

Conglomeration with Bankruptcy Costs: Separate or Joint Financing?*

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Abstract

The paper analyzes the determinants of the optimal scope of incorporation in the presence of bankruptcy costs. Bankruptcy costs alone generate a non-trivial tradeoff between the benefit of *coinsurance* and the cost of *risk contamination* associated to financing projects jointly through debt. This tradeoff is characterized for projects with binary returns, depending on the distributional characteristics of returns (mean, variability, skewness, heterogeneity, correlation, and number of projects), the structure of the bankruptcy cost, and the tax rate advantage of debt relative to equity. Our predictions are broadly consistent with existing empirical evidence on conglomerate mergers, spin-offs, project finance, and securitization.

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1 Introduction

Consider a firm that needs to finance a number of risky projects from a competitive credit market. The firm has the choice of financing the projects either separately with a number of independent loans or jointly with a single loan. With either financing regime, part of the returns are lost to bankruptcy costs when creditors do not obtain full repayment. When is joint financing more profitable than separate financing? By answering this question we can determine the profitability of:

- mergers, which combine cash flows and the financing of otherwise separate corporations;
- holding companies, which protect the assets of individual subsidiaries from creditors' claims against one of the subsidiaries;
- spin-offs, whereby divisions or subsidiaries are set up as independent corporations;
- project finance and securitization, according to which projects are financed through separate special purpose vehicles.

At least since Lewellen (1971), conventional wisdom in corporate finance has largely settled on the view that bankruptcy costs always generate positive financial synergies, so that joint financing is more profitable than separate financing. According to this view, conglomeration brings about a reduction in the probability of bankruptcy by allowing a firm to use the proceeds of a successful project to save an unsuccessful one which would have otherwise failed. By aggregating imperfectly correlated cash flows, the argument goes, joint financing should reduce expected bankruptcy costs and increase borrowing capacity. As aptly summarized by Brealey, Myers, and Allen's (2006, page 880) textbook, "merging decreases the probability of financial distress, other things equal. If it allows increased borrowing, and increased value from the interest tax shields, there can be a net gain to the merger."

In this paper, we amend this conventional view by revisiting the purely *financial* effects of conglomeration. We argue that bankruptcy costs alone create a non-trivial tradeoff for conglomeration, even abstracting from tax considerations and changes in borrowing capacity. While the literature has mostly focused on the coinsurance benefits of conglomeration, we characterize the risk contamination losses that result when the logic of the conventional argument is turned on its head. Through risk contamination, the failure of one project can drag down another successful project that is financed jointly, thus increasing the probability of bankruptcy. The additional bankruptcy costs associated with the project that is driven

into bankruptcy would have been saved if the projects had been financed separately. Precisely because the dragged down project is healthy, risk contamination can be substantial.

To illustrate the value of breaking up a conglomerate to avoid risk contamination losses, consider the spin-off of R. J. Reynolds Tobacco Company from Nabisco's food business in 1999. As Steven F. Goldstone, chairman and chief executive officer of RJR Nabisco, commented in the official news release, this sale "paves the way for us to separate the domestic tobacco business from the rest of our organization on a sound and prudent financial basis." Similar considerations led many commentators to favor a split of UBS during the recent financial crisis, as the troubled investment banking unit was dragging down the highly profitable private banking business. As suggested by the Financial Times, UBS benefited from coinsurance gains ("the main reason its investment bank had access to such cheap funding during the boom that led to such huge losses was because UBS had a high credit rating, supported by its private banking business") but ended up suffering the effects of risk-contamination ("the losses have prompted clients to withdraw cash from UBS's core wealth management business").¹

To best understand the determinants of the tradeoff between coinsurance and risk contamination, we focus on the simplest setting in which each project has two possible realizations of returns, either low or high. In the baseline model we constrain financing to be obtained only through standard debt. The low return realization is insufficient to cover the initial investment outlay, thus generating the possibility of bankruptcy. Separate financing involves a number of nonrecourse loans. This means that when the repayment obligation on one loan is not met, creditors do not have access to the returns of other projects. Through joint financing, instead, the returns of multiple projects are conglomerated, so that bankruptcy costs are only incurred when the sum of the returns of the projects falls below the overall repayment obligation required by the creditors.

To develop an initial intuition, consider a setting with two ex ante identical and independent projects, as in the baseline specification of our model. If the projects are financed separately, each loan defaults when the corresponding project yields a low return. If, instead, the projects are financed jointly, default occurs if the per-project repayment obligation is higher than the average return of the two projects. Similar to the case of separate financing, default occurs if the returns of both projects are low (bottom-left realization of the joint

¹See "UBS does not have luxury of time before it splits up," Financial Times, March 17, 2008, and "Integration loses its attraction," Financial Times, August 13, 2008.

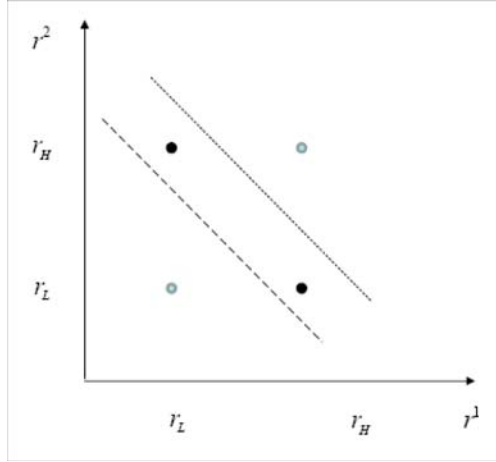


Figure 1: **Joint distribution of returns.** Each project $i = 1, 2$ yields an independent random return r^i with a binary distribution. The return is either low, $r^i = r_L > 0$, with probability $1 - p_i$, or high, $r^i = r_H > r_L$, with probability p_i .

distribution of returns in Figure 1) and does not occur if the returns of both projects are high (top-right realization). The key to the comparison with separate financing is whether or not the required repayment obligation can be met when one project yields a low return and the other project yields a high return, as illustrated by the top-left and bottom-right realizations in Figure 1. In turn, the repayment obligation is endogenously determined and depends on the financing regime. In either regime, competition forces creditors to set the repayment obligation at a level that allows the firm to obtain the projects' present value net of the expected bankruptcy costs.

There are two scenarios. First, suppose that the repayment obligation is below the average of the high and the low return, as illustrated by the dashed diagonal line in the figure. In this case, the probability of bankruptcy is reduced with joint financing. Ex post, a low-return project, which would have defaulted if it had been financed separately, is saved if the other project yields a high return. Ex ante, the two projects coinsure each other and the expected inefficiency associated with bankruptcy is reduced. A higher probability of full repayment forces creditors to reduce the repayment rates below the levels required under separate financing. This coinsurance effect drives the classic logic of “good” conglomeration stressed by Lewellen (1971).

This result is reversed if the per-project repayment obligation is above the average of the high and the low return, as illustrated by the dotted diagonal line in the figure. In this second

scenario, the probability of bankruptcy is actually higher under joint financing! Ex post, a high-return project, which would have stayed afloat if it had been financed separately, is now dragged into bankruptcy when the other project has a low return. Ex ante, projects risk-contaminate each other and the expected inefficiency associated with bankruptcy is increased with joint financing. If the bankruptcy recovery rate is low, competing creditors are forced to increase repayment rates above the level that results under separate financing because the loan will be repaid in full less often with joint financing. In this case, conglomeration is “bad” due to risk contamination.

The thrust of our analysis consists in deriving conditions on the exogenous parameters for good and bad conglomeration to arise. To this end, we first solve for the equilibrium repayment obligations that result in the two financing regimes, and then determine the region of parameters for which the borrower finds separate financing more profitable than joint financing. In the context of the baseline model with two identical and independent projects, we illustrate that separate financing can be optimal for empirically plausible parameter values and derive a number of testable comparative statics predictions, such as the following:

- A reduction in the bankruptcy recovery rate decreases the profitability of joint financing. Given that the amount available to creditors following bankruptcy is lower when bankruptcy costs are higher, the repayment obligation associated with joint financing increases with the level of bankruptcy costs. It is then more difficult for the repayment obligation to be below the average of the high and the low return. Thus the profitability of joint financing is reduced. Consistent with this theoretical prediction, Rossi and Volpin (2004) show that improvements in judicial efficiency and creditor rights significantly increase M&A activity, while Subramanian, Tung, and Wang (2009) find that project finance is more likely than corporate finance in countries with less efficient bankruptcy procedures and weaker creditor rights.
- Joint financing is also less profitable when the projects are riskier, consistent with project finance being more widespread in riskier countries, as shown empirically by Kleimeier and Megginson (2000) among others.
- A mean-preserving increase in the negative skewness of the distribution of returns reduces the attractiveness of joint financing. This result explains why projects with negatively skewed returns, due for example to expropriation risk, are likely to be financed on a project basis (see Esty, 2003).

In the discussion so far we have compared the profitability of separate and joint financing when both financing regimes are feasible. In the paper, we also characterize situations in which it is feasible to finance projects with positive net present value either only separately or only jointly. When the coinsurance effect prevails, joint financing increases the borrowing capacity, resulting in projects that can be financed jointly but cannot be financed separately. When risk contamination prevails, instead, joint financing decreases the borrowing capacity, so that there are projects that can be financed separately but not jointly.

The paper also derives a number of useful managerial implications. We show that a rule of thumb that prescribes adopting the financing regime associated with the lowest interest rate can result in suboptimal decisions. We illustrate situations in which it is more profitable for a firm to finance projects separately, even though joint financing at a lower interest rate is feasible. Indeed, joint financing can have lower repayment rates despite having a higher probability of bankruptcy. When the bankruptcy recovery rate is sufficiently high (or equivalently, bankruptcy costs are sufficiently low), at any given exogenous repayment rate, creditors expect to obtain more with joint financing than with separate financing because bankruptcy occurs more frequently. As a result, competition forces creditors to offer a lower repayment rate to firms that finance projects jointly. This theoretical finding can explain the widespread use of project finance despite the fact that “project debt is often more expensive than corporate debt,” solving one of the “apparently counterintuitive features [of project finance]” (Esty, 2003).

Next, we turn to the case of projects with heterogeneous distributions of returns. Coinsurance and risk contamination may then be present simultaneously when two heterogeneous projects are financed jointly. We characterize situations in which a first project either saves or drags down a second project, depending on whether the first project succeeds or fails. This situation arises when projects differ in their riskiness, measured by second-order stochastic dominance. We show that separate financing is relatively more profitable if the differences in terms of risk profiles of the two projects are higher. This is in line with empirical findings by Gorton and Souleles (2005) that riskier originator banks are more likely to securitize.

As we show, an increase in correlation in project returns tends to favor separate financing. When projects’ return are perfectly negatively correlated, the risk contamination effect is absent and the coinsurance effect is so strong so as to eliminate bankruptcy altogether when projects are financed jointly, as it is also intuitive.

In an extension of the model to allow for multiple projects, we characterize situations in which partial conglomeration of projects into subgroups of intermediate size is optimal. By grouping subset of projects into small conglomerates some of the benefits of coinsurance can be obtained while also containing the costs of risk contamination. Exploiting the logic of the law of large numbers, we also show that full conglomeration results when the number of independent projects becomes arbitrarily large; in the limit, the risk contamination effect vanishes and it becomes optimal to finance jointly all the projects.

We also investigate the robustness of our results to different specifications for the structure of bankruptcy costs. In the baseline model, bankruptcy costs are proportional to the value of the assets under bankruptcy, as is commonly assumed in the theoretical and empirical literature. In a more general model with variable returns to scale in bankruptcy costs, we show that economies of scale (according to which per-project bankruptcy costs are lower when projects are financed jointly) favor joint financing, while diseconomies of scale favor separate financing. Importantly, we demonstrate that our main result on the optimality of separate financing is robust to the introduction of mild economies of scale in bankruptcy costs. We also show that logic of risk contamination also applies when bankruptcy costs depend on the number of projects that go bankrupt rather than on the value of assets under bankruptcy. Actually, we show that separate financing is then optimal for a larger set of parameters because it becomes easier to obtain joint financing, but only at the rate for which intermediate bankruptcy occurs.

In the context of a version of the model with normal returns, we identify a simple sufficient condition for the optimality of separate financing. We also show that our comparative statics predictions on the optimal scope of conglomeration are robust to a continuous specification of returns. Nevertheless, our baseline specification with binary returns allows us to investigate the role played by asymmetries in the distribution of returns as well as to reach a more thorough understanding and characterization of the determinants of the optimal scope of conglomeration.

In the baseline model, we restrict financing to be obtained through debt. Note that debt is the optimal contractual arrangement if returns are privately observed by the borrower and can be verified by creditors only by inducing bankruptcy and incurring the bankruptcy costs. The optimality of debt follows immediately from a classic result derived by Townsend (1979), Diamond (1984), and Gale and Hellwig (1985) in the context of the costly state verification model.

We also extend the analysis to allow financing through equity in addition to debt. As in the tradeoff theory of capital structure, equity saves on bankruptcy costs but is subject to higher taxation. We show that if the incremental tax on equity (or, equivalently, the tax advantage of debt) is sufficiently low, joint financing is inconsequential because bankruptcy can be avoided altogether with joint as well as with separate financing. More interestingly, if the tax differential is intermediate, joint financing is profitable in more cases than in the baseline model because, through the use of equity it becomes possible to obtain a repayment rate that avoids default when one project yields a high return and the other project yields a low return. Finally, if the tax differential is sufficiently high, separate and joint financing are profitable in the same situations as in the baseline model, because then no equity is used in either financing regime. In our model, when financing projects separately is strictly more profitable than financing them jointly, financing is made exclusively through debt. Equity is more expensive and it is only used if it helps to obtain a repayment rate that decreases the probability of default, in which case joint financing is optimal. The exclusive use of debt in separate finance, which arises endogenously in our model, is consistent with the many empirical studies that find that a large proportion of funding in project finance is in the form of debt (see e.g. Kleimeier and Megginson, 2000).

In terms of the theoretical literature in corporate finance, we depart from Modigliani and Miller's (1958) world without financial synergies by introducing bankruptcy costs. In the absence of bankruptcy costs, joint and separate financing are equivalent (see, for example, Levy and Sarnat, 1970). By clarifying the conditions for the value of conglomeration in the presence of bankruptcy costs, this paper contributes to a voluminous literature on the analysis of purely financial motives for mergers. In his discussion to Lewellen (1971), Higgins (1971) notes that joint financing also affects the riskiness of the lender's returns; hence we abstract from risk concerns by assuming risk neutrality. Scott (1977) and Sarig (1985) show that if cash flows can be negative, a firm can exploit the shelter of limited liability by financing projects through separate corporations. In our analysis we explicitly abstract from this limited liability effect, so that the financing regime affects only the payoff of the firm and its creditors, without having any impact on the payoff of third parties.²

²A number of papers (e.g., Higgins and Schall, 1975, and Kim and McConnell, 1977) have analyzed the effect of the current capital structure on merger incentives. These papers noted that, while mergers may increase total firm value, bondholders may gain at the expense of shareholders. We abstract from such a distributional conflict among (cashless) stakeholders, by considering the ex ante choice of corporate structure

Our results are most closely related to three more recent contributions. In the context of a bank lending model, Winton (1999) is the first to uncover the possibility of bad conglomeration. Our contribution is a systematic analysis of the trade-off between coinsurance and risk contamination. We derive a rich set of comparative statics predictions depending on the distributional characteristics of returns, the structure of bankruptcy costs, and the tax rate advantage of debt relative to equity.

Second, Inderst and Müller (2003) (and Faure-Grimaud and Inderst, 2005) analyze financial conglomeration in a two-project version of Bolton and Scharfstein's (1990) dynamic model of debt. In their dynamic setting, financing two projects within the same corporation can reduce the firm's ability to borrow when the firm is able to finance follow-up investments internally without returning to the external capital market. In our model, bad conglomeration arises because of bankruptcy costs, a channel that is absent in Inderst and Müller (2003).

Third, Leland (2007) compares numerically the profitability of separate and joint financing for a borrower who trades off bankruptcy costs with tax shields by adjusting the mix of debt and equity. In the baseline version of our model, we consider fixed-investment projects that must be financed only with debt and thus we explicitly rule out the possibility of increasing leverage and re-optimizing the capital structure. Our analysis uncovers situations in which separate financing is optimal even when the amount borrowed through debt does not depend on whether projects are financed jointly or separately. This result was not envisioned by Leland's (2007) numerical analysis, as we show in a comparable version of the model with continuous returns.³

The paper proceeds as follows. Section 2 formulates the model. Focusing on the baseline version of the model with two identically and independently distributed projects, financed with debt only, and with bankruptcy costs proportional to returns, Section 3 analyzes the conditions setting apart good from bad conglomeration and performs comparative statics with respect to the distribution (mean, variance, and skewness) of returns and the bankruptcy recovery rate. Section 4 turns to the case of projects with heterogeneous returns. Section 5 shows that an increase in the correlation of returns favors separate financing. Gen-

by shareholders and forcing bondholders to compete and therefore obtain no surplus.

³Our results are also very different from those of Shaffer (1994), who studies the effect of joint financing on the probability of joint failure. Instead, we compare the firm's expected payoff when the interest rate is endogenously determined by competition among creditors.

eralizing the optimal conglomeration conditions to a setting with multiple projects, Section 6 characterizes situations in which partial conglomeration is profitable and demonstrates that joint financing is optimal when the number of projects is sufficiently large. Section 7 argues that our results are robust to different specifications of bankruptcy costs, also allowing for some economies of scale. Section 8 extends the analysis to a setting in which equity is also available and is taxed at a higher rate than debt. Section 9 characterizes conditions for bad conglomeration to result when projects' returns are normally distributed. Section 10 concludes with a summary of the main predictions of our theory and a discussion of avenues for future research. The Appendix collects the proofs omitted from the text.

2 Baseline Model

This section formulates the simplest possible model to analyze how multiple projects should be optimally financed in the presence of bankruptcy costs. In the rest of the paper we derive results for special cases or extensions of this baseline scenario.

A risk-neutral firm has access to n projects. Project i requires at $t = 1$ an investment outlay normalized to $I = 1$ and yields at $t = 2$ a random payoff or return r^i with a binary distribution: the return is either low, $r^i = r_L^i > 0$, with probability $1 - p_i$, or high, $r^i = r_H^i > r_L^i$, with probability p_i . Each project has a positive net present value, $(1 - p_i)r_L^i + p_i r_H^i - 1 > 0$. The low return is insufficient to cover the initial investment outlay, $r_L^i < 1$. Returns are possibly correlated across projects. In Section 9 we allow the returns to be normally distributed and show that our main results are robust to continuous distributions.

Before raising external finance, the firm chooses how to group projects into corporations, or equivalently into separate nonrecourse loans. This means that investors in each corporation have access to the returns of all projects in that corporation, but they do not have access to the returns of the projects in the other corporations set up by the firm. Financing for each corporation can be obtained in a competitive credit market. For notational simplicity, we stipulate that the firm seeks financing only when expecting to obtain a strictly positive expected payoff.

Creditors are risk neutral and lend money through standard debt contracts. Without loss of generality we normalize the risk-free interest rate to $r_f = 0$.⁴ Therefore, creditors expect

⁴To see that $I = 1$ and $r_f = 0$ are innocuous normalizations, suppose that the investment outlay is equal to I and that the investors' required interest rate is $r_f > 0$. Denoting the random cash flow by R_i ,

to make zero expected profits. This is equivalent to assuming that each corporation makes a take-it-or-leave-it repayment offer to a single creditor for each loan j , promising to repay r_j^* at $t = 2$ for each unit borrowed at $t = 1$.⁵ Thus r_j^* denotes the promised repayment per project. According to our accounting convention, the corporation's repayment obligation comprises the amount borrowed as well as net interest.⁶

Creditors are repaid in full when the total realized return of the projects pledged is sufficient to cover the promised repayment. If instead the total realized return falls short of the repayment obligation, the corporation defaults and the ownership of the projects' realized returns is transferred to the creditor. Following default, the creditor is only able to recover a fraction $\gamma \in [0, 1]$ of the realized returns r , so that the bankruptcy costs following default are equal to $B(r) = (1 - \gamma)r$. The bankruptcy recovery rate γ measures the efficiency of the bankruptcy process and is industry specific.⁷ In Section 7, we show that our results hold robustly with a more general structure of bankruptcy costs, provided that economies or diseconomies of scale in bankruptcy are not too extreme.

For the baseline specification of the model we restrict external financing to be obtained through debt. Note that debt is the optimal contractual arrangement if we assume that returns are privately observed by the borrower and can be verified by creditors only at a cost. As shown by Gale and Hellwig (1985), in the context of the classic analyses of the costly state verification model (see Townsend, 1979, and Diamond, 1984), the verification of returns can be interpreted as a costly bankruptcy process. In this context, they show that the optimal contract turns out to be the standard debt contract under which returns are observed if and only if the borrower cannot repay the loan in full.

In Section 8, we extend the model to also allow for financing through tax-disadvantaged equity and we show that full debt financing is optimal if taxes are sufficiently high. In the context of that more general model, we characterize how the tradeoff between debt and equity influences the decision of whether projects should be financed separately or jointly.

the project's return in the model can be reinterpreted in terms of percentage gross return for each unit of period-2 equivalent outlay: $r_i = R_i/[I(1 + r_f)]$. Thus without loss of generality we can set $I = 1$ and $r_f = 0$.

⁵Thus, for the case in which each loan (or corporation) is financed by multiple creditors, we implicitly assume that there are no coordination failures across the creditors who syndicate the same loan.

⁶The net interest rate i satisfies $1 + i = r_j^*$ and therefore the repayment obligation can be interpreted as the gross interest rate.

⁷For estimates of bankruptcy costs and other costs of financial distress across industries see, for example, Warner (1977), Weiss (1990), and Korteweg (2007).

Until then we mute the choice between equity and debt and thus focus on the conglomeration decision when debt is the only source of financing that is available.

3 Two Identical and Independent Projects

This section analyzes the simplest possible specification of the model to develop our main insight. The firm has access to two identically and independently distributed projects. Each project i yields a low return $r_L^i \equiv r_L$ with probability $1 - p_i \equiv 1 - p$ and a high return $r_H^i \equiv r_H > r_L$ with probability $p_i \equiv p$.

In Section 3.1 we proceed to examine the conditions for when the borrower is able to finance the two projects separately and jointly. In Section 3.2 we compare the profitability of separate and joint financing, when they are both feasible. In Section 3.3 we illustrate that separate financing can be optimal for empirically plausible parameter values. In Section 3.4 we characterize the effect of conglomeration on the firm's borrowing capacity. In Section 3.5 we derive a set of comparative statics predictions for the occurrence of joint and separate financing. Finally, in Section 3.6 we show that the financing option with the lowest repayment rate is not necessarily optimal.

3.1 Financing Conditions

Consider first the possibility of financing the two projects through two separate nonrecourse loans or, equivalently, through two different limited liability corporations. Given that the two projects are ex ante identical, financing of each project, if possible, takes place at the same rate. Such rate r_i^* must satisfy $r_L < 1 < r_i^* < r_H$, so that there is a positive probability that the loan is not repaid in full. Indeed, the firm would not accept to be financed at a rate above r_H , because this would result in zero payoff for the firm. Also, the rate must be above 1 because at rates at or below 1 the creditor would make negative expected profits (by obtaining a return never above the investment outlay of 1 and strictly below 1 with strictly positive probability) and therefore would not be willing to extend the loan.

Given that the credit market is competitive, creditors must make zero expected profits. Thus the repayment requested by the creditor, r_i^* , is such that the gross expected proceeds, $pr_i^* + \gamma(1 - p)r_L$, are equal to the initial investment outlay 1. As a result, each project can

be financed through a separate loan if and only if

$$r_i^* := \frac{1 - \gamma(1-p)r_L}{p} \leq r_H. \quad (1)$$

The repayment obligation, which is fully paid only in the case of a high return, is equal to the investment outlay, 1, less the expected proceeds from bankruptcy, $\gamma(1-p)r_L$, divided by the probability of staying afloat, p . Intuitively, the creditor needs to recover the expected shortfall in the event of bankruptcy from the event in which the project yields a high return. As a result, $r_i^* > 1$.

Next, consider joint financing of the two projects through a single loan or, equivalently, within the same corporation. Denote by r_m^* the equilibrium repayment obligation per unit of investment, so that $2r_m^*$ is the total repayment promised to the creditor in return for the initial financing of the two projects, $2I = 2$. Two cases need to be distinguished, depending on whether or not the required repayment rate induces bankruptcy in the case with “intermediate returns,” when one project yields a high return while the other project yields a low return.

Suppose first that the equilibrium repayment rate r_m^* is such that $r_L \leq r_m^* \leq \frac{r_H + r_L}{2}$, so that there is no default with intermediate returns. As a result, the probability of default is reduced to $(1-p)^2$. Substituting again in the expected creditor profits, the borrower would only be able to obtain this rate in a competitive market if and only if

$$r_m^* := \frac{1 - \gamma(1-p)^2 r_L}{1 - (1-p)^2} \leq \frac{r_H + r_L}{2}. \quad (2)$$

Suppose now that the equilibrium rate r_m^{**} is such that $\frac{r_H + r_L}{2} \leq r_m^{**} \leq r_H$ and therefore the borrower defaults in the event of a high and a low return. Hence, default occurs with probability $1 - p^2$. In a competitive credit market, this rate can be obtained if and only if

$$r_m^{**} := \frac{1 - \gamma(1-p)(pr_H + r_L)}{p^2} \leq r_H. \quad (3)$$

Since the borrower’s expected profits for a given distribution are decreasing in the equilibrium rate, if both conditions (2) and (3) are satisfied, the borrower prefers rate r_m^* to rate r_m^{**} .⁸ Summarizing the results so far, we have the following proposition.

Proposition 1 (Financing conditions) Two independent and identical projects can be financed separately if and only if condition (1) is satisfied, in which case the equilibrium rate

⁸It is straightforward to show that if $r_m^* > (r_H + r_L)/2$, then $r_m^{**} > (r_H + r_L)/2$. Therefore, if it is not possible to obtain r_m^* , then we can disregard the $r_m^{**} > (r_H + r_L)/2$ constraint.

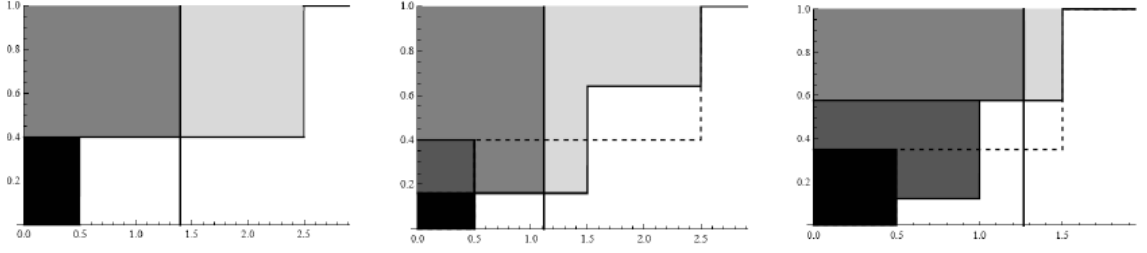


Figure 2: Financing Conditions and Optimal Conglomeration. In all panels, the net expected return for the borrower corresponds to the light gray areas. The per-project creditor's gross expected return is the sum of (i) the medium gray areas and (ii) a fraction γ of the dark gray and black areas. The equilibrium rate r^* (vertical line) is such that the creditor's gross expected return is equal to 1. In panel (a), projects can be financed separately because the creditor's gross expected return at a rate equal to r_H is greater than 1. In panel (b), projects can be financed jointly at a rate below the crossing point because the per-project creditor returns at $(r_H + r_L)/2$ are greater than 1. The reduction in expected bankruptcy costs with joint rather than separate financing (co-insurance effect) is equal to the dark gray area. In panel (c), projects cannot be financed jointly at a rate below the crossing point. The increase in expected bankruptcy costs (risk contamination effect) is equal to the dark gray area. The parameters used in panels (a) and (b) are $p = 0.6$, $r_L = 0.5$, $r_H = 2.5$ and $\gamma = 0.8$ and in (c) $p = 0.65$, $r_L = 0.5$, $r_H = 1.5$ and $\gamma = 0.9$.

is r_i^* . Projects can be financed jointly if and only if conditions (2) or (3) are satisfied. If condition (2) is satisfied, the equilibrium rate is r_m^* , and if it is not satisfied, the rate is r_m^{**} .

Figure 2 depicts how per-project expected returns (equal to the area above the distribution function up to 1) are divided between the borrower and the creditor in the three scenarios described by Proposition 1.⁹ In all panels, the net expected return for the borrower corresponds to the light gray area. The gross expected return of the creditor is the sum of (i) the medium gray area, corresponding to the profits when the project stays afloat, and (ii) a fraction γ of the dark gray and black areas, corresponding to the expected proceeds in case of bankruptcy. The remaining fraction $1 - \gamma$ of the dark gray and black areas is equal to the expected bankruptcy costs.

The equilibrium rate r^* in the three panels is such that the gross expected return of

⁹Joint financing steepens the average return distribution around the center, $(r_H + r_L)/2$ and as a result the distribution of (per-project) returns with separate financing is a mean-preserving spread of the distribution with joint financing.

the creditor is equal to 1. In panel (a), projects can be financed separately because the per-project creditor's gross returns at a rate equal to r_H are greater than 1. In panel (b), projects can be financed jointly at a rate that avoids intermediate bankruptcy because creditor returns at the crossing point, $(r_H + r_L)/2$, are greater than 1. In panel (c), projects can be financed jointly only at a rate that does not avoid intermediate bankruptcy, because creditor returns at $(r_H + r_L)/2$ are lower than 1 and at r_H they are greater than 1.

3.2 Good and Bad Conglomeration

When both separate and joint financing are feasible, which regime is more profitable and thus optimal for the borrower? Obviously, in the absence of bankruptcy costs (i.e., when $\gamma = 1$) the borrower is indifferent between financing the projects separately or jointly. The next proposition states the gains and losses when $\gamma < 1$.

Proposition 2 (Separate v. joint financing) When the borrower can finance two independent and identical projects separately as well as jointly:

- (a) If condition (2) is satisfied, it is optimal to finance the projects jointly to enjoy the coinsurance gains: $p(1 - p)(1 - \gamma)r_L$.
- (b) If condition (2) is not satisfied, it is optimal to finance the projects separately to avoid the risk contamination losses: $p(1 - p)(1 - \gamma)r_H$.

Intuitively, when the borrower obtains a rate that avoids intermediate bankruptcy, the probability of default under joint financing is lower than under separate financing. The low-return project is saved from default when the other project yields a high return, thereby reducing the inefficiency associated with bankruptcy. Per-project expected savings when the projects are financed jointly rather than separately—the “coinsurance effect”—are equal to the probability that the first project yields a low return while the second project yields a high return, $p(1 - p)$, multiplied by the losses avoided due to bankruptcy costs, $(1 - \gamma)r_L$. Graphically, per-project savings due to the coinsurance effect associated with joint financing are represented by a fraction $(1 - \gamma)$ of the dark gray area in Panel (a) of Figure 2.

If, instead, the borrower obtains a joint rate that does not avoid intermediate bankruptcy, a project with low return drags down the other project, increasing the probability of default. Per-project expected losses when projects are financed jointly rather than separately—the “risk contamination effect”—are equal to the probability that the first project yields a high return while the second project yields a low return, $p(1 - p)$, multiplied by the additional

losses in bankruptcy costs incurred, $(1 - \gamma)r_H$. Graphically, the per-project costs due to the risk contamination effect associated with joint financing are represented by a fraction $(1 - \gamma)$ of the darker gray area in Panel (c) of Figure 2.

The key is whether the equilibrium repayment rate for joint financing is below or above the crossing point, $(r_H + r_L)/2$. Notice that the crossing point is not necessarily at the mean. In particular, if $p > 1/2$, so that the distribution is skewed to the left (i.e., returns are negatively skewed), the crossing point is below the mean. As a result, equilibrium rates above the crossing point are consistent with a probability of default below 50%. The resulting default probabilities are then $1 - p$ for separate financing and $1 - p^2$ for joint financing, which for a high enough p may be very low, as illustrated in the following numerical example.

3.3 Illustration

We now illustrate how conglomeration can result in an increase in expected bankruptcy costs for empirically plausible parameter values under the maintained assumption that returns are binary. To this end, we perform a calibration of the four parameters (r_H , r_L , p , and γ) of the baseline version of the model with separate financing. As representative values, we set:

- (i) the probability of bankruptcy at 10% (parametrized by $1 - p = 0.1$) by using Longstaff, Mittal, and Neis (2005) estimate for bonds by BBB rated firms with a five-year horizon;
- (ii) the internal rate of return at 17.5%, so that $[pr_H + (1 - p)\gamma r_L - 1]/1 = 0.175$;
- (iii) the bankruptcy recovery rate at $\gamma = 65\%$ from Alderson and Betker (1995); and
- (iv) bankruptcy costs as a fraction of a firm's value at 10% (so that $(1 - \gamma)r_L/[pr_H + (1 - p)\gamma r_L] = 0.1$), at the low end of Altman's (1984) estimate of 11–17% for bankruptcy costs as a fraction of firm value up to three years before default and more conservative than Korteweg's (forthcoming) estimate of 15–30% of firm value at the point of bankruptcy.

The calibrated values are then $r_H = 1.28$; $r_L = 0.34$; $p = 0.9$; $\gamma = 0.65$, for which it is feasible to finance the projects separately, since $r_i^* = 1.09 < 1.28 = r_H$, as well as jointly, since $r_m^{**} = 1.12 < 1.28 = r_H$, but not at the rate below the crossing point, because $r_m^* = 1.01 > 0.81 = (r_H + r_L)/2$. Thus, separate financing is more profitable than joint financing. In this illustration, the risk contamination effect identified in Proposition 2 is $p(1 - p)(1 - \gamma)r_H = 0.04$, 4% of the investment outlay $I = 1$, corresponding to $0.04/0.175 \approx 23\%$ of the project's net present value.

3.4 Borrowing Capacity

So far we have compared the profitability of separate and joint financing when both financing regimes are feasible. As we have seen in Section 3.1, there are situations in which it is feasible to finance projects with positive net present value either only separately or only jointly. Thus, conglomeration does not necessarily increase the firm's ability to finance projects.

Proposition 3 (Borrowing capacity) Consider two identical and independent projects:

- (a) If condition (2) is satisfied, there are projects that can be financed jointly but not separately.
- (b) If condition (2) is not satisfied, any project that can be financed jointly can be financed separately and there are projects that can only be financed separately.

When the coinsurance effect prevails, there are projects that can be financed jointly but cannot be financed separately. In this first case, conglomeration increases the firm's borrowing capacity, as in Lewellen (1971). However, when risk contamination prevails, joint financing decreases the firm's borrowing capacity, so that there are projects that can be financed separately but not jointly.

3.5 Testable Predictions

We now derive comparative statics predictions with respect to changes in the characteristics of the projects: the recovery rates and the distribution of returns (mean, variability, and skewness). For each attribute, we study whether separate or joint financing is optimal for a larger range of the remaining parameters. At the same time, we contrast our predictions with those from existing theories and discuss how our predictions on joint and separate financing match existing empirical evidence. Support for the desirability of joint financing is given by the occurrence of mergers, especially conglomerate mergers, whereas support for separate financing can be found in spin-offs. As also argued by Leland (2007), structured finance, and in particular asset securitization and project finance, are other means to separate activities from originating or sponsoring organizations to bankruptcy-remote special purpose entities or vehicles (SPE or SPV). From an analytical perspective, these entities have the key features of separate corporations.

Prediction 1 (Bankruptcy costs) For higher bankruptcy costs (lower γ) then (a) both joint and separate financing can be obtained for a smaller region of parameters and (b) joint financing is optimal for a smaller region of the remaining parameters.

Higher bankruptcy costs decrease pledgeable returns, since the recovered returns in case of default are lower (higher discount in the black area). Since bankruptcy costs do not affect the crossing point, $(r_H + r_L)/2$, financing at a rate that avoids intermediate bankruptcy is more difficult and thus joint finance is less likely. To the best of our knowledge, this prediction had not been formulated before.

Still, this prediction is consistent with empirical evidence indicating that merger activity is less likely and project finance is more likely in countries with weaker investor protection. Rossi and Volpin (2004) show that improvements in judicial efficiency and creditor rights significantly increase M&A activity. Comparing the incidence of bank loans for project finance with regular corporate loans for large investments, Subramanian, Tung, and Wang (2009) show that project financing is more frequent in countries with less efficient bankruptcy procedures and weaker creditor rights. Increases in these two measures of investor protection decrease bankruptcy costs and should favor, according to our model, joint financing (mergers) over separate financing (project finance).

Prediction 2 (Probability of high return) For higher probability of a high return (higher p) then (a) both joint and separate financing can be obtained for a larger region of parameters and (b) joint financing is optimal for a larger region of the remaining parameters.

If the probability of a high return increases, it becomes easier to finance projects as well as to finance at a repayment rate that avoids intermediate bankruptcy. Graphically, all the horizontal lines in Figure 2 are then lowered, thereby increasing the expected value (equal to the area above the distribution function) without affecting the crossing point. Financing is eased and, in particular, financing at a rate that avoids intermediate bankruptcy, because the expected return pledgeable to creditors also increases.

This prediction contrasts with that of Inderst and Müller (2003). In their model, it is optimal to keep better projects separate to avoid self-financing and thus commit to return to the capital market. These two contrasting effects might explain the conflicting empirical evidence on the productivity of conglomerate firms. While Maksimovic and Phillips (2002) find that conglomerate firms, for all but the smallest firms in their sample, are less productive

than single-segment firms, Schoar (2002) finds that the productivity of plants in conglomerate firms is higher than in stand-alone firms.¹⁰

During booms, projects might have a higher expectation across-the-board. Our prediction would then be consistent with a large body of empirical evidence that shows that merger activity usually heats up during economic booms and slows down in recessions (see, for example, Maksimovic and Phillips, 2001). Similarly, Cantor and Demsetz (1993) show that off-balance sheet activity (separate financing) grows following a recession.

Prediction 3 (Mean-preserving spread) Consider the effect of a mean-preserving spread in the project's return consisting of an increase in the high return r_H and a reduction in the low return r_L so as to maintain the mean return constant. Then, there exists $\bar{p} < 1/2$ such that the region of parameters for which joint financing is optimal decreases if and only if $p > \bar{p}$.

That is, a mean preserving spread in the distribution of returns favors separate financing as long as the distribution of returns is not too positively skewed. If the distribution is symmetric ($p = 1/2$), a mean preserving spread increases r_H by as much as it reduces r_L . While the crossing point is unaffected, the joint financing rate that avoids intermediate bankruptcy becomes more difficult to obtain because the low return is even lower and the pledgeable returns before the crossing point are lower. In the graph, the black area shrinks. If the distribution of returns is negatively skewed ($p > 1/2$), the crossing point is decreased and it becomes even more difficult to obtain joint financing below the crossing point.¹¹

This prediction is consistent with a similar prediction obtained by Leland (2007). Empirical support can be found in the project finance literature. Kleimeier and Megginson (1999), for example, find that project finance loans are far more likely to be extended to borrowers in riskier countries, particularly countries with higher political and economic risks. They

¹⁰Still, Shoar (2002) finds that conglomerates are less valued than focused firms (the so-called market diversification discount), and argues that the discrepancy can be attributed to conglomerates leaving more rents to workers. A number of papers have also argued that the diversification discount could also be spurious, because of measurement problems and selection biases. For example, Graham, Lemmon, and Wolf (2002) show that acquirers' excess values decline because the business units acquired are already discounted, thus explaining the diversification discount with a self-selection argument. See also Campa and Kedia (2002), Villalonga (2004), and Custodio (2009).

¹¹To maintain the mean constant, a given increase in r_H must be combined with a larger decrease in r_L , resulting in a reduction in the crossing point. Formally, from $r'_H = r_H + \varepsilon$ and $r'_L = r_L - \varepsilon p / (1 - p)$, we have $(r'_H + r'_L) / 2 = (r_H + r_L) / 2 - \varepsilon (2p - 1) / 2 (1 - p)$.

claim that: “As a whole, these geographic lending patterns are consistent with the widely held belief that project finance is a particularly appropriate method of funding projects in relatively risky (non-OECD) countries.”

Prediction 4 (Skewness) Consider the effect of a mean-preserving increase in negative skewness in the project’s return consisting of a reduction in the low return level r_L and an increase in the probability of high return p so as to maintain the mean return constant. Then, it becomes optimal to finance the projects jointly for a smaller region of parameters if and only if the high return level r_H is sufficiently large.

An increase in the negative skewness has two conflicting effects. On the one hand, as r_L decreases the crossing point is reduced and the returns in case of bankruptcy are lower, so that joint financing at the rate that avoids intermediate bankruptcy becomes more difficult. On the other hand, as p increases so as to keep the mean constant, the probability that both projects’ returns are low is reduced, so that it becomes easier to finance the projects at the rate avoiding intermediate bankruptcy. Graphically, the black area (creditor’s expected returns in case of default) becomes less wide and less high and the gray area (creditor’s expected returns if staying afloat) becomes less wide but also higher at the crossing point. If r_H is sufficiently high, the first effect dominates and separation becomes optimal for a larger set of parameters. Indeed, for a given increase in p , one needs a higher reduction in r_L to ensure a constant mean.

We can find support for this prediction in the literature on project finance. For example, Esty (2003) shows that project finance is widespread when it is possible to lose the entire value due to expropriation. This type of risk generates returns with large negative skewness, as opposed to more symmetric risks such those affecting exchange rates, prices, and quantities.

3.6 Managerial Implications

We now show that the financing regime with the lowest repayment rate does not necessarily entail the lowest likelihood of bankruptcy and is thus not necessarily optimal. Thus borrowers would be misguided by choosing the scope of conglomeration by choosing the option with lowest interest rate. The following proposition characterizes when it is more profitable to finance projects separately, even though joint financing is available at a lower rate.

Proposition 4 (Separate financing at higher repayment rate) Separate financing is optimal even though it results in higher repayment rates if and only if (i) condition (3) is satisfied but condition (2) is not satisfied and (ii) $\gamma [pr_H + (1 - p)r_L] > 1$.

Suppose that the borrower has the choice of financing the projects independently and jointly, although only at a rate with intermediate bankruptcy. In this region, the low return project drags down the high-return project and it is more profitable to finance projects separately. Joint financing, however, can result in a lower repayment rate despite having a higher probability of bankruptcy. When the bankruptcy recovery rate is sufficiently high (or equivalently, when bankruptcy costs sufficiently low), at any given exogenous repayment rate creditors expect to obtain more with joint financing because bankruptcy happens more often and they thus offer lower rates. The borrower might feel tempted to finance the projects jointly, but this is suboptimal. Low interest rate associated with joint financing here are deceptively attractive. Even though conglomeration looks good, it is unambiguously bad.

The logic can be further illustrated by Panel (c) of Figure 2. For an (exogenous) repayment rate above the crossing point, $r > (r_H + r_L)/2$, as the one depicted, the creditor's expected returns might be higher if projects are financed jointly in spite of the increased occurrence of bankruptcy. Indeed, with joint financing, the creditor obtains the part of the gray area above the dashed line as well as a fraction γ of the dark gray and black areas. With separate financing, the creditor obtains the gray area, the upper part of the dark gray area and a fraction γ of the black area. Subtracting, the creditor's returns are higher if proceeds from the fraction γ of the dark gray area, $p(1 - p)\gamma r_H$, are greater than the sum of the upper part of the dark gray area and the part of the gray area below the dashed line, $p(1 - p)r$. That is, if and only if $\gamma r_H > r$. If this condition is satisfied by the equilibrium rate with separate financing, $\gamma r_H > r_i^*$ (as in the statement of the proposition), the equilibrium rate with joint financing must be lower, $r_m^{**} < r_i^*$, despite a higher probability of bankruptcy. Intuitively, creditors obtain higher proceeds from a bankrupt high value project than what they can charge for separate loans. So, they are forced by competition to offer lower interest rate. Thus, the borrower obtains a higher expected payoff with separate financing at a higher interest rate.¹²

¹²Note if the distribution of returns was continuous (as in the extension considered in Section 9), the extra losses from higher probability of bankruptcy if the equilibrium rate with joint financing were marginally above the crossing point would always be compensated by the increased proceeds from bankruptcy. Therefore, interest-rate reducing but profit-reducing conglomeration always appears when the project's returns are

4 Heterogeneous Projects

So far, we have assumed that projects are ex ante symmetric. In this section, we extend the baseline setup to allow for heterogeneity across projects. Project i , $i = 1, 2$, yields (independent) returns r_H^i with probability p_i and r_L^i with probability $1 - p_i$. Without loss of generality, assume that $r_H^1 + r_L^2 > r_L^1 + r_H^2$, interchanging the indices if necessary. With heterogeneous projects, four (rather than three) levels of combined returns are possible, adding an extra case to the conditions for joint financing. Now, the possibility arises that default is avoided if project 1 yields a high return and project 2 a low return, whereas default is not avoided if the reverse occurs.

4.1 Financing Conditions

In the case of joint financing, there are now three possible rates and therefore three financing conditions. As in the baseline setup, there exists r'_m such that bankruptcy can be avoided if one project's return is high and the other is low, $r'_m \leq (r_L^1 + r_H^2)/2$.¹³ If projects are heterogenous, there exists r''_m such that bankruptcy can be avoided if project 1's return is high and project 2's is low but not viceversa, $r''_m \leq (r_H^1 + r_L^2)/2$. Finally, as in the baseline case, there exists r'''_m such that bankruptcy cannot be avoided if any of the two projects' return is low, which can be obtained if and only if $r'''_m \leq (r_H^1 + r_H^2)/2$. The dotted and dashed lines in Figure 3 depict the cumulative distribution of returns of two heterogeneous projects, whereas the thick line depicts the distribution of the average returns of the two projects. The three possible types of rates correspond to the three flat parts of the average distribution.

4.2 Good and Bad Conglomeration

Wis(s)-(e)-25(three)]TJ/TT71JJ426.4080TD0Tc()Tj/TT51Ttf54230Tf-0161f0brroc-28irrog(o)4(ersp)6.e

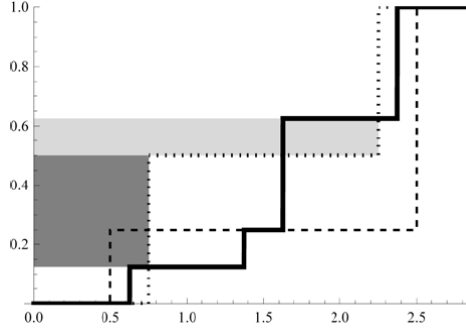


Figure 3: **Heterogeneous Projects:** The dotted and dashed lines depict the cumulative distribution of returns of two heterogeneous projects, whereas the thick line depicts the distribution of the average returns of these two projects. The three possible types of rates in joint financing correspond to the three flat parts of the thick line. Assuming that the second rate (r_m'') is obtained, the reduction in expected bankruptcy costs obtained with joint rather than separate financing (coinsurance effect) is equal to the darker gray area whereas the increase in expected bankruptcy costs obtained with joint rather than separate financing (risk contamination effect) is equal to the lighter gray area. The parameters used in the graph are $p_1 = 0.75$, $p_2 = 0.5$, $r_L^1 = 0.5$, $r_L^2 = 0.75$, $r_H^1 = 2.5$, $r_H^2 = 2.25$.

more favorable intermediate situation, then both coinsurance and contamination effects are present at the same time. On the one hand, project 1, when it yields a high return, saves project 2 when project 2 yields a low return; on the other hand, project 1, when it yields a low return, contaminates project 2 when project 2 yields a high return. The optimality of separate or joint financing depends on whether the gains from coinsurance dominate the losses from risk contamination.

Proposition 5 (Separate v. joint financing with heterogeneous projects) When the borrower can finance two heterogeneous projects separately as well as jointly, there exist r_m' , r_m'' and r_m''' such that

- (a) If $r_m' \leq (r_L^1 + r_H^2)/2$, it is optimal to finance the projects jointly to enjoy the coinsurance gains: $(1 - p_1)p_2(1 - \gamma)r_L^1 + p_1(1 - p_2)(1 - \gamma)r_L^2$.
- (b) If $r_m'' \leq (r_H^1 + r_L^2)/2$ but $r_m' > (r_L^1 + r_H^2)/2$, it is optimal to finance the projects separately if and only if the risk contamination losses dominate the coinsurance gains: $(1 - p_1)p_2(1 - \gamma)r_H^2 > p_1(1 - p_2)(1 - \gamma)r_L^2$.
- (c) If $r_m''' \leq (r_H^1 + r_H^2)/2$ only is satisfied, it is optimal to finance the projects separately to

avoid the risk contamination losses: $p_1(1 - p_2)(1 - \gamma)r_H^1 + (1 - p_1)p_2(1 - \gamma)r_H^2$.

In the new case (b), the probability of default with joint financing is (i) increased by $(1 - p_1)p_2$, because a successful project 2 is dragged down by a failing project 1, but (ii) decreased by $p_1(1 - p_2)$, because a failing project 2 is saved by a successful project 1. Project 2, however, is saved when it yields a low return but it is dragged down following a high return. Thus if the two projects have the same probability of success, the risk contamination effect always dominates the coinsurance effect.

The tradeoff between coinsurance and risk contamination in the new case (b) is depicted in Figure 3. The risk-contamination losses, equal to $(1 - p_1)p_2(1 - \gamma)r_L^2$, are represented by the light gray area and correspond to the added bankruptcy costs on the high-return project 2 that is dragged down when project 1 has a low return. The coinsurance gains, equal to $p_1(1 - p_2)(1 - \gamma)r_H^2$, are represented by the gray area and correspond to reduced bankruptcy costs on the low-return project 2 that is saved when project 1 has a high return. For the numerical value used in the figure, it is more profitable to finance the projects separately because the risk-contamination losses are larger than the coinsurance gains.

When projects are heterogeneous, separation has the additional advantage of allowing for project-specific loans. We now characterize situations in which it is better to finance a high-expected return separately rather than jointly with a low-expected return project. In some cases, separate financing is optimal even if a rate that avoids bankruptcy for all intermediate returns can be obtained.

Proposition 6 (Stochastic dominance) Suppose that project 1 *first-order stochastically dominates* project 2, and in particular, $r_H^1 = r_H^2$ and $r_L^1 = r_L^2 = r_L$ and $p_1 > p_2$, and only the high-mean project 1 can be *financed* separately. Then it is optimal for the borrower to only *finance* project 1 unless (i) $r'_m \leq (r_L^1 + r_H^2)/2$ and (ii) the ex post net present value of *financing* the low-mean project 2 separately is compensated by the coinsurance effects, i.e., if and only if $[(1 - p_2)p_1 + (1 - p_1)p_2](1 - \gamma)r_L > 1 - p_2r_H - (1 - p_2)\gamma r_L$.

4.3 Testable Predictions

For the case in which one project is a mean preserving spread of the other, the next result establishes that more risk typically induces more separation.

Prediction 5 (Spread) If project 2 second-order stochastically dominates project 1, and therefore $p_1 = p_2$ and $r_H^1 = r_H^2 + \varepsilon$ and r_L^1

if (i) $r_m'' \leq (r_H^1 + r_L^2)/2$ but $r_m' > (r_L^1 + r_H^2)/2$, (ii) the risk-contamination losses dominates coinsurance gains: $(1 - p_1)p_2(1 - \gamma)r_H^2 > p_1(1 - p_2)(1 - \gamma)r_L^2$, and (iii) the probability of risk contamination is lower than the probability of coinsurance: $p_1 > p_2$.

As shown in Proposition 5, if $r_m'' \leq (r_H^1 + r_L^2)/2$, we have that (i) if project 1 yields a low return, it drags down project 2's high return (whereas project 2 would have stayed afloat with separate financing) and (ii) if project 1 yields a high return, it saves project 2's low return (whereas project 2 would have defaulted with separate financing). Projects should then be financed separately if the expected benefits from coinsuring the second project dominate the expected losses from risk-contaminating it. Even when the second project is more likely to be saved ($p_1 > p_2$ and therefore $p_1(1 - p_2) > p_2(1 - p_1)$), the risk contamination effect might still dominate because the potential losses from dragging down a successful project are higher than the potential gains from saving an unsuccessful one ($r_H^2 > r_L^2$). Therefore, separation is optimal even if it entails a higher probability of bankruptcy. Figure 3 is an example in point.

5 Correlated Projects

To allow for correlation, we now modify the distribution of joint returns for the baseline case with two identical projects. Suppose that the probability of two high returns result is equal to $p[1 - (1 - p)(1 - \rho)]$, the probability of two low returns is equal to $(1 - p)[1 - p(1 - \rho)]$, and the probability that one of the projects yields a high return whereas the other yields a low one is equal to $p(1 - p)(1 - \rho)$. Thus ρ is the correlation coefficient between the two projects. For the joint probability distribution to be well defined, it is necessary to assume that $\rho \geq \max\langle -(1 - p)/p, -p/(1 - p) \rangle$. Clearly, if $\rho = 0$ we are back to the baseline scenario with independent returns.

Prediction 7 (Correlation) If the correlation between the projects increases (ρ is larger), then separate financing is optimal for a larger set of parameters.

The probability of having two high returns and the probability of having two low returns increase simultaneously with ρ . As a result, the repayment rate when intermediate bankruptcy is avoided is higher because the probability of two low returns is higher. When

intermediate bankruptcy cannot be avoided, the repayment rate is lower because the probability of two high returns also increases. As a consequence, the financing conditions avoiding intermediate bankruptcy are tighter and those not avoiding it looser.

The effects of correlation on the optimality conditions are also intuitive. In the extreme case with perfect negatively correlation (i.e., if $\rho = -1$ and $p = 1/2$), when one project has a high return the other necessarily has a low one, so that projects can always be jointly financed at a rate that avoids intermediate bankruptcy.¹⁴ Thus, it is clearly optimal to always finance projects jointly when the negative correlation is perfect. As correlation increases above $\rho = -1$, conglomeration is optimal for a smaller region of parameters. If projects have perfect positive correlation ($\rho = 1$), the conditions for joint and separate financing are identical and the firm is clearly indifferent between them. This prediction is similar to the one obtained by Inderst and Müller (2003) and by Leland (2007).

6 Multiple Projects

In this section, we consider a borrower with access to a general number of identical projects with independent returns. In Section 6.1, we characterize the size (and the number) of the groups that it is optimal to finance jointly, thereby identifying conditions for partial conglomeration. In Section 6.2, we show that if the number of independent projects is sufficiently large, it becomes possible and optimal for the borrower to finance all of them jointly, so that full conglomeration results.

Consider a group with k projects. Generalizing our baseline analysis for a group with two projects, the per-project repayment rates depend on the number of projects with high return m ($1 \leq m \leq k$) that are necessary to avoid bankruptcy,

$$r_k(m) := \frac{1 - \gamma \left[\sum_{s=0}^{m-1} h(s) \frac{sr_H + (k-s)r_L}{k} \right]}{1 - H(m-1)}, \quad (4)$$

where $h(s)$ is the probability that s out of the k projects yield a high return,

$$h(s) := \binom{k}{s} p^s (1-p)^{k-s} \text{ for } s = 0, 1, \dots, k, \quad (5)$$

and $H(s)$ is the corresponding probability distribution, $H(s) := \sum_{t=0}^s h(t)$.¹⁵ As before, the

¹⁴This is not true for $p \neq 1/2$ because either the probability of two high realizations or the probability of two low realizations is greater than 0, even when the correlation is at the lowest possible level.

¹⁵For notational convenience we define here the density and distribution functions at the number of projects

equilibrium repayment rate is the one which requires the minimum number of high returns, i.e. $r_k^* = r_k(m')$ where m' is the lowest m that satisfies $r_k(m) < [(m-1)r_L + r_H]/m$.

6.1 Partial Conglomeration

To simplify the comparison in this section, assume that the firm can only form groups of symmetric sizes. Then, the number of available projects n is such that $n = 2^z$ for some $z \in \mathbb{N}$. In this context, the firm should choose the size of the group k , where $k = 2^w$ for $w = 0, 1, \dots, z$. If k is the size of the groups then n/k is the number of groups. The following result generalizes Proposition 2 to the case with n projects.

Proposition 8 (Partial conglomeration) Suppose that there are n projects that can be financed in symmetric groups. If the probability of high return is sufficiently small, $p \leq p^*$, then it is optimal to finance the projects in groups of size k^* , where k^* is the largest k that satisfies $r_k^* < [(k-1)r_L + r_H]/k$.

First, if a rate that satisfies $r_{k^*}^* < [(k^*-1)r_L + r_H]/k^*$ can be obtained by financing the projects in groups of size k^* , then it is better to finance the projects in groups of size k^* rather than in smaller groups. In this case, a single high return and $k^* - 1$ low returns allow all the projects in the group to stay afloat, so that a single project coinsures the rest of the group. Groups of smaller size cannot be better because one high-return project would save, at most, only the low-return projects of the smaller group. If all the projects in the other groups yield a low return, they will go bankrupt and the bankruptcy losses would be higher.

Second, if p is small, forming a group of size k^* also dominates forming groups of larger size k' in which $r_{k'}^* > ((k'-1)r_L + r_H)/k'$. In this case, if k' projects are financed jointly and $k' - 1$ low returns are realized, risk-contamination would result. Instead, if the projects had been financed in smaller groups of size k^* , the groups with a single high-return realization would have been saved. To illustrate, consider the case with four projects ($n = 4$), with $(3r_L + r_H)/4 < r_4^* < (r_L + r_H)/2$ and $r_2^* < (r_L + r_H)/2$. If all four projects are financed jointly, three low returns risk-contaminate the fourth, while two high returns coinsure the other two projects. If the projects are financed in two groups composed of two projects each, a high return in one project saves the other project in the same group. The advantage of financing projects in two partial conglomerates with two projects each is that in the event of

with high return s rather than at the corresponding return, $\hat{r}(s) := [sr_H + (k-s)r_L]/k$.

three low returns, one of the partial conglomerates is saved through coinsurance, while risk contamination is contained. The disadvantage is that if one group yields two low returns and the other group yields two high returns, it would have been possible to save the two projects with low returns through coinsurance if all projects had been financed jointly in a full conglomerate. If p is small (below $p^* = 2/3$ if $n = 4$), the first effect dominates and it is optimal to finance projects in groups of two. If, instead, p is larger ($p > 2/3$), the second effect dominates and it is optimal to finance all four projects jointly. Thus an increase in the probability of high return favors larger groups, extending the logic of Prediction 2 beyond the baseline model.

For the case in which projects are symmetric ($p = 1/2$), we derive a sufficient condition on full separation and full conglomeration, given that the distribution function of the average of any symmetric group of projects always cross at the mean.

Proposition 9 (Joint v. separate financing of multiple projects) If there are n symmetric projects which have to be financed in symmetric groups, then they should all be financed jointly if $r_n^* < [(n-1)r_L + r_H]/n$ and all separately if $r_k^* > (r_H + r_L)/2$ for any $k \geq 2$.

All projects should be financed jointly if with a full conglomerate it is possible to obtain a rate such that a single high-return project coinsures all the other projects. On the other hand, if in any group of projects it is not possible to obtain a rate such that half of the projects save the other half, then the projects should be financed separately. These sufficient conditions become necessary and sufficient if $n = 2$, thus also generalizing Proposition 2.

6.2 Large Number of Projects

We now show that if the number of independent projects is sufficiently large, it always becomes possible for the borrower to finance all the projects jointly. This result exploits the law of large numbers. Namely, as the number of projects n increases, the probability that the average number of projects with high returns differs from p , the probability of a high return, by more than a small amount ε tends to zero. We can then construct a rate offer to finance all projects jointly that is acceptable to the creditors. The borrower's returns when financing all projects jointly is then arbitrarily close to the first-best as the number of projects increases. Therefore, when the number of projects is large, financing all the projects jointly is approximately optimal for the borrower because the resulting payoff is close to the highest possible level.

Proposition 10 (Many projects) There exists n' and $q \in (0, p)$ such that when the number of projects satisfies $n > n'$, joint financing of all projects can be obtained at a repayment rate that avoids bankruptcy when nq projects have high returns. The resulting per-project return approaches the net present expected value of each project as n grows.

This result leads to our next prediction:

Prediction 8 (Full conglomeration) If there is a large number of independent projects, it is optimal to finance them jointly.

7 Structure of Bankruptcy Costs

In line with most of the theoretical and empirical literature, in our baseline specification bankruptcy costs are proportional to realized returns, with $B(r) = (1 - \gamma)r$. Note that this baseline specification entails constant returns to scale: $B(2r) = (1 - \gamma)2r = 2B(r)$. To investigate the robustness of our results to the structure of the bankruptcy costs, this section consider a general specification that allows for economies or diseconomies of scale in bankruptcy. We retain the feature that bankruptcy costs are larger for higher levels of realized returns, so that $B(r)$ is increasing in r .

As is intuitive, economies of scale in bankruptcy favor joint financing whereas diseconomies of scale favor separation. As demonstrated in the next result, if economies of scale are sufficiently strong, so that $B(2r_L) - 2B(r_L)$ is negative enough, then joint financing is optimal. Separate financing is optimal if, instead, there are sufficiently strong diseconomies of scale, so that $B(2r_L) - 2B(r_L)$ is positive enough. In the intermediate case, which includes constant returns to scale as well as weak economies and diseconomies of scale, separation is optimal if the rate that avoids intermediate bankruptcy cannot be obtained.

Proposition 11 (Scale economies in bankruptcy costs) With a general structure of bankruptcy costs, there exist thresholds $\underline{S} < 0$ and $\overline{S} > 0$ such that

- (i) If $B(2r_L) - 2B(r_L) < \underline{S}$, joint financing is always optimal;
- (ii) If $\underline{S} < B(2r_L) - 2B(r_L) < \overline{S}$, separate financing is optimal if and only if

$$\frac{1 - (1 - p)^2 [r_L - B(2r_L)/2]}{1 - (1 - p)^2} > \frac{r_H + r_L}{2};$$

- (iii) If $B(2r_L) - 2B(r_L) > \overline{S}$, separate financing is always optimal.

To further characterize the thresholds independently of the level of returns, consider bankruptcy costs given by $B(r) \equiv (1-\gamma)r + \alpha(r-r_L)r$. This specification allows for economies ($\alpha < 0$) and diseconomies of scale ($\alpha > 0$) and includes our baseline case with constant returns to scale as a special case ($\alpha = 0$). Following the procedure set out in the previous proposition, if $\alpha < \underline{\alpha} := -(1-\gamma)pr_H / [(1-p)r_L^2 + pr_H^2 + pr_Lr_H]$, joint financing is optimal (case (i)); if $\underline{\alpha} < \alpha < \bar{\alpha}$, separate financing is optimal if and only if the rate that avoids intermediate bankruptcy cannot be obtained (case (ii)); and if $\alpha > \bar{\alpha} := p(1-\gamma) / [(1-p)r_L]$, separate financing is optimal (case (iii)).

An alternative specification with constant returns to scale consists in assuming a fixed per-project bankruptcy cost b ($< r_L$), so that $B(r) = b$ for $r = r_H, r_L$ and $B(r) = 2b$ for $r = 2r_H, r_H + r_L, 2r_L$. Thus, we have $B(2r_L) - 2B(r_L) = 0$, so that case (ii) always results. In addition, it can be shown that with per project bankruptcy costs separate financing is optimal for a relatively larger set of parameters than in our baseline case with proportional bankruptcy costs.¹⁶ Next, if there is a fixed recovery rate per project w ($< r_L$), case (ii) also results.

In sum, joint financing is optimal if there are significant economies of scale in bankruptcy, while separate financing is optimal if there are sufficiently strong diseconomies of scale. For weak economies or diseconomies of scale, as well as for several specifications with constant returns of scale in bankruptcy, separate financing is optimal as long as intermediate bankruptcy cannot be avoided. Higher bankruptcy costs then favor separate financing more generally, as in our baseline specification.

8 Debt, Equity, and Taxes

This section allows the firm to use equity, as well as debt, to finance part of the initial investment. As in the standard tradeoff theory of capital structure, equity payments are subject to corporate taxation, whereas debt payments are tax deductible and are therefore

¹⁶To see this, suppose that the per-project bankruptcy costs are set at the same level as the proportional losses of the low-return project, $b = (1-\gamma)r_L$. Then, the proceeds from a bankrupt high-return project are relatively higher in the per-project case compared to the proportional case. Rates r_i^* and r_m^* are the same as those resulting from proportional bankruptcy costs, so that conditions (1) and (2) do not change. However, the rate r_m^{**} is now lower and it becomes easier to satisfy condition (3). As a result, it becomes easier to obtain joint financing, but only at the rate for which intermediate bankruptcy occurs. Therefore, when both separate and joint financing are feasible, separate financing is optimal for relatively larger set of parameters.

exempt from taxes.

8.1 Model Extension

Financing for each corporation can be obtained in competitive credit and equity markets. As in the basic model, the availability of a competitive credit market is equivalent to assuming that each corporation makes a take-it-or-leave-it offer to a single creditor. Corporation j , consisting of n_j projects, promises to repay $n_j r'_j$ at $t = 2$ in exchange of $n_j D_j$ at $t = 1$. Thus, the promised per-project repayment r'_j now depends on the part of the initial investment outlay of each project that is financed through debt, $D_j \leq 1$.

A competitive equity market is equivalent to assuming that each corporation makes a take-it-or-leave-it offer to a single outside equity investor. We denote the fraction of the equity sold by corporation j as α_j , and the equity value of the corporation, if it consists of n_j projects, as $n_j E_j$. For all the projects to be financed, the sum of debt and equity financing per-project must cover the initial investment outlay of each project, $D_j + \alpha_j E_j = 1$. We also assume that, while debt payments are tax deductible and therefore exempt from taxes, equity payments are subject to a corporate tax of τ , which captures the tax penalty of equity relative to debt.¹⁷

8.2 Financing Conditions

For the case of separate financing, we now need to distinguish two cases, because there are situations in which it is possible to obtain a rate r'_i that avoids bankruptcy altogether, $r'_i \leq r_L$, by selling a fraction α of the corporation. If this rate exists, it should satisfy

$$\alpha(1 - \tau) [p(r_H - r'_i) + (1 - p)(r_L - r'_i)] = \alpha E_i \quad \text{and} \quad r'_i = D_i.$$

Since there is no bankruptcy, the net interest rate is zero and the principal is equal to the debt value. Substituting into the total financing condition, $D_i + \alpha E_i = 1$, this rate can be

¹⁷Leland (2007) makes the more realistic assumption that only interest expenses are tax deductible. This, however, creates an endogeneity problem. When interest only is deductible, the fraction of debt service attributed to interest payments depends on the value of the debt, which in turn depends on the fraction of debt service attributed to interest payments. Instead of relying in numerical techniques to find debt values and optimal leverage, we follow Kale, Noe, and Ramirez (1991) and assume that both interest and principle are tax deductible. We also assume away personal taxes.

obtained if and only if

$$r'_i(\alpha) := \frac{1 - \alpha(1 - \tau) [pr_H + (1 - p)r_L]}{1 - \alpha(1 - \tau)} \leq r_L. \quad (6)$$

If the firm uses no equity ($\alpha = 0$), then $r'_i = 1$ and the condition is never satisfied ($r_L < 1$), as in the baseline debt-only case. But, as more equity is offered, the debt repayment is lower ($r'_i(\alpha)$ is decreasing) and, if taxes are low, the condition can be satisfied. Equity, however, is costly because of taxes. It is optimal for the firm to sell the lowest equity stake α'_i satisfying condition (6), $r'_i(\alpha'_i) = r_L$. Still, if taxes are high enough, it is not possible to obtain this rate, not even by selling all the equity.

Following the same procedure, a rate such that $r''_i < r_H$ can be obtained if and only if

$$r''_i(\alpha) := \frac{1 - \alpha(1 - \tau)pr_H - (1 - p)\gamma r_L}{[1 - \alpha(1 - \tau)]p} \leq r_H, \quad (7)$$

which generalizes condition (1) of the baseline setup to $\alpha > 0$, as $r''_i(0) = r^*_i \leq r_H$. This condition is satisfied precisely as long as condition (1) is satisfied, independently of the level of equity sold. Given that the firm prefers to sell the lowest possible fraction of equity, no equity at all is sold in the optimum, $\alpha''_i = 0$. In this case, equity does not help in reducing the probability of bankruptcy.

The following proposition characterizes which of these two rates is optimally chosen when they are both available.

Proposition 12 (Equity and taxes: separate financing) Suppose that both rates r'_i and r''_i are available. There exists $\bar{\tau}_i$ such that the optimal rate and fraction of equity sold are, respectively, r_L and $\alpha'_i > 0$ if $\tau \leq \bar{\tau}_i$, and r^*_i and $\alpha = 0$ if $\tau > \bar{\tau}_i$.

If taxes are sufficiently high, the projects are financed at the same rate as in the baseline case without equity. Moreover, it is then optimal to finance the projects entirely with debt. When taxes are lower, however, it becomes optimal to finance the projects at a rate that avoids bankruptcy altogether ($r'_i(\alpha'_i) = r_L$) by selling a positive amount of equity, $\alpha'_i > 0$.

For the case of joint financing, there are three potential rates. The first rate, which avoids bankruptcy altogether, $r'_m \leq r_L$, is the same as (and can be obtained under the same circumstances as) the rate resulting with separate financing, $r'_m = r'_i$. Indeed, if bankruptcy can be avoided, then the corporate structure does not matter.

Second, a rate that avoids bankruptcy if one realized return is high and the other is low

can be obtained if and only if

$$r_m''(\alpha) := \frac{1 - \alpha(1 - \tau) [p^2 r_H + 2p(1 - p) \frac{r_H + r_L}{2}] - (1 - p)^2 \gamma r_L}{[1 - \alpha(1 - \tau)] [1 - (1 - p)^2]} \leq \frac{r_H + r_L}{2}, \quad (8)$$

which is again a generalization for $\alpha \geq 0$ of condition (2) of the baseline case, $r_m''(0) = r_m^* \leq (r_H + r_L)/2$. Again, it is optimal for the firm to choose the minimum amount of equity that satisfies condition (8). If condition (2) is satisfied, the firm does not need to sell any equity at all, $\alpha_m'' = 0$. If condition (2) is not satisfied, this rate can still be obtained, however, by selling some equity.

Third, a rate that avoids bankruptcy only if both realized returns are high can be obtained as long as

$$r_m'''(\alpha) := \frac{1 - (1 - p)^2 \gamma r_L - 2p(1 - p) \gamma \frac{r_H + r_L}{2} - \alpha(1 - \tau) p^2 r_H}{[1 - \alpha(1 - \tau)] p^2} \leq r_H, \quad (9)$$

which again, generalizes the condition of the baseline case for $\alpha \geq 0$, i.e. $r_m'''(0) = r_m^{**} \leq r_H$. As in the highest rate for separate financing, this condition is satisfied precisely as long as condition (3) is satisfied, independently of the equity sold. Given that the firm prefers to sell the lowest possible fraction of equity, the resulting level is $\alpha_m''' = 0$.

Proposition 13 (Equity and taxes: joint financing) Suppose more than one rate (r'_m, r_m'', r_m''') is available. There exist $\bar{\tau}_m^a, \bar{\tau}_m^b$, and $\bar{\tau}_m^c$ such that:

- (i) If condition (2) is satisfied, the optimal rate and fraction of equity sold are, respectively, r_L and $\alpha'_m > 0$ if $\tau \leq \bar{\tau}_m^a$, and r_m^* and $\alpha = 0$ if $\tau > \bar{\tau}_m^a$.
- (ii) If condition (2) is not satisfied, the optimal rate and fraction of equity sold are, respectively, r_L and $\alpha'_m > 0$ if $\tau \leq \bar{\tau}_m^b, \frac{r_H + r_L}{2}$ and $\alpha''_m > 0$ if $\bar{\tau}_m^b < \tau \leq \bar{\tau}_m^c$, and r_m^{**} and $\alpha = 0$ if $\tau > \bar{\tau}_m^c$.

8.3 Good and Bad Conglomeration

The profitability of joint financing depends on the cases identified in Proposition 13. In case (i), joint financing is always profitable at least weakly. This case is equivalent to the case of good conglomeration in the baseline model. The condition is exactly the same as the condition enabling the firm to obtain r_m^* in Section 3. In case (ii), conglomeration is bad in the baseline model. And, if taxes are sufficiently high, conglomeration is still bad here. If taxes are lower, however, financing with equity allows the firm to finance the projects with rates that avoid bankruptcy in the case with intermediate returns and even with rates that

avoid bankruptcy altogether.

Proposition 14 (Equity and taxes: joint v. separate financing) When both separate and joint financing are feasible:

- (i) If condition (2) is satisfied, both financing regimes are equally profitable if $\tau \leq \bar{\tau}_m^a$, whereas joint financing dominates if $\tau > \bar{\tau}_m^a$.
- (ii) If condition (2) is not satisfied, both financing regimes are equally profitable if $\tau \leq \bar{\tau}_m^b$, joint financing dominates if $\bar{\tau}_m^b < \tau \leq \bar{\tau}_i$, and separate financing dominates if $\tau > \bar{\tau}_i$.

In sum, if taxes are sufficiently high, only debt is used and the same situation analyzed in the baseline model arises. That is, joint financing is profitable in case (i) and separate financing is profitable in case (ii). The condition setting apart joint and separate financing is exactly the same as in the baseline model without equity. If taxes are intermediate, joint financing can be profitable in cases in which it is not profitable in the baseline model with only debt (case ii). This is because, by financing jointly and using equity, it becomes possible to obtain a rate that avoids intermediate bankruptcy or bankruptcy altogether. Finally, if taxes are sufficiently low, joint financing is inconsequential because bankruptcy can be avoided altogether with joint as well as with separate financing.

The exclusive use of debt in separate finance is consistent with the many empirical studies that find that a disproportionate proportion of funding in project finance is in the form of debt. Kleimeier and Megginson (2000), for example, find that projects funded with project finance loans have an average loan-to-project value ratio of 67%. Esty (2003) shows that the average (respectively median) project company has a book value debt-to-total capitalization ratio of 70% (respectively 70%) compared to 33.1% (respectively 30.5%) for similar-sized firms.

9 Normal Returns

This section analyzes the model when returns are normally distributed rather than binary. The purpose of this extension is twofold. First, we show that our results on bad conglomeration and the main comparative statics predictions are robust to continuous distributions. Second, this specification of returns allows us to make a precise comparison with results obtained through numerical simulations by Leland (2007); our analytical results clarify that the optimality of separate financing holds even when the capital structure mix (in terms of

debt and equity) is not adjusted to the scope of incorporation. As part of the analysis, we also provide an easy-to-verify sufficient condition for the optimality of separate financing.

9.1 Model Extension

A firm has access to n symmetric, normally distributed projects, $r_i \sim N(\mu, \sigma^2)$ for $i = 1, \dots, n$, with symmetric correlation coefficient ρ . As in the binary case, the distribution function of the average returns lies below the distribution of a single return until a unique crossing point (here equal to the mean because of symmetry), after which the ordering is reserved. Indeed, the average of two normal random variables is also normal with a density that is more peaked around the mean than the original normal density. To retain analytical tractability, we assume (i) that there is a fixed per-project recovery rate w ($w < 1 < \mu$);¹⁸ (ii) that the firm can only form symmetric groups of projects (and therefore $n = 2^z$ for some $z \in \mathbb{N}$), as in Section 6.1; and (iii) that projects need to be financed exclusively with debt.

9.2 Financing Conditions

As in Section 6.1, the firm should choose the size of the groups k , where $k = 2^w$ for $w = 0, 1, \dots, z$. The per project repayment requested by a creditor in a competitive market to finance a group of size k , r_k^* , is defined by

$$kr_k^*[1 - G(kr_k^*)] + wkG(kr_k^*) = k, \quad (10)$$

where G is the distribution function of the sum of k normal random variables. Noting that the distribution of the sum computed at kr is

$$G(kr) = \Pr(r_1 + \dots + r_k \leq kr) = \Pr\left(\frac{r_1 + \dots + r_k}{k} \leq r\right) =: H(r), \quad (11)$$

where H is the distribution of the average of r_1, \dots, r_k , this condition is equivalent to

$$r_k^*[1 - H(r_k^*)] + wH(r_k^*) = 1. \quad (12)$$

¹⁸Given that returns are normally distributed, with positive probability there realized return is lower than the recovery rate. For simplicity, we disregard this problem, given that the probability of these realizations can be made arbitrarily small with an appropriate choice of parameters. Alternatively, the proof of Proposition 15 holds for the general class of log-concave symmetric distributions, which allow for positive support and recovery rates below the support. The key property driving the result is that the density of the average of n random variables is more peaked around the mean compared to the original density. As shown by Proschan (1965), this property holds generally for log-concave symmetric distributions.

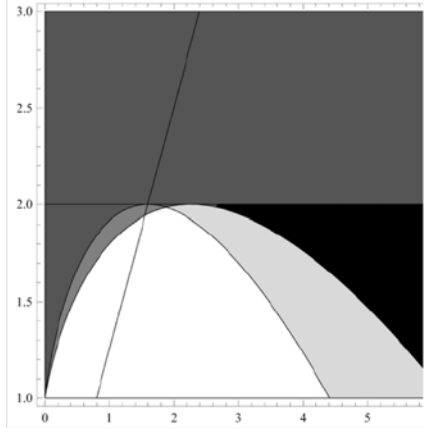


Figure 4: **Financing and Optimality Regions with Normally Distributed Returns.**

In the (σ, μ) combinations of the light gray area financing is only possible separately. In the medium gray area financing is only possible jointly. In the dark gray and black areas both separate and joint financing is possible. In the black area (delimited by the two straight lines depicting the two conditions in Proposition 12) separate financing is optimal. In this picture, we take $w = 0$ and $\rho = 0$.

The firm's per-project payoff is then

$$\int_{r_k^*}^{+\infty} \frac{r_1 + \dots + r_k}{k} dH - r_k^* [1 - H(r_k^*)] = \int_{r_k^*}^{+\infty} \left(\frac{r_1 + \dots + r_k}{k} - r_k^* \right) dH. \quad (13)$$

Given that this payoff is a decreasing function of r_k^* , it is optimal for the firm to select the lowest r_k^* at which condition (12) is satisfied, if such a r_k^* exists. Financing is obtained in such a case. Figure 4 represents the mean-variance parameters allowing projects to be financed separately ($k = 1$) and in groups of two ($k = 2$).

9.3 Good and Bad Conglomeration

We now turn to the question of when is it optimal to finance the projects separately when there are multiple options available.

Proposition 15 (Optimality of separate financing) If it is feasible to finance separately n normally distributed projects with mean μ and standard deviation σ , separate financing is optimal if

$$\mu + w < 2 \text{ and } \mu - w < \sigma \sqrt{[1 + \rho(n - 1)] \pi / 2n}. \quad (14)$$

These conditions identify the region of parameters for which separate financing is optimal in Figure 4. We obtain the same comparative statics as in the baseline model. Separation holds for a larger region of parameters if the mean returns are low (Prediction 2) and if the variance is high (Prediction 3). Indeed, it is more difficult to satisfy both conditions in (14) if μ increases, and it is easier to satisfy the second condition if σ increases. As in Prediction 7, when the coefficient of correlation increases the region for which separate financing is optimal increases. Similar to the binary case in Section 6.2, when the number of projects increases the region for which separation is optimal shrinks. In the limit, if there is a large number of independent projects the second condition is never satisfied, in accordance with Prediction 10.

Similarly, an increase in the recovery rate favors the optimality of joint financing (Prediction 1). To see this, consider the mean-variance parameter combinations for which joint and separate financing are both feasible for two levels of recovery rates, $w = w_1$ and $w = w_2$ where $w_1 < w_2$. Then, the region for which separate financing is optimal is smaller for $w = w_2$ than for $w = w_1$. Indeed, an increase from w_1 to w_2 makes it more difficult for the first condition in (14) to be satisfied, thereby shrinking the region in which separate financing is optimal. Even though it becomes easier to satisfy the second condition, the new parameter values for which separate financing is optimal belongs to a region in which it is not feasible to finance the projects.

Through a set of numerical results, Leland (2007) argues that the separate financing becomes optimal because of the endogenous choice between debt and equity. Instead, our analytical characterization identifies conditions under which separate financing is optimal when financing is restricted only to debt and in the presence of bankruptcy costs alone. We conclude that bad conglomeration arises even when the capital structure mix in terms of debt and equity is not reoptimized with the scope of conglomeration. This result and the simple logic that underlies it were not envisioned by Leland's (2007) analysis.

10 Conclusion

This paper analyzes the simple economics of conglomeration with bankruptcy costs. Our results qualify the long-standing claim that joint financing generates financial benefits by economizing on bankruptcy costs. By turning on its head the classic logic that generates coinsurance savings from conglomeration, we characterize instances in which expected bank-

ruptcy costs increase because of risk contamination. For projects with binary returns we provide a complete characterization of the tradeoff between coinsurance and risk contamination. The theory predicts that:

- An increase in the bankruptcy recover rate and an increase in the probability of a high return favor joint financing;
- An increase in the riskiness of (sufficiently negatively skewed) projects favors separate financing;
- An increase in the negative skewness of projects (with sufficiently high return) favors separate financing;
- An increase in the differences in terms of risk profiles of two heterogeneous projects favors separate financing;
- An increase in the correlation of projects favors separate financing;
- Joint financing of a sufficiently large number of independent projects is optimal;
- Economies of scale in bankruptcy costs favor joint financing.

In addition, we show that separate financing can be optimal even when joint financing involves paying a lower repayment rate or results in a lower probability of bankruptcy.

Our modeling framework is tractable and can be extended in many further directions. In our setup, investors in both projects have recourse to the returns of the other project (with joint financing) or none of them have access to the returns of the other (with separate financing). In reality, an asymmetric, intermediate situation could also arise whereby investors in one (recourse) project have access to the returns of the other (nonrecourse) project, but not conversely. In this case, one of the diagonal entries in Figure 1 would be akin to separate financing. That is, if the project with nonrecourse has a low return and the one with recourse has a high return, the former project would go bankrupt while the latter project would stay afloat. In the other diagonal entry, however, both projects would stay afloat if the recourse project can be saved by the nonrecourse project. If so, this intermediate solution would dominate separate financing, but if not, it would be worse. A complete analysis for the resulting tradeoff is left to future research; see Nicodano and Luciano (2009) for an investigation in this direction in a setting with both bankruptcy costs and taxes.

Saving an unsuccessful project might sometimes be optimal for reputational reasons, even if it has been financed with (nonrecourse) debt and the firm is under no legal obligation to

save it. Gorton (2008), for example, points out that securitization issuers retain substantial implicit exposure even after mortgages are securitized. In the credit card asset-based securities (ABS) market, for example, Higgins and Mason (2004) document instances in which issuers of credit card ABS have taken back non-performing loans despite not being contractually required to do so. Similarly, Gorton and Souleles (2006) show that prices paid by investors in credit card ABS take into account issuers' ability to bail out their ABS. To capture this tradeoff, one could extend our static model to a dynamic framework. It is also natural to extend the model to allow for multiple (and possibly risk-averse) investors, as in Winton's (1995) and Bond's (2004) analysis of intermediation in the costly state verification model.

Finally, our model can also be extended to analyze the public policy problem of optimal conglomeration in the presence of systemic spillovers; a topic that has recently attracted attention (see, for example, Acharya, 2009, and Ibragimov, Jaffee, and Walden, 2010). In this case, bankruptcies create significant negative externalities and the borrower should minimize the probability of bankruptcy instead of maximizing net returns. For the case with normally distributed returns, it can be shown that the two conditions identified in Proposition 15 then become necessary and sufficient. On the one hand, if the equilibrium repayment rate lies above the crossing point (the two conditions in (14) are satisfied), then the equilibrium rates and the probability of bankruptcy are lower with separate financing. On the other hand, if the repayment rate is below the crossing point (either or both conditions in (14) are not satisfied), joint financing reduces the equilibrium rates and the probability of bankruptcy. We leave the development of this extension to future research.

Appendix

Proof of Proposition 1: The proof follows from the analysis reported in the text. Q.E.D.

Proof of Proposition 2: If projects can be financed separately, i.e. condition (1) is satisfied, the entrepreneur obtains a per-project return of $p(r_H - r_i^*)$, which is equal to the ex post net present value

$$pr_H + \gamma(1 - p)r_L - 1. \quad (15)$$

Similarly, if condition (2) is satisfied, the entrepreneur obtains a per-project return of $p^2(r_H -$

$r_m^*) + 2p(1-p)[(r_H + r_L)/2 - r_m^*]$, or

$$p^2 r_H + 2p(1-p)(r_H + r_L)/2 + \gamma(1-p)^2 r_L - 1, \quad (16)$$

and, if condition (3) but (2) is not satisfied, she obtains $p^2(r_H - r_m^{**})$, or

$$p^2 r_H + \gamma 2p(1-p)(r_H + r_L)/2 + \gamma(1-p)^2 r_L - 1. \quad (17)$$

Subtracting (16) from (15), we obtain $(1-\gamma)p(1-p)r_L$ and therefore joint financing is more profitable than separate financing. Instead, subtracting (15) from (16), we obtain $(1-\gamma)(1-p)pr_H$ and therefore separate financing is more profitable than joint financing. Q.E.D.

Proof of Prediction 1: The statements follow from the fact that the derivatives of the left-hand of (1), (2), and (3) with respect to γ are negative. Q.E.D.

Proof of Prediction 2: The statements follow from the fact that the derivatives of the left-hand of (1), (2), and (3) with respect to p are negative. Q.E.D.

Proof of Prediction 3: Letting ε be such that $\hat{r}_H = r_H + \varepsilon$, we have that, in order to have a mean preserving spread, $\hat{r}_L = r_L - \frac{p}{1-p}\varepsilon$. Substituting into condition (2), the derivative of the left-hand side less the derivative of the right-hand side is equal to

$$\frac{1-p}{2-p}\gamma + \frac{1}{2(1-p)} - 1,$$

which is positive if and only if $p > \bar{p}$, where $\bar{p} \equiv \left[1 + 4(1-\gamma) - \sqrt{1 + 8(1-\gamma)}\right] / 2(1-\gamma)$. Therefore, condition (2) is less likely to be satisfied following an increase in ε if and only if $p > \bar{p}$. It can be easily checked that $\bar{p} < 1/2$ for any γ . Q.E.D.

Proof of Prediction 4: Letting ε be such that $\hat{r}_L = r_L - \varepsilon$, we have that, in order to have a mean preserving spread, $\hat{p} = p - \frac{(1-p)\varepsilon}{r_H - r_L + \varepsilon}$. Following the same procedure as in the proof of the previous prediction, there exists \bar{r}_H , such that condition (2) is less likely to be satisfied following an increase in ε if and only if $r > \bar{r}_H$. Q.E.D.

Proof of Proposition 3: (i) Suppose that γ and r_L are arbitrarily close to 1, condition (2) is arbitrarily close to $\frac{r_H + r_L}{2} > 1$ whereas condition (1) simplifies to $r_H > 1$. Clearly there are situations in which condition (2) is satisfied, and therefore projects can be financed jointly, but condition (1) is not satisfied, and therefore projects cannot be financed separately.

(ii) If condition (2) is not satisfied, projects can only be financed jointly if condition (3) is satisfied. Condition (3) can be rewritten as

$$pr_H - p(1-p)r_H(1-\gamma) + (1-p)\gamma r_L > 1.$$

This implies that $pr_H + (1-p)\gamma r_L > 1$, which implies that projects can be financed separately. Of course, the opposite is not true, if the parameters are such that $pr_H + (1-p)\gamma r_L$ is arbitrarily close to 1, then condition (3) is not satisfied. Q.E.D.

Proof of Proposition 4: Suppose first that a rate below the crossing point can be obtained.

We have that

$$r_m^* = \frac{1 - (1-p)^2 \gamma r_L}{1 - (1-p)^2} < \frac{1 - (1-p)\gamma r_L}{p} = r_i^*,$$

because $1 > \gamma r_L$. Next, suppose that only a rate r_m^{**} above the crossing point can be obtained and therefore the probability of bankruptcy is higher with joint financing. Nevertheless, the rate r_m^* associated with joint financing is lower than r_i^* associated with separate financing whenever

$$r_m^{**} = \frac{1 - (1-p)\gamma(pr_H + r_L)}{p^2} < \frac{1 - (1-p)\gamma r_L}{p} = r_i^*,$$

or equivalently when

$$\gamma r_H > \frac{1 - (1-p)\gamma r_L}{p} = r_i^*,$$

as claimed. Q.E.D.

Proof of Proposition 5: We first derive the financing conditions. Following the same procedure as in the symmetric case, the repayment rate should satisfy $1 < r'_i < r_H^i$. The creditor's zero profit condition is now

$$pr'_i + (1-p_i)\gamma r_L^i - 1 = 0, \tag{18}$$

and project i can be financed (at r'_i) if and only if

$$r'_i := \frac{1 - (1-p_i)\gamma r_L^i}{p_i} < r_H^i. \tag{19}$$

There are three cases in which joint financing is feasible depending on whether bankruptcy can be avoided in both cases with intermediate returns, or only when project 1 yields a high return and project 2 yields a low return, or in neither case. If bankruptcy can be avoided in both cases with intermediate returns, competition in the credit market results in

$$[1 - (1-p_1)(1-p_2)]2r'_m + (1-p_1)(1-p_2)\gamma(r_L^1 + r_L^2) - 2 = 0, \tag{20}$$

so that this case is possible if and only if

$$r'_m := \frac{1 - (1 - p_1)(1 - p_2)\gamma \frac{r_L^1 + r_L^2}{2}}{1 - (1 - p_1)(1 - p_2)} < \frac{r_L^1 + r_H^2}{2}. \quad (21)$$

If bankruptcy can be avoided with high intermediate returns but not with low intermediate returns, then

$$p_1 p_2 2r''_m + p_1(1 - p_2)2r''_m + (1 - p_1)p_2\gamma(r_L^1 + r_H^2) + (1 - p_1)(1 - p_2)\gamma(r_L^1 + r_L^2) - 2 = 0, \quad (22)$$

and therefore this case is possible if and only if

$$\frac{r_L^1 + r_H^2}{2} < r''_m := \frac{1 - (1 - p_1)p_2\gamma \frac{r_L^1 + r_H^2}{2} - (1 - p_1)(1 - p_2)\gamma \frac{r_L^1 + r_L^2}{2}}{p_1} < \frac{r_H^1 + r_L^2}{2}.$$

If bankruptcy cannot be avoided with either intermediate returns, then

$$p_1 p_2 2r'''_m + p_1(1 - p_2)\gamma(r_H^1 + r_L^2) + (1 - p_1)p_2\gamma(r_L^1 + r_H^2) + (1 - p_1)(1 - p_2)\gamma(r_L^1 + r_L^2) - 2 = 0, \quad (23)$$

and therefore this is possible if and only if

$$\frac{r_H^1 + r_L^2}{2} < r'''_m < \frac{r_H^1 + r_H^2}{2}, \quad (24)$$

where

$$r'''_m := \frac{1 - p_1(1 - p_2)\gamma \frac{r_H^1 + r_L^2}{2} - p_2(1 - p_1)\gamma \frac{r_L^1 + r_H^2}{2} - (1 - p_1)(1 - p_2)\gamma \frac{r_L^1 + r_L^2}{2}}{p_1 p_2}.$$

Again, since the borrower obtains all the ex post net present value, rate r'_m is preferred to r''_m and r''_m is preferred to r'''_m . To complete the proof we only need to show that the lower bound conditions for r''_m and r'''_m are irrelevant. From (20) and (22), and rearranging, we have

$$p_1(r'_m - r''_m) = p_2(1 - p_1) \left[\gamma \left(\frac{r_L^1 + r_H^2}{2} \right) - r'_m \right],$$

and therefore if $r'_m > \frac{r_L^1 + r_H^2}{2}$ then the right-hand side is negative. As a consequence, we have $r''_m > r'_m > \frac{r_L^1 + r_H^2}{2}$. Similarly, from (22) and (23) and rearranging, we have

$$p_2(r''_m - r'''_m) = (1 - p_2) \left[\gamma \left(\frac{r_H^1 + r_L^2}{2} \right) - r''_m \right]$$

and therefore if $r''_m > \frac{r_H^1 + r_L^2}{2}$ then the right-hand side is negative. As a consequence, we have $r'''_m > r''_m > \frac{r_H^1 + r_L^2}{2}$.

We now turn to the choice between joint and separate financing. Substituting r'_m in the right-hand side of (20) and r'_i in the right-hand side of (18) and subtracting the latter from

the former, we have

$$p_2(1-p_1)(1-\gamma)r_L^1 + p_1(1-p_2)(1-\gamma)r_L^2 (> 0).$$

Similarly, substituting r_m'' in the right-hand side of (22) and subtracting again the ex post net present value of financing the two projects separately from this, we obtain

$$-(1-p_1)p_2(1-\gamma)r_H^2 + p_1(1-p_2)(1-\gamma)r_L^2,$$

which can be positive or negative. Lastly, substituting r_m''' in the right-hand side of (23) and subtracting the ex post net present value of financing the two projects separately from this, we have

$$-p_1(1-p_2)(1-\gamma)r_H^1 - p_2(1-p_1)(1-\gamma)r_H^2 (< 0),$$

as desired. Q.E.D.

Proof of Proposition 6: If only one project can be financed separately, we have that $p_i r_H + (1-p_i)\gamma r_L > 1$ and $p_j r_H + (1-p_j)\gamma r_L < 1$ for $i \neq j$, where we denote again $r_H := r_H^i$ and $r_L := r_L^i$ for $i = 1, 2$. Then the expected surplus from financing the project separately is $p_i r_H^i + (1-p_i)\gamma r_L^i - 1$. Subtracting this from the expected surplus from joint financing in the case in which the repayment rate r_m' can be obtained and simplifying, we have

$$p_j r_H + (1-p_j)\gamma r_L - 1 + [(1-p_j)p_i + (1-p_i)p_j](1-\gamma)r_L.$$

On the other hand, if we subtract this from the expected surplus from joint financing in the case in which the repayment rate r_m' can be obtained and simplifying, we obtain

$$p_j r_H + (1-p_j)\gamma r_L - 1 - [(1-p_j)p_i + (1-p_i)p_j](1-\gamma)r_H,$$

so that separate financing is optimal because both terms are negative. Q.E.D.

Proof of Prediction 5: Given that one project is obtained from an elementary increase in risk from the other and returns should still be binary, we must have that $p_1 = p_2 \equiv p$. Letting ε be such that $r_H^1 = r_H^2 + \varepsilon$, we have $r_L^1 = r_L^2 - \frac{p}{1-p}\varepsilon$. Indeed, $p(r_H^2 + \varepsilon) + (1-p)r_L^1 = pr_H^2 + (1-p)r_L^2$. We can also check that $r_L^1 + r_H^2 = r_L^2 - \frac{p}{1-p}\varepsilon + r_H^2 < r_L^2 + \varepsilon + r_H^2 = r_H^1 + r_L^2$.

As shown in the previous proposition, given that the probabilities of success are equal, we have that, when both projects can be financed separately as well as jointly, joint financing is only optimal if a rate r_m' can be obtained. Moreover, the region for which joint financing is optimal shrinks as the repayment rate r_m' is more difficult to obtain if ε increases. Indeed,

the left-hand side of condition (21) decreases in ε and the repayment rate (the right-hand side) increases in ε .

On the other hand, the region for which separate financing is possible expands if ε increases. Indeed, the derivative of the left-hand side of condition (19) is equal to γ whereas the right-hand side is equal to 1. Hence, this condition is more easily satisfied as ε increases. Q.E.D.

Proof of Prediction 6: From the proof of Proposition 5, if $r_L^1 = r_L^2$ and $r_L^1 = r_L^2$, we have that, when both projects can be financed separately as well as jointly, joint financing is only optimal if a rate r'_m can be obtained. The statement follows from the fact that the derivatives of the left-hand of (21) with respect to p_1 and p_2 are negative. Q.E.D.

Proof of Proposition 7: Clearly, from Proposition 5, if statements (i) and (ii) are satisfied, separation is optimal. The probability of default of project 1 is the same in both financing regimes. With separate financing, the probability of default of project 2 is (i) reduced by $(1 - p_1)p_2$, as a successful project 2 would not be dragged down if project 1 fails, but (ii) increased by $p_1(1 - p_2)$, as a failing project 2 would not be saved if project 1 is successful. Given that, according to (iii), $p_1 > p_2$, we have that $p_1(1 - p_2) > (1 - p_1)p_2$. As a result, the probability of default with separate financing is higher. Q.E.D.

Proof of Prediction 7: Clearly, separate financing is not affected by correlation. The joint financing repayment rates, r_m^* and r_m^{**} in Proposition 1, and the corresponding financing conditions, are now replaced by $r_{m,\rho}^*$ and $r_{m,\rho}^{**}$, respectively, where

$$r_{m,\rho}^* := \frac{1 - (1 - p) [1 - p(1 - \rho)] \gamma r_L}{1 - (1 - p) [1 - p(1 - \rho)]} < \frac{r_H + r_L}{2},$$

and

$$r_{m,\rho}^{**} := \frac{1 - (1 - p) \gamma r_L}{p [1 - (1 - p) (1 - \rho) (1 - \gamma)]} < r_H.$$

Note that $r_{m,\rho}^*$ and $r_{m,\rho}^{**}$ are respectively increasing and decreasing in ρ . Q.E.D.

Proof of Proposition 8: We first show that if $r_k^* < ((k - 1)r_L + r_H)/k$ then it is better to form groups of k projects rather than smaller groups. If this condition is satisfied, the per-project expected bankruptcy losses are given by $(1 - p)^k \gamma r_L$. In groups of smaller size, $m < k$, the minimum loss is given by $(1 - p)^m \gamma r_L$, which is larger.

We now show that if $r_k^* < ((k - 1)r_L + r_H)/k$ and $r_m^* > ((m - 1)r_L + r_H)/m$ for $m > k$ then it is better to form groups of size k rather than groups of larger size. Indeed, the

bankruptcy losses of a group of size $m > k$, are greater or equal to $(1 - p)^m \gamma r_L + m(1 - p)^{m-1} p \gamma ((m - 1)r_L + r_H)/m$ which is in turn greater than $[(1 - p)^m + m(1 - p)^{m-1} p] \gamma r_L$. Then, subtracting the per-project bankruptcy costs of a group of size k , the difference is given by $(1 - p)^k w(p)$ where $w(p) := (1 - p)^{m-k-1} (1 + mp) - 1$. But, $w(0) = 0$, $w(1) < 0$ and $w'(p) := (1 - p)^{m-k-2} [-(m - k)mp + k + 1]$. Given that $w'(0) > 0$ and $w'(1) \leq 0$ there exists a unique $0 < p' \leq 1$ such that $w'(p) > 0$ if $p < p'$ and $w'(p) < 0$ if $p > p'$. Therefore, there exists a unique $0 < p^* < 1$, such that $w(p) > 0$ if $p < p^*$ and $w(p) < 0$ if $p > p^*$. As a result, the per-project bankruptcy costs for a group of m projects are larger than for a group of k projects if $p < p^*$. Q.E.D.

Proof of Proposition 9: The first statement follows from the same argument as in the first part of the proof of Proposition 8.

With respect to the second statement, note that for any discrete symmetric distribution $H((r_H + r_L)/2 - \varepsilon) = 1 - H((r_H + r_L)/2 + \varepsilon)$ for any $\varepsilon > 0$, where the density and distribution functions are defined at the average returns (rather than at the number of projects with high return s as in the text). Given that $H(\cdot)$ is increasing and $h((r_H + r_L)/2) > 0$, we have that $H((r_H + r_L)/2 - \varepsilon) < H((r_H + r_L)/2 + \varepsilon)$ and substituting $H((r_H + r_L)/2 - \varepsilon) < 1/2$ and therefore $H((r_H + r_L)/2 + \varepsilon) > 1/2$. Hence, we have that $H(r) > 1/2$ for any $(r_H + r_L)/2 < r \leq r_H$ and therefore the distribution of the average return of any group of projects for $(r_H + r_L)/2 < r \leq r_H$ is above that of the returns of the projects financed separately, which is equal to $1/2$ for $r_L \leq r < r_H$.

Now, if $r_k^* > (r_H + r_L)/2$ for any group of k projects, then we have that the per project bankruptcy losses is greater than $H((r_H + r_L)/2 + \varepsilon) \gamma r_L$, which is in turn greater than the losses in the case of separate projects, $(1/2) \gamma r_L$. Q.E.D.

Proof of Proposition 10: First statement. Define $g(\delta) := \delta r_H + (1 - \delta) r_L$. We have that $g(p) > 1$ because of the positive net present value condition, and trivially $g(0) = r_L < 1$ and $g'(\delta) > 0$. Then there exists a unique $\delta^* \in (0, p)$ such that $g(\delta^*) = 1$. For a fixed rational number ε (small) define $q := \delta^* + \varepsilon$. Clearly, $q r_H + (1 - q) r_L > 1$.

Take any number of projects n such that nq is an integer number. Suppose that we were to finance all these n projects jointly at an interest rate that avoids bankruptcy when at least nq of them have high returns. This is possible if and only if the per-project repayment satisfies

$$r_n^* \leq q r_H + (1 - q) r_L.$$

Given that the returns recovered in the event of bankruptcy are positive, we have that the equilibrium repayment rate in (4) satisfies

$$r_n^* \leq \frac{1}{1 - H(nq - 1)} < \frac{1}{1 - H(nq)}.$$

From the law of large numbers we have that $H(nq)$ tends to 0 as n grows large (remembering that $q < p$). Therefore r_n^* is bounded above by a number that is arbitrarily close to 1. Given that $qr_H + (1 - q)r_L > 1$, there exists n' such that for all $n > n'$ then r_n^* is such that

$$r_n^* \leq qr_H + (1 - q)r_L,$$

as was to be shown.

Second statement: From the loan described above, the borrower obtains a per-project gross profit

$$\pi_n = \gamma \sum_{k=0}^{nq-1} h(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n} \right) r_L \right] + \sum_{k=nq}^n h(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n} \right) r_L \right].$$

Fix a small rational number ε and an integer n such that $n(p - \varepsilon)$ and $n(p + \varepsilon)$ are integer numbers. Then, given that $q < p - \varepsilon$, and that all terms in the first and in the second sum are positive, we have that

$$\pi_n \geq \sum_{k=n(p-\varepsilon)}^{n(p+\varepsilon)} h(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n} \right) r_L \right].$$

Given that the terms in the second factor in the sum are larger for larger k , the sum is reduced by replacing the summand of a given k by that of $n(p - \varepsilon)$, the smallest term. Then, rearranging, we obtain

$$\pi_n \geq [(p - \varepsilon)r_H + [1 - (p - \varepsilon)]r_L] [H[n(p + \varepsilon)] - H[n(p - \varepsilon)]].$$

From the law of large numbers, $H[n(p + \varepsilon)] - H[n(p - \varepsilon)]$ tends to 1 as n grows. Indeed from Chebyshev's inequality we know that

$$H[n(p + \varepsilon)] - H[n(p - \varepsilon)] \geq 1 - \frac{(p + \varepsilon)(1 - p)}{n\varepsilon^2} - \frac{(1 - p + \varepsilon)p}{n\varepsilon^2} = 1 - \frac{2p(1 - p) + \varepsilon}{n\varepsilon^2}$$

and therefore

$$\pi_n \geq [pr_H + (1 - p)r_L - \varepsilon(r_H - r_L)] \left(1 - \frac{2p(1 - p) + \varepsilon}{n\varepsilon^2} \right).$$

That is for n large, the gross per-project profit differs from the (gross) present value of each

project by an amount that is arbitrarily small, $\varepsilon(r_H - r_L)$. Similarly,

$$\frac{\pi_n}{\pi^*} \geq \left(1 - \frac{\varepsilon(r_H - r_L)}{pr_H + (1-p)r_L}\right) \left(1 - \frac{2p(1-p) + \varepsilon}{n\varepsilon^2}\right)$$

where π^* is equal to first-best gross profits, $\pi^* = pr_H + (1-p)r_L$. Q.E.D.

Proof of Prediction 8: The proof follows directly from Proposition 10. Q.E.D.

Proof of Proposition 11: With separate financing the rate satisfies $pr_i^* + (1-p)[r_L - B(r_L)] = 1$ and therefore the condition is

$$r_i^* := \frac{1 - (1-p)[r_L - B(r_L)]}{1 - (1-p)} < r_H,$$

and the per-project net present value is $pr_H + (1-p)r_L - (1-p)B(r_L) - 1$. Similarly, the condition for obtaining a rate for joint financing that saves both projects when one has low return is given by

$$r_m^* := \frac{1 - (1-p)^2[r_L - B(2r_L)/2]}{1 - (1-p)^2} < \frac{r_H + r_L}{2}, \quad (25)$$

and the net present value is $pr_H + (1-p)r_L - (1-p)^2B(2r_L)/2 - 1$. Finally, the rate for joint financing that saves both projects only when both give high returns is given by

$$r_m^{**} = \frac{1 - (1-p)(p[r_L + r_H - B(r_L + r_H)] - (1-p)[r_L - B(2r_L)/2])}{p^2} < r_H.$$

and the per-project net present value is $pr_H + (1-p)r_L - (1-p)[pB(r_L + r_H) + (1-p)B(2r_L)/2] - 1$.

As in the baseline case, if it is possible to choose, the second rate is better than the third, as the net present value is larger. Separate financing is therefore optimal if the first rate (and therefore also the second) is better than the second, that is if $(1-p)B(2r_L)/2 > B(r_L)$ or equivalently

$$B(2r_L) - 2B(r_L) > \frac{2p}{1-p}B(r_L) \equiv \overline{S}.$$

Similarly, joint financing is optimal if the third rate (and therefore also the second) is better than the first, that is if $pB(r_L + r_H) + (1-p)B(2r_L)/2 < B(r_L)$, or equivalently

$$B(2r_L) - 2B(r_L) < -\frac{2p}{1-p}[B(r_L + r_H) - B(r_L)] \equiv \underline{S}.$$

Finally, if the first rate is better than the third but worse than the second we have, as in our baseline case, that joint financing is optimal if and only if the second rate can be obtained in joint financing, i.e. if condition (25) is satisfied. Q.E.D.

Proof of Proposition 12: We proceed by computing the payoff obtained when using each of the two rates and then we compare the payoffs. If the firm uses $r'_i(\alpha)$ (specified in (6)), the firm obtains, substituting into $(1 - \alpha)E_i$,

$$\frac{(1 - \alpha)(1 - \tau)}{\tau + (1 - \alpha)(1 - \tau)} [pr_H + (1 - p)r_L - 1]. \quad (26)$$

This payoff is decreasing in α , as the firm obtains a fraction of the net present value that corresponds to the (after-tax) equity holding; the remaining part is retained by the government through taxes. Therefore the firm should use the smallest level of equity possible. But, as explained in the text, the firm should use a positive level of equity to satisfy condition (6). Optimally, we have

$$\alpha'_i := \frac{(1 - r_L)}{(1 - \tau) [pr_H + (1 - p)r_L - 1 + (1 - r_L)]}.$$

Provided that $\alpha'_i \leq 1$ ($r'_i(\alpha)$ can be obtained), the firm obtains, substituting into (26),

$$[pr_H + (1 - p)r_L - 1] - \tau p(r_H - r_L). \quad (27)$$

As argued in the text, if the firm uses $r''_i(\alpha)$ (specified in (7)), the optimal amount of equity is $\alpha''_i = 0$. The borrower then obtains

$$(1 - \tau) [pr_H + (1 - p)\gamma r_L - 1]. \quad (28)$$

Comparing the payoffs in each case, (27) and (28), it is optimal for the firm to choose the first over the second rate if and only

$$\tau < \bar{\tau}_i := 1 - \frac{(1 - r_L)}{(1 - \gamma)(1 - p)r_L + (1 - r_L)},$$

that is if bankruptcy costs $(1 - \gamma)$ are high enough and/or taxes are small. Q.E.D.

Proof of Proposition 13: Following the same procedure as in the proof of Proposition 12, we first compute the per-project payoff of the firm when using each of the three types of rate and then we compare these payoffs. If the projects are financed at a rate that avoids bankruptcy altogether, $r'_m(\alpha)$, which is equal to $r'_i(\alpha)$ (as specified in (6)), the firm obtains the same payoff as in the case of separate financing, equal to (27). As in the case of separate financing, the firm needs to use a positive level of equity to obtain this rate, and therefore uses the minimum amount $\alpha'_m > 0$ such that $r'_m(\alpha'_m) = r_L$.

If projects are financed at a rate $r_m''(\alpha)$ (as specified in (8)), the firm obtains

$$(1 - \tau) \left[p^2 r_H + 2p(1 - p) \left(\frac{r_H + r_L}{2} \right) + (1 - p)^2 \gamma r_L - 1 \right] \quad (29)$$

if condition (2) is satisfied and

$$(1 - \tau) \left(p^2 r_H - p^2 \frac{r_H + r_L}{2} \right) + [1 - (1 - p)^2] \frac{r_H + r_L}{2} + (1 - p)^2 \gamma r_L - 1 \quad (30)$$

if condition (2) is not satisfied. If condition (2) is satisfied, the firm does not need to use any equity to obtain r_m'' , and therefore $\alpha_m'' = 0$ and $r_m''(0) = r_m^*$. If condition (2) is not satisfied, the firm needs to use a positive level of equity to obtain r_m'' , and therefore uses the minimum amount $\alpha_m'' > 0$ such that $r_m''(\alpha_m'') = \frac{r_H + r_L}{2}$.

Finally, if projects are financed at a rate $r_m'''(\alpha)$ (specified in (9)), the firm obtains

$$(1 - \tau) \left[p^2 r_H + 2p(1 - p) \gamma \frac{r_H + r_L}{2} + (1 - p)^2 \gamma r_L - 1 \right], \quad (31)$$

and no equity is used, $\alpha_m''' = 0$, as it does not help to reduce the probability of bankruptcy.

We now compare the payoffs in each case. Suppose first that condition (2) is satisfied (part (i) in the statement of the proposition). Then, the payoff when using r_m'' is given by (29). It can be easily checked that this is always greater than the payoff that can be obtained when using r_m''' , (31). Comparing the payoffs when using r_m' with those of using r_m'' , r_m' is optimal if and only if

$$\tau < \bar{\tau}_m^a := 1 - \frac{(1 - r_L)}{(1 - r_L) + (1 - p)^2(1 - \gamma)r_L}.$$

Suppose second that the condition (2) is not satisfied (part (ii) in the statement of the proposition). Then, the payoff when using r_m'' is given by (30). In this case, r_m' is preferred to r_m'' as long as

$$\tau < \bar{\tau}_m^b := 1 - \frac{[1 - (1 - p)^2] \frac{r_H + r_L}{2} + (1 - p)^2 \gamma r_L - r_L}{(1 - \frac{p}{2})p[r_H - r_L]},$$

r_m'' is preferred to r_m''' as long as

$$\tau < \bar{\tau}_m^c := 1 - \frac{1 - [1 - (1 - p)^2] \frac{r_H + r_L}{2} - (1 - p)^2 \gamma r_L}{1 - p^2 \frac{r_H + r_L}{2} - 2p(1 - p) \gamma \frac{r_H + r_L}{2} - (1 - p)^2 \gamma r_L},$$

and r_m' is preferred to r_m''' as long as

$$\tau < \bar{\tau}_m^d := 1 - \frac{(1 - r_L)}{(1 - \gamma) [2p(1 - p) \frac{r_H + r_L}{2} + (1 - p)^2 r_L] + (1 - r_L)}.$$

It can be easily shown that the order of these cutoffs is given by $\bar{\tau}_m^b < \bar{\tau}_m^d < \bar{\tau}_m^c$. Therefore,

we have the optimal choices claimed in the text. Q.E.D.

Proof of Proposition 14: In this proof, we need to compare the payoffs of joint and separate financing. Suppose first that condition (2) is satisfied (part (i) in the statement of the proposition). If r'_m is used for joint financing ($\tau < \bar{\tau}_m^a$), then the payoff with joint and separate financing are the same. If r''_m is used ($\tau > \bar{\tau}_m^a$), then the payoff with joint financing is larger than the payoff of separate financing.

Suppose now that condition (2) is not satisfied (part (ii) in the statement of the proposition). Comparing the cutoffs for joint and separate financing, it is easy to show that $\bar{\tau}_m^b < \bar{\tau}_i < \bar{\tau}_m^c$. Then, we can compare the payoffs under joint and separate financing. First, when r'_m is optimal with joint financing ($\tau < \bar{\tau}_m^b$), the payoffs under joint and separate financing are the same. When r''_m is optimal with joint financing ($\bar{\tau}_m^b < \tau < \bar{\tau}_m^c$), it is straightforward to check that the payoffs are higher under joint financing if r'_i is obtained with separate financing, i.e. $\tau < \bar{\tau}_i$, but are lower if r''_i is obtained with separate financing, i.e. $\tau > \bar{\tau}_i$. Finally, when r'''_m is optimal with joint financing ($\tau > \bar{\tau}_m^c$), joint financing yields a lower payoff than separate financing (under separate financing r''_i would be optimal because $\bar{\tau}_i < \bar{\tau}_m^c$). As in the baseline model, the bankruptcy costs are higher under joint financing because of the risk shift.

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(r_i^*) and jointly (r_n^*) satisfy $r_i^*, r_n^* > w$. Indeed, if $r_n^* < w$, the creditor's profits would be $r_n^* [1 - H(r_n^*)] + wH(r_n^*) < w$. Given that by assumption $w < 1$, the creditor would not be able to recover the initial investment. Applying the same reasoning, we conclude also that $r_i^* < w$ cannot hold. From now on, we thus restrict to repayment rates $r_i^*, r_n^* > w$.

If r_n^* , the lowest r such that $r[1 - H(r)] + wH(r) = 1$, is such that $r_n^* < \mu$, then $r_n^* < r_i^*$. Indeed, even though r_i^* exists by assumption, it is not possible that $r_i^* < r_n^*$ because, by (34) and single-peakedness of the profit function, we have that for $r < r_n^*$, $r[1 - F(r)] + wF(r) < r[1 - H(r)] + wH(r) < r_n^*[1 - H(r_n^*)] + wH(r_n^*) = 1$. As a result, from (32) and monotonicity of F , we conclude that the probability of bankruptcy is lower with joint financing, $H(r_n^*) < F(r_n^*) < F(r_i^*)$.

On the other hand, if r_n^* is such that $r_n^* > \mu$, then $r_n^* > r_i^*$. Indeed, given that the creditor's proceeds at $r = w$ are equal to $w < 1$ and they are higher than 1 at $r = r_n^*$, as $r_n^*[1 - F(r_n^*)] + wF(r_n^*) > r_n^*[1 - H(r_n^*)] + wH(r_n^*) = 1$, by the intermediate value theorem there exists some $r_i^* < r_n^*$ at which $r_i^*[1 - F(r_i^*)] + wH(r_i^*) = 1$. As a result, from (32) and monotonicity of H , we have that the probability of bankruptcy is lower with separate financing, $F(r_i^*) < H(r_i^*) < H(r_n^*)$. Since $F(r) < H(r)$ for $r > \mu$, the net surplus of the borrower is then

$$\int_0^{r_n^*} [1 - H(x)] dx < \int_0^{r_n^*} [1 - F(x)] dx < \int_0^{r_i^*} [1 - F(x)] dx.$$

Therefore financing the two groups separately is optimal.

By single-peakedness, r_n^* is such that $r_n^* > \mu$ if and only if the following two conditions hold

$$r[1 - H(r)] + wH(r)|_{r=\mu} < 1 \text{ and } \left. \frac{\partial}{\partial r} (r[1 - H(r)] + wH(r)) \right|_{r=\mu} > 0,$$

which are equivalent to

$$\mu + w < 2 \text{ and } (\mu - w)h(\mu) < \frac{1}{2}.$$

Substituting for the density h of a normal distribution with mean μ and variance $[1 + \rho(n - 1)]\sigma^2/n$, we obtain

$$\mu + w < 2 \text{ and } \mu - w < \sigma \sqrt{[1 + \rho(n - 1)]\pi/2n}.$$

Note that if this condition is satisfied for n then it is better to finance each half of the n available projects separately rather than all the n projects jointly. Now, if this condition is satisfied for n then it is also be satisfied for $n/2$. As a result, it is optimal to finance each half of the $n/2$ projects separately rather than jointly. Iterating this reasoning, we conclude

that it is optimal to finance all projects separately, as claimed. Q.E.D.

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