Information Aggregation and Investment Decisions*

Elias Albagli
USC Marshall

Christian Hellwig
Toulouse School of Economics

Aleh Tsyvinski
Yale University

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Abstract

We study a role of asset prices in aggregating information and guiding real investment. First, we develop a tractable, yet flexible class of noisy REE models of a financial market with a general specification of asset's payoffs. We show that the interplay between noisy information aggregation and firm decisions leads to a systematic wedge between asset prices and expected firm value, conditional on the information contained in market prices. From an ex ante perspective, the expected wedge, i.e. the difference between the expected price and the expected dividend, is positive (negative) if the dividend function is convex (concave). The expected wedge is zero if the dividend function is linear, as is typically assumed in standard REE models. Second, we apply our framework to study the interaction between information aggregation in financial markets and firms investment decisions. The option value inherent in firm's use of the information contained in market prices convexifies the payoff. This implies that expected share prices exceed expected dividends. Third, we show that linking managerial compensation to share prices gives managers an incentive to manipulate the firm's decisions to their own benefit at the share-holders' expense. By conditioning their decisions excessively on the information conveyed through market prices, managers can inflate the share price by taking inefficient investment decisions, which reduces expected dividends. This further amplifies the wedge between share price and expected firm value.

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1 Introduction

A key role played by asset prices is aggregation of information about the value of firms. By pooling together the dispersed knowledge of individual actors, prices provide information that helps shape investor expectations and portfolio decisions. To the extent that the information conveyed in prices is not already known within the firm, prices also affect the firm's assessment of the value of investment projects and guide real allocations. If information aggregation is perfect, asset prices fully reflect current expectations of future dividends and provide a parsimonious way of conveying information and guiding real investment. If managerial compensation is linked to share prices, the price mechanism also aligns the managers' incentives with the best interest of shareholders.

This paper reconsiders the role of asset prices in aggregating information and guiding real investment, when information aggregation is imperfect, and share prices offer only a noisy signal of the underlying fundamentals. We consider a setting in which firm's shares are traded in a financial market, in which the share price emerges as a noisy signal pooling the dispersed information investors may have about the firm's fundamental value. The firm then takes an investment decision based on the information conveyed by the share price. The investors in turn anticipate the firm's decision in their trading strategies. In equilibrium, the resulting feedback determines the firm's share price, investment decision, and its dividend value.

As our main results, we show that the interplay between noisy information aggregation and firm decisions leads to a systematic wedge between asset prices and expected firm value, conditional on the information contained in market prices. From an ex ante perspective, this wedge has a positive expected value, i.e. expected prices exceed expected firm value. Moreover, linking managerial compensation to share prices gives managers an incentive to manipulate the firm's decisions to their own benefit at the share-holders' expense. By conditioning their decisions excessively on the information conveyed through market prices, managers can inflate the share price, while reducing expected dividends. This further amplifies the wedge between share price and expected firm value.

To develop these results, we propose a tractable, yet flexible model of noisy information aggregation in financial markets. We consider a market structure in which informed traders are risk-neutral and face constraints on portfolio holdings. This environment allows us substantially more flexibility in specifying the firm's dividends, its investment decision, and the nature of informational asymmetries between investors than the canonical models of noisy information aggregation with CARA preferences and normally distributed dividends.[1]

More specifically, the paper proceeds as follows. We first develop our financial market model assuming that dividends are an exogenous function of underlying fundamentals. We show that a systematic wedge emerges between the share price and the expected dividends, whenever the share price aggregates dis-

[1] Among many others, see Grossman and Stiglitz (1980), Hellwig (1980) and Diamond, and Verrecchia (1981), as well as recent textbook treatments by Brunnermeier 2001, Vives 2008, and Veldkamp 2011.

persed shareholder information with noise: the expected dividends conditional on the information conveyed by the price always increase less than one-for-one with the price. In other words, prices respond more to shocks in the underlying environment than would be justified by their information content about expected future dividends. The share price then is higher (lower) than the expected dividends when expected fundamentals conditional on the price are high (low). We label this wedge between share price and expected firm value the information aggregation wedge.

The intuition for the wedge is as follows. Any shareholder's expectation of future dividends (and, hence, his trading decisions) is based both on his private information and on the information conveyed through the price. The expectation of future dividends conditional on market information, on the other hand, only reflects the information conveyed by the price. From the perspective of any individual shareholder, the noise in market information is uncorrelated with the noise in private signals. In the aggregate, however, the trading equilibrium induces a positive correlation between the noise in the private signal of those shareholders who end up holding the shares, and the noise in the market signal provided by the share price. As long as the shareholder's private information gets reflected in the share price, the price then responds more strongly to fundamental shocks than would be implied solely by its effect on expected dividends.

We show that from an ex ante perspective, the expected information aggregation wedge between the share price and the firm value may be positive or negative, depending on whether the dividend is a convex or a concave function of the underlying fundamentals. The slope of the dividend function determines shareholder's exposure to fundamental shocks, i.e. how much the dividends vary with changes in fundamentals. The shareholders' exposure determines the absolute magnitude of the information aggregation wedge, for given realizations of the share price. When dividends are a linear function of fundamentals, the shareholders' exposure is symmetric around the prior expectation, and the positive wedge on the upside exactly offsets the negative wedge on the downside. From an ex ante perspective, the expected wedge is zero. When instead the dividends are a convex function of fundamentals, the firm's upside risk is larger than its downside risk, and the positive wedge on the upside exceeds the negative wedge on the downside. From an ex ante perspective, the expected wedge is positive. The opposite reasoning applies with concave dividend functions, where the downside risk exceeds the upside risk in dividends, and the expected wedge is negative.[2]

Second, we endogenize the dividend allowing the firm to make an investment decision after observing its own share price. We solve the model in a closed form

[2]The information aggregation wedge also emerges in a canonical CARA-normal setting. However, the restrictions imposed in the CARA normal setting only allow for the case of a linear dividend payoff and a symmetric wedge. If on average the asset is in zero net supply, the ex ante expected price then coincides with ex ante expected dividends. When the average net supply of the asset is not zero, a difference between price and expected dividends emerges due to risk premia, which are absent in our model. We further analyze this issue in the appendix.

and derive a simple expression for the average wedge across states of nature.[3] We show that the dividend function in our model of endogenous investment is convex, resulting in a positive expected wedge. We further decompose the expected wedge into two key components. First, the reaction of market prices to expected fundamentals determines the magnitude of the conditional wedge for a fixed level of investment. Second, the variability of firm's posterior expectation about fundamentals captures the value of market information for firm's investment problem. Our expression for the wedge illustrates the role of two central elements that originate the wedge in our model: the dispersed nature of information and the value of market information for firm's decision.

The option value inherent in firm's ex post use of market information convexifies its expected dividends (e.g., Dixit and Pindyck 1994). As the firm takes on more exposure to the fundamentals in good states than in bad states, the information aggregation wedge is asymmetric and larger in absolute value, when it is positive. The feedback from information aggregation to firm decisions leads to share prices that are higher than expected dividends from an ex ante perspective. This positive information aggregation wedge emerges even when firms' management acts in the best interest of its shareholders, and when shareholders are perfectly rational. Moreover, the investment decisions of the firm are efficient.

Thirdly, we consider a model in which managerial compensation is tied to market valuations. Specifically, we assume manager's objective is to maximize a weighted sum of expected dividends and the price. Because of the information aggregation wedge, investment decisions that maximize firm's share price need not maximize its expected dividend. The manager can manipulate the share price to his benefit at the shareholder's expense, by responding more aggressively to the information conveyed through the price. The manager overinvests and increases the exposure in high states when the wedge is positive, but underinvests and reduces exposure in low states when it is negative. In contrast to the case in which the manager acts in the shareholder's best interest, compensation tied to the price results in higher share prices and lower expected dividends. The extent of this manipulation is increasing in the degree to which compensation is linked to share prices.

Finally, we extend the model in several dimensions. First, we generalize our noise trading assumption to include uninformed traders who trade partly for exogenous motives, and partly in response to the perceived wedge between expected dividends and prices. The price elasticity of uninformed trader's demand determines the price impact of private information by the informed shareholders. The information aggregation wedge is largest in illiquid markets when informed traders have a large price impact. Thus, the better uninformed traders are able to arbitrage the discrepancy between expected dividends and price, the smaller is the information aggregation wedge and the resulting investment distortions. Second, we consider an environment in which a signal about market-specific in-

[3] Our model with endogenous investment can be solved in a standard framework of noisy information aggregation only under a restrictive assumption that the firms have no proprietary information and firm's decisions are perfectly anticipated by the market.

formation is also observed by the firm. This decreases firm's reliance on market prices, reducing the option value component of market information as well as the temptation to manipulate the wedge when managerial incentives are tied to prices. The resulting information aggregation wedge is smaller, and firm's investment decision becomes more efficient. Thirdly, we also explore the role of assumptions on the dividend structure and investment costs, study a variant in which firm-specific proprietary information is partially known to the market, and a variant in which firm's investment decision is perfectly anticipated by the market.

Related Literature

Our model of information aggregation shares similar features with a large literature on noisy information aggregation in rational expectations models, including the papers cited above. Our alternative formulation of the asset market draws on Hellwig, Mukherji, and Tsyvinski (2006), and may prove convenient for other applications that require a more flexible specification of dividends than the canonical setting with CARA preferences and normal distributions allows for. The price for this gain in flexibility comes in form of the assumption of risk neutrality, which imposes strong restrictions on the shareholders' preferences.

The information aggregation wedge appears to have received little attention by the literature - to our knowledge, the only explicit discussion of the excess sensitivity of prices relative to fundamentals appears in Vives (2008), and is by its nature limited to the linear, symmetric case. In contrast, the added flexibility of our model offers new results linking asset over- or under-valuation to the shape of the asset's payoff, without any reference to compensation for risk on the downside, or leverage, speculation, other frictions or behavioral trading motives on the upside.

Our model relating firm investment to market information is most closely related to the literature on REE models with the feedback effect in which real decisions depend on the information contained in the price (e.g., Goldstein and Guembel 2008, Dow and Rahi 2003, Dow, Goldstein, and Guembel 2010). These models build primarily on environments in which there is only market-specific information, and focus more on the implications for investment efficiency and firm value than on asset pricing consequences. Dow and Rahi (2003) study risk-sharing and welfare in a setting with endogenous investment. Their CARA-Normal setup imposes restrictions on the information structure, which imply that the firm's decision can be directly inferred from the share price. Goldstein and Guembel (2008) focus on the strategic aspect of the feedback effect. They show that when traders exploit the impact of their demand on prices and investment, manipulative short-selling strategies that distort the firm's investment decision can be profitable. Dow, Goldstein, and Guembel (2010) study a setup with endogenous information production. Since speculators' incentives to produce information increase with the ex-ante likelihood of an investment opportunity, the authors find that small changes in fundamentals can cause large shifts in investment and firm value. There are two important differences with those models. First, in our model of endogenous investment we derive an ex-

plicit characterization of the environment in which both the firm and market have private information. Second, we focus more on the resulting asset pricing implications of our model, as well as the link between asset prices, expected dividends and managerial incentives.

Our analysis of managerial compensation tied to share prices is most closely related to Benmelech, Kandel, and Veronesi (2010). They build a dynamic REE setting where managerial effort postpones the decline in growth opportunities of a firm. When growth rates slow down, share price compensation incentivizes managers to conceal the true state by over-investment in negative NPV projects. Price-based incentives thus imply a tradeoff: while inducing high effort in early stages, it leads to concealment and suboptimal investment in later part of the firm's life-cycle. The central difference of our model is that investment distortions in our setup do not relate to misreporting, but rather arise from the excessive weighing of market information in the signal extraction problem of managers.

Our paper is related more broadly to a large literature on REE models with investment. Leland (1992) addresses efficiency considerations in a model with insider trading, where information aggregation affects the level of available funding to the firm. Dow and Gorton (1997) study a dynamic model of feedback effects in a setup where prices accurately reflect public information, and distortions arise from differences in horizons between managers and shareholders. Subrahmanyam and Titman (1999) endogenize investment on share prices, but assume it affects a growth option independent of the dividend of shareholders: share prices thus convey information about fundamentals, but do not internalize their impact on investment. Angeletos, Lorenzoni, and Pavan (2010) model the interaction between early investment choices by entrepreneurs and the later transfer of firm property to traders. An informational advantage that originates from the dispersed nature of entrepreneur information induces a speculative motive that causes excess non-fundamental volatility in real investment and asset prices. Goldstein, Ozdendoren, and Yuan (2010) discuss efficiency considerations and trading commonality (frenzies) arising from the socially suboptimal weighting of private and exogenous public signals about fundamentals. An important difference with the latter two papers is that we allow all agents to simultaneously condition on equilibrium prices when making trading and real investment decisions. As a result, there are no strategic complementarities that deviate the weighting of public and private sources of information from the optimal signal extraction problem of a future price, which is at the heart of the mechanism highlighted by these authors.

The rest of the paper is structured as follows. Section 2 describes a simple financial market model with exogenous dividends to illustrate the basic mechanism behind the information aggregation wedge. Section 3 introduces the model with endogenous investment decisions, highlighting the central role of this mechanism for generating a positive average wedge between share prices and firm's dividends. This section also includes the analysis of the effects of tying managerial incentives to share prices. In section 4, we analyze various extensions and robustness exercises to our baseline model. Section 5 concludes.

2 A Model with Exogenous Dividend

In this section, we develop a simple model of noisy information aggregation in a financial market in which an asset's dividends are exogenously given. This model serves as a building block for later sections in which we consider endogenous investment decisions, as well as more general payoff specifications and information environments.

2.1 Agents, Information Structure, and Financial Market

We formulate the trading environment as a Bayesian game between a unit measure of risk-neutral, privately informed traders, and a 'Walrasian auctioneer'.

Initially, nature draws a stochastic "fundamental" θ, which is normally distributed according to $\theta \sim \mathcal{N}(\mu, \lambda^{\ 1})$, with mean μ, and unconditional variance $\lambda^{\ 1}$,

Each informed trader is endowed with one share of a firm, which pays a final dividend $\pi(\theta)$ to its shareholders. The function $\pi(\cdot)$ is strictly increasing and twice continuously differentiable.

Each informed trader i then receives a noisy private signal x_i about firm's fundamental. This signal is normally distributed according to $x_i \sim \mathcal{N}(\theta, \beta^{\ 1})$, with a mean θ and a variance $\beta^{\ 1}$, and is i.i.d. across traders (conditional on θ). Traders then participate in an asset market in which they decide whether to hold or sell their share at the market price, P. Specifically, trader i submits a a price-contingent supply schedule $s_i(\cdot) : \mathbb{R} \to [0, 1]$, to maximize her expected wealth $w_i = (1 - s_i) \cdot \pi(\theta) + s_i \cdot P$. By restricting supply to $[0, 1]$, we assume that traders can sell at most their endowment, and cannot buy a positive amount of shares. Individual trading strategies are then a mapping $s : \mathbb{R}^2 \to [0, 1]$ from signal-price pairs (x_i, P) into the unit interval. Aggregating traders' decisions leads to the aggregate supply function $S : \mathbb{R}^2 \to [0, 1]$,

$$S(\theta, P) = \int s(x, P) d\Phi(\sqrt{\beta}(x - \theta)), \tag{1}$$

where $\Phi(\cdot)$ denotes a cumulative standard normal distribution, and $\Phi(\sqrt{\beta}(x - \theta))$ represents the cross-sectional distribution of private signals x_i conditional on the realization of θ.[4]

Nature then draws a random demand of shares by "noise traders". We assume the demand shock has the form $D(u) = \Phi(u)$, where u is normally distributed with mean zero and variance $\delta^{\ 1}$, $u \sim \mathcal{N}(0, \delta^{\ 1})$, independently of θ. This specification is from Hellwig, Mukherji, and Tsyvinski (2006).[5] This functional form allows us to preserve the normality of posterior beliefs and retains the tractability of Bayesian updating.

[4] We assume that the Law of Large Numbers applies to the continuum of traders, so that conditional on θ the cross-sectional distribution of signal realizations ex post is the same as the ex ante distribution of traders' signals.

[5] We generalize this demand specification in Section 4.2 allowing for price-elastic demands by noise traders.

Once informed traders have submitted their orders and the exogenous demand for shares is realized, the auctioneer selects a price P that clears the market. Formally, the market-clearing price is a function $P : \mathbb{R}^2 \to \mathbb{R}$ that selects, for all realizations (θ, u) a price P from the correspondence $\hat{P}(\theta, u) = \{P \in \mathbb{R} : S(\theta, P) = D(u, P)\}$ of market-clearing prices.[6]

Finally, after all trades have occurred, firm's dividends $\pi(\theta)$ are realized and disbursed to the final shareholders.

Let $H(\cdot|x, P) : \mathbb{R} \to [0, 1]$ denote the shareholders' posterior cdf of θ, conditional on observing a private signal x, and conditional on the market price P. A *Perfect Bayesian Equilibrium* consists of supply functions $s(x, P)$ for informed traders, a price function $P(\theta, u)$, and posterior beliefs $H(\cdot|x, P)$ such that (i) $s(x, P)$ is optimal given $H(\cdot|x, P)$; (ii) the asset market clears for all (θ, u); and (iii) $H(\cdot|x, P)$ satisfies Bayes' rule whenever applicable, i.e., for all p such that $\{(\theta, u) : P(\theta, u) = p\}$ is non-empty.

2.2 Characterization

We begin by characterizing informed traders' supply of shares. With risk-neutrality, supply decisions are equal to either 0 or 1 almost everywhere – an order to hold ($s_i = 0$) or sell the share ($s_i = 1$) at P. The trader's expected value of holding the share is $\int \pi(\theta) dH(\theta|x, P)$. Since private signals are log-concave, posterior beliefs $H(\cdot|x, P)$ are first-order stochastically increasing in x, for any P that is observed in equilibrium. Since $\pi(\cdot)$ is increasing in θ, this implies that the traders' decisions are monotone in x, and characterized by a signal threshold function $\hat{x} : \mathbb{R} \to \mathbb{R} \cup \{\pm\infty\}$, such that

$$s(x_i, P) = \begin{cases} 1 & \text{if } x_i < \hat{x}(P), \\ 0 & \text{if } x_i > \hat{x}(P), \\ \in [0,1] & \text{if } x_i = \hat{x}(P), \end{cases} \tag{2}$$

so a trader sells if $x_i < \hat{x}(P)$ and holds if $x_i > \hat{x}(P)$. We call the informed trader who observes the signal equal to the threshold, $x = \hat{x}(P)$, and who is therefore indifferent, the *marginal trader*. The supply threshold is uniquely defined by

$$\begin{aligned} \hat{x}(P) &= +\infty \quad \text{if } \lim_{x \to +\infty} \int \pi(\theta) dH(\theta|x, P) \quad P, \\ \hat{x}(P) &= -\infty \quad \text{if } \lim_{x \to -\infty} \int \pi(\theta) dH(\theta|x, P) \geq P, \\ P &= \int \pi(\theta) dH(\theta|\hat{x}(P), P) \ \text{ otherwise.} \end{aligned} \tag{3}$$

Expression (3) illustrates three cases: (*i*) if the most optimistic trader's expected dividend is lower than the price, all traders sell so the signal threshold becomes $+\infty$; (*ii*) if the most pessimistic trader's expected dividend exceeds the price, all traders keep the share and the threshold for selling is $-\infty$;

[6] If the function $\pi(\cdot)$ is bounded, we can restrict the range of $P(\cdot)$ to coincide with the range of $\pi(\cdot)$ without loss of generality.

(iii) only some traders sell, and the threshold $\hat{x}(P)$ takes an interior value at which the marginal trader's posterior expectation of the dividend must equal the price. Aggregating the individual supply decisions, the market supply is $S(\theta, P) = \int_{1}^{\hat{x}(P)} 1 \cdot d\Phi(\sqrt{\beta}\,(x-\theta)) = \Phi(\sqrt{\beta}\,(\hat{x}(P)-\theta))$, which equals 1 if $\hat{x}(P) = +\infty$, and 0 if $\hat{x}(P) = -\infty$.

Next, we analyze the market-clearing condition. Since $D(u) \in (0,1)$, in equilibrium, $\hat{x}(\cdot)$ must be finite for all P on the equilibrium path, and satisfy the third condition in (3). Equating demand and supply, we characterize the correspondence of market-clearing prices:

$$\hat{P}\,(\theta, u) = \left\{ P \in \mathbb{R} : \hat{x}(P) = \theta + \frac{1}{\sqrt{\beta}} u \right\}. \tag{4}$$

From now on, we focus on equilibria in which the price is conditioned on (θ, u) through the state variable $z \equiv \theta + 1/\sqrt{\beta} \cdot u$. The equilibrium beliefs are characterized in the lemma that follows. All proofs are provided in the appendix.

Lemma 1 (Information Aggregation) *(i) In any equilibrium with conditioning on z, the equilibrium price function $P(z)$ is invertible. (ii) Equilibrium beliefs for price realizations observed along the equilibrium path are given by*

$$H\,(\theta|x, P) = \Phi\left(\sqrt{\lambda + \beta + \beta\delta}\left(\theta - \frac{\lambda\mu + \beta x + \beta\delta\hat{x}(P)}{\lambda + \beta + \beta\delta}\right)\right). \tag{5}$$

Part (i) of Lemma 1 shows that in any equilibrium, the price function must be invertible with respect to z, implying that the observation of P is equivalent to observing z. If the price function is not invertible, then some price realization P would be consistent with multiple realizations of the state z. It then follows immediately from (4) that P cannot be consistent with market clearing in all these states simultaneously.

Part (ii) of the Lemma exploits the invertibility to arrive at a complete characterization of posterior beliefs $H(\cdot|x, P)$. With invertibility, we can summarize information conveyed by the price through z, and note that conditional on θ, z is normally distributed with mean θ and variance $(\beta\delta)\ ^{1}$. Thus, the price is isomorphic to a normally distributed signal of θ, with a precision that is increasing in the precision of private signals, and decreasing in the variance of demand shocks.

Using Lemma 1 we rewrite (3), the indifference condition that defines the signal threshold $\hat{x}(P)$:

$$P = \int \pi(\theta) d\Phi\left(\sqrt{\lambda + \beta + \beta\delta}\left(\theta - \frac{\lambda\mu + \beta\,(1+\delta)\,\hat{x}(P)}{\lambda + \beta + \beta\delta}\right)\right). \tag{6}$$

This condition equates the price (on the left-hand side of (6)) to the marginal trader's expectation of dividends on the right-hand side. Notice that the latter is also influenced by P, through its effect on posterior beliefs. Using the market-clearing condition, we uniquely characterize the equilibrium price, $P(z)$, and the expected dividend conditional on public information, $V(z)$, as a function of z.

Proposition 1 (Asset market equilibrium) *Define $P(z)$ as*

$$\begin{aligned} P(z) &= \mathbb{E}\left(\pi(\theta)|x=z,z\right) \\ &= \int \pi(\theta) d\Phi\left(\sqrt{\lambda+\beta+\beta\delta}\left(\theta-\frac{\lambda\mu+(\beta+\beta\delta)z}{\lambda+\beta+\beta\delta}\right)\right), \end{aligned} \tag{7}$$

The asset market equilibrium is characterized by the price function $P(z)$ and the traders' threshold function $\hat{x}(p) = z = P\ ^{1}(p)$. The expected dividend conditional on public information z, denoted $V(z)$, is

$$V(z) = \mathbb{E}\left(\pi(\theta)|z\right) = \int \pi(\theta) d\Phi\left(\sqrt{\lambda+\beta\delta}\left(\theta-\frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}\right)\right). \tag{8}$$

Proposition 1 characterizes the asset market equilibrium. If (and only if) the price function is invertible, traders infer the state z from the price. In our case, this directly follows from the strict monotonicity of $\pi(\theta)$. The resulting price function $P(z)$ is uniquely defined.[7]

The price $P(z)$, and the expected dividend conditional on public information, $V(z)$, differ in how expectations of θ are formed. The price equals the dividend expectation of the marginal trader who is indifferent between keeping or selling her share. This trader conditions on the market signal z, as well as a private signal, whose realization must equal the threshold $\hat{x}(P)$ in order to be consistent with the trader's indifference condition. The trader treats these two sources of information as mutually independent signals of θ. At the same time, the market-clearing condition implies that $\hat{x}(P)$ must equal z in order to equate demand and supply of shares. The marginal trader's expectation $\mathbb{E}\left(\pi(\theta)|x=z,z\right)$ thus behaves as if she received one signal z of precision $\beta(1+\delta)$ instead of $\beta\delta$. In contrast, the expected dividends $\mathbb{E}\left(\pi(\theta)|z\right)$ conditional on P (or equivalently z) weighs z according to its true precision $\beta\delta$.

Alternatively, we can view the difference in the responsiveness of price and the expected dividend conditional on the price, as the result of a compositional change in the identity of the traders holding the shares. An increase in z raises the price and expected dividend through a direct effect on all traders' expectations. This is reflected in the weight $\beta\delta$ attributed to z in both $P(z)$ and $V(z)$. In addition, an increase in z changes the identity of the marginal trader: since the random demand for shares is larger (on average) for a higher z, market clearing requires more selling from the original shareholders. This implies that the new marginal trader must have higher expectations about the dividend, which holds in equilibrium since her private signal now corresponds to the higher realization of z.[8] The resulting extra shift in prices is captured by the additional weight β attributed to z in the price function.

[7] Notice that this only implies the uniqueness of the equilibrium that conditions on the summary statistic z, not overall uniqueness of the equilibrium characterized in proposition 1.

[8] If z increases because of θ, the distribution of private signals shifts up, decreasing supply for a given P. If instead z increases because of u, the distribution of signals and hence supply remains unchanged, but the demand for shares has gone up. Of course, in equilibrium traders cannot disentangle these two possibilities from observing the price.

2.3 The Information Aggregation Wedge

The main asset pricing implication of Proposition 1 is that at the interim stage –when the share price is observed but before dividends are realized– the equilibrium price generally differs from the expected dividend, conditional on the public information. We label this difference the *information aggregation wedge*, $W(z) \equiv P(z) - V(z)$. Our first theorem characterizes how the expectation of this wedge depends on the shape of the dividend function $\pi(\theta)$:

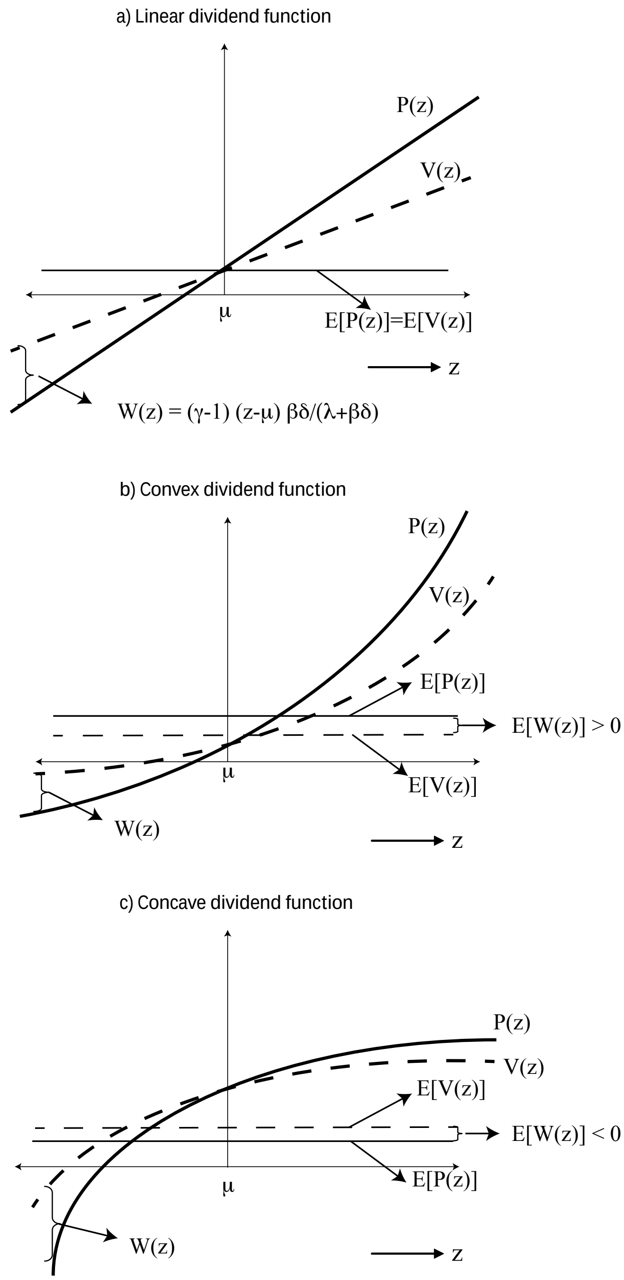
Theorem 1 ***(Average wedge):*** *In the equilibrium of Proposition 1, the sign of the unconditional information aggregation wedge depends on the convexity of $\pi(\theta)$;*

$$\begin{aligned} \pi^{00}(\cdot) &> 0 \Longrightarrow \mathbb{E}(W(z)) > 0 \\ \pi^{00}(\cdot) &= 0 \Longrightarrow \mathbb{E}(W(z)) = 0 \\ \pi^{00}(\cdot) &< 0 \Longrightarrow \mathbb{E}(W(z)) < 0 \end{aligned}$$

This theorem states that the unconditional expectation of the gap between prices and expected dividends is determined by the second derivative of the dividend function, as a function of the underlying state. As a starting point, suppose that $\pi(\cdot)$ is linear, $\pi(\theta) = \theta$. Panel a) of figure 1 plots the price (the thick solid line) and expected dividend (the thick dashed line) as a function of the state variable z. In this case, we can compute the price, the expected dividend and the wedge as

$$P(z) = \frac{\lambda\mu + (\beta + \beta\delta)z}{\lambda + \beta + \beta\delta}, \quad V(z) = \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}, \quad W(z) = \qquad z$$

Figure 1: Price, Expected Dividend and Wedge with Exogenous Dividend



in the downside exceeds the absolute value of the wedge in the upside. The resulting expected wedge is negative. These two cases are plotted in panels b) and c) in figure 1.

The magnitude of the wedge at a given realization of z depends on the shareholders' exposure (the net effect in the dividend) to changes in the underlying fundamentals, which locally is captured by the derivative $\pi^{0}(\cdot)$. With convex payoffs, this exposure is increasing in θ, and therefore higher on the upside. With concave payoffs, this exposure is decreasing in θ and hence higher on the downside.

In the appendix, we compare our model to the information aggregation wedge derived for a model with CARA preferences and normally distributed signals and shocks.[9] This model remains tractable as long as the traders' terminal wealth is normally distributed, conditional on their available information. Importantly, this restricts dividends to be a linear function of fundamentals. Consequently, if the average supply of the asset is zero the expected price equals the expected dividend ex ante and the expected wedge is zero. More generally any difference between unconditional expectations of prices and dividends results from risk premia when the average supply of shares differs from zero.

It is important to note that our results on differences between expected prices and dividends are not a consequence of irrational trading strategies, behavioral biases of investors, or agency conflicts. Nor such differences are accounted for by risk premia (since traders are risk neutral). Our model, and theorem 1 in particular, offers a theory in which expected prices and expected dividends can generally differ as a result of the interplay between the dividend structure and the partial aggregation of information into prices, in a context where traders hold heterogeneous beliefs in equilibrium.[10] To our knowledge, this result is new to the literature.

In the next section, we use the insights offered by our general model to study the informational feedback to endogenous investment decisions by firms. We show that the option value of responding to market information endogenously generates a convex dividend function, which results in a positive wedge from an ex ante perspective.

3 A Model with Endogenous Investment

We now consider a model in which the dividend depends on a real investment decision by firm's manager. We augment the setting in section 2 to include an additional stage in which the firm's manager takes a decision after observing the market price, as well as some private information that influences dividends. The asset market stage is modeled as before, but its outcome is now influenced

[9] See, e.g., Hellwig (1980), Diamond and Verrecchia (1981), and Grossman and Stiglitz (1980).

[10] Note that our specific limits of arbitrage assumption (individual supplies bounded within $[0, 1]$) is not crucial to the result. Belief heterogeneity and hence the information aggregation wedge remains in equilibrium as long as risk-neutral traders face finite portfolio constraints.

by the trader's anticipation of firm's manager's response to the market price.

3.1 General Formulation

Formally, we suppose that firm's dividend function takes the form $\pi : \Theta \times Y \times A \to \mathbb{R}$, where $\theta \in \Theta = \mathbb{R}$ denotes firm's fundamental, $y \in Y \subseteq \mathbb{R}$ denotes firm's private information, and $A \subseteq \mathbb{R}$ denotes a compact set from which the firm chooses an action $a \in A$ after observing y and its share price.

As before, nature initially draws the stochastic fundamental θ and the demand shock u, which are independent of each other and distributed according to $\theta \sim \mathcal{N}(\mu, \lambda^{-1})$ and $u \sim \mathcal{N}(0, \delta^{-1})$. In addition, nature draws y, which we assume is distributed according to the conditional cdf. $G : Y \times \Theta \to [0,1]$, where $G(\cdot|\theta)$ denotes the the cdf of y, conditional on θ, and $g(\cdot|\theta)$ the corresponding pdf.

At this stage, and before the market opens, the firm's manager commit to a decision rule $a : Y \times \mathbb{R} \to A$, which is selected to maximize manager's expectation of an objective function $\tilde{\pi}(\theta, y, a, P)$. This objective allows, in particular, for a manager's compensation to depend on the market price.

Each trader i then receives a noisy private signal $x_i \sim \mathcal{N}(\theta, \beta^{-1})$. Traders decide whether to hold or sell their share at the market price P. Individual trading strategies are then a mapping $s : \mathbb{R}^2 \to [0,1]$ from signal-price pairs (x_i, P) into the unit interval. The aggregate supply function $S : \mathbb{R}^2 \to [0,1]$ satisfies $S(\theta, P) = \int s(x, P) d\Phi(\sqrt{\beta}(x - \theta))$. The demand for shares takes the form $D(u) = \Phi(u)$. Once the orders to hold or sell are submitted, and the demand for shares is realized, the price P is selected to clear the asset market. The a market-clearing price function is $P : \mathbb{R}^2 \to \mathbb{R}$. This asset market stage is unchanged from the previous section.

Finally, the firm observes the price, its private information, and implements its decision $a(y, P)$.

Let $H(\cdot|x, P)$ denote the traders' posterior cdf of θ, conditional on observing a private signal x, and a market-clearing price P. A *Perfect Bayesian Equilibrium* of the augmented game consists of a shareholder's supply function $s(x, P)$, a price function $P(\theta, u)$, a decision rule $a(y, P)$ for the firm, and posterior beliefs $H(\cdot|x, P)$, such that (i) the supply function is optimal given the shareholder's beliefs $H(\cdot|x, P)$ and the anticipated investment rule $a(y, P)$; (ii) $P(\theta, u)$ clears the market for all (θ, u); (iii) $a(y, P)$ solves manager's decision problem; and (iv) $H(\cdot|x, P)$ satisfies Bayes' Rule whenever applicable.

Discussion: The general formulation of our endogenous investment model embeds several important special cases which we will analyze to discuss the role of specific assumptions.

1. If $\pi(\theta, y, a) = \tilde{\pi}(\theta, y, a, P)$, then manager's and final shareholder's objective coincide. The manager maximizes expected dividends. Under this benchmark, manager's decisions will make ex post efficient use of the information conveyed by prices. Moreover, the assumption that the firm pre-commits to a

rule is innocuous in this case, as the ex post optimal choice of manager corresponds to the rule to which the firm commits ex ante.

2. If $\pi(\theta, y, a) \neq \tilde{\pi}(\theta, y, a, P)$, then there is a conflict of interest between managers and shareholders. Such a conflict may result from agency problems, moral hazard considerations, or from the structure of manager's compensation contracts. This formulation also allows for compensation contracts explicitly tied to observable market prices. The pre-commitment assumption plays a role for our results in the case of price-based incentives. An ex post choice of the investment decision takes the price P as given, whereas the prior commitment to a rule allows the manager to internalize the effect of its investment decision on market prices.[11]

3. If $\pi(\theta, y, a) = \pi(\theta, y^{\prime}, a)$, for all y, $y^{\prime}$, and all (θ, a), then y is a noisy signal of the underlying fundamental θ which has no direct payoff implications for the firm.

4. If $G(y|\theta) = G(y)$, for all θ, then the firm has no additional private information about θ. However, the information contained in y is relevant to firm's decision problem. This is the case we focus on in the next section. The firm decides on an investment project whose cost is known to firm's manager. The returns are determined by the fundamental θ and are observed with noise only by shareholders.

The novelty of our investment model is that it allows for feedbacks through the price in presence of both market- and firm-specific information. Traders condition dividend expectations on the price since it provides information about θ, and also because it affects the investment choice of the firm. This is what we call a two-way feedback effect: the price aggregates information and affects investment. The investment decision also affects traders' dividend expectations and trading decisions, which in turn determine the equilibrium price. In the CARA-normal setup, endogenous investment has been modeled assuming that either i) the marginal effect of investment in the dividend does not enter traders' payoffs (Subrahmanyam and Titman (1999); Goldstein, Ozdenoren and Yuan (2010)), or ii) the share price is a sufficient statistic for firm's investment choice (Dow and Rahi (2003)). Our model allows to characterize the two-way feedback effect under more general payoff specifications and information structures. This allows us to discuss a richer set of implications on the relation between managerial incentives, share prices, and investment decisions.

Equilibrium Characterization: The equilibrium characterization proceeds in two stages. The second stage —financial market— proceeds along the same lines as in the previous section. Suppose that firm's manager's decision is characterized by an arbitrary decision rule $a(y, P)$. If $\int \pi(\theta, y, a(y, P))\, dG(y|\theta)$ is monotone in θ, the trader's strategies are characterized as before by threshold

[11] Perhaps a simple way to justify this pre commitment is that the firm's decision making is based on internal reporting, compensation rules and decision making procedures that are updated less frequently than the decisions themselves. In a dynamic environment, the design of such procedures then internalizes the impact of such decisions on future market prices.

rule $\hat{x}(P)$ such that they choose to sell whenever their private signal $x \quad \hat{x}(P)$. Lemma 1 extends immediately to this more general model. The observation of P is equivalent to observing $z = \hat{x}(P) = \theta + 1/\sqrt{\beta} \cdot u$, $z \sim \mathcal{N}(\theta, (\beta\delta) \;\; ^{1})$. Along the equilibrium path the shareholders' posterior beliefs are

$$H\left(\theta|x, P\right) = \Phi\left(\sqrt{\lambda + \beta + \beta\delta}\left(\theta - \frac{\lambda\mu + \beta x + \beta\delta\hat{x}(P)}{\lambda + \beta + \beta\delta}\right)\right).$$

We continue to focus on equilibria in which the price is conditioned on the (θ, u) through $z = \theta + 1/\sqrt{\beta} \cdot u = \hat{x}(P)$. Equilibrium price and expected dividend solve:

$$P\left(z\right) = \mathbb{E}\left(\pi\left(\theta, y, a\left(y, P\left(z\right)\right)\right)|x = z, z\right), \tag{10}$$

$$V\left(z\right) = \mathbb{E}\left(\pi\left(\theta, y, a\left(y, P\left(z\right)\right)\right)|z\right), \tag{11}$$

where expectations are taken both with respect to θ and y. This condition implicitly defines the price function. On the right of equation (10), $P(z)$ appears both through the public signal z that it conveys about θ and through its impact on firm's decision $a(y, P(z))$. For given $a(y, P)$, the market equilibrium exists if and only if there exists a strictly monotone solution to the condition for $P(z)$ in (10). In what follows, we disregard the monotonicity requirement at first, and then verify ex post whether it holds at the proposed equilibrium price function.

The manager's first-stage problem —before the financial market opens— is formulated as follows:

$$\max_{a(y;P);P(z)} \int \tilde{\pi}\left(\theta, y, a\left(y, P\left(z\right)\right), P\left(z\right)\right) dG\left(y|\theta\right) d\Phi\left(\sqrt{\beta\delta}\left(z - \theta\right)\right) d\Phi\left(\sqrt{\lambda}\left(\theta - \mu\right)\right)$$

s.t.

$$P\left(z\right) = \mathbb{E}\left(\pi\left(\theta, y, a\left(y, P\left(z\right)\right)\right)|x = z, z\right).$$

That is, manger chooses a price-contingent decision rule, subject to the constraint that the price function is an equilibrium price function.

Using the fact that $P(z)$ must be invertible, we reformulate manger's decision rule as a function of y and z. After changing the order of integration between θ, y and z, the optimal decision is characterized by the solution to the pointwise optimization problem as follows,

$$\max_{a(y;z);P(z)} \int \tilde{\pi}\left(\theta, y, a\left(y, z\right), P\left(z\right)\right) dH_F\left(\theta|y, z\right)$$

$$\begin{aligned} s.t.\ P\left(z\right) &= \mathbb{E}\left(\pi\left(\theta, y, a\left(y, z\right)\right)|x = z, z\right), \\ &= \int \pi\left(\theta, y, a\left(y, z\right)\right) dG\left(y|\theta\right) d\Phi\left(\sqrt{\lambda + \beta + \beta\delta}\left(\theta - \frac{\lambda\mu + \beta\left(1 + \delta\right) z}{\lambda + \beta + \beta\delta}\right)\right). \end{aligned}$$

Firm's posterior conditional on firm-specific information y and market information z, $H_F(\cdot|y, z)$, is given as follows,

$$H_F\left(\theta|y, z\right) = \frac{\int_{\;\;1} g\left(y|\theta\right) d\Phi\left(\sqrt{\lambda + \beta\delta}\left(\theta - \frac{+ \quad z}{+}\right)\right)}{\int_{\;\;1}^{1} g\left(y|\theta\right) d\Phi\left(\sqrt{\lambda + \beta\delta}\left(\theta - \frac{+ \quad z}{+}\right)\right)}.$$

As long as manager's payoff does not depend directly on P, i.e. when $\tilde{\pi}(\theta, y, a, P) = \tilde{\pi}(\theta, y, a)$, there is no commitment issue. The rule $a(y, z)$ that is chosen by the manager ex ante also corresponds to the rule that is ex post optimal from manager's perspective, once the price is taken as given. Moreover, when $\tilde{\pi}(\theta, y, a) = \pi(\theta, y, a)$, manager's and final shareholder's incentives are perfectly aligned, and the resulting investment decisions make efficient use of the information conveyed by z. The manager's incentive to manipulate the price to his own benefit and the shareholders' detriment by committing to an investment rule is thus directly linked to an incentive scheme that rewards managers for the share price performance. In the remainder of this section, we compare investment incentives under dividend value maximization with those induced by price-based incentives.

3.2 A Binary Action Model

In this section, we discuss the implications of the feedback effect and the role of managerial incentives in an environment where the manager makes a binary choice. In section 4, we use this model as a benchmark for a series of extensions with different assumptions regarding the information structure, asset dividends, and the asset demand.

Firm's manager makes a binary investment decision $a \in \{0, 1\}$, where $a = 1$ denotes the decision to invest, and $a = 0$ denotes the decision not to invest. The dividend of the firm is given by:

$$\pi(\theta, F; a) = \rho \cdot \theta + a \cdot (\theta - F), \tag{12}$$

where $\rho > 0$. The dividend has two components. The first is an exogenous effect $\rho \cdot \theta$ of the fundamental, θ, on payoffs.[12] The second component is endogenous. If the firm chooses to invest and to incur the cost F, the dividend increases by $(\theta - F)$. We assume that F is independent of θ, distributed with cdf $G(\cdot)$ and density $g(\cdot)$. Let $\underline{F}$ denote the lower bound of the distribution of F ($\underline{F}$ can be equal to $-\infty$). The cost F is observed by the manager before choosing investment. Traders do not observe signals about F. That is, F corresponds to firm's private information. The information structure captures the following environment. The *firm-specific* cost F summarizes characteristics of the project about which the firm holds precise information (for example, proprietary technical specifications). The *market-specific* fundamental θ relates to conditions about which knowledge is dispersed throughout the market (for example, demand for a new product).[13]

Suppose for now that firm's investment decision is characterized by a threshold rule $\tilde{F}(P)$:

$$a(F, P) = \begin{cases} 1 \text{ if } F \quad \tilde{F}(P); \\ 0 \quad \text{otherwise.} \end{cases} \tag{13}$$

[12] We further discuss the role of the exogenous component in Section 4.1.

[13] See Miller and Rock (1985) and Rock (1986) for further discussions on market- and firm-specific sources of information.

The firm invests if and only if the investment cost is below the threshold $\tilde{F}(P)$. Below we consider two specifications of manager's objective $\tilde{\pi}(\theta, y, a, P)$ which are consistent with the threshold investment rule in (13). For now, we leave $\tilde{F}(P)$ deliberately general to characterize the market equilibrium.

Following the same steps as above, trader's supply decisions are characterized by a threshold rule $\hat{x}(P)$, which satisfies:

$$\begin{aligned} P &= \rho \int \theta dH(\theta|\hat{x}(P), P) + \int [a(F, P) \cdot (\int \theta dH(\theta|\hat{x}(P), P) \; - F)] dG(F) \\ &= \left(\rho + G\left(\tilde{F}(P)\right)\right) \int \theta dH(\theta|\hat{x}(P), P) - \int_{\underline{F}}^{\tilde{F}(P)} F dG(F). \end{aligned} \tag{14}$$

The first integral in the upper line of equation (14), $\rho \int \theta dH(\theta|x, P)$, is the marginal trader's expectation of the dividend if the firm does not invest. The second term in the first line is the expected impact of investment on the dividend. For each pair (F, P), the marginal trader considers the difference between the posterior expectation of the fundamental and the investment cost, $\int \theta dH(\theta|x, P) \; - F$. Since the trader does not observe F, the expectation is an integral over the investment range $F \leq \tilde{F}(P)$. Equation (14) compares the cost of holding the share, P (the left-hand side) to the expected dividends (the right hand side). The price enters the expected dividend through its impact on marginal trader's expectation of θ, and by its influence on firms' investment threshold $\tilde{F}(P)$.

With invertibility of the price function, we redefine the investment threshold as a function of z: $\tilde{F}(z) = \tilde{F}(P)$. Using the market-clearing condition $z \equiv \hat{x}(P)$, and the characterization of shareholder beliefs in Lemma 1, we characterize the equilibrium of the endogenous investment model:

Proposition 2 (Equilibrium with endogenous investment) *For an investment threshold $\tilde{F}(z)$, define $P(z)$ by:*

$$P(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \frac{\lambda\mu + \beta(1+\delta) z}{\lambda + \beta + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} F dG(F). \tag{15}$$

If $P(z)$ is strictly increasing, the asset market equilibrium is characterized by the price function $P(z)$ and traders' threshold function $\hat{x}(p) = z = P^{-1}(p)$. The expected dividend conditional on public information z is given by

$$V(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} F dG(F). \tag{16}$$

The equilibrium price and expected dividend in Proposition 2 can be decomposed into three terms. First, $\rho \cdot \frac{\lambda\mu + \beta(1+\delta) z}{\lambda + \beta + \beta\delta}$ and $\rho \cdot \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}$ denote the expected dividend if the firm does not invest, from the marginal trader's and manager's perspective. Second, $G\left(\tilde{F}(z)\right) \cdot \frac{\lambda\mu + \beta(1+\delta) z}{\lambda + \beta + \beta\delta}$ and $G\left(\tilde{F}(z)\right) \cdot \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}$ are the additional expected payoff if the firm invests. The third term $\int_{\underline{F}}^{\tilde{F}(z)} F dG(F)$ is the expected investment cost.

In the next two subsections, we consider two separate cases for manager's objective and compare the resulting threshold functions $\tilde{F}(z)$, equilibrium prices, and expected dividend values.

3.3 The Benchmark Case: Dividend Maximization

We now characterize the equilibrium in which manager's objective is to maximize the expected dividend: $\tilde{\pi}(\theta, F; a) = \pi(\theta, F; a)$. The manager invests if (and only if) the realization of the cost F is (weakly) lower than the posterior of the fundamental $\mathbb{E}(\theta|P)$. The investment threshold is given by

$$\tilde{F}(P) = \tilde{F}(z) = \int \theta dH(\theta|z) = \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}. \tag{17}$$

The equilibrium is given by Proposition 2 after replacing $\tilde{F}(z) = \mathbb{E}(\theta|z)$. For the discussion below, it is convenient to redefine the state z in terms of the posterior expectation $Z \equiv \mathbb{E}(\theta|z)$. We then rewrite the price, the expected dividend, and the wedge in terms of this posterior expectation Z:

$$P(Z) = (\rho + G(Z))(\mu + \gamma(Z - \mu)) - \int_{\underline{F}}^{Z} F dG(F), \tag{18}$$

$$V(Z) = (\rho + G(Z)) Z - \int_{\underline{F}}^{Z} F dG(F), \tag{19}$$

$$W(Z) \equiv P(Z) - V(Z) = (\gamma - 1)(Z - \mu)(\rho + G(Z)). \tag{20}$$

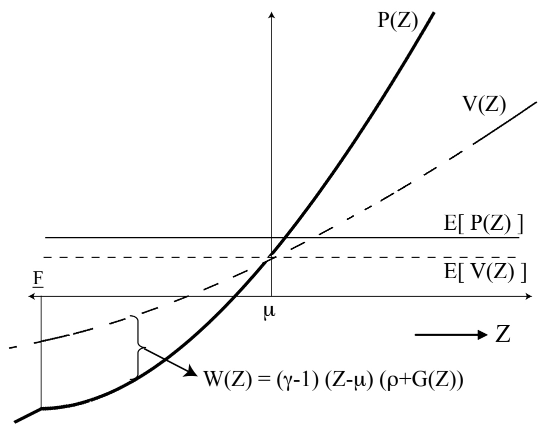
The parameter $\gamma > 1$ is given by expression (9), which corresponds to the ratio of Bayesian weights assigned to the market signal z by the marginal trader, and manager (or an uninformed outsider). The next lemma establishes a sufficient condition for the invertibility of the price function:

Lemma 2 (Invertibility of the Price Function) *The price function is invertible, if $\rho + G(F) + g(F)(F - \mu) > 0$, for all $F \geq \underline{F}$.*

Lemma 2 states a sufficient condition for price invertibility in the case of expected dividend maximization. Price non-invertibility, which is caused by price non-monotonicity, can arise if the marginal trader's valuation of investment is locally decreasing in z. This is inevitably the case whenever $G(F)/g(F)$ is non-decreasing and converges to 0 as $F \to -\infty$. The condition in Lemma 2 imposes a lower bound on the sensitivity of the dividend to the fundamental through the exogenous payoff component $\rho \cdot \theta$. We further expand on this issue in Section 4.1 and Appendix B.

The wedge $W(Z)$ can be decomposed as a product of two terms. The first term given by equation (Info agg wedge) corresponds to the difference between marginal trader's and manager's posterior beliefs about θ: $(\gamma - 1)(Z - \mu)$. This term determines the sign of the wedge and follows from our discussion in Section 2. The price, $P(z)$, is the expectation of dividends by the marginal trader who

Figure 2: Price, Expected Dividend and Wedge with Endogenous Investment



observes z both as private *and* public information of θ. Thus, the price reacts more strongly to the market information relative to the dividend expectation of the manager, $V(Z)$.

The second term is the marginal effect of the fundamental, θ, on the expected dividend: $(\rho + G(Z))$. This term includes the exogenous effect of the fundamental (ρ) and the endogenous effect from investment ($G(Z)$). It captures the value of market information for the investment decision: the market signal determines manager's posterior belief of θ and the probability of investment $G(Z)$, which determines the net effect of θ on the dividend and, hence, the absolute magnitude of the wedge.

Figure 2 plots the price (solid line), the expected dividend (dashed line), and the wedge as a function of the state variable Z. The expected dividend $V(Z)$ is increasing and convex, and the price function is also increasing given the condition imposed in Lemma 2. As in the case of the exogenous dividend model of Section 2, the information aggregation wedge is negative for $Z < \mu$, zero at $Z = \mu$, and positive for $Z > \mu$. The key insight is that the model with investment gives rise to an *endogenously* convex dividend function. Therefore, we can apply the results of Theorem 1 (part (ii)):

$$\mathbb{E}(W(Z)) = (\gamma - 1)\, Cov\,(G(Z), Z) > 0, \tag{21}$$

since $G(\cdot)$ is increasing.

Expression (21) explicitly shows that two necessary conditions for a positive unconditional wedge are belief heterogeneity ($\gamma > 1$) and endogenous investment ($G^0(\cdot) > 0$). A violation of either condition results in an unconditional information wedge equal to zero. To illustrate the role of belief heterogeneity, suppose instead that traders observe an identical signal about the fundamental.

The price then equals the common posterior of the dividend, and all traders are indifferent between selling or keeping the share. The price also reveals the signal to firm's manager. The expected dividend by all participants equals the price, and no wedge arises in equilibrium.

The endogenous investment ($G^{0}(\cdot) > 0$) contributes to the wedge by changing the marginal effect of θ on dividends as a function of z. The higher is z, the higher the probability that the firm invests, and the higher is the net effect of θ on the dividend. If investment were chosen before the observation of the price, the investment probability would be constant. The wedge would be symmetric around the prior mean $Z = \mu$, and its ex ante expectation equal to zero. This corresponds to the linear dividend case in Theorem 1.

Expression (21) also shows that, for a given cost distribution $G(\cdot)$, the unconditional wedge is increasing in the variance of firm's posterior Z, $\sigma_Z^2 = \beta\delta / (\lambda + \beta\delta) \cdot \lambda^{\ 1}$. This variance measures how strongly information conveyed by the market affects beliefs about θ; σ_Z^2 represents the value of market information for the investment decision and is increasing in the precision of the market signal $\beta\delta$ and the prior variance of the fundamental $\lambda^{\ 1}$. Intuitively, precise private information (high β), or low variance of noise trading (high δ) increase the likelihood that movements in z are due to innovations in θ. This makes z a more reliable signal and increases the sensitivity of manager's posterior Z to changes in z. Also, learning about θ is more important the larger its ex-ante variance ($\lambda^{\ 1}$).

The value of market information for the firm highlights the close relation of our setup with real options (e.g., Dixit and Pindyck, 1994). Firm's payoff depends on the realization of a random variable (θ) and on an endogenous choice of investment (a). The price is a public signal of the variable θ for the firm. The firm can better match a good realization of θ by investing, while limiting the negative effects of a low realization by not investing. The value of the investment option depends on the precision of information, and the prior uncertainty about the random variable. An important novel result of our model is that the option value of information also leads to expected prices to be higher than expected dividends when traders hold heterogenous beliefs in equilibrium.

We now discuss how the parameters of the model affect the size of the information aggregation wedge, the expected price, and the dividend.

Proposition 3 (Comparative Statics) *(i) For a given value of γ, $\mathbb{E}(P(Z))$, $\mathbb{E}(V(Z))$, and $\mathbb{E}(W(Z))$ are increasing in σ_Z^2. (ii) For given value of σ_Z^2, $\mathbb{E}(V(Z))$ does not depend on γ; $\mathbb{E}(P(Z))$ and $\mathbb{E}(W(Z))$ are increasing in γ.*

The unconditional price, dividend, and wedge are all increasing in the prior uncertainty about the firm's posterior, σ_Z^2. Recall that the unconditional expected dividend is larger when the manager learns more information from the market (higher σ_Z^2). Moreover, since the marginal trader's posterior is more sensitive to z than manager's, the impact of σ_Z^2 on the average price is stronger than on the expected dividend. This raises the unconditional wedge. The unconditional dividend is independent of the difference between manager's and

marginal trader's expectations (γ). The expected price and, hence, the wedge, scale up γ.

The primitive parameters β, δ, and λ affect expected dividends $\mathbb{E}\left(V\left(Z\right)\right)$ through σ_Z^2, which is increasing in both the precision of the market signal ($\beta\delta$), and in the prior uncertainty ($\lambda\ \ ^1$). Both better market information and a more variable prior increase the value of the real option to invest, raising the unconditional expected dividend. The same parameters affect the unconditional information aggregation wedge $\mathbb{E}\left(W\left(z\right)\right)$ through both σ_Z^2 and γ. Notice, however, that the effects go in opposite directions: γ is decreasing in $\lambda\ \ ^1$, decreasing in β, and decreasing in δ. The overall comparative statics on the unconditional wedge and price are ambiguous, and the unconditional wedge is largest at intermediate values. Prior uncertainty $\lambda\ \ ^1$ must be sufficiently high to generate option value from investment, the precision of market information $\beta\delta$ must be large enough so that the manager wants to respond to it, and private information precision β needs to be large to create belief dispersion. At the same time, $\lambda\ \ ^1$ must remain sufficiently small relative to $\beta\delta$ so that the market information does not completely crowd out the prior and eliminate the wedge.

3.4 Tying Managerial Incentives to Share Prices

In the benchmark model of endogenous investment of Section 3.2, we assumed the manager's and shareholders' objectives coincide: $\tilde{\pi}(\cdot) = \pi(\cdot)$. This assumption lead to an investment rule that maximizes $V\left(Z\right)$: the expected dividend conditional on the share price. We now discuss the effects of managerial incentives tied to stock market performance. A well known result in economies with complete markets and common information is that $P\left(Z\right) = V\left(Z\right)$. In all states, maximizing prices or expected dividends is equivalent.

This result does not hold in our model because of the wedge between the price and the expected dividend. Moreover, the wedge is directly affected by the investment decision that determines the net impact θ on the dividend. If a manager has incentives to maximize the share price as opposed to the dividend, the size of the wedge can be changed by choosing investment to increase market prices. Since investment in the benchmark model maximized expected dividends, any deviations from the investment rule defined by the threshold $\tilde{F}\left(Z\right) = Z$ reduces the expected payoffs of shareholders.

We now consider the case where manager's objective function is given by $\tilde{\pi}(\theta, F, a; P) = (1-\alpha)\pi(\theta, F; a) + \alpha P$, $\alpha \in [0, 1]$. This objective function is a linear combination of the share price, $P\left(Z\right)$, and the dividend $\pi(\theta, F; a)$. More specifically, we assume that the manager chooses an initial decision rule $a\left(F, P\right)$, which he commits to prior to the market opening.[14] For a given value of Z,

[14] In this context, the assumption of pre-commitment is important, as the ex post optimization (taking P as given) results in the same maximization of expected dividends as before. The influence of firm's decisions on market prices (and the role of price-based incentives) results precisely from the market's anticipation of the firm's investment choices.

manager's problem can then be stated as follows:

$$\max_{\tilde{F}(Z)} \ \mathbb{E}(\tilde{\pi}(\theta, F, a; P)|Z) = \alpha P(Z) + (1-\alpha) V(Z),$$

where $P(Z)$ and $V(Z)$ are given by equations (18) and (19), and $\alpha \in [0,1]$ measures how strongly incentives are based on the price relative to expected dividends.

The equilibrium is characterized by Proposition 2: threshold functions $\hat{x}(P)$ for traders and $\tilde{F}(P)$ for manager, and an invertible price function $P(Z)$. The investment threshold is found by maximizing $\mathbb{E}(\tilde{\pi}|Z)$ pointwise, for all Z, and then checking that the resulting price function is invertible. Formally, we have

$$\begin{aligned}
\tilde{F}(Z) &\in \arg\max_{\tilde{F}} \{\alpha P(Z) + (1-\alpha) V(Z)\} = \arg\max_{\tilde{F}} \{V(Z) + \alpha W(Z)\} \\
&= \arg\max_{\tilde{F}} \left\{ \left(\rho + G\left(\tilde{F}\right)\right)[\mu + k(Z-\mu)] - \int_{\underline{F}}^{\tilde{F}} F dG(F) \right\}. \qquad (22)
\end{aligned}$$

The parameter $k = 1 + \alpha(\gamma - 1)$ measures excess weighting of market information by the manager and captures the strength of the distortion introduced by price-based incentives. The variable $k \in [1, \gamma]$ depends on the size of the information aggregation wedge through γ and the weight given to the prices in the manager's objective function, α. At one extreme, $\alpha = 0$ and $k = 1$ correspond to our benchmark model of dividend maximization (section 3.2). At the other extreme, $\alpha = 1$ and $k = \gamma$: the manager's incentives are based only on the share price.

Taking first-order conditions to determine the investment threshold $\tilde{F}(Z)$ (and checking price invertibility) yields the following equilibrium characterization.

Proposition 4 (Equilibrium with price-based incentives) *In the PBE with price-based incentives, the investment threshold $\tilde{F}(Z)$, price $P(Z)$, and expected dividend $V(Z)$ are given by*

$$\tilde{F}(Z) = \mu + k(Z-\mu), \qquad (23)$$

$$P(Z) = \rho\tilde{F}(Z) + \int_{\underline{F}}^{\tilde{F}(Z)} G(f)\,df + (\gamma - k)(Z-\mu)\left(\rho + G\left(\tilde{F}(Z)\right)\right), \qquad (24)$$

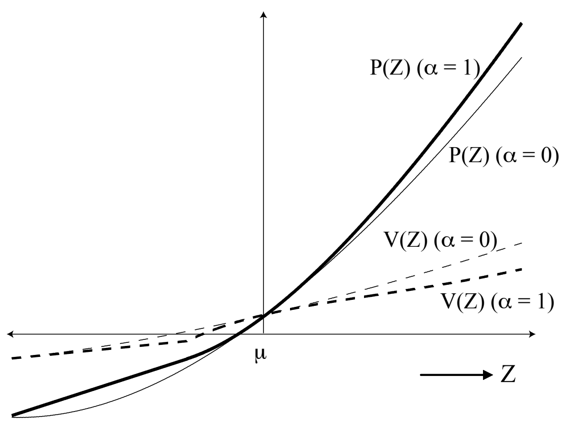
$$V(Z) = \rho\tilde{F}(Z) + \int_{\underline{F}}^{\tilde{F}(Z)} G(f)\,df - (k-1)(Z-\mu)\left(\rho + G\left(\tilde{F}(Z)\right)\right). \qquad (25)$$

Equations (24) and (25) decompose the effect that the information aggregation wedge has on the price and the expected dividend. Without price-based incentives ($\alpha = 0$; $k = 1$), the stronger reaction of the price to market signals has no impact on the expected dividend. When $\alpha > 0$ ($k > 1$), the price increases with α at an efficiency cost that reduces the expected dividend in (25). We formalize the result in the next theorem.

Theorem 2 (Tying managerial incentives to share-prices) *In the PBE with price-based incentives, (i) the volatility of investment is increasing in k:* $\tilde{F}^0(Z) = k > 1$*; (ii) the share price* $P(Z)$ *is increasing in* k*, and expected dividends* $V(Z)$ *are decreasing in* k *for all* $Z \neq \mu$*.*

Theorem 2 states that investment becomes the more volatile, the more managerial incentives are based on the share price. The manager reacts more strongly to the information conveyed by the price, as captured by the parameter $k > 1$. When Z is higher than μ, firm's investment threshold is too high. The firm invests in some states in which the cost F exceeds the expected gains from investment, Z. When Z is lower than μ, the firm's investment threshold is too low. The firm foregoes investment in some states in which Z exceeds the cost F. This results in higher prices but lower expected dividends, for all $Z \neq \mu$.

Figure 3: Effect of Price-based Incentives



Price-based incentives induce the manager to align investment with the beliefs of those traders who have the largest impact on the price. Figure 3 plots the price and expected dividend functions for the extreme cases $\alpha = 1$ (the thick lines) and $\alpha = 0$ (the thin lines). When $\alpha = 1$ ($k = \gamma$) the manager is purely concerned with price maximization. The investment rule in (23) exactly matches the marginal trader's expectation of θ. The firm behaves as if it were run by the marginal trader and achieves the largest share price for each realization of the state Z to the detriment of expected dividends. Indeed, figure 3 shows how the price in this case (the thick, solid line) is always above the one attained under dividend maximization, or $\alpha = 0$ (the thin, solid line). The expected dividend under priced-based incentives (thick, dashed line) is everywhere below its counterpart in the benchmark case (the thin, dashed line). The conditional information aggregation wedge is thus exacerbated and so is the unconditional wedge.

The analysis above gives an argument against tying executive compensation too closely to market valuations. When dispersed information drives a wedge between prices and expected dividends, and when the wedge responds to firm's endogenous investment decision, price-based incentives lead to inefficient investments that drive up prices but lower firm value.

4 Discussion and Extensions

We now study the robustness of our main results by considering several extensions of our benchmark model of endogenous investment. We explore alternative specifications of dividends, exogenous asset demand, and informational environments.

4.1 Exogenous Dividend Component: $\rho > 0$

In the setup of endogenous investment introduced in section 3.2, we assumed that the firm generates a dividend $\rho \cdot \theta$ if it doesn't invest, and an additional payoff $(\theta - F)$ if it invests. Under this assumption, the dividend depends on θ irrespective of the investment choice, guaranteeing that traders always have an incentive to trade on their private information about θ. Hence, the price always reveals z provided that ρ is large enough to ensure the price function is invertible (Lemma 2).

In Appendix B we characterize equilibria in an environment in which there is no exogenous component in the dividend function: $\rho = 0$. This model admits a rich set of possible equilibrium outcomes including the potential for multiplicity, indeterminacy, and non-existence of equilibria.

Price multiplicity and indeterminacy arise from a feedback in the informativeness of the price. We separately consider two possibilities. If at a price $P = 0$ there is a positive probability of investment, traders respond to their private signals, and prices fully reveal Z through the price function $P(Z)$ (which has the same features as above, with $\rho = 0$). Alternatively, if at the price $P = 0$ investment occurs with zero probability, then traders do not condition on private signals, and the price conveys no information. This, in turn, sustains no investment as an equilibrium at the price of $P = 0$. $P = 0$ is then, at most, partially revealing of Z. It is possible to sustain (almost) arbitrary selections from the correspondence $\{0, P(Z)\}$ as equilibrium prices.

Non-existence is linked to the non-invertibility of the price function.[15] If we eliminate the exogenous dividend component from our benchmark model, non-invertibility occurs with unbounded support of F and a thin-tail assumption on the distribution $G(\cdot$ F

of our paper. Our baseline model with an exogenous dividend component allows us to focus on the cases in which there is a unique equilibrium.

4.2 Price Impact of Information

We now generalize our assumption about exogenous asset demand (noise trading) by assuming it comes from uninformed traders: they trade partly for exogenous motives, and partly in response to gaps between the price and their dividend expectation, conditional on the price. This demand specification allows an additional comparative static on the information aggregation wedge that relates naturally to the concept of market liquidity.

Specifically, we consider the following formulation for asset demand:

$$D(u, P) = \Phi\left(u + \eta\left(\mathbb{E}\left(\pi|P\right) - P\right)\right), \tag{26}$$

with $u \sim \mathcal{N}\left(0, \delta \;\; ^{1}\right)$. Uninformed traders' demand is increasing in the expected return conditional on the price, $\mathbb{E}\left(\pi|P\right) - P$, with an elasticity given by η. This specification of demand generalizes our previous formulation to allow for a response of uninformed traders to perceived excess returns on the asset, as well as stochastic trading motives which are unrelated to dividend expectations (for example, liquidity or hedging needs). The parameter η captures the responsiveness of uninformed traders to the expectation of dividends in excess of prices, or in other words, the extent to which they are willing or able to arbitrage away the difference between expected price and dividend value. Equivalently, η measures the price impact of private information which relates naturally to the concept of market depth.

We keep the dividend specification as in the binary action model of section 3.2. We then follow our previous characterization of equilibrium strategies and asset market clearing with minor changes to account for the endogeneity of demand to asset prices. We show that all the previous effects, regarding over-valuation and the role of price based incentives, are inversely related to the magnitude of η. When η is higher the uninformed traders are better able to arbitrage the difference between expected value and price. This reduces the absolute value of the information aggregation wedge for all realizations of the state Z, mitigating the distortions induced by price-based incentives on investment decisions.

We now characterize the equilibrium to formalize this intuition. For a given investment threshold $\tilde{F}\left(P\right)$, we characterize the equilibrium threshold $\hat{x}\left(P\right)$ for the informed traders and the market-clearing price function $P\left(z\right)$. Market-clearing implies

$$\Phi\left(\sqrt{\beta}\left(\hat{x}\left(P\right) - \theta\right)\right) = \Phi\left(u + \eta\left(\mathbb{E}\left(\pi|P\right) - P\right)\right),$$

or

$$z = \hat{x}\left(P\right) - \eta/\sqrt{\beta} \cdot \left(\mathbb{E}\left(\pi|P\right) - P\right).$$

Observing P is isomorphic to observing $z \sim \mathcal{N}\left(\theta, \left(\beta\delta\right) \;\; ^{1}\right)$, and Lemma 1 applies without any changes. We use the state variable $Z = \frac{+ \quad z}{+}$, the manager's

posterior expectation of θ, which now coincides with the uninformed traders' expectations. For an investment threshold $\tilde{F}(Z)$, and after substituting

$$\hat{x}(P) = z + \eta/\sqrt{\beta} \cdot (\mathbb{E}(\pi|P) - P) = z + \eta/\sqrt{\beta}\,(V(Z) - P(Z)),$$

we obtain the expected dividend, the price, and the wedge functions:

$$\begin{aligned}
V(Z) &= \left(\rho + G\left(\tilde{F}(Z)\right)\right) Z - \int_{\underline{F}}^{\tilde{F}(Z)} f dG(f), \\
P(Z) &= \left(\rho + G\left(\tilde{F}(Z)\right)\right)(\mu + \gamma(Z - \mu)), \\
&\quad + \frac{\sqrt{\beta}\eta\left(\rho + G\left(\tilde{F}(Z)\right)\right)}{\lambda + \beta + \beta\delta}(V(Z) - P(Z)) - \int_{\underline{F}}^{\tilde{F}(Z)} f dG(f), \\
W(Z) &= \frac{\lambda + \beta + \beta\delta}{\lambda + \beta + \beta\delta + \sqrt{\beta}\eta\left(\rho + G\left(\tilde{F}(Z)\right)\right)}(\gamma - 1)\left(\rho + G\left(\tilde{F}(Z)\right)\right)(Z - \mu).
\end{aligned}$$

The information aggregation wedge is thus inversely related to the uninformed traders' demand elasticity η. Higher η lowers the price impact of private information, and thus the magnitude of the information aggregation wedge. At the extreme with infinite elasticity, price equals expected dividends and the wedge disappears. The other extreme ($\eta = 0$) corresponds to our baseline setup of section 3.2.

If manager's objective is the maximization of $\alpha P(Z) + (1 - \alpha) V(Z)$, first-order conditions lead to

$$\tilde{F}(Z) = Z + \alpha \left\{ \frac{\lambda + \beta + \beta\delta}{\lambda + \beta + \beta\delta + \sqrt{\beta}\eta\left(\rho + G\left(\tilde{F}(Z)\right)\right)} \right\}^2 (\gamma - 1)(Z - \mu).$$

As long as $\alpha = 0$, investment remains undistorted. When $\alpha > 0$, there is overinvestment for $Z > \mu$ and underinvestment for $Z < \mu$, but the inefficiency is reduced by the demand elasticity; η. For all η, $\tilde{F}(Z) \in [Z, \mu + k(Z - \mu)]$, with $\lim_{\to \infty} \tilde{F}(Z) = Z$ and $\lim_{\to 0} \tilde{F}(Z) = \mu + k(Z - \mu)$. Efficient investment arises as the uninformed demand becomes infinitely elastic. In this case, it is the uninformed traders who price the shares, arbitraging away the discrepancy between price and expected dividends, conditional on market information.

4.3 Alternative Information Environments

In this section we consider three environments with alternative information structures.

4.3.1 Market-speci c Information Observed by the Manager

We now augment the manager's information set by assuming that the manager observes a private signal y about the fundamental: $y \sim N(\theta, \kappa^{-1})$. The manager's posterior of θ now corresponds to $Z(y, z) = \frac{+\ y +\ z}{+\ +}$. Although y is

not observed by traders, it is convenient to define the (hypothetical) marginal trader's posterior including the manager's signal y as:

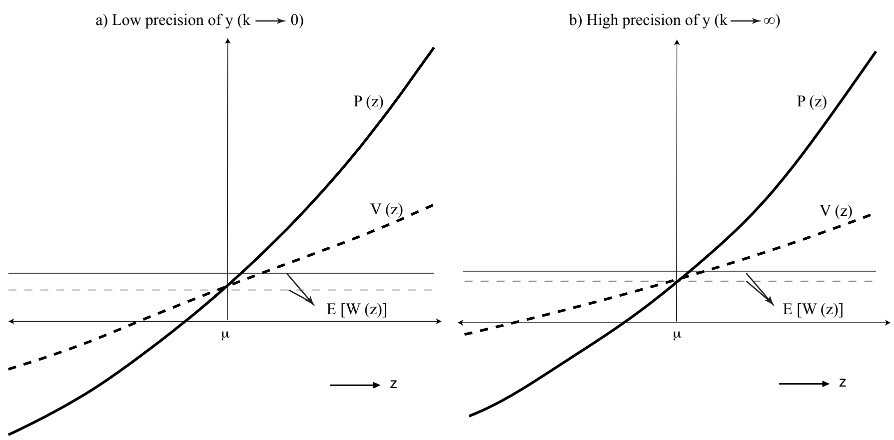
$$\hat{Z}(y,z) \equiv \frac{\lambda\mu + \kappa y + \beta(1+\delta)z}{\lambda + \kappa + \beta(1+\delta)}.$$

The price and expected dividend for an investment threshold $\tilde{F}(Z)$ are

$$\begin{aligned} V(z) &= \int_y (\rho Z + \int_{\underline{F}}^{\tilde{F}} (Z-F)dG(F))d\Phi(y|z) \equiv \int_y v(y,z)d\Phi(y|z), \\ P(z) &= \int_y (\rho \hat{Z} + \int_{\underline{F}}^{\tilde{F}} (\hat{Z}-F)dG(F))d\Phi(y|x=z,z) \equiv \int_y p(y,z)d\Phi(y|x=z,z), \end{aligned}$$

where the (y,z)-contingent information wedge is $w(y,z) \equiv p(y,z) - v(y,z) = (\hat{Z} - Z)(\rho + G(\tilde{F}(Z)))$.

Figure 4: Price, Expected Dividend and Wedge with Manager's Private Signal



Assuming the more general objective function of Section 3.3, the manager's investment rule follows from the pointwise maximization of $v(y,z) + \alpha \cdot w(y,z)$, which leads to the f.o.c. $\tilde{F} = (1-\alpha)Z + \alpha\hat{Z}$. In what follows, it will be convenient to redefine the state variable by $\tilde{F}$. Characterizing the distribution of $\tilde{F}$ then allows analyzing the unconditional price and the information aggregation wedge. Some algebra (Appendix C) allows to rewrite $W(\tilde{F}) \equiv P(\tilde{F})$ and $V(\tilde{F})$ as:

$$W(\tilde{F}) \quad = \quad (\rho + G(\tilde{F}))\frac{\beta(z-\tilde{F})}{\lambda + \kappa + (1-\alpha)\beta + \beta\delta},$$

We can now find an expression for the unconditional information wedge from the difference between the ex-ante price and expected dividend by computing $\mathbb{E}(z|\tilde{F}) - \tilde{F}$ and the variance of $\tilde{F}$ using Bayes rule. This yields

$$\mathbb{E}(W) = [\frac{(\gamma_z/(\beta\delta) + \gamma\ /\lambda - \sigma^2_{\tilde{F}})\beta}{\sigma^2_{\tilde{F}} \cdot (\lambda + \kappa + (1-\alpha)\beta + \beta\delta)}] \cdot Cov(G(\tilde{F}), \tilde{F}) > 0$$

where the coefficients γ_z, γ and the variance $\sigma^2_{\tilde{F}}$ depend on the underlying parameters of the model.

The wedge now also depends on the precision of the manager's signal, κ. The higher its value, the less investment responds to the information contained in the price and the lower the resulting unconditional wedge. Figure 4 plots the comparative statics with respect to the manager's private signal precision; κ. The wedge is large when the precision of the manager's signal is low (panel a), in comparison with the case where the signal is very precise (panel b).

4.3.2 Firm-specific Information Observed by the Market

We now assume that at $t = 0$ traders observe $\Omega_T = \{x, f^*\}$, where f^* is a common signal about F. The equilibrium characterization closely follows Section 3. Manager's decision is described by (13) since investment is chosen after observing the price and the actual realization of the cost F. Trader's supply decisions are monotone in x and can be characterized by a threshold function $\hat{x}(P; f^*) \in \mathbb{R} \cup \{\pm\infty\}$, such that

$$s(x_i, P; f^*) = \begin{cases} 1 & \text{if } x_i < \hat{x}(P, f^*), \\ 0 & \text{if } x_i > \hat{x}(P, f^*), \\ \in (0,1) & \text{if } x_i = \hat{x}(P, f^*). \end{cases}$$

This supply schedule for traders imply the following market clearing condition:

$$\int_{-\infty}^{\hat{x}(P;f^*)} 1 \cdot d\Phi(\sqrt{\beta}\,(x-\theta)) = \Phi(\sqrt{\beta}\,(\hat{x}(P, f^*) - \theta)) = \Phi(u),$$

or $\hat{x}(P, f^*) = \theta + u/\sqrt{\beta}$, for all $(\theta, u; f^*)$. The conditions for the existence of equilibria and its characterization are summarized in the next proposition.

Proposition 5 (Equilibrium with cost signal) *For a given investment threshold $\tilde{F}(z)$, if $\rho + G(F|f^*) + (F-\mu)g(F|f^*) > 0\ \forall F$ and f^*, a unique equilibrium exists, characterized by the price function*

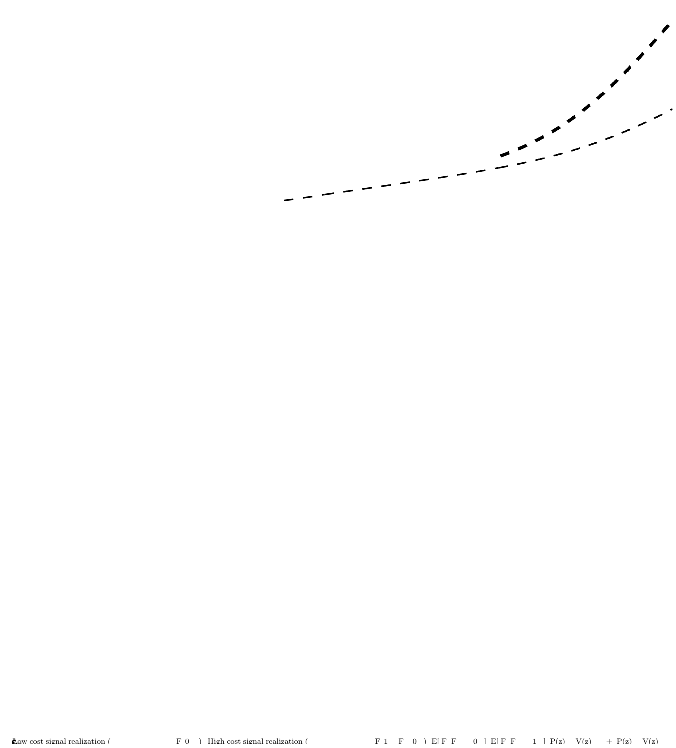
$$P(z, f^*) = \left(\rho + G\left(\tilde{F}(z)\,|f^*\right)\right) \frac{\lambda\mu + \beta(1+\delta)z}{\lambda + \beta + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} F dG(F|f^*), \quad (27)$$

and the traders' signal threshold $\hat{x}(p, f^*) = z = P^{-1}(p, f^*)$. The expected dividend conditional on z and f^* is given by

$$V(z, f^*) = \left(\rho + G\left(\tilde{F}(z)\,|f^*\right)\right) \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} F dG(F|f^*). \quad (28)$$

We analyze a tractable example assuming that F is normally distributed with mean $\bar{f}$ and variance $\lambda_f{}^{\;1}$, and that the signal $f\;$ is drawn from a normal distribution with mean F and variance $\chi\;{}^{1}$. We plot the price and expected dividend function under this particular cost distribution in Figure 5. The figure captures two comparative statics: changes in the posterior mean $\mathbb{E}(F|f\;)$ due to different cost signal realizations $f\;$, and changes in the precision of the posterior mean due to different signal precision; $\chi\;$.

Figure 5: Price and Expected Dividend with Cost Signal



In panel a) we compare the price and expected dividend functions for a high (dashed lines) and a low (solid lines) realization of the investment cost: $f_0 < f_1$. The figure shows that a higher cost signal lowers the price (thick dashed line) and expected dividend (thin dashed line) since it raises the the posterior mean $\mathbb{E}(F|f\;)$. This has two effects on the valuation of dividends. It makes investment less likely (the probability that $F \quad \tilde{F}(z)$ falls), and also reduces net payoff from the project, conditional on investment. However, a higher $f\;$ need not necessarily reduce the price and/or the expected dividend, for all z. Recall

that investment defined by $\tilde{F}(z)$ need not coincide with the marginal trader's posterior of θ, and hence the reduction in investment probability could have a positive impact on the price. This is the case if her expectation of θ is below $\tilde{F}(z)$, as she would prefer no investment to take place. The result is formalized in the next lemma.

Lemma 3 *(i) Under the investment rule $\tilde{F}(z)$ implied from the firm manager's objective function $\alpha P(z, f\ \ ) + (1-\alpha)V(z, f\ \ )$, $\forall \alpha \in (0, 1]$, there exists $z^0 < \mu$ such that $\partial P(z, \cdot)/\partial f\ \ < 0\ \forall\ z \geq z^0$. (ii) For $\alpha = 0$, $\partial V(z, \cdot)/\partial f\ \ < 0 \forall z$.*

In panel b) of Figure 5 we vary the precision of the public signal χ. The dashed lines plot the price and expected dividend for a low signal precision ($\chi = 0.00001$). In the limit ($\chi \to 0$), this case corresponds to the baseline model with no cost signal. The solid lines plot the respective functions when the public signal f has higher precision ($\chi = 1$).[16] Around the value $\hat{z} \equiv \tilde{F}\ ^{1}(\hat{f})$, the price and expected dividend functions become steeper. Intuitively, a higher signal precision raises the likelihood that F corresponds to its posterior mean, so the probability that the firm invests increases at a faster rate as z approaches $\tilde{F}\ ^{1}(\hat{f})$. This is made precise in the next lemma.

Lemma 4 *For any strictly increasing threshold $\tilde{F}(z)$, if $\chi_0 > \chi_1$ and $\mathbb{E}(F|f_0; \chi_0) = \mathbb{E}(F|f_1; \chi_1) \equiv \hat{f}$, there exists $\epsilon > 0$ such that $\forall\ \ \tilde{F}\ ^{1}(\hat{f}) - \epsilon < z < \tilde{F}\ ^{1}(\hat{f}) + \epsilon$, $P^0(\cdot, f_0; \chi_0) > P^0(\cdot, f_1; \chi_1)$ whenever $\mathbb{E}(\theta|x = z, z) > \tilde{F}(z)$, and $V^0(\cdot, f_0; \chi_0) > V^0(\cdot, f_1; \chi_1)$ whenever $\mathbb{E}(\theta|z) > \tilde{F}(z)$.*

4.3.3 Predetermined Investment Cost

In this section, we consider a special case in which the investment cost F is predetermined at the fixed value $\bar{f}$, which is common knowledge for all traders. The manager then invests if (and only if) $\tilde{F}(z) < \bar{f}$. This case can be nested in our general formulation so apply the solution method directly, characterizing the equilibrium in the next proposition.

Proposition 6 (Equilibrium with predetermined investment cost) *For a given investment threshold $\tilde{F}(z)$; (i) If $\tilde{F}\ ^{1}(\bar{f}) \geq \frac{(\bar{f}\quad)}{(1+\ )} + \bar{f}$, there exists a unique equilibrium with conditioning on z characterized by the price function*

$$P(z) = \begin{cases} \rho \cdot \frac{\ +\ (1+\ )\, z}{\ +\ (1+\ )} & \text{if} \quad z < \tilde{F}\ ^{1}(\bar{f}) \\ (\rho + 1) \cdot \frac{\ +\ (1+\ )\, z}{\ +\ (1+\ )} - \bar{f} & \text{otherwise,} \end{cases} \tag{29}$$

and the traders' signal threshold $\hat{x}(p) = z = P\ ^{1}(p)$. The expected dividend conditional on z is

$$V(z) = \begin{cases} \rho \cdot \frac{\ +\ \ z}{\ +\ } & \text{if} \quad z < \tilde{F}\ ^{1}(\bar{f}) \\ (\rho + 1) \cdot \frac{\ +\ \ z}{\ +\ } - \bar{f} & \text{otherwise.} \end{cases} \tag{30}$$

[16] We have chosen parameters so that in both cases the posterior mean $\mathbb{E}(F|\cdot) \equiv \hat{f}$ are the same in order to separate this effect from the previous comparative static.

(ii) If $\tilde{F}\ ^{1}(\bar{f}) < \frac{(\bar{f} \quad)}{(1+ \quad)} + \bar{f}$*, an equilibrium with conditioning on* z *does not exist.*

Proposition 6 is a restatement of Proposition 2 under the special case of the distribution $G(F) = 0$ for $F < \bar{f}$, and 1 otherwise. The condition in part (i) specifies the parameter space for which the price function is invertible, allowing an equilibrium with conditioning on z. When the price function is invertible, the equilibrium found is unique. To understand the condition for existence, note that a predetermined cost implies investment is perfectly forecastable, conditional the price. The price function can then be separated into two segments. For $z < \tilde{F}\ ^{1}(\bar{f}) \equiv \bar{z}$, the firm does not invest and the price equals the marginal trader's posterior of θ times the exogenous constant ρ. The slope is given by $\rho \cdot \partial\mathbb{E}(\theta|x = z, z)/\partial z$. For $\bar{z} \quad z$, investment is undertaken with probability 1. The slope of the price function becomes $(\rho + 1) \cdot \partial\mathbb{E}(\theta|x = z, z)/\partial z$. There is a discrete level change in the price at $P(z = \bar{z})$ equal to the posterior expectation of θ of the marginal trader, net of the fixed investment cost $\bar{f}$. Depending on the particular investment rule $\tilde{F}(\cdot)$ and the rest of the parameters, the discontinuity can be negative, yielding a non-invertible price function. Formally, the derivative at $z = \bar{z}$ is given by

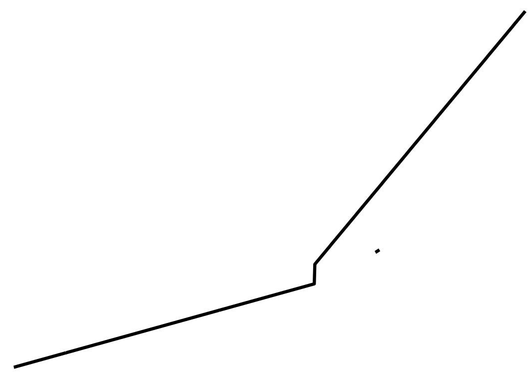
$$\begin{aligned} P^{0}(\cdot)|_{z=z} \quad &= \quad (\rho + G(\tilde{F}(\bar{z}))) \cdot \frac{\partial\mathbb{E}(\theta|x = \bar{z}, \bar{z})}{\partial z} \\ &\quad + g(\tilde{F}(\bar{z})) \cdot \tilde{F}^{0}(\bar{z})[\mathbb{E}(\theta|x = \bar{z}, \bar{z}) - \tilde{F}(\bar{z})] \\ &= \quad (\rho + 1) \cdot \frac{\beta(1+\delta)}{\lambda + \beta(1+\delta)} + \infty \cdot [\frac{\lambda\mu + \beta(1+\delta)\bar{z}}{\lambda + \beta(1+\delta)} - \bar{f}], \end{aligned} \tag{31}$$

which follows from the fact that the investment decision is perfectly forecastable, conditional on the share price $(g(\tilde{F}(\bar{z})) \to \infty)$. Replacing $\bar{z} = \tilde{F}\ ^{1}(\bar{f})$ yields the condition in part (i).

Figure 6 plots the price (solid line) and expected dividend (dashed line) for a predetermined investment cost when the firm manager maximizes the expected dividend ($\tilde{F}(z) = Z$). The price invertibility condition in (i) reduces to $\mu \quad \bar{f}$. Indeed, for $\mu \quad \bar{f}$, the extra payoff θ from undertaking investment at $z = \bar{z} \equiv \tilde{F}\ ^{1}(\bar{f})$ is larger than the investment cost according to the marginal trader's beliefs, since the information aggregation wedge is positive in this region. It follows that the investment triggered at $\bar{z}$ with probability 1 has a (weakly) positive discrete impact on the price.

Another interesting case corresponds to the investment rule analyzed in Section 3.4, for the extreme case in which the manager of the firm maximizes the share price; $\alpha = 1$. Since $\tilde{F}(z) = \mu + \gamma(Z - \mu) = \mathbb{E}(\theta|x = z, z)$ in this case, the term inside the square bracket in (31) is zero, so there is no discontinuity in the price function. Intuitively, to maximize the price the manager decides on investment as if she observed the same information as the marginal trader. At $\bar{z}$, the minimum value of z for which the firm invests, it must then be the case that the investment cost exactly equals the posterior of θ of the marginal trader. For $\alpha = 1$ then, the condition in (i) holds and the price function is invertible for all remaining parameter values.

Figure 6: Price and Expected Dividend with Predetermined Investment Cost



5 Concluding Remarks

This paper examines the role of asset prices in aggregating information about fundamentals and providing guidance for real investment decisions. We first develop a model that allows to characterize a broad set of heterogenous information environments in which dividends are an (exogenous) nonlinear function of the fundamentals. We then develop a model in which a firm's payoffs depends on fundamentals and the choice of investment, thus endogenizing the dividend. Information about the fundamentals is dispersed among traders in a financial market and partially aggregated in firm's share price, upon which the firm conditions its investment choice.

We find that market-generated information enhances firm's value by encouraging investment when high prices communicate good fundamentals, but limiting the loses by discouraging investment when low prices signal poor realizations. However, the interaction between dispersed information and endogenous investment is also the source of a systematic departure between equilibrium share prices and expected dividends –*the information aggregation wedge.* The wedge originates from a higher weight assigned to market-generated signals by the marginal informed trader. This higher weight arises because of the positive correlation between the noise in the public signal revealed by the price, and the noise in private signals of those traders who end up holding (and therefore pricing) the share, and is perfectly consistent with individual rationality. Moreover, because the firm responds to the information conveyed by the price investing more in good states than in bad ones, it exacerbates the price overreaction on the upsides more than the price underreaction on the downsides. As a result, the information aggregation wedge is asymmetric, and the share price exceeds on average the expected dividend value of the firm.

We then discuss the role of price-based managerial incentives. We find that

compensation tied to share prices may enhance share overvaluation and induce excess volatility in investment, as managers try to cater investment policies to those traders who have the largest impact on market prices.

Our model has two predictions that align with empirical evidence. First, the model suggests the value enhancing effects of market-specific information in guiding real investment importantly depend on the extent of informed trading activity. This fits the evidence provided by Chen, Goldstein and Jiang (2007) who study the impact of informed trading in the sensitivity of real investment to price changes. The authors find stronger sensitivity in shares with more informed trading activity, as measured by PIN (*probability of informed trading* – Easley et al. (1996)).[17] Second, Polk and Sapienza (2009) provide support to our findings regarding the impact of stock-based compensation. They test a "catering" theory using discretionary accruals as a proxy for mispricing,[18] finding a positive relation between share overvaluation and excess investment after controlling for Tobin's Q. This relation is stronger for firms with higher share turnover, which could proxy for traders' short-term horizons. Moreover, they find that firms with high excess investment subsequently have low share returns, the more so the larger is their measure of mispricing. This suggests that such investment behavior is indeed inefficient.

While our model has taken the manager's objective as given, the design of optimal incentive structures in the presence of a wedge between expected dividend and prices remains an important question for future research. Our model of financial markets with information aggregation appears to provide a promising building block for future work in this direction, as well as for other questions that require a flexible payoff structure for analyzing the interplay between managerial incentives, corporate decisions, and market prices.

[17] Roll, Schwartz and Subrahmanayam (2009) provide related evidence arguing that developed options markets for a firm's share stimulate the entry of informed traders. They find that firms with deeper options markets have higher sensitivity of corporate investment to share prices, which translates into higher values of Tobin's Q.

[18] Discretionary accruals measure the extent to which a firm has abnormal non-cash earnings. Firms with high discretionary accruals typically have relatively low share returns in the future, suggesting that discretionary accruals artificially drive up prices temporarily.

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Appendix A: Proofs

Proof of Lemma 1. Part (i): By market-clearing, $z = \hat{x}(P(z))$ and $\hat{x}(P(z^0)) = z^0$, and therefore $z = z^0$ if and only if $P(z) = P(z^0)$.

Part (ii): Since $P(z)$ is invertible, observing P is equivalent to observing $z = \hat{x}(P(z))$ in equilibrium. But $z|\theta \sim \mathcal{N}\left(\theta, (\beta\delta)^{\ 1}\right)$, from which the characterization of $H(\cdot|P)$ and $H(\cdot|x, P)$ follows immediately from Bayes' Law. ■

Proof of Proposition 1. Substituting the market-clearing condition $\hat{x}(P) = z$, a price function $P(z)$ is part of an equilibrium if and only if it satisfies (7) and is invertible. $P^0(z) > 0$ is immediate because $\pi(\cdot)$ is strictly increasing, and an increase in z represents a first-order stochastic shift in the posterior over θ. Since the market-clearing price is uniquely defined for each z, and the price function is monotone and continuous over its domain and spans its entire range, all prices are observed in equilibrium (and hence out-of-equilibrium beliefs play no role). Thus, the characterization in proposition 1 defines the unique equilibrium in which prices are conditioned only on z. ■

Proof of Theorem 1. (i): $P^0(z) = -\beta(1+\delta)\int \pi(\theta)\phi^0\left(\sqrt{\lambda+\beta+\beta\delta}\left(\theta - \frac{+(\ +\ )z}{+\ +}\right)\right)d\theta$, while $V^0(z) = -\beta\delta\int \pi(\theta)\phi^0\left(\sqrt{\lambda+\beta\delta}\left(\theta - \frac{+\ \ z}{+}\right)\right)d\theta$. Since
$\int \pi(\theta)\phi^0\left(\sqrt{\lambda+\beta+\beta\delta}\left(\theta - \frac{+(\ +\ )z}{+\ +}\right)\right)d\theta < \int \pi(\theta)\phi^0\left(\sqrt{\lambda+\beta\delta}\left(\theta - \frac{+\ \ z}{+}\right)\right)d\theta$
for any increasing function $\pi(\theta)$, it follows that $P^0(z) - V^0(z) > 0$. Now, since $Var(P(z)) = Var(V(z)) + cov(V(z), W(z)) + Var(W(z))$, it suffices to show that $cov(V(z), W(z)) > 0$. But this follows immediately since $V^0(z) > 0$ and $W^0(z) = P^0(z) - V^0(z) > 0$.
(ii) Using the fact that $\pi(\theta) = \pi(\mu) + \pi^0(\mu)(\theta-\mu) + \int\ (\pi^0(\theta^0) - \pi^0(\mu))\,d\theta^0$, we write $V(Z)$ and $P(Z)$ as follows: (with $Z = (\lambda\mu + \beta\delta z)/(\lambda+\beta\delta)$)

$$\begin{aligned}
V(Z) &= \pi(\mu) + \pi^0(\mu)(Z-\mu) + \int_{1}^{1}\int\ (\pi^0(\theta^0) - \pi^0(\mu))\,d\theta^0 d\Phi\left(\sqrt{\lambda+\beta\delta}\,(\theta - Z)\right) \\
P(Z) &= \pi(\mu) + \pi^0(\mu)\gamma(Z-\mu) \\
&\quad + \int_{1}^{1}\int\ (\pi^0(\theta^0) - \pi^0(\mu))\,d\theta^0 d\Phi\left(\sqrt{\lambda+\beta+\beta\delta}\,(\theta - \mu - \gamma(Z-\mu))\right)
\end{aligned}$$

Taking expectations and integrating out the variable Z, we have

$$
\begin{aligned}
&\mathbb{E}\left(V\left(z\right)\right)=\pi\left(\mu\right) \\
&+\int_{-\infty}^{\infty}\int\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\int_{-\infty}^{\infty}\sqrt{\lambda+\beta\delta}\,\phi\left(\sqrt{\lambda+\beta\delta}\left(\theta-Z\right)\right)\frac{1}{\sigma_Z}\phi\left(\frac{Z-\mu}{\sigma_Z}\right)dZd\theta \\
&=\pi\left(\mu\right)+\int_{-\infty}^{\infty}\int\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\sqrt{\lambda}\phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)d\theta, \\
&\mathbb{E}\left(P\left(z\right)\right)=\pi\left(\mu\right) \\
&+\int_{-\infty}^{\infty}\int\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\int_{-\infty}^{\infty}\frac{1}{\sigma_P}\phi\left(\frac{\theta-\mu-\gamma\left(Z-\mu\right)}{\sigma_P}\right)\frac{1}{\sigma_Z}\pi\left(\frac{Z-\mu}{\sigma_Z}\right)dZd\theta \\
&=\pi\left(\mu\right)+\int_{-\infty}^{\infty}\int\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\frac{1}{\tilde{\sigma}_P}\phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)d\theta,
\end{aligned}
$$

where $\sigma_P^2=\left(\lambda+\beta+\beta\delta\right)^{-1}$, and $\tilde{\sigma}_P^2=\sigma_P^2+\gamma^2\sigma_Z^2$. Therefore,

$$
\begin{aligned}
\mathbb{E}\left(W\left(z\right)\right) &= \int_{-\infty}^{\infty}\int\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\left\{\frac{1}{\tilde{\sigma}_P}\phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)-\sqrt{\lambda}\phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)\right\}d\theta \\
&= \int^{\infty}\int\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\left\{\frac{1}{\tilde{\sigma}_P}\phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)-\sqrt{\lambda}\phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)\right\}d\theta \\
&\quad+\int_{-\infty}\int\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\left\{\frac{1}{\tilde{\sigma}_P}\phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)-\sqrt{\lambda}\phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)\right\}d\theta \\
&= \int^{\infty}\left(\pi'\left(\theta\right)-\pi'\left(\mu\right)\right)\left\{\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)-\Phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)\right\}d\theta,
\end{aligned}
$$

where we have proceeded by integration by parts. Since

$$
\begin{aligned}
\tilde{\sigma}_P^2 &= \frac{1}{\lambda+\beta+\beta\delta}+\left(\frac{\beta+\beta\delta}{\lambda+\beta+\beta\delta}\frac{\lambda+\beta\delta}{\beta\delta}\right)^2\frac{\beta\delta}{\lambda+\beta\delta}\lambda^{-1} \\
&= \frac{1}{\lambda}\left(1+\frac{\beta+\beta\delta}{\lambda+\beta+\beta\delta}\left(\gamma-1\right)\right)>\frac{1}{\lambda},
\end{aligned}
$$

it follows that $\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)-\Phi\left(\left(\theta-\mu\right)/\tilde{\sigma}_P\right)>0$, when $\theta<\mu$, and $\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)-\Phi\left(\left(\theta-\mu\right)/\tilde{\sigma}_P\right)>0$, when $\theta>\mu$.

Thus, clearly, when $\pi''\left(\cdot\right)=0$, i.e. when $\pi\left(\cdot\right)$ is linear, $\mathbb{E}\left(W\left(z\right)\right)=0$. When instead $\pi''\left(\cdot\right)>0$, $\pi'\left(\theta\right)-\pi'\left(\mu\right)$ is increasing in θ, and positive (negative), when $\theta>\mu$ ($\theta<\mu$), so $\left(\pi'\left(\theta\right)-\pi'\left(\mu\right)\right)\left\{\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)-\Phi\left(\frac{}{\tilde{}_P}\right)\right\}>0$ for all $\theta\neq\mu$, and hence $\mathbb{E}\left(W\left(z\right)\right)>0$. Finally, when $\pi''\left(\cdot\right)<0$, $\pi'\left(\theta\right)-\pi'\left(\mu\right)$ is decreasing in θ, and positive (negative) when $\theta<\mu$ ($\theta>\mu$), so $\left(\pi'\left(\theta\right)-\pi'\left(\mu\right)\right)\left\{\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)-\Phi\left(\frac{}{\tilde{}_P}\right)\right\}<0$ for all $\theta\neq\mu$, and hence $\mathbb{E}\left(W\left(z\right)\right)<0$. ■

Proof of Proposition 2. Substituting the market-clearing condition $\hat{x}(P) = z$ and the investment threshold $\tilde{F}(z) = \tilde{F}(P(z))$ into (14), a price function $P(z)$ is part of an equilibrium if and only if it satisfies (15) and is invertible. Given the investment threshold $\tilde{F}(z)$, firm's expected dividend value is $V(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right)\mathbb{E}(\theta|z) - \int_{\underline{F}}^{\tilde{F}(z)} f dG(f)$. ■

Proof of Lemma 2. Taking the derivative with respect to Z in equation (18) gives

$$P^{0}(\cdot) \quad = \quad \gamma \cdot (\rho + G(Z)) + g(Z)(Z - \mu)(\gamma - 1).$$

Since $\gamma > 1$, we can write the following inequality for values $Z < \mu$,

$$P^{0}(\cdot) \quad > \quad \rho + G(Z) + g(Z)(Z - \mu),$$

which is positive for all Z whenever the distribution $G(F)$ satisfies the condition stated in the Lemma. ■

Proof of Proposition 3. (i) Since $V^{0}(Z) = \rho + G(Z) > 0$ and $V^{00}(Z) = g(Z) > 0$, $V(Z)$ is increasing and convex, so an increase in σ_Z^2 increases $\mathbb{E}(V(z))$. Moreover, $\mathbb{E}(Z - \mu)(\rho + G(Z)) = (\rho + G(\mu))\mathbb{E}(Z - \mu)$ $+ \mathbb{E}(G(Z) - G(\mu))(Z - \mu) = \mathbb{E}(G(Z) - G(\mu))(Z - \mu)$. Since $(G(Z) - G(\mu))(Z - \mu)$ is strictly positive for $Z \neq \mu$, and increasing in absolute value as $|Z - \mu|$ increases, it follows that an increase in σ_Z^2 shifts the variance of firm's posterior to higher values of $|Z - \mu|$, and therefore increases $\mathbb{E}(G(Z) - G(\mu))(Z - \mu)$. The result for $\mathbb{E}(P(z))$ then follows from the statements about $\mathbb{E}(V(z))$ and $\mathbb{E}(W(z))$.

(ii) is immediate given that γ does not affect $V(Z)$, but linearly scales up $W(Z)$. ■

Proof of Proposition 4. Equation (23) follows immediately from (22). Substituting this into the pricing equation, we find:

$$\begin{aligned}
P(Z) &= (\rho + G(\mu + k(Z - \mu)))(\mu + \gamma(Z - \mu)) - \int_{\underline{F}}^{+k(Z\quad)} f dG(f) \\
&= (\rho + G(\mu + k(Z - \mu)))(\mu + k(Z - \mu)) \\
&\quad - \int_{\underline{F}}^{+k(Z\quad)} f dG(f) + (\gamma - k)(Z - \mu)(\rho + G(\mu + k(Z - \mu))) \\
&= \rho(\mu + k(Z - \mu)) + \int_{\underline{F}}^{+k(Z\quad)} G(f)\, df \\
&\quad + (\gamma - k)(Z - \mu)(\rho + G(\mu + k(Z - \mu))).
\end{aligned}$$

To check that this is an equilibrium, we check that the price function is invertible:

$$\begin{aligned}
P^{0}(Z) &= \gamma(\rho + G(\mu + k(Z - \mu))) + (\gamma - k)k(Z - \mu)g(\mu + k(Z - \mu)) \\
&= k(\rho + G(\mu + k(Z - \mu))) \\
&\quad + (\gamma - k)(\rho + G(\mu + k(Z - \mu)) + k(Z - \mu)g(\mu + k(Z - \mu))),
\end{aligned}$$

which is strictly positive, because of our initial assumption on ρ and G. The expected dividend function (25) then follows from rearranging $V(Z) = (\rho + G(\mu + k(Z-\mu)))Z - \int_{\underline{F}}^{+k(Z \quad)} f dG(f)$ along the same lines as the price function. ■

Proof of Theorem 2. Part (i) is immediate. For part (ii) notice that

$$\begin{aligned}
\frac{\partial V}{\partial k} &= g(\mu + k(Z-\mu))(Z-\mu)(Z-(\mu + k(Z-\mu))) \\
&= g(\mu + k(Z-\mu))(Z-\mu)^2(1-k) < 0, \text{ and} \\
\frac{\partial P}{\partial k} &= g(\mu + k(Z-\mu))(Z-\mu)(\mu + \gamma(Z-\mu) - (\mu + k(Z-\mu))) \\
&= g(\mu + k(Z-\mu))(Z-\mu)^2(\gamma - k) > 0, \text{ for } Z \neq \mu.
\end{aligned}$$

■

Proof of Proposition 5.

Follows the same argument as in proposition 2, but using the market-clearing condition $\hat{x}(P, f \quad) = z$. The particular condition for price invertibility is now stated in terms of the conditional distribution $G(F|f \quad)$. ■

Proof of Lemma 3. With our assumption of normally distributed cost F and cost signal f , the price function can be rewritten as

$$\begin{aligned}
P(z, f \quad) &= [\rho + \Phi(\sqrt{\lambda_f + \chi}(\tilde{F} - \frac{\bar{f}\lambda_f + f \ \chi}{\lambda_f + \chi}))] \cdot \mathbb{E}(\theta|x = z, z) \\
&\quad - \int_1^{\tilde{F}(z)} f\phi(\sqrt{\lambda_f + \chi}(\tilde{F} - \frac{\bar{f}\lambda_f + f \ \chi}{\lambda_f + \chi}))\sqrt{\lambda_f + \chi} df, \\
&= [\rho + \Phi(\sqrt{\lambda_f + \chi}(\tilde{F} - \frac{\bar{f}\lambda_f + f \ \chi}{\lambda_f + \chi}))] \cdot (\mathbb{E}(\theta|x = z, z) - \tilde{F}(z)) \\
&\quad - \int_1^{\tilde{F}(z)} (f - \tilde{F}(z))\phi(\sqrt{\lambda_f + \chi}(\tilde{F} - \frac{\bar{f}\lambda_f + f \ \chi}{\lambda_f + \chi}))\sqrt{\lambda_f + \chi} df
\end{aligned}$$

Taking the partial w.r.t. f gives (to a constant of proportionality)

$$\begin{aligned}
P^0(z, \cdot) &\propto -\phi(\sqrt{\lambda_f + \chi}(\tilde{F} - \frac{\bar{f}\lambda_f + f \ \chi}{\lambda_f + \chi}))] \cdot (\mathbb{E}(\theta|x = z, z) - \tilde{F}(z)) \\
&\quad + \sqrt{\lambda_f + \chi}\int_1^{\tilde{F}(z)} (f - \tilde{F}(z))\phi^0(\sqrt{\lambda_f + \chi}(\tilde{F} - \frac{\bar{f}\lambda_f + f \ \chi}{\lambda_f + \chi}))df.
\end{aligned}$$

When the firm manager maximizes the linear combination $\alpha P(z, f \quad) + (1 - \alpha)V(z, f \quad)$ for all $\alpha \in [0, 1)$, the first term has the opposite sign as the information aggregation wedge: it is positive $\forall\ z < \mu$, negative for $\forall\ z > \mu$, and strictly

decreasing in z (and it is zero $\forall z$ if $\alpha = 1$). The second term is always negative, and strictly decreasing $\forall\ z < \mu$. It follows that there always exist $z^0 < \mu$ such that $\partial P(z, \cdot)/\partial f \quad < 0\ \forall z^0 \quad z$, which proves part (i) of the lemma.

Part (ii) addresses the effect of the cost signal on the expected dividend:

$$
\begin{aligned}
V^0(z,\cdot) \quad \propto \quad & -\phi(\sqrt{\lambda_f + \chi}(\tilde{F} - \frac{\bar{f}\lambda_f + f\ \chi}{\lambda_f + \chi}))] \cdot (\mathbb{E}(\theta|z) - \tilde{F}(z)) \\
& + \sqrt{\lambda_f + \chi} \int_{1}^{\tilde{F}(z)} (f - \tilde{F}(z))\phi^0(\sqrt{\lambda_f + \chi}(\tilde{F} - \frac{\bar{f}\lambda_f + f\ \chi}{\lambda_f + \chi}))df.
\end{aligned}
$$

The first term is zero for $\alpha = 0$; i.e., the firm investment threshold is the posterior of θ conditional on the price. For $\alpha \in (0, 1]$ however, the first term is negative and strictly increasing in $z < \mu$, and positive for $z > \mu$. The whole expression can therefore be positive, zero or negative for $z > \mu$, depending on specific parameter values. We can therefore only ensure $\partial V(z, \cdot)/\partial f \quad < 0$ for each $z > \mu$ when the sufficient condition $\mathbb{E}(\theta|z) > \tilde{F}(z)$ holds, which is true for all arbitrary, strictly increasing investment rules $\tilde{F}(z)$. ■

Proof of Lemma 4. Taking the partial w.r.t. z in the price functions $P^{(}\cdot, f_0; \chi_0)$ and $P^{(}\cdot, f_1; \chi_1)$ and subtracting leads to

$$
\begin{aligned}
P^0(\cdot, f_0; \chi_0) - P^0(\cdot, f_1; \chi_1) = & \frac{\partial \mathbb{E}(\theta|x = z, z)}{\partial z}(G(F|f_0; \chi_0) - G(F|f_1; \chi_1)) \\
& + \tilde{F}^0(z) \cdot (\mathbb{E}(\theta|x = z, z) - \tilde{F}(z)) \cdot (g(F|f_0; \chi_0) - g(F|f_1; \chi_1)),
\end{aligned}
$$

Since we have assumed equal posterior means; $\mathbb{E}(F|f_0; \chi_0) = \mathbb{E}(F|f_1; \chi_1) \equiv \mu^0_f$, the normal distribution implies the first term is zero at $z = \tilde{F}\ \ ^1(\mu^0_f)$, while the second term has the same sign as $\mathbb{E}(\theta|x = z, z) - \tilde{F}(z)$, since $\tilde{F}^0(z) > 0$ and $g(F|f_0; \chi_0) > g(F|f_1; \chi_1)$ for $\chi_0 > \chi_1$. By continuity of the normal density, it follows that there exist an open neighborhood $z \in (\tilde{F}\ \ ^1(\mu^0_f) - \epsilon, \tilde{F}\ \ ^1(\mu^0_f) + \epsilon)$ such that whenever $\mathbb{E}(\theta|x = z, z) > \tilde{F}(z)$, $P^0(\cdot, f_0; \chi_0) - P^0(\cdot, f_1; \chi_1) > 0$. A similar calculation for the difference in slopes of the expected dividend finds that the sufficient condition in this case is $\mathbb{E}(\theta|z) > \tilde{F}(z)$, which completes the proof. ■

Proof of Proposition 6. It is immediate from Proposition 2, since it is a special case of the general model of section 3. The particular condition for price invertibility follows from the discussion in the text. ■

Appendix B: The model without exogenous dividends

need to revise

In this appendix, we discuss the model without exogenous dividends: $\rho = 0$. In this case, market prices aggregate information about only the endogenous investment component, while other exogenous dividends are already known to the market, and thus fully reflected in prices. As a result, there is also a feedback in the informativeness of prices: when the firm doesn't invest with probability 1, private signals are uninformative of dividends, and hence so is the price. This feedback can lead to multiple equilibria.

More specifically, two different possibilities arise, depending on whether $\tilde{F}(P) \quad \underline{F}$ or $\tilde{F}(P) > \underline{F}$, i.e. whether there is a positive probability of investment, and fundamentals are partitioned into two sets, one for which the investment probability is zero, and one for which it is positive. If $\tilde{F}(P) \quad \underline{F}$, the firm does not invest, regardless of F, and the expected dividends are zero, regardless of private signals. Therefore, the traders' supply takes the form

$$s(x_i, P) = \begin{cases} 0 \text{ if } P < 0 \\ [0,1] \text{ if } P = 0 \\ 1 \ P > 0 \end{cases} .$$

That is, all traders are just indifferent between holding the asset or not, when $P = 0$. Since $\Phi(u) \in (0,1)$, we never observe a price on the equilibrium path for which either all traders sell or don't sell their share. This implies that $P = 0$ clears the market provided that $\tilde{F}(0) \quad \underline{F}$, whereas any price $P \neq 0$ is ruled out as a market-clearing price with $\tilde{F}(P) \quad \underline{F}$.

If instead $\tilde{F}(P) > \underline{F}$, there is a positive probability of investment, which makes the dividends uncertain, and the private signals informative. In this case, the same analysis as in Proposition 2 applies, with $\rho = 0$. A trader with private signal x keeps the share, whenever $x \geq \hat{x}(P)$, where $\hat{x}(P)$ is implicitly defined by

$$P = G\left(\tilde{F}(P)\right) \int \theta d\Phi \left(\sqrt{\lambda + \beta + \beta\delta} \left(\theta - \frac{\lambda\mu + (\beta + \beta\delta)\,\hat{x}(P)}{\lambda + \beta + \beta\delta} \right) \right) - \int_{\underline{F}}^{\tilde{F}(P)} f dG(f),$$

and the market clears, if $\hat{x}(P) = z$. Now, given an investment threshold function $\tilde{F}(z) = \tilde{F}(P(z))$, we have the following equilibrium characterization:

Proposition 7 *Define* $\tilde{P}(Z)$

$$\begin{aligned} \tilde{P}(Z) &= \int_{\underline{F}}^{\tilde{F}(z)} G(f)\, df + (\gamma - 1)(Z - \mu)\, G\left(\tilde{F}(z)\right), \\ \textit{where } Z &= \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}. \end{aligned}$$

If $P(Z)$ *characterizes an equilibrium, then: (i)* $P(Z) \in \left\{0, \tilde{P}(Z)\right\}$ *for all* Z*, and* $P(Z) \neq 0$ *only if* $\tilde{F}(Z) > \underline{F}$*, whereas* $P(Z) = 0$ *only if* $\tilde{F}(Z) \quad \underline{F}$*, and (ii)* $P(Z)$ *is invertible over* $\mathcal{S}$*, where* $\mathcal{S} = \{Z \in \mathbb{R} : P(Z) \neq 0\}$.

Proof. The characterization of $\tilde{P}(Z)$ follows from the same arguments as above, and the supplementary conditions then take into account that P must be invertible when it's different from 0, and $P(Z) = 0$ must occur only if the firm doesn't invest, with probability 1. ■

Next, we characterize firm's optimal behavior. As before, we define the manger's objective as a weighted average of dividend value and price (with weights $1-\alpha$ and α, respectively, and find that, as before, that conditional on Z, the investment threshold is

$$\tilde{F}(Z) = \mu + k(Z-\mu),$$

and the resulting maximized objective is

$$(1-\alpha)V(Z) + \alpha P(Z) = \int_{\underline{F}}^{\ +k(Z\quad)} G(f)\,df.$$

This function is strictly positive whenever $\mu + k(Z-\mu) > \underline{F}$, and 0 otherwise. The resulting price function $\tilde{P}(Z)$ on the other hand is

$$P(Z) = \int_{\underline{F}}^{\ +k(Z\quad)} G(f)\,df + (\gamma - k)(Z-\mu)(G(\mu + k(Z-\mu))).$$

In addition, when $P(Z) = 0$, it must be optimal for the firm not to invest, which requires that $\mathbb{E}(\mu + k(Z-\mu) | Z \in \mathbb{R}\setminus\mathcal{S}) \quad \underline{F}$. Summarizing, the equilibrium imposes the following requirements on the set $\mathcal{S}$, for which the firm invests with positive probability:

Proposition 8 *Any equilibrium is characterized by a price function* $P(Z) \in \left\{0, \tilde{P}(Z)\right\}$*, and a set* $\mathcal{S} \subseteq \mathbb{R}$*, such that*

$$P(Z) = \begin{cases} 0 \text{ if } Z \in \mathbb{R}\setminus\mathcal{S} \\ \tilde{P}(Z) \text{ if } Z \in \mathcal{S} \end{cases}$$

firm's investment threshold satisfies

$$\tilde{F}(Z) = \begin{cases} \underline{F} \text{ if } Z \in \mathbb{R}\setminus\mathcal{S} \\ \mu + k(Z-\mu) \text{ if } Z \in \mathcal{S} \end{cases}$$

The set $\mathcal{S}$ *satisfies the following necessary and sufficient conditions for equilibrium: (i)* $\tilde{P}(Z)$ *for* $Z \in \mathcal{S}$*, (ii)* $\mathbb{E}(\mu + k(Z-\mu) | Z \in \mathbb{R}\setminus\mathcal{S}) \quad \underline{F}$*, and (iii)* $Z \in \mathbb{R}\setminus\mathcal{S}$ *if* $\mu + k(Z-\mu) \quad \underline{F}$*.*

Proof. If $P(Z)$ is an equilibrium price function, $P(Z) \in \left\{0, \tilde{P}(Z)\right\}$ for all Z, where $\tilde{P}(Z)$ is defined as above. $P(Z)$ must be invertible over the range in which $P(z) \neq 0$, i.e. the set $\mathcal{S}$ for which $P(z) = \tilde{P}(Z)$. When $P = 0$, the firm must find it optimal not to invest, requiring $\mathbb{E}(\mu + k(Z-\mu) | Z \in \mathbb{R}\setminus\mathcal{S}) \quad \underline{F}$.

And finally, whenever $\mu + k(Z - \mu) \quad \underline{F}$, the firm would find it optimal not to invest, if it knows Z, requiring $\tilde{P}(Z) = 0$, so that necessarily $z \in \mathbb{R} \setminus \mathcal{S}$. Moreover, by construction, for any set $\mathcal{S} = \{Z \in \mathbb{R} : P(Z)\, b \neq 0\}$ that satisfy these conditions, the corresponding price function and threshold functions for traders and the firm constitute an equilibrium. ■

Thus, without exogenous dividends, the realizations of Z are divided into two sets $\mathcal{S}$, and $\mathbb{R} \setminus \mathcal{S}$, such that when $z \in \mathbb{R} \setminus \mathcal{S}$, the price is zero and the firm does not invest, and when $Z \in \mathcal{S}$, the firm invests with positive probability, the price differs from zero, and aggregates private information. To satisfy the equilibrium conditions, $P(Z)$ must be invertible over the range with positive probability of investment $\mathcal{S}$, and the firm does not find it optimal to invest, when $Z \in \mathbb{R} \setminus \mathcal{S}$. Moreover, when $\mu + k(Z - \mu) \quad \underline{F}$, the information the market price conveys is so negative that the firm would find it optimal not to invest. In these cases it is also necessary that $z \in \mathbb{R} \setminus \mathcal{S}$.

We next examine what these restrictions imply for the equilibrium set. Taking derivatives, we have

$$\tilde{P}^{\prime}(Z) = \gamma G(\mu + k(Z - \mu)) + (\gamma - k)\, k(Z - \mu)\, g(\mu + k(Z - \mu)).$$

Thus, when $\underline{F} \geq \mu$, $\tilde{P}^{\prime}(Z) > 0$ for all $\mu + k(Z - \mu) \geq \underline{F}$, so $\tilde{P}(Z)$ is invertible over its entire range, and therefore any subset $\mathcal{S}$ of $\{\mu + k(Z - \mu) \geq \underline{F}\}$. Moreover, for any $\mathcal{S} \subseteq \{Z : \mu + k(Z - \mu) \geq \underline{F}\}$, $\mathbb{E}(\mu + k(Z - \mu) | Z \in \mathcal{S}) \geq \underline{F} \geq \mu$, and therefore $\mathbb{E}(\mu + k(Z - \mu) | Z \in \mathbb{R} \setminus \mathcal{S}) \quad \mu \quad \underline{F}$, implying that whenever $P = 0$ is observed, it is indeed optimal not to invest, regardless of F. Thus, when $\underline{F} \geq \mu$, the additional requirements are automatically satisfied, and impose no additional restrictions: any set $\mathcal{S}$ that includes $\{Z : \mu + k(Z - \mu) \quad \underline{F}\}$ characterizes an equilibrium. Notice that this includes a 'least informative' equilibrium, in which $\mathcal{S} = \varnothing$, corresponding to an equilibrium in which the firm never invests, and a 'most informative' equilibrium, in which $\mathcal{S} = \{Z : \mu + k(Z - \mu) \quad \underline{F}\}$, and whenever $\mu + k(Z - \mu) \geq \underline{F}$, the equilibrium price is positive, and there is a positive chance of investment. The condition that $\underline{F} \geq \mu$ says that the prior mean is sufficiently pessimistic so that the firm would not want to invest if it didn't receive any further information about the profitability of the investment.

When instead $\underline{F} < \mu$, the additional restrictions have some bite: there now exists $\underline{Z} < \mu$ at which $\tilde{P}(Z)$ reaches a minimum; for Z, such that $\mu + k(Z - \mu) \in (\underline{F}, \mu + k(\underline{Z} - \mu))$, $\tilde{P}^{\prime}(Z) < 0$, while for $Z > \underline{Z}$, $\tilde{P}^{\prime}(Z) > 0$.[19] In addition, there exists $\bar{Z} < \infty$ such that $\mathbb{E}\left(\mu + k(Z - \mu) | Z \quad \bar{Z}\right) = \underline{F}$.

Thus both the equilibrium with $P(Z) = \tilde{P}(Z)$ whenever $\mu + k(Z - \mu) > \underline{F}$, and the equilibrium in which $P(Z) = 0$ for all Z fail to exist; the former because the resulting price function is not invertible, and the latter because the firm would want to invest with positive probability if it obtained no additional information. The invertibility requirement then implies that $\mathcal{S}$ must include some of the domain of Z over which $\tilde{P}(Z)$ is not invertible, so as to restore

[19]To avoid some technicalities, we assume that $G(F)/g(F)$ is non-decreasing and that $\gamma > k$. When $\gamma = k$, $\mu + k(\underline{Z} - \mu) = \underline{F}$, and non-invertibility is not an issue.

invertibility over $\mathbb{R}\diagdown\mathcal{S}$. For example, this is satisfied by any set $\mathcal{S}=\{Z \quad Z^0\}$, for any $Z^0 \geq \underline{Z}$. In addition, we need that conditional on $P(Z)=0$, it is optimal not to invest. For any set $\mathcal{S}=\{Z \quad Z^0\}$, this optimality condition is satisfied whenever $Z^0 \quad \bar{Z}$, and if $\underline{Z}<\bar{Z}$, we have thus constructed a continuum of monotone equilibria for any set $\mathcal{S}=\{Z \quad Z^0\}$, for $Z^0 \in \left[\underline{Z},\bar{Z}\right]$. As $\underline{Z}$ becomes larger, or $\bar{Z}$ smaller, these additional requirements on $\mathcal{S}$ become more and more restrictive, until the point where $\underline{Z}=\bar{Z}$ (at which point there is a unique threshold equilibrium). When instead $\underline{Z} \geq \bar{Z}$, the requirements of invertibility at $P \neq 0$ and no investment at $P=0$ become so restrictive that they are mutually contradictory, and hence an equilibrium fails to exist.

Depending on parameters, any of these three scenarios is possible: Non-existence occurs whenever the distribution of F is unbounded below and has sufficiently thin tails, or $\underline{F}$ is finite, but very low. The intermediate case in which $\underline{F}<\underline{Z}<\bar{Z}<\infty$ occurs when $\underline{F}$ is sufficiently high, but lower than μ. These statements are summarized in the following proposition:

Proposition 9 (Equilibrium existence and multiplicity) *Define*

$$\begin{aligned}
\underline{Z} &= \inf\left\{Z:\mu+k\left(Z-\mu\right)\geq\underline{F}:\tilde{P}^0\left(Z^0\right)>0 \text{ for all } Z^0>Z\right\} \\
\text{and } \bar{Z} &= \sup\left\{Z:\mu+k\left(Z-\mu\right)\geq\underline{F}:\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z \quad \bar{Z}\right) \quad \underline{F}\right\}.
\end{aligned}$$

(i) if $\underline{F}\geq\mu$, then $\mu+k\left(\underline{Z}-\mu\right)=\underline{F}$, and $\bar{Z}=+\infty$, and any selection $P(Z)\in\left\{0,\tilde{P}(Z)\right\}$ characterizes an equilibrium.

(ii) if $\underline{F}<\mu$, and $\underline{Z} \quad \bar{Z}$, then for any $Z^0\in\left[\underline{Z},\bar{Z}\right]$, there exists an equilibrium, in which $P(Z)=0$ if and only if $Z \quad Z^0$.

(iii) if $\underline{Z}>\bar{Z}$, an equilibrium does not exist.

Proof. (i) and (ii) are proved in the text.

(iii) If $\underline{Z}>\bar{Z}$, we show that the two equilibrium conditions on $\mathcal{S}$ are mutually contradictory, i.e. $\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z\in\mathbb{R}\diagdown\mathcal{S}\right)>\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z \quad \bar{Z}\right)=\underline{F}$ for any $\mathcal{S}$, such that $\tilde{P}(z)$ is invertible over $\mathcal{S}$. To show this consider an arbitrary $\mathcal{S}$, such that $\tilde{P}(z)$ is invertible over $\mathcal{S}$. Let $\bar{\mathcal{S}}=\left(\mathbb{R}\diagdown\mathcal{S}\right)\cap\left\{Z>\bar{Z}\right\}$ and $\underline{\mathcal{S}}=\mathcal{S}\cap\left\{Z \quad \bar{Z}\right\}$, and notice that

$$\begin{aligned}
&\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z\in\mathbb{R}\diagdown\mathcal{S}\right)=\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z \quad \bar{Z}\right) \\
&+\int_{\bar{\mathcal{S}}}\left(\mu+k\left(Z-\mu\right)-\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z \quad \bar{Z}\right)\right)\psi(Z)\,dZ \\
&-\int_{\underline{\mathcal{S}}}\left(\mu+k\left(Z-\mu\right)-\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z \quad \bar{Z}\right)\right)\psi(Z)\,dZ,
\end{aligned}$$

where we use $\Psi(\cdot)$ to denote the prior distribution of Z, which is normal with mean μ (and $\psi(\cdot)$ the corresponding density).

Now, for each $Z\in\underline{\mathcal{S}}$, there exists $Z^0(Z)\in\bar{\mathcal{S}}$ such that $\tilde{P}(Z)=\tilde{P}\left(Z^0(Z)\right)$. Moreover, $\mu>Z^0(Z)>Z$, so, $\psi(Z)<\psi\left(Z^0(Z)\right)$ and

$$\mu+k\left(Z-\mu\right) \quad \mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z \quad \bar{Z}\right)<\mu+k\left(Z^0(Z)-\mu\right),$$

where the first inequality uses the fact that $\mu + k(Z - \mu) \geq \underline{F}$ for $Z \in \underline{\mathcal{S}}$, and the second the fact that $Z^0(Z) > \bar{Z}$. But then,

$$
\begin{aligned}
& \left(\mu + k(Z - \mu) - \mathbb{E}\left(\mu + k(Z - \mu) \,|Z \quad \bar{Z}\right)\right) \psi(Z) \\
< \quad & \left(\mu + k\left(Z^0(Z) - \mu\right) - \mathbb{E}\left(\mu + k(Z - \mu) \,|Z \quad \bar{Z}\right)\right) \psi\left(Z^0(Z)\right),
\end{aligned}
$$

or

$$
\begin{aligned}
& \int_{\underline{S}} \left(\mu + k(Z - \mu) - \mathbb{E}\left(\mu + k(Z - \mu) \,|Z \quad \bar{Z}\right)\right) \psi(Z)\, dZ \\
< \quad & \int_{\underline{S}} \left(\mu + k\left(Z^0(Z) - \mu\right) - \mathbb{E}\left(\mu + k(Z - \mu) \,|Z \quad \bar{Z}\right)\right) \psi\left(Z^0(Z)\right) dZ \\
& \int_{\bar{S}} \left(\mu + k(Z - \mu) - \mathbb{E}\left(\mu + k(Z - \mu) \,|Z \quad \bar{Z}\right)\right) \psi(Z)\, dZ
\end{aligned}
$$

which implies$\mathbb{E}\left(\mu + k(Z - \mu) \,|Z \in \mathbb{R} \setminus \mathcal{S}\right) > \mathbb{E}\left(\mu + k(Z - \mu) \,|Z \quad \bar{Z}\right) = \underline{F}$ ■

The model without exogenous dividend thus admits a rich set of possible equilibrium outcomes, including the potential for multiplicity, indeterminacy and non-existence. Multiplicity and indeterminacy are a consequence of the feedback on the signals' informativeness: when the firm is certain not to invest, private signals are worthless and traders are indifferent at a price of $P = 0$. Since $P = 0$ is then at most partially revealing of Z, it is possible to sustain (almost) arbitrary selections from the correspondence $\left\{0, \tilde{P}(Z)\right\}$ as equilibrium prices.

Assuming that the exogenous dividend $\pi(\theta)$, with $\pi^0(\theta)$ strictly positive introduces an exogenous motive for information aggregation and hence trading based on private signals. This breaks the feedback cycle that is responsible for multiplicity and indeterminacy in our benchmark model.

Non-existence on the other hand is linked to the non-invertibility of the price function, and hence the non-monotonicity of $\tilde{P}(\cdot)$. In our benchmark model, this non-invertibility occurs with unbounded support of F, and a thin-tail assumption on the distribution $G(\cdot)$. By removing the possibility of no information aggregation and no investment at a price of 0 from the set of possible market-clearing outcomes, the exogenous dividend $\pi(\theta)$ makes non-invertibility even more of an issue, unless $\pi^0(\theta)$ is bounded sufficiently far away from zero so that the resulting price function is guaranteed to be monotone.

Appendix C: The model with private manager's signal about θ

We have defined (y, z)-contingent manager's (Z) and marginal trader's $(\tilde{Z})$ beliefs by

$$
Z(y, z) \equiv \frac{\lambda\mu + \kappa y + \beta\delta z}{\lambda + \kappa + \beta\delta}, \quad \hat{Z}(y, z) \equiv \frac{\lambda\mu + \kappa y + \beta(1 + \delta)z}{\lambda + \kappa + \beta(1 + \delta)}.
$$

The price and expected dividend for an investment threshold $\tilde{F}(Z)$ are:

$$
\begin{aligned}
V(z) &= \int_y (\rho Z + \int_{\underline{F}}^{\tilde{F}} (Z - F) dG(F)) d\Phi(y|z) \equiv \int_y v(y,z) d\Phi(y|z) \\
P(z) &= \int_y (\rho \hat{Z} + \int_{\underline{F}}^{\tilde{F}} (\hat{Z} - F) dG(F)) d\Phi(y|x = z, z) \equiv \int_y p(y,z) d\Phi(y|x = z, z)
\end{aligned}
$$

The manager's pointwise maximization of the objective $v + \alpha \cdot w$ leads to the f.o.c. $\tilde{F} = (1-\alpha)Z + \alpha\hat{Z}$. In what follows we redefine the state variable by $\tilde{F}$, as computing the distribution of $\tilde{F}$ will then allow to analyze the unconditional price, dividend and information aggregation wedge. With some algebra we can derive the following relations:

$$
\begin{aligned}
Z - z &= \frac{\lambda + \kappa + \beta(1+\delta)}{\lambda + \kappa + (1-\alpha)\beta + \beta\delta}(\tilde{F} - z), \\
\hat{Z} - z &= \frac{\lambda + \kappa + \beta\delta}{\lambda + \kappa + (1-\alpha)\beta + \beta\delta}(\tilde{F} - z),
\end{aligned}
$$

which we can use to rewrite $v(y,z)$ and $p(y,z)$ in terms of $\tilde{F}$ and z;

$$
\begin{aligned}
v(\tilde{F}, z) &= \rho Z + \int_{\underline{F}}^{\tilde{F}} G(F)\, dF + G(\tilde{F})(Z - z - \tilde{F} + z) \\
&= -\rho(\frac{\lambda + \kappa + \beta(1+\delta)}{\lambda + \kappa + (1-\alpha)\beta + \beta\delta}(z - \tilde{F}) + z) \\
&\quad + \int_{\underline{F}}^{\tilde{F}} G(F)\, dF - G(\tilde{F})\frac{\alpha\beta}{\lambda + \kappa + (1-\alpha)\beta + \beta\delta}(z - \tilde{F}), \\
p(\tilde{F}, z) &= -\rho(\frac{\lambda + \kappa + \beta\delta)}{\lambda + \kappa + (1-\alpha)\beta + \beta\delta}(z - \tilde{F}) + z) \\
&\quad + \int_{\underline{F}}^{\tilde{F}} G(F)\, dF + G(\tilde{F})\frac{(1-\alpha)\beta}{\lambda + \kappa + (1-\alpha)\beta + \beta\delta}(z - \tilde{F}).
\end{aligned}
$$

Now we can integrate over z to write

$$
\begin{aligned}
V(\tilde{F}) &= -\rho(\frac{\lambda+\kappa+\beta(1+\delta)}{\lambda+\kappa+(1-\alpha)\beta+\beta\delta}(\mathbb{E}(z|\tilde{F})-\tilde{F})+\mathbb{E}(z|\tilde{F})) \\
&\quad + \int_{\underline{F}}^{\tilde{F}} G\,(F)\,dF - G(\tilde{F})\frac{\alpha\beta}{\lambda+\kappa+(1-\alpha)\beta+\beta\delta}(\mathbb{E}(z|\tilde{F})-\tilde{F}), \\
P(\tilde{F}) &= -\rho(\frac{\lambda+\kappa+\beta\delta}{\lambda+\kappa+(1-\alpha)\beta+\beta\delta}(\mathbb{E}(z|\tilde{F})-\tilde{F})+\mathbb{E}(z|\tilde{F})) \\
&\quad + \int_{\underline{F}}^{\tilde{F}} G\,(F)\,dF + G(\tilde{F})\frac{(1-\alpha)\beta}{\lambda+\kappa+(1-\alpha)\beta+\beta\delta}(\mathbb{E}(z|\tilde{F})-\tilde{F}).
\end{aligned}
$$

so we just need to to compute $\mathbb{E}(z|\tilde{F})-\tilde{F}$ and the variance of $\tilde{F}$. To do so, note that we can write

$$\tilde{F}-\mu = \gamma_y(y-\theta)+\gamma_z(z-\theta)+\gamma\ \ (\theta-\mu)$$

where

$$
\begin{aligned}
\gamma_y &= (1-\alpha)\frac{\kappa}{\lambda+\kappa+\beta\delta}+\alpha\frac{\kappa}{\lambda+\kappa+\beta(1+\delta)}, \\
\gamma_z &= (1-\alpha)\frac{\beta\delta}{\lambda+\kappa+\beta\delta}+\alpha\frac{\beta+\beta\delta}{\lambda+\kappa+\beta(1+\delta)}, \\
\gamma &= (1-\alpha)\frac{\kappa+\beta\delta}{\lambda+\kappa+\beta\delta}+\alpha\frac{\kappa+\beta(1+\delta)}{\lambda+\kappa+\beta(1+\delta)}.
\end{aligned}
$$

Using the joint normal distribution between $\tilde{F}-\mu$ and $Z-\mu$, we can write (using Bayes rule)

$$\mathbb{E}(z|\tilde{F})-\tilde{F} = \frac{\gamma_z/(\beta\delta)+\gamma\ \ /\lambda-\sigma_F^2}{\sigma_F^2}(\tilde{F}-\mu)$$

where $\sigma_F^2 = \gamma_y^2/\kappa+\gamma_z^2/(\beta\delta)+\gamma^2/\lambda$. We can now write the wedge $W(\tilde{F})$ as

$$W(\tilde{F}) \equiv P(\tilde{F})-V(\tilde{F}) = \quad (G(\tilde{F})+\rho)(\tilde{F}-\mu)\frac{}{{}^2_F}\frac{{}_z=(\quad)+\ {}_\theta=\qquad {}^2_F}{+\ +(1\quad)\ +}.$$

which yields an unconditional wedge of

$$\mathbb{E}(W) = \int_{\tilde{F}}(W(\tilde{F}))d\Phi(\tilde{F}) = [\frac{(\gamma_z/(\beta\delta)+\gamma\ \ /\lambda-\sigma_{\tilde{F}}^2)\beta}{\sigma_{\tilde{F}}^2\cdot(\lambda+\kappa+(1-\alpha)\beta+\beta\delta)}]\cdot Cov(G(\tilde{F}),\tilde{F}) > 0.$$