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Consumer Demand for Variety: Intertemporal Effects of Consumption, Product Switching and Pricing Policies

Ricardo Ribeiro (Universidade Católica Portuguesa)

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Ricardo Ribeiro*

Universidade Católica Portuguesa

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Abstract

The consumption of a good typically creates temporal satiation that diminishes the marginal utility of consuming more. Satiation induces consumers to increase their stimulation level by seeking variety and substitute towards other goods (substitutability across time) or towards differentiated versions of the same good with different attributes (substitutability across attributes). The literature on variety-seeking has developed along two strands, each focusing on only one type of substitutability. I specify a demand model that attempts to link these two strands of the literature. Consumers are assumed to make rational purchase decisions and take into account, not only current and future satiation levels, but also prices and product attribute choices. This issue is economically relevant for (i) competition policy since neglecting such forward-looking behaviour introduces a bias on the demand estimates, central in antitrust analysis, and (ii) firms since both types of consumer substitutability are important in designing intertemporal price discrimination strategies. Consumers are found to be forward-looking with respect to the duration since the last purchase, to price expectations and product attribute choices. In particular, I find evidence that consumption has a lasting effect on utility that induces substitutability across time and that the median consumer has a taste for variety in her product decisions. I also find that failing to account for such forward-looking behaviour overestimates state-dependence, hindering antitrust analysis. I then use the model to evaluate the demand implications of a major pricing policy change from hi-low pricing to an everyday low pricing strategy. In particular, the pricing policy simulations suggest that retailers may increase revenue by reducing the variance of prices, but that lowering the everyday level of prices may be unprofitable.

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1 Introduction

"Rachel: Hi!

Chandler: Another cheesecake came! They delivered it to the wrong address again!

Rachel: So just bring it back downstairs, what's the problem?

Chandler: I can't seem to say goodbye.

Rachel: Are you serious?! Chandler, we ate an entire cheesecake two days ago and you want more?"

Friends, Episode 7-11, The one with All The Cheesecakes

The concept of diminishing marginal utility is a cornerstone of economic theory. The consumption of a good typically creates satiation that diminishes the marginal utility of consuming more. The length of time that the marginal utility is diminished is likely to vary across goods and, as Rachel and Chandler's cheesecake episode illustrates, across consumers. While it only took Chandler two days for his cheesecake marginal utility to return to pre-consumption levels, Rachel seemed still satiated (suggesting that the utility provided by her prior cheesecake consumption had not yet faded).

Temporal satiation induces consumers to increase their stimulation level by seeking variety and therefore substitute towards consumption alternatives with different attributes. In this paper, I define a forward-looking dynamic discrete choice model of demand that, similarly to Hartmann (2006), allows consumption to have an intertemporal effect: consuming produces a consumption capital stock that provides utility over time until it gradually depreciates. However, unlike Hartmann (2006), I consider a differentiated products setting that following McAlister (1982) allows consumers to switch not only towards other goods, but also towards other differentiated versions of the same good with different attributes.

In each shopping trip, consumers decide whether or not to purchase the good, and in case they decide to purchase, which quantity and product (and implicitly attributes) to buy. Consumers are assumed to make rational purchase decisions by taking into account, not only current and future satiation levels, but also prices and product attribute choices. Price expectations are an important determinant of intertemporal substitution. If prices are expected to be higher in the future, consumers may anticipate their purchase decisions and vice-versa. Another important feature is consumer product attribute choice. The marginal utility of the attributes of a product is allowed to change over consumption occasions, depending on the taste for attribute variety (or habit persistence/switching costs) of the individual. Con-

sumers with a strong taste for attribute variety (or with low habit persistence/switching costs) may anticipate their purchase decisions whenever they alter their product attribute choice.

This paper relates to the literature on variety-seeking developed from Jeuland (1978) and McAlister (1982). Jeuland (1978) explains variety-seeking behaviour by proposing that prior experience with a good decreases the consumer's utility for that good, which constitutes a direct application of the diminishing marginal utility concept. This explanation is predictive of the consumer's tendency to switch away from the most recently consumed good. McAlister (1982) then refined the explanation by proposing that prior experience with the *attributes* of a good decreases the consumer's utility for goods with similar attributes, which refocuses the diminishing marginal utility concept over attributes rather than goods. These two explanations have governed the subsequent development of an extensive literature centered on the implications of habit persistence/switching costs for consumer choice (see, for example, Keane (1997)). Recently, Hartmann (2006) extended the variety-seeking literature by allowing intertemporal effects of consumption, that is, by allowing consumption to have a lasting effect that diminishes the marginal utility of future consumption. However, the homogeneous nature of the good studied, golf, did not allow him to focus on product attribute switching.

This paper attempts to link the two strands of the literature on variety-seeking by allowing consumers to substitute towards other goods (substitutability across time), as well as to other differentiated products of the same good (substitutability across attributes). This issue is economically relevant for (i) competition policy since neglecting to account for such forward-looking behaviour introduces a bias on the demand estimates, central in antitrust analysis, and (ii) firms since both types of consumer substitutability are important for retailers and manufacturers in designing intertemporal price discrimination strategies. I specify a consumer demand model which allows consumption to have an enduring effect and allows the marginal utility of the different product attributes to vary over consumption occasions. The main contribution of the paper is to study (i) the importance of such dynamic considerations for demand estimation, and (ii) how different pricing policies affect consumer demand for goods with both enduring effects of consumption and a high degree of differentiation. In particular, the model can be used to simulate the demand implications of major pricing policy changes like a shift from hi-low pricing to everyday low pricing. To my knowledge, there is only one study that structurally addresses consumer response to such major policy changes, Erdem *et al.* (2003). However, they studied storable goods and do not allow for attribute habit persistence/switching costs. I find similar patterns deriving from an entirely

different source of dynamics, the stock of past consumption.

The state space implied by a dynamic problem where forward-looking consumers make optimal decisions in light of current and future satiation levels, prices and product attribute choices is, in a differentiated products setting, extremely large for practical estimation. In order to reduce the dimensionality of the state space, I adopt a multi-stage budgeting approach that decomposes the consumers decision into a quantity choice and a product choice (see Aguirregabiria (2002) and Hendel and Nevo (2006a) for similar dynamic applications of Gorman (1971)'s approach). Under the set of assumptions discussed below, I show that the consumer product (and implicitly attribute) choice *conditional on the quantity purchased* does not depend on dynamic considerations. This simplifies the estimation of many of the parameters of the demand model, while the remaining ones are estimated solving a simplified dynamic problem that involves only quantity and timing decisions.

I estimate the different stages of the model by maximum-likelihood and solve the dynamic programming problem by using value function parametric approximation with policy function iteration in the lines of Benitez-Silva *et al.* (2000). In order to control for unobserved heterogeneity, I incorporate a rich specification. This is important since in both stages unobserved consumer heterogeneity may confound the inference of true state-dependence effects. As Heckman (1981) points out, if households have different preferences "and if these differences are not properly controlled, previous experience may appear to be a determinant (...) of future experience solely because it is a proxy for temporally persistent unobservables that determine choices." The state dependence in the product (and implicitly attribute) decision arises because the marginal utility of the attributes of a product is allowed to change over consumption occasions, while the state dependence in the quantity decision is induced by the consumption capital stock. For reasons I discuss below, I incorporate *observable* heterogeneity in the product choice and a *continuous distribution* of consumer heterogeneity in the quantity decision.

I apply the model to an indulgence good: ice cream (and related frozen desserts). The reason is twofold. First, ice cream constitutes the textbook illustration of the diminishing marginal utility concept and the industry is characterized by a high degree of product differentiation. Second, the temptation nature of the good can (and in fact does, for the empirical application considered) make stockpiling limited in relevance (and in particular duration). This is important because in a context where temporary price promotions are a key marketing tool, if consumers respond to temporary price cuts by accelerating (anticipating) purchases and hold inventories for future consumption (i.e. stockpile), the separate identification of satiation and stockpiling would be somewhat problematic. I show below that, even though

consumers do anticipate purchases in response to temporary price promotions, they do not stockpile, maybe because of the temptation feature of the good.

I find evidence that consumption has a lasting effect on utility that induces substitutability across time and that the median consumer has a taste for variety in her product decisions. Consumers are found to be forward-looking with respect to the duration since the last purchase, to price expectations and product choices. Pricing policy simulations suggest that retailers may increase revenue by reducing the variance of prices, but that lowering the everyday level of prices may be unprofitable.

2 Data Description and Preliminary Analysis

I use Information Resources Inc. (IRI) scanner data collected from June 1991 to May 1993 in two separate submarkets of a large Midwest city. The dataset covers 24 different product categories at both the store and household levels. The former includes weekly sales, prices, and promotional activities for each universal product code (UPC) in nine supermarkets, belonging to different chains, while the latter tracks the store visits of 548 households and includes when and how much each household spent in her shopping trips.

I estimate the model for an indulgence good category: ice cream and related frozen desserts. Ice creams come in a limited number of package sizes, with the top four sizes accounting for more than 99% of the market: 64 oz. (72.0%), 160 oz. (11.6%), 16 oz. (10.9%) and 32 oz. (4.8%). The choice set available to the households is substantial. The average supermarket in the sample carries 181 different frozen dessert products (from 21 brands) on a weekly basis. I define a product on the basis of three attributes: segment, brand and flavour so that, for example, Häagen-Dazs Vanilla Ice Cream, Häagen-Dazs Chocolate Ice Cream, and Häagen-Dazs Vanilla Frozen Yoghurt are classified as distinct products.

Frozen desserts are offered in four segments: regular ice cream, diet ice cream, frozen yoghurt and ice milk. Regular ice creams account for 67% of the volume sold at the supermarket level, with diet ice creams and frozen yoghurt roughly splitting the remaining of the market. The market share of ice milk is less than one percent. Kemps is the dominant brand with 23% volume market share, followed by Breyers and Wessanen's Value Pack (both with 12%), Dreyer's (10%) and Häagen-Dazs (6%). Store private labels account for 3.9% of the market. The most popular flavours are vanilla (21%), chocolate (9%), neapolitan (6%), strawberry (5%) and chocolate chip (5%), although a typical supermarket would carry an average of 86 different flavours, each week. In contrast with the moderated brand

TABLE 1
*Household Demographics**

	Mean	Percentile				
		5	25	50	75	95
Household size	2.214	1	1	2	3	5
Household without a male	0.447	0	0	0	1	1
Household without a female	0.087	0	0	0	0	1
Household income	5.806	1	3	6	8	11
Female head works 35+ hours	0.272	0	0	0	1	1

* An observation is a household. Household size denotes the number of people in the household. Household without a male (female) denotes an indicator variable that takes the value 1 if there is no male (no female) within the household. Household income denotes the household income group: (in thousand dollars) 1- [0,10), 2-[10,12), 3-[12,15), 4-[15,20), 5-[20,25), 6-[25,35), 7-[35,45), 8-[45,55), 9-[55,65), 10-[65,75), 11-[75,+inf). Female head works 35+ hours denotes an indicator variable that takes the value 1 if the female head (or the male head in the case there is no female within the household) works more than 35 hours per week.

and favour concentration, there is substantial market fragmentation at the product level. Breyers Vanilla Ice Cream is the market leader with a 2.6% volume market share.

I conduct the subsequent analysis using a subset of the 517 sampled households that purchase frozen desserts. The subset was selected on the basis of three criteria. First, I eliminate households recorded purchasing in supermarkets for which no price data is available. An alternative approach could have been to include those households and either (i) eliminate the purchases in unsampled stores as if they never occurred, or (ii) assume some cross-store price pattern and generate price data to be imputed for those purchases. All solutions potentially could introduce bias in the analysis. I opted for the elimination after ensuring the subset sample was representative, an issue I discuss below. Second, since multiple-item purchases account for 21% of all consumer purchases, computational barriers compel me to eliminate households that made purchase decisions involving a non-representative combination of number of items and total volume, which I consider to be those that account for less than one percent of purchases. Their inclusion would increase the dimensionality of the state space to a degree that made the structural estimation computationally infeasible. Finally, I eliminate households that made less than 10 purchases of ice creams over the sample period considered (by far the most important reason for the reduction in the number of households included in the analysis) since they are likely to be either (i) not regularly in the market, or (ii) purchasing in alternative stores.² This reduces the sample size from 517 to 103 households, who made a total of 16,966 supermarket trips and 2,917 ice-cream purchases. Table 1 presents

²For reasons I discuss below, I partition the sample period in two subperiods using the first 12 weeks to generate an initial purchase for each household and conducting the analysis on the remaining 92 weeks. For completeness, I consider households that made at least 10 purchases of ice-creams over the latter sample period, conditional on having made at least 1 purchase over the former sample period.

summary statistics about the selected household demographics. The median household has two members - both a male and a female, an annual income between 25,000 and 35,000 dollars, with the female head working less than 35 hours.

An important question is obviously whether the subset sample is representative of the whole population buying at these supermarkets. Table 2 addresses this question by reporting, for the different samples, the top-8 products, brands and flavours in terms of their volume market share. The simple comparison of the columns show that, with minor exceptions, the product, brand and flavour market shares in the different samples are very similar, which is suggestive that the subset sample is reasonably representative.

2.1 Substitutability Across Time

In this section, I analyze the shopping behaviour of households and the frequency of their purchasing patterns for the ice cream category as a whole. Table 3, Panel A examines the supermarket trips of households by percentile. Although there is evidence of substantial heterogeneity across households with regard to their shopping behaviour, the median household in the sample visits a supermarket every three days to a total of 125 times over the observed sample. This household shops in two different supermarkets, but concentrates her purchases on a single one. In order to compute the households intertrip duration, I use the first six months in the sample to generate an initial trip for each household. I will discuss below that these first six months will also be instrumental in generating an initial product choice for each household to avoid spurious switching.

Table 3, Panel B examines the ice-cream purchasing patterns of households by percentile. The results suggest substantial heterogeneity also at this level, with the median household making a single-item purchase of 64 oz. of ice-cream every 14 days to a total of 18 purchases over the sample period.

I now move on to examine the hypothesis that consumption has a lasting effect that diminishes the marginal utility of future consumption. If the magnitude of this effect is such that induces households to vary their choice of dessert, the probability of purchase will be related to how long it has been since their last purchase. If, on the other hand, the magnitude of the effect is small, then the probability of purchase will not depend on interpurchase duration. Figure 1 displays the purchase hazard rate by no-purchase spell duration in days. The hazard rate denotes here the probability that you purchase if you have not purchased up to now. The pattern illustrated by the figure provides some support for

TABLE 2
*Volume Market Shares**

Panel A: Product level		Household					
		All		Subset		Store	
		S	CS	S	CS	S	CS
1	Wessanen's value pack NY vanilla IC	3.85	3.85	6.31	6.31	2.03	2.03
2	Wessanen's value pack vanilla IC	2.71	6.56	3.17	9.48	2.58	4.61
3	Wessanen's value pack neapolitan IC	2.06	8.62	2.81	12.29	1.78	6.39
4	Fieldcrest vanilla IC	2.05	10.66	2.48	14.77	1.22	7.61
5	Kemps vanilla FY	1.97	12.64	4.61	19.38	1.13	8.74
6	Kemps vanilla IC	1.68	14.32	0.83	20.22	2.15	10.89
7	Breyers vanilla IC	1.51	15.83	2.06	22.28	2.60	13.49
8	Wessanen's value pack chocolate IC	1.41	17.24	0.75	23.03	1.14	14.62
Panel B: Brand level		Household					
		All		Subset		Store	

TABLE 3
*Household Category Purchasing Patterns**

Panel A: Supermarket trips	Mean	Percentile				
		5	25	50	75	95
Number of trips	144.5	50	86	125	186	297
Days from previous trip	4.461	0	2	3	6	13
Number of stores visited	2.146	1	2	2	3	3
Store HHI	0.858	0.51	0.72	0.95	1.00	1.00
Panel B: Ice cream purchases	Mean	Percentile				
		5	25	50	75	95
Number of purchases	24.27	10	12	18	28	58
Volume	67.07	16	64	64	64	128
Multiple-item purchases	0.155	0	0	0	0	1
Days from previous purchases	25.61	3	7	14	30	90

* For number of trips, number of stores visited, store HHI and number of purchases, an observation is a household. For days from previous trip, an observation is a trip instance. For all other statistics, an observation is a purchase instance. Volume is measure in ounces. Store HHI denotes the household's Herfindahl-Hirschman index for store level ice-cream volume purchases.

the duration dependence argument: there is evidence of a non-linear relationship between the probability of purchase and the duration since the last purchase. The hazard rate is quite low immediately after a purchase, then gradually increases until day 14, after what it exhibits a gradual, although rather jagged, downward trend (an interesting aspect of this hazard rate relates to its recurring spikes, an issue I address below). However, the downward trend of the hazard rate suggests that, in contrast to the initial argument of this chapter, the probability of purchase seems to decrease (and not increase) with the duration since last purchase. There are two possible explanations for this behaviour (and that illustrate the well known problem that unobserved heterogeneity can be confounded with state-dependence). Either the utility from consuming ice cream does in fact decrease with duration (positive state-dependence), or alternatively, the utility increases with duration from last purchase (negative state-dependence), but there exists an over-representative group of low-demand consumers who are more likely to exhibit longer interpurchase durations (heterogeneity). In order to evaluate the degree of consumer unobserved heterogeneity, I re-compute the hazard rate at the *household-level*. The household-level hazard rate denotes here the probability that a household purchases if she has not purchased up to now. Figure 2 displays, for each duration spell, a scatter of the 103 household-level hazard rates. The figure illustrates substantial heterogeneity across households, which is suggestive of the importance of controlling for unobserved heterogeneity in the structural estimation.

While the duration dependence of ice cream purchases implied by Figures 1 and 2 can be consistent with variety-seeking behaviour induced by a diminishing marginal utility, it can

FIGURE 1
Purchase Hazard

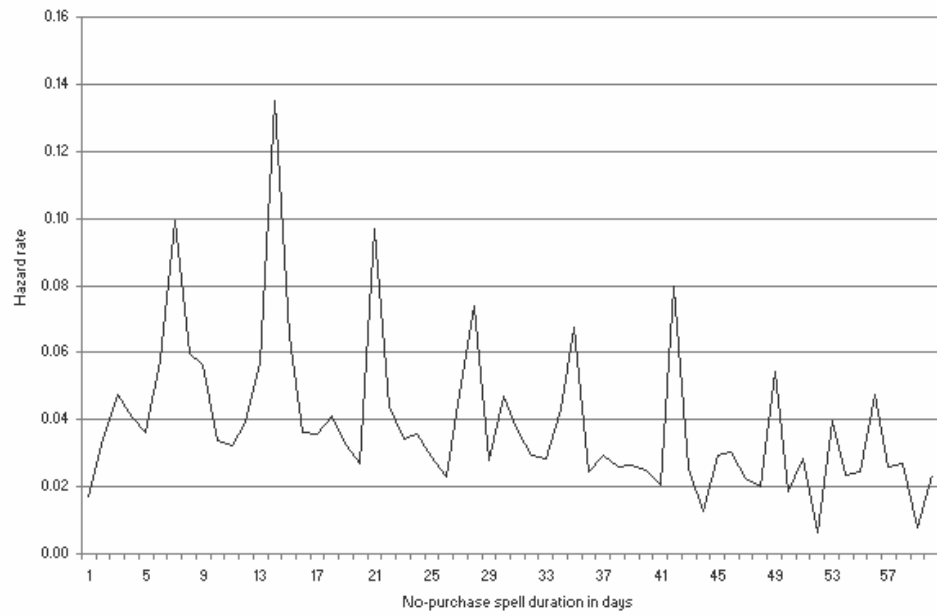


FIGURE 2
Household-level Purchase Hazard

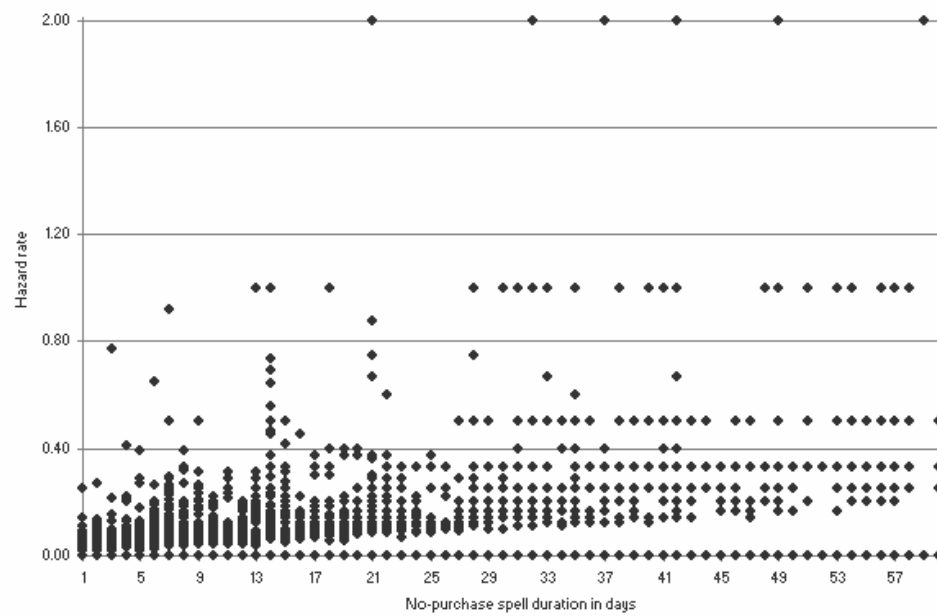
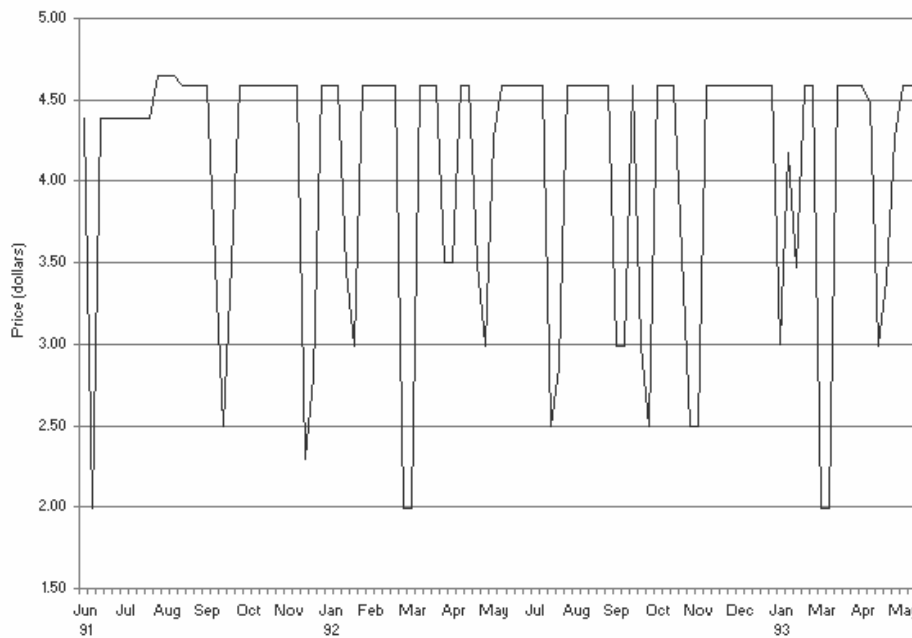


FIGURE 3
Price Example: Dreyer's Vanilla Ice Cream



also be consistent with the main alternative theory: if households respond to temporary price cuts by accelerating (anticipating) purchases and hold inventories for future consumption (i.e. stockpile), the probability of purchase will also be duration-dependent. Furthermore, if storage costs vary substantial across households, the household-level probability of purchase will also exhibit high heterogeneity. Temporary price promotions are an important marketing tool in the pricing strategy of many nondurable goods and ice creams are no exception. The ice cream prices in the sample display a classic high-low pattern: products have a "regular (modal) level" that remains constant for long periods of time with occasional temporary reductions. Figure 3 displays, as an illustration, the price of Dreyer's Vanilla Ice Cream 64 oz. over the sample weeks in a typical supermarket. The price is at the "regular level" (\$4.59) for 57% of the weeks. Defining a sale to be a price reduction of at least 5% (below the modal level), it is on sale for 31% of the time, with the average price discount being \$1.61. If we consider the sample as a whole, untabulated statistics show that prices are, on average, 66% at the "regular level" and 26% on sale (with an average discount of \$0.70). In such an environment, households may respond to temporary price cuts by accelerating (anticipating) purchases and stockpile.

Table 4 addresses the purchase acceleration effect by comparing household level sale and nonsale purchasing patterns. The first column displays averages during nonsale purchases.

TABLE 4
*Category Purchasing Patterns: Comparison between Sale and Nonsale**

	Average during Nonsale	Difference during Sale			Week F.E.
		Total	Households		
			Within	Between	
Volume	63.9 (0.90)	6.99 (1.34)	5.41 (1.08)	18.0 (9.84)	
		7.59 (1.39)	5.82 (1.12)	41.1 (20.1)	yes
Units	1.13 (0.01)	0.06 (0.02)	0.07 (0.02)	0.10 (0.08)	
		0.06 (0.02)	0.07 (0.02)	0.09 (0.12)	yes
Average package size	56.6 (0.60)	2.83 (0.90)	0.96 (0.66)	11.5 (7.46)	
		3.21 (0.93)	1.19 (0.68)	28.8 (17.5)	yes
Days from previous purchase	25.9 (0.87)	-0.57 (1.30)	-2.72 (1.33)	3.57 (6.48)	
		-0.50 (1.33)	-3.30 (1.37)	26.9 (11.0)	yes
Days to next purchase	23.7 (0.84)	1.42 (1.25)	-0.33 (1.29)	5.17 (6.16)	
		1.81 (1.29)	-0.50 (1.33)	22.8 (15.0)	yes

* An observation denotes a purchase instance. Standard errors in parentheses. The dependent variable is volume (measured in ounces), units, average package size, days from previous purchase and days to next purchase. Sale denotes an indicator variable that takes the value one if the purchase involves at least one product on sale, where a sale is defined as any price at least 5% below the modal price (over the observed period) for each store-UPC combination.

The following columns examine the difference towards purchases that involve sales, decomposing the total difference into a within and a between households effects. As before, a sale is defined as any price at least 5% below the modal price (over the observed period) for each store-UPC combination. I focus the analysis on the within column,³ that compares the household purchasing patterns over time. For each dimension, I consider the results for two specifications: with and without week fixed effects since the former controls for eventual seasonality, an issue I address below. The evidence seems to indicate that households do respond to temporary price cuts. Unsurprisingly, the results suggest that households tend to shorten the duration from previous purchase (between 2-3 days) and to increase their volume purchases (by roughly 8.5%), whenever they buy products on sale.

An interesting question is whether this response by households translates into a consumption effect or merely represents a demand-anticipation effect with households stockpiling for future consumption. In order to examine this question, I follow Hendel and Nevo (2006a) and examine the household's interpurchase duration to the next purchase. The idea here is that if consumers do stockpile, then the duration to next purchase is expected to be longer for large volume purchases (like purchases on sale). The results from Table 4 show that there

³The results from the between column in Table 2.3 suggest substantial heterogeneity in how consumers respond to temporary price cuts, with households that purchase more frequently on sale, buying larger volumes and less frequently. This reinforces the need for the structural model to control for consumer heterogeneity.

is no significant difference in the duration forward to next purchase between sale and nonsale purchases. The comparison of the quantity and duration effects seem therefore to indicate that stockpiling may not be a relevant feature for ice-cream demand and, as a consequence, that the quantity effect induced by temporary price reduction substantiates a consumption effect. In order to examine the robustness of this conclusion, I also compared the duration forward to the next purchase when consumers buy an above average volume. The results of those regressions (which are untabulated) are consistent with the above conclusion. The difference in interpurchase duration is again not significantly different from zero. The alternative theory that state-dependence in the probability of purchase is due to stockpiling can not explain these results. Furthermore, the analysis of how the additional quantity is bought is consistent with the variety-seeking theory since, when purchasing on sale, consumers seem to increase the number of units of ice cream purchased (6.2%) relatively more than the average package size (1.7%). This supports the variety-seeking story because if the increased volume translates into increased consumption, then purchasing multiple-items is a sensible strategy to deal with the diminishing marginal utility from consumption.

Having addressed the issue of eventual stockpiling behaviour, I now move on to address another somewhat problematic issue. In this chapter, I model consumption to have a lasting effect that diminishes the marginal utility of future consumption. However, I do not observe the actual time and magnitude of consumption. So, I am forced to infer it from purchase choices. The results from Table 4 provide some evidence that, not only consumers do not anticipate purchases to hold inventories for future consumption, as already discussed, but also that utility does not depend on the stock of past consumption. If consumption did create a stock, then duration to next purchase would increase with the size purchased, which it does not. That said, following Table 4, the only relevant variable that may affect the marginal utility of future consumption is the timing of current consumption. As I discuss below, due to the temptation nature of the good, assuming that the time of consumption coincides with the time of purchase is not unreasonable. At least for most people, in line with what Erdem *et al.* (2003) argue, ice creams are technologically, but not practically storable over more than a few days. It may seem inconsistent to assume that consumption has a lasting effect that induces intertemporal substitution in purchases while assuming that the good held in inventory has a temptation feature. These assumptions are however consistent with observed behaviour, since consumers seem to depreciate the costs of goods they have in inventory (see Gourville and Soman (1998) and Prelec and Loewenstein (1997)).

I now move on to describe two other timing aspects of households ice cream category purchasing patterns. I begin by addressing seasonality. If the decisions of households are

TABLE 5
*Category Purchasing Patterns: Seasonality**

	Average during Autumn	Within Household Difference during		
		Winter	Spring	Summer
Volume	65.3 (1.28)	-0.34 (1.35)	1.97 (1.31)	2.64 (1.38)
Units	1.16 (0.01)	0.01 (0.02)	0.01 (0.02)	0.01 (0.02)
Average package size	56.8 (0.85)	-1.22 (0.81)	0.92 (0.79)	1.33 (0.84)
Days from previous purchase	24.5 (1.22)	4.55 (1.65)	-1.80 (1.61)	-3.45 (1.70)

* An observation denotes a purchase instance. Standard errors in parentheses. The dependent variable is volume (measured in ounces), units, average package size, days from previous purchase and days to next purchase.

seasonal, then the structural model must reflect this feature. Table 5 addresses this question by comparing household level purchasing patterns for the different seasons. The first column displays averages during Autumn purchases while the following columns examine the within household difference towards purchases in the remaining seasons. The results seem to suggest that seasonality does not induce a significant difference in the purchasing patterns of households that relate to volume, units or average package size. However, seasonality does seem to have a significant impact in interpurchase duration. In particular, the results seem to suggest that households tend to significantly increase the duration from previous purchase in the Winter (between 4-5 days), while shorten that duration in the Summer (between 3-4 days). As a consequence, seasonality seems to be an important feature in the purchasing decision of ice cream and related frozen desserts and therefore must be incorporated into the structural model.

Another timing aspect of consumers ice cream category choice patterns relates to the purchase day. If households are more likely to purchase on a particular day of the week or weekend, then the structural model must somehow incorporate it. In order to address this issue, I examine the within household likelihood of purchase (both in number of purchases and volume) for each day of the week. The results of this analysis (which is untabulated) seems to suggest that households exhibit a higher likelihood of purchase on Fridays, Saturdays and Sundays. I interpret this result as illustrating potentially reduced transaction costs of households purchasing on those days. This constitutes a feature of household behaviour that must be incorporated into the structural model.

TABLE 6
*Household Product Concentration Choice Behaviour**

	Mean	Percentile				
		5	25	50	75	95
Number of different products	10.32	3	7	10	13	20
CR1	0.371	0.14	0.22	0.31	0.53	0.76
CR5	0.793	0.50	0.68	0.81	0.94	1.00
HHI	0.245	0.08	0.13	0.18	0.34	0.58

* An observation is a household. CR m and HHI denote the household's m -product volume concentration ratio and Herfindahl-Hirschman index, respectively.

2.2 Substitutability Across Products

Having described the shopping behaviour and purchasing patterns of households for the frozen desserts category as a whole, I now move on to examine their product choice patterns. Table 6 displays household-level product concentration measures by percentile. The first row of Table 6 presents the number of different products a household is ever observed to purchase. The median household purchases 10 different products over the sample period, with 75% of households purchasing at least 7 different products. Rows two and three of Table 6 show volume concentration ratios for each household-specific $m = 1, 5$ top ranked products, CR m . The top one and five ranked products account for 31% and 68% of the median household purchased volume, respectively, which suggests that there is no single (or set of) product(s) that dominate(s) this household's purchases of frozen desserts. The last measure is the Herfindahl-Hirschman index, HHI, computed as the sum of the square of the volume shares for each household-specific purchased product. The HHI for the median household is 0.18, with 75% of households having a HHI of at most 0.34, which is indicative that households fragment their volume purchases of frozen desserts considerably.

In order to examine whether the fragmented household product choice described above is consistent with a household taste for variety seeking or whether, in contrast, it is consistent with habit persistence, I follow Dubé *et al.* (2010) and examine the household's purchase and conditional repurchase frequencies for each product, where the latter denotes the frequency at which, conditional on a purchase, each household repeat purchases the same product. Note that if the household is variety seeking, the purchase of a product on a particular occasion tends to have a negative impact on the likelihood of repurchasing that same product on the next occasion, and therefore her purchase frequency for any given product tends to be considerably higher than the corresponding conditional repurchase frequency. In contrast, if the household is habit persistent, the purchase of a product on a particular occasion tends to increase (reinforce) the likelihood of repurchasing that same product

on the next occasion, and therefore her purchase frequency for any given product tends to be considerably smaller than the corresponding conditional repurchase frequency. As discussed previously, I use the first six months in the sample to generate an initial product choice for each household. This approach is similar to Shum (2004) and Pozzi (2009). Figure 4 shows the frequency distribution for the difference between repurchase and purchase frequencies for each household-specific product. The analysis seems to suggest substantial heterogeneity across households in how previous purchases impact current choices, which is indicative of the importance of controlling such heterogeneity in the structural estimation. Moreover, there is also evidence that the marginal purchase frequency for the median household-specific product exceeds the corresponding conditional repurchase frequency (the median difference is -0.043) which seems to suggest the relevance of variety seeking behaviour.

One problem with inferring variety seeking from the comparison of purchase and repurchase frequencies is that unobserved heterogeneity can be confounded with structural state-dependence. The identification problem arises because a household may repeatedly alternate products in her purchases (and exhibit a conditional repurchase frequency that is considerably smaller than the purchase frequency) either because of a weak unobserved, idiosyncratic preference for the different products or because she has in fact a structural taste for variety. In order to address this issue, I replicate the analysis above for the subset of households that have a household-level product HHI above 0.25. Figure 5 shows the corresponding frequency distribution for the difference between repurchase and purchase frequencies for each household-specific product. Households with high product HHI are assumed, in the line of Simonson and Winer (1992), to have strong product preferences, given the high product-level concentration of their shopping basket. In this manner, the initial product choice may not merely reflect a weak unobserved, idiosyncratic preference for the different products. The results are consistent with the variety seeking pattern found above. The marginal purchase frequency for this subset median household-specific product exceeds the corresponding conditional repurchase frequency (the median difference is -0.036) which seems to indicate the variety seeking pattern described above to be structural and not only due to persistent, household-specific differences.

Having established a structural search for variety, I now move on to characterize it in more detail. First, I quantify its extent. In particular, I follow Menon and Kahn (1995) and examine two product-level switching concepts: *successive* product switches and *exploration* product switches. The former defines a switch as occurring each time the product(s) chosen on a purchase occasion is different from those chosen on the immediately preceding purchase instance. This definition is consistent with Faison (1977) and Venkatesan (1973). The latter

FIGURE 4
*Frequency Distribution for the Difference between Household-Specific
 Product Repurchase and Purchase Rates
 (All Households)*

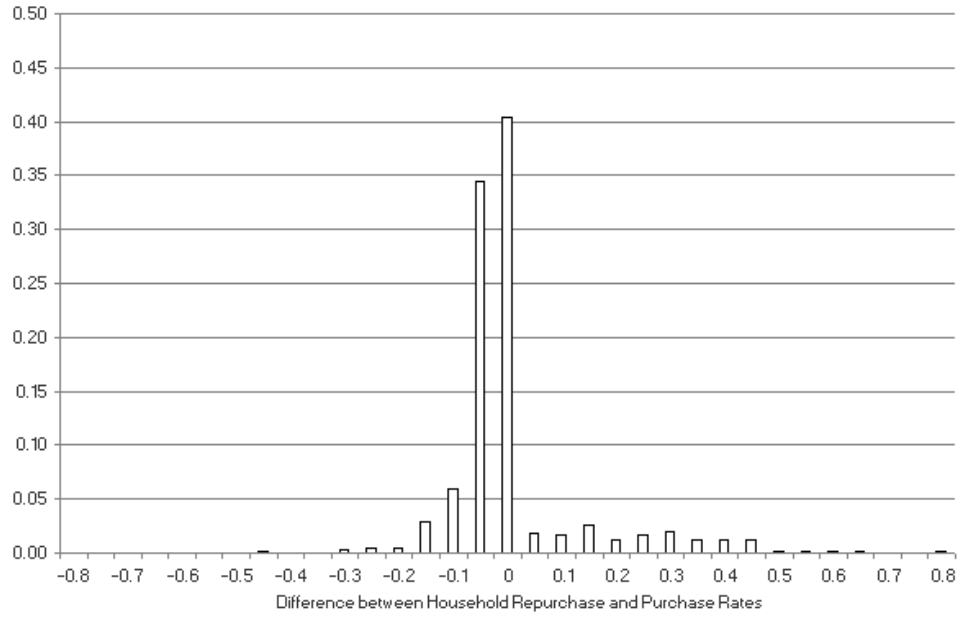


FIGURE 5
*Frequency Distribution for the Difference between Household-Specific
 Product Repurchase and Purchase Rates
 (Household-level Product HHI above 0.25)*

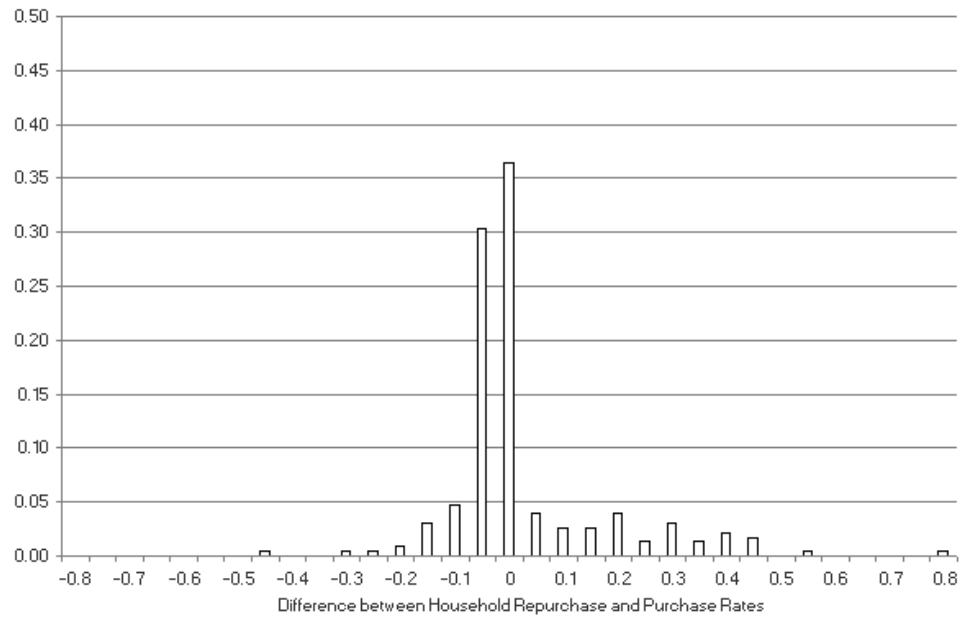


TABLE 7
*Purchase-level Product Switching Behaviour**

	Mean	Percentile				
		5	25	50	75	95
Number of successive product switches	0.770	0	0	1	1	2
Number of exploration product switches	0.353	0	0	0	1	1

* An observation is a purchase instance. The number of successive product switches denotes, for each purchase instance, the number of products purchased that involve a successive product switch from the immediately preceding purchase. The number of exploration product switches denotes, for each purchase instance, the number of products purchased that involve a exploration product switch from the preceding choices.

presents an alternative approach by defining a switch as occurring each time the product(s) chosen on a purchase occasion is different from any of the preceding choices. This definition is consistent with Faison (1977) and Pessemier (1985). The two definitions differ in the idea of variety-seeking that may capture. While the latter definition implicitly assumes that the level of stimulation of a household can only be increased by exploring new products, i.e. products that the consumer never tried before, the successive switching definition assumes that the level of stimulation of a household can be increased by alternating from one product to another, even if the products in the switching set are all familiar. As discussed previously, I use the first six months in the sample to generate an initial product choice for each consumer. This avoids the spurious switching that would be generated if I counted switching from the beginning of the sample period. Table 7 presents summary statistics for purchase-level measures of the two product switching concepts described above. The number of successive product switches denotes, for each purchase instance, the number of products purchased that involve a successive product switch from the immediately preceding purchase, while the number of exploration product switches denotes, for each purchase instance, the number of products purchased that involve a exploration product switch from the preceding choices. The median purchase involves switching to *one* product that differs from those chosen on the immediately preceding purchase instance, but with which the household is already *familiar*. This is suggestive of the relative importance of switching that involves familiar products. For this reason, in the remainder of the paper I focus on successive switches.

Second, I decompose successive product switching between segment, brand and flavour switching. In order to examine the fraction of successive product switching that is due to segment switching, I begin by running an OLS regression of the number of successive product switches on the number of successive segment switches. The untabulated results show that segment switching explains only 9.0% of the variation in the purchase-level number of successive product switches. This suggests that households do not tend to increase their level of stimulation by switching to other product segments. Next, I examine the fraction

of successive product switching that is due to brand switching. I run an OLS regression of the number of successive product switches on the number of successive brand switches, controlling for the number of successive segment switches. The untabulated results show that brand switching explains an additional 24.3% of the variation in the purchase-level number of successive product switches, which is suggestive of the relative importance of different brands in increasing the level of stimulation of households. I then investigate flavour switching. I run an OLS regression of the number of successive product switches on the number of successive flavour switches, controlling for both the number of successive segment and brand switches. The untabulated results suggest that flavour is the most important driver in increasing the level of stimulation of households since flavour switching explains an additional 46.4% of the variation in the purchase-level number of successive product switches.

Finally, I examine whether there is an association between product switching behaviour and marketing mix strategies, household purchase decisions and household demographics. In particular, I run a set of Poisson regressions where the dependent variable is the number of successive product switches. I choose to do so because the dependent variable is a count. The different specifications vary on the covariates included. Table 8 presents the results of this analysis, with the different columns reporting distinct specifications that vary on the covariates included. Specification (1) examines the association between product switching and total price, where total price denotes the sum of prices paid by the household for all products purchased in each purchase instance. The price coefficient is positive and statistically significant, which suggests that when households switch, they tend to purchase more expensive products. Specification (2) controls for promotional strategies by including the number of products on sale, feature and display as covariates. A sale denotes an indicator variable that takes the value 1 if a given price is at least 5% below the modal price (over the observed period), while feature and display denote an indicator variable that takes the value 1 if there is any type of product advertising and special store display, respectively.⁴ The number of products on sale, feature and display denote the cumulative sum of the corresponding variables across all products purchased in each purchase instance. The results suggest that households do respond to promotional activities as the coefficients on those controls are positive and statistically significant. Specification (3) controls for household purchase decisions by including the number of units and total volume of each purchase instance as additional covariates. The coefficient on the number of units is positive and statistically significant, while the coefficient on total volume is statistically insignificant. This suggests that purchase instances associated with switching are not significantly different in terms of total volume,

⁴ The data includes several categories of feature and display. I aggregate across the different categories to feature/no feature and display/no display.

but do involve a higher number of units. Moreover, the inclusion of the additional controls decreases the coefficient on total price almost by half, which is suggestive of an endogeneity issue. Total price is positively correlated with the number of units purchased, which overestimates the coefficient on total price if the latter is not controlled for. Despite this fact, the price coefficient is still estimated to be positive and statistically significant. I interpret this positive association to be capturing switches towards luxury or quality products, which are naturally more expensive. Finally, specification (4) examines the association between product switching behaviour and household demographics. The results suggest that larger households, households without a male or a female and households with higher income are more likely to engage in more successive product switching. The coefficient on female head works more than 35 hours is statistically insignificant. Under the assumption that work hours proxies for spare time and the propensity to search, this finding is consistent with product switching that does not require significant search time since it involves mostly familiar products.

A final problem with inferring variety-seeking from product switching is that product unavailability may generate spurious switching. In order to address this concern, I need to separate *true* product switching from switching *induced* by product unavailability. However, as the IRI dataset does not include information on product availability, I have to infer it from store product sales. I consider that a product was available in a given week and supermarket combination if the store sold at least one unit of the product in that week.⁵ Next, I compute, for each household, the proportion of *true* product switching, i.e. switching that occurs despite the products chosen on the immediately preceding purchase instance being available (as measured by the proxy). This analysis (which is untabulated) seems to suggest that most product switching is not induced by unavailability: the median proportion of *true* product switching is 90%, the average is 86%, the 25th percentile is 76%, and the 75th percentile is 100%.

Having described the important features of the purchasing behaviour of consumers, I now move on to specify the demand model and the proposed estimation procedure.

3 Demand Model

This section introduces the utility function and the assumptions of the model. I study the demand for a temptation good in a setting similar to Hartmann (2006) where consumption

⁵The product availability proxy will obviously overestimate the induced product switching.

TABLE 8
*Purchase-level Successive Product Switching Poisson Regression**

	(1)	(2)	(3)	(4)
Constant	-0.745 (0.054)	-0.848 (0.056)	-1.092 (0.070)	-1.404 (0.118)
Total price	0.165 (0.016)	0.158 (0.016)	0.081 (0.022)	0.087 (0.023)
Number of products on sale		0.147 (0.045)	0.080 (0.046)	0.089 (0.046)
Number of products on feature		0.118 (0.051)	0.058 (0.051)	0.061 (0.051)
Number of products on display		0.157 (0.087)	0.095 (0.086)	0.065 (0.087)
Number of units			0.550 (0.092)	0.541 (0.093)
Total volume			-0.001 (0.001)	-0.002 (0.001)
Household size				0.068 (0.025)
Household without a male				0.132 (0.070)
Household without a female				0.203 (0.109)
Household income				0.022 (0.010)
Female head works 35+ hours				-0.054 (0.063)
Log likelihood	-2,482	-2,457	-2,440	-2,432

* An observation is a purchase instance. Standard errors in parentheses. The dependent variable is the number of products chosen on a purchase occasion that differ from those chosen on the immediately preceding purchase instance. Total price is measured in dollars. A sale denotes an indicator variable that takes the value 1 if a given price is at least 5% below the modal price (over the observed period). Feature and display denote an indicator variable that takes the value 1 if there is any type of product advertising and special store display, respectively. The number of products on sale, feature and display denote the cumulative sum of the corresponding variables across all products purchased in each purchase instance. Volume is measured in ounces. Household size denotes the number of people in the household. Household without a male (female) denotes an indicator variable that takes the value 1 if there is no male (no female) within the household. Household income denotes the household income group: (in thousand dollars) 1- [0,10), 2-[10,12), 3-[12,15), 4-[15,20), 5-[20,25), 6-[25,35), 7-[35,45), 8-[45,55), 9-[55,65), 10-[65,75), 11-[75,+inf). Female head works 35+ hours denotes an indicator variable that takes the value 1 if the female head (or the male head in the case there is no female within the household) works more than 35 hours per week.

creates a stock that diminishes over time. This creates in the consumer an incentive to variety-seeking and thus intertemporal substitute consumption for the good. Unlike Hartmann (2006), I extend the analysis to address the differentiated nature of the good and examine not only substitution across time, but also substitution across products. In order to do so, I adapt Aguirregabiria (2002) and Hendel and Nevo (2006a) multi-stage budgeting approach.

3.1 The Setup

There are I consumers who are indexed by i . In each shopping trip t , consumer i chooses whether or not to purchase the good, and in case she decides to purchase, which product and size to buy. Let $j = 1, \dots, J$ index the inside product alternatives to the consumer, with each product alternative being (possibly) offered in a variety of different sizes x . Multiple-item purchases are included by expanding the choice set to allow for bundles. If in a particular trip a consumer buys, for example, both 64 oz. of Häagen-Dazs Vanilla Ice Cream *and* 16 oz. of Häagen-Dazs Chocolate Ice Cream, the purchase size is given by $x = 80$ oz. and product j denotes the bundle of the two products. The no purchase choice (outside alternative) is indexed by $j = 0$.

3.2 Consumer Flow Utility

The consumer flow utility is expressed in terms of the indirect utility from each of the available alternatives. I begin by specifying the indirect utility from not purchasing (the outside option). I follow Hartmann (2006) and relax the common assumption of additively-separable utility in consumption by considering a frequency of purchase model where past choices affect current utility. In particular, I assume the utility of the outside option to be a function of the depreciated stock of past consumption:

$$u_{i0t}(y_{it}, \varepsilon_{i0t}) = z_{it} + \varepsilon_{i0t}, \quad (1)$$

where z_{it} denotes the stock of past consumption of individual i at time t and ε_{i0t} is a random shock to consumer choice. The depreciated stock of past consumption will, in full generality, depend on both the time elapsed since the previous consumption and the magnitude (or size) of past consumption. However, because the stock of past consumption is intangible and unobservable, I am required to infer it from (observed) past purchase choices. In order to do so, I make the following assumptions.

Assumption 2.1 *Consumption takes place at the time of purchase.*

Assumption 2.1 is motivated by the temptation nature of the good. Since the data description analysis has shown that consumers do not anticipate purchases to hold inventories for future consumption, inferring that individuals do consume their purchased ice cream *before* their next purchase occasion is not unreasonable. However, the actual time of consumption is unobserved. Due to the temptation nature of the good, I assume that the time of consumption coincides with the time of purchase. At least for most people, in line with what Erdem *et al.* (2003) argue, ice creams are technologically, but not practically storable over more than a few days. As discussed above, it may seem inconsistent to assume that consumption has a lasting effect that induces intertemporal substitution in purchases while assuming that the good held in inventory has a temptation feature. These assumptions are, however, consistent with observed behaviour, since consumers seem to depreciate the costs of goods they have in inventory (see Gourville and Soman (1998) and Prelec and Loewenstein (1998)).

Assumption 2.2 *The stock of past consumption fully depreciates after a new consumption occasion.*

Assumption 2.2 implies that the stock of past consumption does not accumulate across multiple consumption occasions and that only the last consumption occasion is relevant. The motivation behind this assumption is twofold. First, it significantly reduces the state space since, instead of keeping track of all past choices, only the last consumption (which is observable under Assumption 2.1) is relevant to the decision of consumers. An alternative approach would consist of constructing an accumulated stock index, a strategy that would also have the advantage of a simplified state space. However, it carries a disadvantage related to the second justification for Assumption 2.2. Under this assumption, the initial stock of past consumption is observable and does not need to be inferred, which would not be true if I allowed the stock to accumulate across multiple consumption occasions.

One concern with the simplification implied by Assumption 2.2 is that it comes at a cost: the stock of past consumption is measured with error. Although measurement error is a potentially troublesome problem, it may not be too problematic here. The error introduced will, at best, underestimate the incidence of intertemporal substitution, rather than falsely induce spurious intertemporal substitution. In order to understand why this is the case, note that because the estimated stock of past consumption will not exceed the *true* stock of past consumption, the utility of the outside option will be underestimated (in a setting where this

alternative is the most common choice made by consumers). As a consequence, intertemporal substitution is underestimated.

Assumption 2.3 *The stock of past consumption is independent of the quantity purchased.*

Assumption 2.3 relates to a previous discussion since the descriptive analysis of the data has shown that the consumer interpurchase duration is not affected by the quantity purchased. Even though consumers respond to price promotions by increasing their purchased volume, the effect on the duration to the next purchase is not significantly different from zero.

A consequence of Assumptions 2.1 to 2.3 is that only the time elapsed from the last purchase is relevant to infer the depreciated stock of past consumption. The utility of the outside option can then be specified as:

$$u_{i0t}(H_{it}, \varepsilon_{i0t}) = \varphi(H_{it}) + \varepsilon_{i0t}, \quad (2)$$

where H_{it} is the number of days since the last purchase occasion and $\varphi(H_{it})$ denotes the function that allows me to infer the (unobserved) stock of past consumption from (observed) past purchase choices.

I now move on to specify the indirect utility from choosing an inside alternative. I assume the utility to individual i in time period t from choosing a product j of size $x > 0$ that belongs to h_{it-1} is:

$$\begin{aligned} u_{ijxt}(p_{jxt}, a_{jxt}, h_{it-1}, \varepsilon_{ijxt}) &= \bar{u}_{ijxt}(p_{jxt}, a_{jxt}, h_{it-1}) + \varepsilon_{ijxt} \\ &= \gamma_{ix} + \alpha_i p_{jxt} + \xi_{ijt} + \beta_i a_{jxt} + \lambda_i m_{jxt} + \eta_i y_{ijt-1} + \varepsilon_{ijxt}, \end{aligned} \quad (3)$$

where h_{it-1} indicates the set of products purchased by consumer i in her previous purchase event, γ_{ix} denotes the (dis)utility from making a purchase of size x , which could be interpreted as a carrying cost associated with that particular purchase, p_{jxt} is the price of product j in size x , ξ_{ijt} is consumer i taste for product j that could be a function of product characteristics (like, for example, size), a_{jxt} denotes a vector of indicator variables that control for other promotional activities, m_{jxt} is an indicator variable that takes the value 1 if product j denotes a multiple-item purchase, and ε_{ijxt} is a random shock to consumer choice. The variable y_{ijt-1} keeps track of the number of products that do not belong to the set h_{it-1} if consumer i purchases product j in purchase event t .

The term η_i accounts for state-dependence effects. A positive η_i implies that consumer i has a taste for variety-seeking, since switching to products different from those included in the h_{it-1} set increases the consumer's utility (see McAlister and Pessemier (1982)). The marketing literature provides several explanations for such variety seeking behaviour. Consumers may have an internal desire for change due to satiation or need for stimulation, or they may be balancing the different tastes within the household (see Kahn, 1995, for a comprehensive review of the variety seeking literature). A negative η_i , on the other hand, implies consumer i is habit persistent/incurs in a switching cost, since switching to products that do not belong to the set h_{it-1} decreases the consumer's utility (see Pollack (1970) and Spinnewyn (1981)). Klemperer (1995) provides a number of possible reasons for switching costs. Consumers may have shopping search costs and, therefore, do not reoptimize the set of products purchased at every purchase occasion, or they may keep repurchasing the same product as part of a learning process. I do not attempt to distinguish here between these alternative explanations. Rather, I focus on whether state dependence in fact exists and can be identified from observed purchasing behaviour. This approach is similar to Osborne (2007).

3.3 Consumer Dynamic Optimization Problem

Consumers in each period decide if or not to purchase, and in case they opt to purchase, which product or products to choose. I make the following assumptions about how consumer expectations of the future affect current period decisions.

Assumption 2.4 *Consumers are forward-looking with regard to their purchase decisions, but myopic with respect to their product choices.*

The myopic assumption implies that consumers maximize their per-period expected utility when making their product choices and is motivated by solely pragmatism. A forward-looking consumer, who experiences state-dependence in her choices of product, considers the future consequences of those choices. The state space of the dynamic problem without this assumption would be extremely large, making the structural estimation computationally infeasible. The development of a framework that incorporates such forward-looking behaviour into a feasible computational estimation procedure is a very interesting potential area for future research. That said, the myopic assumption seems a reasonable assumption about consumer formation of expectations with regard to product choice. While some consumers

may plan the whole sequence of product decisions accounting for the consequence of state-dependence in future periods, I tend to believe such forward-looking behaviour to be rare. I should note, however, that Assumption 2.4 does not imply that dynamics are absent from product choice. As I discuss below, current product choices impact the expected future flow utility of the different inside alternatives and, as a consequence, influence the purchase size decision. In other words, even though consumers are myopic with regard to product choice, their decisions have dynamic implications for current and future purchase size choices.

Assumption 2.4 implies a multi-stage budgeting approach to model the purchase and product decisions of consumers (see Aguirregabiria (2002) and Hendel and Nevo (2006a) for similar dynamic applications of Gorman (1971)'s approach). The consumer's expected discounted utility in purchase occasion t can therefore be represented as:

$$V(s_{it}) = \max_{\Pi_i} \sum_{\tau=t}^{\infty} \delta^{\tau-t} E \left[\sum_{x>0} \max_{\Phi_i} \sum_j d_{ix\tau} d_{ij/x\tau} u_{ijx\tau}(p_{jx\tau}, a_{jx\tau}, h_{i\tau-1}, \varepsilon_{ijx\tau}) \right. \\ \left. + d_{i0\tau} u_{i0\tau}(H_{i\tau}, \varepsilon_{i0\tau}) | s_{it}, \Pi_i, \Phi_i \right], \quad (4)$$

where s_{it} denotes the state at time t and $\delta > 0$ the discount factor. The state s_{it} in each period consists of the vector of current prices and promotional activities for all products and sizes, the set of products purchased by consumer i in her previous purchase event, the stock of past consumption as measured by the time since the last purchase, and the vector of random shocks to consumer choices, $s_{it} \equiv (p_t, a_t, h_{it-1}, H_{it}, \varepsilon_{it})$. For convenience, I define also the state space s_t^* that consists only of the vector of current prices and promotional activities for all products and sizes, and the set of products purchased by consumer i in her previous purchase event, $s_{it}^* \equiv (p_t, a_t, h_{it-1})$.

Π_i and Φ_i denote a set of decision rules mapping states, s_{it} , to choices, d_{ixt} and $d_{ij/xt}$, respectively, where d_{ixt} is an indicator variable equal to 1 if the choice of consumer i is a purchase of size x (with $x = 0$ standing for no purchase) and $d_{ij/xt}$ is an indicator variable equal to 1 if consumer i chooses to buy product j when purchasing size x . The product of the two indicator variables, $d_{ijxt} = d_{ixt} d_{ij/xt}$, denotes the purchase of product j and size x . I assume that $\sum_{x,j} d_{ijxt} = 1$.

At every state, s_{it} , the consumer faces the same infinite-horizon maximization problem. The value function $V(s_{it})$ defined in equation (4) above is, therefore, the solution to the

following Bellman's equation:

$$V(s_{it}) = \max_{d_{ixt}} \left\{ \sum_{x>0} \max_{d_{ij/xt}} \sum_j d_{ixt} d_{ij/xt} u_{ijxt}(p_{jxt}, a_{jxt}, h_{it-1}, \varepsilon_{ijxt}) \right. \\ \left. + d_{i0t} u_{i0t}(H_{it}, \varepsilon_{i0t}) + \delta E[V(s_{it+1}) | s_{it}, d_{ixt}, d_{ij/xt}] \right\}. \quad (5)$$

In order to complete the specification of the demand model, I make the following assumptions about the beliefs of consumers regarding the uncertain future prices (and promotional activities) and future utility random shocks.

Assumption 2.5 *Consumers have rational expectations.*

The assumption of rational expectations implies that consumers take all available information into account in forming expectations. Though expectations may turn out incorrect, they will not be systematically wrong. In particular, Assumption 2.5 implies that consumers know both the true transition probability of prices and promotional activities, and the true distribution of the utility random shocks.

Assumption 2.6 *The transition probability of prices and promotional activities are exogenous from the point of view of consumers. Furthermore, they follow a first-order Markov process.*

Assumption 2.6 is consistent with the view that retailers inter-temporal price discriminate by playing mixed strategies that are exogenous from the point of view of consumers (Conslik *et al.* (1984), Sobel (1984), Varian (1980), Pesendorfer (2002)). This assumption implies that, conditional on the control variables, price and promotional activities are independent of the unobserved random shocks, which might be unreasonable if consumers stockpile and inventories are not accounted for. If prices are persistent over time and consumers anticipate purchases in order to hold inventories for future consumption, then unobserved inventories will be correlated with current prices causing an endogeneity problem. Another concern with this assumption might be seasonality. If the likelihood of a temporary promotion is affected by seasonality and it is not accounted for into the transition probability, then unobserved random shocks will be correlated with current prices causing (again) an endogeneity problem. However, as discussed in the previous section, both issues are probably not a concern here.

The first-order Markov process assumption reduces the state space and, although probably inconsistent with equilibrium prices, it is not unreasonable with regard with observed

consumers' memory and formation of expectations. The assumption can be relaxed to allow higher order processes, with an increase in the associated computational burden.

Assumption 2.7 ε_{ixjt} is independently and identically distributed extreme value type 1.

Assumption 2.7 is motivated by pragmatism as it significantly reduces the computational burden. The main concern with this type of assumption might be to preclude correlation between products. This is not probably a concern here since the model accounts for product heterogeneity and product state-dependence. Incorporating correlation between the unobserved random shocks of different products can, in principle, be allowed, but at a significant increase in the computational costs of the estimation procedure.

4 Maximum Likelihood Estimation

This section presents the estimation details. I estimate the parameters of the model via maximum likelihood. The standard approach would begin by specifying the probability of observing consumer i 's choices at time t , which is given by the following likelihood function:

$$l_{it}(d_{it}|s_{it}) = \prod_{x,j} [\Pr(d_{ijxt} = 1) | s_{it}]^{d_{ijxt}}, \quad (6)$$

where $d_{it} \equiv \{d_{ijxt}\}$ denotes the vector of her choices. The likelihood of consumer i choices across all time periods would then be:

$$L_i(s_{i1}, \dots, s_{iT}, d_{i1}, \dots, d_{iT} | s_{i0}, d_{i0}) = \int \prod_t l_{it}(d_{it}|s_{it}) dF(s_{it}|s_{it-1}, d_{it-1}), \quad (7)$$

where s_{i0} and d_{i0} denote the initial conditions, which are observed, and $F(s_{it}|s_{it-1}, d_{it-1})$ is the transition probability.

The problem with the standard approach relates to the computation of $\Pr(d_{ijxt} = 1)$, the probability of observing consumer i purchasing product j and size x in period t , due to the dimensionality of the state space. In order to understand why this is the case, note that given the extreme value assumption on the unobserved utility random shocks (Assumption 2.7) this probability can be defined as:

$$\Pr(d_{ijxt} = 1 | s_{it}) = \frac{\exp \{ \bar{u}_{ijxt}(p_{jxt}, a_{jxt}, h_{it-1}) + \delta E[V(s_{it+1}) | s_{it}, d_{ixt}, d_{ij/xt}] \}}{\sum_{y,k} \exp \{ \bar{u}_{iky}(p_{kyt}, a_{kyt}, h_{it-1}) + \delta E[V(s_{it+1}) | s_{it}, d_{ixt}, d_{ij/xt}] \}}, \quad (8)$$

where the summation is over all products from all sizes. The state space includes the vector of current prices and promotional activities for all products and sizes, the set of products purchased by consumer i in her previous purchase event, the stock of past consumption as measured by the time since the last purchase, and the vector of random shocks to consumer choices. Given the multitude of products and sizes available to consumers, the state space is extremely large for practical estimation of $\Pr(d_{ijxt} = 1|s_{it})$.

In order to simplify the estimation procedure, I propose a three-stage budgeting approach in the lines of Aguirregabiria (2002) and Hendel and Nevo (2006a).

Step 1 *Estimation of Product Preferences*

I begin by noting that $\Pr(d_{ijxt} = 1|s_{it})$ can, in full generality, be decomposed into the product of two components: the probability of choosing product j conditional on the size x purchased *and* the probability of choosing a purchase of size x :

$$\Pr(d_{ijxt} = 1|s_{it}) = \Pr(d_{ij/x} = 1|s_{it}, d_{ixt}) \Pr(d_{ixt} = 1|s_{it}). \quad (9)$$

The myopia of consumers with regard to product choice (Assumption 2.4), implies that consumers maximize their per-period expected utility when making their product decisions. As a consequence, $\Pr(d_{ij/x} = 1|s_{it}, d_{ixt})$ can be computed without solving the full dynamic problem. Furthermore, given the extreme value assumption on the utility shocks (Assumption 2.7), that probability can be defined as:

$$\begin{aligned} \Pr(d_{ij/x} = 1|s_{it}, d_{ixt}) &= \frac{\exp[\bar{u}_{ijxt}(p_{jxt}, a_{jxt}, h_{it-1})]}{\sum_k \exp[\bar{u}_{ikxt}(p_{kxt}, a_{kxt}, h_{it-1})]} \\ &= \frac{\exp(\gamma_{ix} + \alpha_i p_{jxt} + \xi_{ijt} + \beta_i a_{jxt} + \lambda_i m_{jxt} + \eta_i y_{ijt-1})}{\sum_k \exp(\gamma_{ix} + \alpha_i p_{kxt} + \xi_{ikt} + \beta_i a_{kxt} + \lambda_i m_{kxt} + \eta_i y_{ikt-1})} \\ &= \Pr(d_{ij/x} = 1|p_{xt}, a_{xt}, h_{it-1}), \end{aligned} \quad (10)$$

where the summation is now only over the products of size x .

The parameters in $\bar{u}_{ijxt}(p_{jxt}, a_{jxt}, h_{it-1})$ - with the exception of γ_{ix} that cancels out - can therefore be recovered by maximizing the likelihood of consumer product choice conditional on the size purchased. Let $L_i^{step1}(h_{i1}, \dots, h_{iT-1}, d_{ij/x1}, \dots, d_{ij/xT}|h_{i0})$ denote the likelihood

of consumer i 's conditional choices across all time periods:

$$L_i^{step1} (h_{i1}, \dots, h_{iT-1}, d_{ij/x1}, \dots, d_{ij/xT} | h_{i0}) = \prod_t \prod_{j/x} [\Pr (d_{ij/xt} = 1 | p_{xt}, a_{xt}, h_{it-1})]^{d_{ij/xt}}, \quad (11)$$

where h_{i0} denotes the initial set of products purchased by consumer i , which is observed. Taking the product of this likelihood function across consumers yields the likelihood function to be maximized in step 1:

$$L = \prod_i L_i^{step1} (h_{i1}, \dots, h_{iT-1}, d_{ij/x1}, \dots, d_{ij/xT} | h_{i0}). \quad (12)$$

In making the utility of choosing a given product state-dependent from the set of products bought in the previous purchase occasion, h_{it-1} , I introduce an identification problem since unobserved consumer heterogeneity may confound the inference of true state-dependence effects. As Heckman (1981) points out, if households have different preferences "and if these differences are not properly controlled, previous experience may appear to be a determinant (...) of future experience solely because it is a proxy for temporally persistent unobservables that determine choices."

State-dependence is usually identified by testing the null hypothesis that the current choice, after accounting for consumer-level heterogeneity, is independent of the previous choice. One approach to introduce heterogeneity is to include *observed* consumer heterogeneity. This approach assumes the existence of a finite number of types or segments, with each type consisting of a set of consumers with identical overall choice preferences (Kamakura and Russell (1989)). As the number of types assumed increases, so will the degree of heterogeneity accounted for under this approach. Goldfarb (2006) presents the extreme case where the number of types exactly coincides with the number of consumers. He makes use of a rich dataset containing nearly 1,000 observations per household to estimate a fully flexible model of consumer preferences, by allowing for consumer-specific regressions.

Another approach is to introduce heterogeneity by considering consumer preferences to be realizations of *random variables*. These random variables are assumed in the literature to follow a multitude of distributional assumptions. For example, Chintagunta *et al.* (1991), Gonul and Srinivasan (1993) and Keane (1997) consider preferences to follow a continuous probability distribution, while Jain *et al.* (1994) consider a discrete probability distribution approximation. An intermediate assumption is presented by Dubé *et al.* (2006) that considers a flexible semi-parametric, but continuous model of consumer heterogeneity.

The estimation procedure in step 1 can not allow for random effects in the lines of

the latter approach. If consumer preferences are assumed to be realizations of random variables that follow a probability distribution (either parametric or semi-parametric), then computing $\Pr(d_{ij/xt} = 1 | p_{xt}, a_{xt}, h_{it-1})$ requires integration over the assumed distribution. Although conditional on the type of consumer, this probability will still be independent of the dynamic purchase decision, computing this probability unconditional on the type of consumer requires integration over the distribution of types conditional on the size bought. And working out this distribution requires solving the dynamic problem.

Consumer-level heterogeneity can, however, be allowed in the lines of the former approach: either by using observable household demographics to segment consumers into types or, in the lines of Goldfarb (2006) and Hendel and Nevo (2006a), by considering household-level product and state-dependence fixed effects. One concern with the latter solution might be the dimensionality of the parameters to estimate. However, since the likelihood function in equation (12) is well behaved, the estimation of a considerable number of consumer-level fixed effects is feasible and involves very slight increases in computational costs. Furthermore, the consumer-product fixed effects need only to include those products that belong to each consumer shopping history. Consumer-level product preferences can not be estimated for products never purchased by the household. This reduces the number of consumer-product fixed effects substantially since each household typically purchases a relatively small number of products when compared with the full supermarket assortment. Another concern might be the standard incidental parameters problem. However, given the large number of consumer shopping trips in the typical scanner panel datasets, this issue is probably not a concern and therefore assuming T grows asymptotically is not unreasonable.

Step 2 *Estimation of the Inclusive Values Transition Process*

Having outlined the procedure to estimate the probability of choosing product j conditional on the size x purchased, I now move on to specify the two remaining steps required to estimate the probability of choosing a purchase of size x .

The consumer decision with regard to purchase size (whether and what quantity to purchase) is the solution to the dynamic problem characterized by Bellman's equation (5). However, instead of solving this problem, I follow Hendel and Nevo (2006a) and consider a simplification of the state space that makes use of the extreme value assumption on the utility shocks (Assumption 2.7). This simplification involves summarizing a *subset* of the consumer state space, $s_{it}^* \equiv (p_t, a_t, h_{it-1})$, into a single index per size, an index representing the utility expected by the consumer, *before* seeing the realization of the utility shocks, from

all products of each size. Under Assumption 2.7, this expected utility is given by the inclusive value w_{ixt} (McFadden (1981a)):

$$w_{ixt} = \log \left[\sum_k \exp (\alpha_i p_{kxt} + \xi_{ikt} + \beta_i a_{kxt} + \lambda_i m_{kxt} + \eta_i y_{ikt-1}) \right], \quad (13)$$

which can be computed with the parameter estimates from step 1.

In order to show that the original dynamic problem can be written in terms of the simplified state space, I make the following additional assumption, where w_{it} denotes the vector of inclusive values at time t :

Assumption 2.8 $F(w_{it}|s_{t-1}^*)$ can be summarized by $F(w_{it}|w_{it-1})$.

Solving the consumer dynamic programming decision requires solving the associated Bellman's equation, which in turn involves working out the expectation of the value function. In order to compute such expectation, I need to specify the transition probabilities for the different state variables. Assumption 2.8 simplifies these processes. The motivation is twofold. First, the transition probabilities of prices and promotional activities from a multitude of different products of the same size are summarized into the transition probability of a single index. Second, it also simplifies the transition probabilities of product state-dependence. Although consumer product choice is, by Assumption 2.4, myopic (which means that current product choices do not impact future product choices), it does not mean that dynamics are absent. Current product choices impact the expected future flow utility of the different inside alternatives and, as a consequence, impact the expected future inclusive values that influence the purchase size decision. In other words, even though consumers are myopic with regard to product choice, their decisions have dynamic implications for current and future purchase size choices. Assumption 2.8 summarizes the transition probabilities regarding product choice into the inclusive values processes. Because product-choices are consumer-specific, the inclusive values and their transition processes will necessarily be consumer-specific, requiring that the Bellman's equation is solved separately for each consumer

One concern with Assumption 2.8 might be that shopping trips involving different prices, promotional activities and/or previous period product choices can be reflected in a same inclusive value, which in turn yields the same future transition probabilities. This restriction can, to some extent, be relaxed, although at a substantial computational cost.

Step 3 *Estimation of the Intertemporal Effects of Consumption*

Step 3 addresses the computation of the probability of choosing a purchase of a given size x . Not by solving the dynamic problem characterized by Bellman's equation (5), but by solving a simplified problem, where the subset of the consumer state space, s_{it}^* , is summarized into the vector of each single size indexes, w_{it} . In this simplified problem, the consumer observes only H_{it} and w_{it} and decides whether and how much to purchase.

I now move on to specify the details of this simpler problem. The utility of consumer i in time period t is given by:

$$\begin{aligned} u_{i0t}^{step3}(H_{it}, \varepsilon_{i0t}) &= \varphi(H_{it}) + \varepsilon_{i0t}, & \text{if } x = 0 \\ u_{ixt}^{step3}(w_{ixt}, \varepsilon_{ixt}) &= \gamma_{ix} + w_{ixt} + \varepsilon_{ixt}, & \text{if } x > 0, \end{aligned} \quad (14)$$

where, as before, $x = 0$ stands for no purchase. The consumer is assumed to be forward-looking and, therefore, to maximize the expected discounted utility:

$$\begin{aligned} V_{step3}(H_{it}, w_{it}, \varepsilon_{it}) &= \max_{\Pi_i^{step3}} \sum_{\tau=t}^{\infty} \delta^{\tau-t} E \left[\sum_{x>0} d_{ix\tau} u_{ix\tau}^{step3}(w_{ix\tau}, \varepsilon_{ix\tau}) \right. \\ &\quad \left. + d_{i0\tau} u_{i0\tau}^{step3}(H_{i\tau}, \varepsilon_{i0\tau}) \mid H_{it}, w_{it}, \varepsilon_{it}, \Pi_i^{step3} \right], \end{aligned} \quad (15)$$

where Π_i^{step3} denotes a set of decision rules mapping states to choices, d_{ixt} . The Bellman's equation associated with the consumer's simpler dynamic problem is given by:

$$\begin{aligned} V_{step3}(H_{it}, w_{it}, \varepsilon_{it}) &= \max_{d_{ixt}} \left\{ \sum_{x>0} d_{ixt} u_{ixt}^{step3}(w_{ixt}, \varepsilon_{ixt}) + d_{i0t} u_{i0t}^{step3}(H_{it}, \varepsilon_{i0t}) \right. \\ &\quad \left. + \delta E[V_{step3}(H_{it+1}, w_{it+1}, \varepsilon_{it+1}) \mid H_{it}, w_{it}, \varepsilon_{it}, d_{ixt}] \right\}. \end{aligned} \quad (16)$$

It remains to be shown that the probability of purchasing size x computed from the simplified problem is equivalent to the one computed from the original problem. Establishing this equivalence involves two steps. In the first step, I show that the Bellman's equations associated with the original and simplified problems have the same solution. The second step involves actually showing the equivalence of the probability of purchasing size x from the two problems.

The two-step proof adapts the one presented in Hendel and Nevo (2006a) to this variety-seeking framework.

Proposition 2.1 *The Bellman's equations associated with the original and simplified problems have the same solution.*

Proof. I begin by addressing the original dynamic problem. The Bellman's equation associated with this problem is given in equation (5), reproduced here for convenience:

$$V(s_{it}) = \max_{d_{ixt}} \left\{ \sum_{x>0} \max_{d_{ij/xt}} \sum_j d_{ixt} d_{ij/xt} u_{ijxt}(p_{jxt}, a_{jxt}, h_{it-1}, \varepsilon_{ijxt}) + d_{i0t} u_{i0t}(H_{it}, \varepsilon_{i0t}) + \delta E[V(s_{it+1}) | s_{it}, d_{ixt}, d_{ij/xt}] \right\}.$$

Given Assumption 2.7, the expected value of $V(s_{it+1} | s_{it}, d_{ixt}, d_{ij/xt})$ will be a function of $H_{it}, s_{it}^*, d_{ixt}$ and $d_{ij/xt}$. Recall that s_{it}^* denotes the state space that consists only of the vector of current prices and promotional activities for all products and sizes, and the set of products purchased by consumer i in her previous purchase event. Let $V^e(H_{it}, s_{it}^*)$ denote such function to simplify notation: $V^e(H_{it}, s_{it}^*) = E[V(s_{it+1}) | s_{it}, d_{ixt}, d_{ij/xt}]$.

Computing the expected value of $V(s_{it})$ conditional on the information available at time $t-1$ yields:

$$V^e(H_{it-1}, s_{it-1}^*) = \int \left\{ \max_{d_{ixt}} \left\{ \sum_{x>0} \max_{d_{ij/xt}} \sum_j d_{ixt} d_{ij/xt} u_{ijxt}(p_{jxt}, a_{jxt}, h_{it-1}, \varepsilon_{ijxt}) + d_{i0t} u_{i0t}(H_{it}, \varepsilon_{i0t}) + \delta V^e(H_{it}, s_{it}^*) \right\} dF(s_{it} | s_{it-1}, d_{it-1}) \right\}.$$

The myopic assumption with regard to consumer product choice together with the extreme-value assumption allows this expected value to be re-written in terms of the inclusive values defined in equation (13):

$$V^e(H_{it-1}, s_{it-1}^*) = \int \log \left\{ \sum_{x>0} \exp(\gamma_{ix} + w_{ixt} + \delta V^e(H_{it}, s_{it}^*)) + \exp[\varphi(H_{it}) + \delta V^e(H_{it}, s_{it}^*)] \right\} dF(s_{it}^* | s_{it-1}^*, d_{it-1}).$$

where the expression inside the integral represents integration over the vector of random utility shocks. From the analysis of the above equation, it is possible to conclude that $V^e(H_{it-1}, s_{it-1}^*)$ can, under Assumption 2.8, be iterated using the following Bellman's equation rewritten in terms of w_t instead of s_{it-1}^* :

$$V^e(H_{it-1}, w_{it-1}) = \int \log \left\{ \sum_{x>0} \exp[\gamma_{ix} + w_{ixt} + \delta V^e(H_{it}, w_{it})] + \exp[\varphi(H_{it}) + \delta V^e(H_{it}, w_{it})] \right\} dF(w_{it} | w_{it-1}, d_{it-1}).$$

I now address the simplified problem. The Bellman's equation associated with this prob-

lem is given in equation (16). After substituting for $u_{i0\tau}^{step3}(H_{it}, \varepsilon_{i0t})$ and $u_{ixt}^{step3}(w_{ixt}, \varepsilon_{ixt})$ yields:

$$V_{step3}(H_{it}, w_{it}, \varepsilon_{it}) = \max_{d_{ixt}} \left\{ \sum_{x>0} d_{ixt} (\gamma_{ix} + w_{ixt} + \varepsilon_{ixt}) + d_{i0t} [\varphi(H_{it}) + \varepsilon_{i0t}] + \delta E[V_{step3}(H_{it+1}, w_{it+1}, \varepsilon_{it+1}) | H_{it}, w_{it}, \varepsilon_{it}, d_{ixt}] \right\}.$$

Taking expectations given the information available at time $t - 1$ and integrating out the utility random shocks making use of the extreme-value assumption (Assumption 2.7) allows me to write the expected value, $V_{step3}^e(H_{it-1}, w_{it-1})$, as:

$$V_{step3}^e(H_{it-1}, w_{it-1}) = \int \log \left\{ \sum_{x>0} \exp[\gamma_{ix} + w_{ixt} + \delta V^e(H_{it}, w_{it})] + \exp[\varphi(H_{it}) + \delta V^e(H_{it}, w_{it})] \right\} dF(w_{it} | w_{it-1}, d_{it-1}).$$

Thus, as the proposition claims, the solution to the Bellman's equations associated with the original and simplified problems is the same. ■

I now address the second step of the proof, showing the equivalence between the probability of purchasing size x computed from the original problem to the one computed from the simplified problem.

Proposition 2.2 $\Pr(d_{ixt} = 1 | H_{it}, s_{it}^*) = \Pr(d_{ixt} = 1 | H_{it}, w_{it})$.

Proof. The probability of purchasing size x computed from the simplified problem is given by:

$$\Pr(d_{ixt} = 1 | H_{it}, w_{it}) = \frac{\exp[\gamma_{ix} + w_{ixt} + \delta V^e(H_{it}, w_{it})]}{M_{i0t} + \sum_{y>0} \exp[\gamma_{iy} + w_{iyt} + \delta V^e(H_{it}, w_{it})]},$$

where for notational simplicity $M_{i0t} = \exp[\varphi(H_{it}) + \delta V^e(H_{it}, w_{it})]$. If, on the other hand, this probability is computed from the original problem, it is given by:

$$\Pr(d_{ixt} = 1 | H_{it}, s_{it}^*) = \frac{\sum_j \exp\{\bar{u}_{ijxt}(s_{it}^*) + \delta V^e(H_{it}, s_{it}^*)\}}{M_{i0t}^* + \sum_{y,k} \exp\{\bar{u}_{iky}(s_{it}^*) + \delta V^e(H_{it}, s_{it}^*)\}},$$

where $M_{i0t}^* = \exp[\varphi(H_{it}) + \delta V^e(H_{it}, s_{it}^*)]$. The summation in the numerator is over all products of size x , and the summation in the denominator is over all products of all sizes. As discussed in Proposition 2.1, $V^e(H_{it}, s_{it}^*)$ can, under Assumption 2.8, be re-written in terms of w_{it} instead of s_{it}^* . This implies that the expected value function depends on the purchase size chosen, but not on the particular product choice. Furthermore, $M_{i0t} = M_{i0t}^*$.

As a consequence, the above probability can be decomposed and simplified as follows:

$$\begin{aligned}
\Pr(d_{ixt} = 1 | H_{it}, s_{it}^*) &= \\
&= \frac{M_{ixt} \sum_j \exp(\alpha_i p_{jxt} + \xi_{ijt} + \beta_i a_{jxt} + \lambda_i m_{jxt} + \eta_i y_{ijt-1})}{M_{i0t} + \sum_{y>0} M_{iyt} \sum_k \exp(\alpha_i p_{kyt} + \xi_{ikt} + \beta_i a_{kyt} + \lambda_i m_{kyt} + \eta_i y_{ikt-1}^e)} \\
&= \frac{M_{ixt} \exp \left\{ \log \left[\sum_j \exp(\alpha_i p_{jxt} + \xi_{ijt} + \beta_i a_{jxt} + \lambda_i m_{jxt} + \eta_i y_{ijt-1}) \right] \right\}}{M_{i0t} + \sum_{y>0} M_{iyt} \exp \left\{ \log \left[\sum_k \exp(\alpha_i p_{kyt} + \xi_{ikt} + \beta_i a_{kyt} + \lambda_i m_{kyt} + \eta_i y_{ikt-1}^e) \right] \right\}} \\
&= \frac{\exp(w_{ixt} + M_{ixt})}{M_{i0t} + \sum_{y>0} \exp(w_{iyt} + M_{iyt})} \\
&= \frac{\exp[\gamma_{ix} + w_{ixt} + \delta V^e(H_{it}, w_{it})]}{M_{i0t} + \sum_{y>0} \exp[\gamma_{iy} + w_{iyt} + \delta V^e(H_{it}, w_{it})]} \\
&= \Pr(d_{ixt} = 1 | H_{it}, w_{it}),
\end{aligned}$$

where $M_x = \exp[\gamma_{ix} + \delta V^e(H_{it}, w_{it})]$.

Thus, as the proposition claims, the probabilities computed from the original and simplified problems are equivalent. ■

Having established the equivalence of the probability of a purchase of size x between the two problems, I move on to specify the estimation procedure. I estimate the remaining parameters by maximizing the likelihood of consumer purchase choices. Let the likelihood of consumer i 's purchase choices across all time periods be denoted by:

$$L_i^{step3}(H_{i1}, \dots, H_{iT}, d_{ix1}, \dots, d_{ixT} | H_{i0}) = \prod_t \prod_x [\Pr(d_{ixt} = 1 | H_{it}, w_{it})]^{d_{ixt}} dF(w_{it} | w_{it-1}, d_{it-1}), \quad (17)$$

where H_{i0} denotes the initial stock of past consumption as measured by the time since the initial purchase.

In making the utility of purchasing a given size state-dependent from the duration since the last purchase, I introduce an identification problem similar to the one discussed previously for product switching: unobserved consumer heterogeneity may confound the inference of true state-dependence effects. The identification problem arises from the fact the interpurchase duration may be long either due to a low taste for the good or a strong state-dependence effect. In order to control for unobserved heterogeneity, I assume preferences vary across consumers using a random effects specification:

$$\lambda_i = \lambda + \Gamma v_i, \quad (18)$$

where λ_i denotes the vector of the remaining parameters to be estimated (and includes the

duration dependence and the size-specific (dis)utilities parameters) and v_i is a independently and identically distributed standard normal. The vector λ denotes the mean values of the different coefficients, while Γ denotes the Cholesky decomposition of the variance-covariance matrix, Σ , which for computational simplicity is assumed diagonal.

I should note that although one may argue that consumer-specific inclusive-values already control for unobserved consumer heterogeneity in their taste for the good, a random effects specification for the size-specific (dis)utilities is required in practice. The justification relates to an unfortunate property of the conditional logit model used in step 1. The model is unable to estimate an intercept since it plays no role in determining the product-choice probability conditional on the size purchased. As a consequence, in order to estimate the consumer-level product preferences in step 1, a normalization is required for each consumer and size (since including dummy variables for all products in the conditional choice would amount to estimate a size-specific intercept). The random effects specification for the size-specific (dis)utilities in step 3 is instrumental in making the inclusive values across sizes and consumers, each estimated using a different normalization, comparable.

With the introduction of the random effects, the likelihood of consumer i 's purchase choices across all time periods is now given by:

$$L_i^{step3}(H_i, d_{ix}|H_{i0}) = \int \prod_t \prod_x [\Pr(d_{ixt} = 1|H_{it}, w_{it}, v_i)]^{d_{ixt}} dF(v_i) dF(w_{it}|w_{it-1}, d_{it-1}). \quad (19)$$

I follow Pakes (1986), Pakes and Pollard (1989), and McFadden (1989) and draw n_s pseudo-random consumers to approximate the integral using a (smooth) simulator estimator:

$$L_i^{step3}(H_i, d_{ix}|H_{i0}) = \int \prod_t \prod_x \left[\frac{1}{n_s} \sum_s \Pr(d_{ixt} = 1|H_{it}, w_{it}, v_s) \right]^{d_{ixt}} dF(v_i) dF(w_{it}|w_{it-1}, d_{it-1}). \quad (20)$$

Taking the product of this likelihood function across consumers yields the likelihood function to be maximized in step 3:

$$L = \prod_i L_i^{step3}(H_i, d_{ix}|H_{i0}). \quad (21)$$

4.1 Bellman's Equation Solution

The structural estimation is based on Rust (1987)'s algorithm that nests the solution of the consumer's dynamic programming within the estimation parameter search. In this section, I address the computational details of the strategy used to solve the functional equation

(16) associated with the consumer's simpler dynamic problem. One strategy to solve dynamic programming problems is by *discrete approximation*. In this type of approach, the value function is solved for numerically by discretizing continuous state spaces into a finite number of n grid points. However, in high-dimensional problems, discretization results in a curse of dimensionality, since n increases exponentially fast in the dimension of the state space. Another approach is to solve dynamic programming problems by *parametric approximation*, where the value function is approximated by a smooth parametric function with k unknown parameters. The latter approach is superior to the former whenever the number of parameters k required to obtain a good global approximation (according to some metric) under parametric approximation is smaller than the value n of grid points required to obtain a comparable ...t by discrete approximation.

I follow Hendel and Nevo (2006a) and solve the functional equation (16) by using value function parametric approximation with policy function iteration in the lines of Benitez-Silva *et al.* (2000). Policy function iteration consists of an alternating sequence of *policy improvement* and *policy valuation* steps:

Policy Valuation

The policy valuation step computes the value function, $V_{step3}(H_{it}, w_{it}, \varepsilon_{it})$, for a given initial guess of the consumer decision, d_{it} . Under a parametric approximation approach, the value function is approximated by a linear combination of k basis functions (ρ_1, \dots, ρ_k) :

$$V_{step3}(H_{it}, w_{it}, \varepsilon_{it}) \simeq \sum_k \theta_k \rho_k(H_{it}, w_{it}, \varepsilon_{it}). \quad (22)$$

Substituting $V_{step3}(H_{it}, w_{it}, \varepsilon_{it})$ in functional equation (16) by the polynomial approximation yields a linear equation in k unknown parameters θ :

$$\begin{aligned} \sum_k \theta_k \rho_k(H_{it}, w_{it}, \varepsilon_{it}) &= \sum_x d_{ixt} u_{ixt}^{step3}(w_{ixt}, \varepsilon_{ixt}) + d_{i0t} u_{i0t}^{step3}(H_{it}, \varepsilon_{i0t}) \\ &+ \delta \int \sum_k \theta_k \rho_k(H_{it+1}, w_{it+1}, \varepsilon_{it+1}) dF(\varepsilon_{it+1}) dF(w_{it+1}|w_{it}, d_{it}). \end{aligned} \quad (23)$$

This can be solved by ordinary least squares when evaluated at a finite set of $m \geq k$ sample points in the state space $(H_{it}, w_{it}, \varepsilon_{it})$. In order to understand why this is the case, define

the $(m \times k)$ matrices r and Er , as well as the $(m \times 1)$ vector u with the following elements:

$$\begin{aligned} r_{mk} &= \rho_k(H_m, w_m, \varepsilon_m) \\ Er_{mk} &= \int \rho_k(H_{m+1}, w_{m+1}, \varepsilon_{m+1}) dF(\varepsilon_{m+1}) dF(w_{m+1}|w_m, d_m) \\ u_m &= \sum_x d_{xm} u_{xm}^{step3}(w_{xm}, \varepsilon_{xm}) + d_{0m} u_{0m}^{step3}(H_m, \varepsilon_{0m}). \end{aligned} \tag{24}$$

Equation (23) can then be re-written as a system of linear equations: $u = X\theta$, where $X = (r - \delta Er)$. The solution to this system of equations, which is given by $\hat{\theta} = (X'X)^{-1} X'u$, can then be used to evaluate the approximated value function.

Policy Improvement

The policy improvement step updates the guess of the consumer decision, d_{it} , using the value function approximation from the policy valuation step. The updated consumer decision (purchase size) can be performed analytically by maximizing the sum, evaluated at the same m sample points, of current utility and the expected discounted future utility:

$$\begin{aligned} d_{xm} &= \arg \max \left\{ \sum_{x>0} d_{xm} u_{xm}^{step3}(w_{xm}, \varepsilon_{xm}) + d_{0m} u_{0m}^{step3}(H_m, \varepsilon_{0m}) \right. \\ &\quad \left. + \delta \int \sum_k \hat{\theta}_k \rho_k(H_{m+1}, w_{m+1}, \varepsilon_{m+1}) dF(\varepsilon_{m+1}) dF(w_{m+1}|w_m, d_m) \right\}. \end{aligned} \tag{25}$$

The two steps are then iterated until convergence of the parameters of the value function approximation. The consumer decision that it converges to, and the corresponding value functions are approximated solutions to the Bellman's equation. See Puterman and Shin (1978) for sufficient conditions for policy iteration to converge in continuous state spaces.

4.2 Identification

In this section, I provide an informal discussion of identification. I begin by addressing the identification of step 1 parameters. The identification of the non-dynamic product preference parameters is standard, with the coefficients being identified through the effect of *current* period's variation in those exogenous variables on *current* period's probability of choosing a given product. Temporary price and non-price promotions provide variation to identify sensitiveness to price and other promotional activities. The (dis)utility from multiple-item

purchasing is identified by the share of multiple-item purchases across trips. Consumer-level product effects are identified from variations in consumer shares across products.

Product choice state-dependence is identified, as argued by Chamberlain (1985), through the effect of *previous* period's variation in exogenous variables on *current* period's probability of choosing a given product. If a temporary promotion for product j at time $t - 1$ decreases the probability of a given consumer choosing product j at time t , then the consumer may be a variety-seeker. If, on the other hand, such promotion increases that probability, the consumer may be habit persistent/incurred in switching costs. Given a long enough consumer-level price (and other promotional activities) time series, variation in previous period's promotions identifies product-choice state-dependence.

Step 2 parameters are identified through the effect of previous period's variation in each consumer inclusive values on her current period's inclusive values.

I now move on to address the identification of step 3 parameters. The purchase size coefficients help with the infrequent incidence of purchase across observed trips and are naturally identified from each consumer's propensity to purchase the different sizes. The intertemporal effect of purchasing on the utility of the outside alternative is identified by each consumer's interpurchase duration in days. Because in a discrete choice demand model only the relative utilities are identifiable, an identifying normalization is required. I normalize the utility of the outside option to zero when $H_{it} \leq 1$. Finally, I note that in this frequency of purchase model, the discount factor is not identifiable. I assume it to equal 0.995.

5 Empirical Analysis

5.1 Step 1: Estimation of Product Preferences

Step 1 estimates product preferences by maximizing the likelihood of observing the sequence of household product choices, conditional on the size purchased. The choice set includes, therefore, only products of the same size as the actual purchase. Table 9 presents the results of this analysis, with the different columns reporting distinct specifications that vary on the covariates included. Specification (1) examines the impact of price on household demand. The price coefficient is of the expected sign and statistically significant suggesting that the average household is price sensitive. Specification (2) controls for promotional activities by including feature and display indicator variables as additional covariates. The coefficients on these controls are positive and statistically significant suggesting that consumers do re-

spond to promotional activities. However, the comparison of the price coefficient in the two specifications is suggestive of an endogeneity issue. Prices are negatively correlated with promotional activities since promoted products sell at lower prices and, as a consequence, not including these controls will overestimate consumer price sensitiveness. In specification (3), I include product dummy variables in order to control for market-level unobserved product characteristics. The product dummy variables are interacted with size so that the preference for each specific product is proportional to the package size purchased. The effect of including these controls on the price coefficient is again suggestive of an endogeneity issue. Products with higher unobserved characteristics sell at higher prices inducing a positive correlation that will underestimate consumer price sensitiveness if not accounted for.

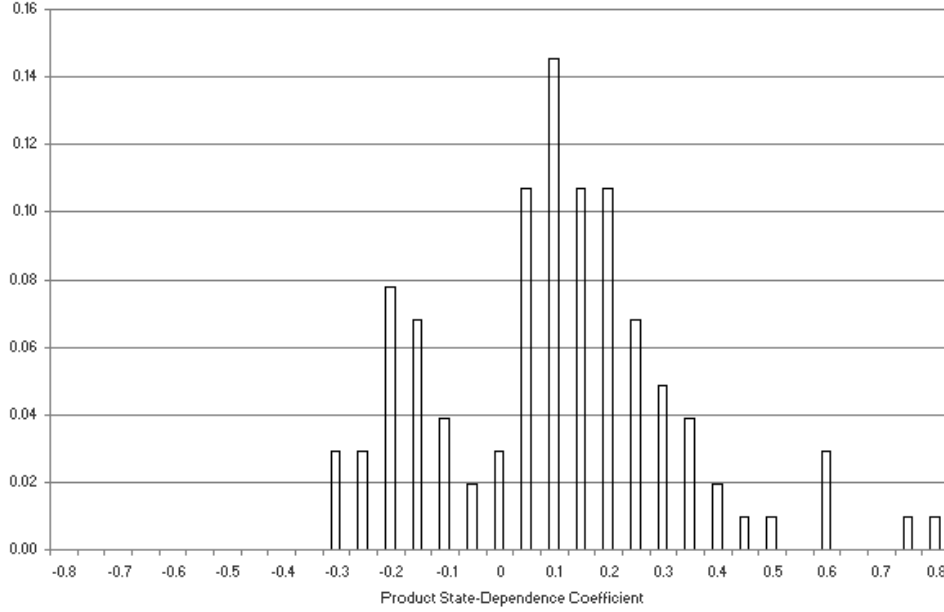
Specification (4) addresses the question of whether households are habit persistent/incur in switching costs or in variety-seeking by including as covariate the number of products that, in each alternative choice, do not belong to the set of products bought in her previous purchase event. The coefficient is negative and statistically significant suggesting that the average consumer incurs in a cost when switching products in successive purchase occasions. The problem with this specification is that unobserved household heterogeneity will confound the inference of true state-dependence effects. The identification problem arises because a consumer may repeatedly purchase a particular product either because of a strong unobserved, idiosyncratic preference for it or because she dislikes switching. In order to identify true state dependence, I control for household product heterogeneity in specification (5). I do so by introducing household-level product dummy variables. I assume, as before, preference for each specific product to be proportional to the package size purchased. No dimensionality problem arises with this introduction because (i) I only consider the products that belong to each household shopping history (I can not expect to estimate household product preferences for products never purchased by the household), (ii) each household buys a relatively small number of products, and (iii) has a relatively long time sequence of purchases. The results seem to indicate that controlling for household heterogeneity matters. Most household-level product dummy variables are statistically significant. Moreover, just as the endogeneity story predicts, the degree of habit persistence previously found decreases significantly. In fact, the coefficient on product state-dependence statistically insignificant when we control for household-level unobserved product preferences. No habit persistence nor taste for variety is supported by the data. The interpretation of this finding is familiar: households are more likely to repeat purchase products for which they have stronger unobserved, idiosyncratic preferences. Specification (6) replicates the previous specification but decomposes the product state-dependence in their different attributes: segment, brand and flavour state-dependence. The coefficients on segment and brand state-dependence are

TABLE 9
*Step 1: Estimation of Product Preferences**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Price	-0.740 (0.038)	-0.658 (0.040)	-0.712 (0.060)	-0.690 (0.061)	-0.699 (0.065)	-0.677 (0.066)	-0.921 (0.292)
Household size							0.088 (0.060)
Household without a male							-0.031 (0.195)
Household without a female							0.104 (0.302)
Household income							0.010 (0.029)
Female head works 35+ Hours							-0.082 (0.178)
Feature		0.456 (0.068)	0.469 (0.082)	0.485 (0.083)	0.667 (0.094)	0.705 (0.095)	0.651 (0.094)
Display		0.464 (0.118)	0.553 (0.126)	0.515 (0.129)	0.617 (0.156)	0.604 (0.156)	0.661 (0.155)
Product state-dependence				-0.648 (0.053)	0.050 (0.068)		-0.621 (0.291)
Household size							0.142 (0.071)
Household without a male							0.502 (0.201)
Household without a female							0.698 (0.388)
Household income							0.016 (0.027)
Female head works 35+ hours							0.029 (0.186)
Segment state-dependence						-0.179 (0.102)	
Brand state-dependence						-0.447 (0.079)	
Flavour state-dependence						0.161 (0.062)	
Product dummy variables			yes	yes			
HH Product dummy variables					yes	yes	yes
Log likelihood	-5,137	-5,105	-4,606	-4,529	-4,024	-3,998	-4,014

* An observation is a purchase instance. Standard errors in parentheses. Household size denotes the number of people in the household. Household without a male (female) denotes an indicator variable that takes the value 1 if there is no male (no female) within the household. Household income denotes the household income group: (in thousand dollars) 1- [0,10), 2-[10,12), 3-[12,15), 4-[15,20), 5-[20,25), 6-[25,35), 7-[35,45), 8-[45,55), 9-[55,65), 10-[65,75), 11-[75,+inf). Female head works 35+ hours denotes an indicator variable that takes the value 1 if the female head (or the male head in the case there is no female within the household) works more than 35 hours per week.

FIGURE 6
Frequency Distribution for Product State-Dependence Coefficient



estimated to be negative. The former is statistically significant at 7.9% of significance level, while the latter is measured precisely enough to be highly significant. The coefficient on favour state-dependence is estimated to be positive and statistically significant. These results suggest that households habit persist in segment (although on statistically significant and brand choice, while variety seek in favour. Finally, in specification (7), I introduce household heterogeneity in the price and product state-dependence covariates. I do so by interacting those covariates with observable household demographics. The results suggest that observable demographics are not important in explaining price sensitiveness since the interactions introduced in price are not significantly different from zero. However, the results do provide some support for the hypothesis of an association between product state-dependence and observable household demographics, with larger households and households without a male being significantly estimated to be less habit persistent. The coefficient on the interaction with households without a female is estimated to be positive and statistically significant at 7.2% of significance level. The remaining interactions are not significantly different from zero. Figure 6 plots the household-level coefficient on product state-dependence frequency distribution computed using the estimates from specification (7). The results provide support for the variety seeking hypothesis. The median product state-dependence coefficient is 0.079, the average is 0.082, the 25th percentile is -0.096 , and the 75th percentile is 0.188.

[THE RESULTS BELOW ARE PRELIMINARY]

5.2 Step 2: Estimation of the Inclusive Values Transition Process

Step 2 estimates the transition process for the inclusive values, which were computed for the purchase sizes observed in the data (16 oz., 32 oz., 64 oz., 80 oz., 128 oz. and 160 oz.) using step 1's estimates from specification (7) above. I follow Hendel and Nevo (2006a) and assume the following first-order Markov process for the transition probability of the inclusive values:

$$w_{ixt} = \theta_{ix0} + \sum_{s \in \mathfrak{S}} \theta_{ixs} w_{ist-1} + \varsigma_{ixt}, \quad (26)$$

where the summation is over the set of package sizes $\mathfrak{S} = (16 \text{ oz.}, 32 \text{ oz.}, 64 \text{ oz.}, 160 \text{ oz.})$ and ς_{ixt} is distributed normal with mean zero and standard deviation σ_{ix} . Multicollinearity precludes the transition process of being defined over the set of all possible purchase sizes (since consumers that are observed purchasing 80 oz. and 128 oz., do so by buying multiple items: 80 oz. = 16 oz. + 64 oz. while 128 oz. = 64 oz. + 64 oz.). Finally, the transition process parameters are index by i because the inclusive values are consumer-specific.

The assumption that the inclusive values are normally distributed may seem somewhat problematic given the evolution of the state variables they summarize. In order to test this assumption, I performed the Shapiro-Wilk W test for normality on the different consumer-level inclusive values. In the untabulated tests, the null hypothesis that the inclusive values are distributed normal is accepted only for a small fraction of consumers: 16 oz. (3%), 32 oz. (3%), 64 oz. (17%), 80 oz. (3%), 128 oz. (13%) and 160 oz. (1%). This assumption can be relaxed, although with a substantial increase in the computational burden.

Table 10 reports the estimated transition probabilities. Table 10, Panel A presents the point estimates (and associated standard errors) under the constraint that all consumers face the same transition probabilities. The results suggest that the lagged inclusive value of own size (or of the two own sizes for those cases that involve multiple-item purchases) is the most important in predicting its future variation. In Table 10, Panel B the estimated transition probabilities are consumer-specific, with the results displaying the mean and standard deviation across the different consumer-level estimates. There is evidence of substantial heterogeneity across consumers, as suggested by the large standard deviations, which supports the option for the individual-level transition processes.

TABLE 10
*Step 2: Estimation of the Inclusive Values Transition Process**

	Panel A				Panel B			
	Same Process for All Consumers				Consumer-Level Process			
	ω_{16t-1}	ω_{32t-1}	ω_{64t-1}	ω_{160t-1}	ω_{16t-1}	ω_{32t-1}	ω_{64t-1}	ω_{160t-1}
ω_{16t}	0.97 (0.09)	0.14 (0.06)	-0.05 (0.03)	0.54 (0.08)	0.21 (0.34)	-0.19 (2.61)	-0.20 (6.51)	0.36 (3.20)
ω_{32t}	0.02 (0.11)	1.08 (0.07)	-0.09 (0.04)	0.15 (0.10)	0.31 (1.06)	0.08 (0.21)	0.08 (5.30)	0.11 (4.25)
ω_{64}	0.60 (0.08)	-0.27 (0.05)	0.96 (0.03)	0.00 (0.07)	-0.02 (0.43)	-0.01 (0.11)	0.25 (0.26)	0.01 (0.06)
ω_{80t}	2.21 (0.12)	-0.46 (0.07)	0.48 (0.04)	0.27 (0.10)	0.17 (0.56)	-0.19 (2.61)	-0.05 (6.56)	0.39 (3.41)
ω_{128t}	0.63 (0.08)	-0.28 (0.05)	1.76 (0.03)	0.16 (0.07)	-0.03 (0.43)	-0.02 (0.20)	0.45 (0.47)	0.02 (0.13)
ω_{160t}	0.33 (0.07)	-0.10 (0.04)	-0.03 (0.02)	1.24 (0.06)	-0.00 (0.00)	0.00 (0.01)	0.12 (5.98)	0.11 (0.25)

* An observation is a shopping trip instance by a household. Also included are a constant and size indicator variables to control for unavailability of a package size at a given shopping trip. Panel A displays point estimates and standard errors in parentheses. Panel B displays the mean and standard deviation across the different consumer estimates.

5.3 Step 3: Estimation of the Intertemporal Effects of Consumption

Step 3 maximizes the likelihood of observing the sequence of consumer purchase choices after solving the consumer-specific Bellman's equations associated with the simplified dynamic programming problem. Even though I solved the Bellman's equation separately for each consumer, the random effects specification for the parameters allowed me to pool the likelihoods across consumers. As discussed previously, I approximated the value function by a linear combination of k basis functions (ρ_1, \dots, ρ_k) , with the approximation basis used being a polynomial in the natural logarithm of the duration in days since the consumer's last purchase and in the levels of the remaining state variables.

In order to estimate the model, I have to specify a functional form for $\varphi(H_{it})$. I assume the following:

$$\varphi(H_{it}) = \kappa_0 \ln(H_{it}) + \kappa_1 H7_{it},$$

where $H7_{it}$ denotes an indicator variable that takes the value 1 if the shopping trip at time t corresponds to the consumer seven days cycle as suggested by the purchase hazard rate.

Table 11 reports the results for different specifications of step 3. In specification (1) I do

not allow for heterogeneity or forward-looking behaviour. The state-dependence results suggest that the utility of the outside option decreases with the duration since the last purchase, which supports the intertemporal substitution argument. This result should not come as a surprise, despite the opposite suggestion from the raw data (recall the slight downward trend of the hazard rate), because the consumer-specific inclusive values do control in some extent for unobserved heterogeneity. The coefficient on the indicator variable $H7_{it}$ is negative, which suggests that once every 7 days, the value of the outside option decreases. I interpret this result as illustrating potentially reduced transaction costs of consumers purchasing in their main shopping trip. The estimates for the size-specific effects are statistically significant at standard significance levels. Econometrically, they help sort each size frequency of purchase. However, as discussed previously, the magnitude and ordering of these estimates can not be directly interpreted as they capture the different normalizations required for step 1 estimation.

Specification (2) introduces dynamic considerations into the consumers decisions, with this introduction substantively reducing state-dependence. The reason is that the static specification omits price expectations from the consumers purchase decisions. When facing a price promotion, the typical consumer expectation is that the price will go up in the future. This induces her, as I discussed in the descriptive analysis section, to typically take advantage of the price promotion by anticipating purchases. The static specification, by omitting price expectations, bias the results since it interprets this shorter interpurchase durations as stronger state-dependence. The addition of the forward-looking behaviour also impacts the coefficient on the indicator variable, which becomes (significantly) positive. This result is unexpected and hard to interpret. The estimates for the size-specific effects maintain the same ordering and magnitude.

Finally, specification (3) estimates the version of the model described in the previous sections that allows for both heterogeneity (via random coefficients) and forward-looking behaviour. The results for the mean estimates do not change substantively. However, the fit of the model, as measured by the log-likelihood, increases significantly. This illustrates the importance of accounting for heterogeneity, not only to control for different degrees of state-dependence (the results suggest substantial heterogeneity at this level), but also to control for the different *consumer*-size normalizations required for step 1 estimation.

TABLE 11
*Step 3: Estimation of the Intertemporal Effects of Consumption**

	Static	Dynamic		
	Standard Logit Estimate (1)	Standard Logit Estimate (2)	Random Coefficients Mean Estimate (3)	Standard Deviation
No Purchase (Outside Alternative)				
κ_0	-2.42 (0.00)	-1.63 (0.00)	-1.83 (0.02)	4.57 (0.03)
κ_1	-2.37 (0.02)	0.43 (0.02)	-0.22 (0.19)	0.26 (0.15)
Purchase Size				
16 oz.	-36.87 (0.02)	-36.84 (0.02)	-38.14 (0.34)	8.76 (0.18)
32 oz.	-40.52 (0.02)	-40.27 (0.02)	-40.56 (0.22)	6.08 (0.12)
64 oz.	-25.56 (0.01)	-25.25 (0.01)	-35.88 (0.29)	13.95 (0.21)
80 oz.	-43.39 (0.20)	-41.57 (0.04)	-41.61 (0.27)	4.29 (0.43)
128 oz.	-49.43 (0.02)	-49.60 (0.02)	-42.89 (0.13)	6.09 (0.08)
160 oz.	-39.42 (0.05)	-39.48 (0.05)	-39.52 (0.26)	5.42 (0.40)
Log-likelihood	-45,501	-36,294	-14,244	

* An observation is a shopping trip instance by a household. Standard errors in parentheses.

5.4 Simulation Algorithm and Goodness of Fit

In this section, I present an algorithm to simulate the several dimensions of the model and examine its fit. The need to specify a simulation algorithm arises because the estimation algorithm decomposes the likelihood of the consumer choices into two components: the choice of whether to purchase (and what size) and the decision of which product to buy if purchasing a positive amount. However, the choice of product influences the purchase decision and vice-versa. In order to address this issue, I propose the following consumer-level algorithm:

1. Solve the simplified dynamic programming problem and simulate the sequence of purchase decisions (whether to purchase and not, and what size if they decide to purchase) conditional on the *observed* inclusive values.
2. For each shopping trip that the consumer decides to purchase a positive amount, simulate her product choice(s).
3. Using the sequence of simulated product choices, I then *simulate* the corresponding inclusive values and update the associated transition probabilities.
4. Iterate the previous steps until convergence of the coefficients of the inclusive values transition processes.

I do not provide here a convergence proof for this algorithm. I note, however, that convergence was, in practice, achieved for *all* consumers after a small number of iterations. The remaining section examines several dimensions of the fit between the simulated and observed sequence of consumer choices.

The simulated probability that a consumer makes a purchase in any given week is 13.65%, which fits the observed probability (15.55%) reasonably well. Figure 7 analyzes how the model fits the purchase decision dynamics, by comparing the simulated and observed distribution of inter-purchase duration in days. Overall, the fit is very good, although it slightly underestimates the frequency of purchases for duration spells between 3 and 4 days, at the expense of slightly overestimating the frequency of purchases for durations of 7 days. Other than that, the model is quite accurate in simulating this interpurchase duration. Figure 8 examines the hazard rate of purchasing by duration in days from the last purchase, i.e. the probability that the consumer purchases a positive amount given that she has not purchased up to now. Again, the model predicts the pattern of the hazard rate quite accurately only

FIGURE 7
Observed and Simulated Interpurchase Duration Distribution

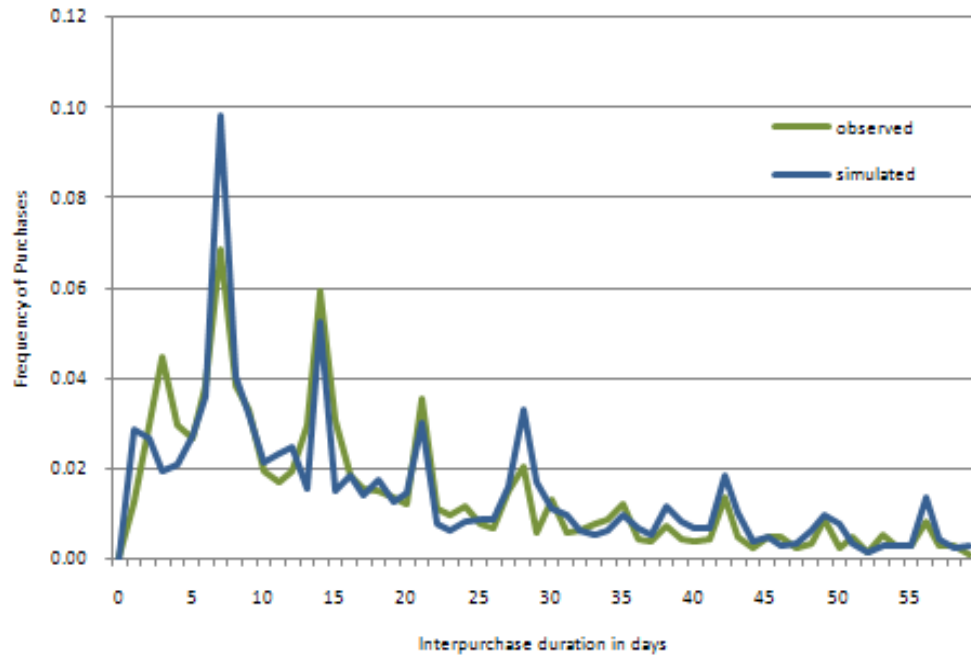


FIGURE 8
Observed and Simulated Purchase Hazard

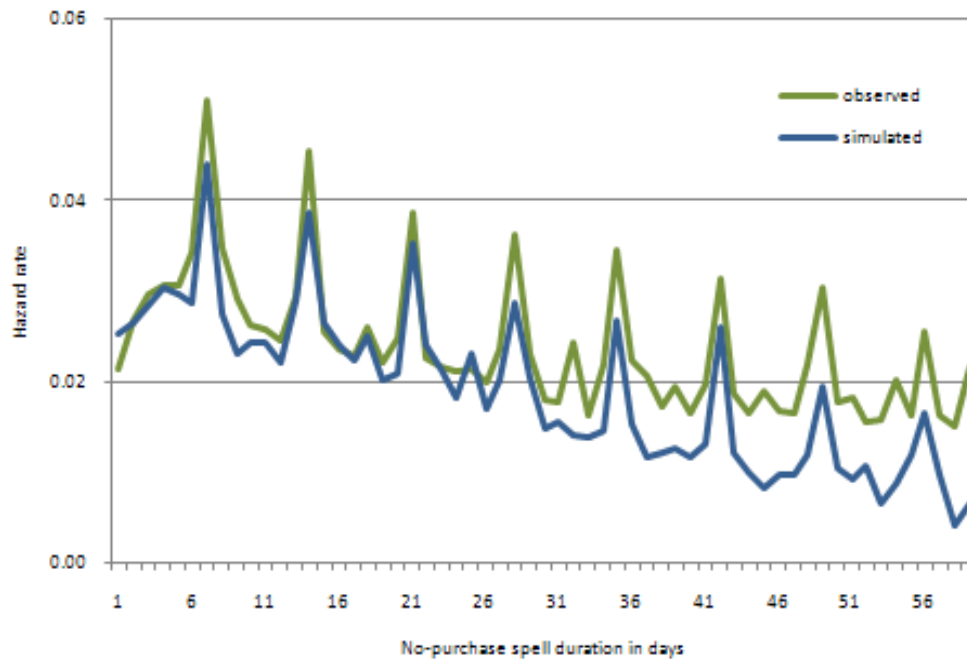
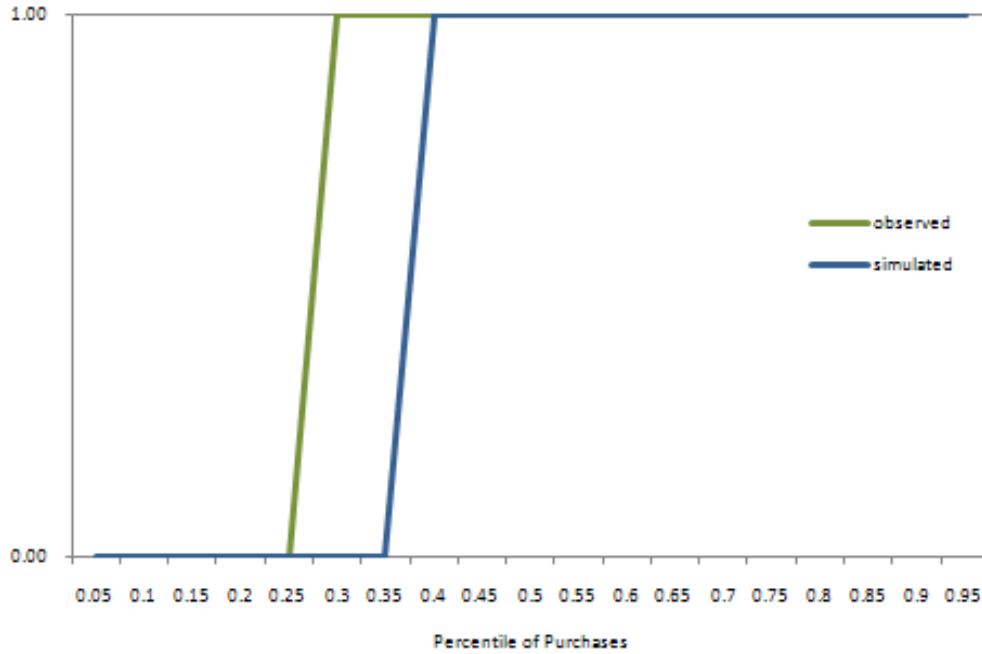


FIGURE 9
Observed and Simulated Product Switching Distribution



very slightly underestimating the purchase probabilities for longer durations of no purchase spells, due to the low frequency of purchases with such duration.

Having addressed the purchase decision dynamics, I now move one to examine how the model ...ts the product switching decision dynamics. The simulated probability that a consumer exhibits a product switch from a purchase instance to the next is 63.79%, which only slightly underestimates the observed probability (72.81%). Figure 9 presents the distribution of purchases with regard to product switching. Although there is a slight underestimation of product switching, the ...t is reasonably good.

6 Policy Implications

The pricing decision is one of the most critical for retailers. In this section, I discuss the implications of a major pricing policy change from hi-low pricing strategy to everyday low pricing (EDLP). In a pure EDLP policy, retailers charge a constant everyday price with no temporary price discounts. In contrast, in a hi-low pricing policy, prices have a higher regular level that remains constant for long periods of time, but then retailers run frequent promotions that lower the price below the EDLP level. In practice, however, pure EDLP

TABLE 12
*Simulated Effects of Pricing Policy Changes **

Panel A: 0% Price Reduction	Variance Reduction		
	25%	50%	75%
Average Interpurchase Duration (Days)	-0.02	-0.13	-0.25
Proportion of Product Switching	-0.24	-1.35	-1.25
Total Volume Purchased	-0.07	-0.33	-0.43
Total Revenue	1.84	2.95	4.07
Panel B: 5% Price Reduction	Variance Reduction		
	25%	50%	75%
Average Interpurchase Duration (Days)	-0.28	-0.73	-0.77
Proportion of Product Switching	-0.30	-1.22	-1.48
Total Volume Purchased	0.14	0.45	0.62
Total Revenue	0.83	1.12	1.82
Panel C: 10% Price Reduction	Variance Reduction		
	25%	50%	75%
Average Interpurchase Duration (Days)	-0.47	-1.14	-1.31
Proportion of Product Switching	-0.36	-1.30	-1.30
Total Volume Purchased	0.35	1.01	1.22
Total Revenue	-0.17	-0.57	-1.31

* The table reports the percentage changes implied by the different policy changes when compared to the actual pricing strategy.

strategies rarely exist (see Information Resources, Inc. (1993)). EDLP retailers typically charge lower prices on an everyday basis, but do engage in some temporary price discounts.

The pricing policy choice is an empirical question. Hi-low pricing policies have been prevalent in the industry since it allows retailers to price discriminate between consumers that are heterogenous in their price sensitiveness (Pigou (1920)), the degree of price information (Varian (1980)), the level of inventory costs (Blattberg *et al.* (1981), Jeuland and Narasimhan (1985)), or the extent of store loyalty (Narasimhan (1988)), just to mention a few dimensions. However, the success of retailers like Wal-Mart, Home Depot and Toys R Us has increased the popularity of EDLP policies. There are various rationales for adopting EDLP. On the supply side, EDLP is assumed to lower operating costs through (i) better inventory control, warehouse handling and lower in-store personnel costs due to less variability in demand, and (ii) lower advertising expenses due to a focus on image rather than price. On the demand side, EDLP is assumed to restore price credibility with consumers disenchanted with constant changing prices.

Table 12 examines this empirical question. I evaluate the *demand* implications of various degrees of a policy change from high-low pricing towards EDLP in four dimensions: the

average interpurchase duration, the proportion of product switching, total volume purchased and total revenue. I should note that I do not compute market equilibrium prices, which is beyond the scope of this chapter (although providing such a framework constitutes a very interesting potential area for future research). I consider only ad-hoc changes in the observed pricing strategy. The table reports the percentage changes implied by different policy changes when compared to the actual pricing strategy.

Table 12, Panel A addresses the implications of changing *only* the extent of the hi-low pricing policy by simulating a reduction in price variance, while keeping the mean price for each supermarket-product-size combination constant. The results suggest that a pricing policy that exhibits lower price variance slightly decreases the average interpurchase duration. This is the outcome of two opposite effects. On one hand, the magnitude of the promotion

(by only reducing price variance, while keeping the mean price constant) may be revenue-increasing. However, not knowing the cost function, I can't determine the general impact on profits.

7 Conclusions

In this paper, I attempt to link two strands of the literature on variety-seeking: one focusing on substitutability across time and another on substitutability across products. This issue is economically relevant because both types of substitutability are important for retailers and manufacturers in designing intertemporal price discrimination strategies. I specify a consumer demand model which allows consumption to have an enduring effect and allows the marginal utility of the different products to vary over consumption occasions. I then use the model to evaluate the demand implications of a major pricing policy change from hi-low pricing to an everyday low pricing strategy

I find evidence that consumption has a lasting effect on utility that induces substitutability across time and that the median consumer has a taste for variety in her product decisions. Consumers are found to be forward-looking with respect to the duration since the last purchase, to price expectations and product choices. Pricing policy simulations suggest that retailers may increase revenue by reducing the variance of prices, but that lowering the everyday level of prices may be unprofitable.

This chapter leaves many estimation issues yet to be explored. The development of a framework that allows consumers to be forward-looking in their product choice or incorporates the supply side of the market to derive equilibrium pricing strategies seem very interesting potential areas for future research.

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