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Credit Cycles in a Dynamic Duopoly

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PRELIMINARY. COMMENTS WELCOME.

Abstract

This article examines the impact of a credit crunch on market structure. I construct and simulate a dynamic model of a duopolistic industry in which firms' investment in capacity are constrained by credit availability. In such an industry the dynamic interaction of credit limits and the competitive responses of firms turn out to be a powerful transmission mechanism by which effects of shocks amplify and persist. I show that a small, temporary shock to one firms' capacity can lead to its market exit - even if it is equally productive as the remaining incumbent in the beginning. Consequently, if a recession is accompanied by a credit crunch, its cleansing effect might lead to monopolization of markets and welfare losses.

JEL Codes: L13, G32, C73

Keywords: Dynamic Oligopoly, Endogenous Financial Structure, Credit Rationing, Ericson-Pakes Framework, Applied Markov Equilibrium

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1. Introduction

In 2007 and 2008 there existed a widespread fear that several OECD countries were suffering from a credit crunch. Loan losses and lower asset prices ate significantly into the equity of the banking sector, which many believed would cause banks to ration credit. According to standard macroeconomic models less available credit leads to a reduction in investment resulting in lower production and less welfare in consecutive periods (e.g. Bernanke, Gertler, and Gilchrist 1999). However, these models do not take into account the change in market structure which might result from the financial frictions. For welfare analysis, market structure is important because it directly influences prices and the available choices for consumer. For example, if in a duopoly a smaller competitor is unable to replace its broken machinery because of a lack of credit, he might exit and leave the consumer with a monopoly supplier.

Including financial frictions in any oligopoly model is challenging because investment, financing decisions and market competition are inherently interdependent and dynamic: Past investment decisions determine today's market structure which in turn influences current investment decisions. But a firm can only invest if enough means are available. Investment funds can either come from current profits (determined by today's market structure), retained cash (determined by past financing decisions) and its debt capacity (determined by future profits). Additionally, a firm cannot retain more cash or pay back more debt than its current available funds. To complicate matters further, firms rationally anticipate future investment needs and shortages in funding.

This article contributes to the literature on financial constraints by proposing a computational feasible model which takes all these factors into account for the case of a dynamic duopoly. To integrate financing and investment decisions, I introduce firms with an endogenous capital structure and an optimizing bank into an Ericson-Pakes framework. To deal with the large state space, I use the algorithm and Applied Markov Equilibrium framework introduced by Fershtman and Pakes (2009).

Each firm is characterized by three state variables: capacity, debt level, and cash reserves. Firms accumulate capacity over time and compete repeatedly in the product market to earn profits. In every period, they aim to maximize the net present value of dividend payments. For this purpose, they optimally choose production, investment, the amount of cash to retain and the sum of debt repayments. Whatever is left of the profits after subtracting all incurred costs

is distributed as a dividend to the shareholders. Firms can apply for a loan if the current cash flow is insufficient to cover expenses. The loan is provided by a risk-neutral bank given that its expected return exceeds an exogenously set minimum threshold. This threshold parameterizes the amount of credit rationing prevalent in the market.

Using this model, I show that credit rationing serves as a propagation mechanism which amplifies small idiosyncratic shocks on capacity. This mechanism can lead to the monopolization of the market. If a firm looses production capacity through a depreciation shock, lower current profit is available for financing investment. With well-functioning credit markets, the firm can compensate this loss in cash flow by increasing the amount of credit financing. By contrast, if credit is rationed, firms might be unable to cover the cost of capacity addition. Without investment, the firm remains at the lower capacity level which is associated with less funds. If the firm is hit by another depreciation shock which further tightens the credit constraints, its ability to react by an increase in investment is reduced even more. Eventually, this process can lead to the exit of the firm - even though it has the same total factor productivity as the remaining incumbent. A recession which is accompanied by a credit crunch might be not only "cleansing", i.e. destroy unproductive firms (as in Caballero and Hammour (1994)), but also force viable competitors out of the market.

The monopolization of the market is made permanent by two other effects: First, with credit rationing entrants face financing constraints, too. Therefore new firms cannot enter because they do not obtain sufficient credit to finance initial outlays. Second, the competing firm can expand its own capacity and market share. Increased capacity translates into higher profits which eases credit rationing in the competitor's investment process. In the consecutive play the monopolist can then finance itself through cash retainment.

□ **Related literature:** This article contributes to the growing literature on modeling imperfect competition with heterogeneous firms. It is the first model to introduce financial frictions in an Ericson-Pakes framework.[1] I extend the dynamic duopoly model outlined in Besanko and Doraszelski (2004) and Besanko, Doraszelski, Lu, and Satterthwaite (2010) by firms with an endogenous capital structure and an optimizing bank. To model credit rationing, I employ a variant of the credit crunch model of Holmstrom and Tirole (1997). The larger state space resulting from the endogenous capital structure makes it is necessary to use the new stochastic

[1]For a survey on this literature please refer to Doraszelski and Pakes (2007).

algorithm of Fershtman and Pakes (2009) to numerically compute the equilibrium. With this algorithm, I can solve the game much faster than with the commonly used Pakes and McGuire (1994) or Pakes and McGuire (2001) algorithm. To the best of my knowledge, the only other applications of the Ericson-Pakes framework to finance is Kadyrzhanova (2009), which models the effect of corporate control imperfections on industry structure.

However, others have worked on financial frictions in dynamic firm models using the alternative framework of Hopenhayn (1992). In contrast to Ericson-Pakes type models, this framework considers only aggregate firm dynamics by assuming an infinite number of firms with an infinitely small market share. Therefore it is impossible to consider oligopoly behavior. In addition, in the model of Hopenhayn (1992), all dynamics are driven by permanent firm specific shocks, because temporary shocks average out. In contrast to that, temporary idiosyncratic shocks are amplified through the capital structure and competitive behavior in my model. The application of the modeling framework of Hopenhayn (1992) to finance are numerous: For instance Cooley and Quadrini (2001) investigate the effect of financial frictions on firm growth. Gomes (2001) explains the effect of financial frictions on investment. Hennessy and Whited (2005) consider a dynamic trade-off model of leverage, corporate saving and real investment to explain debt dynamics.[2]

My results are qualitatively similar to the effects described in Kiyotaki and Moore (1997), which characterize the emergence of credit cycles. Their main idea is that in downturns both earnings and the liquidation value of collateral are low because potential buyers are cash-strapped. Due to the lower collateral value credit constraint firms cannot borrow for investment which further reduces their future earnings and collateral value. In contrast, in my article, the firms cannot borrow further money because banks are cash strapped and the effect is transmitted via the expectations of the banking sector and oligopoly behavior.

The remainder of this article is organized as follows. I set up the model in section 2 . Sections 3 and 4 present the results and robustness checks. Section 5 concludes.

[2]Other articles modeling the intersection of investment and financial policy are e.g. Acharya, Almeida, and Campello (2007), Almeida and Campello (2007), Almeida, Campello, and Weisbach (2009) Moyen (2004), Titman and Tsyplakov (2007) and Adam, Dasgupta, and Titman (2007). Please refer to Hubbard (1998) and Stein (2003) for reviews on this literature.

2. A duopoly with endogenous capital structure

2.1. Static framework: The optimization of the firm

Assume that there are two firms which compete repeatedly in the same market and there exists one risk-neutral bank. Each firm $i \in \{1,2\}$, is fully characterized by its state variables: its capacity ($\bar{q}_i$), its debt level (d_i), and the cash reserves (c_i). The combined state variables of firm 1 and 2 is called the industry state $s = (\bar{q}_1, \bar{q}_2, d_1, d_2, c_1, c_2)$ which is common knowledge.

Each firm can take actions to change these state variables: A firm can choose to add one unit of capacity ($INV = 1$) by incurring the expansion costs η or remain inactive ($INV = 0$). It can pay back the amount Δ_{debt} of debt, or it can increase its cash reserves by Δ_{cash}. As all these actions might yield interdependent results firms decide among actions sets, $a = (INV, \Delta_{debt}, \Delta_{cash})$. Every action set is associated with an instant payoff - the dividend payments, $div(a,s)$, and with costs, $cost(s) = (\eta \cdot INV_i + \Delta_{debt,i} + \Delta_{cash,i})$. The dividend payments are the current profits ($\pi(\bar{q}_i, \bar{q}_j)$) reduced by the interest payments ($r \cdot d_i$) and the expenses associated with the action set:

$$div(a,s) = \max\left\{ \pi(\bar{q}_i, \bar{q}_j) - r \cdot d_i - \underbrace{(\eta \cdot INV_i + \Delta_{debt,i} + \Delta_{cash,i})}_{\text{expenses}}, 0 \right\}.$$

r is the interest rate paid by the firm.

To carry out an action set all expenses ($cost(a,s)$) associated with the action set must be covered by the funds available, i.e. the current profits (reduced by interest payments), the cash reserves and the available line of credit ($credit(a,s)$). The line of credit is determined by the bank and described in section 2.3. An action is in the set of affordable actions ($A(s)$), i.e. $a_i = (INV_i, \Delta_{debt,i}, \Delta_{cash,i}) \in A(s)$ iff

$$\underbrace{\pi(\bar{q}_i, \bar{q}_j) - r \cdot d_i + c_i + credit(a,s)}_{\text{available funds}} - cost(a,s) \geq 0. \tag{1}$$

Each firm i acts in every period in the interest of its shareholders and chooses the action set

which maximizes the expected discounted value of dividends, $V(s)$.[3] $V(s)$ is defined by

$$\begin{aligned} V(s) \equiv \quad & \max_{\{a_t, a_{t+1}, \ldots\}} E\left[\sum_{t=0}^{\infty} \beta^t div(a_t, s_t) | s\right] \\ & \text{s.t. } a_k \in A(s_k)\ \forall k \in \{t+n, n \in \mathbb{N}_0\} \end{aligned} \tag{2}$$

where $E(\cdot)$ is the expectation operator and $0<\beta<1$ is the discount factor. The value function in (2) is known to be the unique solution of the Bellman equation

$$V(s) = \max_{a \in A(s)} div(a, s) + \beta E_{a'_*, s'}[V(a'_*, s') | a, s] \tag{3}$$

where s' is next periods state and a'_* next periods optimal action. The law of motion for the state variables are defined in section 2.2.

Let $W(s, a)$ be the expected value of dividends if firm i chooses action a in state s

$$W(a, s) = div(a, s) + \beta E_{a'_*, s'}[V(a'_*, s') | a, s]$$

Then I can rewrite the value function in (3) as

$$V(s) = \max_{a \in A(s)} W(a, s). \tag{3'}$$

and solve the optimization problem of the firm by choosing the action tuple $a_* = (INV^*, \Delta^*_{debt}, \Delta^*_{cash})$, which delivers the highest continuation value $W(\cdot)$ among all feasible action tuples

$$a_* = \arg \max_{a \in A(s)} W(a, s).$$

The optimal action tuple a_* is a trade-off between costs today (by reducing the current dividend) and an improved state tomorrow.

2.2. Dynamic framework: State to state transition

The model contains three state variables: Capacity, debt and cash reserves. In the following, I outline the law of motion for each state variable in turn. At the end of this section I describe what happens if firms are bankrupt, exit or enter the market.

[3] For the sake of readability I surpress the subscript i in the following.

□ **Capacity:** A firm can choose to add one capacity unit ($INV = 1$) or remain inactive ($INV = 0$). With an exogenous probability δ, the current capacity is reduced by one unit because of depreciation. Therefore the next period's capacity $\bar{q}'$ of a firm with capacity $\bar{q}$ is determined by:

$$\bar{q}' = \begin{cases} \bar{q}+1 & \text{with probability } (1-\delta) \text{ if } INV = 1. \\ \bar{q} & \text{with probability } \delta \text{ if } INV = 1 \text{ and with probability } (1-\delta) \text{ if } INV = 0. \\ \bar{q}-1 & \text{with probability } \delta \text{ if } INV = 0. \end{cases}$$

If the firm decides to add capacity and no depreciation shock takes place, the capacity is increased by one. The capacity is decreased if there is no investment and a depreciation shocks hits the firm. It stays constant in all other cases.

Therefore investment is treated as lumpy. This is line with (s,S) modeling tradition of capacity adjustment (e.g. Caballero and Engel 1999, Caplin and Leahy 2010), prior work on Ericson-Pakes models (e.g. Besanko and Doraszelski 2004, Besanko, Doraszelski, Lu, and Satterthwaite 2010) and empirical evidence. For example, Doms and Dunne (1998) show that in U.S. census data a significant amount of investment adjustment takes place in a relatively short period of time, while most periods are characterized by only minor changes. For example 25% percent of total investment derives from firms that adjust their capital stock in a given year by more than 30% percent.

□ **Debt and Cash reserves** The costs, $cost(a,s) = (\eta \cdot INV_i + \Delta_{debt,i} + \Delta_{cash,i})$, associated with every action set $a_i = (INV_i, \Delta_{debt,i}, \Delta_{cash,i})$ are financed (in that order) by current profits, a reduction of the cash reserves and/or with new debt. The reduction in the cash reserve (c_{used}) is the difference of all costs and current cash flow, but not more than the current cash in stock.

$$c_{used} = \min[\max(\overbrace{\eta \cdot INV + \Delta_{debt,i} + \Delta_{cash,i}}^{\text{expenses } cost(a,s)} - \underbrace{\pi(\bar{q}_i, \bar{q}_j) - r \cdot d_i}_{\text{current cash flow}}, 0), c]$$

If current profits and cash reserves are not sufficient to cover all costs, the firm can finance itself with new debt, d_{new}. The amount of new debt is given by the the positive difference of expenses and current cash flow given that it is smaller than the maximum available line of

credit.

$$d_{new} = \min(\max(cost(a,s) - \overbrace{(\pi(\bar{q}_i, \bar{q}_j) - r \cdot d_i + c)}^{\text{funds without new debt}}, 0), credit(a,s)).$$

Although this hierarchy of finance looks strict, it is not: For example firms can at the same time use cash and increase their cash reserves by choosing a high Δ_{cash}, thus increasing the percentage of debt financing. The only thing that is not possible is, to rely on cash reserves and debt financing without using all current cash flow. The hierarchy of finance approach is line with the prominent pecking order theory of Myers (1984). Due to adverse selection, firms prefer internal to external finance.

If c_{used} and d_{new} are computed for an action set, the law of motions of the cash and debt state are simple: The debt level in the next period (d'), is the current debt (d) increased by the new loans (d_{new}) and reduced by the debt repayments Δ_{debt}

$$d' = d + d_{new} - \Delta_{debt}$$

Analogously, the cash reserve in the next period (c') is the current cash level (c) reduced by the cash used for the action and increased by the amount put in storage (Δ_{cash}).

$$c' = c - c_{used} + \Delta_{cash}$$

□ **Market exit and entry** There are two exemptions for the laws of motions outlined above: The exit and the entry of a firm. A firm exits the market, if it is either bankrupt or all its capacity is depreciated. Firm i is bankrupt if it is unable to pay its due interest payments out of current profit and retained cash, i.e.

$$\pi(\bar{q}_i, \bar{q}_j) + c - r \cdot d_i < 0.$$

The remainder of the cash reserve is given to the bank and the firm vanishes from the market. The bankruptcy process imposes an upper bound on the total amount of debt, precludes Ponzi games and limits the size of the state space. Furthermore a firm exits if all its capacity is depreciated. In that case, the cash reserve is first used to pay back debt and the rest is distributed to the shareholder. If a firm exits, a possibility arises for an entrant to become the

second player in the market. The new player has no capacity, no debt but the amount c^e of cash from equity investors.

2.3. The optimization of the financial intermediary

The bank collects savings from depositors and gives out corporate loans. It earns profits on the interest rate spread, i.e. by paying savers an interest rate of r_{Bank} and charging the firms a higher interest rate $r > r_{Bank}$. A firm pays back the amount Δ_{debt} of debt, if it chooses to do so.

The bank has rational expectations about the discounted sum of payments ($V_{Bank}(s,a)$) it will receive from a firm, if the firm takes action a in industry state s.

$$V_{Bank}(a,s) \equiv E\left[\sum_{t=0}^{\infty} \beta_{Bank}^{t}(r \cdot d_t + \Delta_{debt,t} - d_{new,t})|a,s)\right]$$

$\beta_{Bank} = \frac{1}{1+r_{Bank}}$ is the discount factor and r^{Bank} is the risk-free interest rate of the savers. $r \cdot d_t$ is the interest income and Δ_{debt} is the amount the firm repays of its loan. To account for the repayment to the savers, I subtract the net present value of all interest payments and the repayment of the principal at the time the loan is granted. Through construction, this is exactly the value of the newly obtained credit, d_{new}.

In every period the bank offers the firm i a line of credit ($credit_i(a,s)$) conditionally on the action taken by the firm and the industry state. This credit line is determined by the difference of discounted sum of payments the bank receives from the firm with the credit ($V_{Bank}(a,s)$) and the amount if the credit is not granted ($V_{Bank}(\tilde{a}_*,s)$) adjusted by the return R. $\tilde{a}_*$ is the optimal action the firm would take if no credit is given.

$$credit(a,s) = \frac{V_{Bank}(a,s) - V_{Bank}(\tilde{a}_*,s)}{R}, \tag{4}$$

Credit rationing is more severe if bank demand an higher return R on their loans given. The rational for this functional form stems from a simplified version of the model of Holmstrom and Tirole (1997). Assume that there exist three types of agents: a continuum of firms, uninformed investors and a continuum of banks. In each period, every firm has one project which differs in its return. The investor would like to invest in these projects, but is unable to prevent the firm

from diverting the funds to other, only privately profitable projects. The bank can perfectly rule out such bad projects by monitoring the firms' efforts, but cannot credibly commit to do so because monitoring entails non-verifiable private costs. Consequently, the uninformed investor is only willing to employ the bank as monitor, if it invests a fixed amount of its own capital in the firm, too. This makes it privately optimal for the bank to control the firms. Because the bank has only a finite amount of equity, it can only fund a limited number of firms.

To choose which project to fund, the intermediary sorts the projects from the highest return to the lowest. Starting with the most profitable project, the bank gives loans to projects with lower and lower returns until all its equity is pledged. The excess return on the loan (corrected for the costs), which just attracts funding is denoted R. This return therefore entails a scarcity rent which might be high, if a credit crunch has eaten up all of the bank's equity.

In my model, I set the return R exogenously and time invariant to parameterize the amount of credit rationing. This implies that throughout the economy, the return distribution of projects is stable and the banks do not raise equity. If a firm in the considered duopoly can deliver in expected value the return R, it gets the loan, otherwise the loan is given to some other firm in the economy.

2.4. Timing

At the beginning of each period, the bank decides how much credit it offers to each firm and action. Furthermore, if a firm is unable to pay its due interest payment out of its current cash flow, the firm declares bankruptcy and exits. Next, each firm is privately informed about its cost of capacity addition η. Conditional on these investment costs and the amount of available credit, each firm takes its optimal action. Then both firms compete in the product market. At the end of the period, capacity is subject to depreciation and all decisions are implemented.

2.5. Equilibrium concept and computation

I focus my attention on a symmetric Applied Markov Equilibrium (AME). An AME consists of (i) a subset of the set of possible states (the recurrent class), (ii) a vector of strategies which is optimal given the equilibrium continuation values from (iii), and (iii) a vector of continuation values for every state which is consistent with optimal actions defined in (ii). The concept of

AME as defined in Fershtman and Pakes (2009) is similar, but weaker concept than Markov Perfect Equilibrium because it is sufficient to calculate optimal policies on the recurrent class of states. States are member of the recurrent class if they are visited infinitely often in infinite time.

To solve for the AME, I use a variant of the reinforcement learning algorithm outlined in Fershtman and Pakes (2009). I describe the computation and outline the merits and problems of this algorithm in appendix D.

2.6. Parameterization

☐ **State space & choice set:** To enable computation, I discretize the state space: I distinguish 10 different capacity states, 40 different debt states and 20 different cash states per firm. The value of the first state in all three dimensions is set to zero, the second state signifies 5 units, the third state 10 units and so on. Consequently, I restrict maximum capacity to 45, debt to 195 and cash to 95 units. These bounds are arbitrary but high enough, such that they are never reached in equilibrium play. Furthermore, I could use smaller unit steps than five between the different states. This improves the results but also increases the time to compute the equilibrium tremendously. To ensure that firms stay within the state space, I have to restrict the potential choices of Δ_{debt} and Δ_{cash} to multiples of five with a finite upper bound.
☐ **Single period profit:** Firms compete in quantities, which are less or equal than the firm's capacity. Consequently, I use the profit function ($\pi(\bar{q_i}, \bar{q_j})$) for capacity constraint quantity competition with the same parameters as Besanko and Doraszelski (2004). The derivation is outlined in appendix A and illustrated in figure (1). This specification is inessential because the same results ensues with capacity constraint price competition.
☐ **Investment:** The investment costs η are random and private information to the firm as in Besanko, Doraszelski, Lu, and Satterthwaite (2010). They are determined by

$$\eta_{i,t} = \phi + \epsilon\psi_{i,t},$$

where

- ϕ are minimum construction costs which are the same for both firms and over time and
- $\epsilon\psi_{i,t}$ are project specific costs. $\psi_{i,t}$ is a random variable, drawn anew from a Beta(3,3)

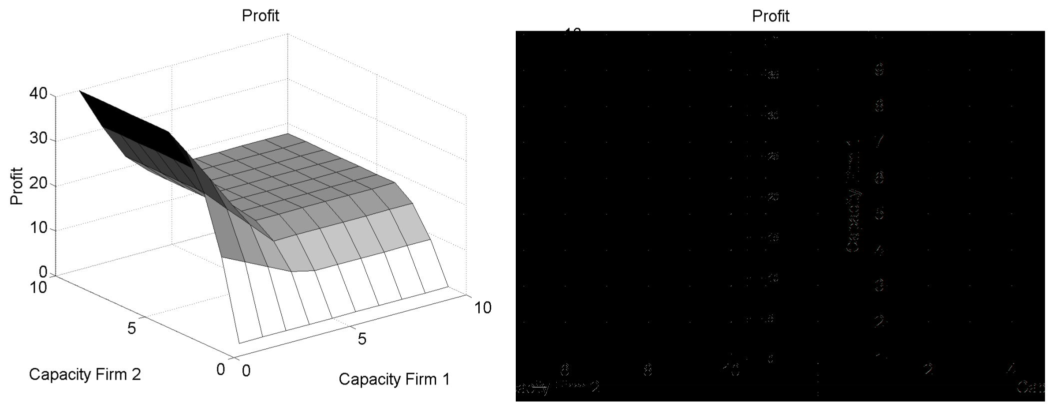


Figure 1: Profit of Firm 1

distribution with support [0,1] independently for each firm and each period. $\psi_{i,t}$ is private information for firm i and captures the idea that project opportunities are not the same for both firms and change over time.

Incorporating random investment costs and incomplete information is now common practice in the simulation of Ericson-Pakes models (e.g. Ryan 2009, Besanko, Doraszelski, Lu, and Satterthwaite 2010). It is realistic that firms do not know exactly the expansion costs of the rival. Furthermore it makes it possible to use the purification techniques of Doraszelski and Satterthwaite (2010) to ensure the computability of the equilibrium.

ϕ is set to 50 and ϵ is set to 5. Therefore, the support of the expansion cost is [50, 55]. These expansion costs are high enough that a firm has to rely to some degree on external financing. Following Gomes (2001), who matches the investment and capital data obtained from Compustat, I set the probability of depreciation to $\delta = 12\%$.

□ **Entry:** The amount of start-up financing is random because some entrepreneurs have a better connection to investors than others.[4] The value of the cash state of the entrant is determined by

$$c^e = \phi^e + \epsilon^e \psi^e_{i,t},$$

where ϕ^e is a minimum amount available to every entrant, ϵ^e is a scale variable and ψ^e_i is a random variable drawn from a Beta(3,3) distribution with support [0,1] in every period.

[4]As with random investment costs, the random start-up costs ensure computability.

ϕ^e is set to 50% of ϕ and ϵ^e is 50% of ϵ. ψ^e is a random variable, drawn anew from a Beta(3,3) distribution with support [0,1] independently for each firm and each period. Therefore the expected value of c^e is set to 50% of the average investment costs η. The support of start-up financing is [20,25]. The shareholder finance around a half capacity block up front and a credit between 25 and 30 is necessary to enter the market. Unfortunately there is not enough data to calibrate start-up financing adequately. The only study I know of is Baker and Wurgler (2002) which find that book debt to assets was around 66% pre-IPO and 43% post-IPO in a large sample of initial public offerings recorded in COMPUSTAT from 1970 to 1999. Consequently, a 50% debt ratio upon market entry might be a good choice.

□ **Financial parameters:** In the analysis, I set the yearly interest rate to $r = 6.5\%$ and the discount factor accordingly to $\beta = \frac{1}{1+0.065}$. This is about the real interest rate over the last century (Gomes 2001). In my simulation the interest rate for savers to $r_{Bank} = 4.5\%$ so the discount factor of the bank is $\beta_{Bank} = \frac{1}{1+0.045}$. Thus I obtain an interest rate spread of 2 percent, which is the average spread between six-month CDs (lending rate) and the prime rate (borrowing rate) for the period from 1968 with 1997 (Gomes 2001). I consider two degrees of credit rationing: $R = 5\%$ and $R = 120\%$. Without credit rationing the loan must deliver at least 5% return in net present value terms compared with the case that the loan is not given. The firm must be able to pay interest for 2 years and to return the principal to obtain such a loan. In the case of credit rationing, R is set to 120%. Such a return can not be met by interest payments on the given credit alone, but there must be an additional future value for the bank: example given, the loan could ensure that a debt laden firm survives and pays back more of its debt. Another possibility is, that the loan helps a new firm to enter which relies heavily on the bank in future play.

□ **Informational assumptions:** In the main setting, I allow for complete information, i.e. both firm know the complete industry state $s = (\bar{q}_1, \bar{q}_2, d_1, d_2, c_1, c_2)$. To check for robustness, I also consider imcomplete information in section 4, where each firm only knows its own debt and cash status, $s_i = (\bar{q}_i, \bar{q}_j, d_i, c_i)$.

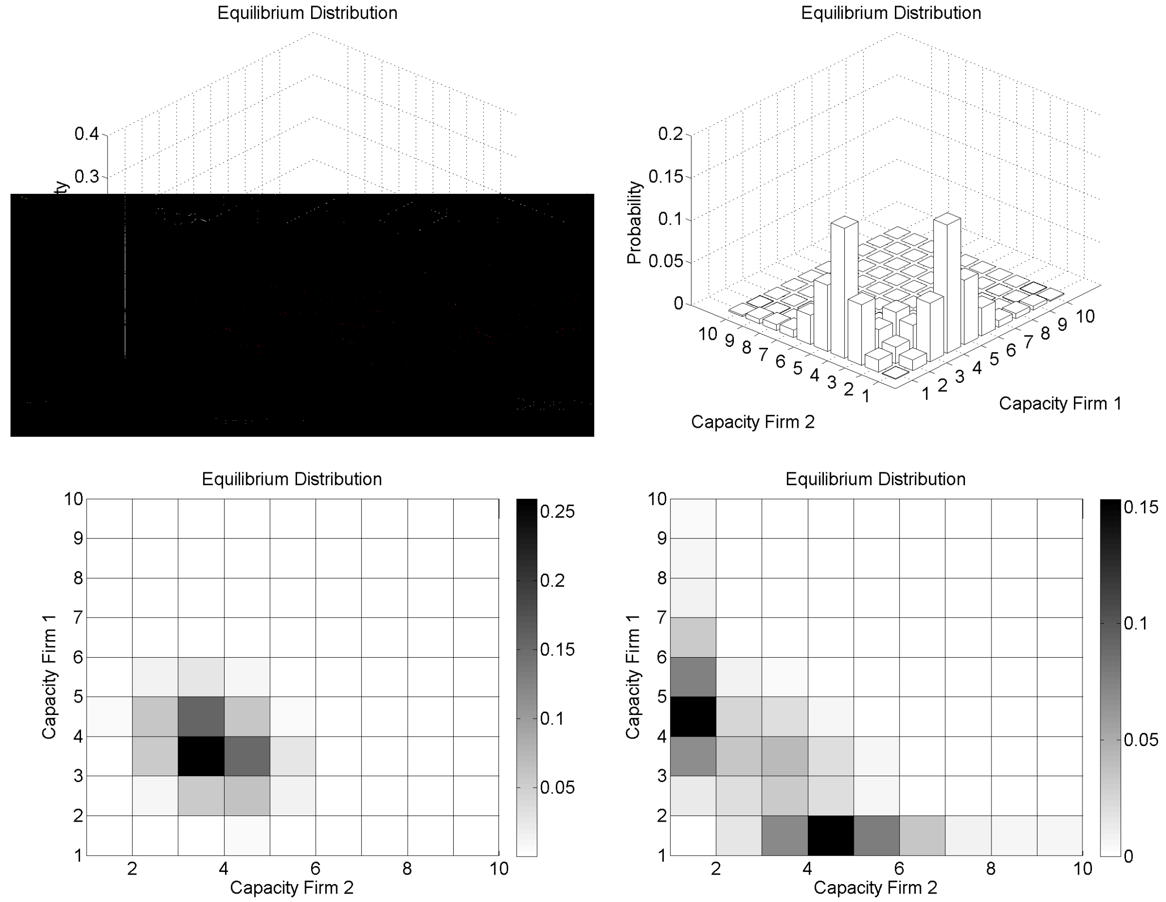
3. The e□ects of financial frictions on the equilibrium capacity distribution

In this section, I show that credit rationing leads to the monopolization of markets and a loss in welfare. The following section is concerned with the propagation mechanism which leads to the exit of one player, starting from a symmetric duopoly. Finally, I consider how credit rationing makes the monopolization permanent by preventing market entry.

3.1. Equilibrium distribution and welfare results

Table 1 and figure 2 shows the invariant equilibrium capacity distribution for the case without credit rationing (R=5%) on the left hand side and with credit rationing (R=120%) on the right hand side. Credit rationing causes the equilibrium distribution to become skewed: One firm exits the market and the (equally productive) competitor becomes the sole monopolist. Without financial frictions, both firms have an equal size with three capacity blocks. With credit rationing, one firm becomes large and the other firm exits the market. The large firm has four capacity blocks and the small firm has no capacity. There is some probability mass in between these two configurations, indicating that leadership changes from time to time.

These findings complement the results of Caballero and Hammour (1994) on the cleansing effect of recessions: They find that a fall in demand during a recession leads to job destruction what they conjecture is due to the exit of unproductive firms. A recession is therefore ”cleansing” for an economy. In my model, credit rationing (which often accompanies a fall in demand) leads to the exit of an a firm which is equally productive as the remaining incumbent. Consequently a recession with a credit crunch might only increase market power without the merits of increased average productivity. My effect bears some resemblance with the "scarring" effect of recessions outlined in Ouyang (2009). In this article, firms learning is reduced by a recession what kills potentially good firms already in their infancy. In contrast, in my article firms are only constrained by financial factors for an economy.



Note: The capacity states of the two firms are depicted on the x and y axis. In the upper panel the probability of a state is displayed on the z-axis. In the lower panel the probability is shown by different colors.

Figure 2: Equilibrium distribution for R=5% (left) and R=120% (right)

The monopolization of the market leads to a loss in welfare (table 2) which is mainly borne by the consumers and the banks. The firm surplus stays approximately the same. The welfare loss of the consumer originates from lower capacities and higher prices (table 3). In case of credit rationing, the average capacity is lower because the expected quantity of a monopolist is smaller than the quantity of two firms in duopoly. Lower capacity leads mechanically to higher prices and to the described welfare losses.

All other statistics in table 3 are in line with expectations: The average debt level is lower and the amount of retained cash is higher when credit rationing is present. If the financial market is not working properly firms get less and smaller loans and try to finance themselves

		j=1 $\bar{q}_j = 0$	j=2 $\bar{q}_j = 5$	j=3 $\bar{q}_j = 10$	j=4 $\bar{q}_j = 15$	j=5 $\bar{q}_j = 20$	j=6 $\bar{q}_j = 25$
i=1	$\bar{q}_i = 0$	0	0	0	0	0	0
i=2	$\bar{q}_i = 5$	0	1	5	6	1	0
i=3	$\bar{q}_i = 10$	0	5	25	15	2	0
i=4	$\bar{q}_i = 15$	0	6	15	6	0	0
i=5	$\bar{q} = 20$	0	1	2	0	0	0
i=6	$\bar{q}_i = 25$	0	0	0	0	0	0

(a) R=5%

		j=1 $\bar{q}_j = 0$	j=2 $\bar{q}_j = 5$	j=3 $\bar{q}_j = 10$	j=4 $\bar{q}_j = 15$	j=5 $\bar{q}_j = 20$	j=6 $\bar{q}_j = 25$
i=1	$\bar{q}_i = 0$	0	1	6	15	7	3
i=2	$\bar{q}_i = 5$	1	1	3	2	0	0
i=3	$\bar{q}_i = 10$	6	3	4	2	0	0
i=4	$\bar{q}_i = 15$	15	2	2	0	0	0
i=5	$\bar{q}_i = 20$	7	0	0	0	0	0
i=6	$\bar{q}_i = 25$	3	0	0	0	0	0

(b) R=120%

Table 1: Probability that a state is played in equilibrium (in %)

Surplus	Consumer	Producer	Bank	Total
Model with credit rationing	14.72	35.95	0.61	51.28
Model without credit rationing	24.19	35.03	2.38	61.60
Difference	-9.46	0.92	-1.77	-10.32

Note: This is the expected welfare over all states. For the calculation of these measures please refer to appendix B.

Table 2: Welfare effects of credit rationing

	Capacity	Price	Debt	Cash
Model with credit rationing	8.90	2.35	7.44	18.37
Model without credit rationing	11.25	1.83	26.53	13.20
Difference	-2.35	0.52	-19.09	5.17

Table 3: Summary statistics

through the retainment of cash.[5]

3.2. Credit rationing as propagation mechanism

Approximately, every twelve periods a depreciation shock hits each firm. This initial small shock sets a process in motion which results in a skewed equilibrium distribution given that

[5]Almeida, Campello, and Weisbach (2004) use a similar reasoning to justify cash flow sensitivities of cash as a sensible measure for financial constraints.

credit rationing is present.

The propagation mechanism works as follows: A shock reduces the capacity of firm i. Lower capacity translates in lower current profits. Because of credit rationing, this loss in profit cannot be compensated by taking out more loans. With lower cash flow and insufficient available credit, the probability that a firm can afford the cost of capacity expansion is reduced. A reduction in investment, together with an unaltered probability of depreciation results in less capacity, which again triggers less investment. The competitor benefits from this mechanism: The original shock takes capacity from the market and increases the price level. Therefore, he has more profit available for retainment and investment. With this additional profit, he can increase his investment to further tighten the credit constraints of the smaller firm.

To illustrate, assume that in a market with credit rationing a firm in capacity state (3,3) is hit by a depreciation shock. The capacity is reduced by one unit so the firm finds itself in the state (2,3). On average, it has only 35 for investment instead of 40 as shown in table 5. The investment probability is reduced from 66% (table 4a) to 27% (table 4b). After the first shock, the competitor has on average 48 available for investment what results in an investment probability of 27%. If the competitor invests (and the other firm fails to do so), the capacity state evolves to (2,4), putting the smaller firm at an additional disadvantage with only 30 units to invest. The competitor therefore tries to tip the smaller firm toward market exit.

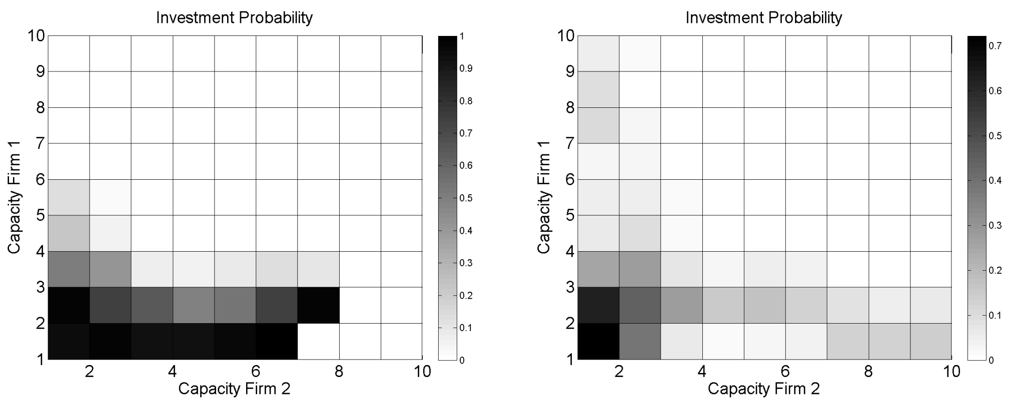


Figure 3: Investment probabilities for Firm 1, R=5% (left) and R=120% (right)

The effect is driven by the differing optimal probabilities that investment is carried out with and without credit rationing (table 4). Because the analysis is ceteris paribus, this difference can only originate from the amount of credit available to the firms: With $R = 5\%$, abundant

		j=1 $\bar{q_j}=0$	j=2 $\bar{q_j}=5$	j=3 $\bar{q_j}=10$	j=4 $\bar{q_j}=15$	j=5 $\bar{q_j}=20$	j=6 $\bar{q_j}=25$
i=1	$\bar{q_i}=0$	94	98	94	92	96	100
i=2	$\bar{q_i}=5$	98	74	66	49	54	73
i=3	$\bar{q_i}=10$	51	41	7	5	8	13
i=4	$\bar{q_i}=15$	23	6	1	0	0	0
i=5	$\bar{q_i}=20$	14	2	1	0	0	0
i=6	$\bar{q_i}=25$	0	0	1	0	0	0

(a) R=5%

		j=1 $\bar{q_j}=0$	j=2 $\bar{q_j}=5$	j=3 $\bar{q_j}=10$	j=4 $\bar{q_j}=15$	j=5 $\bar{q_j}=20$	j=6 $\bar{q_j}=25$
i=1	$\bar{q_i}=0$	72	39	7	2	3	4
i=2	$\bar{q_i}=5$	63	44	27	15	18	14
i=3	$\bar{q_i}=10$	25	27	8	3	5	4
i=4	$\bar{q_i}=15$	6	10	1	0	0	0
i=5	$\bar{q_i}=20$	5	5	1	0	0	0
i=6	$\bar{q_i}=25$	3	3	1	0	0	0

(b) R=120%

Note: This is the average investment probability in each capacity state. Investment probabilities also depend on the debt state and the amount of retained cash. States written in grey are played in equilibrium with a probability below 0.1 % and are likely calculated with error.

Table 4: Investment probability in %

		j=1 $\bar{q_j}=0$	j=2 $\bar{q_j}=5$	j=3 $\bar{q_j}=10$	j=4 $\bar{q_j}=15$	j=5 $\bar{q_j}=20$	j=6 $\bar{q_j}=25$
i=1	$\bar{q_i}=0$	55	40	**30**	28	28	29
i=2	$\bar{q_i}=5$	59	39	**35**	30	30	24
i=3	$\bar{q_i}=10$	49	**48**	**40**	33	31	29
i=4	$\bar{q_i}=15$	45	49	43	32	30	27
i=5	$\bar{q_i}=20$	44	52	45	43	0	0
i=6	$\bar{q_i}=25$	44	61	48	42	0	0

Note: This is the average amount available for investment in each capacity state. This amount is also dependent on the debt state and the amount of cash retained. States written in grey are played in equilibrium with a probability below 0.1 % and are likely calculated with error.

Table 5: Sum of current profits, retained cash and credit with R=120%

credit is extended in any state (table 6a) and the firm does not need to delay any investment. With credit rationing, only 6 units of credit are offered by the financial intermediary (table 6b). This lack of credit drives down the investment probability.

The described qualitative results are akin to Kiyotaki and Moore (1997), however, the mech-

		j=1	j=2	j=3	j=4	j=5	j=6
		$\bar{q}_j = 0$	$\bar{q}_j = 5$	$\bar{q}_j = 10$	$\bar{q}_j = 15$	$\bar{q}_j = 20$	$\bar{q}_j = 25$
i=1	$\bar{q}_i = 0$	899	436	338	236	229	308
i=2	$\bar{q}_i = 5$	338	228	198	76	66	37
i=3	$\bar{q}_i = 10$	47	44	14	4	6	4
i=4	$\bar{q}_i = 15$	44	8	0	0	0	0
i=5	$\bar{q}_i = 20$	27	1	1	0	0	0
i=6	$\bar{q}_i = 25$	1	0	0	0	0	0

(a) R=5%

		j=1	j=2	j=3	j=4	j=5	j=6
		$\bar{q}_j = 0$	$\bar{q}_j = 5$	$\bar{q}_j = 10$	$\bar{q}_j = 15$	$\bar{q}_j = 20$	$\bar{q}_j = 25$
i=1	$\bar{q}_i = 0$	27	12	2	0	0	1
i=2	$\bar{q}_i = 5$	20	11	6	3	3	2
i=3	$\bar{q}_i = 10$	3	3	1	0	1	0
i=4	$\bar{q}_i = 15$	1	1	0	0	0	0
i=5	$\bar{q}_i = 20$	1	0	0	0	0	0
i=6	$\bar{q}_i = 25$	1	0	0	0	0	0

(b) R=120%

Note: This is the average amount of credit offered in each capacity state. The amount of credit is dependent also on the debt state and the amount of cash retained. States written in grey are played in equilibrium with a probability below 0.1 % and are likely calculated with error.

Table 6: Credit

anism is different. In their model a negative productivity shock reduces the net worth of the credit constrained firm. This leads to a reduction in investment in the productive factor which is also a collateral for credit. The resulting shortfall in demand for the productive factor reduces its value as collateral. Consequently, the firm cannot get as much credit as before. This reduces the demand for the productive factor further, drives down asset prices again and the firm enters a reinforcing credit cycle.

In my model, firms hit by a depreciation shocks are unable to tap the credit market to finance investment. The bank does not offer enough credit because the expected net present value of the investment is not high enough to satisfy the return requirements. Without investment, firms capital stock depreciates further what results again in reduced credit and reduced current profits. In the end this mechanism can lead to the exit of one firm.

3.3. Credit rationing as entry barrier

If one firm exits, the possibility arises for an entrant to become the second firm in the market. The monopolization of the market is (relatively) stable because credit rationing also serves as an barrier to entry. Thus financial frictions lead to lower entry rates what results in a skewed capacity distribution.

In the basic configuration, investment costs are uniformly distributed between 50 and 55 and the amount of start-up financing provided by the equity markets is between 20 and 25. Therefore the firm has to take at least 25 as credit to enter the market.[6] If credit rationing is present the entrant gets only such a credit if the competitor firm is out of the market or small (one or two capacity blocks - table 6). Accordingly, the investment probabilities are only high in these states, but not if the competitor has more capacity (table 4). The investment probability is 72 % for firm 1, given no firm is in the market. With a competitor's capacity of one capacity block the investment probability decreases to 38 %. The investment probability becomes neglegible, if the capacity of the competitor is higher. On the equilibrium path the monopolist is out of the market or small only with a probability of one percent (table 1b). Credit rationing therefore serves as viable barrier to entry.

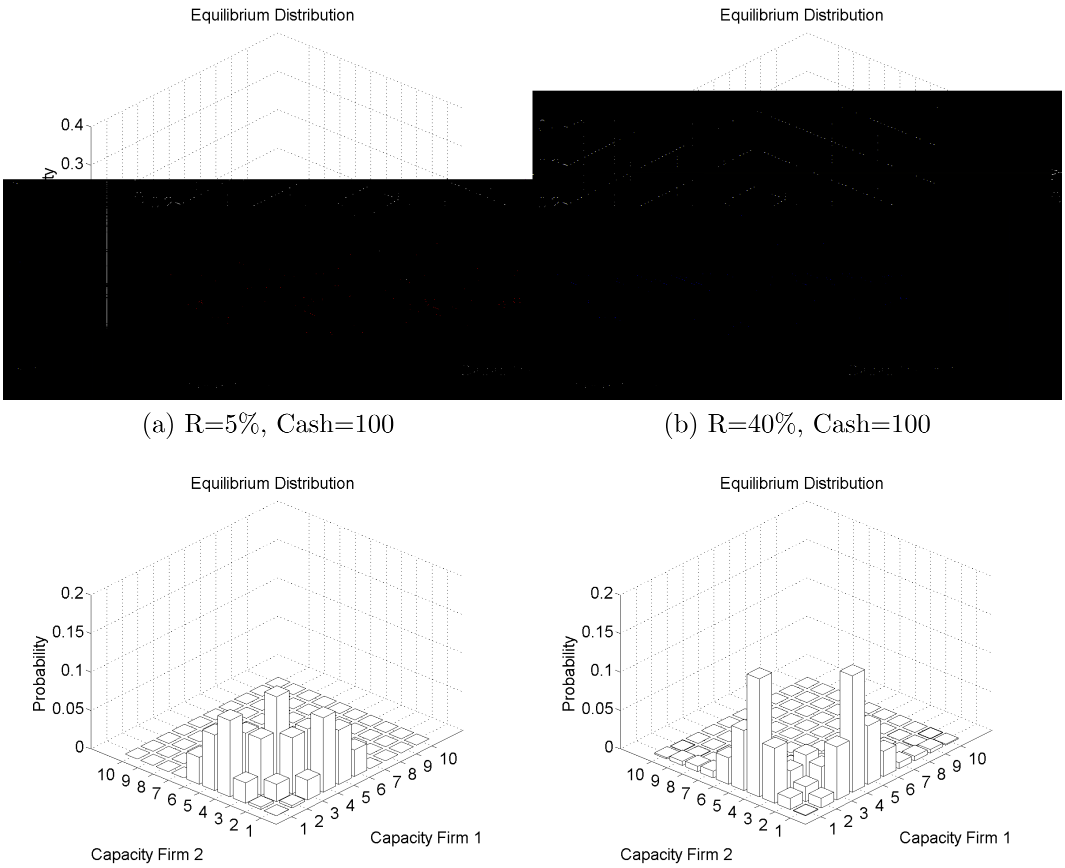
In contrast, without credit rationing: The probability to invest for an entrant is always above 90 % irrespective of the capacity of the competitor (table 4). Consequently, a firm with zero capacity always enters the market immediately.

4. Robustness checks

To show that the outlined results are stable despite the multiplicity of maintained assumptions, I vary three key parameters in my model: the severeness of credit rationing R, the maximum possible amount of retained cash and the amount of start-up financing c^e. Furthermore, I explore the implications of incomplete information on the equilibrium capacity distribution.

□ **Severeness of credit rationing:** In figure 4, I gradually increase the severeness of credit rationing. With an increase in the minimum return R, the probability of an asymmetric equilibrium capacity distribution rises. This is intuitive: The more banks tighten the credit constraint, the more adverse is the effect.

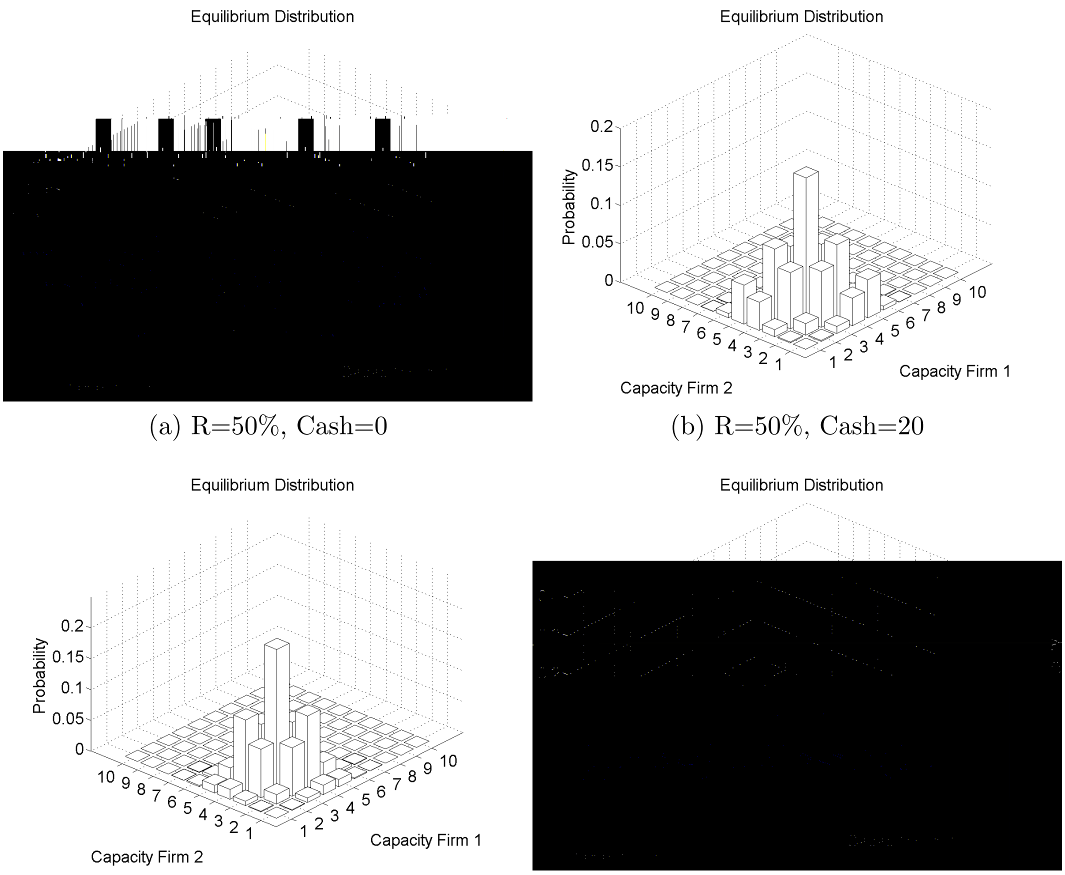
[6]Profits in the outside state are zero.



(a) R=5%, Cash=100 (b) R=40%, Cash=100

(c) R=80%, Cash=100 (d) R=120%, Cash=100

Figure 4: Equilibrium distribution with varying amount of credit rationing



(a) R=50%, Cash=0

(b) R=50%, Cash=20

(c) R=50%, Cash=80

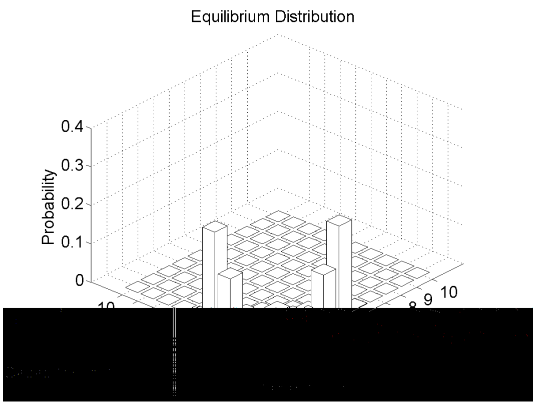
(d) R=50%, Cash=100

Figure 5: Equilibrium distribution with varying amount of retainable cash

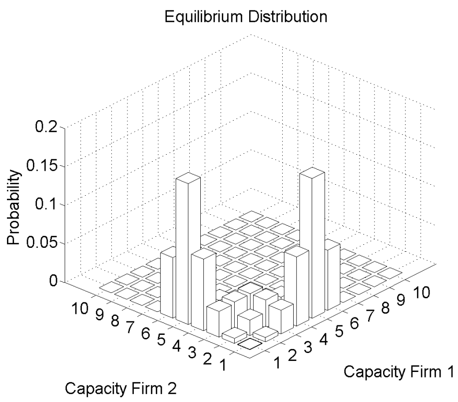
□ **Maximum amount of retainable cash:** In the preceding analysis, firms were able to accumulate a large amount of cash. The limit was set to 100, what is the equivalent of two capacity blocks or approximately 5 periods of profit. However, in reality shareholders might have the incentive to limit the amount of cash a company can hold to mitigate moral hazard problems: If a manager must regularly apply for funds, the capital market controls their proper use (Jensen 1986, Easterbrook 1984).

Figure 5 presents the equilibrium distribution with varying amount of maximum retainable cash. The effects of credit rationing become more severe if a company can retain less cash.

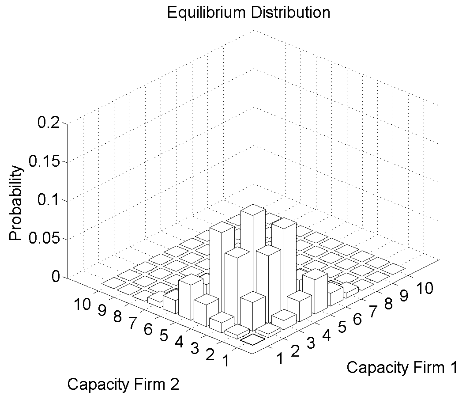
□ **Increase in start-up financing** Throughout the main part of the analysis, entrants only had a limited amount ϕ^e of start-up financing. This can be thought of as the amount a start-up can acquire on the equity market. This small scale is intuitive because the bank as monitor



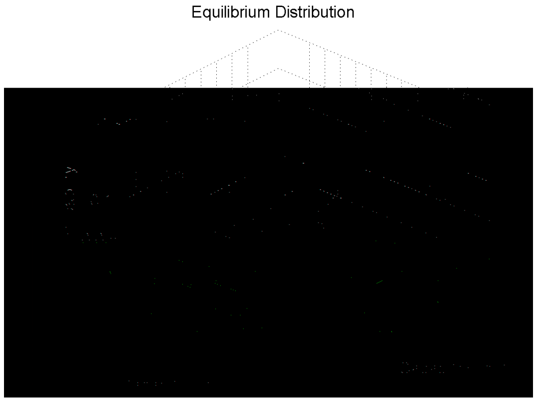
(a) R=120%, ϕ^e=20



(b) R=120%, ϕ^e=25



(c) R=120%, ϕ^e=35



(d) R=120%, ϕ^e=50

Figure 6: Equilibrium distribution with varying start-up financing

due to severe moral hazard. Consequently, naive investors are not willing to finance start-ups on a large scale in such a market.

In my analysis, the size of start-up financing is 50% of the investment an entrant needs to enter the market. If I increase this proportion, the effects of credit rationing is diminished as can be seen in figure 6.

□**Incomplete information:** In the whole analysis, I allow the firms to condition on the complete industry state $s = (\bar{q}_i, \bar{q}_j, d_i, d_j, c_i, c_j)$. The assumption that each firm knows the exact financial structure of its competitor might be a rather extreme. Fortunately, the Fershtman and Pakes (2009) algorithm allows to introduce incomplete information in the Ericson Pakes framework. If I let firms condition their strategy only on their own financial structure and the two capacity states, the results are qualitatively similar to the full information case (figure 7).

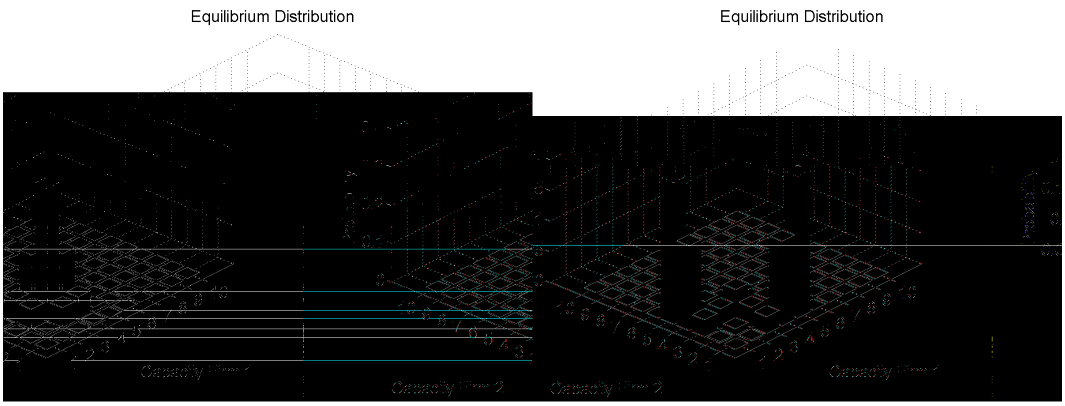


Figure 7: Equilibrium distribution with incomplete information for R=5% (left) and R=120% (right)

5. Conclusion

In this article, I identify the effects of credit rationing on the equilibrium market structure in duopoly. I employ the Applied Markov Equilibrium framework presented in Fershtman and Pakes (2009) to extend the model of Besanko and Doraszelski (2004) by an optimizing bank and firms which actively choose their capital structure.

In my model, firms can retain cash, borrow from banks or use current cash flow to finance themselves. Due to a shortage of capital, banks might be unable to fund every profitable project. Therefore, credit rationing might prevail. If then a small shock reduces the capacity of a firm, this firm might find itself unable to finance capacity expansion. If a firm then stays at a lower capacity level, it has also less funds to finance investment in future periods. Eventually, this lack of investment can lead to the exit of one firm and monopolization of the market. The monopolization is made permanent because new entrants also suffer from credit rationing and cannot enter to fill the void.

This article shows, that in equilibrium the exit of firms during recession might not be driven by insufficient productivity but by a lack of credit financing. Therefore policy maker should put emphasis on the functioning of the credit market during recession to prevent welfare losses through an increase in market power.

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Appendices

A. Single period profit function

The mode of product market competition is capacity constraint quantity competition as in Besanko and Doraszelski (2004).

The inverse demand function $P(q_1, q_2)$ with P as market price and q_i as quantity produced by firm i is given by

$$P(Q) = \frac{a}{b} - \frac{Q}{b}.$$

Suppose that firm i and firm j's capacities are given by $(\bar{q}_i, \bar{q}_j)$ and that they compete in the product market by setting quantities (q_i, q_j).

The profit-maximization problem for firm i with $i, j \in (1, 2), i \neq j$ is then given by

$$\max_{0<q_i<\bar{q}_i} P(q_i + q_j)q_i$$

This maximization problem for i and the symmetric problem for j leads to symmetric reaction functions which are known to have a unique Nash Equilibrium (Vives (2001)). The single period profit function of firm i in the Nash equilibrium of the capacity constrained quantity setting game is therefore

$$\pi_i(\bar{q}_i, \bar{q}_j) = P(q_i^* + q_j^*)q_i^*.$$

The demand parameters used in the simulation are $a = 40$ and $b = 10$ and thus the same as in Besanko and Doraszelski (2004). These parameters ensure that a company can have more capacity than the entire market demand.

		j=1	j=2	j=3	j=4	j=5	j=6
		$\bar{q}_j = 0$	$\bar{q}_j = 5$	$\bar{q}_j = 10$	$\bar{q}_j = 15$	$\bar{q}_j = 20$	$\bar{q}_j = 25$
i=1	$\bar{q}_i = 0$	0	0	0	0	0	0
i=2	$\bar{q}_i = 5$	18	15	13	10	9	9
i=3	$\bar{q}_i = 10$	30	25	20	15	15	15
i=4	$\bar{q}_i = 15$	38	30	23	18	18	18
i=5	$\bar{q}_i = 20$	40	31	23	18	18	18
i=6	$\bar{q}_i = 25$	40	31	23	18	18	18

Table 7: Profit

B. Welfare measures

To evaluate the implication of credit rationing on welfare, I calculate the expected consumer surplus and the expected producer surplus of the firms and of the bank.

Expected consumer surplus is calculated by integrating the demand function

$$CS = E\left[\int_{p_{max}}^{p_{Market}(s)} D(t)dt\right]$$

where $D(\cdot)$ is the demand function, p_{max} is the choke price and p_{Market} is the prevailing market price. The expectation is taken with respect to the probability of the state in equilibrium.

As marginal costs are normalized to zero, expected producer surplus for every state is calculated as sum of profits minus the financing costs:

$$PS = E\left[\pi(\bar{q}_1, \bar{q}_2) + \pi(\bar{q}_2, \bar{q}_1) - d_1 \cdot r - d_2 \cdot r\right].$$

Expected bank surplus is the interest rate differential multiplied with the sum of debt:

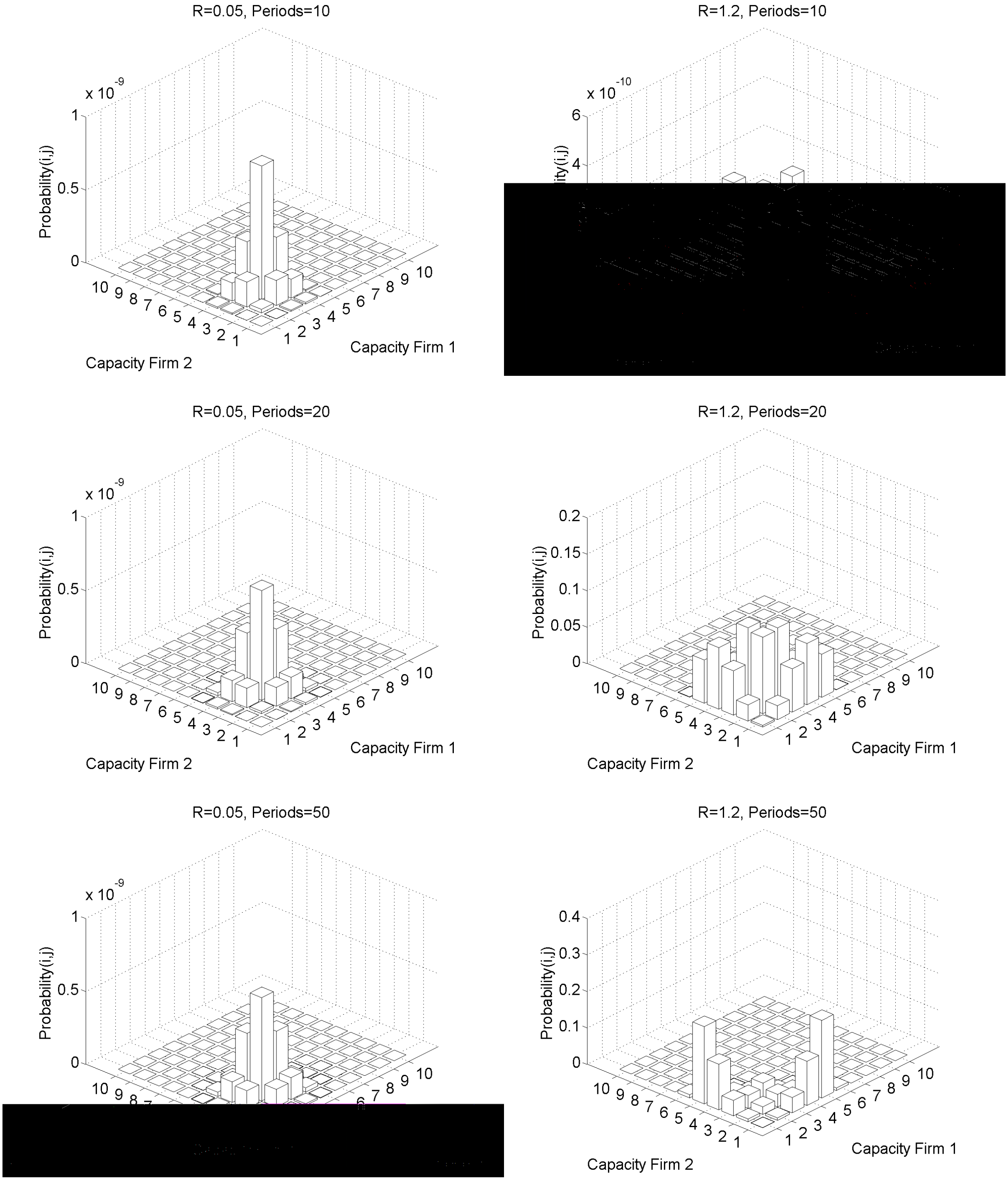
$$BS = E\left[(r - r_{Bank}) \cdot d_1 + d_2\right].$$

C. Transitory dynamics

The Figure 8 shows the distribution after 10, 20 and 50 periods starting from the initial value of zero capacities for both players. The distribution evolves without credit rationing directly towards symmetry and stays there forever. On the other, with credit rationing, first an symmetric configuration with equal capacity for both firms is reached and then the distribution becomes asymmetric. There is a large probability that one firm exits the market on the equilibrium path.

D. The reinforcement learning algorithm

In this section, I outline the reinforcement learning algorithm used in the paper. It is a variant of the algorithm described in Fershtman and Pakes (2009). Please refer to this article for an extensive description. Intuitively the employed algorithm works as follows: A firm starts in



Note: The capacity states of the two firms are depicted on the x and y axis. The probability of a state is displayed on the z-axis. The left hand side the evolution without credit rationing is pictured. On the right hand side severe credit rationing prevails.

Figure 8: Transitory Dynamics for R=5% (left) and R=120% (right)

state s and time t. For every potential action a and state s, the firm holds beliefs $W(s_t, a_t)$ about the expected discounted sum of cash flows the action would yield. The firm then chooses the best action a^* according to its beliefs and gets an instant payoff of $\Gamma(s_t, a_t^*)$. The actions together with the law of motions of the state variables prescribe the next state. In the next state, the firm chooses again its best actions according to its belief $W(s_{t+1}, a_{t+1})$. At this point, the algorithm can update the belief $W(s_t, a_t^*)$ because $\Gamma(s_t, a_t)$ and $W(s_{t+1}, a_{t+1}^*)$ are part of the discounted sum of cash flows originating in s_t if a_t is chosen. This procedure is repeated for $Iter$ periods. The optimal actions a_t in every period are stored in memory for use in the equilibrium testing procedure. To test for a equilibrium, the algorithm simulates a large number of periods with the stored optimal actions and checks if the resulting beliefs $W^{test}(s, a)$ are the same as the beliefs $W(s, a)$ which justified the actions in the first place.

A tentative example: Assume that the player is in state s. Assume further that the player is under the impression that storing 5 more units of cash starting from state s gives him - for sake of illustration - a continuation value of 2000. This is better than not do so, as he believes that this gives him an continuation value of 1500. He stores 5 more units and then finds out through the play, that this decision resulted only in a continuation value of 1600. So he adjusts his expectation of storing 5 units in state s downwards to 1800. He does not adjust it downwards to 1600 as he cannot perfectly distinguish if this was just a matter of bad luck or truly the consequence of his actions.

The benefit of this solution algorithm is, that it can accommodate larger state spaces than the commonly used Pakes and McGuire (1994) algorithm. The algorithm only calculates policies on the recurrent state space and therefore ignores states which are never played in equilibrium. To give an idea, the state space in my calculation is about $6.25 \cdot 10^{12}$ states large. By selecting states relevant for the equilibrium I only have to calculate equilibrium policies for around one million states, what is still large but manageable. The idea of calculating policies in an Ericson-Pakes model only for a sample of states is gaining prominence in numerical analysis, e.g. another algorithms using this method is Farias, Saure, and Weintraub (2010).

There are also several known problems for this kind of algorithms and numerical simulations of imperfect competition in general:

1. It is not guaranteed that an equilibrium exits. Even if one exits, the algorithm does not

necessarily converge to it.

2. There might be multiple equilibria for reasonable parameter values. Besanko, Doraszelski, Kryukov, and Satterthwaite (2009) offer a possible solution, however, I did not explore this issue up to now.

3. There might be more than one recurrent class associated with a set of policies.

In line with common practice, I check if the algorithm converges to the same equilibrium for different starting values. Although this appears to be the case, the above mentioned issues should be kept in mind. **Scheme of the algorithm:** The algorithm requires the following inputs

- a set of beliefs about the continuation value for every action in every state $W(s,a)$
- a counter $h(s,a)$ for every state and action which measures how often the action was taken in this
- an arbitrary initial state $\breve{s}$ and an arbitrary initial action $\breve{a}$
- an instant return function $\Gamma(s_t, a_t)$ for every state and action
- a function assigning the next period's state s' conditional on today's state s and actions a, $f(\cdot)$.
- technical parameters: length of iteration ($Iter$), ϵ precision of the approximation, β discount factor

Algorithm for calculating AME

$s_t = \breve{s}$, $a_t = \breve{a}$ {Set initial state and initial actions }

repeat

t:=0 {Set index t for best simulation}

while t<=Iter {Begin learning process, last for T periods}

$t = t + 1$

$s_{t+1} = f(s_t, a_t)$ {Assign next state in t+1 according to the optimal actions and state in t }

Load $W(s_{t+1}, .)$ for all a_{t+1} from memory if already visited, otherwise assign initial values.

$a^*_{t+1} = arg\max_{a_{t+1}} W(s_{t+1}, a_{t+1})$
{Calculate the optimal action}

$h(s_{t+1}, a^*_{t+1}) = h(s, a) + 1$
{Increase the counter of the state s_{t+1} and action a_{t+1} by one. }

$\hat{W}(s_t, a^*_t) = \Gamma(s_t, a^*_t) + \beta W(s_{t+1}, a^*_{t+1})$
{Calculate the continuation value in t according to the next periods action and state. $W(s_{t+1}, a_{t+1})$ is a draw of the integral governing the continuation value. }

$W(s_t, a^*_t) = W(s_t, a^*_t) + \frac{1}{h(s_{t+1}, a^*_{t+1})}[W(s_t, a^*_t) - \hat{W}(s_t, a^*_t)]$
{save the updated belief $W(s_t, a^*_t)$ to memory and store the optimal a^*_t action}

set $s_t = s_{t+1}$ and $a^*_t = a^*_{t+1}$

end

t=0

while t<=T{Begin test procedure}

$s_{t+1} = f(s_t, a^*_t)$ {Assign new state}

Load a^*_{t+1} and $W(s_{t+1}, a^*_{t+1})$ from memory

$h(s_{t+1}, a^*_{t+1}) = h(s, a) + 1$
{Increase the counter of the state s_{t+1} and action a_{t+1} by one. }

$\hat{W}^{test}(s_t, a^*_t) = \Gamma(s_t, a^*_t) + \beta W(s_{t+1}, a^*_{t+1})$
{Calculate the continuation value in t according to the next periods action and state. }

$W^{test}(s_t, a^*_t) = W^{test}(s_t, a^*_t) + \frac{1}{h(s_{t+1}, a^*_{t+1})}[W^{test}(s_t, a^*_t) - \hat{W}^{test}(s_t, a^*_t)]$
{Update the belief about the continuation value. Do the procedure also for the square of $W^{test}(.)$ to calculate the sampling variance.}

store $W^{test}(s_t, a^*_t)$

set $s_t = s_{t+1}$ and $a^*_t = a^*_{t+1}$

end

$Bias(s, a) = \frac{W^{test}(s,a)}{W(s,a)}^2 - Var(\frac{W^{test}(s,a)}{W(s,a)})$
{Calculate for every state and action visited on the equilibrium path a bias statistic.The variance term is used to adjust for sampling variance. }

$T = \left\| \sum_a \frac{h(s,a)}{\sum_a h(s,a)} Bias(s, a)|| \right\|_{L^2_{P(s)}}$
{The test statistics is then a L^2 norm in the bias term where $P(s)$ is a measure for the fraction of time s is visited on the equilibrium path. The test statistic measures if the stored optimal action can replicate the continuation values. }

until $T < \epsilon$ {The algorithm if the T is below the required precision ϵ. }