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Supported by

The Pinhas Sapir Center for Development, Tel Aviv University
Recanati School of Business, Tel Aviv University
Department of Public Policy, Tel Aviv University
Journal of Industrial Economics (JIE)
CEPR

Tel Aviv; 24 May 2011

Bonuses and Managerial Misbehaviour

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May 11, 2011

Abstract

Bonuses and variable compensation of managers have repeatedly been criticized for encouraging misbehaviour and thereby ultimately harming a firm's interest. This argument has in particular been made in the wake of the recent financial crisis, claiming that banker's bonus plans induce overly risky investment choices. We show that large bonuses may in fact discourage agents from "gaming" their incentive schemes and may thus have been an optimal way to reduce excessive risk taking. Our finding has strong policy implications and sheds new light on recent proposals to regulate bonuses in the banking industry.

JEL-Classification: D82, D86, G30, M12

Keywords: contracts, incentives, governance, executive compensation

*I would like to thank Jacques Crémer and Klaus Schmidt for helpful discussions as well as Bruno Biais, Benno Bühler, Florian Englmaier, Augustin Landier and Nico Matouschek for useful suggestions. I also appreciate the comments supplied by participants of presentations at the Universities of Munich and Toulouse and at the ACLE 2009, EEA 2009 and EARIE 2010 conferences. Financial support from the German Science Foundation (DFG) through GRK 801 is gratefully acknowledged.

1 Introduction

In the wake of the recent financial crisis, excessive bonuses for bankers were frequently blamed as a source of irresponsible behaviour: In order to be awarded those tempting rewards, bankers were supposedly prepared to engage in behaviour that was not in the interest of banks themselves. In particular, bankers had incentives to take excessive risks, since they could reap the benefits in case of success and were protected by limited liability in case of failure. This argumentation however suggests that banks did collectively set sub-optimal incentive schemes, an idea which at least seems to deserve some closer scrutiny.[1] In this paper we show that high-powered incentives are in many cases not only robust to potential undesirable behaviour, but the danger of non-compliance may even increase the optimal bonus an agent is offered. Offering large bonuses may hence have been optimal even if banks were aware of their employees ability to take up excessive risks.

Our finding that reducing pay-performance ratios may not be a suitable way to encourage compliance is consistent with the observation by John and Qian (2003) that controlling for leverage,[2] banks offer incentives that are not significantly different from the ones given in firms where misbehaviour is less costly. Comparisons of incentives in a number of different industries such as in Conyon and Murphy (2000), Murphy (1999) and Zhou (2000) paint a similar picture: Pay-performance ratios do not seem to be noticeably lower in sectors where compliance is key, such as the Financial Services or Natural Resources industries. Furthermore, our model is in line with the finding by Fahlenbrach and Stulz (2010) that the size of cash bonuses a bank paid was not negatively correlated with performance during the recent financial crisis.

In this paper we propose a standard moral-hazard model where the agent is risk-neutral but protected by limited liability and enlarge the agent's action space by assuming that he can not only choose how much effort to exert, but also whether (and how much) to engage in undesirable behaviour. This unwanted behaviour (or non-compliance) increases the contractible profit signal and hence the agent's variable compensation, but is nevertheless against the principal's interest. If the agent carries out an undesirable action, the principal finds out about this non-compliance with positive probability and is able to punish the manager.

Whenever bonuses are small, an increase in the bonus raises the level of misbehaviour: Any action that positively affects the profit signal now becomes more attractive. For large

[1] An alternative justification for large bonuses would be that shareholders were confident to be bailed out in case risky investments turn bad. However, given the large losses incurred by equity holders during the recent crisis, this confidence seems at most partially justified ex-post.

[2] John and John (1993) offer a theoretical explanation why pay-performance ratios should depend on the debt ratio of a company.

bonuses on the other hand, the level of non-compliance is decreasing in the bonus: The higher the incentives for effort, the higher the bonus the agent is going to lose out on in case his misbehaviour is discovered, in which case he receives zero wage payments. Hence, with very high incentives he will be less prepared to jeopardize these expected earnings by taking undesirable actions and will be more likely to comply; by offering large bonuses the principal exacerbates the maximal punishment that he can impose on a misbehaving agent. Given that misbehaviour is most pronounced for intermediate incentives, the principal optimally chooses "extreme" incentives: If effort is of little importance he will curtail incentives in order to reduce the level of misbehaviour. But if it is important to motivate effort, the principal will offer lavish bonuses in order to curb non-compliance. In both cases, the principal may find it optimal to complement bonuses with fixed wages that are paid regardless of the firm's profit as long as no evidence of misbehaviour surfaces in order to enhance compliance. Our model takes the somewhat extreme view that all wages the agent receives are given to ensure incentive compatibility. They are neither due to collusive practises in the determination of pay (Bebchuk and Fried, 2006), nor do they reflect scarcity of prospective employees (Tervio, 2008; Gabaix and Landier, 2008). While this view is unlikely to fully describe the reality of executive compensation, it gives nevertheless some interesting insights. In particular, fixed wages can not only be used in order to satisfy participation constraints, but they may also be an additional disciplining device aimed against misbehaviour.

Our results imply that for top-management positions high incentives may in fact be a method to induce compliance, whereas lower ranks in a firm's hierarchy will receive very performance-inelastic pay in order to achieve the same goal. Clearly our analysis does not only apply to the remuneration of executives, but can equally explain the pay of portfolio managers, traders etc., who typically receive very high-powered incentives and are equally able to engage in undesirable behaviour, e.g. excessive risk taking that only becomes evident in adverse states of the world. Our model brings us one step closer towards understanding the strong monetary incentives in these jobs.

Undesirable behaviour plays not only a role in the banking ind-393.832(u)-2.377(-)-4.954(n)31.9206

in order to be awarded a bonus.[3]

Often, undesirable behaviour will not only have negative consequences for the firm itself but will also impose negative externalaties on society as a whole. In a final section we consider the implications our model has for a number of policies that a legislator might consider in order to increase the level of compliance within organizations: While shareholders will typically themselves give their managers incentives to comply, a policy maker may want to reduce misbehaviour even further, and he is able to do so by different means. We conclude that caps on bonuses may be counter-productive. But even if they have positive effects, it is always more efficient to make shareholders liable for misbehaviour of their managers.

In looking at a two-dimensional moral hazard model, our work is clearly related to the multi-tasking literature initiated by Holmstrom and Milgrom (1991). Yet, by explicitly incorporating limited liability constraints that are an important issue when punishing managers for misconduct, we reach quite different conclusions:[4] While in the traditional multi-tasking models the introduction of additional tasks typically reduces optimal incentives, in our setting the opposite can be true: Incentive problems in the second dimension are mitigated by *increasing* incentives in the first dimension. The idea that monetary incentives may trigger undesirable behaviour is not new and has for example been studied in the empirical literature on earnings management (Healy, 1985; Asch, 1989; Holthausen, Larcker and Sloan, 1995; Oyer, 1998; Larkin, 2007) or in the context of the optimal scope of a firm (Fischer and Huddart, 2008). However, this literature does not derive the optimal structure of management contracts. Another related strand of literature looks at the interplay between the incentives for effort, short-termism or risk-taking and a company's financial structure (Stein, 1988, 1989; John and John, 1993; Von Thadden, 1995; Biais and Casamatta, 1999; Bolton, Scheinkman and Xiong, 2006). Finally, Spagnolo (2000; 2005) looks at the question whether or not incentive contracts can make collusion harder to sustain while abstracting from the effect a given incentive has on the agent's choice of effort. The most closely related work is by Inderst and Ottaviani (2009) who look at optimal contracts if sales agents must be induced to search for potential customers, but not to sell to unsuitable customers. When a sales agent has found an unsuitable customer

[3]Our only key assumption is that the agent doesn't fully internalize the negative consequences of his actions. This is due to two reasons: a) The agent's limited liability and b) The imperfect observability of undesirable actions. Imperfect observability may arise because often, the negative consequences of undesirable effort only emerge in the distant future. Alternatively, it may be impossible to condition the agent's remuneration on certain outcomes, e.g. a drop in a firm's reputation, due to verifiability constraints. In this case the principal can only impose punishments in case additional evidence on misbehaviour is found.

[4]For a discussion of the problems involved in imposing large punishments on managers, even in cases of outright fraud, see Bebchuk et al. (2006).

he can *only* earn a bonus by misbehaving. This implies that unlike in our setting higher incentives will never have a disciplining effect and will always increase misbehaviour.

The rest of the paper is organized as follows: Section 2 formulates the model. In Section 3 we will state our main result and show how a firm will optimally incentivize its agent in the light of potential misbehaviour. Section 4 analyzes the agent's decision problem more closely and will validate the assumptions on the agents behaviour that we have made before. Sections 5 and 6 derive testable predictions and show that our results are robust to allowing for more general payment schemes. In Section 7 we will explore the policy implications of our model. Section 8 concludes.

2 The model:

A risk-neutral principal employs a risk-neutral but wealth constrained agent to manage a firm. The firm can make either high profits $\overline{\pi}$ or low profits $\underline{\pi}$ with $\overline{\pi} > \underline{\pi}$. The agent's wealth is initially 0 and has to be non-negative in all states of the world. He can exert unobservable effort which determines the probability $a \in [0, \overline{a})$ that high profits arise. We will denote the agent's effort cost for working in the firm by $C(a)$.

Furthermore the agent has the possibility to unobservably increase the probability of high profits by $u \in [0, \overline{u}]$ at a private cost $K(u)$ if he engages in actions that are seen as undesirable by the principal. Since the overall probability of high profits can not exceed 1 we assume that $\overline{a} + \overline{u} \leq 1$. Misbehaviour imposes an expected, non-verifiable cost of $\phi(u) = \delta\phi(u)$ on the principal where δ is some scalar and $\delta\phi'(u) \geq \overline{\pi} - \underline{\pi}$ for all u. That is, the marginal cost of undesirable effort outweighs the benefit of an increase in the likelihood of high profits from the principal's point of view. With a small probability $p(u)$ the principal gains hard information that the agent has been engaging in undesirable behaviour and can punish him by reducing his wage payments to any level that does not violate the limited liability constraint. A natural way to think of misbehaviour in the the financial industry is to assume that the agent can engage in risky investments that will usually yield high returns but perform very poorly with probability $p(u)$, in which case the principal always learns that the agent has been misbehaving.

In what follows we impose the following assumptions:

Assumption 1.

$$i) C(0) = 0, C'(0) = 0, C''(a) > 0, \lim_{a \to \overline{a}} C'(a) = \infty$$
$$ii) K(0) = 0, K'(0) = 0, K''(u) > 0$$

Part i) of the assumption says that the cost of effort is an increasing and convex function of a and ensures that the agent finds it always optimal to choose some $a < \overline{a}$. Similarly, part ii) says that the cost of undesirable behaviour is increasing and convex in u.

Assumption 2.

$$p'(0) = 0, p''(u) > 0, p(\overline{u}) = 1$$

Assumption 2 guarantees that not only the explicit cost of undesirable behaviour but also the implicit cost, i.e. the risk of being caught, is convex. Moreover, the last part of the assumption makes sure that the agent will always find it optimal to choose an interior level of u.

Assumption 3. *The marginal cost of effort $C'(a)$ is convex and $\frac{C'''(a)}{C''(a)^2}C'(a) < f$ for some $f < 2$.*

This technical assumption ensures that the amount of effort an agent exerts is concave in the bonus he expects to get but is not too concave. Assumption 3 is for example satisfied by all power functions $C(a) = \ a^r$ with $\ \geq 0$ and $r \geq 2$. Moreover, log-concavity of the marginal cost of effort is sufficient for the second part of the assumption to hold.[5]

3 The optimal bonus

The main insight of this paper is that shareholder may find it optimal to increase bonuses in order to curb undesirable behaviour by their managers: While higher bonuses typically encourage misbehaviour, sufficiently large bonuses create a strong incentive for the agent to preserve his good prospective pay and can serve to discourage any behaviour that risks losing those payments. To show our main result first, we will start by considering the shareholders problem. In order to do so, we take as given that the level of effort a that an agent exerts is strictly increasing and concave in his bonus, while the level of undesirable behaviour u follows an inverted-U shape. We will then show in the next section that these properties do indeed always hold. Moreover, in order to simplify our analysis we assume that the agent receives payments only if he is not observed misbehaving and the firm makes high profits. This payment will be called the bonus. We will relax the assumption that the agent is only paid in the "best" state of the world in Section 6 and show that our results are robust to allowing for more general compensation schemes.

[5]Log-concavity implies that $\frac{C'''(a)}{C''(a)^2}C'(a) < 1$.

The principal has two objectives that he pursues when setting a bonus: On the one hand he wants to give the agent incentives to exert effort; on the other hand he doesn't want him to engage in undesirable behaviour. When choosing the bonus b he will have to strike a balance between those two goals. For notational simplicity we will assume that instead of choosing b, the principal (equivalently) chooses $\beta = \big(1 - p(u)\big)b$ which is the bonus the agent can *expect* to earn in case of high profits:[6] The agent's choice of effort depends not only on b per se but also on the probability with which he believes to lose the bonus due to misbehaviour. So the effect any change in b has on the manager's choice of effort will be amplified or dampened by the effect it has on undesirable behaviour u. Assuming that the principal chooses the effective bonus β directly allows us to ignore this issue.

Given that the level of effort an agent chooses is given by the concave function $a = G(\beta)$ we can state the principal's objective function as

$$\Pi(\beta, u(\beta)) = \Big(u + G(\beta)\Big)\Big(\overline{\pi} - \beta\Big) + \Big(1 - u - G(\beta)\Big)\underline{\pi} - c(u) \tag{1}$$

where u is a function of β. From the principal's point of view there are two negative effects of undesirable behaviour: First, undesirable behaviour creates an efficiency loss of $c(u)$ which more than off-sets the positive effects undesirable behaviour has on firm profits. Secondly, undesirable behaviour allows the agent to appropriate additional wage payments. The latter effect explains why a principal wants to reduce misbehaviour even if it is costless from an efficiency point of view, i.e. if $c'(u) = \overline{\pi} - \underline{\pi}$.

The necessary condition for an optimum with $\beta > 0$ is given by

$$\frac{d\Pi(\beta, u(\beta))}{d\beta} = \frac{\partial\Pi}{\partial\beta} + \frac{\partial\Pi}{\partial u}\frac{du}{d\beta} = 0. \tag{2}$$

The first important observation is that the optimization problem the shareholders face is not necessarily concave: The cost of misbehaviour makes very small and very large bonuses more attractive, since those bonuses guarantee high levels of compliance. So for large costs of undesirable behaviour, there will typically be two local maxima: One in which the principal pays very small bonuses and $du/d\beta$ is positive and one in which he offers lavish incentives and $du/d\beta$ is negative. Henceforth, we will just assume that the problem has a unique optimum, abstracting from the non-generic case where two local optima generate exactly the same level of profits.

[6] We will show in Lemma 1 that β is a one-to-one function of b.

Although it is interesting to look at the question of how the optimal bonus changes with the ease with which the agent can game his incentive scheme, we take as given that the agent has the opportunity to engage in undesirable behaviour at some cost. The question we will be chiefly interested in is a different one: How does the optimal bonus change with the damage an agent causes by misbehaving? Consider a salesman who harms a firm's reputation by misselling an excessively sophisticated product to a customer and a fund manager who invests in overly risky assets to increase his expected bonus. Even if the private cost of misbehaviour is the same for both the salesman and the fund manger, the principal will presumably have a much larger incentive to reduce misbehaviour in the second case.

Before we proceed, let us briefly recall that we have defined the cost of undesirable behaviour to the principal as [illegible] $(u) = \delta$ [illegible] (u). So it seems a natural choice to model a rise in the damage misbehaviour causes by an increase in the scaling parameter δ. Furthermore, let us define $\hat{\beta} = \hat{b}\,(1 - p(u))$ as the expected reward beyond which any further increase in the bonus reduces misbehaviour.

Proposition 1. *For any $\delta > 0$ we have $\frac{d\beta}{d\delta} \gtrless 0$ if $\beta \gtrless \hat{\beta}$: Whenever the optimal bonus lies above some threshold $\hat{\beta}$ a marginal increase in the cost of undesirable behaviour will increase the bonus a principal optimally offers (and vice versa).*

In short, this proposition verifies a straightforward intuition: If compliance is higher for very small or very large bonuses, the principal will generally set more "extreme" incentives as his concern for compliance increases. If incentives are high already, a principal will reduce misbehaviour by offering even larger incentives. If bonuses are small, he will optimally choose to reduce performance pay. An obvious question to ask is under which circumstances the principal will choose to offer either the very large or the very low bonuses. Unsurprisingly, this depends on the importance of productive effort a, i.e. on $\Delta = \overline{[illegible]} - \underline{[illegible]}$. If effort is of little importance, then it will not be worthwhile to invest into large bonuses and the principal will refer to mute incentives. If on the other hand effort is very important, setting very low incentives is unattractive and the principal will rather offer extremely generous bonuses.

While Proposition 1 describes how the optimal bonus changes with a marginal change in the cost of misbehaviour, the same does not necessarily hold true for more radical changes in the damage a misbehaving agent causes: Assume that the cost of misbehaviour increases drastically. Even if a principal found it optimal to set high bonuses beforehand, after a large hike in the cost of non-compliance he may decide to scrap incentives altogether and offer very small bonuses, which results in negligible levels of misbehaviour. Since the principal's optimization problem is not globally concave, any large change in

the cost of misbehaviour may render a different local optimum more attractive and lead to a discontinuous change in the optimal bonus.

4 The agent's decision problem

After having analysed at the principal's problem, let us look the mechanism that drives the results above in detail. For now, we will maintain the assumption that managers are only paid a positive wage if the firm makes high profits and no evidence on misbehaviour surfaces. In Section 6 we will show that it is indeed always optimal to pay no wage in case the principal has observed non-compliance. Our assumption that the manager receives no wages in case the firm makes low profits on the other hand allows us to concentrate on the main effects and will be relaxed in Section 6.

The utility an agent receives if offered a bonus of b is given by

$$= (a+u)\Big(1-p(u)\Big)b - C(a) - K(u)$$

So any optimal choice of a and u will have to satisfy the following two first order conditions:

$$\frac{}{a} = (1-p(u))b - C'(a) = 0 \tag{3}$$

$$\frac{}{u} = (1-p(u))b - K'(u) - p'(u)b(a+u) = 0 \tag{4}$$

Given Assumptions 1 and 2 we can show that the optimum will always be unique (see Appendix).

Note that we have assumed the agent's effort cost to be additively separable in the two dimensions and there are hence no technological complementarities between the two tasks. Nevertheless, we see that the two dimensions are strongly intertwined: The principal is left with only one instrument, b, to encourage effort and discourage misbehaviour. Moreover, undesirable behaviour will itself reduce the probability with which a successful manager receives the bonus and will therefore erode incentives for effort as can bee seen in equation (3). Effort on the other hand increases the expected bonus the agent loses out on in case misbehaviour is detected and will increase the level of compliance as determined by (4).

Any optimum is implicitly defined by

$$F \equiv (1-p(u))b - K'(u) - p'(u)b\Big(G\big((1-p(u))b\big) + u\Big) = 0 \tag{5}$$

where the effort level is given by $a = G((1-p(u))b)$ and where $G(\cdot) \equiv C'^{-1}(\cdot)$ is a strictly increasing, concave function. In order to determine the overall effect an increase in the bonus b will have on the agent's choice of u we have to look at

$$\frac{du}{db} = -\frac{\frac{\partial F}{\partial b}}{\frac{\partial F}{\partial u}} = \frac{(1 - p\,(u)) - p'(u)\Big((a + u) + (1 - p(u))bG'\big((1 - p(u))b\big)\Big)}{-\ \ F/\ \ u} \tag{6}$$

with the denominator being positive by strict concavity of the agent's objective function in the optimum.

Let us look in more detail at the numerator in equation (6): The first term captures the idea that an increase in b will raise the returns to undetected misbehaviour, which makes such actions more attractive. The second term corresponds to the fact that the relative harm of being caught is increasing in the bonus the agent is going to lose out on, which discourages misbehaviour. Moreover, not only is the bonus the agent misses out on in case of misbehaviour increasing in b per se, but an increase in the bonus will also lead the agent to exert more effort, making the expected reward the agent jeopardizes if he chooses not to comply even larger. Consider a situation in which $b = 0$ and the principal contemplates marginally increasing the bonus: In this case the second term vanishes, since there is no bonus the agent might miss out on. The marginal effect on the return to misbehaviour on the other hand is still strictly positive, which implies that misbehaviour is increasing in the size of incentives. Employees that hardly get any bonuses at all will not be disciplined by the prospect of losing them, which means that any small bonus will inevitably make misbehaviour more attractive. However, for very large bonuses this logic no longer applies: Now the agent cares about loosing his high (expected) compensation and he is hence very reluctant to breach his fiduciary duties. Indeed, Assumption 3 guarantees that there always exists a threshold such that the level of misbehaviour decreases in b if and only if the bonus is larger than this threshold:

As a first approximation, let us consider how the numerator in equation (6) changes as we increase b and hold u constant: The marginal effect of an increase in b on the return to misbehaviour is constant. In contrast to this, the total wage payments an agent can expect to earn (or lose, in case he is caught misbehaving) are convex in b. So the second terms in the numerator of equation (6) will become increasingly large and the effect an increase in the bonus has on the level of non-compliance will eventually become negative.[7] The most intuitive way to think about the second effect is to say that by setting a higher

[7] To see this note that Assumption 3 implies that $-G''(\beta)\beta < 2G'(\beta)$ for all β. Holding u constant, convexity of the expected wage payments an agent receives requires that $G''(\beta)\beta + 2G'(\beta) > 0$ which is indeed the case.

bonus b the principal effectively relaxes the limited liability constraint of the agent. By using larger bonuses to augment the agent's expected wealth, the principal increases the maximal punishment that he can impose on a demonstrably misbehaving agent, which discourages misconduct.

It will be useful to state some basic properties of the agent's choice of a and u:

Lemma 1. *Both, effort and the probability of the firm making high profits, are increasing in the bonus b:* $\frac{da}{db} > 0$ *and* $\frac{d(a+u)}{db} > 0$.

The first part of the statement corresponds to an upper bound on du/db: While an increase in b may lead to more undesirable behaviour and hence a larger $p(u)$, the expected bonus, $(1 - p(u))b$, will still be increasing in b. The second part of Lemma 1 establishes a lower bound on du/db. Even if larger bonuses lead to less undesirable behaviour, the overall probability of the firm making high profits (which is given by $a + u$) is still increasing in the bonus.

Proposition 2. *There exists some strictly positive threshold* $\hat{b}$ *such that*

- *if* $b < \hat{b}$ *an increase in the bonus b will lead to more misbehaviour:* $du/db > 0$
- *if* $b > \hat{b}$ *an increase in the bonus b will lead to less misbehaviour:* $du/db < 0$.

Employees that receive low bonuses are very amenable to misbehaviour: An increase in their rewards runs the risk of encouraging misdeeds that increase the contractible profit signal. Highly incentivized executives and other high-ranked employees on the other hand can expect to earn extremely high bonuses even without misbehaving. This creates a strong concern for preserving those prospects and any policy that increases wage payments will consequently enhance compliance. Taking those two observations together, the function $u(b)$ that determines the level of misbehaviour follows an inverted-U shape:

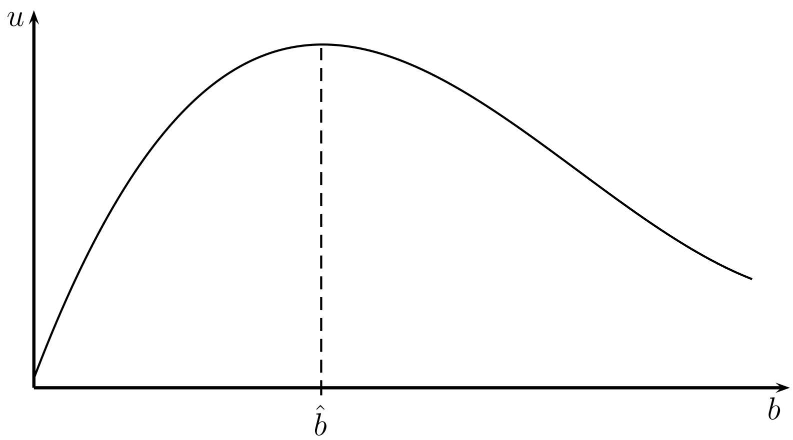


Figure 1: Illustration of Proposition 2

5 Industry Implications

Unfortunately, neither the importance of effort, nor the precise damage that a misbehaving agent causes in a particular firm are likely to be observable in reality. Yet, we would expect the cost of misbehaviour to be industry specific, while even within an industry firms exhibit considerable heterogeneity with respect to the importance of CEO effort (e.g. because of different organizational choices). This allows us to characterize any industry by a distribution of incentive plans. We have seen that the harm done by non-compliance will result in more "extreme" incentives: Those firms that offer very low pay-performance ratios anyway will depress incentives even further as the cost of undesirable behaviour increases, while companies with high-powered incentives will increase bonuses. The bonus distribution will hence be more spread out in an industry where misbehaviour is more costly than in one where it is rather harmless.

Consider an industry where the returns to effort $\Delta = \bar{\pi} - \underline{\pi}$ are distributed in the interval $(0, \infty)$. Furthermore, let us redefine the cost of undesirable behaviour as $\hat{\tau}(u) = (\bar{\pi} - \underline{\pi})u + \delta\hat{\gamma}(u)$ where $\delta\hat{\gamma}'(u) \geq 0$. This specification implies that the principal can not gain form undesirable behaviour no matter how large Δ is: In expectation he will lose any benefits that accrued from misbehaviour plus some additional (net) cost of $\delta\hat{\gamma}(u)$.[8] Since the optimal bonus a principal chooses to offer is an increasing and unbounded function of Δ, there exists some critical value $\hat{\Delta}$ such that the principal offers a bonus of $b \gtreqless \hat{b}$ whenever $\Delta \gtreqless \hat{\Delta}$.[9] So there is always a positive measure of firms which offer small bonuses and will react to an increase in the cost of misbehaviour by reducing incentives. But we will also have firms that offer substantial incentives and choose to raise bonuses if misbehaviour becomes increasingly expensive. This holds true no matter what the cost of undesirable behaviour is, so we get some clear-cut predictions concerning the interquantile ranges of the bonus distribution:[10]

Proposition 3. *Assume there is a continuum of firms which employ one manager each and returns to effort Δ are distributed in the interval $(0, \infty)$. Then the interquantile range $Q_{(1-\epsilon)} - Q_\epsilon$ of the bonus distribution $H(b)$ is strictly increasing in δ for all small values of ϵ. The larger the harm caused by misbehaviour in a particular industry, the more spread out the bonus distribution will be.*

If a rise in the marginal cost of non-compliance prompts principals to reduce bonuses

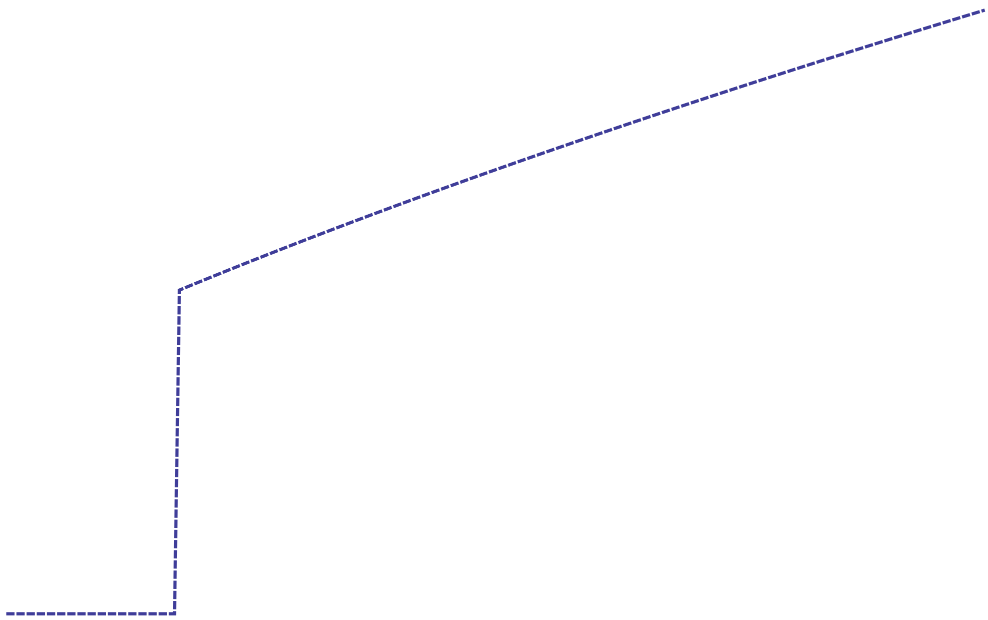
[8]While the specification of $\hat{\tau}(u)$ is mainly for technical convenience, it may represent a situation where the costs of misbehaviour consists of legal fines. Typically, such fines are set as to claim back any benefits the principal may have had from misbehaviour plus an additional deterrent $\delta\hat{\gamma}(u)$.

[9]In this section we look at b instead of the expected bonus β since we believe b to be more easily observable in reality. We could equivalently look at β without changing any of our results.

[10]The interquantile range denotes the difference in value between two given quantiles of a distribution.

that are small already and to increase those bonuses that are generous, this implies that the tails of the bonus distribution will grow further apart.

To illustrate said properties, we can simulate the optimal bonus as a function of Δ for different costs of undesirable behaviour. Consider the example where an agent can exert effort or engage in risky investments which increase the company's chance of making high profits in favourable states of the world, but with a small probability of $p(u) = (u/0.25)^2$ a crisis occurs. In expectation, a crisis will not only destroy any value the investment generates in good states of the world, but it will also lead to a net loss of of T=\$12m (dashed line) or \$25m (dotted line). The cost of undesirable behaviour is hence given by $\hat{\pi}(u) = (\overline{\pi} - \underline{\pi})u + \quad p(u)$. We take the private costs of effort and misbehaviour incurred by the agent to be $C(a) = a^2/(0.75 - a)$ and $K(u) = u^2/(1 - u)$. If we compare the optimal bonuses to a case where misbehaviour does not entail any efficiency loss (solid line), we see that bonuses get more spread out as the cost of misbehaviour increases. It should be noted that in this example, the firm's profit is not concave in the bonus for most values of Δ, which explains why the optimal bonus b is discontinuous in Δ.



the bonus. Section 7 discusses in more detail why we might expect such constraints to play a role in reality.

6 General contracts

So far we have assumed that the manager only receives a positive wage payment in case the firm makes high profits and no undesirable behaviour is detected. However, shareholders can of course decide to employ more sophisticated compensation schemes. In our model the most general contract a principal can offer is characterized by the tuple (w, b, w_p, b_p) where the agent gets a base salary w regardless of profits whenever no evidence on non-compliance is found and he receives an additional bonus b in case of high profits. Similarly, w_p and b_p denote the respective payments in case misbehaviour is detected.

	$\underline{\pi}$	$\overline{\pi}$
No Misbehaviour	w	$w + b$
Misbehaviour	w_p	$w_p + b_p$

Figure 3: General Contracts.

We will now relax the assumption that a manager is only paid in one state of the world and show that it is indeed never optimal to pay positive wages in case of detected misbehaviour. While it may be optimal to pay the agent in case of low profits, the main insights that we have obtained so far still apply if we allow for more general contracts.

Optimal punishments

As noted before, any optimal contract will have $b_p = 0$. The intuition for this result is straightforward: Reducing b_p will always reduce the level of undesirable behaviour the agent chooses, which is beneficial. At the same time a decrease in b_p will lessen the incentives for effort a. However, this effect can be offset by adjusting b in a way that leaves the expected wage payment *given high profits* constant, which leaves us only with the negative impact on u. Also, $w_p > 0$ is never part of an optimum, since it incentivizes misbehaviour without having any effect on the choice of effort a. Finally, it is never optimal to punish the agent in case of high profits: The same a and u could also be obtained by offering the agent no wages at all, strictly reducing the expected wage cost.

Lemma 2. *Any optimal contract that implements some $a > 0$ will pay no wages if undesirable behaviour is detected: $w_p = b_p = 0$. Furthermore, the wage payment in case of high profits will never be smaller than the wage payment in case of low profits: $b \geq 0$.*

Fixed Wages

We have seen that the principal may decide to pay the agent large bonuses in order to increase the wage payments the agent might lose out on if he misbehaves. Alternatively, the principal could achieve the same by offering a fixed wage component w which will be paid out whenever no undesirable behaviour is observed, irrespective of the firm's profits. Paying the agent only in case of high profits has the advantage of motivating effort and increasing the expected punishment for misbehaviour at the same time. Yet, it also increases the returns to undetected misbehaviour. This explains why it may be optimal to pay fixed wages in order to discourage non-compliance. In fact, whenever non-compliance is very costly, the principal will decide to make use of this alternative way of discouraging misbehaviour, since the positive effect of a larger bonus on the agent's effort choice becomes increasingly small.[11] The following proposition characterizes the relationship between w and $\beta = (1 - p(u))b$ when the principal decides to implement a given compliance level u:

Proposition 4. *Assume that $w > 0$. Then the principal will choose β such that*

$$G'(\beta)(\bar{\pi} - \underline{\pi}) = \frac{(1 - p(u))}{p'(u)} \tag{7}$$

Let us compare the optimal bonus as characterized by Proposition 4 with the bonus a principal would choose if the agent did not have the possibility to misbehave. In this case, an optimal incentive $\tilde{\beta}$ would be characterized by $G'(\tilde{\beta})(\bar{\pi} - \underline{\pi}) = \tilde{\beta}G'(\tilde{\beta}) + G(\tilde{\beta})$ where the left hand side represents the benefit of increased effort. The right hand side represents the marginal increment in the agent's compensation and is strictly increasing in $\tilde{\beta}$. Whenever $\tilde{\beta}$ is large, the right hand side of equation (7) will be smaller than $\tilde{\beta}G'(\tilde{\beta}) + G(\tilde{\beta})$ and the principal optimally chooses a bonus above $\tilde{\beta}$. If $\tilde{\beta}$ is small on the other hand the optimal bonus will lie below $\tilde{\beta}$.[12]

[11] Similar reasoning applies if the principal contemplates lowering the bonus to increase compliance. While it is now possible to reduce misbehaviour by paying *lower* rewards, reducing bonuses is costly since the agent chooses less effort. Due to the concavity of $G(\beta)$ this effect gets larger the smaller the bonus.

[12] In this section we can treat $p(u)$ as a constant since we are concerned with the question how a principal optimally implements a given u: Any change in β will be accompanied by a change in w that leaves the incentives for misbehaviour unchanged.

So even if we allow for fixed wages as a way to encourage compliance, the effects we have shown so far will still be present. Since distorting the bonus is costless at the margin, the principal will always choose to do so: Adjusting b to increase compliance is not only a last resort if for some exogenous reason no fixed wages can be paid, but it is indeed part of any optimal contract. In particular, as long as effort is sufficiently important the principal will still set generous bonuses to reduce misbehaviour.

7 Policy Implications

In many cases, undesirable behaviour has negative externalities on society as a whole. To cite our motivating example, excessive risk taking may create systemic risk and require public bail-outs. The same holds true for many other forms of misbehaviour: Cartel agreements reduce consumer surplus, bribes may undermine the rule of law etc. A natural question to ask is how a social planner may want to discourage such behaviour. In order to answer this question, we use our previous results to compare three policy instruments: The first one is a legal cap on bonuses. Such a measure is vividly discussed and therefore merits some closer theoretical examination. An instrument that is applied already in many areas is to impose pecuniary fines on firms if evidence on misbehaviour of their employees surfaces. Finally, we consider whether a policy maker that is free to impose arbitrarily large punishments on misbehaving agents should always choose to do so.

In the following, we will compare the different policy instruments under the assumption that while bonuses can be freely chosen within the legal limits, the principal is constrained in increasing fixed wages. This assumption seems to be warranted given what many have called the "outrage constraint", i.e. the fact that shareholders are unlikely to accept very high levels of fixed wages, which are typically more visible ex-ante than variable compensation. Alternatively, we can think of the constraint on fixed wages as a result of current US tax legislation which treats expenditure on fixed wages unfavourably once it exceeds a certain threshold.

Caps on bonuses

The previous discussion shows that the negative consequences of legal restrictions on the size of bonus payments are potentially twofold: Besides the obvious effect of reducing the incentives for effort, such a policy may even have adverse effects on compliance and encourage misbehaviour.

Corollary 1. *Assume that absent any regulation, the principal sets an expected bonus* $^{*} > \hat{\ }$. *Then a legal cap* $\bar{\ } < {}^{*}$ *on bonuses will i) increase misbehaviour and ii)*

decrease effort as long as $^{*} - \bar{\ } \leq \epsilon$ *for all small* ϵ

Corollary 1 tells us that what would seem to be cautious regulation may in fact be very harmful: By imposing caps on bonuses that are close to the level of bonuses paid in an unregulated labour market, we may destroy incentives for effort *and* increase managerial misbehaviour. Yet, this does not hold for large interventions: If the legal maximum on bonus payments is very small, this will have positive effects on compliance since any cap that is sufficiently close to zero will result in negligible levels of misbehaviour. In fact, even a less stringent cap on bonuses with $\bar{\ } > \hat{\ }$ can potentially increase compliance, since another local optimum with a lower level of non-compliance may now yield higher profits to the principal than setting $= \bar{\ }$. This observation implies that even if caps on bonuses are non-binding in equilibrium, they may nevertheless have an effect since they induce a shift to a different local optimum. However, such interventions clearly erode incentives for managers to work hard and may not increase social welfare.

Corporate Liability

So far we have taken the cost of undesirable behaviour for the principal to be exogenously determined. However, in many cases this cost comprises legal fines. This suggests that a policy maker may choose to increase the cost misbehaviour imposes on the principal in case he wants him to discourage undesirable behaviour. The policy maker has access to a monitoring technology $\mathcal{P}(u)$ with $\mathcal{P}'(u) \geq 0$ and can impose some punishment $\mathcal{T}$ on the principal if he observes illegal behaviour. For simplicity we assume that the states of the world in which the policy maker receives evidence on misbehaviour are a subset of the states in which the principal does so, which implies that $\mathcal{P}(u) \leq p(u)$.

From the principal's point of view, an increase in the corporate fine $\mathcal{T}$ is simply a rise in the marginal cost of misbehaviour. We have already seen that whenever $> \hat{\ }$ the principal will respond to a marginal increase in corporate liability by increasing incentives. Since the principal will always implement effort that is too low from the point of view of a social planner, this is good news:[13] By punishing the principal, the policy maker not only reduces unwanted behaviour, but he also reduces the distortions created by the non-observability of effort. While the reverse is true if $< \hat{\ }$, a policy of punishing the principal is still weakly better in terms of social surplus than a cap on bonuses that implements the same u: A principal that is left to his own devices to achieve a given reduction in u will always do so by choosing (weakly) larger fixed wages and (weakly)

[13] As usual, the first-best effort level would be implemented if the agent would reap the full social benefits from effort, i.e. $\beta = \overline{\pi} - \underline{\pi}$. Instead of choosing $\beta \geq \overline{\pi} - \underline{\pi}$, the principal can always do strictly better by setting $\beta = 0$, which results in $u = 0$.

larger bonuses, which is socially beneficial.

Corollary 2. *Punishing the principal for observed misbehaviour always results in weakly higher social welfare than a cap on bonuses that implements the same* u.

Where social welfare is defined as firm profits before wage payments minus any costs incurred by the agent or the general public. So a policy maker can not do worse by imposing fines on the principal rather than capping bonuses. Furthermore, a marginal increase in the corporate fine will unambiguously increase social welfare whenever ▶ ˆ and the principal's problem has a unique optimum.

Personal Liability

Up to now, we have not considered the possibility that a policy maker might decide to punish the agent directly in case of observed misbehaviour. Indeed, such punishments will usually be ruled out by the fact that the principal already chooses to punish the agent as fiercely as possible in case that undesirable behaviour is detected. So there is only a role for a policy maker in punishing the agent if the legislator disposes of additional options to penalize the manager, e.g. by imposing prison sentences. But even if we allow for the policy maker to impose arbitrarily harsh punishments on the agent, it is unclear whether a welfare-maximizing policy maker will choose to do so: While extremely harsh punishments can ensure that the agent chooses $u = 0$, they also crowd out any compliance-enhancing incentives the principal might otherwise set. In particular, the principal may decide to pay smaller bonuses, which is socially harmful since it aggravates the efficiency loss that is due to the non-observability of effort. Put differently, the policy maker has some incentive to preserve the moral hazard problem in the second dimension in order to reduce the distortions in the first dimension.

8 Conclusion

Our model shows that while bonus schemes generally open up ways for a manager to game them, very large bonus payments may discourage this kind of misbehaviour. Applied to the financial industry this implies that large bonuses should not be mistaken for conclusive evidence that "too big to fail" created meaningful moral hazard problems and motivated banks to readily accept excessive risk taking by their employees. Nor are high bonuses necessarily a feature of suboptimal contracts. According to our model, offering large bonuses may in fact have been an optimal strategy to curb risky behaviour.

Moreover, the finding that there does not exist a monotonic relationship between bonuses and the incentives for misbehaviour is consistent with the observation by Fahlenbrach and Stulz (2010) that the size of previous cash bonuses paid by a bank did not correlate with bad performance during the recent crisis.

Finally, our results shed some new light on the proposal to legally restrict the size of bonuses. We have shown that this may have counterproductive effects and reduce compliance, while at the same time diluting incentives for managers to work hard. In particular this holds true for rather generous legal caps. Highly restrictive caps on the other hand may have some merit in reducing undesirable behaviour. Yet, in order to maximize social surplus, it is always more attractive to give the principal financial incentives to increase compliance. This may even induce the principal to implement higher levels of effort, which is beneficial since it reduces the distortions that are caused by the non-observability of effort.

We have seen that even under fairly general conditions, the overall effect of performance pay on the gaming of incentive schemes is ambiguous. Further empirical results to understand which effects are economically significant would be highly desirable. In particular, we should try to look at the effect performance pay has on compliance in settings where bonuses are substantial and we might conjecture them to have the disciplining effects that we discussed in this paper.

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A Mathematical Appendix

Proof of Proposition 1. For the purpose of this proof we will take as given that $i)$ $\ \ = (1-p(u))b$ is a strictly increasing function of b. So instead of choosing b we can equivalently look at a problem where the principal chooses $\ $. Moreover, for now we will assume that $ii)$ the agent's choice of effort is given by the strictly increasing, concave function $a = G(\ )$ and that $iii)$ undesirable behaviour u is increasing in $\ $ if and only if $\ $ is below some threshold $\hat{\ }$: $-\frac{du}{d\beta} \gtrless 0 \Leftrightarrow \ \gtrless \hat{\ }$. In Section 4 we will verify that this is indeed the case.

By assumption, any optimum of the principal's problem is unique. So a marginal change in the cost undesirable behaviour imposes on the principal will never lead to a discontinuous change in the optimal bonus and we can restrict attention to the neighbourhood around $\ $ where the principal's objective function is strictly concave. By the implicit function theorem the change in the bonus a principal chooses to offer in response to a marginal increase in the cost of misbehaviour is given by

$$\frac{d}{d\delta} = \frac{\ '(u)}{\Theta}\frac{du}{d}$$

where $\Theta = \frac{d}{d\beta}\left[\frac{\partial\Pi}{\partial\beta} + \frac{\partial\Pi}{\partial u}\frac{du}{d\beta}\right] < 0$ by (local) concavity.

So $\frac{d\beta}{d\delta}$ takes the sign of $-\frac{du}{d\beta}$ and we have $\frac{d\beta}{d\delta} \gtrless 0 \Leftrightarrow -\frac{du}{d\beta} \gtrless 0 \Leftrightarrow \ \gtrless \hat{\ }$.

□

Proof of Uniqueness. Let us show that there will always be a unique optimum to the agent's problem and that any local maximum is a global one. In this proof we allow for the possibility that in addition to a bonus b the principal chooses to pay some fixed wage w irrespective of profits whenever no misbehaviour is observed. We only consider situations where $b > 0$: The principal will never choose any $b < 0$ as we show in Lemma 2. Moreover, if $b = 0$, the unique optimum has $a, u = 0$. So any optimum satisfies $(1-p(u))b - C'(a) = 0$ and there is a unique optimal a for any choice of u. This allows us to look at a one-dimensional optimization problem where the agent chooses u and $a(u)$ is given by the above first order condition. Since the agent's utility is continuous on the closed interval $[0, \overline{u}]$, a maximum always exists. Now consider the largest (potentially locally) optimal $\hat{u} = \max\{\arg\max_u \{(a(u)+u)(1-p(u))b - C(a(u)) - K(u) + w(1-p(u))\}\}$. By the necessary condition (4) we must have $(1-p(\hat{u}))b - bp'(\hat{u})a(\hat{u}) > 0$ at the optimum.

Let us now show that for a given b the optimum is unique on the interval $[0, \hat{u}]$. In order to do so, we will need to find a lower bound for $a'(u)$. First, note that $G(\cdot) \equiv C'^{-1}(\cdot)$ is a concave function, since we can use the inverse function rule to show that

$G''(C'(a)) = -\frac{C'''(a)}{[C''(a)]^2}$ is negative for all a. As $a(u) = G((1-p(u))b)$ this implies that $a(u)$ is concave in u. So it suffices to look at $a'(\hat{u}) = -bp(\hat{u})G'((1-p(\hat{u}))b)$. Using the fact that $(1-p(\hat{u}))b - bp'(\hat{u})a(\hat{u}) > 0$ and that $G((1-p(u))b) \geq G'((1-p(u))b)(1-p(u))b$ by concavity of $G(\cdot)$ we get $1 > p'(\hat{u})bG'((1-p(\hat{u}))b)$. It follows that $a'(u) > -1$ for all $u \in [0, \hat{u}]$

Since the agent's utility is given by $\ (u) = (a(u)+u)(1-p(u))b - C(a(u)) - K(u) + w(1-p(u))$ we get $\ ''(u) = -p''(u)b(a(u)+u) - (2+a'(u))\,p'(u)b - K''(u) - p''(u)w$, which is negative for all $u \leq \hat{u}$. So $\ (u)$ is strictly concave over the interval $[0, \hat{u}]$ and the optimum is unique.

$\square$

For notational convenience we will henceforth write $\ = (1-p(u))b$ wherever possible. Moreover, we will no longer stress that $p = p(u)$ in the interest of brevity.

Proof of Lemma 1. First, let us show that $\frac{da}{db} > 0$ which is clearly the case if $b = 0$. If $b > 0$ the condition is equivalent to $\frac{du}{db} < \frac{(1-p)}{bp'}$ since $\frac{da}{db} = (1-p)G'(\ ) - bp'G'(\ )\frac{du}{db}$. Again, we allow for a positive fixed wage w that is paid out whenever no misbehaviour is detected, even in case of low profits..

$$\frac{du}{db} = \frac{(1-p) - p'(G(\ )+u) - p'\ \ G'(\ )}{2bp' + b(G(\ )+u)p'' + wp'' + K''(u) - [bp']^2G'(\ )} < \frac{(1-p)}{bp'}$$

A sufficient condition for this inequality to hold is that

$$\frac{(1-p) - p'\ \ G'(\ )}{2bp' + b(G(\ )+u)p'' + wp'' + K''(u) - [p'b]^2G'(\ )} < \frac{(1-p)}{bp'}$$
$$\Leftrightarrow \quad 0 < bp' + K''(u) + p''b(G(\ )+u) + wp''$$

which is true by Assumptions 1 and 2.

Now we need to show that $\frac{d(a+u)}{db} > 0$, which is equivalent to $(1-p)G'(\ ) + (1 - bp'G'(\ ))\frac{du}{db} > 0$. Again, it is sufficient to consider situations where $b > 0$. From the proof of uniqueness we know that $1 - bp'G'(\ ) > 0$, so the condition can only be violated if $\frac{du}{db} < 0$. Let us hence look for a lower bound for $\frac{du}{db}$:

$$\begin{aligned}\frac{du}{db} =& \frac{(1-p) - p'(G(\ )+u) - p'\ \ G'(\ )}{2bp' + b(G(\ )+u)p'' + wp'' + K''(u) - [bp']^2G'(\ )} > \\ & -\frac{p'\ \ G'(\ )}{2bp' + b(G(\ )+u)p'' + wp'' + K''(u) - [bp']^2G'(\ )} > \\ & -\frac{(1-p)G'(\ )}{1 - bp'G'(\ )}\end{aligned} \tag{8}$$

where the first inequality follows from the fact that $(1-p) > p'(G(\ ) + \ )$ by equation (4) and the second inequality is implied by $1 - bp'G'(\ ) > 0$. Plugging (8) into our initial condition shows that indeed $\frac{d(a+u)}{db} > 0$. ☐

Proof of Proposition 2. We want to show that there is a bonus $\hat{b}$ such that $\frac{du}{db} < 0 \Leftrightarrow b > \hat{b}$ and $\frac{du}{db} > 0 \Leftrightarrow b < \hat{b}$. It can easily be seen that $\frac{du}{db}|_{b=0} > 0$. Now, let us consider the case where $b > 0$ and show that $\frac{du}{db}$ will become negative for very large bonuses. First, note that u and $\lim_{b\to\infty}(u)$ will both belong to the open interval $(0, \overline{u})$. Since the denominator of equation (6) will always be positive, we just have to show that the numerator will eventually turn negative. This in turn is equivalent to showing that $b^{(1-\alpha)}\frac{\partial F}{\partial b}$ will become negative for some $\alpha > 0$. Using equation (4) we get that for all $b > 0$

$$\frac{F}{b} = \frac{K'(u)}{b} - p' \ \ G'(\ \ )$$

Multiplying both sides by $b^{(1-\alpha)}$ and taking the limit we get

$$\lim_{b\to\infty}\left(b^{(1-\alpha)}\frac{F}{b}\right) = \lim_{b\to\infty}\frac{K'(u)}{b^{\alpha}} - \lim_{b\to\infty}\frac{p'}{(1-p)^{(1-\alpha)}}\lim_{b\to\infty} \ ^{(2-\alpha)}G'(\ \ )$$

We can see that the first term is zero and $\lim_{b\to\infty}(p'/(1-p)^2)$ will be strictly positive. Finally, by Assumption 3 we can show that $^{(2-\alpha)}G'(\ \ )$ is increasing in for $2 > 2-\alpha > f$. Since by Lemma 1 is a strictly increasing function of b the expression $^{(2-\alpha)}G'(\ \ )$ is also increasing in b. So the second term of the equation must be strictly positive and $\lim_{b\to\infty} b^{(1-\alpha)}\frac{\partial F}{\partial b} < 0$. This means that for all sufficiently large values of b we get $\frac{du}{db} < 0$.

By continuity of $\frac{du}{db}$ we know that $\frac{du}{db}$ has at least one root. In order to show that it has exactly one root, we are now going to show that it is strictly decreasing in b whenever $\frac{du}{db} \geq 0$, which is sufficient since $\frac{du}{db}$ is differentiable:

$$\frac{d\left(\frac{\partial F}{\partial b}\right)}{db} = \frac{\ ^2F}{b^2} + \frac{\ ^2F}{b \ \ u}\frac{du}{db} \tag{9}$$

Consider the first term of equation (9):

$$\frac{\ ^2F}{b^2} = -p'(1-p)G'(\ \ )\left(2 + \frac{G''(\ \ )}{G'(\ \ )}\right)$$

which is strictly negative by Assumption 3. For the second term we have

$$\frac{^2F}{b \;\; u} = -2p' - p''(G(\;\;) + u) + 2b(p')^2G'(\;\;) - \;\; p''G'(\;\;) + b(p')^2 \;\; G''(\;\;)$$

$$\Rightarrow \quad \frac{^2F}{b \;\; u} < b(p')^2G'(\;\;)\left(2 + \frac{G''(\;\;)}{G'(\;\;)}\right)$$

Using the upper bound on $\frac{du}{db}$ that we derived in the proof of Lemma 1 this implies that (9) will be strictly negative whenever $\frac{du}{db} \geq 0$. This concludes the proof.

□

Proof of Proposition 3. Recall that $\hat{b} \in (0, \infty)$. Furthermore, we have $b \to 0$ as $\Delta \to 0$ and $b \to \infty$ as $\Delta \to \infty$. In between those two extremes, the optimal incentive (and hence b) will be monotonically increasing in Δ: $\frac{d^2\Pi}{d\beta d\Delta} = G'(\;\;) > 0$. This implies that for a given there is some threshold $\hat{\Delta}$ such that $\Delta > \hat{\Delta} \Leftrightarrow b > \hat{b}$ and $\Delta < \hat{\Delta} \Leftrightarrow b < \hat{b}$. Since this is true for any the bonus distribution will always have strictly positive mass on both sides of the threshold $\hat{b}$.

If we want to compare two bonus distributions that result from different costs of misbehaviour, and $\hat{}$, then we can always find some $\hat{} \in (0, 1)$ such that for any $\leq \hat{}$ the -quantile will have $b < \hat{b}$ and the $(1 - \;)$-quantile will have $b > \hat{b}$ for both, and $\hat{}$.

An increase in the marginal cost of misbehaviour from to $\hat{}$ will leave the ranking of the bonuses set by different firms intact and so the identity of the and $(1 - \;)$-quantile will remain unaltered. Any optimal bonus will be either characterized by $\frac{d\Pi}{db} = \frac{\partial\Pi}{\partial b} + \frac{\partial\Pi}{\partial u}\frac{du}{db} = 0$ or by $b = 0$. So an increase in the marginal cost of misbehaviour will weakly reduce the -quantile bonus and strictly increase the $(1 - \;)$-quantile bonus.

□

Proof of Lemma 2. Since we can't make sure that $b_p, b \geq 0$, simply reducing the fix wage components to zero may not be possible without violating limited liability constraints. For this proof it will hence be useful to redefine the wage that is paid in state $[i, j]$ as $w_{i,j}$ where $i \in \{h, l\}$ and $j \in \{p, n\}$ denote whether high profits (h) have been made or not (l) and whether misbehaviour has been detected (p) or not (n) . The utility of the agent

is then given by

$$
\begin{aligned}
&=(a+u)\big(w_{h,n}(1-p)+w_{h,p}p\big)+(1-a-u)\big(w_{l,n}(1-p)+w_{l,p}p\big)-C(a)-K(u)\\
\frac{}{a}\ &=(1-p)w_{h,n}+pw_{h,p}-(1-p)w_{l,n}-pw_{l,p}-C'(a)=0\\
\frac{}{u}\ &=(1-p)w_{h,n}+pw_{h,p}-(1-p)w_{l,n}-pw_{l,p}-K'(u)-p'\Big((a+u)(w_{h,n}-w_{h,p})\\
&\quad+(1-a-u)(w_{l,n}-w_{l,p})\Big)=0
\end{aligned}
$$

In the general case, the expected bonus is given by $=(1-p)w_{h,n}+pw_{h,p}-(1-p)w_{l,n}-pw_{l,p}$. It can never be optimal to have < 0 since the same a and a weakly lower u can be implemented by offering a contract $(0,0,0,0)$ that has a strictly lower wage cost.

Suppose a contract $(w_{l,n},w_{h,n},w_{l,p},w_{h,p})$ implements some $a > 0$ and has $w_{l,p} > 0$ or $w_{h,p} > 0$. Instead, we can choose a contract $(\hat{w}_{l,n},\hat{w}_{h,n},0,0)$ that has $\hat{w}_{l,n}=w_{l,n}+\frac{p}{1-p}w_{l,p}$ and $\hat{w}_{h,n}=w_{h,n}+\frac{p}{1-p}w_{h,p}$ and would implement the same a if the agent were to choose the same level of u (which he is not). Now consider a contract $(\tilde{w}_{l,n},\tilde{w}_{h,n},0,0)$ that does indeed have the same expected wage payments as the initial contract conditional on profits being either high or low. Clearly, this contract implements the same a. Furthermore, assume that it has a probability of detecting misbehaviour $\tilde{p}\geq p$. In this case $(1-\tilde{p})\tilde{w}_{h,n}=(1-p)\hat{w}_{h,n}$ and $(1-\tilde{p})\tilde{w}_{l,n}=(1-p)\hat{w}_{l,n}$ implies that $\tilde{w}_{h,n}\geq\hat{w}_{h,n}$ and $\tilde{w}_{l,n}\geq\hat{w}_{l,n}$. However, this contradicts the assumption that $\tilde{p}\geq p$. So there exists a contract $(\tilde{w}_{l,n},\tilde{w}_{h,n},0,0)$ that has *i*) the same a, *ii*) a strictly lower u and *iii*) the same expected wage payments conditional on the realization of profits. This lets us conclude that any contract involving $w_{l,p} > 0$ or $w_{h,p} > 0$ is dominated. The assumption that $a > 0$ is only needed to ensure that any contract with $w_{l,p} > 0$ or $w_{h,p} > 0$ results in the agent choosing some $u > 0$.

□

Proof of Proposition 4. The principal can either adjust the bonus b or increase fixed wages w in order to increase compliance: Allowing for $w > 0$ the agent's optimal choice of u satisfies

$$-K'(u)-p'b\,(G(\ )+u)-p'w=0$$

So we can show the marginal rate of substitution between b and w which leaves u constant to be

$$\frac{dw}{db}=\frac{(1-p)}{p'}-(G(\ )+u)-\ G'(\ )$$

A contract can only be optimal if for any given u the principal can not increase profits by choosing a different combination of b and w that implements this u. If $w > 0$ (which

can only be the optimal if $b \blacktriangleright 0$) this implies that in an optimum we must have

$$
\begin{aligned}
& \frac{\Pi}{b} + \frac{\Pi}{w}\frac{dw}{db} = 0 \\
\Leftrightarrow \quad & (1-p)G'(\quad)\,(\quad - \quad - \quad) - (1-p)\,(G(\quad) + u) - (1-p)\frac{dw}{db} = 0 \\
\Leftrightarrow \quad & G'(\quad)\,(\quad - \quad) = \frac{(1-p)}{p'}
\end{aligned}
$$

□