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Competition with Exclusive Contracts and Market-Share Discounts

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Abstract

We study the effects of exclusive contracts and market-share discounts (i.e., discounts conditioned on the share a firm receives of the customer's total purchases) in an adverse selection model where firms supply differentiated products and compete in non-linear prices. We show that exclusive contracts intensify the competition among the firms, increasing consumer surplus, improving efficiency, and reducing profits. Firms would gain if these contracts were prohibited, but are caught in a prisoner's dilemma if they are permitted. In this latter case, allowing firms to offer also market-share discounts unambiguously weakens competition, reducing efficiency and harming consumers. However, starting from a situation where exclusive contracts are prohibited, the effect of market-share discounts (which include exclusive contracts as a limiting case) is ambiguous.

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1 Introduction

The effect of exclusive contracts on the intensity of competition is a long debated question in economics and antitrust policy.[1] The debate has recently focused also on market-share discounts (i.e., discounts conditioned on the firm's share of the customer's total purchases), which are often viewed as weaker versions of exclusive-dealing arrangements.[2]

One of the main concerns of antitrust authorities is that these strategies may be used by dominant firms to eliminate competition from equally efficient competitors.[3] However, this view is subject to a two-pronged "Chicago critique:" rational buyers must be compensated for accepting exclusive contracts, and as efficient competitors that are threatened to be foreclosed can in turn respond by offering exclusive contracts or market-share discounts.

In a model that accounts for these effects, Bernheim and Whinston (1998) show that, under complete information, exclusive contracts and market-share discounts are competition neutral. Any equilibrium outcome that can arise when firms use these contracts, that is to say, can also be obtained when firms use non-linear prices that depend only on own sales. This neutrality result provides a natural benchmark to classify post-Chicago theories of exclusive contracts. The negative view of exclusive contracts is typically articulated in models where firms are asymmetric, and an incumbent can contract with the buyers before an entrant enters the market, as in Aghion and Bolton (1987), Rasmusen et al. (1990), and Segal and Whinston (2000). By contrast, the positive view does not focus on the product market competition stage, but rather on firms' incentives, in an earlier stage, to make some kind of relation-specific investments, as in Marvel (1982) and Segal and Whinston (2000a).

In this paper we focus only on the product market competition stage and assume that firms are fully symmetric. However, we contend that Bernheim and Whinston's neutrality result crucially hinges on the fact that, under complete information, non linear prices suffice to price discriminate perfectly (although buyers may obtain a positive rent because of the competition among the sellers). In real life, price discrimination is often impeded by incomplete information about demand. To capture this phenomenon, we propose an adverse selection model where two symmetric firms supply horizontally differentiated products but do not know the buyers' willingness to pay for the products. Thus, buyers obtain informational rents even if firms can use non-linear prices.[4]

In this more realistic framework, we show that exclusive contracts *intensify* the competition

[1] See Rey and Tirole (2007) for an excellent survey of the literature.

[2] See, for instance, Majumdar and Shaffer (2009) and Mills (2004).

[3] A recent example is the *Intel* case, where the European Commission found that "Intel abused its dominant position in the x86 CPU market by implementing a series of conditional rebates." These ranged from conditioning rebates on exclusivity, to requiring buyers to purchase a minimum share of their CPU needs from Intel to qualify for the discount.

[4] In our model, price discrimination is imperfect because of incomplete information, but it is not ruled out at the outset by constraining firms to use linear prices, as in Mathewson and Winter (1987). With their assumption, the competitive effects of exclusive contracts are ambiguous.

among the firms, increasing consumer surplus, improving efficiency, and reducing profits. Firms would actually gain if these contracts were prohibited; however, if exclusive contracts are allowed, they are caught in a prisoner's dilemma and have an unilateral incentive to propose exclusive-dealing arrangements even if this makes competition fiercer.

The intuitive reason why exclusive contracts are pro-competitive is very simple. When firms compete in non-linear prices, the intensity of competition is limited by product differentiation. With exclusive contracts, by contrast, firms compete in utility space, where their products become effectively homogeneous irrespective of the actual degree of differentiation. Nevertheless, competition with exclusive contracts does not drive firms' profits to zero. This is so for two reasons. First, consumers value product variety and hence can be induced to purchase both products at supra-competitive prices even when the exclusionary prices are competitive. As a result, in equilibrium exclusive contracts are offered but are destined not be accepted. This creates the possibility that firms may tacitly "coordinate" their exclusionary offers, raising the exclusionary prices above marginal costs so as to increase their equilibrium payoffs. This is the second reason why equilibrium profits can be positive. However, while such tacit coordination can occur in a non-cooperative equilibrium, its extent is limited. Thus, exclusive contracts always provide consumers with an extra option that disciplines firms' non exclusionary pricing.

As for market-share discounts, their effects depend on the benchmark they are compared to. If exclusive contracts are allowed, permitting firms to offer also market-share discounts unambiguously *weakens* competition, reducing efficiency and harming consumers. The intuition here is that market-share discounts effectively allow firms to impose a "tax" on each other's output. This results in a double marginalization effect, which is so strong that the equilibrium quantities (at least for consumers with a sufficiently large demand) are the same as if the products were perfect complements, even if they are in fact close substitutes.

This means that market-share discounts are not simply weaker versions of exclusive-dealing arrangements after all. However, firms can always use market-share discounts to re-produce exclusive contracts by charging exorbitant high prices when their share of a buyer's total purchases is lower than one. As a result, starting from a situation where exclusive contracts are prohibited, permitting market-share discounts brings about the same pro-competitive effects as exclusive contracts, in addition to the anti-competitive double-marginalization effect mentioned above. The total effect is generally ambiguous. A regime where both exclusive contracts and market-share discounts are all prohibited is more likely to be more competitive than one where they are both allowed, the greater is the degree of tacit coordination among the firms and the less differentiated are their products.

Our analysis of exclusive contracts and market-share discounts builds on some recent advances in the theory of common agency under incomplete information due to Martimort and Stole (2009). They have developed techniques to solve adverse selection models where common agency is *delegated*, not intrinsic as in most of the previous literature. In other words, the buyer has not only the option of purchasing from both firms or not purchasing at all, but also that of purchasing from only one firm. Martimort and Stole (2009) provide a useful characterization of the equilibrium and arrive at an explicit solution for the case of quadratic utility, constant marginal costs, and uniform

distribution of types.

However, Martimort and Stole's analysis is confined to the case of *private* common agency, where firms can condition their prices only on own sales. This corresponds to the case of standard competition in non-linear prices. Using their uniform-quadratic model, we extend the analysis to the case of *public* common agency, where firms can condition their prices not only on their own quantity, but also on their competitor's. We also consider the intermediate case where firms can condition prices only on whether or not the consumer purchases a positive quantity from their competitor (the *semi-public* common agency case). This corresponds to the case where firms can offer exclusive contracts but cannot engage in market-share discounts.[5]

The rest of the paper is organized as follows. Section 2 sets up the model. In section 3, we present some useful benchmarks, including the private, delegated common agency solution developed by Martimort and Stole (2009). In section 4, we analyze the case in which firms can offer exclusive contracts but not market-share discounts. We show that in this case there is a continuum of equilibria, and we characterize the most competitive and the most "cooperative" ones. We show that even the latter is more efficient and favorable to consumers than the private common agency equilibrium. Section 5 develops the analysis of the game where firms can use market-share discounts and thus can condition their prices on their competitor's sales arbitrarily. We show that this extra-flexibility harms consumers if exclusive contracts are already feasible. However, starting from a situation where exclusive contracts are prohibited, the effect of permitting market-share discounts (which include exclusive contracts as a limiting case) is generally ambiguous. Section 6 summarizes the paper, discusses the implications of the results for competition policy, and considers several possible directions for future work.

2 The model

Two symmetric risk-neutral firms, denoted by $i = A, B$, supply differentiated products q and q to a final consumer (a she). Firms' marginal cost is constant and is normalized at zero. The consumer's utility function in monetary terms, $u(q\ ,q\ ,\theta)$, depends on consumption and a parameter, θ, which is the consumer's private information. To get explicit solutions, we assume that θ is uniformly distributed over the interval $[0, 1]$ and posit a quadratic function

$$u(q\ ,q\ ,\theta) = \theta(q\ + q\ ) - \kappa(q^2\ + q^2\ ) - \gamma q\ q\ .$$

With this formulation, the consumer's reservation utility is independent of her type θ and is normalized to zero.

[5] To the best of our knowledge, this is the first paper that analyzes such a model of delegated, public (or semi-public) common agency under incomplete information. Previous analyses focused on models of lobbying, or provision of public goods. In these models, principals are restricted to offer non-negative contributions, and hence it is a (weakly) dominant strategy for the agent to accept all contributions. Then, the interesting question is when principals' offers will be strictly positive: see Martimort and Stole (2009a) for an analysis of this problem.

In an alternative interpretation of the model, A and B are manufacturers that sell their products through a common retailer, and u is the retailer's gross profit. Most of our results apply also to this case, but the results for social welfare hold only under the additional assumption that the retailer can capture the consumer surplus fully, as implicitly assumed in Bork (1978). For example, the retailer must be able to perfectly price discriminate.

The parameter γ captures the degree of substitutability or complementarity between the goods. Following Shubik and Levitan (1980), to prevent changes in γ to affect the size of the market we set

$$\kappa = \frac{1-\gamma}{2}.$$

This implicitly normalizes the efficient quantities of the consumer with the largest demand (i.e., $\theta = 1$) to one for any value of γ. The parameter γ can then vary from $\frac{1}{2}$ (perfect substitutes) to $-\infty$ (perfect complements); the goods are independent when $\gamma = 0$.

Notice that buyers have preference for variety in that purchasing q from both firms instead of purchasing the same total quantity $2q$ from a single firm yields an increase in utility equal to

$$u(q, q, \theta) - u(2q, 0, \theta) = (1 - 2\gamma)q^2 \geq 0.$$

Preference for variety is stronger for higher types, who purchase larger quantities.

Timing

Firms simultaneously and independently offer a menu of contracts, the form of which will be specified presently. The consumer observes the firms' offers and then decides the quantities to purchase. If she refuses to purchase from a firm, no payment is due to it (in other words, common agency is delegated, not intrinsic). Finally, sales and payments are made, and payoffs are realized.

Strategies

We distinguish between three different games according to the type of contracts firms are allowed to offer.

In game G , exclusive contracts and market-share discounts are banned, so each firm can request a payment that depends only on its own quantity. In the jargon of the common agency literature, this is known as the case of *private* common agency. Since in this case the quantity the consumer purchases from the other firm is not contractible, a strategy for firm i is a function[6] $p(q) : \mathcal{Q} \rightarrow \Re_+$ where where $q \geq 0$ is the quantity that firm i is willing to supply, $p \geq 0$ is the corresponding total payment requested, and $\mathcal{Q}$ is a compact subset of $\Re_+$. We shall refer to a quantity-payment pair as a *contract* and to the menu of contracts offered by a firm as a *price schedule*. We assume that firm i can choose the domain $\mathcal{Q}$ of its price schedule, allowing it for certain quantities not to submit any offer. We set almost no restriction on price schedules, assuming only that $p(q)$ is non decreasing (to allow for free disposal) and that $p(0) = 0$. We denote by $\mathcal{S}$ the set of feasible strategies in game G .

[6] This formulation implicitly makes the innocuous assumption that firms offer at most one contract for any quantity level. If they offered more than one, only the contract with the lowest requested payment would matter.

In game G , firms can offer exclusive contracts but cannot engage in market-share discounts (the *semi-public* common agency case).[7] A contract then is a triple $(q\ , I\ , p\ )$, where $I\ \in \{0,1\}$ is an indicator function that is zero when $q\ = 0$ and is one when $q\ > 0$. In this case, a strategy for firm i reduces to a menu of two price schedules, $p\ \ (q\ ) : \mathcal{Q}\ \ \to \Re_+$ and $p\ \ (q\ ) : \mathcal{Q}\ \ \to \Re_+$. The former applies to exclusive contracts ($q\ = 0$), the latter to non exclusive ones ($q\ > 0$).[8] (For future reference, in the semi-public common agency game we define $\mathcal{Q}\ \equiv \mathcal{Q}\ \ \cup \mathcal{Q}\ $.) Each of these schedules must satisfy the condition $p\ (0) = 0$ and be non decreasing. The set of feasible strategies under semi-public common agency is denoted by $\mathcal{S}$.

Finally, in game G firms' requested payments can depend on their competitor's quantity arbitrarily (the *public* common agency case). This formulation captures the case where both exclusive contracts and market-share discounts are allowed. Now a contract is a triple $(q\ , q\ , p\ )$, and firm i's strategy is a bivariate price schedule $p\ (q\ , q\ ) : \mathcal{Q}\ \times \mathcal{Q}\ \ \to \Re_+$.[9] (For future reference, in the public common agency game we define $\mathcal{Q}\ \equiv \mathcal{Q}\ \cap \mathcal{Q}\ $.) Again, we assume that $p\ (0, q\ ) = 0$ and that p is non decreasing in q . The set of feasible strategies in game G is denoted by $\mathcal{S}$.

In all games G with $g \in \{pr, sp, pu\}$, a strategy for the consumer is a function $\mathbf{q}(p\ \ , p\ \ ) : \mathcal{S}^2 \to \mathcal{Q}\ \ \times \mathcal{Q}$ that expresses her consumption as a function of the firms' offers.

For notational convenience, we shall sometimes use the notation $p\ (q\ , q\ )$ to denote also the price schedules under private and semi-public common agency, with the understanding that p cannot depend on q at all under private common agency and can depend only on I under semi-public common agency.

Payoffs

Firm $i = A, B$ maximizes its expected profits

$$\pi\ \ = E[p\ (q\ , q\ )],$$

and the consumer maximizes her net utility

$$U(q\ \ , q\ \ , \theta) = u(q\ \ , q\ \ , \theta) - p\ \ (q\ \ , q\ \ ) - p\ \ (q\ \ , q\ \ ).$$

[7] One interpretation of this case is that firms can observe whether the buyer purchases from their competitor or not, but cannot observe the exact quantity purchased and thus cannot condition their requested payments on it. Another interpretation is that competition policy permits exclusive contracts but prohibits market-share discounts – a policy that now is rarely observed, but is in fact optimal in our model.

[8] This formulation is flexible enough to allow firms to unilaterally enforce exclusivity. This can be accomplished by setting $\mathcal{Q}_i^C = \varnothing$, or by requesting exorbitant high payments $p_i^C(q_i)$. In general, however, whether a consumer of type θ ends up purchasing only one product or both may depend on the strategies of both firms, and possibly also on the consumer's type θ.

[9] The literal interpretation of market-share discounts is that firm i's requested payment depends both on its own quantity q_i and on its market share $s_i = \frac{q_i}{q_i+q_j}$. However, with two firms it is clear that any function $p_i = \phi_i(q_i, s_i)$ can be rewritten as $p_i = \omega_i(q_i, q_j)$, and vice versa. Notice also that we allow firm i to refuse to deal with the consumer unless she purchases from its rival a quantity q_j in a prescribed set $\mathcal{Q}_j^i$.

Equilibrium

Given the timing of the game, it is natural to focus on subgame perfect equilibria where the consumer maximizes U for any possible pair of price schedules submitted by the firms, not only the equilibrium one.[10] Thus, an equilibrium is a triple of feasible strategies $\{\tilde{p}\ ,\tilde{p}\ ,\tilde{\mathbf{q}}(p\ ,p\ )\}$ such that

$$\tilde{\mathbf{q}}(p\ ,p\ ) \in \arg\max_{A\ \ B} U(q\ ,q\ ,\theta) \quad \forall \{p\ ,p\ \} \in \mathcal{S}^2,$$

and

$$E\{\tilde{p}\ (\tilde{q}\ [p\ ,\tilde{p}\ ],\tilde{q}\ [p\ ,\tilde{p}\ ])\} \geq E\{p\ (\tilde{q}\ [p\ ,\tilde{p}\ ],\tilde{q}\ [p\ ,\tilde{p}\ ])\} \quad \forall p\ \in \mathcal{S} \quad i = A, B.$$

3 Non-linear prices

In this section we present three useful benchmarks: the efficient solution, the monopoly solution, and the private common agency solution developed by Martimort and Stole (2009).

The first-best quantities maximize the social surplus $u(q\ ,q\ ,\theta)$ (i.e., the sum of the consumer surplus and firms' profits) under full information, yielding

$$q\ \ (\theta) = q\ \ (\theta) = \theta.$$

In this first-best solution, there is full participation: all types θ trade with both firms.

Next consider a monopolistic firm that supplies both products under asymmetric information. Since high-type consumers will necessarily obtain an informational rent, the monopolist will distort quantities downward so as to reduce the consumer's rent and increase its profits. Using standard techniques one gets:

$$q\ \ (\theta) = q\ \ (\theta) = 2\theta - 1.$$

Clearly, $q\ \ (\theta) \leq q\ \ (\theta)$, with equality only for $\theta = 1$ (no distortion at the top). The marginal consumer is $\theta = \frac{1}{2}$, and types from 0 to $\frac{1}{2}$ do not consume.

The rest of this section derives the private common agency equilibrium first obtained by Martimort and Stole (2009). We adapt their approach so as to make it easier to apply, later, to the case of public and semi-public common agency. Like Martimort and Stole, we start by guessing a specific functional form of the equilibrium price schedules, so that they are fully identified by a few parameters. If the initial guess is correct, the equilibrium of the original game $G\ $ must coincide with that of a restricted game $\hat{G}\ $ where firms can choose only those parameters. We then calculate the equilibrium of the restricted game, which becomes the candidate equilibrium of the original game. Finally, we use direct mechanisms to verify that the candidate equilibrium is, indeed, an

[10] Since the uninformed players move first, imposing subgame perfection is equivalent to adopting Perfect Bayesian Equilibrium as our solution concept.

equilibrium of the game G , where firms' strategy space is unrestricted.[11] The drawback of this procedure is that it fails to locate equilibria that do not conform to the initial guess, if there are any.

The initial guess is, indeed, the critical step in the solution procedure. Fortunately, useful hints can be obtained by analogy with the properties of the monopoly solution, which often are well known or can be determined as a matter of routine.

As we have seen before, in our model monopoly quantities are linear in θ and hence can be supported by quadratic price schedules. This suggests that in a private common agency equilibrium firms will submit quadratic price schedules, i.e.[12]

$$p\ (q\ ) = \alpha_0\ \ + \alpha_1\ q\ + \alpha_2\ q^2 \quad \text{for } q\ \in [0, 1].$$

Each of these price schedules is fully identified by the three coefficients α for $s = 0, 1, 2$. But we can further refine our guess by exploiting two other well known properties of the monopoly solution. The first is the no-distortion-at-the-top property, which in our setting means that type $\theta_{\max} = 1$ must purchase the undistorted quantities $q\ (1) = q\ (1) = 1$. This implies that the equilibrium price schedules must be flat at $q\ = 1$, entailing

$$\alpha_2\ = -\frac{\alpha_1}{2}.$$

The second property is that no fixed fees are charged when the market is uncovered (Wilson, 1994), implying

$$\alpha_0\ = 0.$$

Consider, then, a restricted game where firms are constrained to submit price schedules of the type

$$p\ (q\ ) = \alpha_1\ q\ - \frac{\alpha_1}{2} q^2 \quad \text{for } q\ \in [0, 1].$$

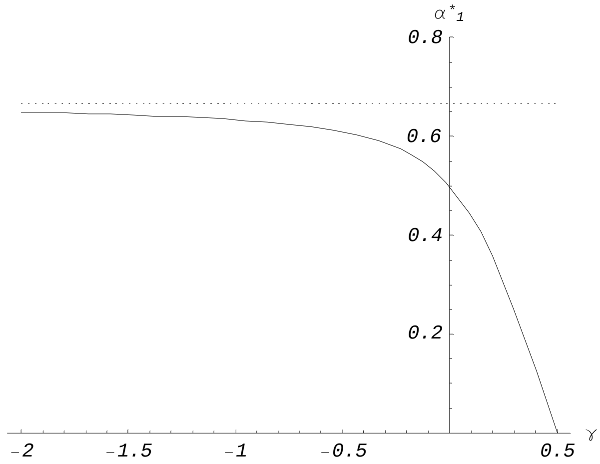
A strategy for firm i then becomes simply a value of α_1 . This restricted game can be solved easily (see Appendix 3.1). There is only one equilibrium, which is symmetric and is given by

$$\alpha_1^* = \frac{1}{4}\left[3(1-\gamma) - \sqrt{1 - 2\gamma + 9\gamma^2}\right]. \tag{1}$$

(The private common agency equilibrium is denoted by a star.) The variable α_1^* plays a key role in all the subsequent analysis. It decreases with γ, vanishes when $\gamma = \frac{1}{2}$ (the perfect substitutes case, where one re-obtains the Bertrand paradox) and converges to $\frac{2}{3}$ as $\gamma \to -\infty$ (the case of perfect complements), as shown in Figure 1.

[11] The difference between this procedure and that used by Martimort and Stole (2009) and Ivaldi and Martimort (1994) is that after making the initial guess, they immediately proceed to construct the firms' best responses using direct mechanisms, whereas we resort to direct mechanisms only after calculating the candidate equilibrium. Our procedure may turn out to be simpler, especially when the number of coefficients to be determined grows large as in the following sections.

[12] These schedules can be extended arbitrarily to $q_i > 1$ provided that the extension is non decreasing.



The coefficient α_1^* as a function of the degree of product differentiation γ.

Our candidate equilibrium then becomes $p^* = \alpha_1^* q \;- \frac{{}^{*}_{1}}{2} q^2$. The final step of the procedure must verify that these price schedules are an equilibrium of the original game G , with the unrestricted strategy set $\mathcal{S}$. The use of direct mechanisms is legitimate in this final step,[13] where one must check that each firm's strategy is a best response to its competitor's. For, given firm j's price schedule, firm i is like a monopolist that faces a consumer with a suitably defined indirect utility function, which accounts for any benefit the consumer can obtain by optimally trading with firm j. As argued by Martimort and Stole, provided that this indirect utility function satisfies appropriate regularity conditions,[14] one can apply the techniques of the monopolistic screening literature. After finding the optimal solution, one can then recover the price schedule that supports it, and verify whether the best response property holds. We relegate this verification to Appendix 3.2, which completes the proof of the following:

Proposition 1 *The price schedules*

$$p^*(q\ ) = \alpha_1^* q\ - \frac{\alpha_1^*}{2} q^2 \qquad \textit{for } 0 \le q\ \le 1 \qquad \textit{and } \ i = A, B, \tag{2}$$

where α_1^* is given by (1), are an equilibrium of the private common agency game G .

Before proceeding, we briefly discuss some important properties of the private common agency equilibrium, which is a fundamental benchmark for our later analysis. The equilibrium quantities

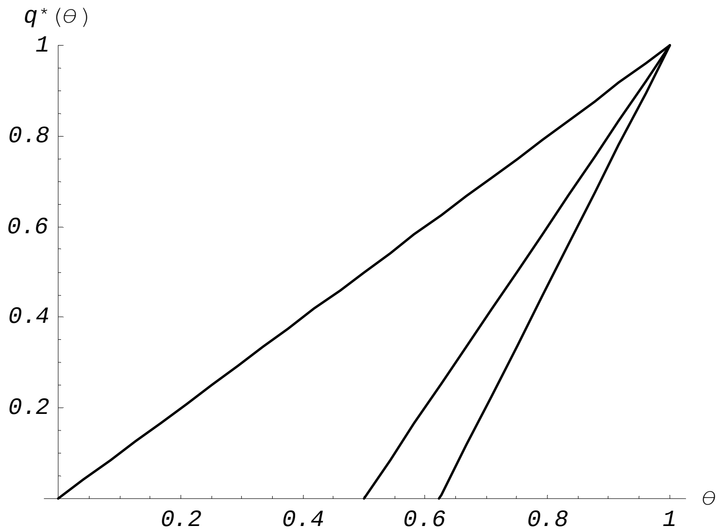
[13] In a direct mechanism, the consumer is asked to reveal her type and firms condition quantities and payments on the announcement. Under monopoly, the revelation principle guarantees that there is no loss of generality in focusing on direct mechanisms only. However, the revelation principle may fail with multiple principals (see e.g. Peters, 2001, Martimort and Stole, 2002 and Pavan and Calzolari 2009), so a game in which firms choose direct mechanisms would generally have different equilibria from the games analyzed in this paper.

[14] In particular, the indirect utility function must satisfy the familiar sorting condition, and the participation constraint must be binding only for the lowest participating type.

are

$$q^*(\theta) = \frac{\theta - \alpha_1^*}{1 - \alpha_1^*}, \tag{3}$$

and are depicted in Figure 2 for representative values of γ. In particular, one re-obtains the efficient solution for $\gamma = \frac{1}{2}$ (the Bertrand paradox) and the monopoly solution for $\gamma = 0$.[15] The equilibrium quantities are smallest, i.e., $q^*(\theta) = 3\theta - 2$, when $\gamma \to -\infty$, since with complementary goods prices are excessively high due to the problem of Cournot complements.[16]



Equilibrium quantities under private common agency for $\gamma = \frac{1}{2}$ (perfect substitutes, the highest line), $\gamma = 0$ (independent good, the intermediate line) and $\gamma = -\infty$ (perfect complements, the lowest line).

Apart from the special case $\gamma = \frac{1}{2}$, the market is uncovered since types θ from 0 to $\alpha_1^* > 0$ do not consume. The consumer's rent is zero for any type $\theta \leq \alpha_1^*$ and is

$$U^*(\theta) = \frac{(\theta - \alpha_1^*)^2}{1 - \alpha_1^*} \tag{4}$$

for types $\theta \geq \alpha_1^*$. For future reference, notice that $\frac{\ ^*}{\ } = q^* (\theta) + q^* (\theta)$. This property, which follows by the envelope theorem, reflects the fact that equilibrium quantities are chosen by the consumer optimally and provides an intuitive explanation for why, in order to reduce the rent obtained by high-type consumers, firms must restrict low-types' consumption.

[15] Thus, under private common agency firms do not effectively interact strategically when the goods are independent. As we shall see below, this property does not carry over to the case of semi-public or public common agency.

[16] As is well known, when several separate firms supply complementary components of the final product and price each component non-cooperatively, the price to the final consumer exceeds the monopoly price, reducing output (and profits) below monopoly output (and profits).

4 Exclusive contracts

In this section we allow firms to use exclusive contracts, but continue to rule out market-share discounts. Thus, each firm i can offer a menu of two price schedules, $p \;\; (q \;)$ and $p \;\; (q \;)$: the former applies to exclusive contracts ($q \;\; = 0$), the latter to non exclusive ones ($q \;\; > 0$). We shall refer to this environment as the semi-public common agency game, $G \;\;$.

4.1 A non neutrality result

In the complete information version of model, which is a special case of Bernheim and Whinston (1998), the same equilibrium outcome is obtained with and without exclusive contracts: in both cases, the allocation is efficient and each firm obtains a profit exactly equal to the incremental value of its product. Thus, exclusive contracts are irrelevant, or competition neutral.

With incomplete information, except in the special case where the products are perfect substitutes ($\gamma = \frac{1}{2}$), permitting exclusive contracts disrupts the private common agency equilibrium.

Proposition 2 *When exclusive contracts are permitted and the goods are not perfect substitutes (i.e., $\gamma < \frac{1}{2}$), in equilibrium exclusive contracts are offered, and no equilibrium re-produces the private common agency equilibrium outcome.*

The Proposition is proved in Appendix 4.1. Assumption $\gamma < \frac{1}{2}$ guarantees that in the private common agency equilibrium the market is uncovered. Some low-type consumers then are not served even if their demand for the goods is positive. The intuitive reason is that in order to serve these consumers, firms should offer contracts that would attract also some high-type consumers, increasing by too much their informational rents. However, starting from the private common agency equilibrium, exclusive contracts allow firms to profitably serve low-type consumers without attracting high-type ones. This is possible because high-type consumers value the opportunity to purchase both products more than low types, as we have seen above, and hence have more to lose from accepting exclusive contracts.

Since exclusive contracts impose on consumers an unnecessary cost, they resemble the strategy of damaging one's goods analyzed by Deneckere and McAfee (1996). Focusing on monopoly, these authors show that the conditions under which damaging goods can be a profitable discriminating strategy are rather restrictive.[17] However, the scope for profitable discrimination expands considerably when firms "damage goods" contractually through exclusionary clauses, since most of the cost of this strategy then falls as a negative externality on their rival. Proposition 2 shows that contractually damaging one's good through exclusionary clauses is unprofitable only if the goods are perfect substitutes, in which case the private common agency equilibrium already entails zero prices, and thus all consumers are already served.

[17] Anderson and Dana (2005) show that a monopolist price discriminates if and only if the condition of *increasing percentage differences* holds. This condition requires that the ratio of the marginal social value from an increase in product quality to the total social value of the good increases with consumers' willingness to pay. This condition is very restrictive when the unit production cost is constant or decreasing in quality, as in the case of damaged goods.

4.2 The most competitive equilibrium

Having shown that permitting exclusive contracts disrupts the private common agency equilibrium, we now turn to the analysis of the new equilibrium with exclusive contracts. We proceed in three steps: we start by guessing the form of equilibrium price schedules, we then calculate a candidate equilibrium, and finally we verify that the candidate equilibrium satisfies the best response property. As we shall see, with exclusive contracts there are many symmetric equilibria; we start, in this subsection, from the most competitive one.

The guess.

When firms offer exclusive contracts, they compete in utility space where their products become effectively homogeneous. As a result, if exclusive contracts involving strictly positive payments were accepted by some consumers, the standard Bertrand logic would imply that firms would try to undercut each other, driving exclusionary prices to cost. This is, indeed, what happens in the models of one-stop shopping of Armstrong and Vickers (2001), Rochet and Stole (2002) and others.

This suggests that we look for an equilibrium in which firms offer competitive exclusionary price schedule, which involve zero payments:

$$p \ \ (q \ ) = 0 \text{ for all } q \ \geq 0,\ i = A, B. \tag{5}$$

Indeed, these contracts can always be part of an equilibrium. To see this, observe that if firm j offers $p \ \ (q \ ) = 0$, firm i will make zero profits with any non negative exclusionary price schedule. Moreover, acting unilaterally firm i cannot affect the outside option that is provided to the consumers by firm j's exclusionary schedule $p \ \ (q \ ) = 0$. This means that given $p \ \ (q \ ) = 0$, firm i is actually indifferent between any exclusionary price schedule, and hence its best reply must include also the schedule $p \ \ (q \ ) = 0$.

With these exclusive contracts, consumer θ's optimal choice is to purchase a quantity

$$q \ \ (\theta) = \frac{\theta}{1-\gamma}$$

of either good,[18] obtaining a strictly positive type-dependent utility

$$U \ \ (\theta) = \frac{\theta^2}{2(1-\gamma)}. \tag{6}$$

Now consider the restricted game in which firms choose only their non exclusionary price schedules $p \ \ (q \ )$, given (5). This restricted game is one of private common agency. Differently from the previous section, however, now there is a strictly positive and type-dependent reservation utility, $U \ \ (\theta)$. This is an important difference, which radically changes the non exclusionary price schedules.[19]

[18] At first, it might seem surprising that under exclusivity demand depends on the product differentiation parameter γ. However, recall that we set $\kappa = (1-\gamma)/2$, so when the consumer purchases only one good, her total demand does depend on γ.

[19] There is another, subtler difference. In the restricted game, either the consumer purchases from both firms, or she obtains $U^E(\theta)$ and both firms make zero profits. Thus, the restricted game can be viewed as a game of *intrinsic* common agency, where both firms must take care that the "participation constraint" is satisfied.

To guess the possible shape of the new equilibrium schedules, we use an important result in the monopolistic screening literature with type-dependent reservation utility due to Armstrong et al. (1994) and Jullien (2000). They show that if the reservation utility is "weakly" convex (in a sense that will be made precise below), then in the monopoly solution there exists a threshold $\hat{\theta}$ such that the participation constraint binds for a set of types $[0, \hat{\theta}]$ and does not bind over the interval $(\hat{\theta}, 1]$. If a similar pattern is to hold under duopoly, then the non exclusionary price schedules must comprise two parts, one intended for consumers who will obtain exactly their reservation utility $U \ (\theta)$, and one for those who will obtain strictly more.

Therefore, we posit non exclusionary price schedules that again depend only on own output but now are piecewise quadratic:

$$p \ (q \ ) = \begin{cases} \underline{\alpha}_0 \ + \underline{\alpha}_1 \ q \ + \underline{\alpha}_2 \ q^2 \ \ (\equiv \underline{p} \ \ (q \ )) & \text{for } 0 \leq q \ \leq \hat{q} \\ \bar{\alpha}_0 \ + \bar{\alpha}_1 \ q \ + \bar{\alpha}_2 \ q^2 \ \ (\equiv \bar{p} \ \ (q \ )) & \text{for } \hat{q} \ < q \ \leq 1 \end{cases} \tag{7}$$

We denote by $\underline{q} \ (\theta)$ the optimal quantities for consumers who choose $q \ \leq \hat{q}$ and by $\bar{q} \ (\theta)$ the optimal quantities for those who choose $q \ > \hat{q}$; $\underline{U}(\theta)$ and $\bar{U}(\theta)$ denote the corresponding net utilities.

The candidate equilibrium.

In the restricted game where firms offer the exclusive contracts (5) and can choose only non exclusionary schedules of type (7), for each firm there are seven coefficients to be determined: the $\underline{\alpha}$'s, the $\bar{\alpha}$'s, and $\hat{q}$. We first calculate the coefficients of the lower part of the non exclusionary price schedules, we then turn to the upper part, and finally we illustrate how the two parts connect.

Step 1. Consider the lower part of the non exclusionary price schedules first. This is intended for consumers who will obtain exactly the reservation utility $U \ (\theta)$ that is guaranteed by the exclusive contracts (5). If consumer $\theta \in [0, \hat{\theta}]$ opted for such contracts, firms would make zero profits on that consumer. However, since consumers value the opportunity to purchase both products, as we have seen above, both firms can obtain positive profits by inducing "common participation". From this perspective, common participation can be viewed as a public good – an indivisible public good that is jointly provided by the two firms, since both must take care that the participation constraint is satisfied.

To induce common participation, firms must charge low enough non exclusionary prices. But since common participation is an *indivisible* public good, each firm stands ready to provide it as long as the residual contribution (in terms of lower prices) that is needed to induce consumers to purchase both products is lower than the benefit. In other words, for any given $\underline{p} \ (q \ )$ set by firm j, firm i will induce the consumer to accept the non exclusive contracts if only it can do so and still charge positive prices $\underline{p} \ (q \ ) > 0$. Anticipating this behavior, firm j may be tempted to free ride by charging high non exclusionary prices and letting the cost of providing "common participation" fall on its rival. And firm i has similar incentives. This creates the possibility of multiple equilibria, as in many other models of private provision of an indivisible public good. In what follows, we shall focus on the unique symmetric equilibrium, where the cost of inducing common participation is divided evenly among the two firms.

In all equilibria, the coefficients $\underline{\alpha}$ must guarantee to low-type consumers a rent equal to their reservation utility U (θ). If the equality is to hold for a non degenerate set of types $[0, \hat{\theta}]$, we must have $\frac{\quad E}{\quad} = \frac{}{}$, which implies

$$\underline{q} \ (\theta) + \underline{q} \ (\theta) = q \ (\theta) \tag{8}$$

for all $\theta \in [0, \hat{\theta}]$. Condition (8) immediately implies (denoting by two stars the semi-public common agency equilibrium)

$$\underline{\alpha}_0^{**} = \underline{\alpha}_1^{**} = 0.$$

Condition (8) also implies

$$\underline{\alpha}_2 \quad \times \underline{\alpha}_2 \quad = \left(\frac{1}{2} - \gamma\right)^2.$$

The non negative solutions to this equation determine the entire set of possible asymmetric candidate equilibria. As discussed above, asymmetric equilibria correspond to uneven divisions among the firms of the cost of inducing consumers to purchase both products. Focusing on the unique symmetric candidate equilibrium, we set[20]

$$\underline{\alpha}_2^{**} = \frac{1}{2} - \gamma.$$

Step 2. Now we turn to the upper part of the non exclusionary price schedule, which is intended for high-type consumers. Since the participation constraint is not binding for them, it is natural to conjecture that the equilibrium quantities are the same as under private common agency. This requires that

$$\bar{\alpha}_1^{**} = \alpha_1^* \text{ and } \bar{\alpha}_2^{**} = -\frac{\alpha_1^*}{2}.$$

Now, however, the fixed fees $\bar{\alpha}_0$ can be different from zero.

This conjecture can be justified as follows. Consider any interval $(\hat{\theta}, 1]$ where the participation constraint does not bind. Given its rival's price schedule $\bar{p}$ $(q$ $)$, firm i's best response will be determined by pointwise maximization of an indirect virtual surplus function that depends only on the consumer's indirect utility and the hazard rate of the distribution function of types (which is $\theta - 1$ with a uniform distribution). For any given $\bar{p}$ $(q$ $)$, the indirect virtual surplus, and hence the best response function, is the same as in the private common agency game. This implies that the equilibrium quantities must be the same as under private common agency, i.e., $\bar{q}$ $(\theta) = q^*(\theta)$.[21]

[20] Notice that $\underline{\alpha}_2^{**}$ tends to infinity when $\gamma \to -\infty$. The intuitive reason is that when the products tend to become perfect complements, exclusive contracts become almost completely unattractive. The reservation utility U^E then is very low, and can be matched by non exclusive contracts with very large prices.

[21] See Calzolari and Scarpa (2008) and Martimort and Stole (2009, Proposition 5) for a more formal argument. These papers show that if the participation constraint does not bind, equilibrium quantities under intrinsic and delegated common agency coincide. However, their argument is more general and effectively implies that in any interval $(\hat{\theta}, 1]$ where the participation constraint does not bind, the equilibrium of the game played by the firms is independent of whatever happens to lower types.

Before proceeding, we note that equality $\bar{q}\ \ (\theta) = q^*(\theta)$ implies also that $\bar{U}(\theta) = U^*(\theta) - \bar{\alpha}_0\ \ - \bar{\alpha}_0\ \ $. This means that the reservation utility $U\ \ (\theta)$ is less strongly convex than $\bar{U}(\theta)$,[22] so it is, indeed, "weakly" convex in the sense of Jullien (2000).

Step 3. It remains to determine the $\bar{\alpha}_0\ \ $'s and the $\hat{q}\ \ $'s. These coefficients must guarantee that the smooth pasting condition holds, that is, that the non exclusionary price schedules be continuous and continuously differentiable at $\hat{q}\ $. Intuitively, the smooth pasting condition ensures that the equilibrium quantities are continuous in θ, a property that must be satisfied by any best response. A formal proof that the smooth pasting condition must hold in equilibrium is provided in Appendix 4.2.

Continuity requires (with a slight abuse of notation):

$$\underline{p}\ \ (\hat{q}\ ) = \bar{p}\ \ (\hat{q}\ ). \tag{9}$$

At $\hat{q}\ $, the left- and right-hand side derivatives of the non exclusionary price schedules coincide if

$$2\left(\frac{1}{2} - \gamma\right)\hat{q}\ \ = \alpha_1^*(1 - \hat{q}\ ).$$

This equation directly yields

$$\hat{q}^{**} = \frac{\alpha_1^*}{1 - 2\gamma + \alpha_1^*}.$$

Substituting into (9) one finally obtains

$$\bar{\alpha}_0^{**} = -\frac{\alpha_1^{*2}}{2\left(1 - 2\gamma + \alpha_1^*\right)}.$$

The fixed fee is negative, since otherwise firms would earn more on the upper part of the non exclusionary price schedules than on the lower part. As long as this is so, firms have an incentive to bribe consumers into the upper part of the schedule by reducing the fixed fees.

Verification of the best response property.

We have thus completed the calculation of a candidate equilibrium of the semi-public common agency game $G\ \ $. Summarizing, both firms offer an exclusive contract

$$p\ \ (q) = 0 \text{ for all } q \geq 0$$

and a non-exclusionary price schedule

$$p\ \ (q) = \begin{cases} \left(\frac{1}{2} - \gamma\right) q^2 & \text{for } 0 \leq q \leq \frac{\ _1^*}{1-2\ \ +\ \ _1^*} \\ -\frac{\ _1^{*2}}{2\left(1-2\ \ +\ \ _1^*\right)} + \alpha_1^* q - \frac{\ _1^*}{2} q^2 & \text{for } \frac{\ _1^*}{1-2\ \ +\ \ _1^*} \leq q \leq 1. \end{cases}$$

[22] That is,

$$\frac{d^2\bar{U}(\theta)}{d\theta^2} = \frac{2}{1 - \alpha_1^*} > \frac{1}{1 - \gamma} = \frac{d^2 U^E(\theta)}{d\theta^2}.$$

Appendix 4.3 verifies that the best-response property is satisfied, implying that this is, indeed, a semi-public common agency equilibrium. In this equilibrium, the market is covered and all consumers purchase both products. Exclusive contracts are offered by both firms, but in equilibrium they are not accepted by any consumer.

However, this last observation calls into question our initial guess that $p\ \ (q\ ) = 0$. That guess was justified by the argument that if an exclusive contract is accepted, the familiar Bertrand undercutting process must drive exclusionary prices to zero. However, in the equilibrium we have just derived, no exclusive contract is actually accepted. This means that firms need not start undercutting each other's exclusionary prices. Although there is always an equilibrium in which both firms set $p\ \ (q\ ) = 0$, as we have argued above, this suggests that there may exist other equilibria, in which firms may offer supra-competitive exclusionary price schedules.

4.3 The most cooperative equilibrium

In common agency models like ours, the multiplicity of equilibria has a very simple explanation. In some cases, firms can offer contracts that are destined not be accepted in equilibrium, but may constrain the payments that their competitors can request in the contracts that will be accepted. Under complete information, for instance, firm i can submit an entire price schedule even if only one contract will be accepted. However, the other contracts firm i offers, in addition to the one that is accepted in equilibrium, may generally affect the consumer's outside option when she deals with firm j, and hence firm j's payoff. Hence, firms can manipulate the offers that are destined not be accepted in order to affect their competitor's equilibrium payoff, whence the multiplicity of possible equilibria. Faced with this multiplicity of equilibria, the literature has typically focused on the equilibrium that is Pareto dominant for the firms: see, for instance, Bernheim and Whinston (1998).[23]

Uncertainty usually mitigates, and sometimes fully solves the indeterminacy problem. In the private common agency model of the preceding section, for instance, the equilibrium price schedules are pinned down fully: since all contracts with $q\ \ \leq 1$ may be accepted with a positive probability, there is no scope for manipulation. With exclusive contracts, however, the problem re-emerges, since exclusive contracts must be offered but are destined not to be accepted in equilibrium. Even if these contracts cannot directly generate any revenue for the firms, they can affect the equilibrium outcome. In particular, the less aggressively firms bid for exclusive contracts, the lower the reservation utility left to consumers, and hence the greater the payments firms can obtain for non exclusive contracts. This suggests that firms can tacitly "coordinate" their exclusive offers in order to increase their equilibrium payoffs.

The equilibrium with $p\ \ (q\ ) = 0$ derived in the preceding subsection is the most competitive equilibrium, in which no such coordination takes place. We are now interested in finding the equilibrium that is Pareto dominant for the firms. What is the maximum degree of coordination

[23] With two firms, Pareto dominance is equivalent to coalition proofness, and hence to truthfulness (Bernheim and Whinston, 1986). With three or more firms, however, the equivalence breaks down, and in the Pareto dominant equilibrium firms obtain greater profits than in the truthful one (Chiesa and Denicolò, 2009).

among firms that is consistent with playing the game non-cooperatively? In other words, supposing that firms tacitly agree to request very large payments in their exclusive contracts, what are the maximum payments that make the tacit agreement stable?

To answer this question, suppose that both firms offer the same strictly positive exclusionary price schedules $p\ \ (q)$ and also the same non exclusionary schedules $\underline{p}\ \ (q)$.[24] In equilibrium, the participation constraint binds for a non-degenerate interval of types $[0, \hat{\theta}]$, so certain low-type consumers must be exactly indifferent between exclusive and non exclusive contracts. This implies that over the interval $[0, \hat{\theta}]$ one must have $q\ \ (\theta) = 2\underline{q}(\theta)$ (for the derivatives of $U\ \ (\theta)$ and $\underline{U}(\theta)$ must be identical over that interval), and also that the following condition must hold:[25]

$$2\underline{p}\ \ (\underline{q}(\theta)) - p\ \ (2\underline{q}(\theta)) = (1-2\gamma)\underline{q}^2(\theta). \tag{10}$$

The economic interpretation of (10) is simple. The left-hand side is the difference between the cost of purchasing quantity $\underline{q}(\theta)$ from both firms and the cost of purchasing the same total quantity, i.e. $q\ \ (\theta) = 2\underline{q}(\theta)$, exclusively from one firm. The right-hand side is the value of variety, i.e., the difference in the utility obtained by consumer θ with those two strategies, as we have seen above. For the consumer to be indifferent between exclusive and non exclusive contracts, the extra cost of common agency must equal the extra benefit of product variety.

Now consider the largest exclusionary price schedules that can be part of an equilibrium. Since consumer $\theta \in [0, \hat{\theta}]$ must be indifferent between exclusive and non exclusive contracts, any arbitrarily small discount must suffice to induce her to switch to exclusive contracts. But no such deviation can be profitable in equilibrium. Since the deviating firm in equilibrium earns $\underline{p}(\underline{q}(\theta))$ on consumer θ while it would earn $p\ \ (2\underline{q}(\theta)) - \varepsilon$ by inducing her to switch, where $\varepsilon > 0$ is arbitrarily small, the no-deviation condition requires $\underline{p}\ \ (\underline{q}(\theta)) \geq p\ \ (2\underline{q}(\theta))$. Using (10), this condition can be rewritten as

$$p\ \ (q) \leq \frac{1-2\gamma}{4} q^2. \tag{11}$$

[24] As we proceed, it will appear that the upper bound on the exclusionary payments is largest when firms behave symmetrically. Thus, given that our goal is to characterize the most "cooperative" equilibrium, there is no loss of generality in assuming symmetry.

[25] To prove equality (10), we begin by noting that in any equilibrium the reservation utility guaranteed by the exclusionary price schedules $p^E(q)$ must be matched by the non exclusionary schedules $\underline{p}^C(q)$; otherwise, some consumers would accept the exclusive contracts and Bertrand competition in utility space would then drive the exclusionary prices to zero. The maximum rent consumer θ could obtain by choosing an exclusive contract is

$$\theta q^E(\theta) - \frac{1-\gamma}{2}\left[q^E(\theta)\right]^2 - p^E(q^E(\theta)),$$

where

$$q^E(\theta) = \arg\max_{q_i}\left[\theta q_i - \frac{1-\gamma}{2} q_i^2 - p_i^E(q_i)\right]$$

On the other hand, the net utility obtained by consumer θ if she purchases $\underline{q}(\theta)$ from both firms is

$$2\theta\underline{q}(\theta) - (1-\gamma)\left[\underline{q}(\theta)\right]^2 - \gamma\left[\underline{q}(\theta)\right]^2 - 2\underline{p}(\underline{q}(\theta)).$$

Equating the net utility consumers obtain with exclusive and non exclusive contracts, equation (10) follows.

This inequality places an upper bound on the maximum payment that can be requested for exclusive contracts.

This upper bound is tight, meaning that for any $0 \leq \mu \leq \frac{1-2}{4}$ there exists a semi-public common agency equilibrium with

$$p \quad (q \quad) = \mu q^2 \qquad i = A, B. \tag{12}$$

The equilibrium where $p \quad (q \quad) = 0$ corresponds to $\mu = 0$ and is, clearly, the most competitive equilibrium. For $\mu = \frac{1-2}{4}$, by contrast, we obtain the least competitive equilibrium, which maximizes firms' profits and minimizes the consumer's rent.

To determine the structure of these and all intermediate equilibria, we proceed as for the case $p \quad (q \quad) = 0$. First of all, note that facing exclusionary schedules (12) consumer θ would purchase

$$q \quad (\theta, \mu) = \frac{\theta}{1 - \gamma + 2\mu},$$

obtaining a reservation utility of

$$U \quad (\theta, \mu) = \frac{\theta^2}{2\,(1 - \gamma + 2\mu)}.$$

With this new and lower reservation utility, the analysis then proceeds as before. The coefficients of the lower part of the non exclusionary price schedules must guarantee to low-type consumers a rent equal to $U \quad (\theta, \mu)$. This requires $\underline{\alpha}_0^{**} = \underline{\alpha}_1^{**} = 0$, as in the most competitive equilibrium, but now we must have

$$\underline{\alpha}_2 \quad \times \underline{\alpha}_2 \quad = \left(\frac{1}{2} - \gamma + 2\mu\right)^2.$$

Focusing again on a symmetric candidate equilibrium, this implies

$$\underline{\alpha}_2^{**} = \frac{1}{2} - \gamma + 2\mu.$$

As for the upper part of the non exclusionary price schedule, we have again $\bar{\alpha}_1^{**} = \alpha_1^*$ and $\bar{\alpha}_2^{**} = -\frac{\overset{*}{1}}{2}$. The $\bar{\alpha}_0$'s and the $\hat{q}$'s must guarantee that the smooth pasting condition is satisfied, or, in other words, that then non exclusionary price schedules are continuous and continuously differentiable at $\hat{q}$. This requires

$$\bar{\alpha}_0^{**} = -\frac{\alpha_1^{*2}}{2\,(1 - 2\gamma + \alpha_1^* + 4\mu)}.$$

and

$$\hat{q}^{**} = \frac{\alpha_1^*}{1 - 2\gamma + \alpha_1^* + 4\mu}.$$

This completes the calculation of the candidate equilibrium for any admissible μ.

Appendix 4.3 verifies that the candidate equilibria satisfy the best response property in the unrestricted strategy space $\mathcal{S}$. Hence, we have:

Proposition 3 *The following are equilibria of the semi-public common agency game G . Firm $i = A, B$ offers an exclusionary price schedule*

$$p^{**}(q) = \mu q^2 \text{ for all } q \geq 0 \tag{13}$$

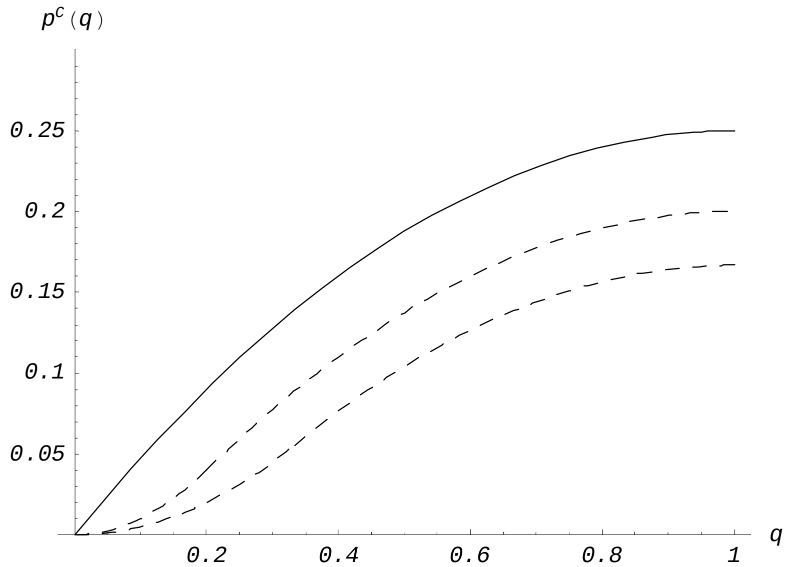
with

$$0 \leq \mu \leq \frac{1 - 2\gamma}{4},$$

and a non-exclusionary price schedule

$$p^{**}(q) = \begin{cases} \left(\frac{1}{2} - \gamma + 2\mu\right) q^2 & \text{for } 0 \leq q \leq \hat{q}^{**} \\ -\dfrac{\alpha_1^{*2}}{2\left(1 - 2\gamma + \alpha_1^* + 4\mu\right)} + \alpha_1^* q - \frac{{}^{*}_{1}}{2} q^2 & \text{for } \hat{q}^{**} \leq q \leq 1. \end{cases} \tag{14}$$

The equilibrium non exclusionary price schedules are depicted in Figure 3. They are at first convex (meaning that consumers pay quantity premia) and then concave (so firms offer quantity discounts eventually, as under private common agency). For any admissible value of μ, these price schedules lie below the private common agency schedule – the dotted curves in Figure 3. This fact suggests that exclusive contracts are pro-competitive. To confirm this intuition, now we turn to the welfare analysis.



Equilibrium price schedules under private common agency (the continuous curve) and with exclusionary contracts (dashed curves, the upper one is the most cooperative equilibrium, the lower one is the most competitive).

4.4 Welfare comparison

In this subsection we compare the equilibria with exclusive contracts to the private common agency equilibrium in terms of consumer surplus, profits, and social welfare. Although multiple equilibria exist with exclusive contracts, we obtain unambiguous predictions irrespective of which equilibrium is selected.

To begin with, notice that since the marginal prices are zero at $q = 0$, now in equilibrium the market is covered and all consumers purchase both products. The equilibrium quantities are

$$q^{**}(\theta) = \begin{cases} \dfrac{\theta}{2\,(1-\gamma+2\mu)} & \text{for } 0 \le \theta \le \hat{\theta}^{**} \\ q^{*}(\theta) \quad \text{for } \hat{\theta}^{**} \le \theta \le 1, & \end{cases} \tag{15}$$

where $\hat{\theta}^{**}$ is the threshold where consumers switch from the lower to the upper part of the non exclusive schedules, and is

$$\hat{\theta}^{**} \equiv \frac{2\alpha_1^{*}(1-\gamma+2\mu)}{1-2\gamma+\alpha_1^{*}+4\mu}.$$

Figure 4 depicts the special case $\gamma = 0$, but the qualitative pattern is more general. The equilibrium quantities with exclusive contracts decrease with μ, but for any value of μ they are at least as large as under private common agency. In particular, consumers of type $\theta \in [0, \alpha_1^{*}]$ now purchase positive quantities whereas without exclusive contracts they would not have purchased at all, and consumers of type $\theta \in (\alpha_1^{*}, \hat{\theta}^{**}]$ increase their consumption.

For $\theta \in [\hat{\theta}^{**}, 1]$, equilibrium quantities stay unchanged. However, these consumers, whose consumption does not change, now enjoy lower prices because of the negative fixed fees. As a result, the net surplus obtained by consumer θ, which is

$$U^{**}(\theta) = \begin{cases} \dfrac{\theta^2}{2\,(1-\gamma+2\mu)} & \text{for } 0 \le \theta \le \hat{\theta}^{**} \\ U^{*}(\theta) + \dfrac{\alpha_1^{*2}}{(1-2\gamma+\alpha_1^{*}+4\mu)} & \text{for } \hat{\theta}^{**} \le \theta \le 1, \end{cases} \tag{16}$$

is everywhere strictly greater than under private common agency (see Figure 5).[26]

[26] By the envelope theorem, this follows immediately from the fact that equilibrium quantities are larger.

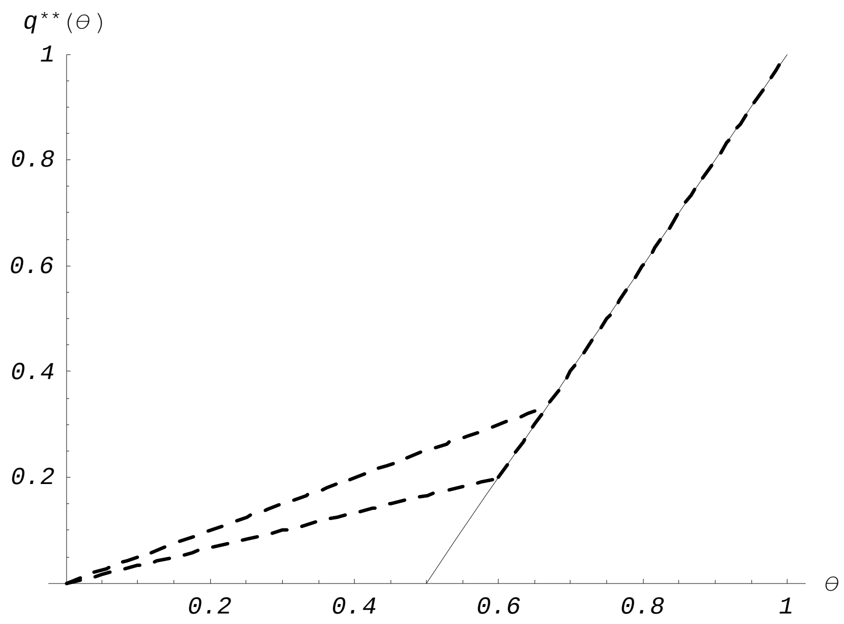
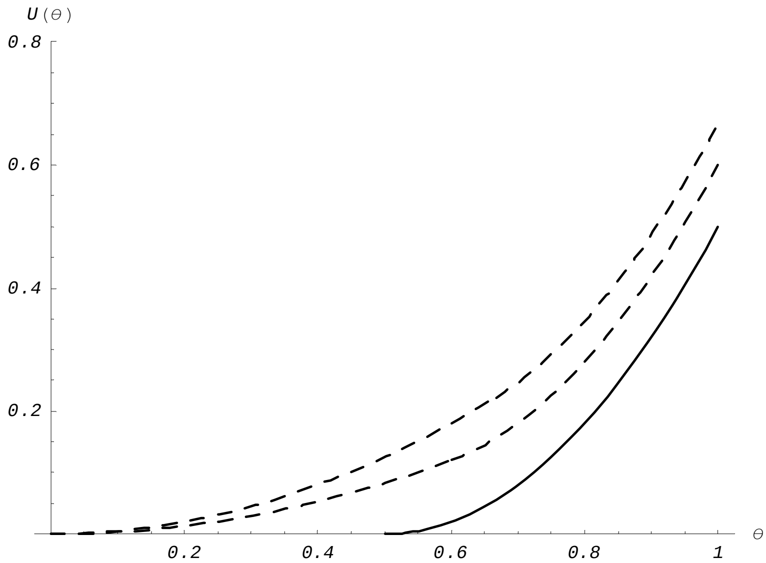


Figure 1: The equilibrium quantities when the goods are independent ($\gamma = 0$) with private common agency (continuous line) and with exclusive contracts (dashed lines) in the most competitive (the upper line) and the most cooperative (the lower line) equilibrium.

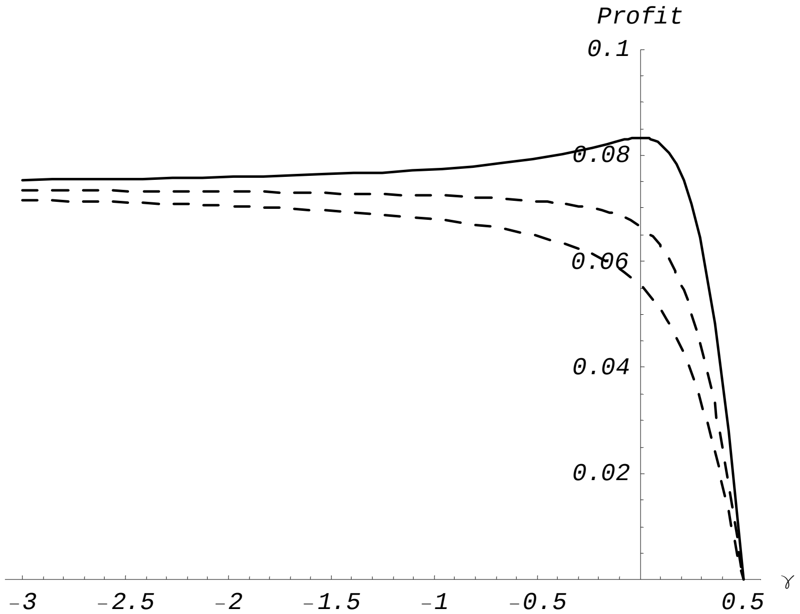


The consumer surplus under private common agency (the continuous curve) and with exclusionary contracts in the most competitive equilibrium (the highest dashed curve) and the most cooperative one (the lowest dashed curve).

Since the new equilibrium quantities are everywhere closer to the first best quantities, and u is concave, social welfare u is always greater with than without exclusive contracts. However, not even in the most competitive equilibrium (i.e., $\mu = 0$) is the efficient solution attained. The intuition is that while in the most competitive equilibrium exclusionary prices vanish, non exclusionary prices are still supra-competitive, and thus inefficient.

Finally, direct calculation shows that for any value of μ and γ, firms' profits are always lower with exclusive contracts than under private common agency (see Figure 6).[27] This means that when exclusive contracts are permitted, firms are caught in a prisoners' dilemma. Both would gain by entering an agreement not to offer exclusive contracts, but if such an agreement is not binding, each has a unilateral incentive to offer exclusionary price schedules (Proposition 2).

[27] Although this results follows from simple algebraic calculations, from an economic point of view it is not obvious when the goods are complements. For permitting exclusionary contracts lowers prices, but when the goods are complements the equilibrium prices under private common agency are too high not only from the social viewpoint, but also from the point of view of joint profit maximization, due to the problem of Cournot complements.



Firms' profits under private common agency (continuous line) and with exclusionary contracts in the most competitive equilibrium (the lower dashed line) and in the most cooperative one (the upper dashed line).

We can summarize the above discussion as follows.

Proposition 4 *Exclusive contracts are pro-competitive for any possible degree of tacit coordination among the firms: they decrease prices, increase consumer surplus, decrease profits, and increase social welfare.*

The more limited is the degree of tacit coordination (i.e., the lower is μ), the more pro-competitive are exclusive contracts.

5 Market-share discounts

Now we turn to the case where both exclusive contracts and market-share discounts are allowed. In this case, each firm i can request a payment $p\ (q\ , q\ )$ that depends not only on its own output, q , but also on its rival's, q . In the jargon of the common agency literature, this is known as the public common agency case.

The guess.

As usual, we start by guessing a specific functional form of the price schedules. At first, it would seem natural to posit quadratic price schedules of the type

$$p\ (q\ ,q\ ) = \alpha_0\ \ + \alpha_1\ q\ + \alpha_2\ q^2 + \alpha_3\ q\ + \alpha_4\ q^2 + \alpha_5\ q\, q\ , \tag{17}$$

and it might seem redundant to allow firms to submit a separate exclusionary price schedule, since the schedules (17) already include exclusive contracts for $q\ = 0$. Notice, however, that the specification (17) may not satisfy the condition that no payment can be due to a firm if nothing is purchased from it, i.e., $p\ (0, q\ ) = 0$. Thus, the schedule $p\ (q\ , q\ )$ may have to be discontinuous at $q\ = 0$. But this discontinuity may entail another discontinuity, this time at $q\ = 0$. For if $p\ (q\ , q\ )$ is discontinuous at $q\ = 0$, one must allow firm i to respond discontinuously at $q\ = 0$. Thus, after all, we must posit that each firm offers also a separate exclusionary price schedule $p\ \ (q\ )$, just as in the previous section.

As we know, given any pair of exclusionary price schedules $p\ \ (q\ )$, the restricted game in which firms must choose only non exclusive contracts can be viewed as a game with type-dependent reservation utility. Again, this suggests that equilibrium non exclusionary price schedules may have two parts, one intended for consumers who will obtain exactly their reservation utility $U\ \ (\theta)$, and one for those who will obtain strictly more.

Another property of the semi-public common agency game that carries over to the public common agency case is the multiplicity of equilibria, which is due to the fact that in equilibrium firms make offers that are destined not to be accepted by any consumer. Since now the set of permitted contracts is broader than under semi-public common agency, the scope for multiple equilibria is even wider. However, it turns out that varying the exclusionary price schedules $p\ \ (q\ )$ suffices to generate the entire set of equilibria. The upper bound on the maximum payment that can be requested for exclusive contracts is again given by (11). The proof is identical to the case of semi-public common agency and is not repeated here.

Thus, suppose that both firms offer exclusionary schedules

$$p\ \ (q\ ) = \mu q^2 \text{ for all } q\ \geq 0$$

with $\mu \in \left[0, \frac{1-2}{4}\right]$. The non exclusionary price schedules now can depend both on own output and the rival's output. Assuming a piecewise quadratic specification, now we conjecture:[28]

$$p\ (q\ , q\ ) = \begin{cases} \underline{\alpha}_0\ \ + \underline{\alpha}_1\ q\ + \underline{\alpha}_2\ q^2 + \underline{\alpha}_3\ q\ + \underline{\alpha}_4\ q^2 . (\equiv \underline{p}\ \ (q\ , q\ )) & \text{for } 0 \leq q\ \leq \hat{q} \\ \bar{\alpha}_0\ \ + \bar{\alpha}_1\ q\ + \bar{\alpha}_2\ q^2 + \bar{\alpha}_3\ q\ + \bar{\alpha}_4\ q^2 . (\equiv \bar{p}\ \ (q\ , q\ )) & \text{for } \hat{q}\ < q\ \leq 1 \end{cases} \tag{18}$$

The candidate equilibria.

[28] We have also analyzed the consequences of adding the term $\alpha_{5,i} q_i q_j$ in both parts of the schedules, but this term turns out to be irrelevant, so the coefficients $\alpha_{5,i}$ can be set equal to zero with no loss of generality.

In the restricted game where firms choose only schedules of type (18), for each firm there are now eleven coefficients to be determined: the $\underline{\alpha}$'s, the $\bar{\alpha}$'s, and $\hat{q}$. As before, we consider the lower part of the non exclusionary price schedules first, then the upper part, and finally the smooth pasting condition.

Step 1. As in the semi-public common agency case, the $\underline{\alpha}$'s are determined by the requirement that the lower part of the schedules must re-produce the reservation utility $U\ (\theta, \mu)$. This requires (using three stars to denote the public common agency equilibrium)

$$\underline{\alpha}_0^{***} = \underline{\alpha}_0^{***} = \underline{\alpha}_1^{***} = \underline{\alpha}_1^{***} = \underline{\alpha}_3^{***} = \underline{\alpha}_3^{***} = 0$$

and

$$\left(\underline{\alpha}_2 \quad + \underline{\alpha}_4 \quad\right) \times \left(\underline{\alpha}_2 \quad + \underline{\alpha}_4 \quad\right) = \left(\frac{1}{2} - \gamma + 2\mu\right)^2. \tag{19}$$

Now, however, we have several degrees of freedom in the choice of the coefficients that appear in equation (19). This is partly due to the fact that a game of private provision of an indivisible public good (i.e., common participation) may have multiple equilibria in which firms contribute asymmetrically, as discussed above. Although setting $\underline{\alpha}_2 = \underline{\alpha}_2$ and $\underline{\alpha}_4 = \underline{\alpha}_4$ now no longer suffices to select a unique equilibrium, it makes the remaining indeterminacy payoff irrelevant. To see why, notice that when $\underline{\alpha}_2 = \underline{\alpha}_2$ and $\underline{\alpha}_4 = \underline{\alpha}_4$, the cost of inducing common participation is divided evenly among the firms. Now, however, within the limits of the total payment it can request, each firm can choose to charge a positive price for its own output or for its rival's. For reasons that will become clear later, we impose a strong form of symmetry, assuming that firms charge equally for both goods. Thus, we set $\underline{\alpha}_2 = \underline{\alpha}_4$. With this convention, we obtain a unique solution

$$\underline{\alpha}_2^{***} = \underline{\alpha}_2^{***} = \underline{\alpha}_4^{***} = \underline{\alpha}_4^{***} = \left(\frac{1}{4} - \frac{\gamma}{2} + \mu\right).$$

Summarizing, in our candidate equilibria the lower part of the non exclusionary price schedules is

$$\underline{p}\ (q\ , q\ ) = \left(\frac{1}{4} - \frac{\gamma}{2} + \mu\right) q^2 + \left(\frac{1}{4} - \frac{\gamma}{2} + \mu\right) q^2.$$

Step 2. Next, consider the upper part of the schedules, $\bar{p}\ (q\ , q\ )$. These are intended for consumers whose participation constraint does not bind. It is therefore natural to conjecture that these parts of the non exclusive schedules must coincide (with the possible exception of the fixed fees) with the equilibrium schedules in an hypothetical game where the consumer's reservation utility is zero. The intuition is similar to the case of semi-public common agency: in any interval $(\hat{\theta}, 1]$ where the participation constraint does not bind, the equilibrium of the game played by the firms must be independent of whatever happens to lower types.

Consider, then, the *fictitious game* in which each firm i chooses the coefficients of a quadratic price schedule

$$p\ (q\ , q\ ) = \alpha_0 \ + \alpha_1\ q\ + \alpha_2\ q^2 + \alpha_3\ q\ + \alpha_4\ q^2 \qquad \text{for all } 0 \leq q\ , q\ \leq 1$$

so as to maximize its expected profits, and the consumer's reservation utility is zero. In searching for the equilibrium of this game, we can exploit the no-distortion-at-the-top property to set

$$\alpha_2 \;=-\frac{\alpha_1}{2} \text{ and } \alpha_4 \;=-\frac{\alpha_3}{2}.$$

In addition, we know that no fixed fee is charged when the market is uncovered, so we set $\alpha_0 \;=0$. As a consequence, now each firm must choose only two coefficients, namely α_1 and α_3 .

Appendix 5.1 shows that this restricted game has a unique equilibrium, which is symmetric and is given by

$$\alpha_1^+ = \alpha_3^+ = \frac{1}{3}.$$

The corresponding quantities are

$$q^+(\theta) = 3\theta - 2.$$

This solution has two remarkable properties. First, for any value of γ the equilibrium quantities are the same as in a private common agency game where the goods are perfect complements (i.e., $\alpha_1^+ = \lim_{\,\to-\infty} \alpha_1^*$). Second, the direct coefficients (resp., α_1^+ and α_2^+) coincide with the cross coefficients (resp., α_3^+ and α_4^+).

These two properties are closely related. The fact that each firm can charge the consumer both for consuming its own output and its rival's creates an externality, which is exactly similar to that arising when the goods are perfect complements. In both cases, the total price faced by the final consumer is the sum of the prices charged by two separate firms, each of which acts non-cooperatively, internalizing only partially the negative consequences of an increase in its own price.

In particular, starting from the private common agency equilibrium where it cannot "tax" the output of good j, firm i always has an incentive to charge a positive price for good j, since this affects negatively only the revenue accruing to firm j. In fact, firm i has an incentive to increase the marginal price it charges on good j precisely to the same extent as firm j has an incentive to increase its own price, so in equilibrium both firms must charge the same marginal price for each good.[29] But this implies that in equilibrium each firm charges equally for both own output and its rival's output.[30]

The upper part of the non exclusionary price schedules must result in the same equilibrium quantities as the hypothetical game, i.e., $q^+(\theta) = 3\theta - 2$. This requires

$$\begin{aligned} \bar{\alpha}_1^{***} &= \bar{\alpha}_3^{***} = \frac{1}{3} \\ \bar{\alpha}_2^{***} &= \bar{\alpha}_4^{***} = -\frac{1}{6}. \end{aligned}$$

[29] This property is related to the "principle of aggregate concurrence" of Martimort and Stole (2009b): since both firms must concur on the choice of the equilibrium quantities, the firms' marginal rates of substitution across q_A and q_B must be identical.

[30] With positive marginal production costs, firms would equalize the price-cost margins. Thus, each firm would charge a greater marginal price on own output (which alone entails a positive marginal cost) than on its rival's.

Step 3. To complete the calculation of the candidate equilibrium, we must determine the $\bar{\alpha}_0$'s and the $\hat{q}$'s. As in section 4, to pin down these coefficients we impose a smooth pasting condition, requiring that the non exclusionary price schedules $p\ (q\ , q\ )$ be continuous and continuously differentiable at $\hat{q}$.[31] Continuity requires

$$\underline{p}\ (\hat{q}\ , \hat{q}\ ) = \bar{p}\ (\hat{q}\ , \hat{q}\ ). \tag{20}$$

On the other hand, equating the right and left derivative of $p\ (q\ , q\ )$ at $q\ = \hat{q}$ we get

$$\left(\frac{1}{2} - \gamma + 2\mu\right)\hat{q}\ = \frac{1}{3} - \frac{1}{3}\hat{q}\ . \tag{21}$$

The solution to the system (18)-(19) is

$$\bar{\alpha}_0^{***} = -\frac{2}{3(5 - 6\gamma + 12\mu)}$$

and

$$\hat{q}^{***} = \frac{2}{(5 - 6\gamma + 12\mu)}.$$

This completes the calculation of the candidate equilibrium.

Verification of the best response property.

The final step of the procedure must verify that these price schedules are an equilibrium of the original game G , with the unrestricted strategy set $\mathcal{S}$. This is done using direct mechanisms, as for the other games, but now the indirect utility function is defined differently. For now each firm can effectively control both q and q . Thus, the indirect utility function for firm i is simply the difference between the consumer's utility, u, and firm j's non exclusionary price schedule. Of course, the consumer has again a type-dependent reservation utility $U\ (\theta, \mu)$. As usual, the details of the verification are relegated to the Appendix.

Proposition 5 *The following are equilibria of the semi-public common agency game. Any firm $i = A, B$ offers an exclusionary price schedule*

$$p^{\ ***}(q\ ) = \mu q^2 \textit{ for all } q\ \geq 0$$

with

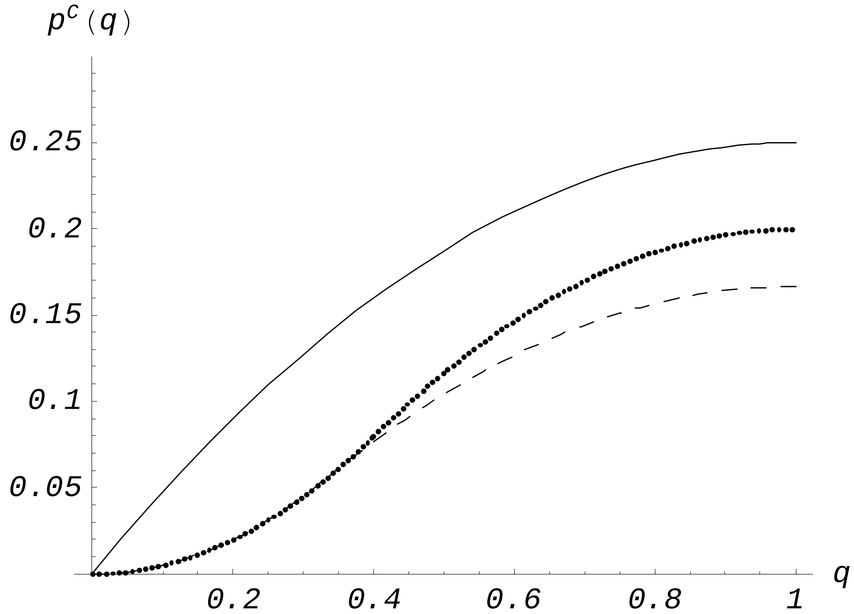
$$0 \leq \mu \leq \frac{1 - 2\gamma}{4},$$

[31] The proof that the smooth pasting condition must be satisfied is similar to the case of semi-public common agency and is not repeated. We notice only that our choice of setting $\underline{\alpha}_{2,A}^{***} = \underline{\alpha}_{2,B}^{***} = \underline{\alpha}_{4,A}^{***} = \underline{\alpha}_{4,B}^{***}$ allows us to obtain a fully symmetric candidate equilibrium after imposing the smooth pasting condition.

and a non-exclusionary price schedule

$$p^{\;***}(q\;,q\;)=\begin{cases}\left(\frac{1}{4}-\frac{}{2}+\mu\right)q^2+\left(\frac{1}{4}-\frac{}{2}+\mu\right)q^2 & \textit{for } 0\le q\;\le \hat{q}^{***}\\[2ex] -\dfrac{2}{3(5-6\gamma+12\mu)}+\frac{1}{3}q\;-\frac{1}{6}q^2+\frac{1}{3}q\;-\frac{1}{6}q^2 & \textit{for } \hat{q}^{***}\le q\;\le 1.\end{cases}$$

Figure 7 compares the public common agency equilibrium price schedules with those obtained under private and semi-public common agency. To make the comparison possible, Figure 7 represents the total payment due by a consumer who purchases the same amounts of both products, $q\;=q\;$, as all consumers do in equilibrium. Equilibrium prices under market-share discounts are greater than in the case when only exclusive contracts are permitted, but they are lower than in the private common agency case.



Equilibrium price schedules under private common agency (the continuos line), with exclusionary contracts (the dashed line) and with market share discounts (the dotted line). In the last two cases, the curves represent the most competitive equilibrium.

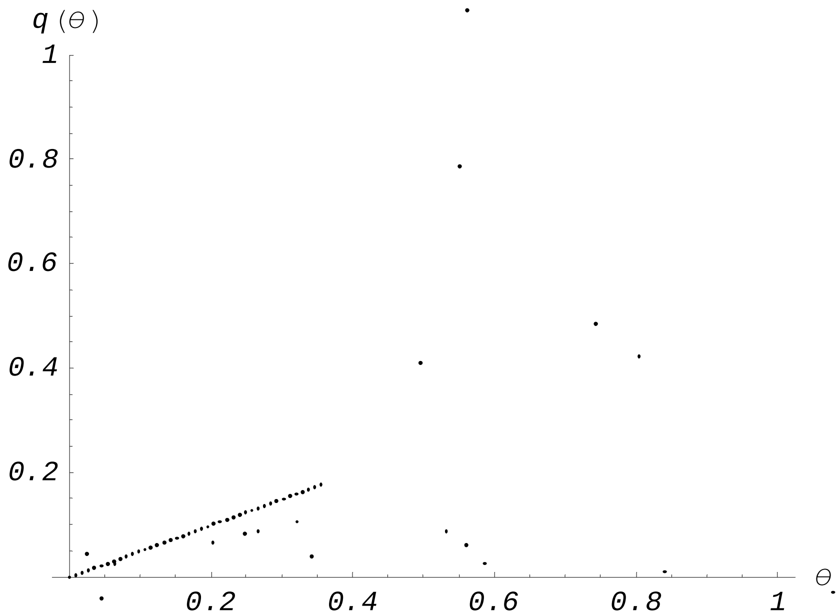
In equilibrium, the market is covered and all consumers purchase from both firms. The equilibrium quantities are

$$q^{***}(\theta)=\begin{cases}\dfrac{\theta}{2\,(1-\gamma+2\mu)} & \text{for } 0\le\theta\le\hat{\theta}^{***}\\[2ex] 3\theta-2 \quad \text{for } \hat{\theta}^{***}\le\theta\le 1, & \end{cases}\tag{22}$$

where the critical threshold $\hat{\theta}$ now is

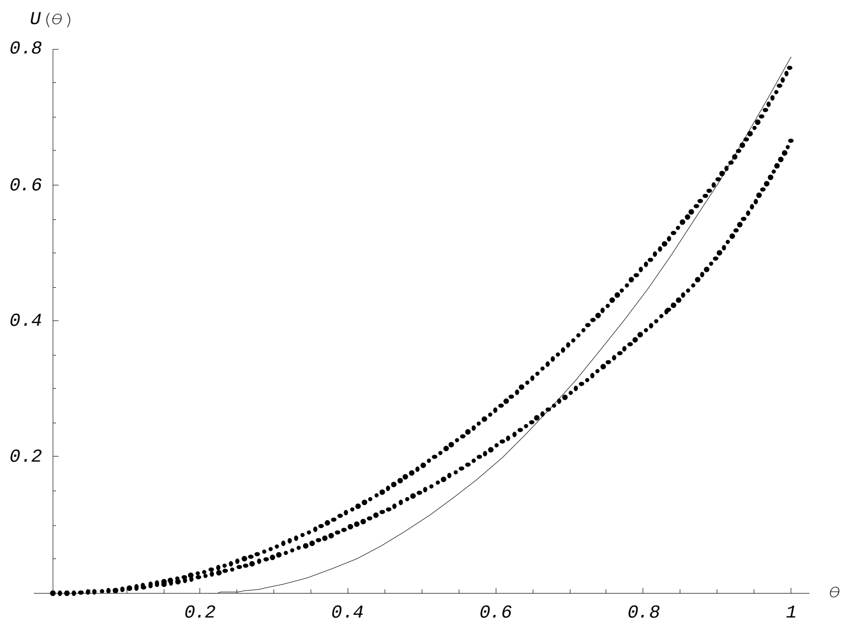
$$\hat{\theta}^{***} = \frac{4(1-\gamma+2\mu)}{(5-6\gamma+12\mu)}$$

They are depicted in Figure 8 for the special case $\gamma = 0$. Equilibrium quantities are lower than in the semi-public common agency case (strictly so when $\theta > \hat{\theta}^{**}$), but the comparison with the private common agency case is ambiguous: equilibrium quantities increase for low-type consumers (i.e., consumers θ such that $\theta < \hat{\theta}^{***}$), but decrease for high-type consumers.



Proposition 6 *For any given degree of tacit coordination among the firms μ, if exclusive contracts are permitted, allowing firms to offer also market-share discounts reduces consumption, consumer surplus, and social welfare.*

The comparison of the public and private common agency equilibria is instead ambiguous. With market-share discounts, the consumption of low-type consumers is less distorted than under private common agency, but the opposite is true for high-type consumers. However, with market-share discounts high-type consumers now obtain a negative fixed fee, which can at least partly offset the negative effect of reduced consumption. Nevertheless, when the goods are substitutes, in the most cooperative equilibrium with market-share discounts some high-type consumers are worse off than in the private common agency equilibrium. Focusing on the most competitive equilibrium makes the comparison more favorable for market-share discounts, but still some high-type consumers necessarily lose when the goods are sufficiently close substitutes (to be precise, the condition is $\gamma > \frac{1}{5}$), as shown in the following figure.



Equilibrium utilities with $\gamma = \frac{1}{3}$ under private common agency (continuous line) and market share discounts (upper dotted line for the most competitive equilibrium and the lower dotted line for the most cooperative one).

Continuing to focus on the most competitive equilibrium, Appendix 5.3 shows that on average, allowing for market-share discounts (starting from a situation where exclusive contracts are prohibited) increases consumer surplus and welfare and reduces profits.

Proposition 7 *In the most competitive equilibrium, permitting both exclusive contracts and market-share discounts increases expected consumer surplus and social welfare, but decreases expected profits.*

When $\mu > 0$, however, even the *ex ante* comparison becomes ambiguous: a move from private to public common agency is pro-competitive if the goods are not too close substitutes, but it can be anti-competitive when γ is sufficiently close to $\frac{1}{2}$.

Proposition 8 *When the degree of tacit coordination among the firms μ and the degree of product substitutability γ are sufficiently large, permitting both exclusive contracts and market-share discounts reduces expected consumer surplus and social welfare.*

The proof of this Proposition is in Appendix 5.4. The intuition is that when the degree of product substitutability increases, $\hat{\theta}^{***}$ goes to zero, so a move from private to public common agency decreases the equilibrium quantities for almost all consumers.

6 Conclusions

We have studied competition in non-linear prices for horizontally differentiated products when firms can offer exclusive contracts and market-share discounts to consumers who are privately informed on their demand. If exclusive contracts are allowed, firms have a unilateral incentive to offer them although this intensifies competition, reducing profits (a prisoner's dilemma) and increasing consumers' surplus and welfare. The reason why exclusive contracts are pro-competitive is that they force firms to compete in utility space where their products are effectively homogeneous, irrespective of the degree of product differentiation. In equilibrium, consumers still purchase both products, but to induce them to do so, firms must reduce their non exclusionary prices so as to match the outside option provided by exclusive contracts. If firms are allowed to offer also market-share discounts, however, competition is weakened, since market-share discounts create a double-marginalization effect by allowing firms to impose a "tax" on each other's output. As a result, a move from a situation where both exclusive contracts and market-share discounts are prohibited to one where they are both allowed has ambiguous effects on consumer surplus and social welfare. Low-type consumers benefit from such a move, but high-type consumers are harmed by the double-marginalization effect created by market-share discounts.

Although to get closed-form solutions we have used a simple specification with uniform distribution of types and quadratic utility function, the arguments we have used to characterize the equilibrium are more general. This suggests that our qualitative results hold under more general conditions, and we are pursuing this extension. Another interesting extension is to the case where the parameter θ is distributed over a support $[\theta_{\min}, 1]$ with $\theta_{\min} > 0$, so that the market can be covered also under private common agency. In the limiting case $\theta_{\min} \to 1$, one then re-obtains a complete information model, where we know from Bernheim and Whinston (1998) that exclusive contracts and market-share discounts are neutral. A preliminary investigation of this case suggests that with exclusive contracts the neutrality result still holds when $\theta_{\min}$ is sufficiently close

to 1, whereas with market-share discounts it is re-obtained only in the limiting case of complete information.

To the best of our knowledge, this is the first paper that analyzes the effects of exclusive contracts and market-share discounts in a model of oligopoly where firms cannot price discriminate perfectly, without making *ad hoc* assumptions on the type of contracts firms can offer and on the timing of offers. Thus, our results have an obvious relevance for competition policy. Although some are clear-cut and others are more ambiguous, the collection taken together raises doubts about the soundness of a *per se* illegality rule, such as that adopted by competition authorities and the courts in Europe. Another implication of our analysis, one that may seem surprising at first, is that competition policy should allow exclusive contracts but prohibit market-share discounts. This conclusion runs against the popular but fallacious view of market-share discounts as weaker versions of exclusive-dealing arrangements.

Our analysis has focused only on the case of symmetric firms. However, the current policy debate suggests that exclusive contracts and market-share discounts may become more dangerous when firms are asymmetric. For a dominant firm might use these contracts to foreclose a rival that is not equally efficient, and yet should stay in the market since it supplies a differentiated product for which there is consumers' demand. We plan to extend our analysis to the case of asymmetric firms in a follow-on paper.

Appendices

Appendix 3.1

The candidate equilibrium under private common agency

Let us consider a restricted game where firms are constrained to submit price schedules of the type

$$p\ (q\ ) = \alpha_1\ q\ - \frac{\alpha_1}{2}q^2 \quad \text{for } q\ \in [0,1].$$

A strategy for firm i then becomes simply a value of α_1. Faced with these price schedules, consumer θ maximizes

$$u\,(q\ \ ,q\ \ ,\theta) - \alpha_1\ \ q\ \ - \frac{\alpha_1}{2}q^2 - \alpha_1\ \ q\ \ - \frac{\alpha_1}{2}q^2\ .$$

The consumer's optimal choice depends on whether α_1 is greater or lower than α_1. Hence, firm A's best response function has two branches, according to whether α_1 is greater or lower than α_1. Since the branching point is $\alpha_1 = \alpha_1$ and the candidate equilibrium is symmetric, either branch can be used to calculate the candidate equilibrium, but the calculations are simpler if one assumes $\alpha_1 \geq \alpha_1$. Focusing on this case we get:

$$\tilde{q}\ \ (\theta) = \max\left[0, \frac{(1-2\gamma-\alpha_1\ \ )\theta-(1-\gamma-\alpha_1\ \ )\alpha_1\ \ +\gamma\alpha_1}{(1-\gamma-\alpha_1\ \ )(1-\gamma-\alpha_1\ \ )-\gamma^2}\right]$$

Firm A's profit then is

$$\begin{aligned}\pi &= \int_0^1 \left[\alpha_1 \ \ \tilde{q} \ \ (\theta) - \frac{\alpha_1}{2}\tilde{q}^2 \, (\theta)\right] d\theta \\ &= \frac{(1-2\gamma)-(1-\gamma)(\alpha_1 \ \ + \alpha_1 \ \ ) + \alpha_1 \ \ \alpha_1}{3(1-2\gamma-\alpha_1 \ \ )}\alpha_1 \ \ ,\end{aligned}$$

and its best response function is

$$\alpha_1 \ \ = \frac{(1-2\gamma)-(1-\gamma)\alpha_1}{2(1-\gamma-\alpha_1 \ \ )} \qquad \text{for } \alpha_1 \ \ \geq \alpha_1 \ \ .$$

Imposing symmetry, $\alpha_1 \ \ = \alpha_1 \ \$, we can solve to get[32]

$$\alpha_1^* = \frac{1}{4}\left[3(1-\gamma) - \sqrt{1-2\gamma+9\gamma^2}\right].$$

It is easy to check that there are no asymmetric equilibria.

Appendix 3.2

Proof of Proposition 1

We must verify that the candidate equilibrium is, indeed, an equilibrium of the original game $G \ \$. That is, we must prove that if firm j offers a price schedule $p \ \ = \alpha_1^* q \ \ - \frac{\overset{*}{1}}{2} q^2$, then firm i's best response in the set of all possible price schedules $\mathcal{S} \ \ $ is indeed $p \ \ = \alpha_1^* q \ \ - \frac{\overset{*}{1}}{2} q^2$.

Given $p \ \ = \alpha_1^* q \ \ - \frac{\overset{*}{1}}{2} q^2$, firm i faces a standard monopolistic screening problem where the consumer's utility is given by the following indirect utility function

$$v^*(q \ , \theta) = \max_{j \geq 0}\left[u(q \ , q \ , \theta) - \left(\alpha_1^* q \ \ - \frac{\alpha_1^*}{2} q^2\right)\right],$$

which is the maximum utility that a consumer of type θ can obtain by purchasing q and trading optimally with firm j. Assuming an interior solution,[33] this indirect utility function can be easily calculated as

$$v^*(q \ , \theta) = A_0 + A_1 q \ \ + A_2 q^2$$

[32] There is also another root, which however does not satisfy the second order condition of the firms' maximization problems.

[33] When $\gamma q_i > \theta - \alpha_1^*$, the consumer finds it optimal to set $q_j = 0$, so the indirect utility function reduces to $\theta q_i - \frac{1-\gamma}{2} q_i^2$. However, notice that (i) in the candidate equilibrium inequality $\gamma q_i > \theta - \alpha_1^*$ never holds, and (ii) even accounting for the corner solution, the indirect utility function is globally concave. These observations imply that the branch of the indirect utility function that we do not consider in the proof is irrelevant.

where

$$\begin{aligned} A_0 &= \frac{(\theta - \alpha_1^*)^2}{2(1-\gamma-\alpha_1^*)} \\ A_1 &= \frac{(1-2\gamma-\alpha_1^*)\theta + \gamma\alpha_1^*}{(1-\gamma-\alpha_1^*)} \\ A_2 &= -\frac{1-\alpha_1^* - 2\gamma + \gamma\alpha_1^*}{2(1-\gamma-\alpha_1^*)} \end{aligned}$$

The indirect utility function is concave. Now, however, the consumer's reservation utility, i.e.,

$$v^*(0,\theta) = A_0,$$

is type dependent. Thus, in order to apply the standard approach of pointwise maximization of the virtual surplus function, we must check not only that the sorting condition is satisfied, but also that the equilibrium rent increases with θ more rapidly than the reservation utility, so that the consumer's participation constraint $v^*(q\ ,\theta) \geq v^*(0,\theta)$ binds only at $\theta = \alpha_1^*$ (see e.g. Laffont and Martimort, 2002). The sorting condition is

$$\frac{\partial^2 v^*}{\partial\theta\partial q} = \frac{1-2\gamma-\alpha_1^*}{1-\gamma-\alpha_1^*} > 0.$$

Substituting (1) into this formula, it can be checked that the inequality always holds. The second condition requires that

$$2q^*(\theta) > \frac{\theta - \alpha_1^*}{1-\gamma+\alpha_1^*},$$

since by the envelope theorem the derivative of the equilibrium rent equals the sum of the quantities purchased. This condition also reduces to $1-2\gamma-\alpha_1^* > 0$, and so it is always satisfied, too. (Calzolari and Scarpa, 2008, show that this is no coincidence: the condition that the equilibrium rent increases with θ more rapidly than the reservation utility is always entailed by the sorting condition in models like ours.)

Thus, firm i's problem reduces to finding a function $q\ (\theta)$ that pointwise maximizes the "indirect

virtual surplus”[34]

$$\begin{aligned} s\ (q\ ,\theta) &= v^*(q\ ,\theta) - (1-\theta)\frac{dv^*}{d\theta} \\ &= v^*(q\ ,\theta) - (1-\theta)\left[\frac{\partial A_0}{\partial\theta} + \frac{\partial A_1}{\partial\theta}q\right]. \end{aligned}$$

The first order condition for a maximum is

$$A_1 + 2A_2 q\ - (1-\theta)\frac{\partial A_1}{\partial\theta} = 0$$

which implies

$$\begin{aligned} q\ (\theta) &= \frac{(1-\theta)\frac{\ _1}{} - A_1}{2A_2} \\ &= \frac{2\theta(1-2\gamma-\alpha_1^*) - (1-2\gamma-\alpha_1^*-\gamma\alpha_1^*)}{1-\alpha_1^*-2\gamma+\gamma\alpha_1^*}. \end{aligned}$$

Using the definition of α_1^* in (1), simple algebra shows that $q\ (\theta) = q^*(\theta)$, so the optimal mechanism for firm i must induce the consumer to choose the candidate equilibrium quantities. Clearly, the only price schedule that supports these quantities is $p\ = \alpha_1^* q\ - \frac{{}^*_1}{2}q^2$. ∎

Appendix 4.1

Proof of Proposition 2

To prove the proposition, it suffices to show that starting from the private common agency equilibrium, each firm can unilaterally increase its profits by offering exclusive contracts. In other words, when exclusive contracts are permitted, if firms offered only non exclusionary price schedules equal to the private common agency equilibrium schedules, then there would exist a profitable deviation.

To construct such a profitable deviation, we focus on a deviation that is targeted to a single type $\tilde{\theta}$. For consumer $\tilde{\theta}$ alone to be induced to purchase exclusively from firm i, firm i must offer a quantity-forcing exclusive schedule that consists of a single contract $(p\ \ ,q\ \ )$, where

$$q\ \ = 2q^*(\tilde{\theta}).$$

[34] Considering also the branch of the indirect utility in which $q_j = 0$ so that $s_i(q_i,\theta) = u(q_i,0,\theta) - (1-\theta)q_i$, both the virtual surplus function and its derivative are discontinuous at the critical value of q_i that makes q_j vanish. However, this discontinuity preserves global concavity, as shown by Martimort and Stole (2009). Since we focus on symmetric equilibria, we can therefore limit our analysis to the case in which both $q_i > 0$ and $q_j > 0$.

When firm i offers the contract $(p\ \ , q\ \ )$, the net utility consumers may obtain by choosing to purchase exclusively from firm i is

$$2\theta q^*(\tilde{\theta}) - 2(1-\gamma)\left[q^*(\tilde{\theta}))\right]^2 - p\ \ .$$

Hence, net utility is linear in θ and has the same slope as $U^*(\theta)$ at $\theta = \tilde{\theta}$. It follows that by appropriate choice of the requested payment p , the two curves can be made tangent, implying that only type $\tilde{\theta}$ is induced to accept the exclusive contract. To achieve this outcome, the requested payment p must be

$$p\ \ = 2\theta q^*(\tilde{\theta}) - 2(1-\gamma)\left[q^*(\tilde{\theta}))\right]^2 - U^*(\tilde{\theta}) - \varepsilon$$

where ε is arbitrarily small. Since the exclusive contract $(p\ \ , q\ \ )$ will be chosen only by type $\tilde{\theta}$, a necessary and sufficient condition for the deviation to be profitable is that the payment p exceeds the revenue firm i obtains from consumer $\tilde{\theta}$ in equilibrium, i.e.,

$$p\ \ > \alpha_1^* q^*(\tilde{\theta}) - \frac{\alpha_1^*}{2}\left[q^*(\tilde{\theta}))\right]^2$$

Now we show that this condition can always be met by suitable choice of $\tilde{\theta}$. The above inequality rewrites as

$$2\tilde{\theta} q^*(\tilde{\theta}) - 2(1-\gamma)\left[q^*(\tilde{\theta})\right]^2 - U^*(\tilde{\theta}) > \alpha_1^* q^*(\tilde{\theta}) - \frac{\alpha_1^*}{2}\left[q^*(\tilde{\theta})\right]^2$$

or, taking into account that $U^*(\tilde{\theta}) = (1-\alpha_1^*)\ \left[q^*(\tilde{\theta}))\right]^2$,

$$\left(2\tilde{\theta} - \alpha_1^*\right) - \left(3 - 2\gamma + \frac{3\alpha_1^*}{2}\right) q^*(\tilde{\theta}) > 0.$$

Since $q^*(\tilde{\theta})$ converges to zero as $\tilde{\theta}$ goes to α_1^*, it is clear that as long as $\gamma < \frac{1}{2}$, and hence $\alpha_1^* > 0$, one can always find $\tilde{\theta} > 0$ close enough to α_1^* that the above inequality holds. ■

Appendix 4.2

The smooth pasting condition

This appendix proves that in equilibrium the non exclusionary price schedules must satisfy the smooth pasting condition, which requires that the schedules be continuous and continuously differentiable at $\hat{q}$. Consider the restricted game in which firms can choose only the $\bar{\alpha}_0$'s and the $\hat{q}$'s, given all the other postulated properties of the price schedules. The candidate equilibrium must be an equilibrium of this restricted game. In such a game, firm i's profit is

$$\pi\ \ = \int_0^{\hat{}} (\frac{1}{2}-\gamma)\left[\frac{q\ \ (\theta)}{2}\right]^2 d\theta + \int_{\hat{}}^1 \left[\bar{\alpha}_{0\ i} + \alpha_1^* q^*(\theta) - \frac{\alpha_1^*}{2}\left[q^*(\theta)\right]^2\right] d\theta,$$

where the first integral is the profit made by selling to consumers who choose contracts on the lower parts of the price schedules (and hence purchase $\underline{q}\ (\theta) = \underline{q}\ (\theta) = \frac{E(\ )}{2}$) and the second integral is the profit on the upper part of the price schedule (where consumers purchase $\bar{q}\ (\theta) = \bar{q}\ (\theta) = q^*(\theta)$). The cutoff $\hat{\theta}$ is implicitly determined by the condition

$$U\ (\hat{\theta}) = U^*(\hat{\theta}) - (\bar{\alpha}_{0\ A} + \bar{\alpha}_0\ ), \tag{A4.1}$$

which states that for the critical type $\hat{\theta}$ it is indifferent to choose the lower or the upper part of the non exclusionary price schedule.

We first show that in any equilibrium of the restricted game, the price schedules must be continuous at $\hat{q}$, i.e.,

$$\underline{p}\ (\hat{q}\ ) = \bar{p}\ (\hat{q}\ ).$$

Suppose to the contrary that this equality does not hold; to fix ideas, let $\underline{p}\ (\hat{q}\ ) > \bar{p}\ (\hat{q}\ )$. Let $\hat{\theta}$ denote the consumer who is just indifferent between choosing a point on the lower or the upper part of the price schedule. Then firm i could slightly increase $\hat{q}$, inducing some consumers (i.e., those with θ just above $\hat{\theta}$) to switch to the lower part of the price schedule, where the per capita profit is larger.

The condition $\underline{p}\ (\hat{q}\ ) = \bar{p}\ (\hat{q}\ )$ implicitly determines $\hat{q}$'s for any given fixed fee $\bar{\alpha}_{0\ i}$. Next consider the first-order condition for a maximum with respect to the fixed fee $\bar{\alpha}_{0\ i}$. This is implicitly given by

$$\frac{d\pi}{d\bar{\alpha}_{0\ i}} = \left(1 - \hat{\theta}\right) + \frac{d\hat{\theta}}{d\bar{\alpha}_{0\ i}} \left[\underline{p}\ (\hat{q}\ ) - \bar{p}\ (\hat{q}\ )\right] = 0,$$

where by implicit differentiation of (A4.1)

$$\frac{d\hat{\theta}}{d\bar{\alpha}_{0\ i}} = -\frac{1}{\left(\frac{dU}{d\theta} - \frac{dU^*}{d\theta}\right) |_{\ =\hat{\ }}}$$

Since the term inside square brackets of the derivative $\frac{\ \ i}{\ \ 0,i}$ vanishes, at equilibrium the denominator of $\left|\frac{d\hat{\theta}}{d\bar{\alpha}_{0\ i}}\right|$ must also vanish. This requires that $\frac{\ E}{\ } = \frac{\ *}{\ }$ at $\hat{\theta}$, so that $\underline{q}\ (\hat{\theta}) = \bar{q}\ (\hat{\theta})$. This in turn requires that

$$\bar{\alpha}_0\ \ + \bar{\alpha}_0\ \ = -\frac{\alpha_1^{*2}}{(1 - 2\gamma + \alpha_1^*)},$$

that is, the aggregate fixed fee must be negative and large enough to make the consumer's net utility on the upper part of the price schedule tangent to the reservation utility. Focusing again on a symmetric equilibrium, we finally get

$$\bar{\alpha}_0^{**} = -\frac{\alpha_1^{*2}}{2\,(1 - 2\gamma + \alpha_1^*)}.$$

Appendix 4.3

Proof of Proposition 3

We must verify that the candidate equilibrium is, indeed, an equilibrium of the original game G . That is, we must prove that if firm j offers the exclusionary and non exclusionary price schedules (13) and (14), then firm i's best response in the set of all possible strategies $\mathcal{S}$ is, indeed, (p^{**}, p^{**}).

The proof is in three steps. First, we show that given $\left(p^{**}, p^{**}\right)$, firm i cannot profitably deviate to any different exclusionary price schedule while sticking to the equilibrium non exclusionary schedule. Second, we show that there isn't any profitable deviation to a non exclusionary price schedule different from p^{**} if firm i sticks to its equilibrium exclusionary schedule. Finally, we show that there is no profitable deviation where firm i simultaneously changes its exclusionary and non exclusionary price schedules.

Step 1. The argument is simplest, and has already been sketched in the text, for the most competitive equilibrium where $\mu = 0$. If firm j offers $p^{**}(q) = 0$, firm i will make zero profits with any exclusionary price schedule. Moreover, firm i's cannot affect the reservation utility that is implicitly provided to the consumers by firm j's offer of $p^{**}(q) = 0$. That means firm i is indifferent between any exclusionary schedule, and hence its best reply includes $p^{**}(q) = 0$.

When $\mu > 0$, the argument is slightly more complex. For any given fixed μ, consider a fictitious situation where firm j offers $p^{**}(q) = \mu q^2$ and both firms offer the lower part of their equilibrium non exclusionary schedules, extended to the entire interval of quantities $[0, 1]$, and let us focus on the deviation to the optimal exclusionary price schedule for firm i. We shall refer to the problem of finding the optimal exclusionary schedule as problem $\mathcal{P}$. Since firm i is like a monopolist, we can apply the revelation principle and focus on direct mechanisms.

If the consumer can purchase only from firm i, her utility function reduces to

$$v(q, \theta) = \theta q - \frac{1-\gamma}{2} q^2.$$

However, the consumer now has a type-dependent reservation utility $U = \frac{2}{2(1- +2)}$ with the associated consumption $q = \frac{}{1- +2}$. Following Jullien (2000), define the virtual surplus function

$$\begin{aligned} \sigma(g, q, \theta) &= v(q, \theta) + (\theta - g) \frac{\partial v(q, \theta)}{\partial \theta} \\ &= (2\theta - g) q - \frac{1-\gamma}{2} q^2 \end{aligned}$$

where the "weight" $g \in [0, 1]$ accounts for the possibility that the participation constraint may bind over any subset of the support of the distribution of types, $[0, 1]$. Pointwise maximization of the

virtual surplus function yields

$$\begin{aligned} \ell \ \ (g,\theta) &= \arg\max_{i} v \ \ (q \ ,\theta) \\ &= \frac{2\theta - g}{1-\gamma}. \end{aligned}$$

Notice that

$$\frac{\partial \ell}{\partial \theta} > \frac{dq}{d\theta}.$$

This inequality implies that problem $\mathcal{P}$ is weakly convex in the sense of Jullien (2000).

Now we show that problem $\mathcal{P}$ satisfies the conditions of Potential Separation (PS), Homogeneity (H) and Full Participation (FP), so that we can apply Proposition 3 of Jullien (2000). PS requires that $\ell \ \ (g,\theta)$ is non-decreasing in θ, which is obviously true. H requires that U can be implemented by a continuous and non decreasing quantity. This is also obviously true, since U is implemented by q. Finally, FP requires that in equilibrium all types participate. Lemma 2 in Jullien (2000) guarantees that FP holds if

$$v \ \ (q \ \ ,\theta) \geq U \ \ (\theta).$$

This condition always holds as an equality, and hence FP is satisfied.

Straightforward application of Proposition 3 in Jullien (2000) then implies that the participation constraint is binding over the interval $\left[0, \frac{1- \ +2}{1- \ +4}\right]$. It follows that over this interval the solution to problem $\mathcal{P}$ is $q \ (\theta) = q \ \ (\theta)$. To implement this solution, firm i must offer a uniform, arbitrarily small discount ε over firm j's exclusionary schedule μq^2. But condition (11) guarantees precisely that this type of deviation is unprofitable, so no profitable deviation exists when $\mu \leq \frac{1-2}{4}$.

To complete this step of the proof, it suffices to consider the restriction of problem $\mathcal{P}$ to the interval $[0, \hat{\theta}^{**}]$, where the lower part of the non exclusionary price schedules actually apply, and note that

$$\hat{\theta}^{**} < \frac{1-\gamma+2\mu}{1-\gamma+4\mu}.$$

Since there are no profitable deviations to attract consumers that would obtain in equilibrium a rent equal to their outside option $U \ \ (\theta)$, it follows a fortiori that there are no profitable deviations that attract types that in equilibrium would obtain a rent strictly larger than $U \ \ (\theta)$. We conclude that no firm can profitably deviate to any exclusionary price schedule different from $p \ \ ^{**}(q)$ while sticking to the equilibrium non exclusionary schedule $p \ \ ^{**}(q)$.

Step 2. We now show that if both firms offer the equilibrium exclusionary schedules $p \ \ ^{**}(q)$, no firm can profitably deviate to a non exclusionary price schedule different from (14). The proof once again builds on Proposition 3 in Jullien (2000), which characterizes the solution to a monopolistic screening problem in which the agent has a type-dependent outside option.

To begin with, let us fix μ so that the exclusive schedules $p \ \ ^{**}$ guarantee to the consumer a reservation utility $U \ \ (\theta) = \frac{2}{2(1- \ +2 \ )}$. When calculating its best response to firm j's non exclusionary schedule $p \ \ ^{**}$, firm i faces a monopolistic screening problem where the consumer's utility

is

$$v^*(q\ ,\theta) = \max_{j \geq 0} \left[u(q\ ,q\ ,\theta) - p\ \ ^{**}(q\ ) \right],$$

and the consumer has a type-dependent reservation utility $U\ \ (\theta) = \frac{2}{2(1- \ +2\ )}$ with the associated consumption $q\ \ = \frac{}{1- \ +2}$.. The "indirect utility function" $v^*(q\ ,\theta)$ is the maximum utility that a consumer of type θ can obtain by purchasing q and then trading optimally with firm j.

Since the price schedule $p\ \ ^{**}(q\ )$ has two branches, the same will be true of the indirect utility function. One can easily calculate[35]

$$v\ (q\ ,\theta) = \begin{cases} B_0 + B_1 q\ + B_2 q^2 & \text{if} \quad \theta - \gamma q\ \leq h \\ \\ A_0 + A_1 q\ + A_2 q^2 & \text{if} \quad \theta - \gamma q\ \geq h \end{cases}$$

where

$$h = \frac{\alpha_1^*(2 - 3\gamma + 4\mu)}{(1 - 2\gamma + \alpha_1^* + 4\mu)}.$$

The first branch of the indirect utility function corresponds to the case where the consumer chooses a contract on the lower part of firm j's non exclusionary price schedule. To be precise, the consumer chooses:

$$q\ (q\ ,\theta) = \frac{\theta - \gamma q}{1 + 2\mu - \gamma} \leq \hat{q}^{**}.$$

The second part of the indirect utility function corresponds to the case where the consumer chooses a contract on the upper part of firm j's non exclusionary price schedule. In this case, the optimal consumption is:

$$q\ (q\ ,\theta) = \frac{\theta - \alpha_1^* - \gamma q}{1 - \alpha_1^* - \gamma} \geq \hat{q}^{**}.$$

We then can calculate

$$\begin{aligned} A_0 &= \frac{(\theta - \alpha_1^*)^2}{2(1 - \gamma - \alpha_1^*)} \\ A_1 &= \frac{(1 - 2\gamma - \alpha_1^*)\theta + \gamma\alpha_1^*}{(1 - \gamma - \alpha_1^*)} \\ A_2 &= -\frac{1 - \alpha_1^* - 2\gamma + \gamma\alpha_1^*}{2(1 - \gamma - \alpha_1^*)} \end{aligned}$$

[35] As in the proof of Proposition 1, the consumer's problem may have a corner solution where $q_j = 0$. Such corner solutions arise when $\theta - \gamma q_i < \alpha_1^*$ and the consumer chooses a contract on the upper part of firm j's non exclusionary price schedule. However, these corner solutions are irrelevant here, since when $q_j = 0$ the relevant price schedule of firm i becomes its exclusionary schedule $p_i^E(q_i)$.

as in the proof of Proposition 1, and

$$\begin{aligned} B_0 &= \frac{\theta^2}{6-10\gamma} \\ B_1 &= \frac{6\,(1-2\gamma)}{6-10\gamma}\theta \\ B_2 &= -\frac{3-4\,(2-\gamma)\,\gamma}{6-10\gamma} \end{aligned}$$

Notice that $v\ (q\ ,\theta)$ is everywhere twice continuously differentiable, since for $\theta - q\ \gamma = h$ we have $A\ \ = B\ \ $ for $k = 0, 1, 2$, and it is globally concave.

Now consider the optimization problem $\mathcal{P}$ of firm i, which faces a uniform distribution of consumers with utility $v\ (q\ ,\theta)$ and reservation utility $U\ \ (\theta)$. Proceeding as in step 1 of the proof, define the virtual surplus function associated with problem $\mathcal{P}$ as

$$\sigma\ (g, q\ ,\theta) = v\ (q\ ,\theta) + (\theta - g)\,\frac{\partial v\ (q\ ,\theta)}{\partial\theta}$$

where the "weight" $g \in [0, 1]$ accounts for the possibility that the participation constraint may not bind only for the lowest type. The solution to problem $\mathcal{P}$ must pointwise maximize the virtual surplus function $\sigma\ (g, q\ ,\theta)$, yielding:

$$\ell\ (g,\theta) = \begin{cases} \frac{(\ -\ )\frac{\partial B_1}{\partial\theta} -\ \ {}_1}{2\ \ {}_2} & \text{if} \quad \theta - \gamma q\ (g,\theta) \leq h \\ \\ \frac{(\ -\ )\frac{\partial A_1}{\partial\theta} -\ \ {}_1}{2\ \ {}_2} & \text{if} \quad \theta - \gamma q\ (g,\theta) \geq h. \end{cases}$$

Notice that

$$\frac{\partial\ell\ (g,\theta)}{\partial\theta} = \begin{cases} -\frac{1}{\ \ 2} & \text{if} \quad \theta - \gamma q\ (g,\theta) \leq h \\ \\ -\frac{1}{\ \ 2} & \text{if} \quad \theta - \gamma q\ (g,\theta) \geq h. \end{cases}$$

and hence is positive, since A_1 and B_1 are positive, while A_2 and B_2 are negative.

Next we show that $\mathcal{P}$ satisfies three conditions: Potential Separation (PS), Homogeneity (H) and Full Participation (FP). PS requires that $\ell\ \ (g,\theta)$ is non-decreasing in θ, which is obviously true. H requires that $U\ \ $ can be implemented by a continuous and non decreasing quantity. This is also obviously true, since $U\ \ $ is implemented by $q\ \ $. Finally, FP requires that in equilibrium all types participate. Lemma 2 in Jullien (2000) guarantees that FP holds if

$$v\ (q\ \ ,\theta) \geq U\ \ (\theta).$$

This condition always holds, since

$$v\ (q\ \ ,\theta) \geq u(q\ \ , 0, \theta) = U\ \ (\theta)$$

by the definition of the indirect utility function. Hence, FP is also satisfied.

To proceed, we must also verify that $\mathcal{P}$ is weakly convex. (The informal argument has been anticipated in the text, but here we offer a more rigorous treatment.) Weak convexity requires that

$$\frac{\partial \ell \ (\hat{g}(\theta), \theta)}{\partial \theta} \geq \frac{dq \ \ (\theta)}{d\theta},$$

where $\hat{g}(\theta)$ is implicitly defined by

$$q \ \ (\theta) = \ell \ (\hat{g}(\theta), \theta).$$

Straightforward calculations show that this inequality always holds.

Proposition 3 in Jullien (2000) then guarantees that the solution to problem $\mathcal{P}$ partitions the set of types into two sets:[36] the set $[0, \hat{\theta}]$, where each consumer obtains an equilibrium payoff equal to $U \ \ (\theta)$, and the set $(\hat{\theta}, 1]$, where consumers obtain a payoff strictly larger than $U \ \ (\theta)$. When the participation constraint binds, the solution to program $\mathcal{P}$ is implicitly defined by the condition

$$q \ (\theta) + q \ (q \ (\theta), \theta) = q \ \ (\theta),$$

that is,

$$q \ + \frac{\theta - \gamma q}{1 - \gamma + 2\mu} = \frac{\theta}{1 - \gamma + 2\mu}.$$

This gives

$$q \ (\theta) = \frac{\theta}{2\left(1 - \gamma + 2\mu\right)} (= q^{**}(\theta)).$$

When the participation constraint does not bind, the solution to program $\mathcal{P}$ is obtained setting $g = 1$ in the virtual surplus function. The optimal quantity then is

$$q \ (\theta) = \ell \ (1, \theta)$$

or

$$\begin{aligned} q \ (\theta) \quad &= \quad \frac{(1-\theta) \frac{\ \ 1}{} - A_1}{2A_2} \\ &= \quad \frac{2\theta(1 - 2\gamma - \alpha_1^*) - (1 - 2\gamma - \alpha_1^* - \gamma\alpha_1^*)}{1 - \alpha_1^* - 2\gamma + \gamma\alpha_1^*} \end{aligned}$$

which, using the definition of α_1^*, turns out to coincide with $q^*(\theta)$, and hence with $q^{**}(\theta)$ over the interval $\theta \in [\hat{\theta}, 1]$.

Finally, $\hat{\theta}$ is implicitly given by the condition

$$q \ \ (\hat{\theta}) = q \ (g, \hat{\theta}) + q \ (q \ (g, \hat{\theta}), \hat{\theta})$$

[36] In general, Proposition 3 of Jullien (2000) accounts also for the possibility that the set $[0, \hat{\theta}]$ may be further split into two sets, $[0, \theta^+]$ and $[\theta^+, \hat{\theta}]$, with the participation constraint binding only in the latter. In our problem, however, the fact that $q^E(\theta) \leq q^{fb}(\theta)$, with a strict inequality whenever $\theta > 0$, implies that $\theta^+ = 0$.

with $g = 1$ and $q \ = \hat{q}^{**}$. Tedious calculations then give $\hat{\theta} = \hat{\theta}^{**}$.

This shows that the solution to problem $\mathcal{P}$ is $q \ = q^{**}(\theta)$. Obviously, this solution can be implemented by firm i using the price schedule $p \ ^{**}$, which therefore is a best response to $\left(p \ ^{**}, p \ ^{**}\right)$.

Step 3. It remains to show that there is no profitable deviation where firm i simultaneously changes its exclusionary and non exclusionary price schedules. This follows immediately from the fact that offering exclusionary prices different from the equilibrium ones is irrelevant if the exclusionary prices are greater than $p \ ^{**}$, and if they are lower it is directly unprofitable (by Step 1 of the proof) and improves the consumer's reservation utility, thereby making any deviation through non exclusive contracts less profitable. ■

Appendix 5.1

The candidate equilibrium under public common agency when the participation constraint does not bind

Consider the hypothetical game in which each firm i offers a quadratic price schedule

$$p \ (q \ , q \ ) = \alpha_1 \ q \ - \frac{\alpha_1}{2} q^2 + \alpha_3 \ q \ - \frac{\alpha_3}{2} q^2 \qquad \text{for all } 0 \leq q \ , q \ \leq 1.$$

Given these schedules and assuming an interior solution, the consumer's optimal choice is

$$\check{q} \ (\theta) = \frac{(\theta - \alpha_1 \ - \alpha_3 \ )(1 - \alpha_1 \ - \alpha_3 \ - \gamma) - \gamma(\theta - \alpha_1 \ - \alpha_3 \ )}{(1 - \alpha_1 \ - \alpha_3 \ - \gamma)(1 - \alpha_1 \ - \alpha_3 \ - \gamma) - \gamma^2}.$$

Hence, firm i's expected profit is

$$\pi \ = \int_{\check{}}^{1} \left[\alpha_1 \ \check{q} \ (\theta) - \frac{\alpha_1^*}{2} [\check{q} \ (\theta)]^2 + \alpha_3 \ \check{q} \ (\theta) - \frac{\alpha_3^*}{2} [\check{q} \ ]^2 \right] d\theta,$$

where $\check{\theta}$ is implicitly defined by the condition $\check{q} \ (\check{\theta}) = 0$.

The first order conditions are

$$\begin{aligned} \frac{\partial \pi}{\partial \alpha_1} &= 0 \\ \frac{\partial \pi}{\partial \alpha_3} &= 0. \end{aligned}$$

After imposing symmetry, i.e., $\alpha_1 \ = \alpha_1$ and $\alpha_3 \ = \alpha_3$, they reduce to

$$\begin{aligned} \gamma(2 - 3\alpha_1 - 3\alpha_3) - (1 - \alpha_1 - \alpha_3)(1 - 2\alpha_1 - \alpha_3) &= 0 \\ \gamma(2 - 3\alpha_1 - 3\alpha_3) - (1 - \alpha_1 - \alpha_3)(1 - \alpha_1 - 2\alpha_3) &= 0. \end{aligned}$$

The unique solution to this system is $\alpha_1 = \alpha_3 = \frac{1}{3}$.

Appendix 5.2

Proof of Proposition 5

We must verify that the candidate equilibrium is, indeed, an equilibrium of the original game G . That is, we must prove that if firm j offers the exclusionary and non exclusionary price schedules (13) and (22), then firm i's best response in the set of all possible strategies $\mathcal{S}$ is indeed (p ***, p ***). The proof is divided into three steps, as the proof of Proposition 3. The first and last steps are in fact identical, and thus they are omitted. We focus only on Step 2.

In this step, we must verify that given firm j's equilibrium strategy $\left(p\ ^{***}, p\ ^{***}\right)$, there is no profitable deviation for firm i to a non exclusionary price schedule different from p ***. As in the proof of Proposition 3, firm i is like a monopolist that faces a consumer with a suitable indirect utility function and a type-dependent reservation utility U (θ, μ). Now, however, each firm can effectively control both q and q , and thus firm i behaves like a multi-product firm that supplies both product A and B. This implies that now the indirect utility function is simply the difference between u and firm j's non exclusionary price schedule. Then, the indirect utility function in program $\mathcal{P}$ is

$$v\ (q\ , q\ , \theta) = \begin{cases} u(q\ , q\ , \theta) - \underline{p}\ ^{***}(q\ , q\ ) & \text{if } q\ \leq \hat{q}^{***} \\ u(q\ , q\ , \theta) - \overline{p}\ ^{***}(q\ , q\ ) & \text{if } q\ \geq \hat{q}^{***} \end{cases}$$

with $\hat{q}^{***} = \frac{2}{(5-6\ +12\ )}$.

To proceed, we invoke symmetry and set $q\ = q\ (= q)$. This implicitly restricts firm i to consider only symmetric deviations, but this does not involve any real loss of generality since, given the symmetry of the utility function and of firm j's non exclusionary price schedule, the most profitable deviation is necessarily symmetric.

The indirect utility function then becomes

$$v\ (q, \theta) = \begin{cases} 2\theta q - \left(\dfrac{3}{2} - \gamma + 2\mu\right) q^2 & \text{if } q \leq \hat{q}^{***} \\ \left(2\theta - \dfrac{2}{3}\right) q - \dfrac{2}{3} q^2 + \dfrac{2}{3(5 - 6\gamma + 12\mu)} & \text{if } q \geq \hat{q}^{***} \end{cases}$$

Proceeding as in the proof of Proposition 3, one easily verifies that the conditions of Potential Separation (PS), Homogeneity (H), and Full Participation (FP) are satisfied. Then, define the virtual surplus as

$$\begin{aligned} \sigma(g, q, \theta) &= v\ (q, \theta) + (\theta - g) \frac{\partial v\ (q, \theta)}{\partial \theta} \\ &= v\ (q, \theta) + 2\,(\theta - g)\, q. \end{aligned}$$

Let

$$\ell(g,\theta) = \begin{cases} \dfrac{4\theta - 2g}{3 - 2\gamma + 4\mu} \\ \\ \dfrac{6\theta - 3g - 1}{2} \end{cases}$$

be the solution to the problem of pointwise maximization of the virtual surplus $\sigma(g, q, \theta)$. Let also $g(\theta)$ be implicitly defined by $q\ \ (\theta) = q(g(\theta), \theta)$, i.e.

$$g(\theta) = \begin{cases} \dfrac{1 - 2\gamma + 4\mu}{2 - 2\gamma + 4\mu}\theta \\ \\ \dfrac{2(2 - 3\gamma + 6\mu)}{3\,(1 - \gamma + 2\mu)}\theta - \dfrac{1}{3} \end{cases}$$

Then, firm i's problem, $\mathcal{P}$, is weakly convex if

$$\frac{dq\ \ (\theta)}{d\theta} \le \frac{\partial \ell(g,\theta)}{\partial \theta},$$

or

$$\frac{1}{1 - \gamma + 2\mu} \le \min\left[\frac{8}{3 - 2\gamma + 4\mu}, 6\right],$$

which obviously always holds.

Applying Proposition 3 in Jullien (2000), we then conclude that:
(a) for types in a set $[0, \hat{\theta}]$ the participation constraint binds, ad hence the optimal solution to program $\mathcal{P}$ is implicitly defined by

$$2q(\theta) = q\ \ (\theta)$$

that is,

$$q(\theta) = \frac{\theta}{2\,(1 - \gamma + 2\mu)}\,(= q^{***}(\theta))\,;$$

(b) $\hat{\theta}$ is such that

$$q\ \ (\hat{\theta}) = \frac{8\hat{\theta} - 4}{3 - 2\gamma + 4\mu}$$

for $q = \hat{q}^{***}$. Simple calculations then give $\hat{\theta} = \hat{\theta}^{***}$;
(c) finally, for types in the set $[\hat{\theta}, 1]$ the participation constraint is not binding, and the optimal solution to program $\mathcal{P}$ is

$$q = \ell(1, \theta),$$

that s $q = 3\theta - 2\,(= q^{***}(\theta))$. This completes the proof of Step 2, and hence the proof of the Proposition. ■

Appendix 5.3

Proof of Proposition 7

The proof of this Proposition is based on the direct calculation of expected profits and the expected consumer surplus (that is, the average of the consumer's rent across consumer types). When $\mu = 0$, we have

$$\begin{aligned} E[\pi^*] &= \frac{(1-\alpha_1^*)\alpha_1^*}{3} \\ E[\pi^{***}] &= \frac{4(1-\gamma)(1-2\gamma)}{3(5-6\gamma)^2} \\ E\left[U^*\right] &= \frac{(1-\alpha_1^*)^2}{3} \\ E\left[U^{***}\right] &= \frac{13-4\gamma(5-\gamma)}{3(5-6\gamma)^2} \end{aligned}$$

Expected social welfare is $2E[\pi\ ] + E\left[U(\theta)\right]$. Using the definition of α_1^* (equation (1)), the Proposition follows immediately. ■

Appendix 5.4

Proof of Proposition 8

Averaging across consumer types, the general formulas for the expected consumer's rent, profits and social welfare with market-share discounts are, respectively:

$$
\begin{aligned}
E\left[U^{***}\right] &= \frac{13 + 4\gamma(\gamma - 2\mu) + 4\left(2\mu - \gamma\right)\left(5 + 2\mu\right)}{3(5 - 6\gamma + 12\mu)^2} \\
E\left[\pi^{***}\right] &= \frac{(1 - \gamma + 2\mu)4(1 - 2\gamma + 4\mu)}{3(5 - 6\gamma + 12\mu)^2} \\
E\left[W^{***}\right] &= \frac{(3 - 2\gamma + 4\mu)(7 - 10\gamma + 20\mu)}{3(5 - 6\gamma + 12\mu)^2}
\end{aligned}
$$

Simple algebraic calculations then prove the following properties:

(i) expected consumer surplus is greater with market-share discounts than under private common agency if and only if $\mu < \mu\ (\gamma)$, where $\mu\ (\gamma)$ is a positive, decreasing function with $\mu\ (\gamma) \leq \frac{1-2}{4}$ only for $\gamma \geq \tilde{\gamma} > 0$. Permitting market share-discounts reduces consumer surplus only when $\mu\ (\gamma) \leq \mu \leq \frac{1-2}{4}$, and this interval is non-empty only if $\gamma \geq \tilde{\gamma}$;

(ii) expected profits are lower with market-share discounts than under private common agency if and only if $\mu < \mu\ (\gamma)$, where $\mu\ (\gamma)$ is a positive, decreasing function with $\mu\ (\gamma) \leq \frac{1-2}{4}$ only for $\gamma \geq \hat{\gamma} > 0$. Permitting market share-discounts reduces consumer surplus only when $\mu\ (\gamma) \leq \mu \leq \frac{1-2}{4}$, and this interval is non-empty only if $\gamma \geq \hat{\gamma}$;

(iii) expected social welfare is greater with market-share discounts than under private common agency if and only if $\mu < \mu\ \ (\gamma)$, where $\mu\ \ (\gamma)$ is a positive, decreasing function with $\mu\ \ (\gamma) \leq \frac{1-2}{4}$ only for $\gamma > 0$. Permitting market share-discounts reduces consumer surplus only when $\mu\ \ (\gamma) \leq \mu \leq \frac{1-2}{4}$, and this interval is non-empty only if $\gamma \geq 0$.■

References

[1] Aghion, P. and P. Bolton (1987), "Contracts as a barrier to entry", *American Economic Review* 77, 388-401.

[2] Anderson E. T. and J. Dana (2005) "Integrating Models of Price Discrimination," WP Kellogg School of Management, Northwestern University.

[3] Armstrong, M., Cowan S. and J. Vickers (1994), "Nonlinear pricing and price cap regulation," *Journal of Public Economics* 58, 33-56.

[4] Armstrong, M. and J. Vickers (2001), "Competitive price discrimination," *RAND Journal of Economics*, 32, 4, 579-605.

[5] Bernheim, D. and M. Whinston (1986), "Menu auctions, resource allocation, and economic influence," *Quarterly Journal of Economic*s 101, 1-31.

[6] Bernheim, D. and M. Whinston (1998), "Exclusive dealing," *Journal of Political Economy*, 106, 64-103.

[7] Bork, R.H. (1987), *The antitrust paradox: A policy at war with itself*, New York: Basic Books.

[8] Calzolari, G. and C. Scarpa (2008), "Delegated and intrinsic common agency with full market participation", WP University of Bologna.

[9] Chiesa, G. and V. Denicolò (2009), "Trading with a common agent: A characterization of the Nash equilibria," *Journal of Economic Theory*, 144, 1, 296-311.

[10] Deneckere and McAfee (1996), "Damaged goods," *Journal of Economics and Management Strategy*, 5, 2, 149-174.

[11] Ivaldi, M. and D. Martimort (1994), "Competition under nonlinear pricing," *Annales d'Economie et de Statistique* 34, 71-114

[12] Jullien, B. (2000), "Participation constraints in adverse selection models," *Journal of Economic Theory*, 93, 1-47.

[13] Laffont J.J. and D. Martimort (2002), *The theory of incentives. The principal-agent model*, Princeton UP.

[14] Majumdar, A. and G. Shaffer (2009), "Market-share contracts with asymmetric information," *Journal of Economics and Management Strategy*, 18, 393-421.

[15] Martimort D. and L. Stole (2002) "The revelation and delegation principles in common agency games," *Econometrica*, 70, 1659–1673.

[16] Martimort, D. and L. Stole (2009), "Market participation in delegated and intrinsic common-agency games," *RAND Journal of Economics*, 40, 1, 78-102.

[17] Martimort, D. and L. Stole (2009a), "Public contracting in delegated common agency games", mimeo.

[18] Martimort, D. and L. Stole (2009b), "A note on aggregate-strategy games", mimeo.

[19] Marvel, H.P. (1982), "Exclusive dealing," *Journal of Law and Economics* 25, 1-25.

[20] Mathewson, F. and A. Winter (1987), "The competitive effects of vertical agreements: Comment," *American Economic Review* 77, 1057-1062.

[21] Mills, D. (2004), "Market-share discounts, mimeo," University of Virginia.

[22] Pavan A. and G. Calzolari (2009) "Truthful revelation mechanisms for simultaneous common agency games", forthcoming *American Economic Journal: Microeconomics.*

[23] Peters, M., (2001) "Common agency and the revelation principle," *Econometrica*, 69, 1349-1372.

[24] Rasmusen, E. , M. Ramseyer and J. Wiley (1991), "Naked exclusion", American Economic Review 81, 1137-1145.

[25] Rey, P. and J. Tirole, (2007), "A Primer on foreclosure", in *Handbook of Industrial Organization*, Mark Armstrong and Rob Porter (eds.), Vol. III, North Holland, p. 2145-2220.

[26] Rochet, J-C. and L. Stole (2002), "Nonlinear pricing with random participation," *Review of Economic Studies*, 69, 277-311.

[27] Segal, I. and M. Whinston (2000),"Naked exclusion: Comment," *American Economic Review* 90, 296-309.

[28] Segal, I. and M. Whinston (2000a), "Exclusive contracts and protection of investments," *RAND Journal of Economics* 31, 603-633.

[29] Shubik, M. and R. Levitan (1980), *Market structure and behavior*, Cambridge, MA: Harvard University Press.

[30] Wilson, R. (1994), *Non linear pricing*, Electric Power Research Institute.