Trading and liquidity with imperfect cognition

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“The perception of the intellect extends only to the few things that are accessible to it and is always very limited.”

René Descartes

Méditations Métaphysiques

Liquidity shock

Reduces ability to hold assets:

- Banks, hedge funds or private equity funds: losses (Khandani Lo, 08)
- Mutual funds: funds outflows (Coval Stafford, 07)
- Insurance companies: downgrades, delistings (Greenwood, 05, Da Gao, 05).

Aggregate: Population of agents simultaneously hit.

Transient: After some time an institution recovers.

Imperfect cognition

Lots of information:

- Fundamentals
- Positions (what has been sold, hedged, netted,…)
- Risks
- Counterparties
- Compliance

Takes time & hard thinking before information collected & processed so that decisions can be reached.

Market mechanism

Trading technology complements investors' limited cognition: make decisions when humans busy doing something else.

Limit order: order executed without intervention of human who placed it

Algo: order placed without human intervention

Trading technology => reaction to shocks

Optimal market mechanism ?

1) Model

Duffie et al (Econometrica 05), Weill (Restud 07)

Mass 1 of risk-neutral competitive institutions

Supply $s < 1$.

Each institution initially endowed with s units.

Discount rate r,

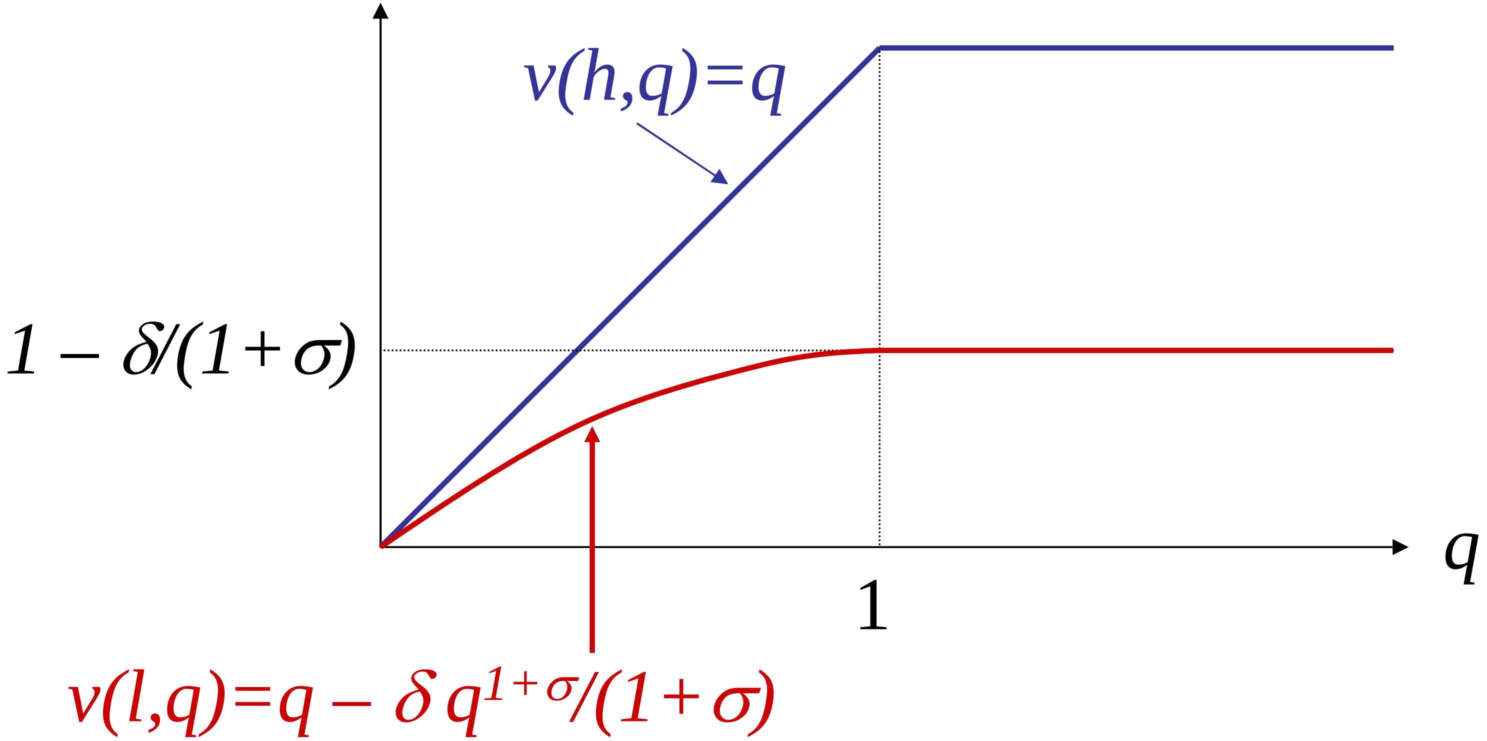
Continuous time, probability space (Ω, F, P).

Utility flow from holding q units of asset at time t.

Before liquidity shock: $\theta = h \Rightarrow v(h,q)$.

When hit by liquidity shock θ = to $l \Rightarrow v(l,q)$

Utility flow



Marginal valuation decreases with quantity held

=> efficient to spread holdings across agents.

Liquidity shock reduces asset holding ability.

Liquidity shock

At t = 0 all institutions hit by shock: low utility flow

Mass of high utility agents at time $t = \mu_{ht}$ (μ_{h0}=0)

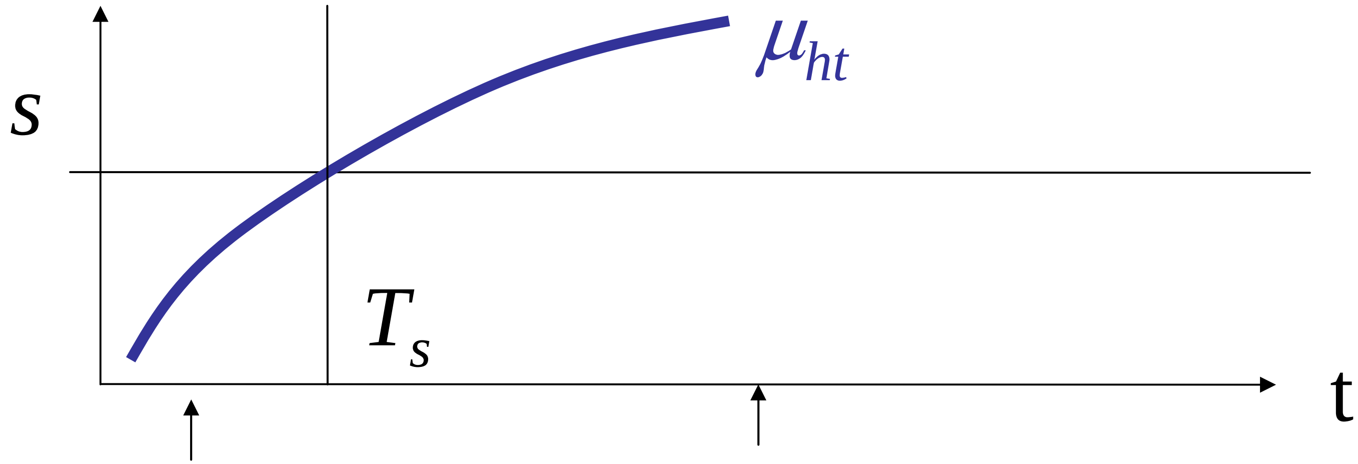
Each institution recovers at first jump of its Poisson process (intensity γ): high utility forever

Recovery processes i.i.d across institutions
=> by LLN aggregate market deterministic:

$$\mu_{ht} = 1 - \exp(-\gamma t)$$

2) Equilibrium with perfect cognition

Trader continuously observes *v* & participates to market



Mass high utility < supply Mass high utility> supply

$t > T_s$: asset held only by high utility

$t < T_s$: also held by low utility

Opportunity cost of holding asset: ξ_t

At t, borrow p_t to buy asset.
At $t+dt$, resell $p_t + p_t' \, dt$, reimburse $p_t(1+rdt)$.

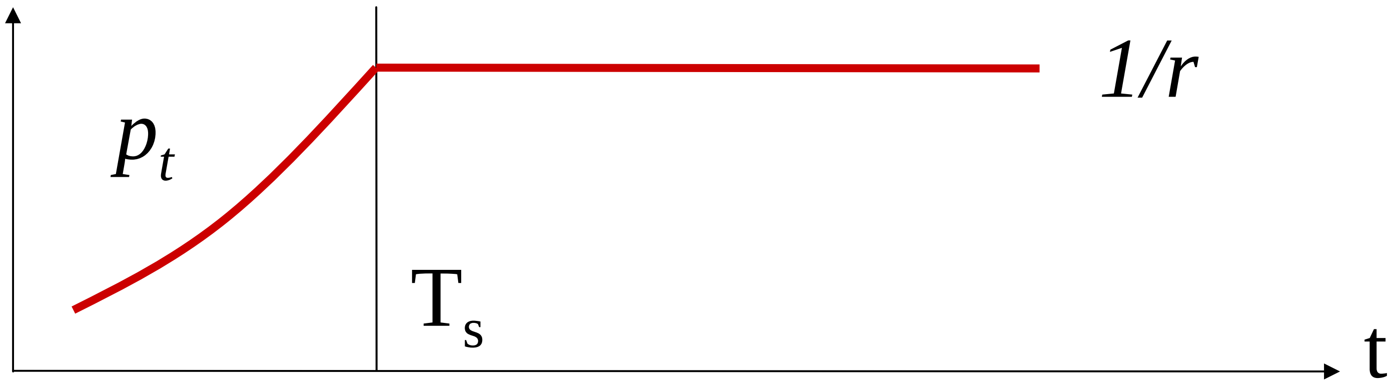
$$\xi_t \overset{d}{=} \underbrace{r\,p_t}_{\text{Time value of money (interest paid)}} - \underbrace{p_t'}_{\text{Capital gain earned}}$$

Pricing with perfect cognition

After T_s asset held only by high utility: $p = 1/r$

Before T_s: marginal agent has low utility: $p < 1/r$

Equilibrium: opportunity cost = marginal valuation of marginal investor



LLN: aggregate market deterministic => price also.

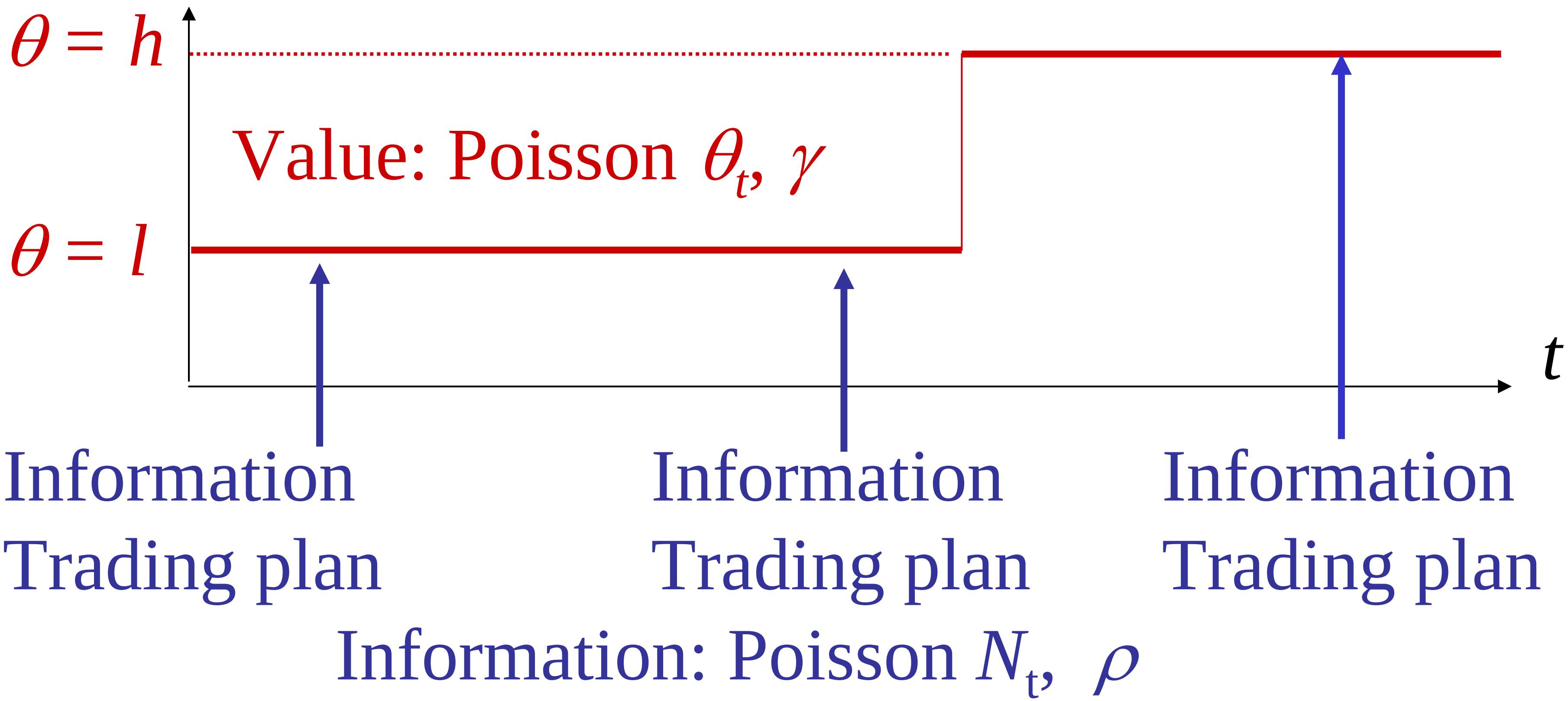
Rise in price = progressive recovery from liquidity shock. Can't be arbitraged.

3) Imperfect cognition

Traders face 2 tasks (can't do both at same time):

i) Find out valuation & optimal strategy: takes time (// Mankiw and Reiss, QJE 2002)

ii) Submit trading plan to market: instantaneous



Market instruments

Asset holding plan:

- $q_{t,u}$ holdings at time u decided at $t < u$
- Adapted to filtration F_t generated by N_t and θ_t
- Bounded variations
- When info process jumps, update plan.

Implemented with market instruments:

- Trading algorithms
- Limit orders

Intertemporal value *V(q)* of holding plan *q(t,u)*

$$E_0\left[\int_{t=0}^{\infty} e^{-rt}\int_{u=t}^{\infty} e^{-(r+\rho)(u-t)}\{E_t[v(\theta_u,q_{t,u})]-\xi_u q_{t,u}\}du\,\rho dt\right]$$

Discounted sum — Expected valuation — Opportunity cost

Algos: choose any $q_{t,u}$ adapted to F_t with bounded variations: contingent on market movements.

Limit orders: less rich conditioning on market movements.

Market clearing

Cross-sectional average holding = per capita supply

By LLN:

$$E_0\left[q_{\tau_u,u}\right] = s$$

Time of last jump of N before u

$$\int_0^u \rho e^{-\rho(u-t)}\left[(1-\mu_{ht})E(q_{\tau_u,u} \mid \theta_t = l) + \mu_{ht}E(q_{\tau_u,u} \mid \theta_t = h)\right]dt = s$$

Demand from low valuation

Demand from high valuation

Supply

4) Trading algorithms

Algo traders choose q (adapted to F_t & with bounded variations) to maximize $V(q)$: pointwise optimization

$$\max_{q_{t,u}} E_t[v(\theta_u, q_{t,u})] - \xi_u q_{t,u}$$

High valuation have uniformly higher utility flow than low valuation traders. If some low valuation holds some asset, all high valuation hold 1 unit.

Residual supply at time *u*

Gross supply of agents with at least 1 info event
- Maximum possible demand from high valuation

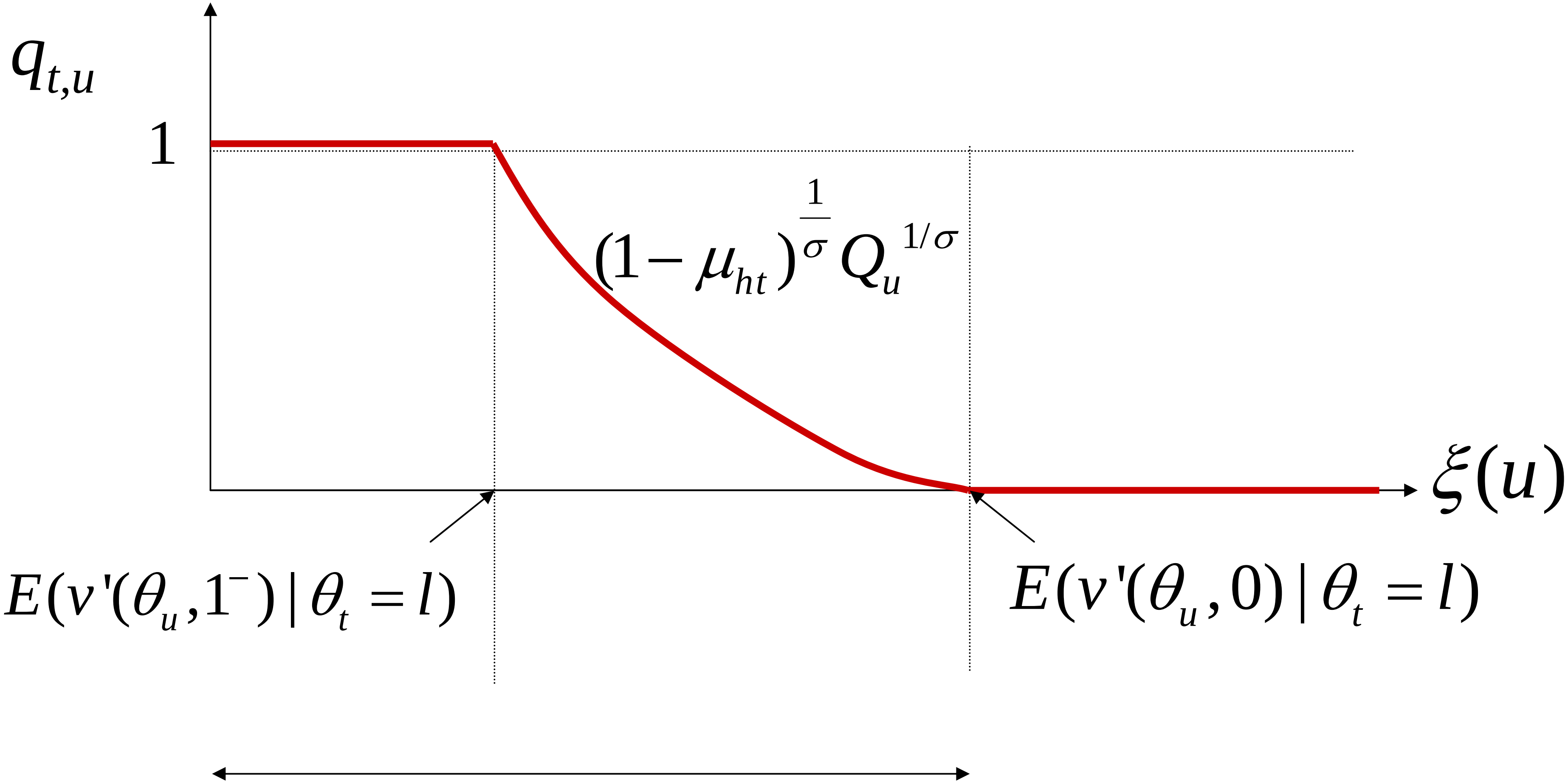
$$\int_0^u \rho e^{-\rho(u-t)}(s-\mu_{ht})dt = S(u)$$

At time T_f virtual supply reaches 0: asset held by high valuation traders only.

Before T_f high valuation traders hold one unit and low valuation traders hold some of the asset.

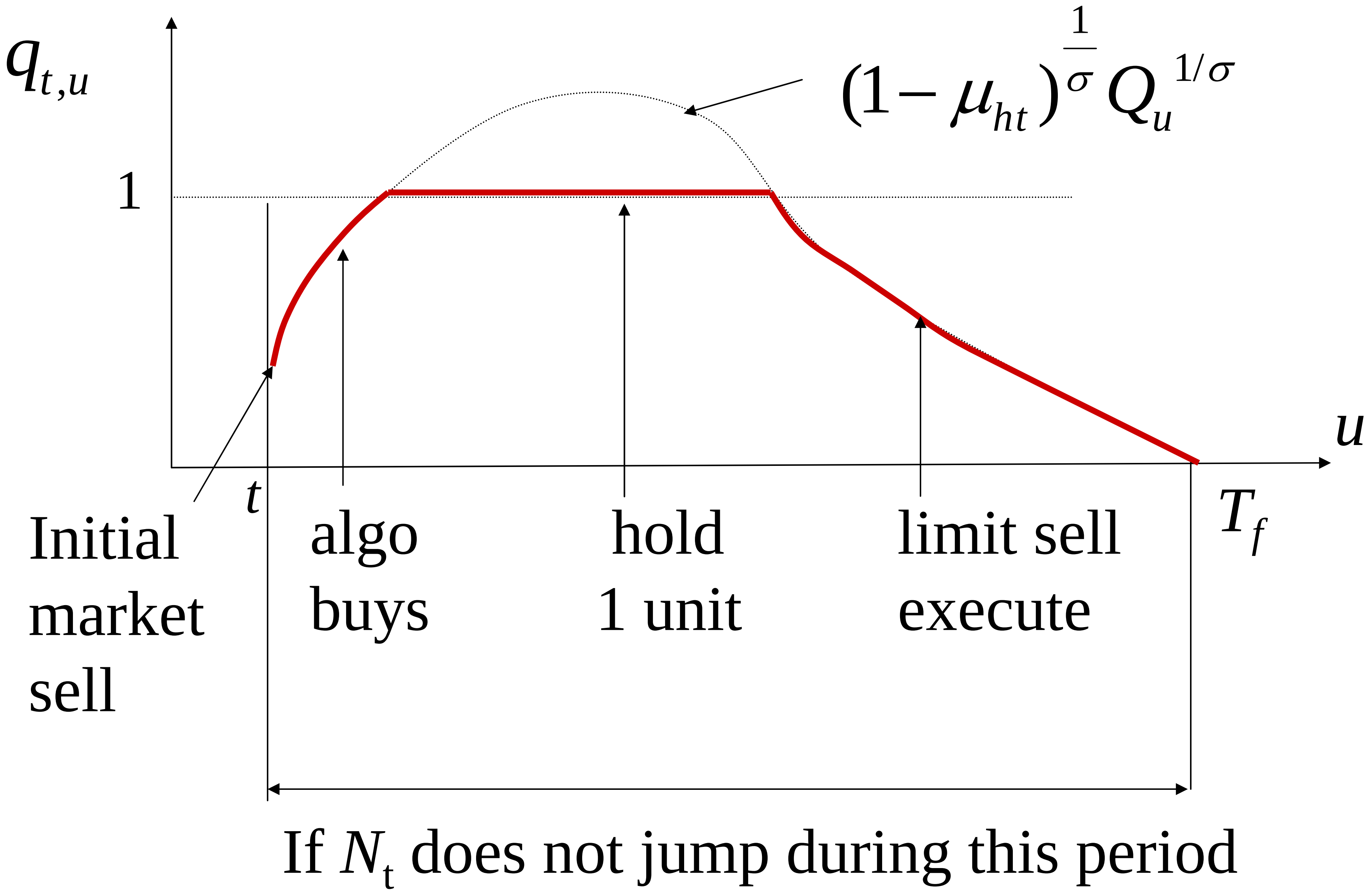
Low valuation traders' holdings

(pointwise maximization of *V(q)*)



Before T_f low valuation must hold some asset

Optimal algo for low valuation trader



Why buy back after selling?

To reap gains from trade !

Trader j observes $\theta = l$ at t.

Trader i observed earlier he had low valuation, but had no information event since, may have switched.

i has higher expected valuation than j at t => i buys from j

Why eventually sell back?

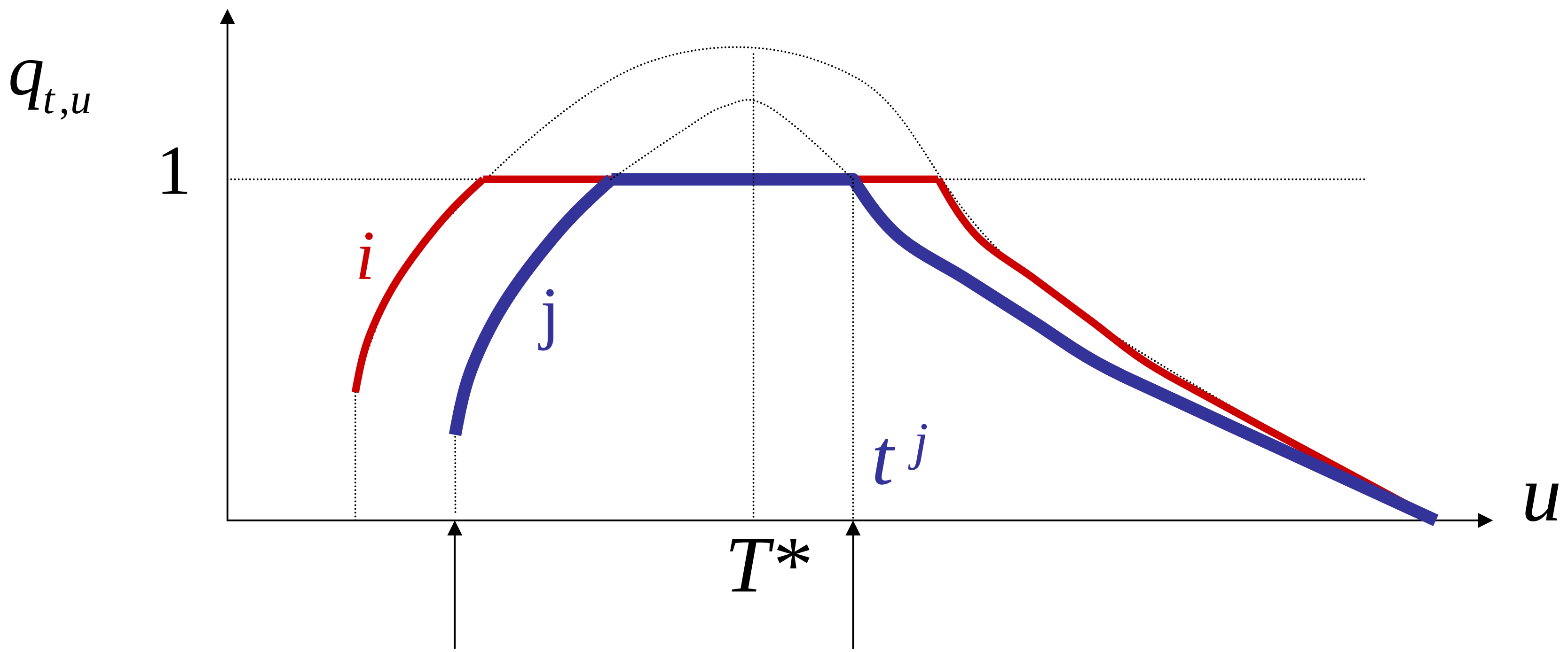
To reap gains from trade !

As time goes buy, more and more traders have recovered from shock.

Mass of high of high valuation traders increases: demand increases.

When demand is high enough: algos sell.

Trading plans at different points in time



Trader coming later … starts selling earlier

j's orders hit earlier than i: at lower prices: j undercut

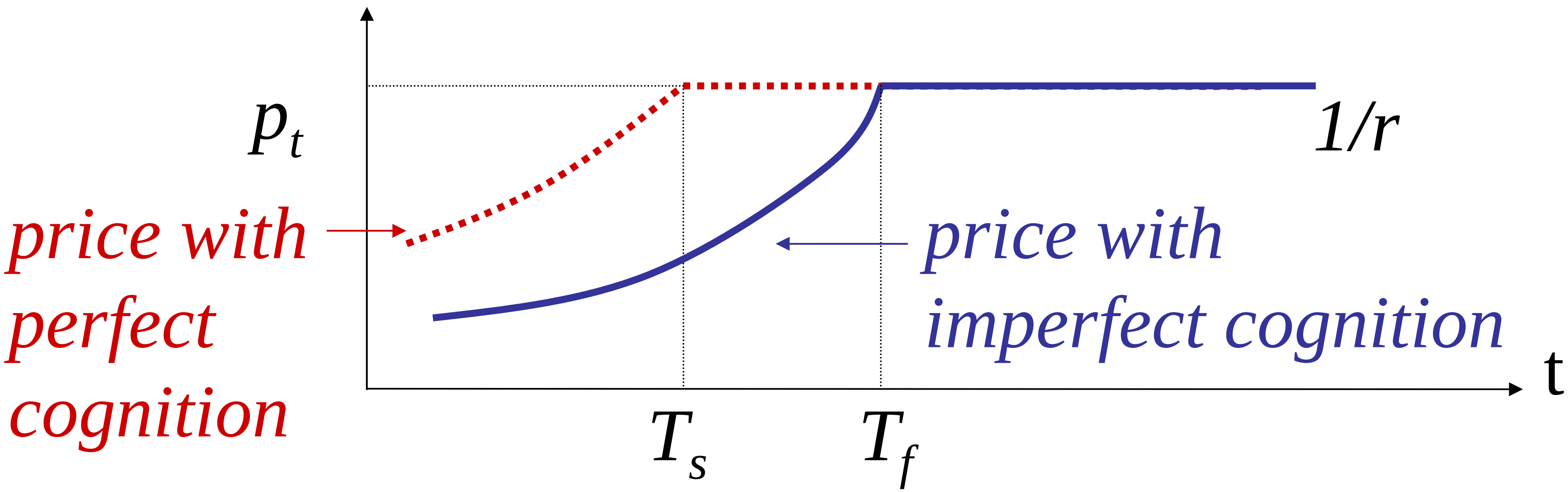
because his expected valuation was lower than i's

Equilibrium price

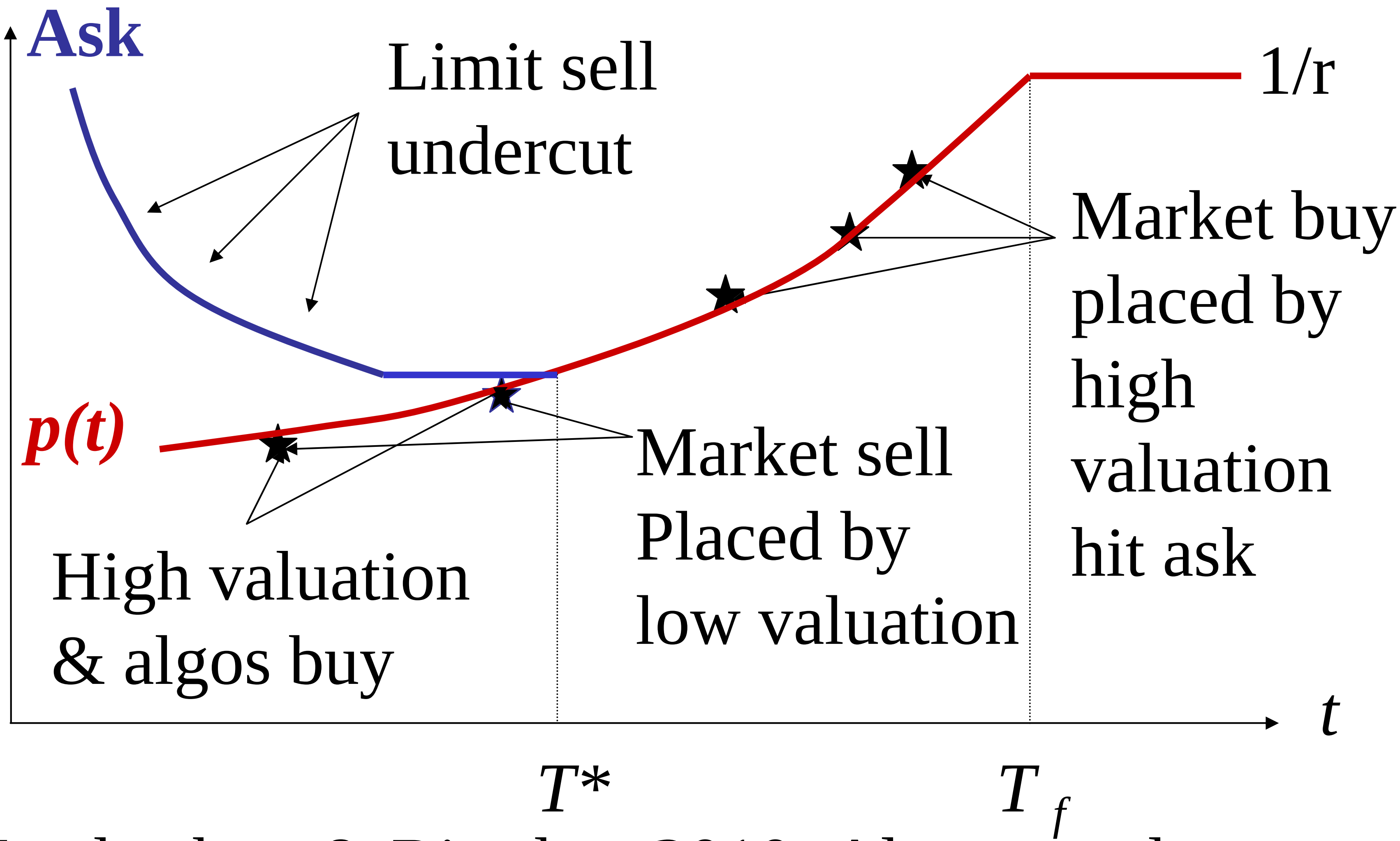
Substitute $q_{t,u}$ in market clearing:

$$rp_u - p_u' = 1 - \delta(1-\mu_{hu})Q_u^{\sigma}$$

Opportunity cost E(v' of marginal agent)



Equilibrium market dynamics



// Hendershott & Riordan, 2010: Algos supply liquidity after shock, sell back when price recovers.

Welfare theorem

Social planner maximizes utilitarian welfare *s.t.* same informational constraints as agents:

=> no trade before first jump of info process

=> trade at u conditional on valuations at τ_u

Information constrained socially optimal allocation = competitive equilibrium with algos.

Algos enable market to allocate asset at t to agents with highest expected valuation given F_t

5) Limit orders

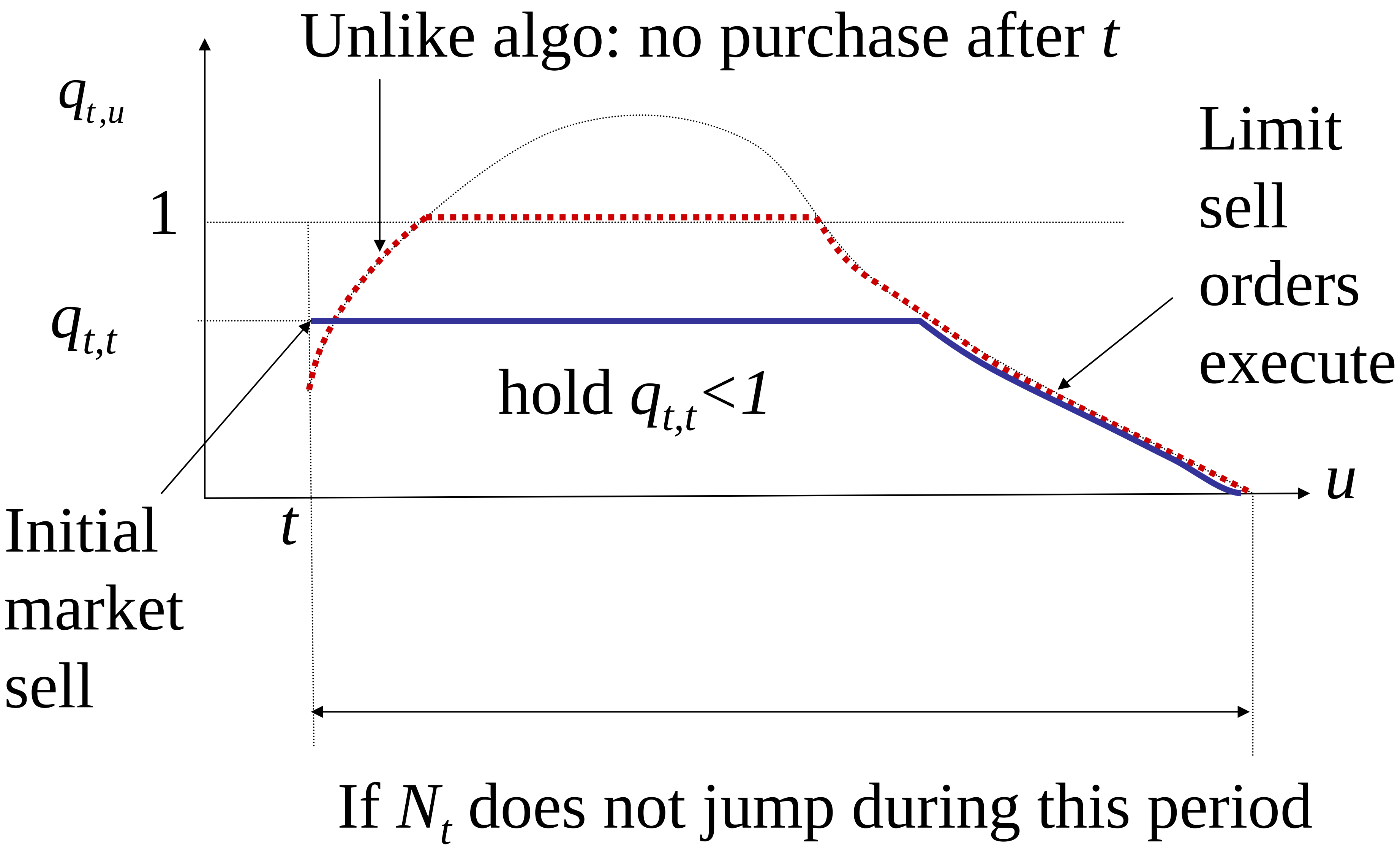
Limit orders can't buy later at higher price

Algos do – except when Q_u always decreasing

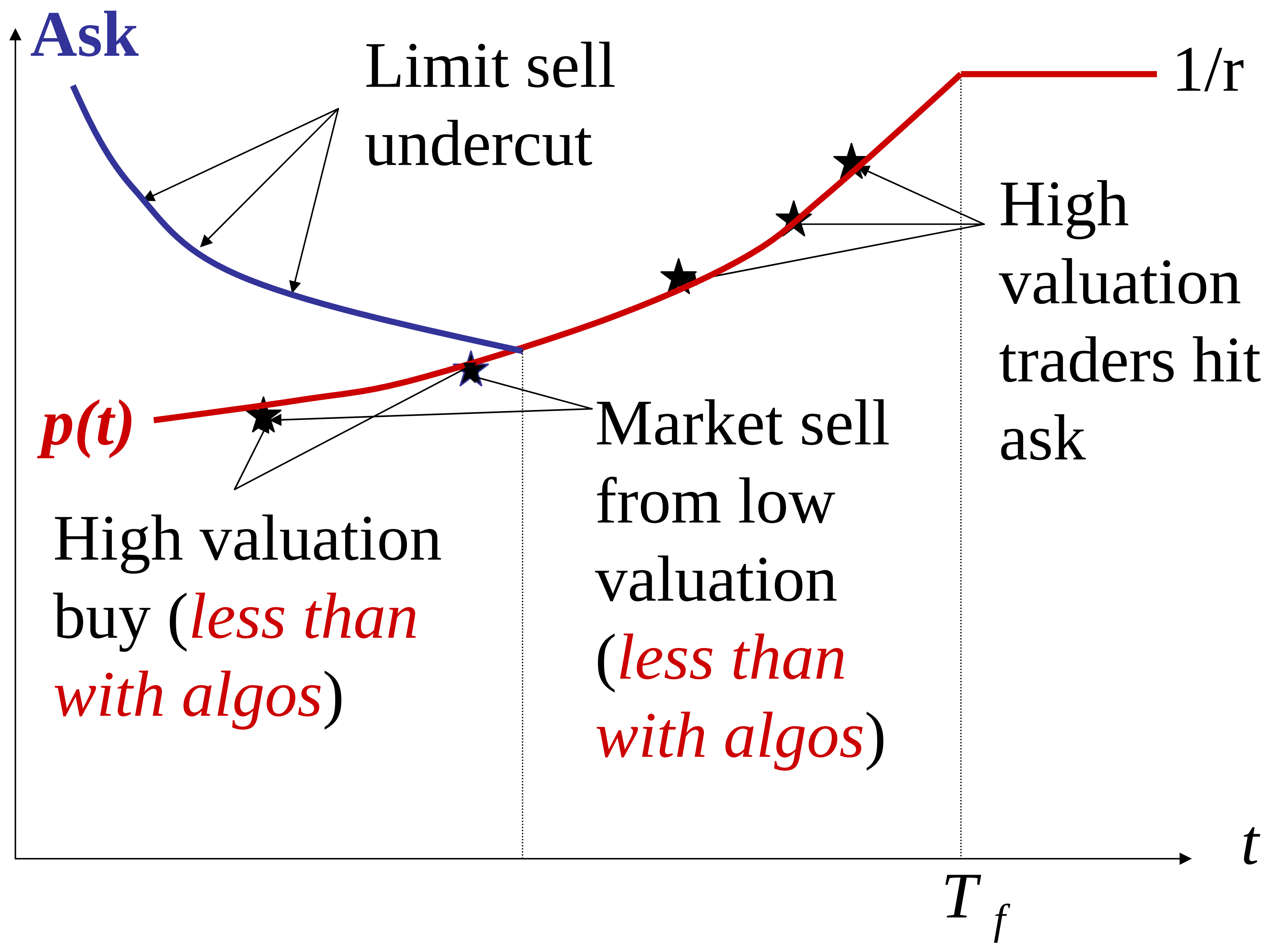
Hence pure limit order market reaches lower allocative efficiency than market with algos

But end of shock still occurs at time T_f when residual supply equals 0. ($S(T_f)=0$)

Optimal limit orders



Equilibrium market dynamics without algos



Algos destabilize markets?

Ambiguous effect on prices:

- low valuation traders sell more initially: market reacts more to shock => price just after shock can be lower with algos than without (and volume higher)
- but then low valuation traders buy back: market recovers faster from shock => after some time price higher with algos than without

Unambiguous effect on welfare.

Open Issues

Here algos mitigate cognition limits.
What could go wrong?

We compare the situation where all traders can use algos to that where nobody can use algos. What if some can use algos & others don't?

In our model individual uncertainty but aggregate market deterministic. What if aggregate shocks?

What if some algos can break up or go mad (operational risk with computer system)?