

# Career Concerns and Free Riding in Teams

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July 12, 2010

- Evolution of cooperation and free-riding in *research* teams.
  - benefits are public, costs are private;
  - success is uncertain;
  - effort is unobservable;
- A model of career concerns in human-capital intensive teams:
  - e.g. scientists, lawyers, consultants;
  - the team's reputation determines clients' willingness to pay.
- Main implications:
  - free riding incentives, effort reduction & procrastination;
  - knowledge within the team is beneficial;
  - better monitoring need not be.

- Moral hazard in teams:
  - Alchian and Demsetz (1972), Holmström (1982).
- Strategic experimentation:
  - Bolton and Harris (1999), Keller, Rady, and Cripps (2005); Bergemann and Hege (1997, 2005).
- Dynamic contributions to public goods:
  - Admati and Perry (1991), Marx and Matthews (2000).
- Career Concerns:
  - Holmström (1999), Dewatripont, Jewitt, and Tirole (1999); Bar-Isaac (2007).

# The Set-Up

$n$  agents are members of a team. Each agent can be **good** or **bad**.

The team is **productive** only if **all** agents are good.

Agents do not know their type.  $\Pr(\text{team is productive}) = \bar{p}$ .

Agent  $i$  exerts **unobservable** effort at rate  $u_{i,t} \in [0, \bar{u}]$  at date  $t$ .

A team generates successes (worth €1) at a Poisson rate

$f(u_{1,t}, \dots, u_{n,t})$  if the team is good, and 0 otherwise.

Most of the talk,  $f(u_{1,t}, \dots, u_{n,t}) = \sum_{i=1}^n (\lambda + u_{i,t})$ .

Agents incur a flow cost  $c_i(u_{i,t}) = \alpha u_{i,t}$ .

The planner's beliefs evolve according to:

$$\dot{p}_t = -p_t(1 - p_t) \sum_{i=1}^n (\lambda + u_{i,t}).$$

After the first success, she knows the team is good ( $p_t = 1$ ).

The first success is worth  $\bar{V} := 1 + (n(\lambda + \bar{u}) - n\alpha\bar{u}) / r$ .

Before the first success, she maximizes

$$\int_0^\infty \left( p_t \sum_{i=1}^n (\lambda + u_{i,t}) \bar{V} - \alpha n u_{i,t} \right) e^{-\int_0^t (r + p_s \sum_{i=1}^n (\lambda + u_{i,s})) ds} dt.$$

- The social planner induces maximal effort ( $u_i = \bar{u}$ ) as long as

$$p_t \geq p^* := \alpha \frac{r + n\lambda}{r\bar{V}},$$

and zero effort afterwards.

# The Strategic Problem

A competitive market pays the team a wage equal to its expected flow output.

Assuming (for now) equal shares within the team, each agent gets

$$w_t = \frac{1}{n} \tilde{p}_t \sum_{i=1}^n (\lambda + \tilde{u}_{i,t}),$$

where  $\tilde{p}_t$  is the market belief, and  $\tilde{u}_{i,t}$  is the expected effort level.

Remember that effort is unobservable.

For the agents then, the wage is just a function of time.

Individual effort must be a best response to other agents' effort, given the wage function  $w_t$ .

# The Strategic Problem

The first success is worth  $\bar{V} = \lambda/r$  to each agent.

Why? After the first success,  $\tilde{p}_t = 1$ , and no effort can be sustained in equilibrium (so  $w_t = \lambda$ ).

Each agent then maximizes

$$\int_0^\infty (p_t \sum_{i=1}^n (\lambda + u_{i,t}) \bar{V} + w_t - \alpha u_{i,t}) e^{-\int_0^t (r + p \sum_{i=1}^n (\lambda + u_{i,s})) ds} dt$$

s.t.  $\dot{p}_t = -p_t(1 - p_t) \sum_{i=1}^n (\lambda + u_{i,t})$ ,

and in equilibrium we require

$$w_t = \frac{1}{n} p_t \sum_{i=1}^n (\lambda + u_{i,t}).$$

# The Basic Trade-Off

Consider  $i$ 's value on the equilibrium path.

Compare the **marginal** effect of effort today ( $u_i = u_{i,t}$ ) and tomorrow ( $u'_i = u_{i,t+dt}$ ).

$$\lim_{dt \rightarrow 0} \frac{dV_i / du_i}{dt} \geq \lim_{dt \rightarrow 0} \frac{dV_i / du'_i}{dt}.$$

Intertemporal transfers of effort must not be profitable:

$$p \cdot \underbrace{\alpha u'_i}_{\text{cost saved}} = \alpha \cdot \underbrace{p(n\lambda + u_{-i} + u_i)}_{\text{Pr. of success at } t} - \underbrace{(p(r\bar{V} - w) - \alpha r)}_{\text{loss due to delay}}.$$

A role for other agents' effort, and for the (prize-wage) gap.



# The Basic Trade-Off

Suppose the equilibrium is symmetric. The indifference condition is

$$(n-1) u^*(p) = \frac{r\bar{V} - w^*(p)}{\alpha} - \frac{r}{p} - n\lambda,$$

where in equilibrium we have

$$w^*(p) = p(\lambda + u^*(p)).$$

Effort and wage as a function of  $p$  are given by:

$$\begin{aligned} u^*(p) &= \frac{\lambda(1-p-\alpha n) - \alpha r/p}{\alpha(n-1) + p} \\ w^*(p) &= \frac{p\lambda(1-\alpha) - \alpha r}{\alpha(n-1) + p}. \end{aligned}$$

- Individual effort  $u^*(p)$  is decreasing in  $n$  and in  $p$  (for  $r$  low).
- The equilibrium wage  $w^*(p)$  is increasing in  $p$ .
- But all this is assuming effort is positive...

# When Does It End?

As  $p_t \rightarrow 0$ , marginal product of effort  $\rightarrow 0$ : effort must stop.

## Lemma 1

There exists a critical  $p_L$  such that  $u^*(p) = 0$  for all  $p < p_L$ .

For all  $\alpha$ ,  $r/\lambda$ , and  $n$ , equilibrium effort stops inefficiently early ( $p^* < p_L$ ).

The threshold belief  $p_L$  is increasing in  $\alpha$ ,  $r/\lambda$ , and  $n$ .

$p_L$  is determined by indifference condition when no one ever works again, and the market pays  $w_t = p_t \lambda$  forever.

Inefficiencies arise because:

- success is worth less (no effort later);
- market wage represents an outside option.
- higher  $n \Rightarrow$  more likely to succeed due to pure talent.

# Symmetric Equilibrium

Before  $p$  reaches  $p_L$ , effort is given by  $u_i = u^*(p)$ .

But recall that effort is also just a function of time.

## Theorem 1

There exists a unique symmetric equilibrium, with effort given by

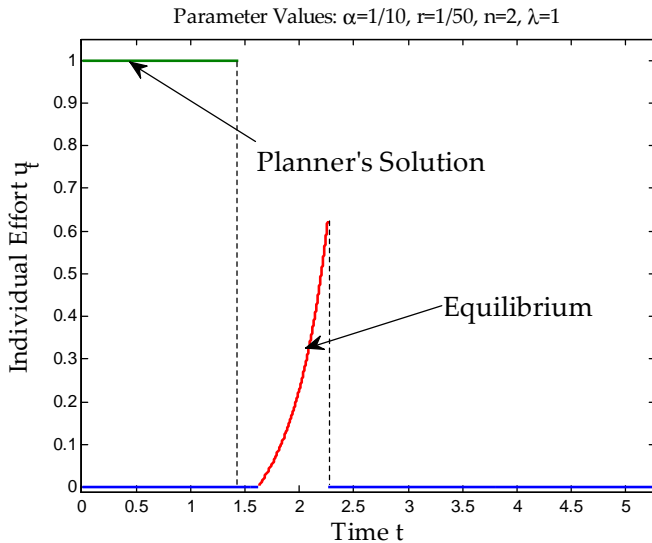
$$u_{i,t}^* = \max \left\{ \frac{\lambda (1 - p_t - \alpha n) - \alpha r / p_t}{\alpha (n - 1) + p_t}, 0 \right\} \text{ for all } p_t \leq p_L.$$

The belief  $p_t$  is the solution to

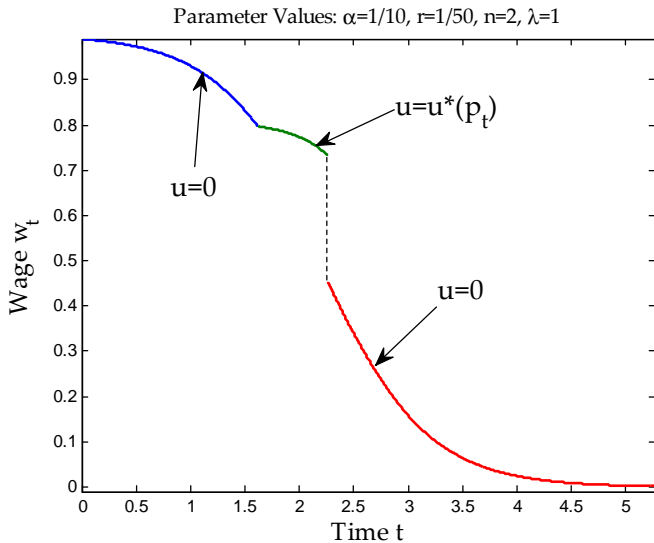
$$\dot{p}_t = -p_t(1 - p_t)n(\lambda + u_{i,t}^*).$$

All effort stops at the time  $T$  where  $p_T = p_L$ .

# Equilibrium Effort



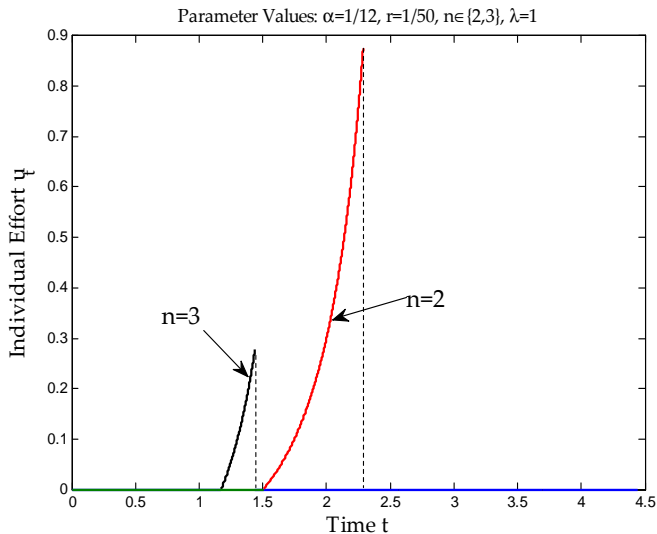
# Equilibrium Wage



## Symmetric Equilibrium: Summary

- Effort “begins” at  $p = p_H$ , where  $u(p_H) = 0$ .
- Effort ends at  $p = p_L$  where  $u(p_L) > 0$  (jump to zero).
- The range of beliefs  $[p_L, p_H]$  is shrinking as  $\alpha$  or  $n$  increase.
- Effort cannot be sustained when beliefs are higher than  $p_H$ .
- Compared to 1st best, both effort reduction and **delay**.

# Equilibrium Effort



# Asymmetric Equilibria

An example: only one agent works, and exerts effort:

$$u_1(p) = n \frac{\lambda(1 - p_t - \alpha n) - \alpha r / p_t}{p_t}.$$

Trade-off between *efficiency* and *fairness* of equilibria: compared to the symmetric equilibrium:

- the thresholds  $p_H$  and  $p_L$  are the same;
- team effort (as a function of  $p$ ) is higher, and it is equal to the cooperative level.

This is the unique equilibrium with asymmetric players or unequal shares.



C.E.S. “production function”

$$f(u_t) = \left( \sum_{i=1}^n (\lambda + u_{i,t})^\rho \right)^{1/\rho}, \text{ where } \rho \in (0, 1].$$

## Theorem 2

There exists a unique symmetric equilibrium, with effort given by

$$\begin{aligned} u_{i,t}^* &= \frac{\lambda n^{\frac{1}{\rho}-1} - \alpha \left( \lambda + (1/p_t) n^{1-\frac{1}{\rho}} \right)}{p_t n^{\frac{1}{\rho}-1} + \alpha (n-1)} - \lambda. \\ \dot{p}_t &= -p_t(1-p_t)n^{1/\rho} (\lambda + u_{i,t}^*). \end{aligned}$$

Effort is decreasing in  $\rho$  for each  $p_t$ .

The range of beliefs  $[p_L(\rho), p_H(\rho)]$  is decreasing in  $\rho$ .

Suppose the team knows its own type.

A bad team will never work.

The market posterior  $\tilde{p}$  declines over time.

Suppose so does the wage.

Then the good team's incentives are increasing over time.

## Theorem 3

There is a unique symmetric equilibrium, in which each member of a good team exerts effort level

$$\begin{aligned} u_{i,t}^K &= \max \left\{ \frac{\lambda - \alpha (\lambda + r)}{\tilde{p}_t + \alpha (n - 1)} - \lambda, 0 \right\}. \\ \tilde{p}_t' &= -\tilde{p}_t (1 - \tilde{p}_t) n (\lambda + u_{i,t}^*). \end{aligned}$$

Effort  $u_{i,t}^K$  (weakly) increases over time, and *never stops*.

Effort  $u_{i,t}^K$  is (weakly) higher than  $u_{i,t}^*$  (in the unknown type case).

Note that (for  $r$  low enough):

- ① expected effort (from the market's perspective) is lower than in the unknown type case;
- ② ... but the market still learns faster.

When effort is observable, consider Markov strategies  $u_i(p, \pi)$ , where  $\pi$  denotes the market belief.

Along the equilibrium path,  $p = \pi$ .

The lower threshold ( $p_L$ ) is unchanged, but  $u(p_L, p_L) = 0$ .

In the limit for  $r \rightarrow 0$ ,  $p_H$  is also unchanged, but effort is lower for all values of  $p$ .

- Strategic substitutes.
- Net present value computations.
- Non-Markov Equilibria?

- ① Unobservable unknown: inefficient threshold, and delay.
- ② Unobservable known: more efficient thresholds, but still delay.
- ③ Observable: same thresholds as in (1), but lower effort ( $\Rightarrow$  more delay).

What happens after a success?

“Juniors” become “seniors”: do they stay together?

Do they go an match with non established partners (whose reputation can still be improved)?

Why? Better monitoring? Better screening? Better mentoring?

In a matching model, what's the equilibrium duration of (unsuccessful) partnerships?

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