Moral Hazard and Debt Maturity

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Abstract

This paper presents a model of the maturity of a bank's uninsured debt. The bank borrows funds to acquire risky assets, and chooses the riskiness of its assets after the borrowing is done. This moral hazard problem leads to an excessive level of risk. Short-term debt may act as a disciplinary device when lenders observe some interim signal of the assets' risk. However, if the signal is noisy short-term debt may lead to inefficient liquidation, so there is a trade-off. The paper characterizes the conditions under which short-term and long-term debt are feasible, and shows circumstances under which only short-term debt is feasible and under which short-term debt dominates long-term debt when both are feasible.

JEL Classification: G21, G32

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"It is difficult to establish any archetype for failure. Banks with high capital ratios imploded while those with lower ratios survived. Plain-vanilla retail banks blew up while some black-box trading shops prospered. Both small and big firms collapsed. Yet there was a common ingredient in most failures: an over-reliance on (short-term) wholesale borrowing." *The Economist*, September 5, 2009.

"Borrowers design their financial structures to their own benefit, and one cannot presuppose that dangerous forms of debt constitute suboptimal liability structures." Jean Tirole, *American Economic Review*, 2003.

1 Introduction

Funding long-term projects with short-term debt risks failure to roll over the debt. Such failure can happen if adverse news about the projects' final payoff arrive at an interim stage, or if short-term lenders have better or more urgent uses for their funds. In that case the projects are liquidated, even when liquidation might be inefficient. Why then fund long-term projects with short-term rather than long-term debt?

Following Diamond and Dybvig (1983), a voluminous literature analyzes the issue focusing on lenders' demand for liquidity. This paper is different. In our model lenders have no demand for liquidity, but they observe some relevant information on the prospects of the project that may lead them to withdraw their funds. But if early liquidation is inefficient, the question about using short-term debt remains. Here is where moral hazard enters the picture. Suppose that borrowers can choose the risk of their projects after the borrowing is done. In such situation, they will have an incentive to take an excessive level of risk. We argue that using short-term may be justified as a way to ameliorate borrowers' risk-shifting incentives.

In our model, the borrowing firm has three attributes of a bank. First, it funds its itself mostly (in the model only) by borrowing. Second, it can easily modify the risk profile of its assets. Third, it invests in financial assets, not real assets that can be redeployed to other

sectors of the economy, which means that their liquidation value is related to (in the model a fraction of) their expected continuation value.

A comparison between short-term and long-term debt entails the analysis of the optimal decision at the outset of the bank's shareholders. At that point they know that if short-term debt is used, they will have to refinance it. We argue that when there is a moral hazard problem in the choice of project risk, the anticipation of the refinancing needs acts as a disciplining device that may render short-term superior to long-term debt financing. So the trade-off is between the disciplining benefits of short-term debt versus the risk of inefficient liquidation that it entails.

The presentation begins with the first-best (equity financing) case, proceeds to the case of a single period debt financing (later re-interpreted as long-term debt), and then presents the case of short-term debt. This model has an intermediate period in which noisy information about the eventual outcome is revealed and, following that revelation, the bank has to repay the initial lenders by issuing another short-term debt. If it cannot refinance the initial debt, the bank is liquidated and the liquidation proceeds go to the lenders.

The main results may be summarized as follows. First, we show that the positive incentive effects of short-term debt only obtain when it is risky, that is, when it implies a positive probability of early liquidation. Second, we show that there are circumstances in which short-term debt may be the only way to secure funding and in which short-term debt may dominate long-term debt, when the latter is feasible. Finally, we show using short-term debt may involve paying an up-front dividend to the bank shareholders.

To explain the intuition for these results is it useful to refer to the seminal paper on credit rationing by Stiglitz and Weiss (1981). They present two models, one based on adverse selection and the other one on moral hazard. In the latter, they show how "higher interest rates induce firms to undertake projects with lower probabilities of success but higher payoffs when successful." We apply this argument to banks instead of firms.

In particular, we build a model in which higher borrowing costs induce banks to undertake investments with lower probabilities of success but higher payoffs when successful. From this perspective, the difference in risk-shifting incentives between long-term and short-term debt

lies in the relevant cost of the bank's borrowing. With debt that matures when the return of the investment is realized (long-term debt) the cost of borrowing reflects the average probability of success, whereas with short-term debt the relevant cost of borrowing reflects the average probability of success conditional on the debt being rolled over at the interim date, because in the other state the bank is liquidated and shareholders get nothing. Since the unconditional probability of success is lower than the probability of success conditional on the good state, it follows that the cost of borrowing will tend to be higher with long-term debt, so the bank will have an incentive to choose riskier investments.

There are two caveats to this argument. First, the information that arrives at the interim stage may be noisy, which reduces the probability of success conditional on the good signal and consequently increases the relevant cost of borrowing with short-term debt. Second, liquidating the investment may be costly. In this case, lenders should be compensated with a higher payoff when the short-term debt is rolled over, which increases the relevant cost of borrowing. Thus, the two key parameters that will determine the optimal maturity structure of the bank's debt are the quality of the lenders' interim information (which in the model will be called q) and the liquidation costs or the complementary recovery rate (which in the model will be called λ). Short-term debt will dominate long-term debt when both q and λ are sufficiently high.

The intuition for the result that short-term debt only makes a difference when it is risky, that is, when it implies a positive probability of early liquidation, should now be clear. If the initial short-term debt is always rolled over, then the unconditional and the conditional probability of success will be the same, and so long-term debt will be equivalent to (safe) short-term debt.

The key role of liquidation at the interim stage also explains the result that using short-term debt may involve paying an up-front dividend to the bank shareholders. Such dividend does not make sense in the case of long-term debt, since it increases the amount due to lenders and consequently worsens the moral hazard problem. But it may be useful in the case of short-term debt in order to guarantee that early liquidation will obtain with positive probability. In other words, paying an up-front dividend raises the hurdle for continua-

tion, which increases the probability of success conditional on continuation and reduces the relevant cost of borrowing for the bank, which accounts for the positive incentive effect.

The financial crisis that began in 2007 has motivated various proposals that aim at reducing banks' short-term wholesale borrowing (such as the Liquidity Coverage Ratio agreed by the Basel Committee on Banking Supervision in December 2010). This paper sounds a word of caution by showing a possible risk-mitigating benefit of short-term debt, which may explain why many banks were increasingly looking to uninsured wholesale funding sources (what Tirole, 2003, calls "dangerous forms of finance") to satisfy funding needs. Such explanation may be important to understand the liability structure of financial institutions and to assess current proposals on the regulation of liquidity risk.

Literature review One risk of short-term funding of long-term projects is that the lenders may find a better or more urgent use of the money and refuse to refinance them. This risk, often called liquidity risk, plays a major role in most papers that analyze short-term funding. The model presented here is an exception in that lenders have no demand for liquidity: we ignore liquidity risk and focus on the possibility that adverse news about the borrower's prospects could make loan rollover impossible. A borrower fails to refinance a short-term loan simply because his project turns out to be weak and its expected payoff is lower than the amount due to the existing lenders. The reason why short-term debt may good is that, aware of the possibility of failure to refinance in the future, the borrower initially chooses a safer project.

Liquidity risk is the focus of the seminal paper by Diamond and Dybvig (1983). They show how banks may efficiently insure this risk, but may be subject to runs by demand depositors suspecting that other depositors may want to withdraw their funds and therefore render the bank illiquid. Our model is closer the work of Jacklin and Bhattacharya (1988) on informationally-based bank runs. But their focus is very different from ours.

There is not much theoretical research on the maturity structure of firms' debt. Diamond (1991) considers an adverse selection model of a firm's choice of the term structure of its borrowing in which variation across borrower quality leads to variation across the optimal

structures of debt maturity. The optimal maturity structure trades off a borrower's preference for short-term debt (due to private information about the future credit rating) against liquidity risk.

Rajan (1992) studies the borrower's choice of creditor between a bank and an arm's-length lender. The bank can lend either short-term or long-term, whereas the arm's-length lender must lend long-term. The preference for bank debt maturity depends on the relative bargaining power of the bank and the borrower after the parties learn the true state of the project. Rajan's main focus is on the effect of lender's type on ex-ante borrower's choice of effort.

Calomiris and Kahn (1991) study a model of bank finance in which the bank can abscond with the funds ex post. The incentive to abscond is greater with lower return realizations. Thus, their focus is on the ex post incentives, the concern being that if the bank does poorly, its shareholders will choose actions that will hurt lenders. In this context it is optimal to use short-term demandable debt, because it gives lenders the option to force liquidation before the absconding is done. In contrast, the focus of our paper is on the role of short-term debt as a disciplinary device on ex ante risk-taking.

Building on Myers (1977), Flannery (1994) points out that financial intermediaries can easily modify the risk profile of their assets. Contracts preventing such modifications are difficult to write and enforce, so a reasonable alternative for the intermediary is to issue short-term debt. The need to roll over the debt will act as a disciplinary device that may restrain the intermediary from increasing the risk of its assets to benefit its shareholders at the expense of its creditors. (A similar intuition is entertained also by Barnea, Haugen and Senbet, 1980.) The formal model in the present paper shows circumstances under which Flannery's intuition is valid and suggests further implications of that intuition.

Leland (1998) offers a dynamic model of the joint determination of capital structure, including debt maturity, and investment risk when debt entails tax benefits in good times and default costs in bad times. An added ingredient is agency costs, the ability of shareholders to choose the risk of the risk at which the firm's asset value evolves. Leland's model abstracts from the possibility that investment risk and expected payoff are related, which is a key

ingredient of our model.

Risky short-term debt is inferior to long-term debt in the model of Diamond and He (2010), due to a debt overhang problem that may lead shareholders to forego positive NPV investments. Similarly to their model, this paper considers the comparison of the investment incentives of short-term and long-term debt. The distinguishing feature of our model is the presence of moral hazard: shareholders choose the riskiness of the bank's assets after the initial debt has been issued. The need to roll over the short-term debt mitigates the moral hazard, thereby conferring special value to short-term financing, to the extent that under some circumstances it may dominate long-term financing.

Huang and Ratnovski (2010) examine the trade-off between the bright side (efficient liquidation) and the dark side (inefficient liquidation) of banks' short-term financing, showing that the dark side dominates when the bank assets are more tradable, leading to more public signals and lower liquidation costs. However, the effect of short-term financing on ex ante risk-shifting incentives is not analyzed.

Structure of paper The paper is structured as follows. Section 2 presents the basic model. Section 3 characterizes the optimal contract with long-term debt. Section 4 introduces an interim date where some public information about the final return of the bank's investment is revealed, and characterizes the optimal contracts with safe and risky short-term debt. Section 5 illustrates the results using numerical solutions for a simple parameterization of the model. Section 6 contains a few interesting extensions of the analysis, and Section 7 concludes.

2 The Basic Model

Consider an economy with two dates ($t = 0, 1$), a risk-neutral *bank*, and a large number of risk-neutral (wholesale) *lenders.* Both the lenders and the bank have a discount rate that is normalized to zero.

At $t = 0$ the bank can invest one unit of funds in a *risky asset* that yields a random

payoff R at $t = 1$. The probability distribution of R is

$$R = \begin{cases} R_0 & \text{with probability } 1-p, \\ R_1 & \text{with probability } p, \end{cases} \tag{1}$$

where $R_0 = 0$ and $R_1 > 1$. Thus, $1 - p$ represents the riskiness of the bank's asset. The bank has no capital and can only fund its investment in the risky asset by borrowing from the lenders. In principle, the bank could raise more than one unit of funds and pay out the excess up-front as a dividend D to the bank shareholders. This possibility turns out to be useful in some circumstances discussed below.

To introduce a moral hazard problem, we assume that $p \in [0, 1]$ is a parameter chosen by the bank at $t = 0$ *after* it has raised the necessary funds, and that the high payoff R_1 is a decreasing function of p, that is

$$R_1 = R(p), \tag{2}$$

with $R'(p) < 0$. Therefore, higher risk (lower p) is associated with a higher success payoff.[1]

We further assume that $R(p)$ is concave and satisfies $R(1)+R'(1) \leq 0$. These assumptions imply that the expected payoff of the risky asset, $E(R) = pR(p)$, reaches a maximum at the *first-best success probability* $p_{FB} \in (0, 1]$ that is characterized by the condition:

$$(p_{FB}R(p_{FB}))' = 0. \tag{3}$$

To see this, notice that the first derivative $(pR(p))' = pR'(p) + R(p)$ equals $R(0) > 0$ for $p = 0$ and $R(1) + R'(1) \leq 0$ for $p = 1$, and the second derivative satisfies $(pR(p))'' = 2R'(p) + pR''(p) < 0$. Moreover, we have $p_{FB}R(p_{FB}) \geq R(1) > 1$, so in the absence of informational problems the bank's investment has a positive NPV.

An example The linear payoff function

$$R(p) = a(2 - p) \tag{4}$$

[1] This setup is borrowed from Allen and Gale (2000) and is essentially the moral hazard model in Stiglitz and Weiss (1981). A possible alternative would be the model in Holmström and Tirole (1997), where the success payoff R_1 is fixed and the bank gets private benefits $\Pi(p)$. The two approaches may be related by assuming that $\Pi(p) = p[R(p) - R_1]$.

satisfies the required properties and will be used to derive the numerical results in Section 5. Parameter a characterizes the profitability of the bank's investment. For this function, we have $(pR(p))' = 2a(1-p)$, which implies $p_{FB} = 1$. Thus, the first-best would be a safe investment with $R(p_{FB}) = a$.

3 Long-term Debt

Suppose that the bank is funded with (long-term) debt that matures at $t = 1$, and let B denote the face value of the debt that lenders receive in exchange for $1 + D$ funds provided at $t = 0$, where $D \geq 0$ is the dividend paid up-front to the shareholders.

Definition 1 *A contract with long-term debt specifies the initial dividend D paid to the shareholders at $t = 0$ and the face value B of the debt payable to the lenders at $t = 1$. Such contract determines the probability of success p chosen by the bank at $t = 0$.*

Definition 2 *An optimal contract with long-term debt is a triple (D_{LT}, B_{LT}, p_{LT}) that solves the problem:*

$$\max_{(D,B,p)} \left[D + p\left(R(p) - B\right)\right] \tag{5}$$

subject to the incentive compatibility constraint:

$$p_{LT} = \arg\max_{p} \left[p\left(R(p) - B_{LT}\right)\right], \tag{6}$$

and the participation constraint:

$$p_{LT} B_{LT} \geq 1 + D_{LT}. \tag{7}$$

The incentive compatibility constraint (6) characterizes the bank's choice of p given the promised repayment B_{LT}, and the participation constraint (7) ensures that the lenders get the required expected return on their investment.

The solution to (6) is characterized by the first-order condition:

$$(pR(p))' = B. \tag{8}$$

Since the left-hand side of (8) is decreasing in p, it follows that higher values of B are associated with lower values of p, that is $dp/dB < 0$. This is the standard risk-shifting effect that obtains under debt finance. Moreover, using the characterization (3) of the first-best probability of success p_{FB}, it follows that $p_{LT} < p_{FB}$, that is the bank will take on more risk than in the first-best.

The following result shows that raising more than one unit of funds and paying out the excess up-front as a dividend D worsens the moral hazard problem and hence it will not be optimal when funding the bank with long-term debt. For this reason, a contract with long-term debt will simply be written as (B_{LT}, p_{LT}).

Lemma 1 *The optimal contract with long-term debt satisfies $D_{LT} = 0$.*

Proof In the optimal contract the participation constraint (7) must be satisfied with equality, for otherwise the dividend D could be increased without changing B and p, improving the bank's payoff (5). Then suppose that $D = pB - 1 > 0$ and consider the effect on the bank's payoff of a change in the face value B:

$$\frac{d}{dB}\left[p\left(R(p) - B\right) + (pB - 1)\right] = \frac{d}{dB}\left[pR(p) - 1\right] = B\frac{dp}{dB} < 0,$$

where we have used the first-order condition (8) and the result $dp/dB < 0$. This means that whenever the constraint $D \geq 0$ is not binding, reducing the face value B would increase the bank's payoff, which proves the result. $\square$

Lemma 1 implies that the participation constraint (7) may be written as $pB = 1$. Solving for B in this expression and substituting it into the first-order condition (8) gives the condition

$$H(p) = 1, \tag{9}$$

where

$$H(p) = p\left(pR(p)\right)'. \tag{10}$$

Since $(pR(p))'$ is positive for $p < p_{FB}$, it follows that the function $H(p)$ is positive for $0 < p < p_{FB}$, and satisfies $H(0) = H(p_{FB}) = 0$.

The equation $H(p) = 1$ may have no solution, a single solution, or multiple solutions. In the first case, financing the bank with long-term debt will not be feasible: the bank's risk-shifting incentives are so strong that the lenders' participation constraint cannot be satisfied. In the second case, the single solution characterizes the optimal contract with long-term debt. And in the third case, the following result shows that the optimal contract is characterized by the solution with the highest probability of success.

Proposition 1 *Financing the bank with long-term debt is feasible if the equation* $H(p) = 1$ *has a solution, in which case* (B_{LT}, p_{LT})*, where* $B_{LT} = 1/p_{LT}$ *and*

$$p_{LT} = \max\{p \in (0, p_{FB}) \mid H(p) = 1\}, \tag{11}$$

is the optimal contract with long-term debt.

Proof Suppose that there exist p_1 and p_2, with $p_1 < p_2$, such that $H(p_1) = H(p_2) = 1$. The contract with $B_1 = 1/p_1$ is dominated by the contract with $B_2 = 1/p_2$ because the fact that the function $pR(p)$ is increasing in p in the interval $(0, p_{FB})$ implies that the corresponding bank's payoffs satisfy:

$$p_1 \left(R(p_1) - B_1\right) = p_1 R(p_1) - 1 < p_2 R(p_2) - 1 = p_2 \left(R(p_2) - B_2\right).$$

Hence if equation (9) has multiple solutions, the optimal contract with long-term debt is characterized by the one with the highest probability of success. □

Summing up, it will be possible to fund the bank with long-term debt if the function $H(p)$ takes values greater than or equal to 1 somewhere in the interval $(0, p_{FB})$. In this case, the bank's payoff will be:

$$\pi_{LT} = p_{LT} R(p_{LT}) - 1. \tag{12}$$

This payoff will be compared with the one corresponding to the optimal contract with short-term debt below.

An example (continued) For the payoff function $R(p) = a(2 - p)$ we have:

$$H(p) = 2ap(1 - p), \tag{13}$$

so solving for the optimal contract with long-term debt gives:

$$p_{LT} = \frac{1}{2}\left(1 + \sqrt{\frac{a-2}{a}}\right) \tag{14}$$

The term inside the square root will be non-negative if $a \geq 2$. Hence, financing the bank with long-term debt requires that the profitability of the bank's investment be sufficiently high. Figure 1 represents the function $H(p)$ and the determination of p_{LT} and p_{FB} for $a = 3.125$ (in which case $p_{LT} = 0.8$). The probability of success p_{LT} is increasing in parameter a , with $\lim_{a\to\infty} p_{LT} = p_{FB} = 1$, and the face value B_{LT} is decreasing in a, with $\lim_{a\to\infty} B_{LT} = 1$.

4 Short-term Debt

To introduce short-term debt we consider an *interim date* $t = 1/2$ at which the initial debt matures and may or may not be rolled over until the terminal date $t = 1$. Moreover, to have some meaningful difference between short-term and long-term debt some information about the prospects of the bank's investment must be revealed at the roll-over date.

In particular, we assume that at $t = 1/2$ the lenders observe a *public signal* $s \in \{s_0, s_1\}$ on the payoff of the bank's risky asset. Based on this signal, they decide whether to refinance the bank. If they do, final payoffs will be obtained at $t = 1$. If they do not, the bank will be liquidated at $t = 1/2$ and the initial lenders will receive the liquidation value L of the bank's asset.

Following Repullo (2005), we assume that the signal s observed by the lenders at the interim date $t = 1/2$ satisfies:

$$\Pr(s_0 \mid R_0) = \Pr(s_1 \mid R_1) = q,$$

where parameter $q \in [1/2, 1]$ describes the quality of the lenders' information.[2,3] This information is only about whether the final payoff R of the bank's risky asset will be low (R_0) or

[2] Note that s is not a signal of the bank's action at $t = 0$ (the choice of p) but of the consequences of such action at $t = 1$ (the final payoff R). The distinction between signals on actions and signals on the consequences of actions is due to Prat (2005).

[3] More generally, we could have $\Pr(s_0 \mid R_0) \neq \Pr(s_1 \mid R_1)$, but then we would have two parameters to describe the quality of the lenders' information.

high (R_1), and not about the particular value $R(p)$ taken by the high payoff. By Bayes' law:

$$\Pr(R_1 \mid s_0) = \frac{\Pr(R_1)\Pr(s_0 \mid R_1)}{\Pr(s_0)} = \frac{p(1-q)}{p+q-2pq}, \tag{15}$$

and

$$\Pr(R_1 \mid s_1) = \frac{\Pr(R_1)\Pr(s_1 \mid R_1)}{\Pr(s_1)} = \frac{pq}{1-p-q+2pq}. \tag{16}$$

When $q = 1/2$ we have $\Pr(R_1 \mid s_0) = \Pr(R_1 \mid s_1) = p$, so the signal is uninformative. When $q = 1$ the posterior probabilities satisfy $\Pr(R_1 \mid s_0) = 0$ and $\Pr(R_1 \mid s_1) = 1$, so the signal completely reveals whether the payoff R will be R_0 or R_1. In general, when $1/2 < q < 1$ (and $0 < p < 1$) one can check that $\Pr(R_1 \mid s_0) < p < \Pr(R_1 \mid s_1)$.[4] For this reason, the states corresponding to observing signals s_0 and s_1 will be called the bad and the good state, respectively.

We assume that the *liquidation value* L of the bank's asset at the interim date $t = 1/2$ satisfies:

$$L = \lambda E(R \mid s),$$

where parameter $\lambda \in [0,1]$ describes the *recovery rate* of the value of the investment. Thus, $(1-\lambda)E(R \mid s)$ are the liquidation costs of the bank's asset. Notice that for any $\lambda < 1$ liquidating the bank at $t = 1/2$ will be inefficient.

Thus, compared to the case of long-term debt, the model of short-term debt involves two additional parameters, namely the quality q of the lenders' interim information and the recovery rate λ of the bank's investment when it is liquidated early.

Suppose that the bank is funded with short-term debt that matures at the interim date $t = 1/2$, and let M denote the face value of the debt that lenders receive in exchange for $1 + D$ funds provided at $t = 0$, where as before $D \geq 0$ is the dividend paid up-front to the shareholders.

At $t = 1/2$ the bank will try to issue new debt, payable at $t = 1$, in order to repay the initial lenders. The face value of this debt will naturally depend on the signal s observed by the lenders at the interim date. Let N_s denote the face value of the debt that lenders receive in exchange for funding the repayment M of the initial debt when the signals is s.

[4] Note that both inequalities are satisfied if $p(1-p)(2q-1) > 0$.

The decision to roll over the initial debt depends on the corresponding conditional probability of success of the investment, $\Pr(R_1 \mid s_0)$ or $\Pr(R_1 \mid s_1)$. As stated in (15) and (16), these posterior probabilities depend on the quality of the signal q, which is known, and the prior probability p, which is not. Hence, the interim lenders will have to decide on the basis of the value $\widehat{p}$ that they conjecture the bank chose at $t = 0$. Under rational expectations, the conjectured $\widehat{p}$ must be equal to the value of p chosen by the bank. Let $\widehat{\Pr}(R_1 \mid s_0)$ or $\widehat{\Pr}(R_1 \mid s_1)$ denote the posterior probabilities corresponding to the prior probability $\widehat{p}$.

At the interim date $t = 1/2$, the lenders will roll over the bank's initial debt M in state s if it satisfies:

$$\widehat{E}(R \mid s) = \widehat{\Pr}(R_1 \mid s)R(\widehat{p}) \geq M,$$

that is, if the conjectured expected value of the bank's asset is greater than or equal to the face value M of the debt to be refinanced. In this case, there exists a face value $N_s(\widehat{p}) \leq R(\widehat{p})$ of the new debt that satisfies the interim lenders' participation constraint:

$$\widehat{\Pr}(R_1 \mid s)N_s(\widehat{p}) = M.$$

From here it follows that, if the initial debt is rolled over in state s, the face value of the debt issued at the interim debt will be:

$$N_s(\widehat{p}) = \frac{M}{\widehat{\Pr}(R_1 \mid s)} \tag{17}$$

It should be noted that when $\widehat{E}(R \mid s) < M$ the initial debt is not rolled over, in which case the initial lenders expect to get $\widehat{L} = \lambda \widehat{E}(R \mid s) < \widehat{E}(R \mid s)$. The expected loss of $(1 - \lambda)\widehat{E}(R \mid s)$ could be avoided if lenders could renegotiate down their claim M, so we are implicitly assuming that such renegotiation is impossible (for example, because lenders are dispersed).

We are assuming, without loss of generality, that no dividend is paid to the shareholders at the interim date $t = 1/2$. As we will see below, paying a dividend at the initial date $t = 0$ changes the face value M of the initial debt, and may have a positive effect on the bank's choice of p. Paying a dividend at the interim date $t = 1/2$ changes the face value N_s of the

interim debt, but at this point p has already been chosen, so there is no incentive effect.[5]

To describe the refinancing decision at $t = 1/2$ it is convenient to introduce the following indicator function:

$$I(x) = \begin{cases} 1 & \text{if } x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

For each state s, variable x will denote the difference between the conjectured expected value of the bank's asset and the face value of the debt to be refinanced:

$$x_s = \widehat{E}(R \mid s) - M.$$

The initial debt M will be rolled over in state s if $x_s \geq 0$, so $I(x_s) = 1$. Otherwise, $I(x_s) = 0$ and the bank will be liquidated.

The initial lenders' participation constraint may be written as

$$\varphi(\widehat{p}, M) = 1 + D, \tag{18}$$

where

$$\varphi(\widehat{p}, M) = \sum\nolimits_{s=s_0,s_1} \widehat{\Pr}(s) \left[I(x_s)M + (1 - I(x_s))\lambda \widehat{E}(R \mid s) \right]. \tag{19}$$

When $I(x_s) = 1$, that is when $\widehat{E}(R \mid s) \geq M$, the initial debt is rolled over and the initial lenders are repaid M. When $I(x_s) = 0$, that is when $\widehat{E}(R \mid s) < M$, the bank is liquidated and the lenders get the liquidation value $L = \lambda \widehat{E}(R \mid s)$. The participation constraint (18) is written as an equality, because otherwise the dividend D could be increased without changing the bank's incentives, improving its payoff.

Taking into account the interim lenders' refinancing decision, the bank's payoff for given values of the dividend D and the face value of the initial debt M may be written as

$$\pi(D, M, p, \widehat{p}) = D + \sum\nolimits_{s=s_0,s_1} \Pr(s) I(x_s) \Pr(R_1 \mid s) \max\left\{R(p) - N_s(\widehat{p}), 0\right\}, \tag{20}$$

where $N_s(\widehat{p})$ is given by (17). When $I(x_s) = 1$, the initial debt is rolled over and the shareholders' payoff equals the initial dividend D plus the expected continuation payoff

[5] Note that we are also assuming that the shareholders cannot inject equity at $t = 1/2$. Such funds could come from saving (part of) a positive initial dividend D.

$\Pr(R_1 \mid s) \max\{R(p) - N_s(\widehat{p}), 0\}$. When $I(x_s) = 0$, the bank is liquidated and the shareholders only get the initial dividend D.

Definition 3 *A contract with short-term debt specifies the initial dividend D paid to the shareholders at $t = 0$ and the face value M of the initial debt payable to the lenders at $t = 1/2$. Such contract determines the probability of success p chosen by the bank at $t = 0$, the roll-over decision $I(x_s)$ at $t = 1/2$, and the face value of the interim debt $N_s(\widehat{p})$ if the initial debt is rolled over in state s.*

Definition 4 *An optimal contract with short-term debt is a triple (D_{ST}, M_{ST}, p_{ST}) that solves the problem:*

$$\max_{(D,M,p,\widehat{p})} \pi(D, M, p, \widehat{p}) \tag{21}$$

subject to the incentive compatibility constraint:

$$p_{ST} = \arg\max_{p} \pi(D_{ST}, M_{ST}, p, \widehat{p}), \tag{22}$$

the initial lenders' participation constraint:

$$\varphi(\widehat{p}, M_{ST}) = 1 + D_{ST}, \tag{23}$$

and the rational expectations constraint:

$$\widehat{p} = p_{ST}. \tag{24}$$

The incentive compatibility constraint (22) characterizes the bank's choice of p given the promised interim repayment M and the roll-over decision implied by the lenders' conjecture $\widehat{p}$ of the value of p chosen by the bank. The participation constraint (23) ensures that the initial lenders get the required expected return on their investment. Finally, the rational expectations constraint (24) requires that the conjectured $\widehat{p}$ equals the value p_{ST} chosen by the bank in the optimal contract.

There are two possible types of optimal contracts with short-term debt: one in which the initial debt is safe, in the sense that the initial lenders are always fully repaid, and another one in which the initial debt is risky, in the sense that the initial lenders are fully repaid in the good state s_1 and the bank is liquidated in the bad state s_0. The next subsections offer characterizations of the optimal contracts with safe and risky short-term debt.

4.1 Safe short-term debt

The easier case to analyze is that of safe short-term debt. This case is also less interesting because, as will be shown below, to every optimal short-term debt contract there corresponds an optimal long-term debt contract with the same success probability p and the same payoff for the bank (but not vice-versa). The safe short-term debt contract is rolled over into another short-term debt contract. The face value of the latter is state-dependent and, regardless of the information revealed at the interim date, the new debt is always risky (since the payoff of the bank's investment is risky).

With safe short-term debt, we have $I(x_s) = 1$, so by (19) we have $\varphi(\widehat{p}, M) = M$. Hence, the initial lenders' participation constraint (18) reduces to:

$$M = 1 + D.$$

Using the definition (17) of $N_s(\widehat{p})$ and the expressions (15) and (16) of $\Pr(R_1 \mid s_0)$ and $\Pr(R_1 \mid s_1)$ the bank's payoff (20) becomes:

$$\begin{aligned}
\pi(D, M, p, \widehat{p}) &= D + p(1-q)\left[R(p) - \frac{\widehat{p} + q - 2\widehat{p}q}{\widehat{p}(1-q)}M\right] + pq\left[R(p) - \frac{1 - \widehat{p} - q + 2\widehat{p}q}{\widehat{p}q}M\right] \\
&= D + p\left[R(p) - \frac{M}{\widehat{p}}\right].
\end{aligned}$$

From here it follows that the first-order condition that characterizes the bank's optimal choice of p is:

$$(pR(p))' = \frac{M}{\widehat{p}},$$

Substituting the participation constraint $M = 1 + D$ into the first-order condition, taking into account the rational expectations constraint $\widehat{p} = p$, and using the definition (10) of $H(p)$ gives:

$$H(p) = 1 + D.$$

For $D = 0$ this is identical to the condition (9) that characterizes the optimal contract with long-term debt. And for the same incentive reasons as before, in the optimal contract with safe short-term debt there should be no up-front dividend, so a contract with safe short-term

debt will simply be written as $(1, p_{ST})$. Therefore, the candidate optimal contract with safe short-term debt is $(1, p_{ST}) = (1, p_{LT})$, where p_{LT} is the probability of success in the optimal contract with long-term debt defined in (11).

However, for $(1, p_{LT})$ to be an optimal contract with safe short-term debt it must be the case that the initial debt is refinanced in the bad state s_0. Using the rational expectations constraint $\widehat{p} = p_{LT}$, this gives the condition:

$$E(R \mid s_0) = \frac{p_{LT}(1-q)}{p_{LT} + q - 2p_{LT}q} R(p_{LT}) \geq 1. \tag{25}$$

Since $E(R \mid s_1) > E(R \mid s_0)$, this condition implies that the initial debt is also refinanced in the good state s_1. For $q = 1/2$ (uninformative signal) the condition reduces to $p_{LT}R(p_{LT}) \geq 1$, which holds if long-term financing is feasible. For $q = 1$ (perfectly informative signal) the condition is never satisfied, because the left-hand side of the inequality is zero. Since $\Pr(R_1 \mid s_0)$ is decreasing in q, there will be an intermediate value of q for which the constraint is satisfied with equality. Solving for q in (25), the condition that guarantees that the initial debt is refinanced in the bad state s_0 becomes $q \leq q(p_{LT})$, where

$$q(p) = \frac{p(R(p) - 1)}{1 + p(R(p) - 2)}. \tag{26}$$

Hence, we have the following result.

Proposition 2 *Financing the bank with safe short-term debt is feasible if financing the bank with long term debt if feasible and $q \leq q(p_{LT})$, where p_{LT} is the probability of success in the optimal contract with long-term debt defined in* (11), *in which case* $(1, p_{ST}) = (1, p_{LT})$ *will be the optimal contract with safe short-term debt.*

Proof By Proposition 1, if financing the bank with long-term debt is feasible, then the optimal contract with long-term debt is characterized by highest solution p_{LT} to the equation $H(p) = 1$. By our previous discussion, this solution also characterizes the optimal contract with safe short-term debt if the initial debt is refinanced in the bad state s_0, that is if $q \leq q(p_{LT})$. If this condition is violated, one can show that no other solution to the equation $H(p) = 1$ will satisfy it. Let $p_1 < p_{LT}$ be one such solution. To prove that $q(p_1) < q(p_{LT})$ it

suffices to show that for $p_1 \leq p \leq p_{LT}$ we have:

$$\frac{dq(p)}{dp} = \frac{(1-p)\,(pR(p))' + pR(p) - 1}{\left[1 + p(R(p) - 2)\right]^2} > 0.$$

But the first term in the numerator of this expression is positive because $(pR(p))' > 0$ for $p < p_{FB}$, and the second term is greater than one because $pR(p) \geq p_1 R(p_1) > 1$ for $p \geq p_1$. Moreover, paying an up-front dividend D will not help with this constraint, since the highest solution to the equation $H(p) = 1 + D$ is decreasing in D. $\square$

Proposition 2 shows that safe short-term debt is viable only if the quality q of the lenders' information is not too high. The intuition for this result is clear. When q is close to 1, observing the bad state s_0 means that the conditional expected value of the bank's asset is close to zero, so the initial debt will not be refinanced. On the other hand, since the upper bound $q(p_{LT})$ is strictly greater than 1/2,[6] when q is close to 1/2 funding the bank with safe short-term debt will be feasible (as long as funding it with long-term debt is).

Summing up, using safe short-term debt does not add anything relative to using long-term debt. Thus, the only possible role of short-term debt is when it is risky.

An example (continued) For the payoff function $R(p) = a(2-p)$, the optimal long-term contract is characterized by the probability of success p_{LT} in (14). This will also characterize the optimal contract with safe short-term debt if the quality of the lenders' information satisfies $q \leq q(p_{LT})$. For example, for $a = 2$ (the minimum value that ensures that the equation $H(p) = 1$ has a solution) we have $p_{LT} = 0.5$ and $q(p_{LT}) = 2/3$. In this case, values of q higher than 2/3 imply that $(1, p_{LT}) = (1, 0.5)$ will not be a feasible contract with safe short-term debt, because the initial debt will not be refinanced in the bad state s_0.

As noted above, the short-term debt issued after the roll-over of the initial debt is no longer safe. For the case $a = 2$, taking $q = 0.6 < 2/3 = q(p_{LT})$, and substituting $M = 1$, $p_{LT} = 0.5$, and $q = 0.6$ into (17) we get $N_{s_0} = \left[\Pr(R_1 \mid s_0)\right]^{-1} = 2.5$ and $N_{s_1} = \left[\Pr(R_1 \mid s_1)\right]^{-1} = 1.67$. Thus, in both states the bank pays a premium over the riskless rate to cover the default risk, which is higher in the bad state s_0.

[6] It can be easily checked that $q(p) > 1/2$ if and only if $pR(p) > 1$.

4.2 Risky short-term debt

When the initial debt is risky, the initial lenders are only repaid in the good state s_1, and the bank is liquidated in the bad state s_0, in which case they anticipate getting a fraction λ of the conjectured expected value of the bank's asset $\widehat{E}(R \mid s_0)$.

With risky short-term debt, the initial lenders' participation constraint (18) becomes:

$$\varphi(\widehat{p}, M) = \widehat{\Pr}(s_0)\lambda\widehat{E}(R \mid s_0) + \widehat{\Pr}(s_1)M = 1 + D.$$

But since

$$\widehat{\Pr}(s_0)\widehat{E}(R \mid s_0) = \widehat{\Pr}(s_0)\widehat{\Pr}(R_1 \mid s_0)R(\widehat{p}) = \widehat{\Pr}(s_0 \mid R_1)\widehat{\Pr}(R_1)R(\widehat{p}) = (1-q)\widehat{p}R(\widehat{p})$$

and

$$\widehat{\Pr}(s_1) = 1 - \widehat{p} - q + 2\widehat{p}q,$$

the constraint may be written as:

$$\varphi(\widehat{p}, M) = \lambda(1-q)\widehat{p}R(\widehat{p}) + (1 - \widehat{p} - q + 2\widehat{p}q)M = 1 + D. \tag{27}$$

Using the definition (17) of $N_s(\widehat{p})$ and the expression (16) of $\Pr(R_1 \mid s_1)$, the bank's payoff (20) simplifies to:

$$\pi(D, M, p, \widehat{p}) = D + pq\left[R(p) - \frac{1 - \widehat{p} - q + 2\widehat{p}q}{\widehat{p}q}M\right], \tag{28}$$

From here it follows that the first-order condition that characterizes the bank's optimal choice of p is:

$$(pR(p))' = \frac{1 - \widehat{p} - q + 2\widehat{p}q}{\widehat{p}q}M. \tag{29}$$

Solving for M in the participation constraint (27) and substituting it into the first-order condition (29), taking into account the rational expectations constraint $\widehat{p} = p$, and using the definition (10) of $H(p)$ gives:

$$H(p) = F(p, q, \lambda, D), \tag{30}$$

where

$$F(p, q, \lambda, D) = \frac{1 + D - \lambda(1-q)pR(p)}{q}. \tag{31}$$

Since $pR(p)$

It should be noted that this condition may be written with a weak inequality, because when $E(R \mid s_0) = M$ the face value $N_{s_0}(p)$ of the new debt issued at $t = 1/2$ equals $R(p)$, in which case the shareholders' stake is the same as in the case of liquidation (that is, zero). For $q = 1$ (perfectly informative signal) the condition is always satisfied, because the left-hand side of the inequality is zero. For $q = 1/2$ (uninformative signal) the condition implies $E(R \mid s_1) = E(R \mid s_0) \leq M$, so the bank would also be liquidated in the good state s_1, which contradicts our definition of risky short-term debt.

Solving for M in the participation constraint (27) and substituting it into the no refinancing in the bad state condition (33), and taking into account the rational expectations constraint $\widehat{p} = p$, gives:

$$\frac{p(1-q)}{p+q-2pq}R(p) \leq \frac{1+D-\lambda(1-q)pR(p)}{1-p-q+2pq}.$$

This simplifies to:

$$G(p,q,\lambda) \leq 1+D, \tag{34}$$

where

$$G(p,q,\lambda) = \left[\frac{1}{p+q-2pq} - (1-\lambda)\right](1-q)pR(p). \tag{35}$$

This condition may not be satisfied for $D = 0$ and p equal to the highest solution of the equation $H(p) = F(p,q,\lambda,0)$. The next result shows that in such case it could be possible to finance the bank with risky short-term debt by paying an up-front dividend $D > 0$ and consequently raising the face value M of the initial debt so that it will not be rolled over in the bad state s_0.

Proposition 3 *Financing the bank with risky short-term debt is feasible if the equation $H(p) = F(p,q,\lambda,D)$ has a solution for some $D \geq 0$ that satisfies $G(p,q,\lambda) \leq 1+D$, in which case (D_{ST}, M_{ST}, p_{ST}), where*

$$\begin{aligned} D_{ST} &= \max\{G(p_{ST},q,\lambda)-1,0\}, && (36)\\ M_{ST} &= \frac{1+D_{ST}-\lambda(1-q)p_{ST}R(p_{ST})}{1-p_{ST}-q+2p_{ST}q}, \text{ and} && (37)\\ p_{ST} &= \max\{p \in (0,p_{FB}) \mid H(p) = F(p,q,\lambda,D) \text{ and } G(p,q,\lambda) \leq 1+D\} && (38)\end{aligned}$$

is the optimal contract with risky short-term debt.

Proof Condition $H(p) = F(p, q, \lambda, D)$ characterizes the values of p and D that satisfy the bank's incentive compatibility constraint and the lenders' participation constraint. Condition $G(p, q, \lambda) \leq 1 + D$ characterizes the the values of p and D for which the initial debt will not be rolled over in the bad state s_0. The set of feasible contracts with risky short-term debt are those that satisfy both conditions.

Since the bank's payoff (32) is increasing in p, the optimal contract will be characterized by the highest value of p that satisfies these two conditions, which gives (38). The optimal value of the initial dividend D will be zero when $G(p, q, \lambda) \leq 1$ for the highest value of p that satisfies $H(p) = F(p, q, \lambda, 0)$, and it will be $G(p, q, \lambda) - 1$ when this is not the case. Note that when the dividend D is positive, it cannot be the case that the constraint $G(p, q, \lambda) \leq 1+D$ is satisfied with a strict inequality, because then it would be possible to find a feasible contract with a higher p. Finally, the face value M_{ST} of the initial debt in the optimal contract is obtained by solving for M in the participation constraint (27), taking into account the rational expectations constraint $\widehat{p} = p$. □

Proposition 3 shows that the feasibility of funding the bank with risky short-term debt depends in a somewhat complex manner on the quality of the lenders' information q and the recovery rate λ of the value of the investment when the bank is liquidated at $t = 1/2$. Interestingly, the optimal contract may involve raising more than the unit cost of the investment at $t = 0$ and paying the difference as an up-front dividend $D > 0$.

To explain the characterization of the optimal contract with risky short-term debt, and to derive some analytical results on the effect of changes in parameters q and λ, it is useful to define the following functions:

$$D_{HF}(p, q, \lambda) = qH(p) + \lambda(1 - q)pR(p) - 1, \text{ and} \tag{39}$$

$$D_G(p, q, \lambda) = \max\{G(p, q, \lambda) - 1, 0\}. \tag{40}$$

The function $D_{HF}(p, q, \lambda)$ is obtained by solving for D in the equation $H(p) = F(p, q, \lambda, D)$. By Proposition 3, the function $D_G(p, q, \lambda)$ characterizes the up-front dividend in the optimal

contract. Increases in p have an ambiguous effect on $D_{HF}(p,q,\lambda)$ (since $H(p)$ is satisfies $H(0) = H(p_{FB}) = 0$ and $H(p) > 0$ for $p \in (0, p_{FB})$), and increase the value of $D_G(p,q,\lambda)$ in the range for which $D_G(p,q,\lambda) > 0$ (since the derivative of the function $G(p,q,\lambda)$ in (35) with respect to p is positive).

The results in Proposition 3 can now be restated as follows. The optimal contract with risky short-term debt is characterized by the highest value of p that satisfies $H(p) = F(p,q,\lambda,D)$ and $G(p,q,\lambda) \leq 1 + D$, for some $D \geq 0$. Therefore, finding the optimal contract is equivalent to finding the highest value of p that satisfies $D_{HF}(p,q,\lambda) = D_G(p,q,\lambda)$.

Figure 2 illustrates the equivalence. Let $\overline{p}_{ST}$ denote the largest value of p for which $H(p) = F(p,q,\lambda,0)$ (that is, that satisfies $D_{HF}(p,q,\lambda) = 0$). Panel A shows a case in which $G(\overline{p}_{ST},q,\lambda) \leq 1$, which implies $D_{HF}(\overline{p}_{ST},q,\lambda) = D_G(\overline{p}_{ST},q,\lambda)$. In this case, $\overline{p}_{ST}$ will be the probability of success in the optimal contract with risky short-term debt, and $D_{ST} = 0$. Panel B shows a case in which $G(\overline{p}_{ST},q,\lambda) > 1$ and there exists $p_{ST} < \overline{p}_{ST}$ such that $D_{HF}(p_{ST},q,\lambda) = D_G(p_{ST},q,\lambda)$. In this case, p_{ST} will be the probability of success in the optimal contract with risky short-term debt, and $D_{ST} > 0$.

From here it is easy to derive the following result on the effects on the optimal contract of changes in the recovery rate λ.

Proposition 4 *For any q for which financing the bank with risky short-term debt is feasible there exists $\lambda_0(q)$ and $\lambda_1(q)$, with $0 \leq \lambda_0(q) \leq \lambda_1(q) \leq 1$, such that:*

1. *For $\lambda \in [\lambda_0(q), \lambda_1(q)]$ the optimal contract is characterized by $D_{ST} = 0$, $\partial M_{ST}/\partial\lambda < 0$, and $\partial p_{ST}/\partial\lambda > 0$.*

2. *For $\lambda \in (\lambda_1(q), 1]$ the optimal contract is characterized by $D_{ST} > 0$, $\partial D_{ST}/\partial\lambda > 0$, and $\partial p_{ST}/\partial\lambda = 0$.*

Proof Suppose that financing the bank with risky short-term debt is feasible for (q,λ), and that the corresponding optimal contract is such that $D_{ST} = 0$. By (39) we have $\partial D_{HF}(p,q,\lambda)/\partial\lambda > 0$, which implies that $\overline{p}_{ST}$, the largest value of p for which $D_{HF}(p,q,\lambda) = 0$, will move to the right, which proves that $\partial p_{ST}/\partial\lambda > 0$. By (40) we have $\partial D_G(p,q,\lambda)/\partial p > 0$ and $\partial D_G(p,q,\lambda)/\partial\lambda > 0$. Hence, there might be a $\lambda_1(q) \leq 1$ such that $D_G(\overline{p}_{ST},q,\lambda_1(q)) =$

0, and if $D_G(\overline{p}_{ST}, q, 1) > 0$ we set $\lambda_1(q) = 1$. The lower bound $\lambda_0(q)$ is defined by $\min\{\lambda \in [0,1] \mid D_{HF}(\overline{p}_{ST}, q, \lambda) = 0\}$. Finally, since M_{ST} in (37) is decreasing in p_{ST} and λ, it follows that $\partial M_{ST}/\partial\lambda < 0$.

Suppose next that financing the bank with risky short-term debt is feasible for (q, λ), and that the corresponding optimal contract is such that $D_{ST} > 0$. Since

$$\frac{\partial D_{HF}(p,q,\lambda)}{\partial\lambda} = \frac{\partial D_G(p,q,\lambda)}{\partial\lambda} = (1-q)pR(p) > 0,$$

it follows that the highest p for which $D_{HF}(p,q,\lambda) = D_G(p,q,\lambda)$ will not change, which proves $\partial D_{ST}/\partial\lambda > 0$ and $\partial p_{ST}/\partial\lambda = 0$. Finally, since M_{ST} in (37) is increasing in D_{ST} and decreasing in λ, it follows that the sign of $\partial M_{ST}/\partial\lambda$ is ambiguous. $\square$

In contrast with the results for the recovery rate λ, the effects on the optimal contract of changes in the quality of the lenders' information q are more difficult to derive, because the sign of

$$\frac{\partial D_{HF}(p,q,\lambda)}{\partial q} = H(p) - \lambda pR(p)$$

is in general ambiguous. However, for q sufficiently large condition (34) will always be satisfied (since $\lim_{q\to 1} G(p,q,\lambda) = 0$) and $\overline{p}_{ST}$ will be the probability of success in the optimal contract with risky short-term debt. Substituting $D_{HF}(\overline{p}_{ST}, q, \lambda) = qH(\overline{p}_{ST}) + \lambda(1-q)\overline{p}_{ST}R(\overline{p}_{ST}) - 1 = 0$ into the previous partial derivative gives:

$$\left.\frac{\partial D_{HF}(p,q,\lambda)}{\partial q}\right|_{p=\overline{p}_{ST}} = \frac{1-\lambda\overline{p}_{ST}R(\overline{p}_{ST})}{q},$$

which is negative for sufficiently large values of λ. Thus, in this special case we have $\partial p_{ST}/\partial q < 0$, that is better quality of information reduces the probability the success associated with the optimal contract with risky short-term debt.

Summing up, we have characterized the conditions under which it is possible to fund the bank with risky short-term debt. By (32) the corresponding expected payoff is:

$$\pi_{ST} = [q + \lambda(1-q)]\, p_{ST}R(p_{ST}) - 1. \tag{41}$$

We would like to compare the conditions under which financing the bank with either long-term or risky short-term debt are feasible, and when both are feasible, to compare π_{LT}

defined in (12) with π_{ST} defined in (41) in order to assess which one will be chosen by the bank. This is trivial in the limit case of a perfectly informative signal ($q = 1$), because then the bank will always be liquidated in the bad state s_0, so $D_{ST} = 0$, and then by (31) we have $F(p, 1, \lambda, 0) = 1$ which implies that the condition $H(p) = F(p, 1, \lambda, 0) = 1$ that characterizes the optimal contract with risky short-term debt coincides with the condition $H(p) = 1$ that characterizes the optimal contract with long-term debt. Hence, in this case $\pi_{ST} = \pi_{LT}$. Since cases with $q < 1$ are more complex, in Section 5 we resort to numerical solutions for a simple parameterization of the model.

An example (continued) For the function $R(p) = a(2 - p)$, the function $D_{HF}(p, q, \lambda)$ obtained by solving for D in the equation $H(p) = F(p, q, \lambda, D)$ becomes a concave parabola:

$$D_{HF}(p, q, \lambda) = -a[2q + \lambda(1 - q)]p^2 + 2a[q + \lambda(1 - q)]p - 1, \tag{42}$$

and the function $D_G(p, q, \lambda)$ that characterizes the up-front dividend in the optimal contract becomes:

$$D_G(p, q, \lambda) = \max\left\{\left[\frac{1}{p + q - 2pq} - (1 - \lambda)\right] a(1 - q)p(2 - p) - 1, 0\right\}. \tag{43}$$

To give an example where the optimal contract involves a positive up-front dividend, let $a = 3.125$, $q = 0.8$, and $\lambda = 0.8$. Then it can be shown that $p_{ST} = 0.72$ and $D_{ST} = 0.46$.

5 Numerical Results

This section poses the following questions:

1. Under what conditions are long-term debt and risky short-term debt feasible?

2. Under what conditions is long-term debt dominated by risky short-term debt?

To answer them we resort to numerical solutions for the simple parameterization of the model that we have introduced in our previous example, namely the linear payoff function $R(p) = a(2 - p)$, where parameter a characterizes the profitability of the bank's investment.

As noted in Section 2, for this payoff function we have $p_{FB} = 1$, so the first-best would be a safe investment with $R(p_{FB}) = a$. Hence, banks with $a \geq 1$ would be funded in the absence of moral hazard.

We have shown in Section 3 that for this payoff function the optimal contract with long-term debt is (B_{LT}, p_{LT}), where $B_{LT} = 1/p_{LT}$ and p_{LT} is given by (14). Hence, financing the bank with long-term debt requires $a \geq 2$. This means that the moral hazard problem prevents financing with long-term debt banks with $1 \leq a < 2$.

We next consider whether we can expand the range of values of a for which financing the bank is feasible by resorting to risky short-term debt. To confirm that this is the case, suppose that a is slightly below 2, so the equation $H(p) = 1$ that characterizes the optimal contract with long-term debt has no solution. By the definition (30) of $F(p, q, \lambda, D)$, for $q = 1$ we have $F(p, 1, \lambda, D) = 1 + D$, so the equation $H(p) = F(p, 1, \lambda, D)$ has no solution for any $D \geq 0$. In other words, the equation $D_{HF}(p, 1, \lambda) = 0$ has no solution. But differentiating with respect to q the expression for $D_{HF}(p, q, \lambda)$ in (42), we get:

$$\frac{\partial D_{HF}(p, q, \lambda)}{\partial q} = ap[2(1 - \lambda) - (2 - \lambda)p],$$

which is negative for λ close to 1. Therefore, a reduction in q shifts up the concave parabola $D_{HF}(p, q, \lambda)$ in such a way that it will intersect the horizontal axis (for a sufficiently close to 2). Moreover, for q close to 1 the initial debt will not be rolled over in the bad state s_0. Hence, we conclude that financing the bank with risky short-term debt is feasible for a sufficiently close to 2 and q and λ sufficiently close to 1.

The preceding argument shows that it may be possible to fund the bank with risky short-term debt for a range of values of a below 2. Solving for the optimal contract for different values of the three parameters a, λ, and q, we find that the minimum feasible value of a is 1.65 (with $\lambda = 1$ and $q = 0.63$). We conclude that using risky short-term debt significantly expands the range of values of a for which the bank can be financed. In other words, risky short-term debt ameliorates the moral hazard problem, but obviously it does not fully solve it since investments with $1 \leq a < 1.64$ will still not be funded.

As noted in Section 4, the optimal contract with risky short-term debt may involve paying an up-front dividend $D > 0$. Figure 3 shows for $a = 1.9$ the range of values of the recovery rate $\lambda \in [0, 1]$ and the quality of the lenders' information $q \in [1/2, 1]$ for which funding the bank with risky short-term debt is not feasible (the dark blue region), for which funding the bank with risky short-term debt is feasible (the orange and the red regions). In the orange region the optimal contract has a zero up-front dividend, while in the red region the optimal contract is characterized by a positive up-front dividend. Notice that these regions are such that the bank's liquidation costs are pretty low (a relatively high value of λ) and the lenders' information is quite noisy (a relatively low value of q). Also, notice that up-front dividends are paid when the lenders' information is very noisy. The frontiers between these regions depict the functions $\lambda_0(q)$ and $\lambda_1(q)$ defined in Proposition 4.

It remains to consider what happens when both long-term and risky short-term debt are feasible, that is when $a \geq 2$. Figure 4 shows for $a = 2.1$ the range of values of the recovery rate $\lambda \in [0, 1]$ and the quality of the lenders' information $q \in [1/2, 1]$ for which risky short-term debt dominates long-term debt (the orange and the red regions), long-term debt dominates risky short-term debt (the light blue region), and for which long-term debt is the only way to finance the bank (the dark blue region). As before, the orange region corresponds to the case in which the optimal risky short-term contract has a zero up-front dividend, while in the red region the optimal contract is characterized by a positive up-front dividend. Hence, risky short-term debt is optimal for high values of λ and a fairly wide range of values of q. Figure 5 shows what happens to these regions when we increase the value of the profitability parameter a to 3.125. Although the area for which risky short-term debt is feasible becomes larger (the dark blue region becomes smaller), now risky short-term debt is optimal for small range of high values of λ and high values of q.

We can summarize these results as follows. First, risky short-term debt may be the only to secure funding, which happens when the profitability of the investment a is relatively low and the quality q of the lenders' information is relatively low. Second, risky short-term debt may dominate long-term debt, when the latter is feasible, which happens when the market for the resale of banks' assets is very efficient (high λ). Third, risky short-term

debt may involve paying an initial dividend, which happens when the quality of the lenders' information is relatively noisy (low q). Finally, it should be noted that risky short-term debt may be optimal, even if it entails inefficient liquidation with positive probability, because it ameliorates the bank's risk-shifting incentives.

6 Extensions

6.1 Mixed debt finance

6.2 Liquidity regulation

6.3 Search for yield

7 Concluding Remarks

References

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Figure 1 Characterization of the optimal contract with long-term debt

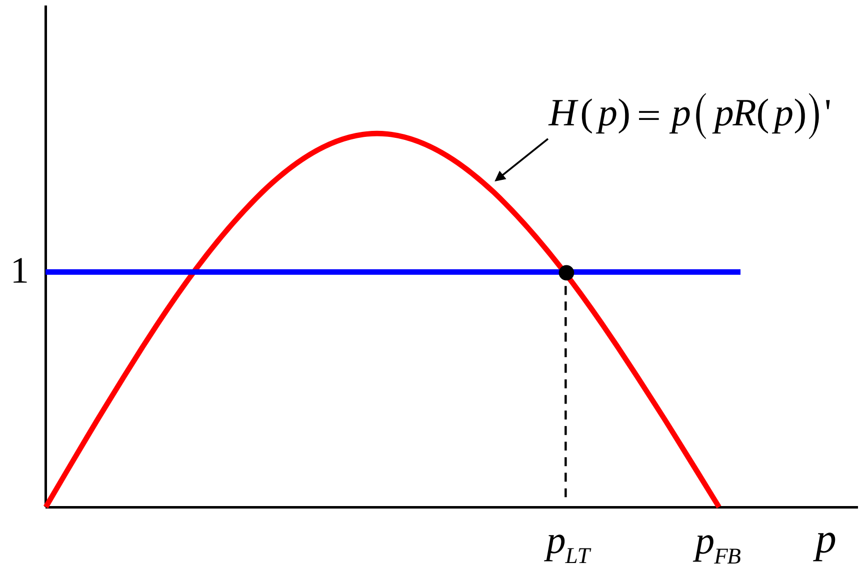

Figure 2 A Characterization of the optimal contract with risky short-term debt and D_{ST} = 0

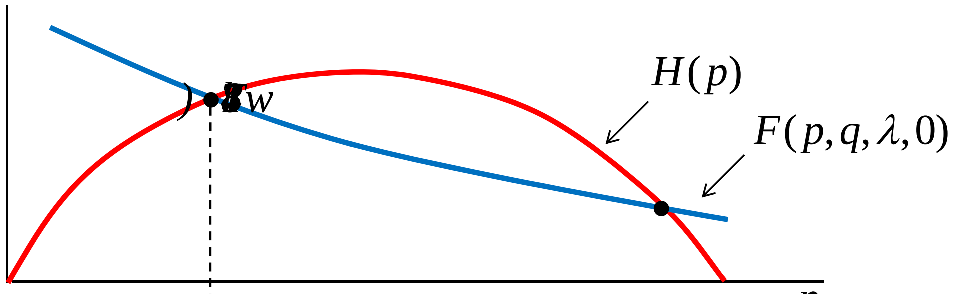

Figure 2 B Characterization of the optimal contract with risky short-term debt and $D_{ST} > 0$

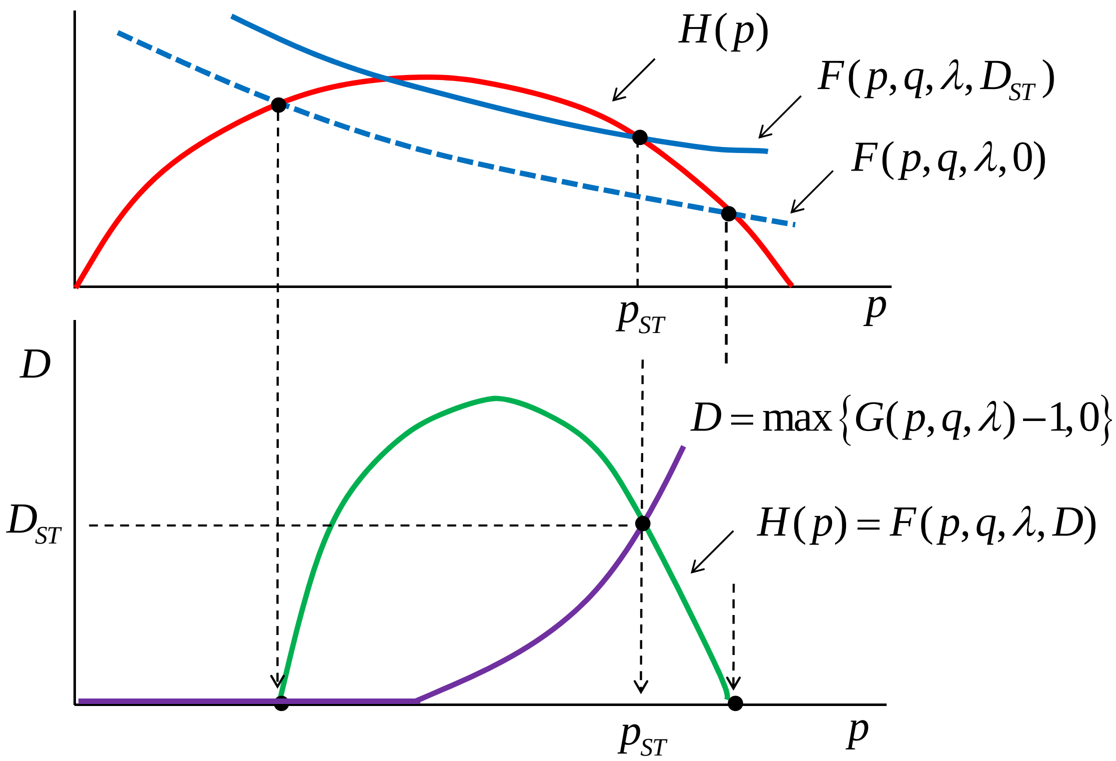

Figure 3 Combinations of the quality of the lenders' information q and the recovery rate λ for which ST debt with a zero dividend is optimal (orange region), ST debt with a positive dividend is optimal (red region), and ST debt is not feasible (dark blue region) for a profitability parameter a = 1.9 (for which long-term debt is not feasible).

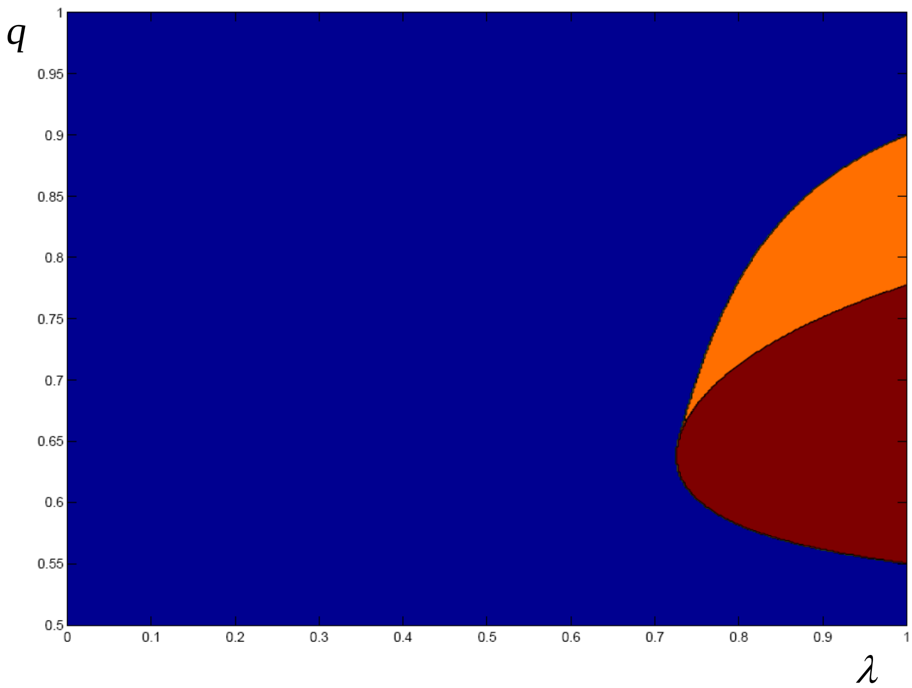

Figure 4 Combinations of the quality of the lenders' information q and the recovery rate λ for which ST debt with a zero dividend dominates LT debt (orange region), ST debt with a positive dividend dominates LT debt (red region), LT debt dominates ST debt (light blue region), and ST debt is not feasible (dark blue region) for a profitability parameter a = 2.1.

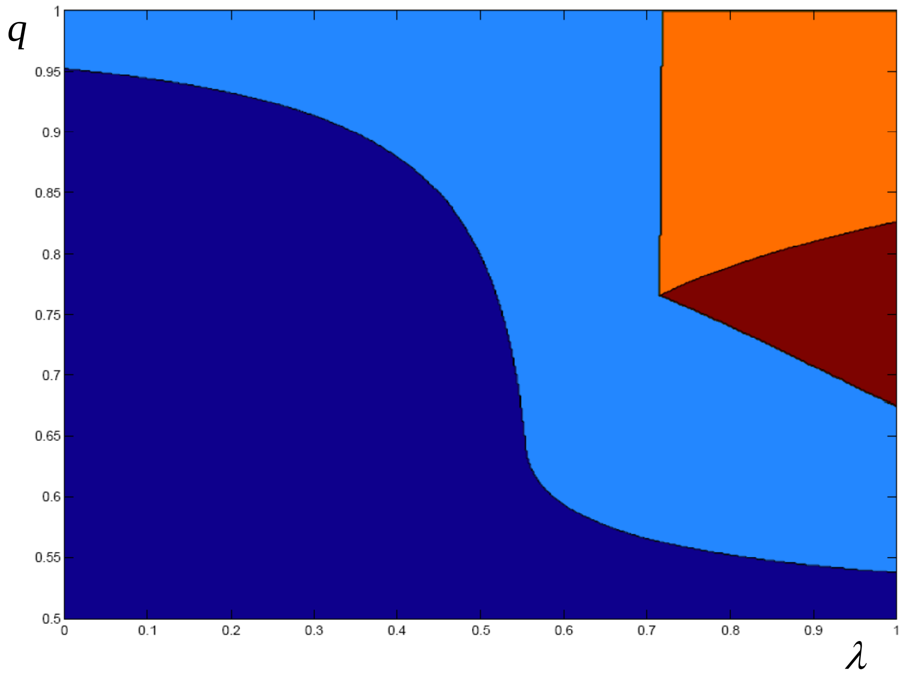

Figure 5 Combinations of the quality of the lenders' information q and the recovery rate λ for which ST debt with a zero dividend dominates LT debt (orange region), ST debt with a positive dividend dominates LT debt (red region), LT debt dominates ST debt (light blue region), and ST debt is not feasible (dark blue region) for a profitability parameter a = 3.125.