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Dynamic Vertical Contracting with Learning-by-Doing

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Preliminary

Abstract

We set-up a dynamic vertical contracting model with learning-by-doing production technologies. There is an upstream duopoly and a downstream monopoly, where the upstream product differentiated manufacturers gain proficiency through the repetition of their production. We study the dynamic interactions in the vertical chain and find that upstream foreclosure may arise in equilibrium when the products are close substitutes and be welfare enhancing. However, the rate of learning is lower that the social optimum and the social planner tends to impose exclusivity more often compared to the downstream monopolist.

JEL Classi...cation: L42; L13; L14; L11; L81

Keywords: Dynamic interactions, learning-by-doing, exclusivity

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1 Introduction

Exclusionary practices in two-tier industries, that is, between upstream and downstream firms, have been drawing increasingly the attention of regulators and anti-trust authorities concerning the implementation of Article 102 of the TOFEU (Treaty on the Functioning of the European Union) on abusive conduct, as well as the interest of academics. In the majority of the cases, exclusionary behavior is viewed with suspicion by the competition policy authorities since they may lead to market foreclosure and hence to reduced competition and product variety and also increased prices by the involved firms. Exclusivity may arise when vertically linked firms sign exclusive dealing contracts or it may arise implicitly through price discrimination by the dominant player (or by offering very disadvantaged contract terms to some firms and thus force them stay out of the market).

While exclusionary practices have been studied in the literature, little attention has been given to the role of exclusivity in a dynamic environment. A standard assumption, in the existing literature, is that firms operate for a single period. In this paper, we introduce dynamic interactions in a vertical chain through the learning-by-doing hypothesis in the production stage. The upstream producers gain proficiency through the repetition of their production while the downstream sector is dominated by a large retailer. The food industry is a leading motivating example to understand the vertical structure models and one that has been attracting the attention of policy makers and even the popular press. In the last decade, large supermarkets have emerged and become powerful in their transactions with the upstream suppliers. Other examples include large tour operators trading with airlines and hotels or general retailers such as Wal-Mart and Carrefour. The issue of exclusionary practices is also very important in the automobile sector (see Brenkers and Verboven, 2005).

To obtain some first insights consider the following setting. Two upstream firms produce differentiated products and face cost functions decreasing to their past accumulated production. A downstream monopolist sets the contract terms, while competition takes place in two periods. Within such a framework several interesting questions arise. How does the final market outcome depend on the learning-by-doing and product differentiation parameters? Does the retailer choose to carry only one product to intensify the learning process? Under what conditions exclusivity and, thus, upstream foreclosure emerges and is beneficial for the final consumers and the firms? Does the market competition favour an outcome with lower prices or with more varieties in the market? Is the presence of the retailer in the market necessary to intensify the learning process? In our model, exclusivity does not arise by a dominant upstream firm's denial to supply some downstream firms the essential input that it produces. Here, the retailer is a large player and the upstream firms can reach the final

consumers only via this retailer.[1]

In the static model, the retailer always chooses to carry both products. In a single period model, there is no learning and exclusivity reduces the varieties in the market without reducing the production costs and the prices. In the dynamic model, exclusivity arises in equilibrium when products are close substitutes. In contrast, when products are complements or not close substitutes, both are purchased in both periods. This result follows from two opposite effects. There is a trade-off between lower prices and more varieties in the market. When product differentiation is low, the "lower prices" effect dominates the "more varieties" effect and the total profits of the chain and the consumers' surplus are higher when one product is excluded from the market. It follows that exclusivity is welfare enhancing. Also, relative to the non-exclusivity case, the product prices in both periods are lower when exclusivity is imposed by the retailer. However, the rate of learning is not equal to the social optimum and the social planner would impose exclusivity more often compared to the retailer. We also obtain that the presence of the intermediary is necessary to intensify the learning-by-doing process when products are complements or when products are not close substitutes and the retailer imposes exclusivity.

Our paper contributes to two different literatures in industrial organization, the vertical contracting and the learning-by-doing hypothesis. Each of these contains very important papers and it is too vast to survey here. We only refer to work that is more closely related to the analysis in this paper. First, numerous contributions highlight the impact of vertical contracting and, in particular, the impact of exclusionary behavior on market competition. For a review and some key results on vertical foreclosure see Rey and Tirole (2007). Krattenmaker and Salop (1986) argue that contracts with input suppliers can be fertile ground for raising competitor's cost and Aghion and Bolton (1987) demonstrate that contracts between buyers and sellers will be signed for entry-prevention purposes. Mathewson and Winter (1987), Besanko and Perry (1993) and O'Brien and Shaffer (1993) derive that exclusive arrangements can have desirable welfare properties.[2] The opportunistic behavior is faced through

[1] Our analysis can be applicable to a multiproduct firm where production is characterized by the learning hypothesis or to a strategic big buyer that can manipulate the learning process through his product choice. However, in this paper we keep thinking about a big downstream firm because we also study whether the existence of this firm intensifies the learning process compared to the case where there are no intermediaries in the market.

[2] Mathewson and Winter (1987) argue that manufacturers under exclusive dealing arangements compete on the basis of wholesale prices for the right to be selected by the retailer. Potential competition replaces actual competition. Besanko and Perry (1993) in a differentiated products oligopoly support that exclusive dealing can eliminate interbrand externalities due to increased promotional investments. O'Brien and Shaffer

exclusivity according to McAfee and Schwartz (1994). Rey and Stiglitz (1995) obtain that when goods are close substitutes producers' profits are higher under exclusive territories. Also, there is a well-known literature on common agency that deals with these issues. See, for example, Martimort (1996), Bernheim and Whinston (1998) and Segal and Whinston (2000). Further, Marx and Shaffer (2005) adopt upfront payments with bargaining power in the downstream level and find that exclusivity arises in equilibrium. Nevertheless, only few papers have examined the vertical chain interactions in a dynamic framework and - to the best of our knowledge - none of these have studied the effect of the learning-by-doing technologies on the product variety and exclusionary practices in a vertical chain.[3]

The second strand of the literature examines the strategic purchases in one-tier industries in a dynamic setting under the learning-by-doing hypothesis. Spence (1981) obtains the dynamic output path of a single and then a multi-firm model and studies the open and closed loop equilibria. Fudenberg and Tirole (1983) derive the precommitment and perfect equilibria with linear demand and learning functions. A price-setting differentiated duopoly with an infinite sequence of heterogeneous buyers and uncertain demands is analyzed by Cabral and Riordan (1994), where they find the Markov perfect equilibrium and study the concept of market dominance. Lewis and Yildirim (2002a) study the trade-off between experience and competition in an industry with privately cost functions and suggest policies that affect the rate of learning, while Lewis and Yildirim (2002b) investigate how incentive regulation should be designed to encourage suppliers to develop and adopt cost-saving technologies. In these models, the learning-by-doing process can be understood as cost-reducing innovations or as the result of economies of scale across multiple market segments where the different time periods play the role of different market segments. In our model, we introduce vertical contracting considerations in such a setting.

In sum, the first strand of the literature examines exclusivity in static vertical contracting models and the second strand of the literature studies dynamic frameworks with learning-by-doing production technologies when firms do not interact in a vertical chain. We combine these two literatures and built a dynamic vertical framework to study the possibility of foreclosure to emerge and be welfare improving.

The remainder of the paper is as follows. Section 2 characterizes the equilibrium when the game lasts for one period. The dynamic model is analyzed in Section 3. Consumers' surplus and total welfare are presented in Section 4. In Section 5, we examine the social

(1993) examine the corner solutions in an oligopolistic vertical setting.

[3]There is some work on vertical contracting with inventories and dynamic considerations or renegotiating contracts, see , for example, Jong-Say Jong (1999) or Taylor and Plambeck (2006).

planning solution. Section 6 studies the role of the intermediary in the learning process and Section 7 concludes.

2 The static model

We begin our analysis by studying the potential for exclusivity to arise in a simple setting where firms compete for a single period. We consider two manufacturers, A and B, who each produce a single, differentiated product. Manufacturers supply the downstrean market and have the same constant marginal production cost, c. In our analysis, we focus on the possibility of exclusivity to emerge as a result of strategic interaction and not due to cost asymmetries in the upstream industry. The fixed production costs are normalized to zero for simplicity.

The downstream market is monopolized by a retailer, R, that has the power to set the contract terms which take the form of linear wholesale prices. One unit of the manufacturers' product becomes one unit of final good at zero marginal cost. The inverse final demand functions take the linear form $p_i = a - q_i - bq_j$, where p_i is the retail price and q_i is the quantity of product i, $i = A, B$, $a \geq c$ and $b \in (-1, 1)$ is the product differentiation parameter.[4] When b equals zero, the two products are independent. When b approaches the unity ($b \rightarrow 1$) the two goods become closer substitutes and when b approaches minus one ($b \rightarrow -1$) the two goods become closer complements. In the extreme case where b is equal to one, the two goods are homogeneous and when b is equal to minus one, the two goods are perfect complements. There is no uncertainty and all contracts are perfect observable. The static framework is presented in Figure 1.

We consider a three-stage game. First, the retailer sets the contract terms. Then, upstream firms either accept or reject the contract offers and in the last stage the retailer sells the products to the final consumers and pays the manufacturers according to their contracts. The game is solved backwards using subgame perfection as the equilibrium concept.

In the first stage of the game, the retailer can set very disadvantaged terms to one of the two suppliers (say B) to force this supplier reject his offer and stay out of the market. The retailer implicitly denies access of the final consumers to that firm's product by offering a wholesale price lower than the production cost of that upstream firm.[5] First, we consider

[4] The representative consumer is characterized by the quadratic and strictly concave utility function $U(q_A, q_B) = a(q_A + q_B) - \frac{q_A^2 + q_B^2 + 2bq_Aq_B}{2}$ (see Singh and Vives (1984)).

[5] An alternative way to foreclosure an input supplier is the retailer to explicitly sign an exclusive dealing contract with the rival upstream firm.

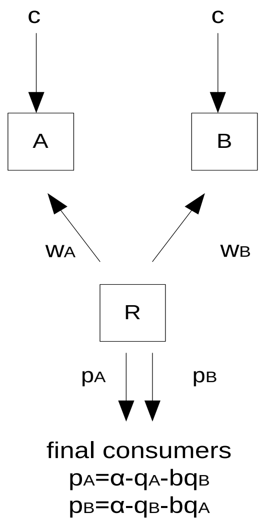


Figure 1: The static model

the case where both products are carried by the retailer (non-exclusivity) and then we study the corner case where the retailer chooses to carry only one product (exclusivity).

Non-exclusivity Consider first the subgames where the retailer chooses to distribute both products. Solving backwards, in the final stage of the game the retailer maximizes its profits in the final goods market:

$$\begin{aligned} \underset{q_A,q_B}{max}\ \Pi_R &= (p_A - w_A)q_A + \ (p_B - w_B)q_B \\ s.t.\ p_i &= a - q_i - bq_j \ \text{ with } i = A, B \text{ and } i \neq j\,. \end{aligned}$$

The demand functions are linear, therefore, the objective function is concave and the optimal quantities are derived by the first order conditions:

$$q_i(w_i, w_j) = \frac{a}{2(1+b)} + \frac{bw_j - w_i}{2(1-b^2)}. \tag{1}$$

Then, the retailer sets the wholesale prices w_i given the individual rational constraints of the upstream firms. The upstream firms accept the offer if they obtain non-negative profits, since at the downstream level there is no alternative retailer which leads to zero outside option for the upstream firms. The maximization now is:

$$\underset{w_A,w_B}{max}\,\Pi_R(q_A(w_A,w_B), q_B(w_A,w_B))\ s.t.\ \Pi_i = (w_i - c)q_i \geq 0 \text{ for } i = A, B.$$

The downstream monopolist sets the wholesale prices equal to the minimum possible level, that is, equal to the unit production cost $w_i = c$ for both manufacturers. This leads to zero upstream profits.[6]

[6]We assume that if the manufacturer is indifferent between obtaining zero profits from production and

Lemma 1 In the static model with both products carried by the retailer, we obtain:

$$\begin{aligned} q_i &= \frac{a-c}{2(1+b)}, \\ p_i &= \frac{a+c}{2},\ w_i = c, \\ \Pi_i &= 0,\ \Pi_R = \frac{(a-c)^2}{2(1+b)}. \end{aligned}$$

Final prices are equal to the monopoly prices and the retailer obtains the monopoly profit. The chain profits are maximized and captured by the retailer.

Exclusivity Assume now that in the first stage of the game, the retailer chooses to distribute only one product, say A. Now it solves:

$$\underset{q_A}{max} \Pi_R = (p_A - w_A) q_A \ s.t.\ p_A = a - q_A.$$

The second order conditions are satisfied, therefore, the first order conditions simply imply

$$q_A(w_A) = \frac{a - w_A}{2}. \tag{2}$$

The retailer then determines the wholesale prices w_A,w_B for the upstream supplier A to accept the offer and for B to reject the offer

$$\underset{w_A}{max} \Pi_R(q_A(w_A))\ s.t.\ \Pi_A = (w_A - c)q_A \geq 0 \text{ and } \Pi_B = (w_B - c)q_B < 0.$$

Following the same logic as before, the retailer charges a wholesale price equal to the unit production cost to manufacturer A, $w_A = c$ and a wholesale price lower than the production cost to manufacturer B, $w_B < c$, to exclude this product from the market. The monopoly results are reached with the retailer to extract the vertical chain's profits.

Lemma 2 In the static model with one product in the market, we obtain:

$$\begin{aligned} q_A &= \frac{a-c}{2}, \\ p_A &= \frac{a+c}{2}, w_A = c,\ w_B < c, \\ \Pi_A &= 0,\ \Pi_R = \frac{(a-c)^2}{4}. \end{aligned}$$

When products are perfect substitutes ($b = 1$) the results from the two subgames coincide, apart from the fact that the total quantity is equally split to the two manufacturers when they are both present in the market. When products are imperfect substitutes ($b \in (0, 1)$), final

---

not producing at all, he will choose to produce. Or you can think that the retailer sets the wholesale prices ε above the production cost (with $\varepsilon \to 0$), therefore, the upstream profits are almost zero.

prices are equal in the two subgames. This is due to the symmetry in the unit production costs of the two differentiated producers. However, the total quantity produced in the market and the profit obtained by the retailer increase as b decreases under non-exclusivity. So, for $b \in (0, 1)$ the retail profits are higher when the retailer distributes both differentiated products in the market compared to the exclusivity case. This happens since consumers prefer to have both varieties in the final market.

When products are complements ($b \in (-1, 0)$), final quantities and profits under non-exclusivity are further increased. The consumption set is larger, thus, consumers' surplus increases. The consumers' surplus when both products are present in the market is equal to $\frac{(a-c)^2}{4(1+b)}$, and when only one product is distributed equals $\frac{(a-c)^2}{8}$.[7] Since profits and consumers' surplus under non-exclusivity are higher, total welfare is also higher in this subgame. In the static model, exclusivity harms both consumers and firms. We summarize our results in the following proposition.

Proposition 1 In the static model, exclusivity does not arise in equilibrium. The retailer purchases both products and sells them at the monopoly price in the final goods market. The total profits of the chain, the consumers' surplus and the total welfare when both products are purchased are higher than when one product is excluded from the market.

3 The dynamic model

In this section, we depart from the simple static model by assuming that the game lasts for two periods. We study the dynamic interactions that emerge in the vertical framework by introducing the learning-by-doing hypothesis. Over time upstream producers gain proficiency through the repetition of their production. The unit production cost decreases as the producer gains more experience, that is, the unit cost function is a decreasing function of past accumulated production. Interesting issues arise in this dynamic vertical environment. Does exclusivity emerge in equilibrium and under what conditions? Does the market competition favour an outcome with lower prices or with more varieties in the market?

In our model, the production is characterized by the linear learning-by-doing hypothesis. The unit production cost of the manufacturers in the second period reduces proportionally with the production of the first period. There is no diffusion of learning, thus, the firms learn only from their own production. Specifically, the unit cost functions in the second period are given by:

[7] A more detailed discussion about the consumers' surplus and the total welfare is deferred for Section 4.

Figure 2: The dynamic model

$$c_{i2} = \left\{ \begin{array}{ll} c - \lambda q_{i1} & \text{if } q_{i1} < \frac{c}{\lambda} \\ 0 & \text{if } q_{i1} \geq \frac{c}{\lambda} \end{array} \right\} \text{ for } i = A, B. \tag{3}$$

The first subscript refers to the manufacturer and the second to the time period ($t = 1, 2$), while λ is the learning parameter. All other assumptions remain the same as in Section 2 (see Figure 2).

The timing of the game is the same as in the static game with the difference that now firms interact for two periods. In each period, the retailer first sets the contract terms, then, the upstream firms either accept or reject the offers and, subsequently, the retailer sells the products to the final consumers and pays the manufacturers. Again two types of subgames are examined: both products are purchased by the retailer or only one product is purchased either in period one or in period two.

Interestingly, there is interdependence between the two periods due to the learning-by-doing process. The unit cost functions in the second period are affected by the quantities produced in the first period. Therefore, the retailer maximizes the present value of its profits in each period: $\Pi_{R1} + \delta\Pi_{R2}$ for the first period and Π_{R2} for the second period, where $\delta \in (0, 1)$ is the discount factor. We proceed backwards to solve for the subgame perfect equilibrium.

3.1 Period two

Taking as given the quantities purchased in the first period (therefore, the production costs in the second period), we consider first the case where the retailer does not exclude an upstream supplier and then the case where the retailer purchases only one product in period

two by setting very disadvantaged contract terms to supplier B.

Non-exclusivity We begin our analysis from stage three. The retailer maximizes its profits:

$$\begin{aligned} \max_{q_{A2},q_{B2}} \Pi_{R2} &= (p_{A2}-w_{A2})q_{A2} + (p_{B2}-w_{B2})q_{B2} \\ s.t.\ p_{i2} &= a - q_{i2} - bq_{j2} \text{ with } i = A, B \text{ and } i \neq j. \end{aligned}$$

Similarly to the static model, the upstream firms accept the offers made in the first stage of the game and the retailer sets the wholesale prices equal to the unit production costs, $w_{i2} = c_{i2}$. After some calculations, we obtain

$$\begin{aligned} q_{i2} &= \frac{a}{2(1+b)} + \frac{bc_{j2}-c_{i2}}{2(1-b^2)}, \qquad (4) \\ p_{i2} &= \frac{a+c_{i2}}{2},\ w_{i2} = c_{i2}, \\ \Pi_{i2} &= 0, \\ \Pi_{R2} &= \frac{a(2a-c_{A2}-c_{B2})}{4(1+b)} + \frac{(a-c_{A2})(bc_{B2}-c_{A2}) + (a-c_{B2})(bc_{A2}-c_{B2})}{4(1-b^2)}, \end{aligned}$$

where $q_{A2} \geq 0$ when $b \leq \frac{a-c_{A2}}{a-c_{B2}}$ and $q_{B2} \geq 0$ when $b \leq \frac{a-c_{B2}}{a-c_{A2}}$. We further assume that if the retailer chooses to carry one product in the first period, this product will be product A. So in the second period the more cost efficient firm (if any) is firm A with $c_{A2} \leq c_{B2}$. This means that $q_{A2} \geq 0$ always holds since $b \leq 1 \leq \frac{a-c_{A2}}{a-c_{B2}}$, but $q_{B2} \geq 0$ holds only when $b \leq \frac{a-c_{B2}}{a-c_{A2}} \leq 1$, where the cost asymmetry in the second period is not high enough. Even if the retailer does not strategically exclude one supplier by offering a wholesale price lower than the production cost, product B may be excluded from the market in period two when firm B has a production cost high enough compared to the rival's cost. Taking into account

this possibility of a corner solution in period two due to a high cost asymmetry, we obtain:

$$
\begin{aligned}
q_{A2} &= \begin{cases} \frac{a}{2(1+b)} + \frac{bc_{B2}-c_{A2}}{2(1-b^2)} & if \quad b \leq \frac{a-c_{B2}}{a-c_{A2}} \\ \frac{a-c_{A2}}{2} & if \quad b > \frac{a-c_{B2}}{a-c_{A2}} \end{cases}, \\
q_{B2} &= \begin{cases} \frac{a}{2(1+b)} + \frac{bc_{A2}-c_{B2}}{2(1-b^2)} & if \quad b \leq \frac{a-c_{B2}}{a-c_{A2}} \\ 0 & if \quad b > \frac{a-c_{B2}}{a-c_{A2}} \end{cases}, \\
p_{A2} &= \frac{a+c_{A2}}{2}, \\
p_{B2} &= \begin{cases} \frac{a+c_{B2}}{2} & if \quad b \leq \frac{a-c_{B2}}{a-c_{A2}} \\ - & if \quad b > \frac{a-c_{B2}}{a-c_{A2}} \end{cases}, \\
w_{A2} &= c_{A2}, w_{B2} = \begin{cases} c_{B2} & if \quad b \leq \frac{a-c_{B2}}{a-c_{A2}} \\ - & if \quad b > \frac{a-c_{B2}}{a-c_{A2}} \end{cases}, \\
\Pi_{A2} &= 0, \Pi_{B2} = \begin{cases} 0 & if \quad b \leq \frac{a-c_{B2}}{a-c_{A2}} \\ - & if \quad b > \frac{a-c_{B2}}{a-c_{A2}} \end{cases}, \\
\Pi_{R2} &= \begin{cases} \frac{a(2a-c_{A2}-c_{B2})}{4(1+b)} + \frac{(a-c_{A2})(bc_{B2}-c_{A2})+(a-c_{B2})(bc_{A2}-c_{B2})}{4(1-b^2)} & if \quad b \leq \frac{a-c_{B2}}{a-c_{A2}} \\ \frac{(a-c_{A2})^2}{4} & if \quad b > \frac{a-c_{B2}}{a-c_{A2}}. \end{cases}
\end{aligned} \tag{5}
$$

Exclusivity Now, assume that the retailer purchases only one product in period two (product A) by offering supplier B very disadvantaged contract terms. Thus, retailer maximizes:

$$
\begin{aligned}
\underset{q_{A2}}{max} \Pi_{R2} &= (p_{A2} - w_{A2})q_{A2} \\
s.t. \ p_{A2} &= a - q_{A2}.
\end{aligned}
$$

Then, the upstream firm A accepts the offer and B rejects the offer where $w_{A2} = c_{A2}$ and $w_{B2} < c_{B2}$. Therefore:

$$
\begin{aligned}
q_{A2} &= \frac{a-c_{A2}}{2}, q_{B2} = 0 \\
p_{A2} &= \frac{a+c_{A2}}{2}, \ w_{A2} = c_{A2}, w_{B2} < c_{B2} \\
\Pi_{i2} &= 0, \\
\Pi_{R2} &= \frac{(a-c_{A2})^2}{4}.
\end{aligned} \tag{6}
$$

Outcome in period two Given all the possible values of the two production costs in period two, the retailer decides if it will impose exclusivity in period two or not. By comparing the retailer's profits in period two for the two alternative cases above (by expressions (5) and (6)), we find that it is profitable for the retailer to purchase both products in the final period

of the game since

$$\frac{a(2a-c_{A2}-c_{B2})}{4(1+b)}+\frac{(a-c_{A2})(bc_{B2}-c_{A2})+(a-c_{B2})(bc_{A2}-c_{B2})}{4(1-b^2)}\geq\frac{(a-c_{A2})^2}{4}.$$

This result holds when the cost asymmetry is not high enough ($b\leq\frac{a-c_{B2}}{a-c_{A2}}$) and both products may be produced in period two. The second period is the last period of the game and excluding one supplier from the market by offering disadvantaged terms only reduces the variety in this period without opting for future cost reduction. In period two, both products are always purchased by the retailer when $b\leq\frac{a-c_{B2}}{a-c_{A2}}$. When $b>\frac{a-c_{B2}}{a-c_{A2}}$, products are close substitutes and costs in period two asymmetric enough to exclude product B from the market in both cases (under exclusivity via disadvantaged wholesale pricing or not). Therefore, the equilibrium outcome in period two is given by expressions (5). Note that when products are complements ($b\in(-1,0)$), both products are always purchased in period two.

3.2 Period one

The first question now is whether the retailer has an incentive to exclude one upstream supplier in the first period to intensify the learning process by purchasing higher quantity by only producer A. The second question is, given that the retailer chooses to carry only product A in the first period, whether it will purchase high enough quantity by producer A to make this producer efficient enough in period two too (which will lead to high cost assymetry in the next period). Without exclusion in the first period, producers are equal cost efficient in the future, in contrast to the case where exclusivity is imposed in the first period. There are two alternative cases: the case where the retailer purchases both products and the case where the retailer purchases only one product in period one by offering disadvantaged contract terms to producer B.

Non-exclusivity In this case, given that the suppliers are initially equal cost efficient, the equilibrium in period one will be symmetric and the suppliers equal efficient in the second period. The retailer maximizes the present value of its profits $\Pi_{R1}+\delta\Pi_{R2}$, where Π_{R2} is taken by expression (5). The quantities purchased in period one affect the production costs in period two and subsequently the prices and the profits obtained. The retailer maximizes the present value of its profits:

$$\max_{q_{A1},q_{B1}}\Pi_{R1}+\delta\Pi_{R2}=(p_{A1}-w_{A1})q_{A1}+\ (p_{B1}-w_{B1})q_{B1}$$

$$+\delta\left[\frac{a(2a-c_{A2}-c_{B2})}{4(1+b)}+\frac{(a-c_{A2})(bc_{B2}-c_{A2})+(a-c_{B2})(bc_{A2}-c_{B2})}{4(1-b^2)}\right]$$

$$s.t. \; c_{i2} = c - \lambda q_{i1} \quad \text{for } i = A, B.$$

After some calculations[8] and by the fact that the upstream firms accept the offers and the wholesale prices are set to the marginal cost $w_{i1} = c$, we have:

Lemma 3 In the dynamic model without exclusivity in period one, we obtain:

$$\begin{aligned}
q_{i1}^{NE} &= \frac{(a-c)(2(1+b)+\delta\lambda)}{4(1+b)^2-\delta\lambda^2}, \\
q_{i2}^{NE} &= \frac{a-c}{2(1+b)} + \frac{\lambda(a-c)(2(1+b)+\delta\lambda)}{2(1+b)(4(1+b)^2-\delta\lambda^2)}, \\
p_{i1}^{NE} &= \frac{2(b+1)^2(a+c) - a\lambda^2\delta - \lambda\delta(a-c)(b+1)}{4(1+b)^2-\delta\lambda^2}, \\
p_{i2}^{NE} &= \frac{2(b+1)^2(a+c) - a\lambda^2\delta - \lambda(a-c)(b+1)}{4(1+b)^2-\delta\lambda^2}, \\
w_{i1}^{NE} &= c, \; w_{i2}^{NE} = \frac{4c(1+b)^2 - \lambda^2\delta - 2\lambda(a-c)(1+b)}{4(1+b)^2-\delta\lambda^2}, \\
\Pi_{i1}^{NE} &= \Pi_{i2}^{NE} = 0, \\
\Pi_{R1}^{NE} &= \frac{2(a-c)^2\left(2(1+b)^2 - \lambda^2\delta - \lambda\delta(1+b)\right)(2b+\lambda\delta+2)}{(4(1+b)^2-\delta\lambda^2)^2}, \\
\Pi_{R2}^{NE} &= \frac{2(a-c)^2(2b+\lambda+2)^2(1+b)}{(4(1+b)^2-\delta\lambda^2)^2}, \\
\Pi_{R1}^{NE} + \delta\Pi_{R2}^{NE} &= \frac{2(a-c)^2((\delta+1)(b+1)+\lambda\delta)}{8b-\lambda^2\delta+4b^2+4}.
\end{aligned} \tag{7}$$

Exclusivity Assume now that in the first period of the game, the retailer chooses to distribute only product A. In this case, producer A will be more cost efficient in period two ($c_{A2} < c_{B2}$) since it is benefited by the learning-by-doing process ($q_{A1} > 0$ and $q_{B1} = 0$). This cost reduction determines whether product B will be produced in period two (depending also on the product differentiation parameter). Firm A maximizes:

$$\max_{q_{A1}} \Pi_{R1} + \delta\Pi_{R2} =$$

$$= \begin{cases} (p_{A1} - w_{A1})q_{A1} + \delta\left(\frac{a(2a-c_{A2}-c_{B2})}{4(1+b)} + \frac{(a-c_{A2})(bc_{B2}-c_{A2})+(a-c_{B2})(bc_{A2}-c_{B2})}{4(1-b^2)}\right) & if \quad b \leq \frac{a-c_{B2}}{a-c_{A2}} \\ (p_{A1} - w_{A1})q_{A1} + \delta\left(\frac{(a-c_{A2})^2}{4}\right) & if \quad b > \frac{a-c_{B2}}{a-c_{A2}} \end{cases}$$

$$s.t. \; c_{A2} = c - \lambda q_{A1} \text{ and } c_{B2} = c.$$

[8] The second order conditions are satisfied when $\left(-8b - \lambda^2\delta + 4b^2 + 4\right)\left(8b - \lambda^2\delta + 4b^2 + 4\right) > 0$ and $\lambda^2\delta + 4b^2 - 4 < 0$.

The wholesale price for supplier A is set equal to the production cost in period one and for supplier B lower than the production cost to make this firm do not operate in the first period. So we have $w_{A1} = c$ and $w_{B1} < c$. When products are complements or when products are not so close substitutes, $b \leq \frac{a-c_{B2}}{a-c_{A2}}$ (or equivalently when the quantity of firm A in period one is not high enough, $q_{A1} \leq \frac{(a-c)(1-b)}{\lambda b}$), both products are purchased in period two. However, when $q_{A1} > \frac{(a-c)(1-b)}{\lambda b}$ the intense learning process leads to high cost asymmetry in period two and only product A can be purchased in the second period.

For $q_{A1} \leq \frac{(a-c)(1-b)}{\lambda b}$ the retailer's maximand is

$$(p_{A1} - w_{A1})q_{A1} + \delta\left(\frac{a(2a - c_{A2} - c_{B2})}{4(1+b)} + \frac{(a - c_{A2})(bc_{B2} - c_{A2}) + (a - c_{B2})(bc_{A2} - c_{B2})}{4(1-b^2)}\right)$$

and is obtained

$$q_{A1} = \begin{cases} \frac{(1-b)(a-c)(2b+\lambda\delta+2)}{4-\lambda^2\delta-4b^2} \text{ interior} & if \quad b \leq \frac{4-\lambda^2\delta}{2(\lambda+2)} \\ \frac{(a-c)(1-b)}{\lambda b} \text{ corner} & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)} \end{cases} \tag{8}$$

with

$$\Pi_{R1} + \delta\Pi_{R2} = \begin{cases} \frac{(a-c)^2\left(-8\delta+8b\delta+\lambda^2\delta^2-4\lambda\delta+4b^2+4b\lambda\delta-4\right)}{4\left(\lambda^2\delta+4b^2-4\right)} \text{ interior} & if \quad b \leq \frac{4-\lambda^2\delta}{2(\lambda+2)} \\ \frac{(a-c)^2\left(-4b^2\lambda-4b^2+4b\lambda+8b+\delta\lambda^2-4\right)}{4b^2\lambda^2} \text{ corner} & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)}. \end{cases}$$

However, for $q_{A1} > \frac{(a-c)(1-b)}{\lambda b}$ the retailer's maximand is

$$(p_{A1} - w_{A1})q_{A1} + \delta\left(\frac{(a - c_{A2})^2}{4}\right)$$

and is obtained

$$q_{A1} = \begin{cases} \frac{(a-c)(1-b)}{\lambda b} \text{ corner} & if \quad b \leq \frac{4-\delta\lambda^2}{2(\lambda+2)} \\ \frac{(a-c)(2+\delta\lambda)}{4-\delta\lambda^2} \text{ interior} & if \quad b > \frac{4-\delta\lambda^2}{2(\lambda+2)} \end{cases} \tag{9}$$

with

$$\Pi_{R1} + \delta\Pi_{R2} = \begin{cases} \frac{(a-c)^2\left(-4b^2\lambda-4b^2+4b\lambda+8b+\delta\lambda^2-4\right)}{4b^2\lambda^2} & if \quad b \leq \frac{4-\delta\lambda^2}{2(\lambda+2)} \\ \frac{(\delta+\lambda\delta+1)(a-c)^2}{4-\delta\lambda^2} & if \quad b > \frac{4-\delta\lambda^2}{2(\lambda+2)}. \end{cases}$$

When products are complements the equilibrium outcome in this subgame is always given by expression (8). However, when products are substitutes, does the retailer purchase high quantity by firm A in period one and intensifies the learning process a lot so as to exclude firm B in the subsequent period? Or does it purchases lower quantity in period one by firm A to keep firm B in the market in period two? By comparing the present values of the retailer for these two cases (expression (8) and (9)), we obtain that when $b \leq \frac{4-\lambda^2\delta}{2(\lambda+2)}$ product B is excluded only in period one, and when $b > \frac{4-\lambda^2\delta}{2(\lambda+2)}$ product B is excluded from the market in both periods.

Lemma 4 In the dynamic model with exclusivity in period one, we obtain[9]

$$
\begin{aligned}
q_{A1}^E &= \begin{cases} \frac{(1-b)(a-c)(2b+\lambda\delta+2)}{4-\lambda^2\delta-4b^2} & if \quad b \le \frac{4-\lambda^2\delta}{2(\lambda+2)} \text{ both in t=2} \\ \frac{(a-c)(2+\delta\lambda)}{4-\delta\lambda^2} & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)} \text{ only A in t=2} \end{cases}, \\
q_{A2}^E &= \begin{cases} \frac{(a-c)(2(1-b)+\lambda)}{4-\lambda^2\delta-4b^2} & if \quad b \le \frac{4-\lambda^2\delta}{2(\lambda+2)} \\ \frac{(a-c)(2+\lambda)}{4-\delta\lambda^2} & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)} \end{cases}, \\
q_{B2}^E &= \begin{cases} \frac{(a-c)\left(4-\delta\lambda^2-2b(\lambda+2)\right)}{2\left(4-\lambda^2\delta-4b^2\right)} & if \quad b \le \frac{4-\lambda^2\delta}{2(\lambda+2)} \\ 0 & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)} \end{cases}, \\
p_{A1}^E &= \begin{cases} \frac{2(1-b)(b+1)(a+c)-\delta\lambda((1-b)(a-c)+a\lambda)}{4-\lambda^2\delta-4b^2} & if \quad b \le \frac{4-\lambda^2\delta}{2(\lambda+2)} \\ \frac{c(\lambda\delta+2)+a(2-\delta\lambda(\lambda+1))}{4-\delta\lambda^2} & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)} \end{cases}, \\
p_{A2}^E &= \begin{cases} \frac{2(1-b)(b+1)(2(a+c)-\lambda(a-c))-\delta\lambda^2(2a-b(a-c))}{2\left(4-\lambda^2\delta-4b^2\right)} & if \quad b \le \frac{4-\lambda^2\delta}{2(\lambda+2)} \\ \frac{c(\lambda+2)+a(2-\lambda(\lambda\delta+1))}{4-\delta\lambda^2} & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)} \end{cases}, \\
p_{B2}^E &= \begin{cases} \frac{a+c}{2} & if \quad b \le \frac{4-\lambda^2\delta}{2(\lambda+2)} \\ - & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)} \end{cases}, \\
w_{A1}^E &= c, \; w_{B1}^E < c, \\
w_{A2}^E &= \begin{cases} \frac{2(1-b)(b+1)(2c-a\lambda+c\lambda)-\delta\lambda^2(a-b(a-c))}{4-\lambda^2\delta-4b^2} & if \quad b \le \frac{4-\lambda^2\delta}{2(\lambda+2)} \\ \frac{c(2\lambda+4)-a\lambda(\lambda\delta+2)}{4-\delta\lambda^2} & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)} \end{cases}, \; w_{A2}^E = c, \\
\Pi_{A1}^E &= \Pi_{A2}^E = \Pi_{B2}^E = 0, \\
\Pi_{R1}^E &= \begin{cases} \frac{(a-c)^2(1-b)(2(1-b)(b+1)-\delta\lambda(1-b+\lambda))(2(1+b)+\lambda\delta)}{\left(4-\lambda^2\delta-4b^2\right)^2} & if \quad b \le \frac{4-\lambda^2\delta}{2(\lambda+2)} \\ \frac{(a-c)^2(2+\delta\lambda)(2-\delta\lambda-\delta\lambda^2)}{(4-\delta\lambda^2)^2} & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)} \end{cases}, \\
\Pi_{R2}^E &= \begin{cases} \frac{(a-c)^2\left(\lambda^4\delta^2-8\lambda^2\delta(1-b)(b+1)+4(1-b)(b+1)(\lambda(\lambda+4(1-b))+8(1-b))\right)}{4\left(4-\lambda^2\delta-4b^2\right)^2} & if \quad b \le \frac{4-\lambda^2\delta}{2(\lambda+2)} \\ \frac{(a-c)^2(2+\lambda)^2}{(4-\delta\lambda^2)^2} & if \quad b > \frac{4-\lambda^2\delta}{2(\lambda+2)} \end{cases}, \\
\Pi_{R1}^E + \delta\Pi_{R2}^E &= \begin{cases} \frac{(a-c)^2\left(-\lambda^2\delta^2+4\delta(\lambda+2)(1-b)+4(1-b)(1+b)\right)}{4\left(4-\lambda^2\delta-4b^2\right)} & if \quad b \le \frac{4-\delta\lambda^2}{2(\lambda+2)} \\ \frac{(\delta+\lambda\delta+1)(a-c)^2}{4-\delta\lambda^2} & if \quad b > \frac{4-\delta\lambda^2}{2(\lambda+2)}. \end{cases}
\end{aligned}
\tag{10}
$$

Characterization of the equilibrium To fully characterize the equilibrium we have to check whether the retailer stategically excludes an upstream supplier in the first period to manipulate the learning process. By comparing the present value of the profits of the retailer when only product A is purchased in period one (take the expressions from (10)) to the present

[9] We need $4 - \lambda^2\delta > 4b^2$ for the second order conditions to be satisfied and to obtain positive quantity in period one. Otherwise no production occurs.

value of the profits when both products are purchased in period one (take the expressions from (7)), we obtain the following proposition.

Proposition 2 The retailer imposes exclusivity and foreclosures supplier B in period one, and subsequently in period two, when $b \in \left(\overline{b}, 1\right)$.[10] Otherwise, it purchases both products in both periods.

By direct comparison, we obtain that for $b \in \left(-1, \frac{4-\lambda^2\delta}{2(\lambda+2)}\right)$ the retailer purchases both products in period one and this also leads to product variety in period two. Thus, it is never an equilibrium outcome to purchase product A in the first period and both products in the second period. For $b \in \left(\frac{4-\lambda^2\delta}{2(\lambda+2)}, 1\right)$ the retailer has to decide whether to purchase both products in period one which also leads to variety in period two or to exclude product B in period one and subsequently in period two. We find that when $b \in (\overline{b}, 1)$, the retailer exclusively carries only product A in both periods. The equilibrium outcome is:

$$q^*_{A1} = \begin{cases} \frac{(a-c)(2(1+b)+\delta\lambda)}{4(1+b)^2-\delta\lambda^2} \text{ both/both} & if \quad b \in \left(-1, \overline{b}\right) \\ \frac{(a-c)(2+\delta\lambda)}{4-\delta\lambda^2} \text{ only A/only A} & if \quad b \in \left(\overline{b}, 1\right) \end{cases},$$

$$q^*_{B1} = \begin{cases} \frac{(a-c)(2(1+b)+\delta\lambda)}{4(1+b)^2-\delta\lambda^2} & if \quad b \in \left(-1, \overline{b}\right) \\ - & if \quad b \in \left(\overline{b}, 1\right) \end{cases},$$

$$q^*_{A2} = \begin{cases} \frac{a-c}{2(1+b)} + \frac{\lambda(a-c)(2(1+b)+\delta\lambda)}{2(1+b)(4(1+b)^2-\delta\lambda^2)} & if \quad b \in \left(-1, \overline{b}\right) \\ \frac{(a-c)(2+\lambda)}{4-\delta\lambda^2} & if \quad b \in \left(\overline{b}, 1\right) \end{cases},$$

$$q^*_{B2} = \begin{cases} \frac{a-c}{2(1+b)} + \frac{\lambda(a-c)(2(1+b)+\delta\lambda)}{2(1+b)(4(1+b)^2-\delta\lambda^2)} & if \quad b \in \left(-1, \overline{b}\right) \\ - & if \quad b \in \left(\overline{b}, 1\right) \end{cases},$$

$$p^*_{A1} = \begin{cases} \frac{2(b+1)^2(a+c)-a\lambda^2\delta-\lambda\delta(a-c)(b+1)}{4(1+b)^2-\delta\lambda^2} & if \quad b \in \left(-1, \overline{b}\right) \\ \frac{c(\lambda\delta+2)+a(2-\delta\lambda(\lambda+1))}{4-\delta\lambda^2} & if \quad b \in \left(\overline{b}, 1\right) \end{cases},$$

$$p^*_{B1} = \begin{cases} \frac{2(b+1)^2(a+c)-a\lambda^2\delta-\lambda\delta(a-c)(b+1)}{4(1+b)^2-\delta\lambda^2} & if \quad b \in \left(-1, \overline{b}\right) \\ - & if \quad b \in \left(\overline{b}, 1\right) \end{cases},$$

[10] We have $\overline{b} = \frac{\sqrt{\lambda^4\delta^4-2\lambda^2\delta^3(\lambda^2+2)+\delta^2(\lambda(24\lambda+\lambda^3+32)+16)+\delta(4(\lambda(8-\lambda)+8))+16}-\lambda\delta(\lambda(1+\delta)+4)}{4(\delta+\lambda\delta+1)} \geq 0$

$$p^*_{A2} = \begin{cases} \frac{2(b+1)^2(a+c)-a\lambda^2\delta-\lambda(a-c)(b+1)}{4(1+b)^2-\delta\lambda^2} & if \quad b \in \left(-1,\bar{b}\right) \\ \frac{c(\lambda+2)+a(2-\lambda(\lambda\delta+1))}{4-\delta\lambda^2} & if \quad b \in \left(\bar{b},1\right) \end{cases}, \tag{11}$$

$$p^*_{B2} = \begin{cases} \frac{2(b+1)^2(a+c)-a\lambda^2\delta-\lambda(a-c)(b+1)}{4(1+b)^2-\delta\lambda^2} & if \quad b \in \left(-1,\bar{b}\right) \\ - & if \quad b \in \left(\bar{b},1\right) \end{cases},$$

$$w^*_{A1} = c, \; w^*_{B1} = \begin{cases} c & if \quad b \in \left(-1,\bar{b}\right) \\ < c & if \quad b \in \left(\bar{b},1\right) \end{cases},$$

$$w^*_{A2} = \begin{cases} \frac{4c(1+b)^2-\lambda^2\delta-2\lambda(a-c)(1+b)}{4(1+b)^2-\delta\lambda^2} & if \quad b \in \left(-1,\bar{b}\right) \\ \frac{c(2\lambda+4)-a\lambda(\lambda\delta+2)}{4-\delta\lambda^2} & if \quad b \in \left(\bar{b},1\right) \end{cases},$$

$$w^*_{B2} = \begin{cases} \frac{4c(1+b)^2-\lambda^2\delta-2\lambda(a-c)(1+b)}{4(1+b)^2-\delta\lambda^2} & if \quad b \in \left(-1,\bar{b}\right) \\ - & if \quad b \in \left(\bar{b},1\right) \end{cases},$$

$$\Pi^*_{A1} = \Pi^*_{B1} = \Pi^*_{A2} = \Pi^*_{B2} = 0,$$

$$\Pi^*_{R1} = \begin{cases} \frac{2(a-c)^2\left(2(1+b)^2-\lambda^2\delta-\lambda\delta(1+b)\right)(2b+\lambda\delta+2)}{(4(1+b)^2-\delta\lambda^2)^2} & if \quad b \in \left(-1,\bar{b}\right) \\ \frac{(a-c)^2(2+\delta\lambda)(2-\delta\lambda-\delta\lambda^2)}{(4-\delta\lambda^2)^2} & if \quad b \in \left(\bar{b},1\right) \end{cases},$$

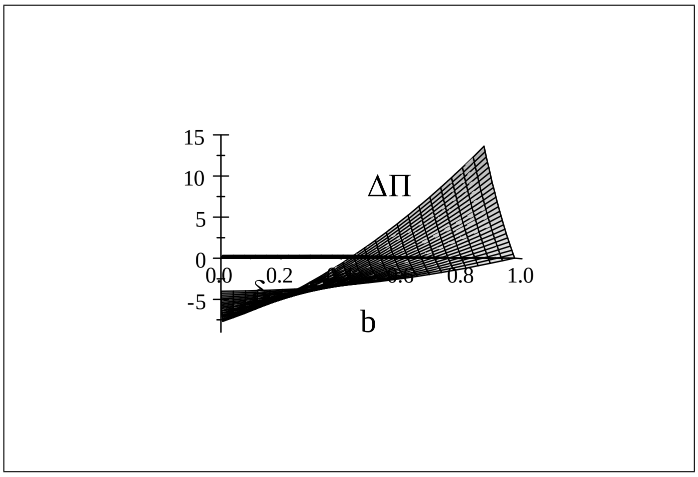
$$\Pi^*_{R2} = \begin{cases} \frac{2(a-c)^2(2b+\lambda+2)^2(1+b)}{(4(1+b)^2-\delta\lambda^2)^2} & if \quad b \in \left(-1,\bar{b}\right) \\ \frac{(a-c)^2(2+\lambda)^2}{(4-\delta\lambda^2)^2} & if \quad b \in \left(\bar{b},1\right) \end{cases},$$

$$\Pi^*_{R1} + \delta\Pi^*_{R2} = \begin{cases} \frac{2(a-c)^2((\delta+1)(b+1)+\lambda\delta)}{8b-\lambda^2\delta+4b^2+4} & if \quad b \in \left(-1,\bar{b}\right) \\ \frac{(\delta+\lambda\delta+1)(a-c)^2}{4-\delta\lambda^2} & if \quad b \in \left(\bar{b},1\right). \end{cases}$$

There is a trade-off between more intensive learning process under exclusivity and more varieties in the market under non-exclusivity. When product differentiation is low (b is high), the "learning" effect dominates the "more varieties" effect and firm B is excluded in period one and subsequently in period two. Intuitively, the more substitutable the products are, the less aggregate demand is foregone if product B is not present in the market and the lower cost is preferred.

An example of the difference in the present value of the profits, $\Delta\Pi = \left(\Pi^E_{R1} + \delta\Pi^E_{R2}\right) - \left(\Pi^{NE}_{R1} + \delta\Pi^{NE}_{R2}\right)$, is useful to understand the equilibrium outcome. Note in Diagram 1 that for high values of the product differentiation parameter ($b \to 1$), the present value of the profits under exclusivity is higher than under non-exclusivity, thus, one product is purchased by the retailer in equilibrium (since $\Delta\Pi > 0$). For a given level of the discount factor δ (axes y), the difference in the present value of the profits starts out negative - for low values of b - and then turns positive as b increases.

Diagram 1



$$\Delta\Pi = \left(\Pi^{E}_{R1} + \delta\Pi^{E}_{R2}\right) - \left(\Pi^{NE}_{R1} + \delta\Pi^{NE}_{R2}\right)$$

$$\lambda = 1$$

We discuss now the properties of the equilibrium we have derived. First, we study the effect of the learning-by-doing process on the quantities produced and the profits. Then, we compare the quanities produced in the first period to the quantities produced in the second period. The consumer surplus and the total welfare is examined separetely in the next section.

Remark 1 As the learning parameter λ increases, all equilibrium quantities increase, the equilibrium pro…ts in the …rst period decrease and the equilibrium pro…ts in the second period and the present value of the total pro…ts increase.

By directly differentiating the equilibrium quantities, we obtain that for both products and for both periods as the learning process becomes more intense, the quantities increase ($\frac{dq^*_{it}}{d\lambda} > 0$ for $i = A, B$ and $t = 1, 2$). Thus, the total production under the learning-by-doing hypothesis exceeds the total production when there are no gains from experience ($\lambda = 0$). Another implication is that the prices in the dynamic model are lower compared to the prices in the static model where there is no learning and production costs do not reduce in time. By directly differentiating the relevant equilibrium expressions at (11), we also obtain that the equilibrium profits in the first period decrease with λ ($\frac{d\Pi^*_{R1}}{d\lambda} < 0$) but the equilibrium profits in the second period increase with λ ($\frac{d\Pi^*_{R2}}{d\lambda} > 0$). A part of the retail profits should be sacrifised in the first period to take advantage of the learning process in the second period. However, the present value of the profits always increase with λ ($\frac{d\left(\Pi^*_{R1} + \delta\Pi^*_{R2}\right)}{d\lambda} > 0$) which means that the dynamic model gives higher total profits compared to the static model.

Remark 2 The quantity produced in the …rst period is lower than the quantity produced in the second period, $q_{i2} \geq q_{i1}$.

The marginal production cost in the first period is higher than the marginal production cost in the second period, which leads to higher prices in the first period ($p_{i1} \geq p_{i2}$) and lower quantities.

4 Consumers' surplus and total welfare

We have already obtained that when products are close substitutes ($b > \bar{b} > 0$) exclusivity emerges in the dynamic setting. Here, we study whether exclusivity is beneficial for the consumers and thus is welfare enhancing too. First we compare the product prices among exclusivity and non-exclusivity.

Remark 3 When exclusivity is imposed by the retailer, the product prices in both periods reduce compared to the non-exclusivity case: $p^E_{At} < p^{NE}_{it}$ for $t = 1, 2$.

When $b > \bar{b}$, consumers face the positive effect of price reduction due to the more intense learning and the negative effect of reduction in the product variety in the market. Which effect dominates? Is exclusivity beneficial for the consumers? We calculate the consumers' surplus that includes both the price reduction effect and the product variety effect and then compare the two cases.[11]

$$\begin{aligned}
CS_1^{NE} &= (1+b)\left(q_{i1}^{NE}\right)^2 = (1+b)\left(\frac{(a-c)(2(1+b)+\delta\lambda)}{4(1+b)^2-\delta\lambda^2}\right)^2, \\
CS_2^{NE} &= (1+b)\left(q_{i2}^{NE}\right)^2 = (1+b)\left(\frac{a-c}{2(1+b)}+\frac{\lambda(a-c)(2(1+b)+\delta\lambda)}{2(1+b)(4(1+b)^2-\delta\lambda^2)}\right)^2, \\
CS_1^{E} &= \frac{1}{2}\left(q_{A1}^{E}\right)^2 = \frac{1}{2}\left(\frac{(a-c)(2+\delta\lambda)}{4-\delta\lambda^2}\right)^2, \\
CS_2^{E} &= \frac{1}{2}\left(q_{A2}^{E}\right)^2 = \frac{1}{2}\left(\frac{(a-c)(2+\lambda)}{4-\delta\lambda^2}\right)^2,
\end{aligned}$$

where the subscript refers to the time period.

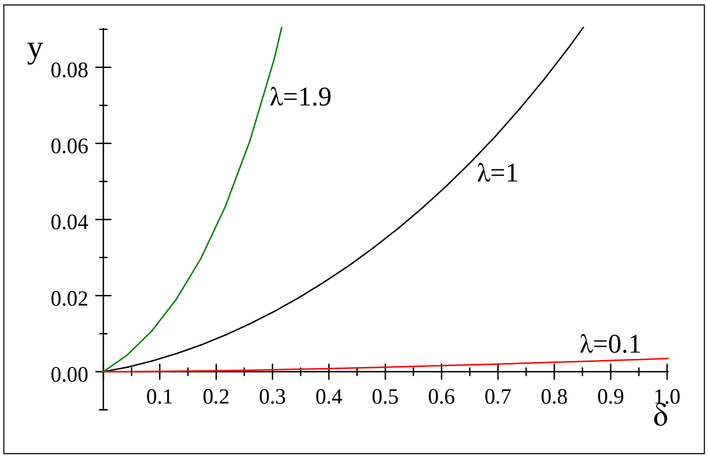
The difference between the consumers' surplus under exclusivity and non-exclusivity for each period, $\Delta CS_t = CS_t^E - CS_t^{NE}$, is increasing in b and there exists a unique and positive b, say $b_t(\lambda, \delta) > 0$, such that $\Delta CS_t = 0$. Above this critical b, the difference in the consumers' surplus is positive. Hence, the following proposition is proved.

Proposition 3 When products are close substitutes - b is positive and takes high values - consumers' surplus is higher under exclusivity compared to non-exclusivity. The price reduction e¤ect outsets the product variety e¤ect.

[11] See Singh and Vives, 1984.

To prove that exclusivity is always beneficial for the consumers (when it is imposed by the retailer) it is sufficient to prove that the difference between the consumers' surplus under exclusivity and non-exclusivity is positive in both periods when $b = \overline{b}$ (that is, $\Delta CS_t \left(b = \overline{b}\right) > 0$). If this difference is positive at point $\overline{b}$, then it is positive for every $b > \overline{b}$ where exclusivity is imposed by the retailer. We have proved numerically this argument and provide an example in the following diagram for the difference in the consumers' surplus in the first period.[12] Analogous results are taken for the second period.

Diagram 2



$$\Delta CS_1 \left(b = \overline{b}\right) = (a-c)^2 * y$$

In Diagram 2, we plot for the various values of the discount factor δ and various values of the learning parameter λ, the difference between the consumers' surplus under E and NE in the first period divided by $(a-c)^2$ when $b = \bar{b}$. We call this expression y, with $y = \frac{\Delta CS_1\left(b=\bar{b}\right)}{(a-c)^2}$, where this is equivalent to plotting $\Delta CS_1 \left(b = \overline{b}\right)$, since $(a-c)^2 > 0$. Observe that y is positive and increasing in δ and λ, therefore, $\Delta CS_1 \left(b = \overline{b}\right)$ is also positive. Thus, when exclusivity is chosen by the retailer this also leads to higher consumers' surplus compared to the non-exclusive case. Consumers prefer lower prices than more varieties in the market when $b > \overline{b}$. The difference in the consumers' surplus increases with δ and λ. Since the discounted stream of profits and consumers' surplus under exclusivity is higher than under non-exclusivity, we have:

Proposition 4 When the retailer imposes exclusivity, total welfare is increased compared to the non-exclusive case.

[12] We have proved this result numerically for several values of the learning parameter λ, however, due to the complexity of the calculations, a formal proof is pending.

5 The social planner's solution

It is important to calculate the social planner's solution and compare this to the equilibrium outcome obtained in Section (3). Is the equilibrium rate of learning the social optimal? The social planner maximizes the total welfare with respect to the quantities produced, that is, the sum of the chain profits plus the consumers' surplus and decides whether to exclude a product to intensify the learning process. The analysis is analogous to the analysis in Section (3). We begin with the static model and proceed with the two-period model.

5.1 Static social optimum

Non-exclusivity When both products are purchased, the social planner maximizes the total welfare - the sum of the chain profits and the consumers' surplus with respect to the quantities:

$$\begin{aligned}
\underset{q_A,q_B}{max} W &= \Pi_A^{total} + \Pi_B^{total} + CS \\
&= ((p_A - w_A)q_A + (w_A - c)q_A) + (\ (p_B - w_B)q_B + (w_B - c)q_B) + \frac{q_A^2 + q_B^2 + 2bq_Aq_B}{2} \\
&= (p_A - c)q_A + \ (p_B - c)q_B + \frac{q_A^2 + q_B^2 + 2bq_Aq_B}{2}.
\end{aligned}$$

This maximization gives:

$$\begin{aligned}
q_i^{NE} &= \frac{a-c}{1+b}, \\
p_i^{NE} &= c, \\
\Pi_i^{NE,total} &= 0, \\
W^{NE} &= CS^{NE} = \frac{(a-c)^2}{1+b}.
\end{aligned}$$

Exclusivity When the social planner chooses to purchase only product A, the problem becomes:

$$\begin{aligned}
\underset{q_A}{max} W &= \Pi_A^{total} + CS \\
&= ((p_A - w_A)q_A + (w_A - c)q_A) + \frac{q_A^2}{2} \\
&= (p_A - c)q_A + \frac{q_A^2}{2},
\end{aligned}$$

with optimum values

$$\begin{aligned} q_A^E &= a - c, \\ p_A^E &= c, \\ \Pi_A^{E,total} &= 0, \\ W^E &= CS^E = \frac{(a-c)^2}{2}. \end{aligned}$$

By direct comparison of the total welfare between the two cases, we obtain:

Lemma 5 The social planner never excludes a product in the static model since there are no gains from experience.

Note also that the final prices are set equal to the production costs, therefore, the total chain profits reduce to zero and the total welfare is equal to the consumers' surplus.

5.2 Dynamic social optimum

In the dynamic model, the social planner maximizes the discounted stream of total welfare. We assume that the discount factor δ of the social planner is the same as the discount factor of the firms. The maximand in the first period is $W_1 + \delta W_2$ and the maximand in the second period is W_2. We proceed backwards to solve for every subgame.

Period two

Taking as given the quantities purchased in the first period (therefore the production costs in period 2), we consider first the case where both products are purchased in period two.

Non-exclusivity The social planner maximizes the total welfare in period two

$$W_2 = (p_{A2} - c_{A2})q_{A2} + \ (p_{B2} - c_{B2})q_{B2} + \frac{q_{A2}^2 + q_{B2}^2 + 2bq_{A2}q_{B2}}{2}.$$

Taking into consideration the cost asymmetries that may exist and the corner solutions we

have

$$
\begin{aligned}
q_{A2} &= \begin{cases} \frac{a}{1+b}+\frac{bc_{B2}-c_{A2}}{1-b^2} & if \quad b\leq\frac{a-c_{B2}}{a-c_{A2}} \\ a-c_{A2} & if \quad b>\frac{a-c_{B2}}{a-c_{A2}} \end{cases}, \\
q_{B2} &= \begin{cases} \frac{a}{1+b}+\frac{bc_{A2}-c_{B2}}{1-b^2} & if \quad b\leq\frac{a-c_{B2}}{a-c_{A2}} \\ 0 & if \quad b>\frac{a-c_{B2}}{a-c_{A2}} \end{cases}, \\
p_{A2} &= c_{A2}, \\
p_{B2} &= \begin{cases} c_{B2} & if \quad b\leq\frac{a-c_{B2}}{a-c_{A2}} \\ - & if \quad b>\frac{a-c_{B2}}{a-c_{A2}} \end{cases}, \\
\Pi_{i2}^{total} &= 0, \\
W_2 &= CS_2=\begin{cases} \frac{a(a-c_{A2}-c_{B2})}{1+b}+\frac{c_{A2}^2+c_{B2}^2-2bc_{A2}c_{B2}}{2(1-b^2)} & if \quad b\leq\frac{a-c_{B2}}{a-c_{A2}} \\ \frac{(a-c_{A2})^2}{2} & if \quad b>\frac{a-c_{B2}}{a-c_{A2}}. \end{cases}
\end{aligned}
$$

Note that the final prices are set equal to the production costs which lead to zero profits and maximum consumers' surplus.

Exclusivity When product B is excluded in period two the social planner maximizes

$$
W_2=(p_{A2}-c_{A2})q_{A2}+\frac{q_{A2}^2}{2},
$$

which gives

$$
\begin{aligned}
q_{A2} &= a-c_{A2}, \\
p_{A2} &= c_{A2}, \\
\Pi_{A2}^{total} &= 0, \\
W_2 &= CS_2=\frac{(a-c_{A2})^2}{2}.
\end{aligned}
$$

Period one

Let us now proceed to the first period of the game. Analogously as in the basic model, we solve for the two cases in period one, under non-exclusivity in the first period which always leads to product variety in the second period and under exclusivity in the first period which might lead to product variety or only one product in the second period.

Non-exclusivity When both products are purchased in the first period, there is no cost asymmetry in the second period of the game and the social planner maximizes

$$
\begin{aligned}
W_1+\delta W_2 &= (p_{A1}-c)q_{A1}+\ (p_{B1}-c)q_{B1}+\frac{q_{A1}^2+q_{B1}^2+2bq_{A1}q_{B1}}{2} \\
&\quad +\delta\left(\frac{a\left(a-c_{A2}-c_{B2}\right)}{1+b}+\frac{c_{A2}^2+c_{B2}^2-2bc_{A2}c_{B2}}{2\left(1-b^2\right)}\right) \\
s.t.\ c_{i2} &= c-\lambda q_{i1}.
\end{aligned}
$$

From the first order conditions we obtain[13]

$$
\begin{aligned}
q_{i1}^{NE} &= \frac{(a-c)(b+\lambda\delta+1)}{2b-\lambda^2\delta+b^2+1}, \\
q_{i2}^{NE} &= \frac{(a-c)(b+\lambda\delta+1)}{2b-\lambda^2\delta+b^2+1}, \\
p_{i1}^{NE} &= \frac{c(b+1)(b+\lambda\delta+1)-a\lambda\delta(b+\lambda+1)}{2b-\lambda^2\delta+b^2+1} < c, \\
p_{i2}^{NE} &= c-\frac{\lambda(a-c)(b+\lambda\delta+1)}{2b-\lambda^2\delta+b^2+1} = c_{i2}, \\
\Pi_1^{NE-total} &= \frac{-2(a-c)^2(b+\lambda\delta+1)\lambda\delta(b+\lambda+1)}{\left(2b-\lambda^2\delta+b^2+1\right)^2} < 0, \qquad (12) \\
\Pi_2^{NE-total} &= 0, \\
CS_1^{NE} &= \frac{(a-c)^2(b+1)(b+\lambda\delta+1)^2}{\left(2b-\lambda^2\delta+b^2+1\right)^2} \geq 0, \\
CS_2^{NE} &= \frac{(a-c)^2(b+1)(b+\lambda+1)^2}{\left(2b-\lambda^2\delta+b^2+1\right)^2} \geq 0, \\
W_1^{NE} &= \frac{(a-c)^2(b+\lambda\delta+1)\left(2b-\lambda\delta-2\lambda^2\delta+b^2-b\lambda\delta+1\right)}{\left(2b-\lambda^2\delta+b^2+1\right)^2}, \\
W_2^{NE} &= \frac{(a-c)^2(b+1)(b+\lambda+1)^2}{\left(2b-\lambda^2\delta+b^2+1\right)^2} \geq 0, \\
W_1^{NE}+\delta W_2^{NE} &= \frac{(a-c)^2(b+\delta+b\delta+2\lambda\delta+1)}{2b-\lambda^2\delta+b^2+1} \geq 0.
\end{aligned}
$$

Note that final prices in the second period are equal to the production costs but prices in the first period are lower than the production costs to intensify the learning process by increasing the production in the first period. Thus, there are some losses in the first period but total welfare is positive and at the maximum level.

Exclusivity When the social planner chooses not to distribute product B in the first period of the game, it maximizes

$$
\begin{aligned}
W_1+\delta W_2 &= \begin{cases} (p_{A1}-c)q_{A1}+\frac{q_{A1}^2}{2}+\delta\left(\frac{a(a-c_{A2}-c_{B2})}{1+b}+\frac{c_{A2}^2+c_{B2}^2-2bc_{A2}c_{B2}}{2(1-b^2)}\right) & if \quad b \leq \frac{a-c_{B2}}{a-c_{A2}} \\ (p_{A1}-c)q_{A1}+\frac{q_{A1}^2}{2}+\delta\frac{(a-c_{A2})^2}{2} & if \quad b > \frac{a-c_{B2}}{a-c_{A2}} \end{cases} \\
s.t.\ c_{A2} &= c-\lambda q_{A1} \text{ and } c_{B2}=c
\end{aligned}
$$

or equivalently

$$
W_1+\delta W_2 =
$$

$$
\begin{cases} (a-q_{A1}-c)q_{A1}+\frac{q_{A1}^2}{2}+\delta\left(\frac{a(a-(c-\lambda q_{A1})-c)}{1+b}+\frac{(c-\lambda q_{A1})^2+c^2-2b(c-\lambda q_{A1})c}{2(1-b^2)}\right) & if \quad q_A \leq \frac{(a-c)(1-b)}{b\lambda} \\ (a-q_{A1}-c)q_{A1}+\frac{q_{A1}^2}{2}+\delta\frac{(a-(c-\lambda q_{A1}))^2}{2} & if \quad q_A > \frac{(a-c)(1-b)}{b\lambda}. \end{cases}
$$

---

[13] To obtain positive quantities and to satisfy the second order conditions, we should have $2b-\lambda^2\delta+b^2+1 \geq 0$, $-2b-\lambda^2\delta+b^2+1>0$ and $1-\lambda^2\delta-b^2>0$.

Following the same steps as in Section (3), the social optimum expressions under exclusion in the first period are

$$
\begin{aligned}
q_{A1}^{E} &= \begin{cases} \frac{(a-c)(1-b)(b+\lambda\delta+1)}{1-\lambda^2\delta-b^2} & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \text{ both in t=2} \\ \frac{(a-c)(1+\lambda\delta)}{1-\lambda^2\delta} & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \text{ only A in t=2} \end{cases}, \\
q_{A2}^{E} &= \begin{cases} \frac{(a-c)(1-b+\lambda)}{1-\lambda^2\delta-b^2} & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ \frac{(a-c)(1+\lambda)}{1-\lambda^2\delta} & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
q_{B2}^{E} &= \begin{cases} \frac{(a-c)\left(1-b-b\lambda-\lambda^2\delta\right)}{1-\lambda^2\delta-b^2} & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ 0 & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
p_{A1}^{E} &= \begin{cases} \frac{c-b^2c-a\lambda\delta+c\lambda\delta-a\lambda^2\delta+ab\lambda\delta-bc\lambda\delta}{1-\lambda^2\delta-b^2} < c & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ \frac{c-a\lambda\delta+c\lambda\delta-a\lambda^2\delta}{1-\lambda^2\delta} < c & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
p_{A2}^{E} &= \begin{cases} \frac{c-a\lambda+c\lambda-b^2c+ab^2\lambda-b^2c\lambda-a\lambda^2\delta+ab\lambda^2\delta-bc\lambda^2\delta}{1-\lambda^2\delta-b^2} = c_{A2} & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ \frac{c-a\lambda+c\lambda-a\lambda^2\delta}{1-\lambda^2\delta} = c_{A2} & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
p_{B2} &= \begin{cases} c & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ - & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
\Pi_1^{E-total} &= \begin{cases} \frac{(a-c)^2(b-1)\lambda\delta(1-b+\lambda)(b+\lambda\delta+1)}{\left(\lambda^2\delta+b^2-1\right)^2} < 0 & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ \frac{-(a-c)^2(\lambda\delta+1)\lambda\delta(\lambda+1)}{\left(\lambda^2\delta-1\right)^2} < 0 & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
\Pi_2^{E-total} &= \begin{cases} 0 & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ 0 & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
CS_1^{E} &= \begin{cases} \frac{(a-c)^2(b-1)^2(b+\lambda\delta+1)^2}{2\left(\lambda^2\delta+b^2-1\right)^2} & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ \frac{(a-c)^2(\lambda\delta+1)^2}{2\left(\lambda^2\delta-1\right)^2} & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
CS_2^{E} &= \begin{cases} \frac{(a-c)^2\left(\lambda^4\delta^2+\left(1-b^2\right)\left(2\lambda-2b-2b\lambda+\lambda^2-2\lambda^2\delta+2\right)\right)}{2\left(\lambda^2\delta+b^2-1\right)^2} & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ \frac{(a-c)^2(\lambda+1)^2}{2\left(\lambda^2\delta-1\right)^2} & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
W_1^{E} &= \begin{cases} \frac{(a-c)^2\left((1-b)(b+2\delta+2\lambda\delta+1)-\lambda^2\delta^2\right)}{2\left(1-\lambda^2\delta-b^2\right)} & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ \frac{(a-c)^2(\delta+2\lambda\delta+1)}{2\left(1-\lambda^2\delta\right)} \geq 0 & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
W_2^{E} &= \begin{cases} \frac{(a-c)^2\left(\lambda^4\delta^2+\left(1-b^2\right)\left(\left(2\lambda-2b-2b\lambda+\lambda^2+2\lambda^2\delta+2\right)-4\lambda^2\delta\right)\right)}{2\left(\lambda^2\delta+b^2-1\right)^2} & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ \frac{(a-c)^2(\lambda+1)^2}{2\left(\lambda^2\delta-1\right)^2} & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda} \end{cases}, \\
W_1^{E}+\delta W_2^{E} &= \begin{cases} \frac{(a-c)^2\left(\lambda^2\delta^2\left(2b-2\lambda+2b\lambda+2\lambda^2\delta+3b^2-5\right)+(b+1)(b-1)^2(b+4\delta+4\lambda\delta+1)\right)}{2\left(\lambda^2\delta+b^2-1\right)^2} & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ \frac{(a-c)^2\left(-2\lambda^3\delta^2-\lambda^2\delta^2+4\lambda\delta+2\delta+1\right)}{2\left(\lambda^2\delta-1\right)^2} & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda}. \end{cases}
\end{aligned} \tag{13}
$$

Again here the social planner intensifies the learning process by setting prices lower than the

production costs in the first period and thus incurring some losses. In the second period, prices are equal to the production costs. We find that exclusivity is imposed in the first period of the game which also leads to exclusivity in the second period of the game when $b > \frac{1-\lambda^2\delta}{1+\lambda}$, however, when $b \leq \frac{1-\lambda^2\delta}{1+\lambda}$ the quantity of product A purchased in the first period of the game is not so high to exclude product B in second period.

Characterization of the social optimum To fully characterize the social planner's solution we have to check when the social planner strategically excludes an upstream supplier in the first period of the game to manipulate the learning process. So we have to compare the present value of the total welfare using expressions (12) and (13) for every product differentiation parameter.

$$\Delta W = \left(W_1^E + \delta W_2^E\right) - \left(W_1^{NE} + \delta W_2^{NE}\right) =$$

$$\begin{cases} \frac{(a-c)^2\left(\lambda^2\delta^2\left(2b-2\lambda+2b\lambda+2\lambda^2\delta+3b^2-5\right)+(b+1)(b-1)^2(b+4\delta+4\lambda\delta+1)\right)}{2\left(\lambda^2\delta+b^2-1\right)^2} - \frac{(a-c)^2(b+\delta+b\delta+2\lambda\delta+1)}{2b-\lambda^2\delta+b^2+1} & if \quad b \leq \frac{1-\lambda^2\delta}{1+\lambda} \\ \frac{(a-c)^2\left(-2\lambda^3\delta^2-\lambda^2\delta^2+4\lambda\delta+2\delta+1\right)}{2\left(\lambda^2\delta-1\right)^2} - \frac{(a-c)^2(b+\delta+b\delta+2\lambda\delta+1)}{2b-\lambda^2\delta+b^2+1} & if \quad b > \frac{1-\lambda^2\delta}{1+\lambda}. \end{cases}$$

When ΔW is positive and $b \leq \frac{1-\lambda^2\delta}{1+\lambda}$, exclusivity is imposed only in period one and in period two both products are purchased. However, when ΔW is positive and $b > \frac{1-\lambda^2\delta}{1+\lambda}$, exclusivity is imposed in both periods. Numerically, we have found that:[14]

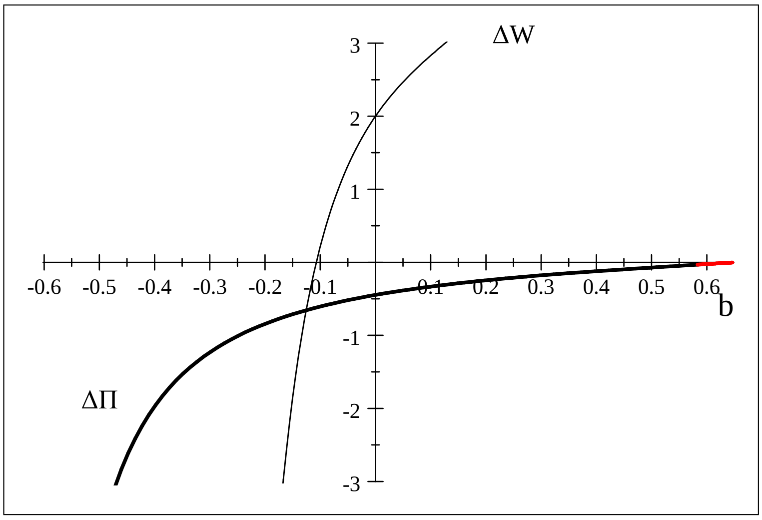
Lemma 6 For low values of the product di¤erentiation parameter the social planner chooses to have both products in the market in both periods. For intermediate values of the product di¤erentiation parameter product B is excluded in the …rst period but not in the second period. Finally, for high values of the product di¤erentiation parameter product B is excluded in both periods.

Also, we have that:

Lemma 7 The social planner tends to impose exclusivity more often than the retailer.

[14]We have proved this result numerically for several values of the parameters, however, a formal proof is pending.

Diagram 3



$\lambda = 1, \delta = 0.5$

To better understand these results, we give a numerical example. For a specific learning and discount factor parameter, we plot in Diagram 3 - for each b - the difference in the present value of the welfare ($\Delta W = \left(W_1^E + \delta W_2^E\right) - \left(W_1^{NE} + \delta W_2^{NE}\right)$, the objective function of the social planner) and the difference in the present value of the profits ($\Delta\Pi = \left(\Pi_{R1}^E + \delta\Pi_{R2}^E\right) - \left(\Pi_{R1}^{NE} + \delta\Pi_{R2}^{NE}\right)$, the objective function of the retailer from Section (3)). We observe that when the products are close complements the social planner never imposes exclusivity and this also holds for the retailer ($\Delta W < 0$ and $\Delta\Pi < 0$ for low and negative b). However, when the products are not close complements (b is negative but not very low) the social planner excludes product B only in the first period which is never an equilibrium choice for the retailer. The retailer never excludes product B in the first period when the products are complements (close or not, $\Delta\Pi < 0$ for $b < 0$). Also, the social planner excludes product B from the market only in the first period when products are not close substitutes and excludes product B from the market in both periods when products are close substitutes. In contrast the retailer never excludes product B only in the first period, it excludes product B in both periods for high and positive values of b.

To summarize, as b increases (from -1 to 1) the social planner first carries both products in both periods, then, carries only product A in the first period and both products in the second period and, finally, it carries only product A in both periods. However, the retailer, first, carries both products in both periods and then carries only product A in both periods. Note also in the diagram that exclusivity happens more often in the social optimum setting compared to the equilibrium outcome where the retailer imposes exclusivity. The area of the parameter b where ΔW is positive is greater than the one where $\Delta\Pi$ is positive.

By direct comparison of the quantities in the first period under the planning solution and the equilibrium outcome of Section (3), we conclude to the following proposition.

Proposition 5 The rate of learning in equilibrium is lower than the social optimum. The production costs in the second period are lower under the social optimal due to the higher quantity produced in the …rst period, compared to the equilibrium outcome without a social planner.

6 The role of the intermediary

In this section we study the role of the retailer in the learning process. Is he necessary to intensify this process? Or do firms take advantage of the learning process equally efficiently without the downstream monopolist. First, we analyze the role of the retailer in the static model where there is no learning and then we study the dynamic model.

We compare the equilibrium outcome when the upstream firms distribute their products to the final market indirectly, via the retailer, to the equilibrium outcome when the upstream firms distribute their products directly to the final consumers. When there is no intermediary, the outcome is the Cournot duopoly outcome with differentiated products and when a retailer exists, the outcome is the monopoly outcome (given in Section (2)). The key parameter is the product differentiation parameter. When the products are substitutes ($b \in (0,1)$) and there is no intermediary, the final prices and total profits are lower and the consumers' surplus and total welfare are higher compared to the case where a retailer exists.

Remark 4 From the social point of view, when the products are substitutes, the presence of the intermediary hurts the market, since the retailer gains the monopoly pro…ts by increasing the …nal prices and, thus, lowering the consumers' surplus and total welfare. In contrast, when the products are complements, the role of the retailer in the market is positive, since it internalizes the externalities from the two complement goods. The prices are lower, the consumers' surplus, pro…ts and total welfare are higher when there exist a retailer in the downstream level.

Then we solve for the dynamic Cournot differentiated duopoly model with learning by doing technology and no intermediary. Both products are purchased in both periods and the equilibrium quantities are given by

$$\begin{aligned} q_{i1}^C &= \frac{(a-c)\left((2-b)(2+b)^2+4\delta\lambda\right)}{(2-b)(2+b)^3-4\delta\lambda^2} \text{ for } i=A,B, \\ q_{i2}^C &= \frac{a-c}{2+b}+\frac{\lambda(a-c)\left((2-b)(2+b)^2+4\delta\lambda\right)}{(2+b)\left((2-b)(2+b)^3-4\delta\lambda^2\right)} \text{ for } i=A,B. \end{aligned}$$

We compare these quantities to the quantities in our equilibrium model with the presence of the retailer

$$q_{A1}^{*} = \begin{cases} \frac{(a-c)(2(1+b)+\delta\lambda)}{4(1+b)^2-\delta\lambda^2} \text{ both/both} & if \quad b \in \left(-1, \bar{b}\right) \\ \frac{(a-c)(2+\delta\lambda)}{4-\delta\lambda^2} \text{ only A/only A} & if \quad b \in \left(\bar{b}, 1\right) \end{cases},$$

$$q_{B1}^{*} = \begin{cases} \frac{(a-c)(2(1+b)+\delta\lambda)}{4(1+b)^2-\delta\lambda^2} & if \quad b \in \left(-1, \bar{b}\right) \\ - & if \quad b \in \left(\bar{b}, 1\right) \end{cases}$$

and obtain that:

Remark 5 For $b \in (-1, 0)$ we have $q_{A1}^{C} < q_{A1}^{*}$, for $b \in \left(0, \bar{b}\right)$ we have ${}_{A1}^{C} > q_{A1}^{*}$ and for $b \in \left(\bar{b}, 1\right)$ we have $q_{A1}^{C} < q_{A1}^{*}$.

The presence of the intermediary intensifies the learning process when the two products are complements or when products are not close substitutes and exclusivity is imposed by the intermediary (i.e. $b \in (-1, 0) \cup \left(\bar{b}, 1\right)$) . The retailer coordinates the purchases of the two products and takes advantage of the learning process in a more efficient way compared to the case where upstream producers sell directly their products to the final consumers. The rate of learning is closer to the social rate of learning - for these parameter values of b - when there is an intermediary in the market.

7 Conclusion and further research

The learning-by-doing effects play a significant role in a vertical chain. In this paper, we examined how the learning-by-doing process affects the final market outcome in a vertical framework. Upstream firms produce differentiated products, either substitutes or complements, and the downstream monopolist sets the linear contract terms. The unit production cost of the upstream firms reduces with the accumulated production. We study how the dynamic interactions between upstream and downstream firms affect the exclusive dealing decisions in a two-period game. The existing literature has either examined vertical contracting without dynamic interactions due to learning effects or the learning-by-doing process in an oligopolistic industry without vertical considerations. While our paper is, to the best of our knowledge, the first paper that studies a dynamic vertical contracting framework with learning-by-doing production technologies.

In the static model, both products are carried by the retailer. Exclusivity never arises in equilibrium, since final consumers like product variety. When we introduce dynamic considerations, the decision on imposing exclusivity depends on the product differentiation

parameter. Close substitutability leads to exclusivity in the dynamic model, since the "lower prices" effect (due to lower production costs) dominates to the "product variety" effect. In contrast, complementarity or not close substitutability leads to an equilibrium where the retailer purchases both products in both periods. More varieties in the market are preferred to lower final prices. Consumers' surplus and total welfare is also higher under exclusivity when it is imposed by the retailer compared to the case where exclusivity is not imposed (for example due to regulatory restrictions). Therefore, exclusivity is welfare improving. Nevertheless, the equilibrium rate of learning is not equal to the social optimum. A social planner would impose more often exclusivity and would reduce more the production costs in the second period compared to the retailer. Finally, when products are complements or close substitutes the presence of the retailer is necessary to coordinate the learning process.

A number of extensions are open for future research. First, one may examine the case where the bargaining power is at the upstream level or firms bargain on the contract terms. What is the Nash bargaining solution in the dynamic game? Does exclusivity arise in equilibrium? When upstream firms set the contract terms and the retailer chooses exclusive dealing or not, exclusivity is never an equilibrium outcome. The downstream monopolist prefers to leave the upstream competition active.

A second interesting extension is to change the vertical structure, that is, assume a different number of firms. Is it more difficult for exclusivity to arise when the number of the upstream suppliers increases? A third extension is to impose different learning parameters and/or different discount factors for the two upstream firms. Is the firm with the higher learning parameter and discount factor favored by the downstream retailer? Another interesting extension is to study the quadratic learning-by-doing formulation or industry-wide learning-by-doing with spillovers between the producers rather than product specific.

The last decades there is a lot criticism on exclusionary practices. Sometimes these are considered as anticompetitive and sometimes as procompetitive. In this paper, we give the conditions under which exclusivity is beneficial for firms and consumers in a dynamic vertical framework with learning-by-doing production technologies. Our analysis enriches the qualitative results concerning the exclusionary practices but further research is, of course, necessary.

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