Search, Choice, and Revealed Preference*

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Abstract

With complete information, choice of one option over another conveys preference. Yet when search is incomplete, this is not necessarily the case. It may instead reflect unawareness that a superior alternative was available. To separate these phenomena, we consider non-standard data on the evolution of provisional choices with contemplation time. We characterize precisely when the resulting data could have been generated by a general form of sequential search. We characterize also search that terminates based on a reservation utility stopping rule. We outline an experimental design that captures provisional choices in the pre-decision period.

Key Words: Revealed preference, search, incomplete information, revealed preference, framing effects, status quo bias, bounded rationality, stochastic choice, decision time

1 Introduction

In principle, incomplete information can explain apparent deviations from utility maximizing behavior: decision makers (DMs) may choose an inferior over a superior alternative if they are not aware that the superior one is available. Yet traditional decision theory focuses exclusively on situations in which choice of one option over another reflects an underlying preference. This "revealed preference" approach breaks down when information is incomplete.

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In contrast with decision theory, search theory is premised on incomplete information [Stigler 1961]. Given the tension between the principle of revealed preference in standard decision theory and search theory, it is understandable that there are few linkages between them.

We develop a unified theoretical and experimental framework to help bridge the gap between search theory and the principle of revealed preference by characterizing models of choice which incorporate the process of information search. We first consider a model of "alternative-based" search (ABS), in which the DM searches sequentially through the available options, comparing searched options in full according to a fixed utility function. We consider also "reservation-based" search (RBS), a refinement of ABS under which the DM searches until an object is identified with utility above a fixed reservation level.

While ABS and RBS represent important classes of search behavior, neither provides testable restrictions for standard choice data. Without additional ad hoc assumptions, any pattern of final choice is rationalizable with either model. We therefore consider a richer data set, which we call "choice process" data, with which to test the models. These data convey not only the final option that the DM selects, but also how their choice changes during the period of contemplation prior to making the final selection.[1] By so enriching the data we are able to characterize whether or not incomplete information and search can explain apparent violations of utility maximization.

The key to the axiomatic characterization of the ABS and RBS models is understanding what type of behavior implies a revealed preference in the context of each model. In neither case does final choice of one object over another necessarily indicate preference as the decision maker may be unaware of the unchosen object. However, in both cases, a DM who changes their choice from one object to another is interpreted as preferring the later-chosen object. The necessary and sufficient condition for the ABS model to hold is that this information must be "consistent", in the sense of being acyclic. Under the RBS model, there may be additional revealed preference information in the final choice itself, as in a set comprising objects all of which are below reservation utility, search must be complete.

The ABS and RBS models both treat search order as unobservable. This makes it natural to develop stochastic variants, given that search order is not a priori fixed and that there is no reason to believe that search from a given set will always take place in the same order. The stochastic versions

[1] This data has previously been considered by Campbell [1978]

of ABS and RBS are developed in section 4. While stochasticity adds to the technical intricacy of the model, there is no conceptual difference between the deterministic and the stochastic cases: the stochastic results are precise analogs of their deterministic counterparts.

The process of information search provides one particular channel by which choice can be affected by seemingly unimportant features of the environment, such as the positioning of objects on the screen, or in a shop. This in turn could lead to behavioral phenomena such as framing effects, status quo bias and stochastic choice. Our models imply that, when driven by search, these phenomena will have distinctive patterns. For example, if stochastic choice is driven by RBS and random search order, choice is random amongst choice sets consisting of above-reservation items, but deterministic in sets containing only below-reservation items. Characterizations in this spirit of framing effects, status quo bias and stochastic choice are in section 5. To be clear, our approach to these phenomena does not well describe several of the most well-studied cases.

The unified approach to theory and experiment that we take in this paper rests on two key premises.

1. **PREMISE 1**: ABS and RBS represent broad styles of search that may be undertaken in a wide variety of different decision making environments.
2. **PREMISE 2**: It is conceptually and experimentally feasible to collect data on the evolution of "intended" choice with contemplation time.[2]

With regard to the first premise, we study ABS and RBS because we see them as broad search modes that are of particular interest. We think that ABS-style search is a natural way to model search behavior in many environments – particularly when there is a cost of switching attention from one alternative to another, or if items can only be understood in their entirety. It is also the canonical model of search within economics: search is alternative-based in most labor market models, as well as Stigler [1961]'s model of price search, and Simon [1955]'s boundedly rational model of search. In addition to its central role in the theoretical canon, there is also experimental evidence suggesting that ABS may be a good description of search in some environments (e.g.

[2] There is a gap between the theoretically ideal data and the data our experiments generate. The model assumes that we can identify not just one, but all best options at each point in time. In contrast, the experiment consider only a single choice at each point in time. A similar gap is encountered in tests of standard rationality axioms.

Reutskaja et al. [2008] and Payne, Bettman and Johnson [1988, 1993]). Similarly, we see RBS as a natural first model of search termination. It is the stopping rule suggested by Simon [1955] in his work on satisficing, and it also bears an interesting relationship with optimal search in certain environments.[3]

With regard to the second premise, in section 6 we outline an experimental design that data on the evolution of provisional choices with contemplation time. Subjects are presented with a collection of objects from which they must choose. They can select an option at any time by clicking on it, and change their selection as many times as they like. The key to the experimental design is that the subject's choice is not recorded at the point at which they press the finish button, but at a randomly selected time unknown to the subject. This ensures that it is in the interest of the subject to always keep selected their currently preferred option. As detailed in section 6, Caplin, Dean, and Martin [2009] conduct a proof-of-principle experiment in which both ABS and RBS are broadly supported.

While important, ABS and RBS are not universally applicable. There are other modes of search available, such as those in which objects are compared on an attribute-by-attribute basis. Hence ABS may be more prevalent in environments in which there are high costs to switching among searched objects (for example, if the items of search were in different physical locations), or where alternatives are best understood holistically (for example a written description of a financial contract). In contrast, if it is easy to compare different alternatives on the same dimension, we might expect ABS to be a poor description of behavior. ABS also appears less intuitively compelling in when objects are easy to identify, yet difficult to compare. In such less favorable contexts, our tests provide formal tools for understanding how the environment impacts search style, which in turn may impact the nature and extent of incomplete information.

We see our approach as complementary to other attempts to use novel data to understand information search based on eye tracking or Mouselab [e.g. Payne, Bettman and Johnson, 1993; Gabaix et al., 2006, Reutskaja et al. 2008]. These approaches make aspects of the search process observable, yet do not connect these intermediate acts of search with their implications for choice.

[3] While we do not explicitly derive ABS or RBS as resulting from optimal search it is true that a reservation based stopping rule is optimal within the class of ABS search behavior for a DM who has fixed costs of search, and is not learning about their environment. Moreover, the optimal reservation level does not depend on the size of the choice set the DM is choosing from, just the cost of search and perceived distribution of object values.

In comparison, choice process data misses out on potentially relevant visual and other cues on search behavior, but captures the moment at which the search that has been undertaken changes the DM's assessment of the best option thus far encountered.[4] The connection of eye tracking and Mouselab data with standard theories of choice has yet to be characterized.

In the theoretical literature, Salant and Rubinstein [2006] also focus on data enrichment. They study choices made from sets presented in "list" order. In their main result, they assume that the order of the list is known to an outside observer, effectively making the order of search observable. In this setting, they characterize a choice procedure by which the list order is only used to break ties in the case of indifference. The tie can be broken either by choosing the first or last of the optimal objects in the list. By contrast, we treat search order as unobservable, and assume that people may not fully examine the available set.

Ours is not the first or only effort to bridge the gap between decision theory and search theory. An alternative approach is to identify restrictions on more standard choice data deriving from particular search procedures. Masatlioglu and Nakajima [2009] characterize choices that result when the search path that is adopted depends only on an initial (externally observable) reference point. Ergin [2003], Manzini and Marrioti [2007], and Ergin and Sarver [2009] also characterize the implications for standard choice of various decision making procedures that produce incomplete information. Masatlioglu, Nakajima and Ozbay [2009] identify objects that a decision maker is actively considering by assuming that the removal of unconsidered objects cannot affect choice. We believe that these various approaches are all worth pursuing, and that the intensification of interest among decision theorists in incomplete consideration of options is overdue.[5]

[4] More broadly, prior experimental work on search and choice has made use of data that is less readily related to choice: the time taken in arriving at a decision [Busermeyer and Townsend, 1992; Rustichini, 2008]; direct observation of the order of information search using Mouselab [Payne, Bettman and Johnson, 1993; Ho, Camerer, and Weigelt, 1998; Johnson et al., 2002; Gabaix et al., 2006]; eye movements [Wang, Spezio and Camerer, 2006]; and verbal responses [Ericsson and Simon, 1984].

[5] In addition to playing an essential role in search theory, the fact that decision makers effectively choose among a small subset of potentially available options is familiar in the marketing literature. One of the central challenges in marketing is how to get an option to be actively considered, rather than being rejected sight unseen. The literature on "consideration sets" reflects this focus on product awareness as a necessary prelude to product choice (e.g. Alba and Chattopadhyay [1985] and Roberts and Lattin [1991]). Eliaz and Speigler [2010] study the behavior of a firm that can use costly marketing devices to manipulate the consideration set of a consumer.

2 Alternative Based Search: The Deterministic Case

2.1 The Choice Process

In order to characterize our models of search, we use an enriched data set we call choice process data. Rather than recording only the alternative that is finally chosen by the DM, choice process data tracks how choice evolves with contemplation time. As such, choice process data comes in the form of sequences of observed choices. Let X be a nonempty finite set of elements representing possible alternatives, with denoting non-empty subsets of X. Let be the set of all infinite sequences from with generic element $Z = \; Z_t \; {}_1^\infty$ with $Z_t \in \; /\emptyset$ all $t \geq 1$. For $A \in$, define ${}_A \subset$ to comprise all such sequences selected from A,

$$_A = \; Z \in \; Z_t \subset A \text{ all } t \geq 1 \; .$$

Definition 1 A (deterministic) ***choice process*** (X, C) comprises a *fi*nite set X and a function, $C : \; \rightarrow$ such that $C(A) \in \; _A \; \forall \; A \in$.

Given $A \in$, choice process data assigns not just final choices (a subset of A), but a sequence of such choices, representing the DM's choices after considering the problem for different lengths of time. We let C_A denote $C(A)$ and $C_A(t) \in A$ denote the t-th element in the sequence C_A, with $C_A(t)$ referring to the objects chosen after contemplating A for t periods. Choice process data represents a relatively small departure from standard choice data, in the sense that all observations represent choices, albeit constrained by time.

2.2 ABS

Our first model captures the process of sequential search with recall, in which the DM evaluates an ever-expanding set of objects, choosing at all times the best object thus far identified. We say choice process data has an alternative-based search (ABS) representation if there exists a utility function and a non-decreasing search correspondence for each choice set such that what is chosen at any time is utility-maximizing in the corresponding searched set. To define this, we introduce ${}^{ND} \subset$, the non-decreasing sequences of sets in ,

$$^{ND} = \; Z \in \; Z_t \subset Z_{t+1} \text{ all } t \geq 1 \; .$$

Definition 2 Choice process (X, C) has an ***ABS*** representation (u, S) if there exists a utility function $u : X \to \mathbb{R}$ and a search correspondence $S : \quad \to \quad ^{ND}$, with $S_A \in \quad_A$ all $A \in \quad$, such that,

$$C_A(t) = \arg \max_{x \in S_A(t)} u(x).$$

The ABS model describes a DM who always chooses the best objects that they have searched. As time passes, objects are either searched, and so in $S_A(t)$, or not searched. All objects that are searched are compared in full according to a fixed utility function. Since the DM is assumed to recall all past searches, $S_A(t)$ is non-decreasing and the choice made by the DM weakly improves over time. It is this assumption that gives the concept of ABS empirical traction. Note that the ABS model makes no assumptions concerning how or why a decision maker decides to stop searching - there is no restriction on how the function S behaves in the limit. There is also no restriction on the first object searched, since it may be the only object identified.

Given that final choice of x over y is unrevealing with incomplete search, the ABS characterization relies on an enriched notion of revealed preference. To understand the required enrichment, it is useful to consider behavioral patterns that contradict ABS. To describe these patterns we use the notation $C(A) = B_1; B_2; ...; B_n!$ with $B_i \subset A$ to indicate that the sets $B_1, ..., B_n$ are chosen sequentially from A, with B_n being the final choice. We can readily identify four patterns of choice process data that contradict ABS.[6]

$$C^{\alpha}(\ x, y\ ) = x; y; x!$$

$$C^{\beta}(\ x, y\ ) = x;\ x, y\ ; y!$$

$$C^{\gamma}(\ x, y\ ) = y; x!;\ C^{\gamma}(\ x, y, z\ ) = x; y!$$

$$C^{\delta}(\ x, y\ ) = y; x!;\ C^{\delta}(\ y, z\ ) = z; y!;\ C^{\delta}(\ x, z\ ) = x; z!$$

C^{α} contains a preference reversal: the DM first switches to y from x. As y has been chosen by the DM, it must be in the searched set when they choose x, implying that x is preferred to y. However, the DM then switches back to y, indicating that y is preferred to x. C^{β} involves y first being revealed indifferent to x, as x and y are chosen at the same time. Yet later y is revealed to be strictly preferred to x as x is dropped from the choice set. In C^{γ} the direction in which preference

[6] We drop the braces around singleton sets: $x; y; x!$ conveys selection of choice sets $\{x\}$, $\{y\}$, and $\{x\}$.

is revealed as between y and x changes between the two element and three element choice set. C^{δ} involves an indirect cycle, with separate two element sets revealing x as preferred to y, y as preferred to z, and z as preferred to x.

As these examples suggest, the appropriate notion of strict revealed preference in the case of ABS is based on the notion of alternatives being replaced in the choice sequence over time. A DM who switches from choosing y to choosing x at some later time is interpreted by the ABS model as preferring x to y. As search is non-decreasing, the DM must be aware of y when they choose x. Thus the choice of x over y indicates revealed preference. Similarly, if we ever see x and y being chosen at the same time, it must be that the DM is indifferent between the two alternatives. We capture the revealed preference information implied by the ABS model in the following binary relations.

Definition 3 Given choice process (X, C), the symmetric binary relation $\sim$ on X is defined by $x \sim y$ if there exists $A \in$ such that $x, y \subset C_A(t)$ some $t \geq 1$. The binary relation ^{C} on X is defined by $x \; ^C \; y$ if there exists $A \in$ and $s, t \geq 1$ such that $y \in C_A(s)$, $x \in C_A(s+t)$ but $y \notin C_A(s+t)$.

For a choice process to have an ABS representation it is necessary and sufficient for the revealed preference information captured in ^{C} and $\sim$ to be consistent with an underlying utility ordering. Our characterization of ABS therefore makes use of Lemma 1, a standard result which captures the conditions under which an incomplete binary relation can be thought of as reflecting some underlying complete pre-order.[7] Essentially, we require the revealed preference information to be acyclic.

Lemma 1 Let P and I be binary relations on a finite set X, with I symmetric, and define PI on X as $P \cup I$. There exists a function $v : X \to \mathbb{R}$ that ***respects*** P and I:

$$\begin{aligned} xPy &\implies v(x) > v(y); \\ xIy &\implies v(x) = v(y); \end{aligned}$$

if and only if P and I satisfy ***OWC*** (only weak cycles): given $x_1, x_2, x_3, .., x_n \in X$ with $x = x_1 PI x_2 PI x_3 .. PI x_n = x_1$, there is no k with $x_k P x_{k+1}$.

[7] Note that Lemma 1 is a direct corollary of Theorem 2.6 in Bossert and Suzumura [2009].

Armed with this result, we establish in theorem 1 that the key to existence of an ABS representation is for $\ ^C$ and $\sim$ to satisfy OWC.[8] This OWC condition is closely related to the standard strong axiom of revealed preference. It is readily testable, and various metrics have been developed to measure how close a data set is to satisfying such conditions (see Dean and Martin [2009] for a review). Corollary 1, which is essentially immediate, characterizes equivalent representations of a choice process for which $\ ^C$ and $\sim$ satisfy OWC.

Theorem 1 Choice process (X, C) has an ABS representation if and only if $\ ^C$ and $\sim$ satisfy OWC.

Proof. By lemma 1, the result is equivalent to establishing that (X, C) admits an ABS representation if and only if there exists a function $v : X \to \mathbb{R}$ that respects $\ ^C$ and $\sim$ in the sense of the lemma. Certainly, if an ABS representation (u, S) exists, $x \sim y$ implies $u(x) = u(y)$ since both achieve the same maximum, while if $x \ ^C y$, then $u(x) > u(y)$ follows from $y \in C_A(s) \subset S_A(s) \subset S_A(s+t)$ with $t \geq 1$ in which $u(x)$ is maximal, while $u(y)$ is not. Conversely, if a function $v : X \to \mathbb{R}$ exists that respects $\ ^C$ and $\sim$ on X, we can define the expanding correspondence $S^* : \quad \mathbb{N} \to$ by,

$$S^*_A(t) = \cup_{s \leq t} C_A(s).$$

To show that (v, S^*) form an ABS representation of (X, C), we show that $C_A(t)$ comprises all elements maximal in $S^*_A(t)$ according to $v : X \to \mathbb{R}$. Note that if $x \in C_A(t)$, then $x \ ^C y$ or $x \sim y$ all $y \in S^*_A(t)$, whereupon $v(x) \geq v(y)$ follows from the fact that v respects $\ ^C$ and $\sim$ on X. Conversely, suppose that we can find $x \in S^*_A(t)$ satisfying $v(x) \geq v(y)$ all $y \in S^*_A(t)$ but with $x \notin C_A(t)$. In this case, all $y \in C_A(t)$ satisfy $y \ ^C x$, implying that $v(y) > v(x)$, which contradiction completes the proof. ■

[8] While their paper has a different set up, there is a natural relation between our OWC condition and the dominating anchor axiom in Masatlioglu and Nakajima [2009]. Under a natural translation between the two settings, OWC implies the dominating anchor axiom but not vice versa. Masatlioglu and Nakajima [2009] consider extended choice problems that map choice sets and a reference point to final choice. The dominating anchor axiom states that, for any set S, there exists a "best" option x such that, if x is the reference point and some element from S is chosen from set T, that element must be x itself. Our axiom implies this if we assume that the starting point is always searched. Under this condition, a violation of the dominating axiom would also lead to a violation of our OWC condition (as every item in the set S would have been revealed inferior to some other element in S). However, the dominating anchor axiom does not imply our OWC condition, as it has nothing to say about intermediate (i.e. non-final) choices.

Corollary 1 Utility function $v : X \to \mathbb{R}$ and search correspondence $S : \quad \to \quad ^{ND}$ form an ABS representation of (X, C) if

1. v respects $\quad ^C$ and $\sim$;
2. $\cup_{s \leq t} C_A(s) \subseteq S_A(t) \subseteq C_A(t) \cup \quad x \in X \; v(x) < v(y), y \in C_A(t)$ for all $A \in \quad$, $t \in \mathbb{N}$.

Note from corollary 1 that there are strong limits to what can be said about search order. It characterizes representations as involving a utility function v that respects $\quad ^C$ and $\sim$ on X, a search correspondence S that must include at least all objects which have been chosen from all sets A at times $s \leq t$, and that may also contain any additional elements that have utility strictly below that associated with chosen objects according to v. Hence all that can be definitely asserted is that items rejected along the path were searched. Items that are never chosen may or may not have been searched. This implies that the more switches there are between objects in the choice process data, the more restricted is the search order.[9]

Given that a utility function $v : X \to \mathbb{R}$ can form the basis for an ABS representation, note that any strictly increasing transform of v will still form an ABS representation in combination with precisely the same set of search correspondences. However, we can also change the function v in non-monotonic ways that do not contradict the information in $\quad ^C$ and $\sim$. For example, if $X = \quad a, b, c \quad$, and $\quad ^C$ contains only $\quad (a, b), (c, b) \quad$, while $\sim$ is empty, the consistent utility functions do not restrict the ranking of a against b, so that non-monotonic changes to the utility function may still form part of an ABS representation. However, corollary 1 states, the upper bound on what may be contained in $S_A(t)$ is determined by the set of objects that have utility lower than those being chosen from A at time t. Thus, non-monotonic changes in the utility function may change the set of permissible search functions.

[9] A reasonable prior, e.g. that search is in list order (Salant and Rubinstein [2008]), may enrich the inferences one can make from choice process data. This theory of search order would be supported if chosen options were only replaced by items higher in the list. Support would be even stronger if the selected options were the successive maxima in list order.

3 Reservation Based Search: The Deterministic Case

Since the ABS model says nothing about the stopping rule for search, we augment it with a simple "reservation utility" stopping rule in which search continues until an object is found which has utility above some fixed reservation level, whereupon it immediately ceases.[10] We believe that RBS is an interesting model in its own right, as many of the search models currently used within economic fall into this category. These include search models in labor economics and industrial organization, as well as the satisficing procedure first introduced by Simon [1955].

The key to the empirical content of RBS is that one can make inferences as to objects that must have been searched even if they are never chosen. Specifically, in any set in which the final choice has below reservation utility, it must be the case that all objects in the set are searched. Hence final choices may contain revealed preference information.

Intuitively, an RBS representation is an ABS representation (u, S) in which a reservation level of utility ρ exists, and in which the above- and below-reservation sets $X_u^\rho = \quad x \in X \ u(x) \geq \rho$ and $X \ \ X_u^\rho$ play critical roles. Specifically, search stops if and only if an above-reservation item is discovered, so that search is complete if there are no above-reservation items in available. In order to capture this notion formally, we define $C_A^L = \lim_{t \to \infty} C_A(t)$, as the final choice the DM makes from a set $A \in \quad$ as well as limit search sets $S_A^L \equiv \lim_{t \to \infty} S_A(t) \in \quad$. Note that, for finite X, the existence of an ABS representation guarantees that such limits are well defined.

Definition 4 Choice process (X, C) has a ***reservation-based search* (*RBS*)** representation (u, S, ρ) if (u, S) form an ABS representation and $\rho \in \mathbb{R}$ is such that, given $A \in \quad$,

R1 If $A \cap X_u^\rho = \emptyset$, then $S_A^L = A$.

R2 If $A \cap X_u^\rho = \emptyset$, then:

(a) there exists $t \geq 1$ such that $S_A(t) \cap X_u^\rho = \emptyset$;

(b) $S_A(t) \cap X_u^\rho = \emptyset \Longrightarrow S_A(t) = S_A(t + s)$ all $s \geq 0$.

[10] One can readily allow for reservation rules that condition on immediately observable features of the choice set, such as its cardinality. Tyson [2007] considers the implications for final choice of a reservation level that decreases as choice sets get larger. However, Tyson assumes that the observable data is the set of all above reservation objects in a particular set.

Condition R1 demands that any set containing no objects above reservation utility is fully searched. Condition R2(a) demands that search must at some point uncover an element of the above-reservation set if present in the feasible set. Condition R2(b) states that search stops as soon as reservation utility is achieved.

It should be noted that the RBS model only refines the behavioral implications of the ABS model by demanding both R1 and R2. With R1 alone, the RBS model imposes no additional behavioral restrictions, as any data that admits an ABS representation would also satisfy R1 if we set the reservation utility ρ such that $X_u^{\rho} = X$. Similarly, data that allows an ABS representation can also trivially satisfy R2 alone by setting ρ such that $X_u^{\rho} = \emptyset$.

As with the ABS model, the key to characterizing the RBS model is to understand the corresponding notion of revealed preference. As RBS is a refinement of ABS, it must be the case that behavior that implies a revealed preference under ABS also does so under RBS. However, the RBS model implies that some revealed preference information may also come from final choice, with sets that contain only below-reservation utility objects being completely searched.

The following cases that satisfy ABS but not RBS illustrate behaviors that must be ruled out:

$$C^{\alpha}(\ x, y\ ) = x; y!;\ C^{\alpha}(\ x, z\ ) = x!;\ C^{\alpha}(\ y, z\ ) = z!$$

$$C^{\beta}(\ x, y\ ) = x; y!;\ C^{\beta}(\ x, y, z\ ) = x!$$

In the first case, the fact that x was replaced by y in x, y reveals the latter to be preferred and the former to be below reservation utility. Hence the fact that x was chosen from x, z reveals z to have been searched and rejected as worse than x, making its choice from y, z contradictory. In the second, the fact that x is followed by y in the choice process from x, y reveals y to be preferred to x, and x to have utility below the reservation level (otherwise search must stop as soon as x is found). The limit choice of x from x, y, z therefore indicates that there must be no objects of above-reservation utility in the set. However, this in turn implies that the set must be fully searched in the limit, which is contradicted by the fact that we know y is preferred to x and yet x is chosen.

These examples indicate the additional revealed preference information inherent in the RBS model. Under an RBS representation, when a unique final choice is made from two objects $x, y \in X$

either of which has below reservation utility, then we can conclude that the chosen object is strictly preferred. To see this, suppose that y has below reservation utility. In this case if it is chosen over x it must be that x was searched and rejected. Conversely, suppose that x is chosen over y. In this case either x is above reservation, in which case it is strictly preferred to y, or it is below reservation, in which case we know that the entire set has been searched, again revealing x superior.

In order to use this insight to characterize when an RBS representation exists, we define a class of binary relations $\ _D^L$ on X for any set $D \in$. These binary relations capture the revealed preference information that would derive from final choice with D as the set of below-reservation utility objects. These binary relations $\ _D^L$ on X are then united with the information from $\ ^C$ to produce the new binary relation $\ _D^R$ which captures the revealed preference information from the RBS model under the assumption that D is the below reservation set.

Definition 5 Given a choice process model (X, C) and set $D \in$, the binary relation $\ _D^L$on X is defined by $x \ _D^L y$ if $x, y \ \cap D = \emptyset$, and there exists $A \in$ with $x, y \in A$, $x \in C_A^L$, yet $y \notin C_A^L$. The binary relation $\ _D^R$ is defined as $\ _D^L \cup \ ^C$, and $\succsim_D^R$ is defined as $\ _D^R \cup \sim$.

To identify conditions for an RBS representation we focus on identifying objects that must be below-reservation utility in any possible representation. As a first step, we know that an object must have utility below the reservation level if we see a DM continue to search even after they have found that object. We call such an object non-terminal.

Definition 6 Given choice process (X, C) define the non-terminal set $X^N \subset X$

$$X^N = \ x \in X \ \exists A \in \quad \text{s.t. } x \in C_A(t) \text{ and } C_A(t) = C_A(t+s) \text{ some } s, t \geq 1$$

Using this concept, Proposition 1 characterizes the below-reservation sets that admit an RBS representation. The result establishes that below reservation sets must satisfy three properties. First, they must contain all non-terminal elements. Second, they must be closed under $\succsim_D^R$: if x is below-reservation, and is revealed at least as good as y, then y must also be below reservation. Third, $\ _D^R$ and $\sim$ must satisfy condition OWC. We prove the proposition in appendix 1.

Proposition 1 A choice process model (X, C) admits an RBS representation with below reservation set D if and only if:

1. $X^N \subset D$.

2. If $x \in D$ and $x \succsim^R_D y$, then $y \in D$.

3. ${}^R_D$ and $\sim$ satisfy OWC.

A necessary and sufficient condition for an RBS representation is therefore that there is some set D that satisfies these conditions. Note that if the third condition is satisfied for some set D, it will be satisfied for any $D^* \subset D$: if $D^* \subset D$, then ${}^R_D$ contains ${}^R_{D^*}$, so that if ${}^R_D$ (along with $\sim$) satisfies OWC, then so will ${}^R_{D^*}$. Thus the relevant necessary and sufficient condition is that the revealed preference information generated by the smallest below-reservation set that satisfies 1 and 2 satisfies OWC.

To identify such a set, we introduce the indirectly non terminal set. This is the set of object in X that are either directly revealed as non-terminal, or are revealed as inferior to a non-terminal object.

Definition 7 Given choice process (X, C) de$fi\phi \sim D$

Corollary 2 A utility function $v : X \to \mathbb{R}$, reservation level ρ, and $S : \quad \to \quad ^{ND}$ form an RBS representation of a choice process if and only if

1. $D = \quad x \in X \; v(x) < \rho$ satisfies the properties of proposition 1.

2. v respects $\; {}^{R}_{D}$ and $\sim$.

3. $\cup_{s \leq t} C_A(s) \subseteq S_A(t) \subseteq C_A(t) \cup \quad x \in X \; v(x) < v(y), y \in C_A(t)$ for all $A \in \quad$, $t \in \mathbb{N}$.

4. $S_A(t) \cap X_u^{\rho} = \emptyset \Longrightarrow S_A(t) = S_A(t+s)$ all $s \geq 0$.

4 The Stochastic Model

The ABS and RBS models both treat search order as unobservable, and characterize the extent to which it is recoverable from choice process data. This makes it natural to develop stochastic variants, since there is no reason to believe that search from a given set will always take place in the same order. We therefore generalize the deterministic models of section 2 and 3 to allow for stochasticity. This allows us to develop stochastic versions of the RBS and ABS models, in which choice is generated f o41-343.7el

Definition 9 A ***stochastic choice process*** $(X, \tilde{C})$ comprises a finite set X and a function $\tilde{C}$: $\rightarrow$ such that $\tilde{C}_A \equiv \tilde{C}(A)$ has support $_A \subset$.

As for the deterministic case, a stochastic choice process has an ABS representation if it can be viewed as resulting from maximization of a utility function in the context of some process of search, with the searched set never shrinking. However we allow the search process to be stochastic. We will use $\tilde{S}$: $\rightarrow$ ND to denote a stochastic search function, where: $^{ND} \subset$ identify probability measures on (,) with support ND, the non-decreasing elements of . Given $A \in$ and $F \in$, let $\tilde{C}_A(F)$, $\tilde{S}_A(F)$ respectively denote the measure assigned to F by $\tilde{C}(A)$, $\tilde{S}(A)$.[11]

Definition 10 Stochastic choice process $(X, \tilde{C})$ has a ***stochastic ABS*** representation $(u, \tilde{S})$ if there exists $u : X \rightarrow \mathbb{R}$ and $\tilde{S}$: $\rightarrow$ ND such that $\tilde{C}$ is the stochastic choice process derived by optimizing u against $\tilde{S}$,

$$\bar{C}_A(F) = \tilde{S}_A\left(\left\{Z \in \quad \left\{\arg \max_{x \in Z_t} u(x)\right\}_{t=1}^{\infty} \in F\right\}\right), \text{ all } A \in \quad , F \in \quad .$$

The theorem that characterizes the stochastic ABS representation is essentially identical to that in the deterministic case. It simplifies notation to define join and replacement sets $J^{xy}, R^{xy} \subset$ for $x, y \in X$, where J^{xy} is the set of choice processes in which x and y are chosen at the same time, while R^{xy} are those in which y is replaced by x.

$$\begin{aligned} J^{xy} &= \quad Z \in \quad x, y \quad \subset Z_t \text{ some } t \geq 1 \quad ; \\ R^{xy} &= \quad Z \in \quad y \in Z_s,\ x \in Z_{s+t},\ y \notin Z_{s+t} \text{ some } s, t \geq 1 \quad ; \end{aligned}$$

Measurability of $J^{xy}, R^{xy} \subset$ is established in appendix 2.

For purposes of establishing the stochastic ABS representation, we define x to be revealed strictly preferred to y if R^{xy} has strictly positive measure, and x to be revealed indifferent to y if the set J^{xy} has strictly positive measure.

Definition 11 Given stochastic choice process $(X, \tilde{C})$, the binary relation $\sim^{\tilde{C}}$ on X is defined by $x \sim^{\tilde{C}} y$ if there exists $A \in$ with $x, y \in A$ and $\tilde{C}_A(J^{xy}) > 0$. The binary relation $^{\tilde{C}}$on X is defined by $x \quad ^{C} y$ if there exists $A \in$ with $x, y \in A$ and $\tilde{C}_A(R^{xy}) > 0$.

[11] That the set of $Z \in \mathcal{Z}$ with $\arg\max_{x \in Z_t} u(x)_{t=1}^{\infty} \in F$ is measurable is demonstrated in appendix 2.

As before, the condition for the characterization is that this revealed preference information is consistent with a fixed underlying utility function.

Theorem 3 Stochastic choice process $(X, \tilde{C})$ has a stochastic ABS representation $(u, \tilde{S})$ if and only if $^{\tilde{C}}$ and $\sim^{\tilde{C}}$ satisfy OWC.

4.2 RBS

As in the deterministic case, the definition of a stochastic RBS representation requires the analysis of limit behavior. Given $B \in$, we define L^B to be the -measurable subset of with limit B,

$$L^B = \left\{ Z \in \quad \lim_{t \to \infty} Z_t = B \right\}.$$

In appendix 2 it is shown that a stochastic choice process model $(X, \tilde{C})$ with stochastic ABS representation $(u, \tilde{S})$ necessarily assigns full measure to the set in which limits exist,

$$\tilde{C}_A \left\{ \cup_{B \in \mathcal{X}} L^B \right\} = 1.$$

Hence, given a stochastic choice process model $(X, \tilde{C})$ with stochastic ABS representation $(u, \tilde{S})$ and $A \in$, we can define limit choice and search probability measures $\tilde{C}^L_A, \tilde{S}^L_A$ on endowed with the discrete sigma-algebra,

$$\tilde{C}^L_A(B) = \tilde{C}_A(L^B) \text{ and } \tilde{S}^L_A(B) = \tilde{S}_A(L^B) \text{ any } B \in \quad .$$

As in the deterministic case, the definition of stochastic RBS involves a utility function $u : X \to \mathbb{R}$ and a level of reservation utility ρ which together identify above reservation set $X^\rho_u \equiv$ $x \in X \; u(x) \geq \rho$. Given $Z \in$, a key random variable in the stochastic RBS representation is the first time that reservation utility is hit. To simplify notation in the stochastic version of RBS, we let $H^\rho_u : \quad \longrightarrow \mathbb{N} \cup \infty$ denote this first hitting time associated with utility function u and reservation utility level ρ,

$$H^\rho_u(Z) = \begin{cases} \inf_{t \geq 1} \quad Z_t \cap X^\rho_u \quad = \emptyset, \text{ if } \quad Z_t \cap X^\rho_u \quad = \emptyset \text{ some } t; \\ \infty \text{ otherwise.} \end{cases}$$

That hitting times are -measurable functions is standard.

We use the notion of hitting times to define the stochastic version of the RBS model.

Definition 12 Stochastic choice process $(X, \tilde{C})$ has a ***stochastic RBS*** representation $(u, \tilde{S}, \rho)$ if $(u, \tilde{S})$ form a stochastic ABS representation and $\rho \in \mathbb{R}$ is such that, given $A \in$,

RS1 If $A \cap X_u^\rho = \emptyset$, then $\tilde{S}_A^L(A) = 1$

RS2 If $A \cap X_u^\rho = \emptyset$, then:

(a) $\tilde{S}_A \quad Z \in \quad H_u^\rho(Z)$ is finite $= 1$;

(b) $\tilde{S}_A \left\{ Z \in \quad \tilde{S}_A^L = \tilde{S}_A\left(H_u^\rho(Z)\right) \right\} = 1.$

As with ABS, the stochastic RBS characterization is the precise analog of the deterministic version, and relies on the identification of directly and indirectly non-terminal sets. We define $\Delta^y \subset$ to be the set of sequences in which $y \in X$ appears at some point, but the sequence changes thereafter. Measurability is established in appendix 2.

Definition 13 Given stochastic choice process $(X, \tilde{C})$, define the ***non-terminal set*** $\tilde{X}^N \subset X$ as,

$$\tilde{X}^N = \left\{ x \in X \ \exists A \in \quad \text{with } x \in A \text{ and } \tilde{C}_A(\Delta^x) > 0 \right\}.$$

Define the ***indirectly non-terminal*** set $\tilde{X}^{IN}$ as $\tilde{X}^N$and elements rejected with positive probability in favor of an element of X^N,

$$\tilde{X}^{IN} = \tilde{X}^N \cup \ \ x \in X \ \ \exists A \in \quad , y \in \tilde{X}^N \text{ with } x, y \in A \text{ and } \tilde{C}_A^L(\ y \ ) > 0 \ .$$

The definition of revealed preference in the stochastic RBS model can now proceed in line with the deterministic case.

Definition 14 Given stochastic choice process $(\tilde{X}, C)$, the binary relation $\quad^{\tilde{L}}$ on X is defined by $x \quad^{\tilde{L}} y$ if $\ x \cup y \ \cap \tilde{X}^{IN} = \emptyset$, and there exists $A \in$ with $x, y \in A$ with $\tilde{C}_A^L \ \ x \ \ > 0$ and $\tilde{C}_A(J^{xy}) = 0$. Binary relation $\quad^{\tilde{R}}$ is defined as $\quad^{\tilde{L}} \cup \quad^{\tilde{C}}$.

Using this definition, the standard application of Lemma 1 characterizes existence of an RBS representation.

Theorem 4 Stochastic choice process $(X, \tilde{C})$ has a stochastic RBS representation $(u, \tilde{S}, \rho)$ if and only if $\quad^{\tilde{R}}$ and $\sim^{\tilde{C}}$ satisfy OWC.

4.3 Sketch of Proofs

The proofs of theorem 3 and of theorem 4 are detailed in appendix 3. We limit ourselves in this discussion to presenting structural elements. Both proofs work by reducing the stochastic case to its deterministic counterpart. The key step involves showing that nothing is lost by "compressing" choice process data by removing time periods in which choice does not change.

Definition 15 Stochastic choice process $(X, \tilde{C})$ is compressed if $\tilde{C}_A(\ ^{COM}) = 1$ for all $A \in$, where,

$$^{COM} \equiv \ Z \in \quad Z_t = Z_{t+1} \Longrightarrow Z_t = Z_{t+s} \text{ all } s \geq 1 \ .$$

In the first step of the reduction, a given stochastic choice process $(X, \tilde{C})$ is associated with a unique compressed choice process by removing all periods of constancy (see appendix 3 for details). The process of compression reduces to equivalence an infinite number of choice processes differing only in the delay between switches.

The first observation that makes compression of value is the invariance of key properties under compression and its inverse, decompression. It is immediate that the orderings $^{\tilde{R}}$, $^{\tilde{C}}$and $\sim^{\tilde{C}}$ are preserved under both operations. It is equally immediate that ABS and RBS survive both under compression and decompression, since one uses exactly the same utility function and reservation utility in the representation of the original process and its transformation, using compression only to change the search correspondence by removing repetition in the case of compression, and inverting suitably in the process of decompression.

The second observation that makes compression of value is that any compressed process that satisfies ABS is "finite", in that only a finite number of sequences have strictly positive probability. Conversely, any compressed stochastic choice process for which $^{\tilde{C}}$ and $\sim^{\tilde{C}}$ satisfy OWC is finite. While the formal definitions and proof are in appendix 3, the intuition is simple. Both ABS and OWC imply that a compressed stochastic choice process must stop changing within a number of periods that matches the cardinality of the power set of .

The bottom line of this reduction process is that the proofs in of theorems 3 and 4, detailed in the appendix, are provided only for finite models, with the extension to the general case being immediate. The critical observation in establishing the finite case is that any finite stochastic

choice processes $(X, \tilde{C})$ can be identified with an appropriately defined convex combinations of deterministic choice processes.

5 RBS and Non-Standard Behavior

The stochastic RBS model allows for two channels by which seemingly unimportant changes in the decision making environment might lead to changes in the choices people make. First, they may impact the probability distribution over paths of search. Second, they may impact the level of reservation utility. These changes can, in turn, lead to framing effects, status quo bias and stochastic choice of a specific form that we now characterize.

5.1 Framing Effects

To model framing effects, let Γ comprise abstract elements $\gamma \in \Gamma$ that we refer to as frames. For example, these frames may represent different ways in which objects are physically displayed to the DM. Let $\Phi : \Gamma \to$ ¯ be a mapping from frames to the class ¯ of stochastic choice processes on (,), with $\Phi(\gamma)$ the process associated with $\gamma \in \Gamma$. We seek to characterize data sets in which all choice processes regardless of frame can be derived from a common underlying utility function but with frame-specific search orders and reservation utilities. Such a characterization is experimentally useful, since it indicates conditions under which one can derive information on preferences in a low search cost (hence high reservation utility) environment that will apply equally in a higher search cost (hence lower reservation utility) frame in which choice process data yields less direct evidence on preferences. It turns out that we need to apply OWC to a binary relation that appropriately unifies revealed preference information across frames. In the statement, ¯ denotes the set of all stochastic search processes on (,).

Definition 16 Define $x \;\; ^{\tilde{R}(\Gamma)} \; y$ if $x \;\; ^{\tilde{R}} \; y$ according to some stochastic choice process $\Phi(\gamma)$ for some $\gamma \in \Gamma$. Similarly define $x \sim^{\tilde{C}(\Gamma)} y$ if $x \sim^{\bar{R}} y$ according to some stochastic choice process $\Phi(\gamma)$ for some $\gamma \in \Gamma$.

Theorem 5 Given finite set X, frames Γ, and $\Phi : \Gamma \to$ ¯, there exists a utility function $u : X \to \mathbb{R}$, a family of reservation utilities $\rho : \Gamma \to \mathbb{R}$, and family of stochastic search processes $\Theta : \Gamma \to$ ¯ such

that $(u, \Theta(\gamma), \rho(\gamma))$ forms a stochastic RBS representation of $\Phi(\gamma)\ \forall\ \gamma \in \Gamma$ if and only if $^{\tilde{R}(\Gamma)}$ and $\sim^{\tilde{C}(\Gamma)}$ satisfy OWC.

5.2 Status Quo Bias

One particular class of framing effect that can be explored using the RBS model is status quo bias - the increased likelihood of selecting a particular object simply because it is the status quo, or currently selected option [Samuelson and Zeckhauser, 1988]. We can model such behavior as a framing model in which each status quo gives rise to its own frame. In order to capture status quo bias, we posit that the status quo object is always the first object searched in any choice environment.

Under this assumption, the stochastic RBS model makes particular predictions about how status quo will affect choice. For above-reservation utility objects, status quo bias will be complete: when such objects are the status quo then they will always be chosen, as the DM is immediately aware of their existence and will indulge in no further search. However, if the status quo object is below reservation utility then it will not be chosen unless it is the highest utility object in the choice set, in which case it will be chosen regardless of the status quo, as the stochastic RBS model implies that search will be complete in such cases. Thus, the RBS model implies a form of status quo bias that has two extremes: either an object will always be chosen when it is the status quo, or the status quo will have no effect.

5.3 Stochastic Choice

It is clear that the stochastic RBS model can give rise to stochastic choice in the form of a probability distribution over final choices. Even with a fixed utility function, final choice will be random if the order of search is random and search is incomplete. However this distribution will be of a particular form: choice may be stochastic among above reservation objects, while objects with below reservation utility are never chosen. In the simplest possible case with all search orders being equally probable, final choice is deterministic and consistent for choice sets made up only of below-reservation items, whereas for choice sets containing above-reservation items, there is an equal chance of choosing any such item. Observed stochasticity in choice will therefore increase as reservation utility falls.

6 Eliciting Choice Process Data in the Laboratory

For the above results to advance our understanding of incomplete search and choice one must be able to experimentally identify the path of provisional choices over the pre-decision period. We sketch the approach that Caplin, Dean and Martin [2009] (CDM) use to generate just this data, and describe results for a highly stylized experiment.

Subjects in the experiment were presented with various subsets of a larger choice set, from each of which they had to make a choice. They were given a fixed time window within which to choose from among each fixed set of available alternatives. They were allowed to select any alternative at any point in a fixed time window.[12] They were informed that they could change the selected alternative whenever they wished. Rather than being based on final choice alone, actualized choice was recorded at a random point in the given time window that was only revealed at the end of the experiment. This incentivized subjects to always have selected their current best option in the choice set. It is for this reason that we interpret the sequence of selections as comprising provisional choices.[13]

Our first experiment using this interface was deliberately stark, missing the conflicting priorities that may typify more intricate decisions. The objects of choice were kept as simple as possible, and subject to clear and universal preferences: all options were deterministic dollar amounts. To render the problem non-trivial, the dollar amount for each option was represented as a sequence of addition and subtraction operations. The simplicity of the setting enabled us to explore the ABS and RBS models in an uncluttered and "friendly" experimental context.

Each experimental round began with the topmost, and worst, option of $0 selected.[14] Subjects could at any time select any of the alternatives on the screen, with the currently selected object

[12] As with tests of standard choice theory, this experiment uncovers only one most preferred element rather than all such elements. This opens some daylight between the theoretical definition of choice process data and the experimental data.

[13] In support of this interpretation, 58 of 76 subjects in a post-experiment survey responded directly that they always had their most preferred option selected, while others gave more indirect responses that suggest similar behavior (e.g. having undertaken a re-calculation before selecting a seemingly superior alternative).

[14] The subjects knew that the $0 option was the worst in the choice set. They therefore had the incentive to immediately change their selection, which is consistent with the ABS model with this being the only object searched. The model is restrictive only when a switch is made, at which point it implies that the object switched to is of higher value.

being displayed at the top of the screen. In each round there was a time constraint, with subjects having up to 120 seconds to complete the choice task (though this constraint was only binding in about 5% of rounds). A subject who finished in less than 120 seconds could press a submit button, which completed the round as if they had kept the same selection for the remaining time. Treatments were run varying in the number of alternatives available and in the complexity of each alternative.

As one might have expected, the experiment provided support for ABS-style search. Subjects made several selections in the course of a round and generally switched from lower value to higher value objects over time. In the context of the experiment this is equivalent to finding positive support for the ABS model of search. A more striking finding was that behavior was well approximated by the RBS model. While behavior did change as the number of available options and their level of complexity was varied, it did so within the RBS framework. The results suggest that choice process data is of more than theoretical interest.

7 Concluding Remarks

Incomplete information may explain many apparent deviations from utility maximizing behavior. Standard choice data does not allow one to pin down when such deviations are caused by changing preferences, and when they result from incomplete information. We develop clean procedures for accomplishing this separation by expanding beyond standard choice data to include data on the evolution of choice with time. We characterize standard alternative-based and reservation-based procedures that are ubiquitous in search theory. Experimental investigation of choice process data is ongoing.

8 Appendix 1: RBS

Proof of Proposition 1 To prove sufficiency, we note from lemma 1 that (3) implies existence of $u : X \to \mathbb{R}$ that respects $\overset{R}{\underset{D}{}}$ and $\sim$ on X. Define

$$\rho = \frac{\max_{x \in D} u(x) + \min_{x \in X \setminus D} u(x)}{2}.$$

Note from (2) that $C^L \;\; x, y \;\; = y$ whenever $y \in X \;\; D$ and $x \in D$, implying $y \;\;\; {}^R_D \; x$ and $u(y) > u(x)$ and hence that $X/D = X^\rho_u$. Mimicking the proof of theorem 1, one can then define a search correspondence such that (u, S) that together form an ABS representation.

$$S_A(t) = \begin{cases} \cup_{s \leq t} C_A(s) \text{ for } t < T(A); \\ \cup_{s \leq T(A)} C_A(s) \cup L(A) \text{ for } t \geq T(A); \end{cases}$$

where $T(A) \equiv \min \;\; t \geq 1 \; C_A(t) = C^L_A \;\;)$ is the time at which choice first achieves its limit and $L(A)$ comprises all elements of A with utility strictly below $\max_{x \in C^L_A} u(x)$. We now show that all requirements for (u, S) and ρ together to form an RBS representation with reservation set $X \;\; D$ are met:

R1: When $A \cap X^\rho_u = \emptyset$, and so $A \subset D$, we know that $x \in C^L_A$, $y \notin C^L_A \Longrightarrow x \;\;\; {}^L_D \; y$, so that $u(x) > u(y)$. Hence $C^L_A = \arg\max_{\{x \in A\}} u(x)$ with $S^L_A = A$ by construction.

R2(a): If $A \cap X^\rho_u = \emptyset$ and so $A \cap X \;\; D = \emptyset$, then $C^L_A \cap D = \emptyset$ since $x \in C^L_A \cap D$, $y \notin C^L_A \Longrightarrow u(x) > u(y)$ contradicting the fact that utility is strictly higher on $X \;\; D$ than on D. Hence there exists $t \geq 1$ such that $C_A(t) \cap X^\rho_u = \emptyset$.

R2(b): If $C_A(t) \cap X^\rho_u = \emptyset$, then $C_A(t) \cap X^N = \emptyset$ by (1), implying directly that $C_A(t+s) = C_A(t)$ all $s \geq 1$, by construction, it is therefore the case that $S_A(t+s) = S_A(t)$ all $s \geq 1$ as required.

That condition (1) of the proposition is necessary for an RBS representation follows directly from property R2(b) of RBS definition, which implies that $X^N \subset D$ is required for D to be a reservation set. Given lemma 1, to prove that (3) is necessary it suffices to show that u represents $\;\; {}^R_D$ and $\sim$ in any RBS representation (u, S, ρ), where $D = X \;\; X^\rho_u$ and X^ρ_u is the corresponding reservation set. The fact that u represents $\;\; {}^C$ and $\sim$ is direct since (u, S) form an ABS representation of (X, C). To see that $\;\; {}^L_D$ is respected, suppose to the contrary that $x \;\;\; {}^L_D \; y$ but $u(y) \geq u(x)$. Note in this case that $x \in D$, since $y \in D \Longrightarrow x \in D$ and $\;\; x \cup y \;\; \cap D = \emptyset$ by definition of $x \;\;\; {}^L_D \; y$. But then by R1, $x \in C^L_A \Longrightarrow C^L_A = \arg\max_{x \in A} u(x)$ hence $u(y) < u(x)$ since $y \notin C^L_A$. This contradiction establishes that u indeed represents $\;\; {}^R_D$ and $\sim$. With this we know that condition (2) of the proposition is necessary, since $x \in D \Longrightarrow u(x) < \rho$ whereupon $x \succsim^R_D y$ implies $u(y) < \rho$, hence $y \in D$, completing the proof.

Proof of Theorem 2 To prove sufficiency, we show that the conditions of the proposition are satisfied in this case for $D = X^{IN}$. For (1) and (3) this is direct. Hence it suffices to establish

that if $x \in X^{IN}$ and $x \succsim^R y$, then $y \in X^{IN}$. By definition $x \in X^{IN}$ implies that we can find $z \in X^N$ with $z \quad {}^{L} x$. Now, if $C^L_{\{y,z\}} = y$, we have that $x \succsim^R y$

Note that:

$$
\begin{aligned}
J^{xy} &= \cup_t \quad {}_{\{x,y\}}(t) \in \quad ; \\
R^{xy} &= \cup_{t=1}^{\infty} \left\{ \quad {}_{\{y\}}(t) \cap \left\{ \cup_{s=1}^{\infty} \left\{ \quad {}_{\{y\}}^{C}(t+s) \cap \quad {}_{\{y\}}(t+s) \right\} \right\} \right\} \in \quad ; \\
\Delta^{x} &= \cup_{t=1}^{\infty} \left\{ \cup_{B \in W_{\{x\}}} \ \cup_{s=1}^{\infty} \ Z \in \quad Z_t = B, Z_{t+s} = B \quad \right\} \in \quad .
\end{aligned}
$$

${}^{NCY} = \quad Z \in \quad Z_{t+1} = Z_t \implies Z_{t+s} = Z_t$ any $s \geq 1$: (see appendix 3). First, index all sets in , $A_1, , A_m, .., A_M$, with M the cardinality of . Define $\Pi(M)$ to be all permutations of the first $m \leq M$ integers. Given $\pi^m \in \Pi(M)$, define the countable set $\Upsilon(\pi^m)$ to comprise all strictly increasing sets of m natural numbers,

$$\Upsilon(\pi^m) = \quad T^m = \quad T_1^m, T_2^m, .., T_m^m) \ T_1^m = 1, \ T_i^m \in \mathbb{N} \text{ and } T_i^m < T_{i+1}^m \text{ all } i \geq 1 \ .$$

That ${}^{NCY} \in$ follows since it is a countable union of cylinder sets,

$$\cup_{\pi^m \in \Pi(M)} \cup_{T^m \in \Upsilon(\pi^m)} \left\{ Z \in \quad Z_t = A_{\pi_i^m} \text{ for } T_i^m \leq t < T_{i+1}^m, \ 1 \leq i \leq m-1; \ Z_t = A_{\pi_m^m} \text{ for } t \geq T_m^m \ \right\}.$$

(Y) : (see appendix 3). Given K non-negative integers s_k define $S_0 = 0$ and partial sums $S_k = \sum_{j=1}^{k} s_j$ enabling the following short definition:

$$(Y) = \cap_{K=1}^{\infty} \left\{ \cup_{s_K=1}^{\infty} .... \left\{ \cup_{s_1=1}^{\infty} \ Z \in \quad Z_\tau = Z_k \text{ for } S_{k-1} + 1 \leq \tau \leq S_k \text{ and } 1 \leq k \leq K \ \right\} \right\} \in \quad .$$

Proposition 2 If $(X, \tilde{C})$ permits of a stochastic ABS representation $(u, \tilde{S})$, then for any $A \in$,

$$\tilde{C}_A \left\{ \cup_{B \in \mathcal{X}} L^B \right\} = 1.$$

Proof. Since $(X, \tilde{C})$ has an ABS representation $(u, \tilde{S})$, we know that $\tilde{S}_A(\quad ^{ND}) = 1$. Note that since is finite, limit elements exist for all $Z \in \quad ^{ND}$, establishing that $\tilde{S}_A \left\{ \cup_{B \in \mathcal{X}} L^B \right\} = 1$. Now note that if $Z \in \cup_{B \in \mathcal{X}} L^B$, then $\arg \ \max_{x \in Z_t} u(x) \ {}_{t=1}^{\infty} \in \cup_{B \in \mathcal{X}} L^B$, as, $Z \in \cup_{B \in \mathcal{X}} L^B$ implies that there must be some t such that $Z_t = Z_{t+s} \ \forall \ s \geq 0$, thus it must be the case that $\arg \max_{x \in Z_t} u(x) = \arg \max_{x \in Z_t} u(x) \ \forall \ s \geq 0$. Hence,

$$\tilde{C}_A \left\{ \cup_{B \in \mathcal{X}} L^B \right\} = \bar{S} \left\{ Z \in \quad \left\{ \arg \ \max_{x \in Z_t} u(x) \right\}_{t=1}^{\infty} \in \cup_{B \in \mathcal{X}} L^B \right\} \geq \bar{S} \left\{ \cup_{B \in \mathcal{X}} L^B \right\} = 1.$$

■

10 Appendix 3: Theorems 3 and 4

We first formally define compression, from which it follows immediately that it is sufficient to prove theorems 3 and 4 for compressed stochastic choice processes. We then show that compressed stochastic choice processes of interest are finite, further simplifying the requirements to establishing 3 and 4 for finite stochastic choice processes. Next, we show that finite stochastic choice processes can be represented as weighted averages of deterministic processes. We close out by proving theorems 3 and 4 for the finite case, which proof is general in light of the earlier results.

10.1 Compression

Definition 17 Given $Z \in$, define the set of times at which Z changes in sequential fashion starting with $\tau_1(Z) = 1$ as follows;

$$\tau_{j+1}(Z) = \begin{cases} \min_{s \geq 1} \ Z_{\tau_j(Z)+s} = Z_{\tau_j(Z)} & \text{if } \exists s \geq 1 \text{ s.t } Z_{\tau_j(Z)+s} = Z_{\tau_j(Z)}; \\ \infty \text{ if } Z_{\tau_j(Z)+s} = Z_{\tau_j(Z)} \text{ all } s \geq 1. \end{cases}$$

Let $J(Z) \in \mathbb{N} \cup \infty$ be the number of distinct points of change, and define the compression of any element $Z \in$, $D(Z) \in \ ^{COM}$, by removing all time indices in which there is repetition and repeating the limit element if there is any repetition,

$$D(Z) = \begin{cases} (Z_{\tau_1(Z)}, ..., Z_{\tau_j(Z)}, ..Z_{\tau_{J(Z)}(Z)}, ...Z_{\tau_{J(Z)}(Z)}, ..Z_{\tau_{J(Z)}(Z)}) \text{ if } J(Z) \text{ is finite}; \\ (Z_{\tau_1(Z)}, ..., Z_{\tau_j(Z)}, ..) \text{ if } J(Z) = \infty. \end{cases}$$

Given $Y \in \ ^{COM}$, define the equivalence classes of compressed elements of $(Y) \subset$ ((the proof that $(Y) \in$ is in appendix 2),

$$(Y) = \ Z \in \quad D(Z) = Y \ .$$

Given a measure $P \in$, we define its compression $D^P \in$ by shifting probabilities onto the compressed representative of each equivalence class,

$$D^P(Y) = \begin{cases} P(\ (Y)) \text{ for } Y \in \ ^{COM}; \\ 0 \text{ for } Y = \quad ^{COM}. \end{cases} .$$

10.2 Compression and Finiteness

Proposition 3 A compressed SCP that has an ABS representation or for which $^{\tilde{C}}$ and $\sim^{\tilde{C}}$ satisfy OWC is ***finite***, in that there exists a finite set $G \in$ such that $\tilde{C}_A(G) = 1$ all $A \in$.

Proof. To show that compression and ABS imply that the SCP is finite, let $M =$ and let $(M) \in$ be sequences that are unchanging after period M :

$$(M) = \quad Z \in \quad Z_t = Z_s \; \forall \; t, s > M \quad .$$

It is intuitive that a compressed choice sequence with an ABS representation satisfies $\bar{C}_A(\quad (M)) = 1 \; \forall \; A \in$. To confirm, consider the union of all cylinder sets with $Z_t = Z_s$ some $t, s > M$. If any element Z in this set is to be in ^{COM}, it must be the case that, for some $r, w < s$, $Z_r = Z_w$ and $r = w \quad 1$. Consider now the cylinder sets defined by,

$$Z \in \quad Z_t = Z_s, \, Z_r = Z_w \quad .$$

Now take any k such that $r < k < w$. and consider the cylinder set

$$Z \in \quad Z_t = Z_s, \, Z_k = Z_r = Z_w \quad .$$

These cylinder sets must have measure zero in any choice process that has an ABS representation, as the set of search sequences such that

$$\arg \max_{x \in S_A(k)} u(x) = \arg \max_{x \in S_A(r)} u(x) = \arg \max_{x \in S_A(w)} u(x),$$

is measure zero (as any such sequence would be non-increasing). As (M) can be obtained by the repeated countable union across $Z \in \quad Z_t = Z_s, \, Z_r = Z_w$, we know that if a choice process is compressed and has an ABS representation $\tilde{C}_A(\quad (M)) = 0 \; \forall \; A \in$, and so $\bar{C}_A(\quad (M)) = 1$. This in turn proves that $(X, \tilde{C})$ is finite.

To prove that a compressed SCP that satisfies for which $^{\tilde{C}}$ and $\sim^{\tilde{C}}$ satisfy OWC is finite, note that this implies that the associated choice process must apply full measure to ^{NCY}, those elements of in which there are no cycles (the proof that ^{NCY} is measurable is in appendix 2),

$$^{NCY} = \quad Z \in \quad Z_{t+1} = Z_t \implies Z_{t+s} = Z_t \text{ any } s \geq 1 \quad \in \quad .$$

To see why $\succsim_{\tilde{C}}$ satisfying OWC implies that $\tilde{C}_A(\quad ^{NCY}) = 1$ for any set $A \in X$, assume to the contrary that there is a set of strictly positive measure according to some $A \in X$ such that $Z_{t+1} = Z_t$, yet $Z_{t+s} = Z_t$ for some $s \geq 1$. There are two possibilities. One is that there is an element $y \in Z_{t+1}$ with $y \notin Z_t$: in this case consider any $x \in Z_{t+1}$, and note that $\tilde{C}_A(R^{xy}) > 0$ due to exit of element y and entry of element x from period $t+1$ to period $t+s$, while also one of the statements $\tilde{C}_A(R^{yx}) > 0$ or $\tilde{C}_A(J^{yx}) > 0$ in consideration of the entry of y in period $t+1$.

In the former case, the contradiction to $\succsim_{\tilde{C}}$ satisfying OWC is that $x \quad ^{\tilde{C}} y$ and $y \quad ^{\tilde{C}} x$, while in the latter case the contradiction is that $x \quad ^{\tilde{C}} y$ and $y \sim^{\tilde{C}} x$. Alternatively, it could be that there is some $y \in Z_t$ and $y \in Z_{t+1}$. A similar argument shows that this violates $\succsim^{\tilde{C}}$ satisfying OWC . This establishes the required finiteness, since elements of $\quad ^{COM} \cap \quad ^{NCY}$ are unchanging after a number of periods no larger than the cardinality of , completing the proof. ■

10.3 Structure of The Finite Case

Proposition 4 A stochastic choice process $(X, \tilde{C})$ is finite if and only if it is the convex combination of a finite number of deterministic choice processes, in that there exist some J deterministic choice processes $\left\{(X, C^j)\right\}_{j=1}^J$ and weight vector $\lambda \in \mathbb{R}^J_{++}$ satisfying $\sum_{j=1}^{J} \lambda_j = 1$, and such that $\tilde{C} = \sum_{j=1}^{J} \lambda_j C^j$: i.e for all $F \in$ and $A \in$,

$$\tilde{C}_A(F) = \sum_{j=1}^{J} \lambda_j C^j_A(F) = \sum_{j=1}^{J} \lambda_j 1 \quad _{C^j_A \in F} \quad .$$

Proof. It is immediate that the convex combination of deterministic choice processes $\left\{(X, C^j)\right\}_{j=1}^J$ is finite, since $\tilde{C}_A \quad Z \in \quad \exists\, j \in \quad 1, .., J$ s.t. $Z = C^j \quad = 1$ all $A \in$. To prove that any finite process $(X, \tilde{C})$ can be decomposed as the proposition asserts, use integers $1 \leq k \leq K$ to index elements Z_k of the finite set G with the property that $\tilde{C}_A(G) = 1 \; \forall \; A \in$: we call these the basic choice processes. Since $\tilde{C}_A(Z_k) \geq 0$ and $\sum_{k=1}^{K} \tilde{C}_A(Z_k) = 1$ we can use indicator functions to record the probability of any set $F \in$ as a convex combination of these basic processes as follows,

$$\tilde{C}_A(F) = \sum_{k=1}^{K} \tilde{C}_A(Z_k) 1_{\{Z_k \in F\}}.$$

We now show that we can use these weights to construct a finite set of choice processes that are able simultaneously to capture such probability information across sets $F \in$ and $A \in$.

First, gather together in the finite set all values taken on by the cumulative distributions taken in order according to k across all $A \in$,

$$= \left\{ x \in (0, 1] \; x = \sum_{i=1}^{k} \tilde{C}_A(Z_i) \text{ for some } A \in \quad , k \in \quad 1, ..K \quad \right\}.$$

We index members of the set by $1 \leq j \leq J$ in increasing order, so that $x_j < x_{j+1}$, with $x_J = 1$. We now define a family of functions $f^A : \ \rightarrow G$ that, for each $A \in$, record which basic choice process is related to each cumulative probability level,

$$f^A(x_j) = \tilde{C}_A(Z_k) \text{ if and only if } x_j \in \left(\sum_{i=1}^{k-1} \tilde{C}_A(Z_i), \sum_{i=1}^{k} \tilde{C}_A(Z_i) \right].$$

We use these objects to construct the finite set of choice processes of interest using the following iteration. The probability assigned to the first deterministic choice process C^1 is x_1 and the actual specification involves using the set specific weights as follow,

$$C^1_A = f^A(x_1).$$

If $J > 1$, we iterate the construction, using at step j weight $x_j - x_{j-1} > 0$ and specifying choice process C^j_A to satisfy,

$$C^j_A = f^A(\sum_{i=1}^{j} x_i).$$

The above construction identifies a finite set of deterministic choice process C^j, $1 \leq j \leq J$ and weights $\lambda_j = x_j - x_{j-1} \geq 0$ and summing to 1. We now such that, for all $A \in$ and $F \in$,

$$\tilde{C}_A(F) = \sum_{j=1}^{J} \lambda_j C^j_A(F) = \sum_{j=1}^{J} \lambda_j 1_{C^j_A \in F}.$$

We consider first the sets $Z_k \in$, noting that,

$$\sum_{j=1}^{J} \lambda_j 1_{C^j_A = Z_k} = \sum_{j=1}^{J} \lambda_j 1_{f^A(\sum_{i=1}^{j} \lambda_i) = Z_k},$$

and that $f^A\left(\sum_{i=1}^{j} \lambda_i\right) = Z_k$ if and only if $\sum_{i=1}^{j} \lambda_i \in \left(\sum_{i=1}^{k-1} \tilde{C}_A(Z_i), \sum_{i=1}^{k} \tilde{C}_A(Z_i)\right]$. Hence we can identify j, l such $\sum_{i=1}^{j} \lambda_i = \sum_{i=1}^{k-1} \tilde{C}_A(Z_i)$ and $\sum_{i=1}^{l} \lambda_i = \sum_{i=1}^{k} \tilde{C}_A(Z_i)$, so that by construction we get,

$$\sum_{j=1}^{J} \lambda_j 1_{C^j_A = Z_k} = \sum_{i=1}^{k} \tilde{C}_A(Z_i) - \sum_{i=1}^{k-1} \tilde{C}_A(Z_i) = \tilde{C}_A(Z_k).$$

That the same is true for any $F \in$ follows directly, since,

$$\tilde{C}_A(F) = \sum_{i=1}^{K} \tilde{C}_A(Z_k) 1_{\{Z_k \in F\}} = \sum_{i=1}^{K} \left(\sum_{j=1}^{J} \lambda_j 1_{C^j_A = Z_k} \right) 1_{\{Z_k \in F\}} = \sum_{j=1}^{J} \lambda_j 1_{C^j_A \in F}.$$

■

10.4 Proof of Theorem 3

Proof. Application of the compression and decompression relations establishes that the finite case is all that needs to be considered. To prove that if $\sim^{\tilde{C}}$ and $\ ^{\tilde{C}}$ satisfy OWC ABS follows, we apply lemma 1 directly to show that $\succsim^{\tilde{C}}$ satisfying OWC implies existence of $\tilde{u} : X \to \mathbb{R}$ that respects the binary relations $\sim^{\tilde{C}}$ and $\ ^{\tilde{C}}$. Moreover, in light of the last proposition, $(X, \tilde{C})$ is the weighted average of deterministic choice processes, $\tilde{C} = \sum_{j=1}^{J} \lambda_j C^j$, which have the property that their corresponding relations $\sim^j$ and $\ ^j$ are all respected by the same $\tilde{u} : X \longrightarrow \mathbb{R}$, since $\sim^{\tilde{C}}$ and $\ ^{\tilde{C}}$ represent the union of these deterministic relations:

$$\begin{aligned}
\tilde{C}_A(J^{xy}) &> 0 \text{ if and only if } x \sim^j y, \text{ some } 1 \leq j \leq J; \\
\tilde{C}_A(F^{xy}) &> 0 \text{ if and only if } x \ ^j\, y, \text{ some } 1 \leq j \leq J.
\end{aligned}$$

Re-application of lemma 1 to each of the deterministic choice processes $\left\{(X, C^j)\right\}_{j=1}^{J}$ implies that $\sim^j$ and $\ ^j$ satisfy OWC for all j, and moreover that the utility function $\tilde{u} : X \to \mathbb{R}$ forms part of some ABS representation of them, further ensuring the existence of deterministic search processes S^j such that $(\tilde{u}, S^j)$ form ABS representations of (X, C^j) for all $1 \leq j \leq J$. Defining the corresponding weighted average search process $\tilde{S} \equiv \sum_{j=1}^{J} \lambda_j S^j$ and $v_A^{S^j} = \left\{\arg\max_{x \in S_A^j(t)} u(x)\right\}_{t=1}^{\infty}$, one can immediately confirm that $(\tilde{u}, \tilde{S})$ form a stochastic ABS representation of $(X, \tilde{C})$, since given $F \in$ and $A \in$,

$$\tilde{C}_A(F) = \sum_{j=1}^{J} \lambda_j 1 \ _{C_A^j \in F} \ = \sum_{j=1}^{J} \lambda_j 1_{\left\{v_A^{S^j} \in F\right\}}.$$

But as $\tilde{S}_A \left\{Z \in \quad Z = S_A^j \text{ for no } j \in \ 1, .., J \ \right\}$, we know that,

$$\begin{aligned}
&\tilde{S}_A \left(\left\{ Z \in \quad \left\{ \arg \max_{x \in Z_t} u(x) \right\}_{t=1}^{\infty} \in F \right\} \right) \\
= &\sum_{j=1}^{j} \tilde{S}_A(\tilde{S}_A^j) 1_{\left\{v_A^{S^j} \in F\right\}} \\
= &\sum_{j=1}^{J} \lambda_j 1_{\left\{v_A^{S^j} \in F\right\}}.
\end{aligned}$$

The last equality follows from the fact that, $\forall\ j \in \ 1, ..J\ $, $\tilde{S}_A(\tilde{S}_A^J) = \lambda_j$.

To prove that ABS implies that $\sim^{\tilde{C}}$ and $\ ^{\tilde{C}}$ satisfy OWC, note that if $(\tilde{u}, \tilde{S})$ form an ABS representation of $(X, \tilde{C})$, Lemma 1 then implies that $\tilde{u}$ respects the orderings $\sim^{\tilde{C}}$ and $\ ^{\tilde{C}}$ on X,

which therefore satisfy OWC. ■

10.5 Proof of Theorem 4

As for ABS, the proof need be given only for the finite case in light of the compression and decompression operations. This finite proof follows from the a generalized version of the RBS characterization precisely as the deterministic result followed from proposition 1. To prove the relevant result we need to generalize the ordering $\succsim^{\tilde{R}}$ of section 4.

Definition 18 Given a stochastic choice process $(\tilde{X}, C)$ and set $D \in$, the binary relation $^{\tilde{L}}_{D}$on X is defined by $x \quad ^{\tilde{L}}_{D}\, y$ if $\ x \cup y \ \cap D = \emptyset$, and there exists $A \in$ with $x, y \in A$ with $\tilde{C}^{L}_{A} \ x \ > 0$ and $\tilde{C}^{L}_{A} \ y \ = 0$. The binary relation $^{\tilde{R}}$ is defined as $^{\tilde{L}}_{D} \cup \ ^{\tilde{C}}$.

Proposition 5 A finite stochastic choice process model $(X, \tilde{C})$ has a stochastic RBS representation $(u, \tilde{S}, \rho)$ with below-reservation set $D \subset X$ if and only if :

1. $\tilde{X}^{N} \subset D$.
2. If $x \in D$ and $x \succsim^{\tilde{R}}_{D} y$, then $y \in D$.
3. Given $x_1, x_2, x_3, .., x_n \in X$ with $x = x_1 \succsim^{\tilde{R}}_{D} x_2 \succsim^{\tilde{R}}_{D} .. \succsim^{\tilde{R}}_{D} x_n = x$, there is no k with $x_k \quad ^{\tilde{R}}_{D}\, x_{k+1}$.

Proof. The proof that conditions (1) - (3) of the proposition are sufficient is constructive, and similar to that in the deterministic case. As there, we define a utility function $u : X \to R$ that respects $^{\tilde{R}}_{D}$ and $\sim$ on X, define reservation utility ρ as the average between the maximum on the set D and the minimum on the set $X \ \ D$, and demonstrate again that $X \ \ D$ is the reservation set associated with the utility function $u : X \to R$ and reservation utility level ρ by noting that $u(x) > u(y)$ whenever $x \in X \ \ D$ and $y \in D$. To see this, note that $x \in X \ \ D$ and $y \in D$ implies by condition (2) above that $C^{L}_{\{x,y\}}(\ x \ ) = 1$, whereupon $x \quad ^{\tilde{R}}_{D}\, y$, so that $u(x) > u(y)$ by construction.

We now consider all deterministic processes C^{j} in the decomposition of the finite stochastic choice process map $\tilde{C}$ that we know by the last proposition to be available. Define X^{N}_{j} as the non-terminal set associated with deterministic choice process (X, C^{j}), and define also the corresponding

binary relations $\sim^j$, $^{C^j}$, $^{L^j}_{D}$, $^{R^j}_{D}$, $\succsim^{C^j}$, $\succsim^{L^j}_{D}$, and $\succsim^{R^j}_{D}$. We show now that any set $D \subset X$ with properties 1-3 above for the stochastic choice process $(X, \tilde{C})$ necessarily satisfies corresponding deterministic properties 1-3 established in theorem 2 to be necessary and sufficient for D to be a reservation set in some RBS representation of each (X, C^j). With respect to the first such property, note directly from the definition that any non-terminal element in (X, C^j) is necessarily so in the stochastic models, so that $X_j^N \subset \tilde{X}^N$, hence $X_j^N \subset D$ as required. The second and third properties follow directly from the fact that, for any $j \in \; 1, ..., J \;$, $x \;\; ^{R^j}_{D} \; y \Rightarrow x \;\; ^{\bar{R}}_{D} \; y$ and $x \sim^j y \Rightarrow x \sim y$. To see this, note first that $x \;\; ^{R^j}_{D} \; y$ implies that either $x \;\; ^{C^j} \; y$ or $x \;\; ^{L^j}_{D} \; y$. The former case indicates that for some $A \in \;$, $\tilde{C}_A(R^{xy}) \geq \lambda_j > 0$, and so $x \;\; ^{C^j} \; y$, while the latter implies that, for some $A \in \;$ and $B \subset A$, $x \in B$, $y \notin B$ and $\tilde{C}^L_A(B) \geq \lambda_j > 0$, so $x \;\; ^{L}_{D} \; y$. In each case, $x \;\; ^{\bar{R}}_{D} \; y$. A similar argument shows that $x \sim^j y$ implies for some $A \in \;$, $\tilde{C}_A(J^{xy}) \geq \lambda_j > 0$ and so $x \sim y$. This result shows that any violation of conditions 2 and 3 at the level of the deterministic choice process j would lead to a violation of the equivalent condition at the level of the stochastic choice function.

Given that the assumptions of theorem 2 are satisfied, we conclude not only that there exists an RBS representation of each (X, C^j) with reservation set D, but also that the utility function $u : X \to R$ and reservation utility level ρ can be utilized in constructing such a representation, given that these are precisely the objects that are constructed in the course of the deterministic proof. Hence, for each j, there exists a search correspondence S^j such that (u, S^j, ρ) represents an RBS representation of (X, C^j). We show now that $(u, \tilde{S}, \rho)$ comprises an RBS representation of $(X, \tilde{C})$, where $\tilde{S}$ is the corresponding convex combination of the deterministic search processes S^j,

$$\tilde{S} = \sum^{J}$$

The proof that conditions 1-3 above are necessary for a finite stochastic choice process $(X, \bar{C})$ to have an RBS representation $(u, \tilde{S}, \rho)$ is essentially identical to that in the deterministic case. We let D be the below reservation set generated by that representation, and establish that the three conditions of the proposition hold. ■

Proof of Theorem 5 Application of Lemma 1 translates the theorem to the statement that there exists $u : X \to \mathbb{R}$, $\rho : \Gamma \to \mathbb{R}$, and $\Theta : \Gamma \to \bar{\ }$ such that $(u, \Theta(\gamma), \rho(\gamma))$ forms a stochastic RBS representation of $\Phi(\gamma)\ \forall\ \gamma \in \Gamma$ if and only if there exists $v : X \to \mathbb{R}$ that respects $^{\tilde{R}(\Gamma)}$ and $\sim^{\tilde{C}(\Gamma)}$. To see that existence of such a function $v : X \to \mathbb{R}$ is necessary, note from theorem 4 that the given function $u : X \to \mathbb{R}$ such that $(u, \Theta(\gamma), \rho(\gamma))$ forms a stochastic RBS representation of $\Phi(\gamma)$ for all $\gamma \in \Gamma$ respects $^{\tilde{R}(\gamma)}$ and $\sim^{\tilde{C}(\gamma)}$ all $\gamma \in \Gamma$ and hence respects $^{\tilde{R}(\Gamma)}$ and $\sim^{\tilde{C}(\Gamma)}$. Conversely, given $v : X \to \mathbb{R}$ that respects $^{\tilde{R}(\Gamma)}$ and $\sim^{\tilde{C}(\Gamma)}$, by definition it respects $^{\tilde{R}(\gamma)}$ and $\sim^{\tilde{C}(\gamma)}$ all $\gamma \in \Gamma$, whereupon theorem 4 implies that there exists an RBS representation of $\Phi(\gamma)$ for all $\gamma \in \Gamma$. In fact the proof of theorem 4 reveals that the given function $v : X \to \mathbb{R}$ that respects $^{\tilde{R}(\gamma)}$ and $\sim^{\tilde{C}(\gamma)}$ can form the basis for an ABS representation with appropriately defined $\rho : \Gamma \to \mathbb{R}$ and $\Theta : \Gamma \to \bar{\ }$, with $(v, \Theta(\gamma), \rho(\gamma))$ therefore forming the required stochastic RBS representation of $\Phi(\gamma)\ \forall\ \gamma \in \Gamma$.

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