a better merger (See Milgrom (1981)). It is easy to check that in this setup the MLRP holds.

a policy threshold  $\hat{s}$ . Then, M chooses the level of precision  $V$  with which he is going to send the signal. A realisation of the signal,  $s$ , is observed by R. Given this realisation, R updates her beliefs about M and clears the merger whenever  $s$  is above  $\hat{s}$ . Finally, the payoffs are realised. We solve this model backwards.

The timing of the game *under no commitment* is as follows. First, M chooses the variance with which he is going to send the signal. A realisation of the signal,  $s$ , is observed by R. Given this realisation, R updates her beliefs about M and decides which policy threshold  $\hat{s}$  to use. Finally, the payoffs are realised. The solution of this model is a Nash equilibrium where the conjectures of each of the players are correct in equilibrium.

### 3 Benchmark: the case of a national merger

#### 3.1 Commitment regime

##### 3.1.1 The problem of the firm

The firm always benefits from the merger, therefore, the multinational is left to choose the level of precision that maximises the expected probability of having the merger cleared, given that he does not know his type. Using the assumption on the form of  $p(s)$ , M is left to maximise the following expression<sup>16</sup>:

$$V(\mu, \hat{s}) = \arg \max_{V \in [V_L, V_H]} \int_{\hat{s}}^{+\infty} g(s) ds$$

This objective function is decreasing in  $V$  whenever the average merger is a good merger (i.e.  $\hat{s} < \mu$ ) and increasing in  $V$  when the average merger is a bad merger (i.e.  $\hat{s} > \mu$ ). Therefore, if  $\hat{s} < \mu$ , all types of mergers will choose the minimum available variance  $V_L$ . Conversely, if  $\hat{s} > \mu$ , all types of mergers will choose the maximum available variance  $V_H$ .

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<sup>16</sup>Using Bayes Rule, the objective function is equivalent to:

$$\int_{-\infty}^{+\infty} f(\theta) \int_{\hat{s}}^{+\infty} g(s, V | \theta) ds d\theta$$

**Proposition 1** *The optimal action taken by  $M$  under commitment is:*

$$V^*(\mu, \hat{s}) = \begin{cases} V_H & \text{if } \mu < \hat{s} \\ V_L & \text{if } \mu > \hat{s} \end{cases}$$

*If  $\hat{s}$  is exactly  $\mu$ , then  $M$  will be indifferent between any variance in  $V^*(\mu, \hat{s}) \in [V_L, V_H]$ .*

**Proof.** See Appendix. ■

For simplicity, we will assume that if  $\hat{s}$  is exactly  $\mu$  the firm will choose  $V_L$ .

The intuition supporting this optimal strategy is that, by increasing the variance, an expected "bad" type (to the left of  $\hat{s}$ ) obtains more chances to be thought "good" (to the right of  $\hat{s}$ ). By contrast, decreasing the variance cuts down an expected "good" type's chance of being considered bad. Therefore, the optimal  $V$  does not depend on how far the particular  $\mu$  is from  $\hat{s}$ , only on whether  $\mu$  lies below or above  $\hat{s}$ . This is due to the "bang-bang" payoff structure created by the cut-off rule. This is in line with the findings of the literature that considers the choice of variability as a strategic variable.

### 3.1.2 The problem of the competition authority

The competition authority chooses  $\hat{s}$  to maximise the ex-ante expected welfare:

$$\begin{aligned} EW(\hat{s}, V^*(\mu, \hat{s})) &= \int_{\hat{s}}^{+\infty} \left( \frac{s\eta^2 + \mu V^*(\mu, \hat{s})}{\eta^2 + V^*(\mu, \hat{s})} \right) g(s, V^*(\mu, \hat{s})) ds \\ \text{subject to} \quad &: V^*(\mu, \hat{s}) = \begin{cases} V_H & \text{if } \mu < \hat{s} \\ V_L & \text{if } \mu \geq \hat{s} \end{cases} \end{aligned} \quad (1)$$

where the term in brackets is the expected welfare given a the realisation of  $s$ <sup>17</sup>.

**Proposition 2** *Under commitment, the optimal threshold,  $\hat{s}^*$ , set by the competition authority is non-monotonic in the expected welfare:*

$$\hat{s}^* = \begin{cases} \frac{-\mu V_H}{\eta^2} & \text{if } \mu < \tilde{\mu} \\ \mu & \text{if } \mu \in [\tilde{\mu}, 0] \\ \frac{-\mu V_L}{\eta^2} & \text{if } \mu > 0 \end{cases}$$

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<sup>17</sup>The objective function is equivalent to  $\int_{-\infty}^{+\infty} \theta f(\theta | s, V^*(\mu, \hat{s})) d\theta$  where  $f(\theta | s, V^*(\mu, \hat{s}))$  is the posterior distribution of  $\theta$ .

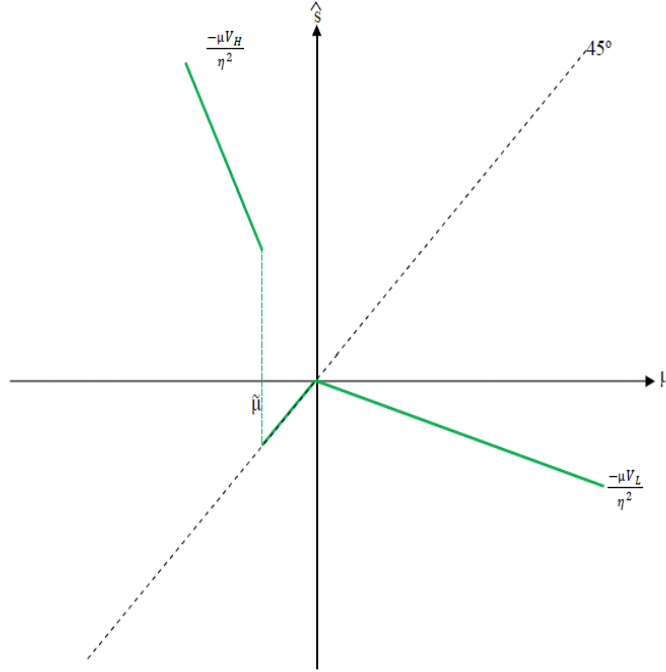


Figure 1: Optimal threshold under commitment

where  $\tilde{\mu}$  is defined in equation (6). The threshold  $\tilde{\mu}$  increases (decreases) with  $V_L$  ( $V_H$ ).

**Proof.** See Appendix. ■

Therefore, if the merger is on average clearly welfare detrimental ( $\mu < \tilde{\mu}$ ), the authority will set a policy threshold above the merger type that leaves welfare unaffected. The intuition behind this is that, if the bulk of mergers are bad mergers, then the authority increases the standard of proof of a good merger by setting the policy threshold above the neutral merger 0. If there is a substantial uncertainty about the undesirability of the average merger ( $\mu \in [\tilde{\mu}, 0]$ ), then the authority decides to commit to a more lenient threshold (not only more lenient than the ex-post optimal threshold but also than the full information threshold) in order to induce the firm to send a very precise signal. Finally, when the average merger is welfare enhancing ( $\mu > 0$ ), the authority will set very lenient standard of proof. This result is depicted in Figure 1.

Note that by Proposition 1, above the  $45^\circ$  line, the firm chooses  $V_H$  and that below, he chooses  $V_L$ . Except for the parameter range ( $\mu \in [\tilde{\mu}, 0]$ ), the optimal threshold,  $\hat{s}^*$ , will have the opposite sign to the average merger and will increase

in absolute value with the mean of the prior and the variance chosen by the firm  $V^*(\mu, \hat{s})$ . Finally, note that if  $V_L$  then the range of mergers for which the authority is more lenient than ex-post optimal,  $[\tilde{\mu}, 0]$ , shrinks as there is less gain from obtaining a precise signal. Similarly, this range will expand if  $V_H$  increases.

## 3.2 No commitment regime

### 3.2.1 The problem of the firm

Under this regime, R cannot commit to a threshold  $\hat{s}$  at the beginning of the game. Therefore, M makes a conjecture about the policy threshold used by R,  $\hat{s}^c$ , where the superscript stands for conjecture. The problem is identical to the one solved in Section 3.1.1 and therefore it is omitted here.

**Lemma 3** *The optimal action taken by M, under no commitment is:*

$$V^*(\mu, \hat{s}^c) = \begin{cases} V_H & \text{if } \mu < \hat{s}^c \\ V_L & \text{if } \mu > \hat{s}^c \end{cases}.$$

*If  $\mu$  is exactly  $\hat{s}^c$ , then M is indifferent between any variance in  $V^*(\mu, \hat{s}^c) \in [V_L, V_H]$ .*

**Proof.** It is omitted (it is the same as Proposition 1). ■

### 3.2.2 The problem of the competition authority

Given the realisation  $s$  and the conjecture that R forms about M's conjecture  $\hat{s}^c$  (and thus, M's strategy  $V^c(\theta, \hat{s}^c)$ ), the authority needs to decide whether to clear the merger or not. In other words, given  $s$  and  $V^c(\theta, \hat{s}^c)$ , the authority needs to choose the threshold  $\hat{s}$  so that the ex-post expected welfare of the merger is maximised:

$$W(s, \hat{s}^c) = \int_{-\infty}^{+\infty} \theta f(\theta | s, V^*(\mu, \hat{s}^c)) d\theta$$

If the expected welfare is positive, then R will clear the merger. Conversely, if the expected welfare is negative, R will block the merger. The authority determines the threshold  $\hat{s}$  so that she is indifferent between blocking or clearing the merger, that is, so that the expected welfare is zero:

$$W(s, \hat{s}^c) |_{s=s(\hat{s}^c)} = \frac{s(\hat{s}^c)\eta^2 + \mu V^*(\mu, s(\hat{s}^c))}{\eta^2 + V^*(\mu, s(\hat{s}^c))} = 0 \quad (2)$$

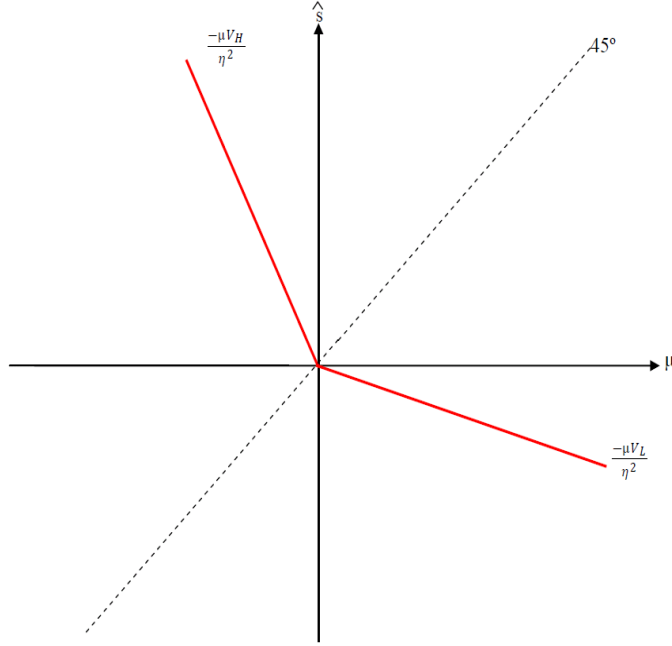


Figure 2: Optimal threshold under no commitment

Denote the solution of equation (2) as  $s(\hat{s}^c)$ . The equilibrium threshold  $\hat{s}^{**}$  is, then, the fixed point of this solution:

$$s(\hat{s}^{**}) = \hat{s}^{**}$$

**Proposition 4** *Under no commitment, the optimal threshold,  $\hat{s}^{**}$ , set by the competition authority is:*

$$\hat{s}^{**} = \left\{ \begin{array}{ll} \frac{-\mu V_H}{\eta^2} & \text{if } \mu < 0 \\ \frac{-\mu V_L}{\eta^2} & \text{if } \mu \geq 0 \end{array} \right\}$$

**Proof.** It is straightfoward to solve equation 2. ■

Thus, whenever the average merger is welfare detrimental, the optimal threshold will be above 0 and thus the firm will choose  $V_H$ . Conversely, if the optimal merger is welfare enhancing, the optimal threshold will be below 0 and the firm will choose  $V_L$ . Figure 2 illustrates these results.

Therefore, when the average merger is welfare detrimental the authority will set a strict standard of proof, while if the average merger is welfare enhancing the authority will set very lenient standard of proof.

The next section compares both regimes.

### 3.3 Comparison of regimes

Because the authority values precision, as this allows her to take more informed decisions, she may choose a different threshold depending on whether she can commit or not to a policy. In particular, when there is uncertainty about how bad a merger can be (that is, when  $\mu$  is negative and close to zero), if under no commitment, the authority sets a threshold  $\widehat{s}^{**}$ , she would gain from committing to a more lenient (lower) threshold  $\widehat{s}^*$  as this would induce M to reduce his equilibrium variance from  $V_H$  to  $V_L$ .

Therefore in some cases, the authority, by gaining commitment, decides to distort his policy by decreasing her standards (in an ex-post non-optimal way) in order to fix the information problem (i.e. to improve the informativeness of the signal).

## 4 The case of a multinational merger

In this section we consider the situation where a multinational wants to undertake the same merger in two different countries (or jurisdictions) and these jurisdictions differ in terms of their policies. This difference in thresholds can be interpreted as Country 1 being in general more lenient in its merger policy than Country 2.<sup>18</sup> For instance, let the welfare function of country  $i$  be  $a_i + b_i\theta$ , where  $i = 1, 2$ . It is easy to check that the resulting optimal threshold under no commitment is:

$$\widehat{s}_i = \left\{ \begin{array}{ll} -\mu \frac{V_H}{\eta^2} - \frac{a_i}{b_i} \frac{\eta^2 + V_H}{\eta^2} & \text{if } \mu < -\frac{a_i}{b_i} \\ -\mu \frac{V_L}{\eta^2} - \frac{a_i}{b_i} \frac{\eta^2 + V_L}{\eta^2} & \text{if } \mu \geq -\frac{a_i}{b_i} \end{array} \right\}$$

where  $a_i$  could be interpreted as how lenient is the authority (or how much competition there is already in the country's markets) and  $b_i$  how sensitive is the authority to the welfare's changes.

Without loss of generality, we can assume that Country 1 will block the merger if the signal is below  $\widehat{s}_1$  and clear it otherwise, while Country 2 will do the same using the threshold  $\widehat{s}_2$ , where  $\widehat{s}_1 < \widehat{s}_2$ .

Since what drives the variance decision is how  $\mu$  compares with the threshold, we will focus on the case where the information-sharing agreement would make a

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<sup>18</sup>Another possible justification is that the merger is expected to be less harmful, for instance because the market concentration levels are lower, or the likelihood of coordinated interaction in an oligopolistic market smaller.

difference in firm's choice. This occurs when there is a conflicting merger in the sense that average merger is good for Country 1 but bad for Country 2, that is,  $\hat{s}_1 < \mu < \hat{s}_2$ .<sup>19</sup>

The goal is to explore the consequences of the introduction of a system that allows the competition authorities to share the information reported by the multinational. In order to do so, we first analyse the benchmark where the competition authorities do not share information. Then we proceed to analyse the consequences that sharing information, i.e. receiving the same realisation of the signal, has on the behaviour of the firm, *keeping the competition authorities policies fixed*, that is, we consider the firm's reaction to  $(\hat{s}_1, \hat{s}_2)$ .

In practice, some mergers may raise competition issues at a national (or sub-national) level that can be solved by a local action (for instance, to impose a remedy that forces the firm to undertake a national divestiture if the merger is cleared somewhere else) without the need to reach an agreement with other competition authorities. In this paper we focus on the type of mergers that do need the agreement of all the jurisdictions in order for the merger to happen. An example of such a merger was the proposed merger between two U.S.-based companies, General Electric and Honeywell, with the E.U. prohibiting the merger,<sup>20</sup> and the U.S. Department of Justice approving it<sup>21</sup> (see Muris (2001) for more detail).

## 4.1 No information-sharing agreement

When the competition authorities do not share information about the firm, the problem of the multinational is separable. The multinational needs to convince each authority individually, regardless of the way by which authorities will reach an agreement ex-post. In other words, the multinational will choose a level of precision for each country so as to maximise the expected probability of having the merger cleared in each country. By Proposition 1, the firm will choose  $V_L$  in Country 1 and  $V_H$  in Country 2.

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<sup>19</sup>Note that if  $\mu < \hat{s}_1$  (or  $\hat{s}_2 < \mu$ ), the firm would choose  $V_H$  (or  $V_L$ ) in both countries and the information-sharing agreement would make no difference.

<sup>20</sup>*GE/Honeywell*, Case No COMP/M.2220, European Commission Decision available at [http://www.ec.europa.eu/competition/mergers/cases/decisions/m2220\\_en.pdf](http://www.ec.europa.eu/competition/mergers/cases/decisions/m2220_en.pdf)

<sup>21</sup>Subject to GE divesting Honeywell's helicopter engine business and licensing a new competitor to maintain and repair certain Honeywell engines.



## 4.2 Information-sharing agreement

If the authorities receive the same realisation of the signal sent by the multinational, the problem of the multinational is no longer separable. The multinational needs to choose a unique level of precision so to maximise the expected probability of having the merger cleared. This probability will depend on the process by which a final decision on the international merger is reached. We consider several possibilities in what follows.

### 4.2.1 Veto power

Consider first the case where the authorities only cooperate in the exchange of information process but not in their decision process, i.e. the authorities use their veto power. The only disagreement that can arise is that Country 1 wants to clear the merger, while Country 2 does not want this. Country 2 using its veto power translates into the multinational only merging if both competition authorities agree that the merger should be cleared (i.e. if the signal is above  $\widehat{s}_2$ ) and as a result the firm will choose  $V_H$  for both countries.

Therefore, Country 2 will be indifferent between signing or not signing the agreement while Country 1 will be strictly worse-off.

### 4.2.2 Cooperation in the decision-making

We turn now to the case where, due to the repeated interaction between authorities, there is a bargaining/persuasion process taking place between the authorities. We model this in a reduced form whereby, from the point of view of the firm, the outcome of this bargaining is a conflicting merger being cleared with some probability. In particular, we model this decision as taking place in two stages: first, the competition authorities decide unilaterally whether they should clear the merger and, then, if the decisions differ, the competition authorities will discuss their arguments with each other until they reach an agreement. Since Country 1 is more lenient towards the merger, the only disagreement that can arise, under the information-sharing regime, is that it wants to clear the merger, while Country 2 does not want this. We assume that if there is disagreement, the merger will be cleared with some particular probability in a way that would be specified below.

**Constant probability** We first analyse the case where the merger is being cleared with probability  $\alpha$  (even though Country 2 does not want this) and being blocked with probability  $1 - \alpha$ . Therefore,  $\alpha$  can be interpreted as a measure of the strength of the authority in Country 1 versus the one in Country 2. If the realisation of the signal lies between  $\hat{s}_1$  and  $\hat{s}_2$ , M will obtain the profits from the merger only with probability  $\alpha$  and if the realisation of the signal lies above  $\hat{s}_2$  with probability one. Therefore, M maximises the following expression:

$$\max_{V \in [V_L, V_H]} \alpha \int_{\hat{s}_1}^{\hat{s}_2} g(s) ds + \int_{\hat{s}_2}^{+\infty} g(s) ds \quad (3)$$

Note that the first part of his objective function is decreasing in  $V$ , while the second part is increasing in  $V$ . In other words, when the firm chooses a high variance, this decreases the mass of the probability distribution of the signal on the interval  $[\hat{s}_1, \hat{s}_2]$ , but at the same time increases the mass in the tails, so the chances that the realisation lies in the interval  $[\hat{s}_2, +\infty)$  increases. Therefore the firm faces a trade-off between choosing  $V$  so as to maximise the probability at the interval  $[\hat{s}_1, \hat{s}_2]$  or at the interval  $[\hat{s}_2, +\infty)$ .

Denote the middle point of the interval  $[\hat{s}_1, \hat{s}_2]$  by  $s_M = \frac{\hat{s}_1 + \hat{s}_2}{2}$ . The behaviour of the multinational for  $\mu \in [\hat{s}_1, \hat{s}_2]$  is summarised in the next Proposition.

**Proposition 5** *For  $\mu \in [\hat{s}_1, \hat{s}_2]$ , the choice of variance may be non-monotonic in  $\mu$ . In particular, for  $\mu \in [\hat{s}_1, s_M)$ , M chooses either the minimum or maximum variance. For  $\mu \in (s_M, \hat{s}_2]$ , M chooses some intermediate variance  $V(\mu, \alpha) = \frac{(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2}{2 \ln\left(\frac{(1-\alpha)(\hat{s}_2 - \mu)}{\alpha(\mu - \hat{s}_1)}\right)} - \eta^2$  (provided  $V(\mu, \alpha) \in (V_L, V_H)$ ).*

**Proof.** See Appendix. ■

The proof relies on the fact that the curvature of the firm's objective function as a function of the variance changes when we consider the different average mergers in the interval  $[\hat{s}_1, \hat{s}_2]$ . In particular, the objective function is locally convex in  $V$  for  $\mu \in (\hat{s}_1, s_M)$  and locally concave in  $V$  for  $\mu \in (s_M, \hat{s}_2)$ . The following two corollaries state the precise variance that the firm chooses for  $\alpha = 0.5$ .

**Corollary 6** *For  $\mu \in (\hat{s}_1, s_M)$  and  $\alpha = 0.5$ , the optimal variance is either  $V_L$  or  $V_H$ . A sufficient condition for the firm to choose  $V_H$  for all the  $\mu$  in the interval is:*

$$\hat{s}_2 - \hat{s}_1 < c(V_L, V_H)$$

where  $c(V_L, V_H)$  is an increasing function of  $V_H - V_L$  defined in (10). Otherwise, the firm will first choose  $V_H$  until  $\mu = \bar{\mu}$  defined in (11) and then switch to  $V_L$ . A sufficient condition for the firm to choose  $V_L$  is:

$$c(V_L, V_H) < \bar{\mu} - \hat{s}_1 < \frac{\hat{s}_2 - \hat{s}_1}{2}.$$

**Corollary 7** For  $\mu \in (s_M, \hat{s}_2)$  and  $\alpha = 0.5$ , the optimal variance is:

$$V(\mu, 0.5) = \max \left\{ \frac{(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2}{2 \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right)} - \eta^2, V_L \right\}$$

and  $\frac{dV(\mu, 0.5)}{d\mu} < 0$ .

The Proof of Corollaries 6 and 7 can be found in the Appendix.

Note that from Corollary 6, we can see that for a given pair of policy thresholds  $(\hat{s}_1, \hat{s}_2)$ , the higher the range of variances  $(V_H - V_L)$  among which the firm can choose, the more likely it is that we are in the case where the firm chooses  $V_H$  for the whole interval  $\mu \in (\hat{s}_1, s_M)$ . In the same way, for a given range of variances  $(V_H - V_L)$ , the larger the conflict between authorities  $(\hat{s}_2 - \hat{s}_1)$ , the more likely it is that we are in the regime where firm chooses first  $V_H$  and then  $V_L$ . Therefore, under the information-sharing regime and if the conflict between authorities is large, the choice of the variance for the "conflicting mergers" is non-monotonic as the average merger increases. Figure 3 depicts this result.

This non-monotonicity is the result of the trade-off between maximising the probability at the interval  $[\hat{s}_1, \hat{s}_2]$  or at the interval  $[\hat{s}_2, +\infty)$ . The intuition behind each choice of variance is the following.

For the average merger  $\mu$  that is between  $\hat{s}_1$  and  $s_M$ , the objective function is convex in  $V$ . Since the firm wants to maximise his objective function, this means that he should choose the variance that is in either one extreme of the interval or the other. First consider the average merger at the first end-point,  $\mu = \hat{s}_1$ , the probability that the realisation of the signal is above  $\hat{s}_1$  is  $\frac{1}{2}$  for all the values of  $V$ . However, the higher the variance, the higher the probability that the realisation of the signal is above  $\hat{s}_2$  where the multinational has the merger cleared with probability 1 rather than  $\frac{1}{2}$ . Therefore, he will choose  $V_H$ . The same intuition holds for the average mergers that are in between  $\hat{s}_1$  and  $\bar{\mu}$ , where  $\bar{\mu}$  is defined in equation (11). If the minimum variance is low enough, then it is optimal for the firm to switch to  $V_L$  after

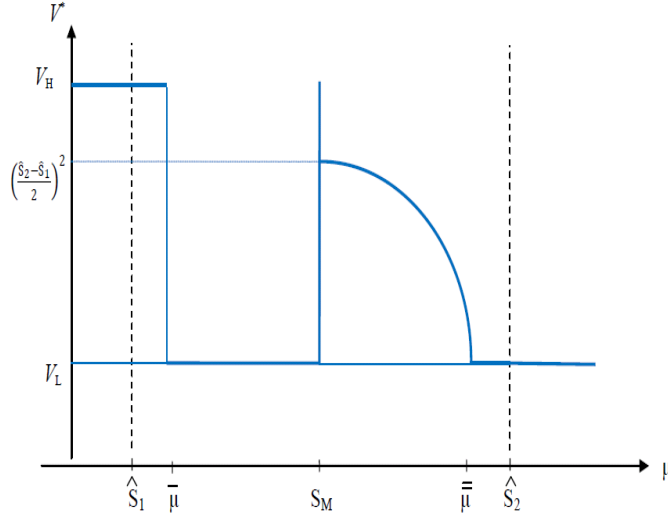


Figure 3: Optimal choice of variance with constant probability

$\bar{\mu}$  in order to maximise the chances of being cleared at least by Country 1.

The average merger that is placed just in the middle,  $\mu = s_M$ , is indifferent between any variance. The reason for this is that the probability of having the merger cleared is  $\frac{1}{2}$  for all possible values of  $V$ .

As  $\mu$  exceeds  $s_M$ , the objective function of the firm becomes concave in  $V$ . Because of the proximity of  $\mu$  with the interval  $[\hat{s}_2, +\infty)$ , it becomes safer to play a riskier strategy by increasing  $V$  (without reaching the maximum level). Finally, as  $\mu$  keeps increasing towards  $\hat{s}_2$ , the value of this optimal intermediate variance decreases. This is because the chances of having the realisation in the interval  $[\hat{s}_1, +\infty)$  are high anyway and, by reducing the variance, the probability of having the realisation in the tail  $(+\infty, \hat{s}_1)$  is decreased. The variance decreases until it hits the minimum level at  $\bar{\mu}$ , defined in equation (12).

Finally, the average merger at the other end-point,  $\mu = \hat{s}_2$ , has a probability  $\frac{1}{2}$  of having a realisation of the signal below  $\hat{s}_2$ ; however the higher the variance, the higher the probability that the realisation of the signal is below  $\hat{s}_1$  where the merger is blocked, rather than cleared with probability  $\frac{1}{2}$ . As a result, he will choose  $V_L$ .

The information-sharing agreement has modified the payoff structure, which is now no longer "bang-bang" as in Section 3. As a result, the firm makes a greater use of intermediate levels of variability in a non-monotonic way.

**Increasing probability** Now we consider the case where the bargaining power increases monotonically with the particular realisation of the signal,  $s$ . This way to model the outcome of the bargaining process intends to reflect that the closer  $s$  is to  $\hat{s}_2$ , the more bargaining power Country 1 has because Country 2 will be less reluctant to clear the merger as compared to a realisation of  $s$  very close to  $\hat{s}_1$ . For simplicity, we consider the case of a linear probability function  $p(s)$ . The expected probability of clearance then becomes:

$$\max_{V \in [V_L, V_H]} \int_{\hat{s}_1}^{\hat{s}_2} \underbrace{\left( \frac{s - \hat{s}_1}{\hat{s}_2 - \hat{s}_1} \right)}_{p(s)} g(s) ds + \int_{\hat{s}_2}^{+\infty} g(s) ds$$

**Proposition 8** *The optimal action taken by  $M$ , under no commitment is:*

$$V^*(\mu, \hat{s}_M) = \begin{cases} V_H & \text{if } \mu < s_M \\ V_L & \text{if } \mu > s_M \end{cases}$$

*If  $\mu$  is exactly  $s_M$ , then  $M$  is indifferent between any variance in  $V^*(\mu, s_M) \in [V_L, V_H]$ .*

**Proof.** See Appendix. ■

From the point of view of the firm, is as if there was a unique threshold but there is uncertainty about where exactly this lies. Because of the linearity, the firm takes the expectation and behaves as in the national merger case with this unique threshold. Therefore, the firm will use high variance whenever he thinks the merger is not going to pass through ( $\mu < s_M$ ) and low variance otherwise. See Figure 4.

### 4.2.3 Cooperation in the policy-making

So far, we have assumed that each authority was fixing its policy threshold unilaterally. In this section, we consider the situation where the authorities fix a unique threshold,  $\hat{s}^J$ , for both countries such that their joint ex-post welfare is maximised:

$$W(s, \hat{s}^J) \big|_{s=s(\hat{s}^c)} = \gamma \left[ a_1 + b_1 \left( \frac{\hat{s}\eta^2 + \mu V^*(\mu, \hat{s})}{\eta^2 + V^*(\mu, \hat{s})} \right) \right] + (1 - \gamma) \left[ a_2 + b_2 \left( \frac{\hat{s}\eta^2 + \mu V^*(\mu, \hat{s})}{\eta^2 + V^*(\mu, \hat{s})} \right) \right]$$

where  $\gamma$  and  $1 - \gamma$  are the weights given to the welfare of Country 1 and 2, respectively. It is easy to check that, given the firm's behaviour in Lemma 3, the optimal threshold

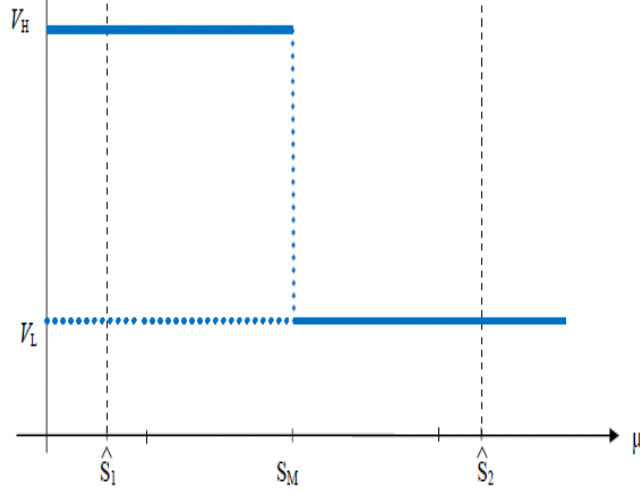


Figure 4: Optimal choice of variance with increasing probability

is:

$$\hat{s}^J = \begin{cases} -\mu \frac{V_H}{\eta^2} - \frac{\gamma a_1 + (1-\gamma)a_2}{\gamma b_1 + (1-\gamma)b_2} \frac{\eta^2 + V_H}{\eta^2} & \text{if } \mu < -\frac{\gamma a_1 + (1-\gamma)a_2}{\gamma b_1 + (1-\gamma)b_2} \\ -\mu \frac{V_L}{\eta^2} - \frac{\gamma a_1 + (1-\gamma)a_2}{\gamma b_1 + (1-\gamma)b_2} \frac{\eta^2 + V_L}{\eta^2} & \text{if } \mu \geq -\frac{\gamma a_1 + (1-\gamma)a_2}{\gamma b_1 + (1-\gamma)b_2} \end{cases}$$

Note that  $\hat{s}^J$  will coincide with  $s_M$  in the increasing probability case if  $\gamma = \frac{b_2}{b_2 + b_1}$ .

## 5 Extension: the case of private information<sup>22</sup>

In this Section, we explore how robust our results are to the assumption that the firm is not more informed than the competition authority about the merger's welfare effects.

If we assume that  $\theta$  is private information of the firm, then, qualitatively, the firm's behaviour does not change. The optimal variance chosen by the firm will depend on the merger's type ( $\theta$ ) in the same way that before it was depending on the average merger ( $\mu$ ). In particular, in the case of the national merger, a firm with a bad merger will choose the risky strategy of high variance while a firm with a good merger will choose low variance to increase the likelihood of obtaining a high realisation of the signal. In the same way, in the case of the multinational merger, the information-sharing agreement will have no impact on the types of merger that are either good or bad for both countries at the same time. However, for those firms whose merger is good for one country but bad for the other, the choice of the variance

<sup>22</sup>The proofs for this section can be obtained from the author upon request.

will be non-monotonic in their type in the same way that we found in Proposition 5.

However, with this new assumption, there is a significant change in the authority's behaviour. First, note that the monotone likelihood ratio property (MLRP) does not generally hold due to the way in which the firm optimally responds to the threshold rule. In particular, a very high realisation of  $s$  is more likely to come from a merger below the threshold due to the fact that the distribution of the signal sent by such a merger has fatter tails. If the MLRP does not hold, then the cut-off rule may not be optimal. However, if nonetheless we restrict the authority to use a one-cut-off rule, then we can show that the equilibrium threshold, when the average merger is welfare detrimental, is stricter than the full information threshold and, contrary to what we found in Section 3, it does not change with the authority's ability to commit.

The intuition for this result is as follows. There are two types of effects following an increase in  $\hat{s}^*$ : the direct and the strategic effect. The direct effect is the result from the trade-off between the benefit of decreasing the clearance probability of a "bad" merger against the cost of decreasing the clearance probability of a "good" merger. The direct benefit can be interpreted as a type II error of clearing a merger that should be blocked. By increasing  $\hat{s}^*$  we make this error less likely. Similarly, the direct cost can be interpreted as a type I error of blocking a merger when in fact it should be allowed and by increasing  $\hat{s}^*$  we make this error more likely.

The strategic (or indirect) effect is the result of the change in the firm's strategy resulting from moving merger types from above to below the threshold (i.e. when the firm switch from  $V_L$  to  $V_H$ , a good type  $\theta$  has less chances of being cleared). Given that the authority values precision, as this allows her to take more informed decisions, the fact that in this new framework she chooses the same threshold regardless of whether she can commit or not to a policy may be puzzling. It seems natural to think that the mechanism highlighted before would still apply (that is, if under no commitment, the authority sets a threshold  $\hat{s}^{**}$ , she would gain from committing to a lower threshold  $\hat{s}^*$  as this would induce the types in the interval  $[\hat{s}^*, \hat{s}^{**}]$  to reduce their equilibrium variance from  $V_H$  to  $V_L$ ). However, when the authority is considering a marginal decrease in the threshold, she only takes into account the strategic effects of the marginal type  $\hat{s}^{**}$  which consists in decreasing the variance from  $V_H$  to  $V_L$ . However, for  $\hat{s}^{**}$  (the type at the margin) the probability that the realisation of the signal is above  $\hat{s}^{**}$  is  $\frac{1}{2}$  in both cases. Therefore, since the marginal change in the firm's behaviour generated by the ability to commit cancels out, the criterion used to set such a threshold is the same under both regimes. In particular,

by lowering the threshold, the authority only trades off the increase in the type II error and the decrease in type I error (i.e. the direct effect).

## 6 Conclusions

The goal of this paper has been to study the impact of the introduction of an information-sharing agreement in the context of an international merger. The strategic variable available to the firm is the level of precision with which he reports the information to the authorities, and the authorities differ in terms of their degree of tolerance towards the merger.

We have showed how a firm dealing with a competition agency individually tries to introduce as much noise as possible in the information transmitted if he knows that the merger is going to raise competition issues on average. The competition authority responds to this strategy by setting stricter standards of proof. We have also shown that, if the authority has the ability to commit and there is substantial uncertainty about the merger's undesirability, the authority will choose a more lenient cut-off in order to obtain better information.

With respect to the consequences of information-sharing on the behaviour of the firm, we find that the agreement has no impact when the average merger is welfare enhancing for both countries at the same time. This would explain why in some cases, where the firm is sure that a merger does not raise competition issues, he voluntarily grants a confidentiality waiver to the authorities. The merger that is on average welfare detrimental for both countries does not change his behaviour either.

However, the agreement does have an impact on the behaviour of the firm whose average merger is welfare enhancing for one country but welfare detrimental for the other. The impact will depend on how authorities reach an agreement in case of disagreement.

If there is no further cooperation and authorities exert their veto power, then sharing information is a bad idea because the firm will send very imprecise information to both authorities making the more lenient authority strictly worse off. If the authorities cooperate further and if the probability of reaching an agreement does not depend on the particular realisation of the report, then the choice of the variance for this firm is non-monotonic in his expected merger and more intermediate levels of precision are chosen in equilibrium (whenever the disagreement between authorities is large enough). Finally, if the probability depends linearly on the particular report



then the firm behaves as in the national merger case where the new cut-off rule is the expectation of the countries' rules.

Given their policies, do the agencies benefit from the agreement? The authorities value the level of precision because this allows them to make more accurate decisions. If there is further cooperation, under the agreement, the "conflicting mergers" make more use of intermediate variances. Therefore, only the less lenient country benefits from the agreement as those mergers that are bad according to her standard but good for the other country will increase on average the level of precision. The more lenient country will not benefit from the agreement because without the agreement she was receiving the information with the maximum level of precision. However, if the lenient countries are not always the same ones, information sharing and cooperation can potentially be beneficial.

A possible way to enrich the setup is to let each authority receive a different realisation from the same random signal<sup>23</sup>. In this context, sharing information will increase the quantity of the information on which to base their decisions. This would allow them to better infer the level of precision used by the firm, which will affect the behaviour of the welfare detrimental type of merger. In this situation, there is room for the more lenient authority to benefit from the agreement.

We compared the no information-sharing regime with the information-sharing regime, keeping the policies of the competition authorities fixed, as we were interested in exploring the changes in the firm's behaviour following the agreement. The other motivation for proceeding this way was that usually these agreements do not contemplate changes in policies. Therefore, one possible direction of future work would be to enrich the model by allowing the competition agencies to optimally adapt their policies to the new regime. This would allow us to assess the total impact of the information-sharing regime, which not only includes the change in the firm's behaviour but also the change in the authority's policy.

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<sup>23</sup>A possible way to interpret this distinction is to think about the realisation of the signal as being the informational content of the report sent to the authority. If the merger is taking place in two similar jurisdictions of the same country or in two countries with similar market conditions, laws, etc., we can think of the competition authorities receiving the same informational content of the report. However, if the countries or their jurisdictions are very different, then the competition authorities will receive different informational contents of the same report. Another possible interpretation for the difference in the signal's realisations could be that it is the information obtained from the particular questions (interviews, questionnaires, etc.) carried out by each authority.

## 7 Appendix

**Proof of Proposition 1.** The first derivative with respect  $V$  is:

$$\frac{\partial}{\partial V} = \int_{\hat{s}}^{+\infty} g(s, V) \frac{(s - \mu)^2 - (\eta^2 + V)}{2(\eta^2 + V)^2} ds$$

Using the following derivatives<sup>24</sup>:

$$\frac{\partial^2 g(s, V)}{\partial s^2} = g(s, V) \frac{(s - \mu)^2 - (\eta^2 + V)}{(\eta^2 + V)^2} \text{ and } \frac{\partial g(s, V)}{\partial s} = g(s, V) \left( \frac{-(s - \mu)}{\eta^2 + V} \right).$$

We can further simplify the derivative with respect to the variance:

$$\frac{\partial}{\partial V} = \frac{1}{2} \left[ g(s, V) \frac{-(s - \mu)}{\eta^2 + V} \right]_{\hat{s}}^{+\infty} = \frac{g(\hat{s}, V)}{2(\eta^2 + V)} [\hat{s} - \mu]$$

Note that when  $\hat{s} = \mu$  this derivative is zero, which implies that M is indifferent between any variance in the interval  $[V_L, V_H]$ . Similarly, when  $\hat{s} > \mu$  the derivative is positive, and therefore the objective function is increasing in the variance. As a result, M will choose the maximum available variance:  $V_H$ . Finally, when  $\hat{s} < \mu$  the derivative is negative and so M will choose  $V_L$ . ■

**Proof of Proposition 2.** If the constraint does not bind, the first order condition for a given variance  $V^*(\mu, \hat{s})$  is:

$$EW'(\hat{s}) = -\frac{\hat{s}\eta^2 + \mu V^*(\mu, \hat{s})}{\eta^2 + V^*(\mu, \hat{s})} g(\hat{s}, V^*(\mu, \hat{s})) = 0 \quad (4)$$

And the second order condition is:

$$\begin{aligned} EW''(\hat{s}) &= \frac{\hat{s}\eta^2 + \mu V^*(\mu, \hat{s})}{\eta^2 + V^*(\mu, \hat{s})} g(\hat{s}, V^*(\mu, \hat{s})) \frac{\hat{s} - \mu}{\eta^2 + V} \\ &\quad - \frac{\eta^2}{\eta^2 + V^*(\mu, \hat{s})} g(\hat{s}, V^*(\mu, \hat{s})) < 0 \end{aligned}$$

The second order condition is locally satisfied because the first term is zero when the first order condition is satisfied and the second term is always negative.

Solving condition (4), give us the optimal  $\hat{s}$ . Note that this expression is zero

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<sup>24</sup>Remember that  $g(s, V)$  is defined as:  $g(s, V) = \frac{\exp\left(\frac{-(s-\mu)^2}{2(\eta^2+V)}\right)}{\sqrt{2\pi(\eta^2+V)}}$ .

only if  $\widehat{s}\eta^2 + \mu V^*(\mu, \widehat{s}) = 0$ , hence:

$$\widehat{s}^* = \frac{-\mu V^*(\mu, \widehat{s})}{\eta^2} \quad (5)$$

R values the level of precision as this allows her to make less mistakes. When  $\mu > \widehat{s}$ , by Proposition 1, the firm chooses  $V_L$ . The signal has the maximum level of informativeness, therefore the constraint in (1) will not bind and the solution will be determined by (5). Conversely, when  $\mu < \widehat{s}$ , the firm chooses  $V_H$ . The unconstrained solution defined in (5) may be dominated by the constrained solution  $\mu = \widehat{s}$  (where the firm will choose  $V_L$  instead of  $V_H$ ) for some parameter range because of the increase in the informativeness of the signal. This is the case whenever the following condition holds:

$$\int_{\frac{-\mu V_H}{\eta^2}}^{+\infty} \frac{s\eta^2 + \mu V_H}{\eta^2 + V_H} g(s, V_H) ds < \int_{\mu}^{+\infty} \frac{s\eta^2 + \mu V_L}{\eta^2 + V_L} g(s, V_L) ds$$

which can be rewritten as follows:

$$\begin{aligned} \int_{\frac{-\mu V_H}{\eta^2}}^{+\infty} -\eta^2 g'(s, V_H) ds + \mu \int_{\frac{-\mu V_H}{\eta^2}}^{+\infty} g(s, V_H) ds &< \int_{\mu}^{+\infty} -\eta^2 g'(s, V_L) ds + \mu \int_{\mu}^{+\infty} g(s, V_L) ds \\ \mu \left[ \frac{1}{2} - G\left(\frac{-\mu V_H}{\eta^2}, V_H\right) \right] &< \eta^2 \left[ \frac{1}{\sqrt{2\pi(\eta^2 + V_L)}} - g\left(\frac{-\mu V_H}{\eta^2}, V_H\right) \right] \end{aligned}$$

We want to determine whether there exists some value of  $\mu$  in the interval  $(-\infty, 0]$  for which this inequality is satisfied. Note that this inequality is trivially satisfied when  $\mu = 0$  as the left hand side is zero and  $g(0, V_L) - g(0, V_H) > 0$ . Conversely, when  $\mu \rightarrow -\infty$ , the left hand side tends to  $+\infty$  while the right hand side tends to  $\frac{\eta^2}{\sqrt{2\pi(\eta^2 + V_L)}}$ . Therefore the inequality is violated. This means that there exists at least one value of  $\mu$ ,  $\tilde{\mu}$ , such that the expected welfare are equal:

$$\int_{\frac{-\tilde{\mu} V_H}{\eta^2}}^{+\infty} \frac{s\eta^2 + \tilde{\mu} V_H}{\eta^2 + V_H} g(s, V_H) ds = \int_{\tilde{\mu}}^{+\infty} \frac{s\eta^2 + \tilde{\mu} V_L}{\eta^2 + V_L} g(s, V_L) ds \quad (6)$$

In order to show that this value is unique, we need to show that the slope of the

difference of expected welfare is strictly decreasing:

$$\begin{aligned}
& \frac{\partial \left( \mu \left[ \frac{1}{2} - G \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \right] - \eta^2 \left[ \frac{1}{\sqrt{2\pi(\eta^2 + V_L)}} - g \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \right] \right)}{\partial \mu} \\
&= \frac{1}{2} - G \left( \frac{-\mu V_H}{\eta^2}, V_H \right) - \mu \frac{\partial G \left( \frac{-\mu V_H}{\eta^2}, V_H \right)}{\partial \mu} + \eta^2 \frac{\partial g \left( \frac{-\mu V_H}{\eta^2}, V_H \right)}{\partial \mu} \\
&= \frac{1}{2} - G \left( \frac{-\mu V_H}{\eta^2}, V_H \right) - \mu g \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \left[ \frac{-V_H}{\eta^2} - 1 \right] + \\
&\quad \eta^2 g \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \left[ \frac{-V_H}{\eta^2} \frac{\left( -\left( \frac{-\mu V_H}{\eta^2} - \mu \right) \right)}{\eta^2 + V_H} + \frac{\left( \frac{-\mu V_H}{\eta^2} - \mu \right)}{\eta^2 + V_H} \right] \\
&= \frac{1}{2} - G \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \leq 0
\end{aligned}$$

This expression is either zero (for  $\mu = 0$ ) or negative for  $\mu < 0$ , therefore there is a unique  $\tilde{\mu}$  for which condition (6) holds.

The comparative statics with respect to  $V_L$ :

$$\frac{d\tilde{\mu}}{dV_L} = \frac{\eta^2}{\left[ G \left( \frac{-\tilde{\mu} V_H}{\eta^2}, V_H \right) - \frac{1}{2} \right] 2(\eta^2 + V_L) \sqrt{2\pi(\eta^2 + V_L)}} \geq 0$$

and with respect to  $V_H$ :

$$\frac{d\tilde{\mu}}{dV_H} = \frac{-\frac{1}{2} g \left( \frac{-\tilde{\mu} V_H}{\eta^2}, V_H \right) \frac{\tilde{\mu}}{\eta^2}}{G \left( \frac{-\tilde{\mu} V_H}{\eta^2}, V_H \right) - \frac{1}{2}} \leq 0$$

■

**Proof of Proposition 5.** The first derivative with respect to the variance is:

$$\begin{aligned}
\frac{\partial}{\partial V} &= \frac{\alpha}{2} \int_{\hat{s}_1}^{\hat{s}_2} \frac{\partial^2 g(s, V)}{\partial s^2} ds + \frac{1}{2} \int_{\hat{s}_2}^{+\infty} \frac{\partial^2 g(s, V)}{\partial s^2} ds \\
&= \frac{\alpha}{2} \left[ \frac{\partial g(s, V)}{\partial s} \right]_{\hat{s}_1}^{\hat{s}_2} + \frac{1}{2} \left[ \frac{\partial g(s, V)}{\partial s} \right]_{\hat{s}_2}^{+\infty} \\
&= \frac{1}{2(\eta^2 + V)} [\alpha g(\hat{s}_1, V) (\hat{s}_1 - \mu) + (1 - \alpha) g(\hat{s}_2, V) (\hat{s}_2 - \mu)]
\end{aligned}$$

note that  $(\hat{s}_1 - \mu)$  is negative while  $(\hat{s}_2 - \mu)$  is positive.

From the first order condition we can single out the variance at which the slope

of the objective function is zero:

$$V = \frac{(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2}{2 \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right)} - \eta^2$$

The second order condition (evaluated at the first order condition) is:

$$\begin{aligned} \frac{\partial^2}{\partial V^2} &= \alpha \frac{\partial g(\hat{s}_1, V)}{\partial V} (\hat{s}_1 - \mu) + (1 - \alpha) \frac{\partial g(\hat{s}_2, V)}{\partial V} (\hat{s}_2 - \mu) \\ &= \alpha g(\hat{s}_1, V) \left( \frac{(\hat{s}_1 - \mu)^2 - (\eta^2 + V)}{2(\eta^2 + V)^2} \right) (\hat{s}_1 - \mu) \\ &\quad + (1 - \alpha) g(\hat{s}_2, V) \left( \frac{(\hat{s}_2 - \mu)^2 - (\eta^2 + V)}{2(\eta^2 + V)^2} \right) (\hat{s}_2 - \mu) \\ &= \frac{1}{2(\eta^2 + V)^2} [\alpha g(\hat{s}_1, V) (\hat{s}_1 - \mu)^3 + (1 - \alpha) g(\hat{s}_2, V) (\hat{s}_2 - \mu)^3] \\ &\quad - \frac{1}{2(\eta^2 + V)} [\alpha g(\hat{s}_1, V) (\hat{s}_1 - \mu) + (1 - \alpha) g(\hat{s}_2, V) (\hat{s}_2 - \mu)] \\ &= \frac{1}{(\eta^2 + V)^2} [\alpha g(\hat{s}_1, V) (\hat{s}_1 - \mu)^3 + (1 - \alpha) g(\hat{s}_2, V) (\hat{s}_2 - \mu)^3] \end{aligned}$$

where the last equality follows from using the first order condition. In order to determine whether the second order condition is locally positive or negative, we need to determine the sign of the expression in brackets, which is proportional to:

$$\begin{aligned} \frac{\partial^2}{\partial V^2} &\propto \alpha \exp \left( \frac{-(\hat{s}_1 - \mu)^2}{2(\eta^2 + V)} \right) (\hat{s}_1 - \mu)^3 + (1 - \alpha) \exp \left( \frac{-(\hat{s}_2 - \mu)^2}{2(\eta^2 + V)} \right) (\hat{s}_2 - \mu)^3 \\ &= -\alpha (\mu - \hat{s}_1)^3 \left( \frac{(1 - \alpha) (\hat{s}_2 - \mu)}{\alpha (\mu - \hat{s}_1)} \right)^{\frac{-(\mu - \hat{s}_1)^2}{(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2}} \\ &\quad + (1 - \alpha) (\hat{s}_2 - \mu)^3 \left( \frac{(1 - \alpha) (\hat{s}_2 - \mu)}{\alpha (\mu - \hat{s}_1)} \right)^{\frac{-(\hat{s}_2 - \mu)^2}{(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2}} \end{aligned}$$

where the last equality follows from plugging in the critical value of  $V$ . By comparing this expression to zero, we find that the ultimate sign of the second derivative will depend on how  $\left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right)^3$  compares to  $\left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right)$ .

The ratio  $\frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1}$  is always positive. When  $\mu$  equals  $s_M$ , the ratio  $\frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1}$  is one and thus the second derivative (as well as the first derivative) is zero. This means that the firm is indifferent between any variance. When  $\mu \in [\hat{s}_1, s_M)$ , the ratio is bigger than one which implies that the cubic function is bigger (i.e. the second derivative is positive). Conversely, when  $\mu \in (s_M, \hat{s}_2]$ , the ratio is smaller than one which implies

that the linear function is bigger (and thus, the second derivative is negative). ■

**Proof of Corollary 6.** First note that for  $\alpha = 0.5$ , the objective function can be rewritten as:

$$0.5 \left[ \int_{\hat{s}_1}^{+\infty} g(s) ds + \int_{\hat{s}_2}^{+\infty} g(s) ds \right]$$

For  $\mu \in (\hat{s}_1, s_M)$ , the optimal variance is either maximum or minimum variance. Whether is one or the other depends on how the ex-ante probability of clearance with  $V_H$  compares to the one with  $V_L$ . In particular, M will choose  $V_L$  whenever:

$$G(\hat{s}_1, V_H) + G(\hat{s}_2, V_H) > G(\hat{s}_1, V_L) + G(\hat{s}_2, V_L) \quad (7)$$

Before we proceed, we state the following useful fact:

If a function  $f(x)$  is symmetric around the point  $x = a$ , then for any  $b \geq 0$

$$\int_{a-b}^{+\infty} f(x) dx = \int_{-\infty}^{a+b} f(x) dx. \quad (8)$$

First, let us now use fact (8) to simplify condition (7). Note that  $\mu$  is the middle point between  $\hat{s}_2$  and  $2\mu - \hat{s}_2$ . Using the fact that the integrand is symmetric around  $\mu$ , by fact (8), the integral from  $-\infty$  to  $\hat{s}_2$  is equal to the integral from  $2\mu - \hat{s}_2$  to  $+\infty$ . We can rewrite (7) as:

$$G(\hat{s}_1, V_H) + 1 - G(2\mu - \hat{s}_2, V_H) > G(\hat{s}_1, V_L) + 1 - G(2\mu - \hat{s}_2, V_L)$$

$$G(\hat{s}_1, V_H) - G(2\mu - \hat{s}_2, V_H) > G(\hat{s}_1, V_L) - G(2\mu - \hat{s}_2, V_L) \quad (9)$$

note that  $2\mu - \hat{s}_2 \leq \hat{s}_1$  for all  $\mu$  in the interval  $\mu \in (\hat{s}_1, s_M)$ .

Because the distribution  $g(s, V_H)$  differs by a mean-preserving spread from the distribution  $g(s, V_L)$ , both distributions will intersect at two values of  $s$ , one would lie below the mean  $\mu$  and the other above. Let us define  $\tilde{s}$  as the smallest of these values:

$$\tilde{s} = \mu - \sqrt{\frac{(V_L + \eta^2)(V_H + \eta^2)}{V_H - V_L} \ln \left( \frac{V_H + \eta^2}{V_L + \eta^2} \right)} \quad (10)$$

Rewrite  $\tilde{s}$  as  $\tilde{s} = \mu - c(V_L, V_H)$ . Replacing  $V_H$  by  $V_L + D$ , it can be shown that sign of the derivative of  $c(V_L, V_H)$  with respect to  $D$  depends on  $D - (V_L + \eta^2) \ln \left( \frac{V_L + D + \eta^2}{V_L + \eta^2} \right)$  which is always positive.

Note that a necessary (but not sufficient) condition for  $V_H$  to be chosen by all

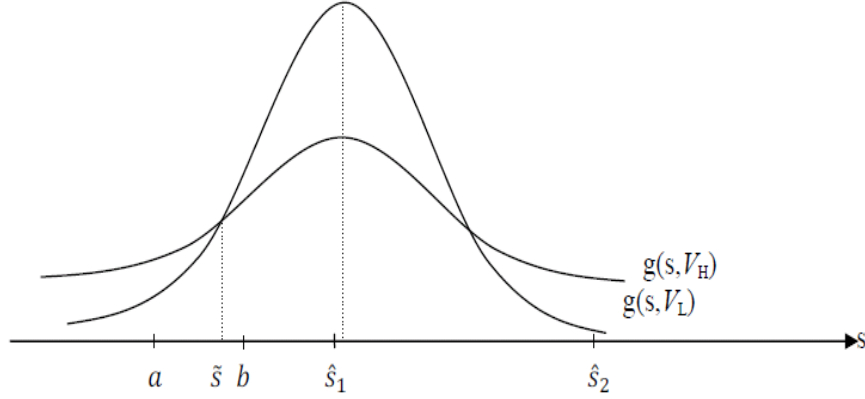


Figure 5:  $\mu = \hat{s}_1 + \Delta$

the expected means is  $\tilde{s} < 2\mu - \hat{s}_2$  for the smallest possible  $\mu$ , which in the limit is  $\hat{s}_1$ :

$$\hat{s}_2 - \hat{s}_1 < c(V_L, V_H)$$

To see why, consider Figure 5 which depicts a value of  $\mu$  at the beginning of the relevant interval:  $\mu = \hat{s}_1 + \Delta$ , where  $\Delta$  is a small positive value. Take the case where  $b = 2\mu - \hat{s}_2$ . Since  $\tilde{s} < 2\mu - \hat{s}_2 < \hat{s}_1$ , the area under  $g(s, V_H)$  is smaller than under  $g(s, V_L)$  for the relevant range of integration (from  $2\mu - \hat{s}_2$  to  $\hat{s}_1$ ) and by condition (9) the firm will choose  $V_H$ .

Conversely, M will choose  $V_L$  if  $\hat{s}_1 < \tilde{s}$ , or equivalently:

$$\hat{s}_1 + c(V_L, V_H) < \mu.$$

Since  $c(V_L, V_H) \geq 0$ , this condition cannot hold for the smallest possible  $\mu$  (i.e.  $\hat{s}_1$  in the limit). In fact, for  $\mu$  very close to  $\hat{s}_1$  M will always choose  $V_H$ . To see why take again Figure 5 where now  $a = 2\mu - \hat{s}_2$ . The area from  $a$  to  $\hat{s}_1$  under  $g(s, V_L)$  would still be larger and the firm would choose  $V_H$  because the area from  $-\infty$  to  $a$  under  $g(s, V_H)$  is larger than under  $g(s, V_L)$  and that the area from  $-\infty$  to  $\hat{s}_1 + \Delta$  should add up to  $\frac{1}{2}$  for both distributions.

Consider now a value of  $\mu$  at the end of the relevant interval:  $\mu = s_M - \Delta$  such as the one depicted in Figure 6. Since  $2\mu - \hat{s}_2 < \hat{s}_1 < \tilde{s}$ , the area under  $g(s, V_H)$  is larger than under  $g(s, V_L)$  for the relevant range of integration and by condition (9) the firm will choose  $V_L$ .

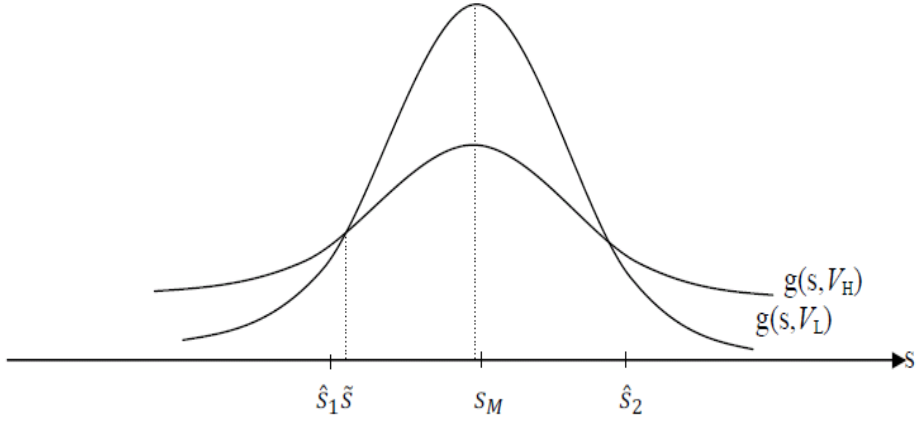


Figure 6:  $\mu = s_M - \Delta$

By continuity, there is a mean,  $\bar{\mu}$ , such that the firm is indifferent between  $V_H$  and  $V_L$ :

$$G(\hat{s}_1, V_H) - G(2\bar{\mu} - \hat{s}_2, V_H) = G(\hat{s}_1, V_L) - G(2\bar{\mu} - \hat{s}_2, V_L) \quad (11)$$

To sum up, in this equilibrium, the firm will choose:

$$V^*(\mu, \hat{s}_1, \hat{s}_2) = \begin{cases} V_H & \text{if } \mu \in (\hat{s}_1, \bar{\mu}) \\ V_L & \text{if } \mu \in (\bar{\mu}, s_M) \end{cases}$$

A sufficient condition of this equilibrium to hold is that this condition:  $\hat{s}_1 + c(V_L, V_H) < \mu$  should hold for  $\bar{\mu}$ . Since we do not have a closed form solution for this mean we will use  $s_M$  instead to get a sense of the lower bound:

$$c(V_L, V_H) < \bar{\mu} - \hat{s}_1 < \frac{\hat{s}_2 - \hat{s}_1}{2}$$

■

**Proof of Corollary 7.** The limit of the optimal variance found in Proposition 5 when  $\mu$  tends to  $s_M$  is:

$$\lim_{\mu \rightarrow s_M} V(\mu) = \frac{(\hat{s}_2 - \hat{s}_1)(\hat{s}_2 + \hat{s}_1 - 2\mu)}{2 \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right)} - \eta^2 = \frac{0}{0}$$



Using l'Hopital rule:

$$\begin{aligned} \lim_{\mu \rightarrow s_M} \frac{-\left(\hat{s}_2 - \hat{s}_1\right)}{\frac{\mu - \hat{s}_1}{\hat{s}_2 - \mu} \left( \frac{-(\mu - \hat{s}_1) - (\hat{s}_2 - \mu)}{(\mu - \hat{s}_1)^2} \right)} &= \frac{\hat{s}_1 - \hat{s}_2}{\frac{\hat{s}_1 - \hat{s}_2}{(\hat{s}_2 - \mu)(\mu - \hat{s}_1)}} = \frac{\hat{s}_1 - \hat{s}_2}{\frac{\hat{s}_1 - \hat{s}_2}{(\hat{s}_2 - \mu)(\mu - \hat{s}_1)}} \\ &= (\hat{s}_2 - \mu)(\mu - \hat{s}_1) = \left( \frac{\hat{s}_2 - \hat{s}_1}{2} \right)^2 \end{aligned}$$

Therefore, in order to focus on the interesting range of equilibrium, we need to assume that:

$$V_L \leq \left( \frac{\hat{s}_2 - \hat{s}_1}{2} \right)^2 \leq V_H$$

The value of the variance at the opposite extreme,  $\hat{s}_2$ , is:

$$\max \{V(\hat{s}_2), V_L\} = \left\{ \frac{-(\hat{s}_2 - \hat{s}_1)^2}{2 \ln(0)} - \eta^2, V_L \right\} = V_L$$

Therefore, there exists a  $\bar{\mu}$  such that:

$$V(\bar{\mu}) = \frac{(\hat{s}_2 - \bar{\mu})^2 - (\bar{\mu} - \hat{s}_1)^2}{2 \ln \left( \frac{\hat{s}_2 - \bar{\mu}}{\bar{\mu} - \hat{s}_1} \right)} - \eta^2 = V_L \quad (12)$$

We now show that  $V(\mu)$  is decreasing in  $\mu$ .

$$\begin{aligned} \frac{dV(\mu)}{d\mu} &= \frac{[-2(\hat{s}_2 - \mu) - 2(\mu - \hat{s}_1)] 2 \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right)}{\left( 2 \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right) \right)^2} \\ &\quad - \frac{2[(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2] \frac{\mu - \hat{s}_1}{\hat{s}_2 - \mu} \left( \frac{-(\mu - \hat{s}_1) - (\hat{s}_2 - \mu)}{(\mu - \hat{s}_1)^2} \right)}{\left( 2 \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right) \right)^2} \\ &= \frac{4(\hat{s}_1 - \hat{s}_2) \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right) - 2[(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2] \left( \frac{\hat{s}_1 - \hat{s}_2}{(\hat{s}_2 - \mu)(\mu - \hat{s}_1)} \right)}{\left( 2 \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right) \right)^2} \\ &= (\hat{s}_1 - \hat{s}_2) \frac{\left[ 2 \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right) - \left( \frac{[(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2]}{(\hat{s}_2 - \mu)(\mu - \hat{s}_1)} \right) \right]}{2 \left( \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right) \right)^2} \end{aligned}$$

The first term is always negative and the denominator is always positive, therefore,

the sign will depend on the term in square brackets. This term can be rewritten as:

$$2 \ln(x) - x + \frac{1}{x}$$

where  $x = \frac{\widehat{s}_2 - \mu}{\mu - \widehat{s}_1}$ . It is easy to see that this expression is positive when  $0 < x < 1$  and negative when  $x > 1$ . Since we focus on  $\mu \in (s_M, \widehat{s}_2]$

## References

- [1] CABRAL, L., (2003), "R&D competition when firms choose variance", *Journal of Economics and Management Strategy*, 12, 139-150.
- [2] DEWAN, T., and MYATT, D.P., (2008), "The Qualities of Leadership: Direction, Communication, and Obfuscation," *American Political Science Review*, Vol. 102, No.3, pp. 351-368.
- [3] GABA, A., and KALRA, A., (1999), "Risk Behavior in Response to Quotas and Contests." *Marketing Science*, Vol. 18, No. 3, Special Issue on Managerial Decision Making, pp. 417-434.
- [4] HOLMSTROM, B., (1982), "Managerial Incentive Schemes—A Dynamic Perspective." In *Essays in Economics and Management in Honour of Lars Wahlbeck*. Helsinki: Svenska Handelshogskolan.
- [5] ———, (1999), "Managerial Incentive Problems: A Dynamic Perspective." *Review of Economic Studies*, Vol. 66, No. 1, pp. 169–182.
- [6] JOHNSON, J.P., and MYATT, D.P., (2006) "On the Simple Economics of Advertising, Marketing, and Product Design." *American Economic Review*, Vol. 96, No. 3, pp. 756-784.
- [7] KOLSTAD, C.D., ULEN, T.S., and JOHNSON, G.V., (1990) "Ex Post Liability for Harm vs. Ex Ante Safety Regulation: Substitutes or Complements?" *American Economic Review*, Vol. 80, No. 4, pp. 888–901.
- [8] KRAKEL, M., NIEKEN, P., and PRZEMECK, J., (2008) "Risk Taking in Winner-Take-All Competition." *Bonn Econ Discussion Papers*, No 07/2008.
- [9] LEWIS, T.R., and SAPPINGTON, D.E.M., (1994) "Supplying Information to Facilitate Price Discrimination", *International Economic Review*, 35, 309-327.
- [10] MARSDEN, P., and WHELAN, P., (2005), "The Contribution Of Bilateral Trade Or Competition Agreements To Competition Law Enforcement Cooperation Between Canada And Costa Rica." Paper prepared for the "Competition Policy Foundations for Trade Reform, Regulatory Reform and Sustainable Development" project, funded by the European Commission under the Sixth

Framework Programme. Paper presented at the OECD Joint Group on Trade and Competition, *COM/DAF/TD(2005)51*.

- [11] MEYER, M., and VICKERS, J., (1997), "Performance Comparisons and Dynamic Incentives." *Journal of Political Economy*, Vol. 105, No. 3, pp. 547-581.
- [12] MURIS, T. J. (2001), "Merger Enforcement in a World of Multiple Arbiters". prepared remarks before the *Brookings Institution Roundtable on Trade & Investment*, Washington, D.C., available at <http://www.ftc.gov/speeches/muris/brookings/pdf>.
- [13] TSETLIN, I., GABA, A., and WINKLER, R.L., "Strategic Choice of Variability in Multi-round Contests and Contests with Handicaps." *Journal of Risk and Uncertainty*, Vol. 29, No. 2, 143-158.
- [14] VALENTINE, D. A. (2000), "Cross Border Canada/US Co-operation in Investigations and Enforcement Actions", remarks before the Canada/U.S. Law Institute, Case Western Reserve University School of Law, available at <http://www.ftc.gov/speeches/other/dvcrossborder.htm>.