RECIPROCAL CONTRACTING

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Abstract. We describe a set of mechanisms we refer to as *reciprocal mechanisms.* These play the same role as direct mechanisms in single mechanism designer problems in that they provide a 'canonical', though abstract, way of representing equilibrium outcomes. We use them to show that the set of outcome functions supportable as Perfect Bayesian equilibrium in regular competing mechanism games is equivalent to the set of outcome functions supportable as Perfect Bayesian equilibria in a reciprocal contracting game. We provide a full characterization of this set of outcome functions. This characterization makes it possible to by pass game theoretic complexities in order to understand the impact of competition in competing mechanism games using a set of inequalities.

1 Introduction

In many interesting environments, competition between rms invo ves much more than simp e price competition Se ers at eBay attract buyers by a owing them to bid in auctions, or to purchase at a xed buy it now price, or to choose between some combination of the two Principa s in a common agency vie for an agents attention with non- inear pricing contracts, the bene ts of which depend on exact y what inds of non- inearities exist in the contracts - ered by other principa s Groups of bidders in procurement auctions co ude in an - ort to compete both with other bidders and with the auctioneer

One comp ication in dea ing with these ind of competitive environments comes from the fact the main too norma y used to thin about contracts - the reve ation princip e - doesn t wor in competitive environments hat it means not to wor is that there are many outcomes that can be supported as equi ibrium in competing contract games that cannot be supported if contract designe

conditions a deviation by one princip e can sometimes be met by a punishment that is meted out by the common agent

This ind of ogic is compounded in a mu tip e agency for examp e Yamashita in which a principa can as his agents direct y whether or not one of his competitors has deviated from some putative equi ibrium then commit himse f to punish such a deviation If there are enough agents each of the agents wi report a deviation truthfu y simp y because he expects the other agents to

The upshot is that competing contract environments shou d norma y be expected to support mu tip e equi ibrium un ess there are severe constraints on the contracts that p ayers are a owed to er

This eads to a second comp ication since the ru es by which p ayers interact in a competing contract environment are often obscured because the co usive outcomes they support are i ega For examp e a bidding ring in an auction supports its co usive behavior by agreeing in secret to some ind of punishment Inte ectua property obbyists negotiate aws enforcing restrictive trade practices with po iticians private y for much the same reason If no one can see exact y what they are doing it is hard to now what shou d be made i ega This ma es it even harder to understand what equi ibrium outcomes oo i e since we do not even now the ru es of the game by which these contracts and outcomes are determined

The objective of this paper is to provide a way around these two obstac es e provide a set of mechanisms ca ed reciproca mechanisms and a game which is referred to as the reciproca contracting game in which p ayers compete by choosing these reciproca mechanisms e provide a fu characterization of the set of outcome functions that can be supported as wea perfect Bayesian equi ibria in this re ative y simp e competing mechanism game e then show that the set of outcome functions supportab e as equi ibrium in some *regular*[2] competing mechanism game is equiva ent to the set of outcome functions supportab e in reciproca contracting games

The advantage of this theorem is two fo d First if we are interested in questions i e whether competition wi support e ciency then we can address this by studying the impact of competition on the inequa ities that characterize equi ibrium rather than reasoning through comp ex game theoretic issues If nothing e se conc usions about the impact of competition wi then be robust to the way that competition is mode ed

Second for prob ems i e co usion and corruption in which the extensive form of the contracting game is un i e y to be understood we can sti use the inequa ities to characterize the equi ibrium outcomes that wou d be most east desirab e from socia point of view then study the behaviora properties of these outcomes

Fina y because our approach can be used to characterize equi ibria in a competing mechanism games it wi a so characterize equi ibrium outcomes when p ayers abi ity to commit is restricted For examp e in a common agency the agent can commit to trade with a particu ar se er for examp e but can t ma e this commitment contingent on any messages at a From our main theorem the set of outcome functions supportab e under there restrictions must be contained in the set characterized here For some of these restrictions the approach here ma es it easy

[2]A regular competing mechanism game is a game in which the introduction of contracts does not exogenously eliminate actions that are available as primitives to players.

to see what additional constraints are needed to understand the new set of equilibrium. We briefly discuss these additional restrictions for problems where some players can't commit at all. Indeed, if no players can commit, it is easy enough to show that the set of outcomes supported by our characterization is simply the set of communications equilibrium (Forges 1986).

We begin by discussing an example that illustrates the approach. We then provided the main theorems in the paper before we return to discuss the relationship with other papers in the literature.

Example

Since modeling collusion is one problem for which the approach is likely to be useful, we consider a simple example in which buyers and sellers bid in a double auction. At this point we are simply interested in whether sellers can use reciprocal contracts to enforce what for them is a collusive outcome.

In this story, there are two sellers and two buyers, i.e. four players in all. Each seller has a single unit of output to which he or she assigns a value of 0. Each buyer has a private valuation, either v_l or v_h ranked the obvious way with $0 < v_l < v_h$. Payoffs to the seller are equal to the money he receives while payoffs to each buyer are equal to their private valuation when they succeed in trading less the money they pay. We assume that valuations are correlated. To make life simple suppose that both valuations are the same with probability $q > \frac{1}{2}$ and that they are equally likely to both be v_h or v_l in that case.

In the double auction that guides the interaction between them, players submit bids. The two available goods are awarded to the two highest bidders at a price equal to the third highest bid with the proviso that if there are more than two highest bidders, then the good is awarded to buyers whenever possible and randomly otherwise. There is a continuum of ex post efficient Bayesian equilibrium outcomes for this game in which all bidders bid $q \in [0, v_l]$ independent of type. The best that sellers can do in any of these equilibrium outcomes is a payoff of v_l, in which case, high value buyers earn $v_h - v_l$ and low valuation buyers earn nothing. In all of these equilibrium outcomes trade occurs for sure.

We are interested in whether there is some kind of collusive outcome in which sellers do better than they do in any of these efficient equilibria. What makes this a conceptually challenging problem is the fact that it is hard to know how sellers would negotiate such an agreement and how they would enforce it.

Whatever this collusive mechanism is, it must ultimately be inefficient. The sellers must restrict their supply somehow in order to extract some of the high value buyer's information rent. Whatever agreement the sellers reach will ultimately determine a trading price in the auction for the three different informational outcomes - both high value, both low value, different values. The argument we are trying to make is that in order to understand what sellers could do, we don't need to model the collusion directly. Instead, the best they can do can be understood by maximizing payoffs subject to a set of inequalities.

To illustrate, suppose the three prices are p_{hh}, p_{ll} and p_{hl}. The sellers have to find a way to share the trading responsibility and to make the trading outcomes incentive compatible. The scheme we implement will have one of the sellers submit a high bid which will result in a trade only if both buyers have high value. The other seller will submit a lower bid that will guarantee him a trade and determine

price when some buyers have

each other type reports and correlating messages in the second round. On the equilibrium path there is also a third round in which players play a cheap talk game. This third round is trivial in the example, so we defer a more complete discussion. The correlating messages are numbers between 0 and 1. The public commitments are messages taken from a measurable set which is to be described. Each message is tied to a very specific commitment which depends on the public messages and the messages that each player privately sees at the end of the second stage.

The actions A_i for each player are bids q_i in some set. To keep a bound on notation, we assume this set is an interval, though in the main body of the paper it is assumed that actions are taken from a finite set. The messages used by players in the first round are proposals about how each player should play, and what each player should do in case there is a disagreement. The rules of the reciprocal contracting game tie each proposal to a very specific commitment. Formally, proposals are lists of direct mechanisms. To describe these, we describe mechanisms that players might use to implement the collusive outcome above.

Second stage reports are all private. The reason for private communication is that we might not want a defecting buyer or seller to know what these type reports are. However, it is important in this story that players make commitments based on messages they send. It is fundamental to mechanism design that messages be verifiable so that commitments can be based on them. This idea is at the heart of inscrutability (Myerson 1983) which allows an informed principal to hide his type information by making commitments based on a type report that he sends after agents have agreed to participate in his mechanism. One difference between the competing mechanisms framework and the usual one is that after the principal sends this message about his type, his agents may use his message as part of their own mechanisms. We are going to adopt the simple assumption that the principal can and does commit himself to send the same message to all of his agents.

Let D_i be the set of measurable mappings from 4-tuples of correlating messages in $[0,1]$ and pairs of type reports into the set of bids. Let $\delta_i = \left\{d_i, \{p_j\}_{j\neq i}\right\}$ be a list of 4 direct mechanisms for player i, with the interpretation that d_i is the mechanism i will use when there is an agreement, while p_j is the mechanism that i will use when player j refuses to participate in the agreement. Let $\delta = \{\delta_i\}_{i=1,\dots,4}$ with Δ the set of all δ. The set Δ represents the set of proposals about how the double auction should be played.

Each proposal that the player makes in the first stage commits the player to a particular action that depends on the proposals of the other players. Specifically, if $\left\{\delta^i\right\}_{i=1,\dots,4}$ is a collection of proposals, then the commitment associated with these proposals for player i is assumed to be

$$\lambda^i\left(\delta^1,\delta^2,\delta^3,\delta^4\right) =$$

$$\begin{cases} d_i & \delta^1=\delta^2=\delta^3=\delta^4 \\ p_j & \delta^j\neq\delta^i,\ \delta^i=\delta^{i'}\forall i'\neq j \\ a_i\in A_i & \text{otherwise} \end{cases}$$

In words, this says that if player i makes a proposal δ^i and the other three proposals agree with his, then i is committed to carry out his part of the proposal. If 2 of the others' proposals agree with his, but there is a single player who disagrees, then i

is committed to carry out his punishment against that p ayer In a other cases, i is simp y a owed to choose whatever action he i es in the third stage

We can now exp ain the mechanisms that support the co usive outcome described above Let $x = \{x_1, x_2, x_3, x_4\}$ be the vector of corre ating messages For examp e, for se er 1, the corre ating message x_1 is the message that he sends to each of the other p ayers As mentioned above, we assume his mechanism commits him to send the same message to each of the other p ayers The message x_2 is what he receives from se er De ne $\gamma\ x\ = \lfloor \sum_i x_1 \rfloor$, i e, the fractiona part of the sum of the corre ating messages For se er 1, de ne the direct mechanism

$$d_1^*\ v, x\ =$$

$$\begin{cases} p_h^* & \gamma\ x\ > \frac{1}{2} \\ v_l & \gamma\ x\ \leq \frac{1}{2} \end{cases}$$

whi e for se er the mechanism

$$d_2^*\ v, x\ =$$

$$\begin{cases} p_h^* & \gamma\ x\ \leq \frac{1}{2} \\ v_l & \gamma\ x\ > \frac{1}{2}. \end{cases}$$

For buyers, de ne direct mechanisms

$$d_i^*\ v, x\ =$$

$$\begin{cases} p_h^* & v_i = v_h \\ v_l & v_i = v_l \end{cases}$$

For punishments, de ne the mechanism

$$\rho_b^*\ v, x\ = v_h$$

when the deviating p ayer j is a buyer, and $\rho_s^*\ v, x\ =$ when the deviating p ayer is a se er

Our c aim is that there is a perfect Bayesian equi ibrium for the reciproca contracting game in which each p ayer ma es the proposa

$$1 \qquad \{\ d_1^*, \rho_s^*, \rho_b^*, \rho_b^*\ ,\ d_2^*, \rho_s^*, \rho_b^*, \rho_b^*\ ,\ d_3^*, \rho_s^*, \rho_s^*, \rho_b^*\ ,\ d_4^*, \rho_s^*, \rho_s^*, \rho_b^*\ \}.$$

The outcome function supported by this equi ibrium is the co usive outcome described above

To see why, notice that if each p ayer ma es this announcement at the rst stage, then each of them is committed to use the mechanism d_i^* to determine their na bid Suppose that each of the p ayers is expected to choose his or her corre ating message uniform y from the interva $[\ , 1]$ and that buyers are expected to report their types to each mechanism truthfu y Since a p ayers commit themse ves to send the same corre ating message to each of the other p ayers, each of the p ayers observes the same va ue $\gamma\ x$ - just as if it were a pub ic signa If the corre ating messages are a uniform, $\gamma\ x$ wi be uniform This device wi ensure that ha f the time se er 1 sets a ow price v_l and trades no matter what the buyer va uations, whi e se er sets a high price p_h^* and trades on y if both buyers va ues are high

The transformation γ has the property that if each of the p ayers chooses his corre ating message uniform y then $\gamma\left(x_i + \sum_{j\neq i} x_j\right)$ is uniform y distributed independent on x_i [3] So it is sequentia y rationa to choose a corre ating message using a uniform distribution

Buyers need to carry out their part of the bargain which commits them to bid p_h^* when they have high types and v_l when their types are ow The consequence is that there are three bids at p_h^* when both buyers have high va ues and both se ers trade at that price If one of the buyers has a ow va ue then the se er who bids v_l trades with the high va ue bidders at price v_l Fina y when both buyers have ow va ues the se er who bid v_l trades at that price

Notice that this is not part of an equi ibrium in the bidding game - buyers are committed to bid p_h^* despite the fact that they rea ize they cou d ower the trading price by bidding ess Their contracts compe them to ma e this bid To to chec sequentia rationa ity it is on y necessary to chec that a high type bidder wou d rather bid p_h^* than to bid v_l By de nition p_h^* is the highest price that has this property so incentive compatibi ity is bui t in by design

The consequence of deviating and announcing some other proposa in the rst stage is to commit the others to a punishment that ma es it impossib e to earn surp us in the doub e auction If a buyer deviates the others a bid v_h If a se er deviates the others a bid

The upshot is that each p ayer is better proposing than they are ma ing some proposa because doing so resu ts in the other p ayers punishing them much in the manner of a repeated game The appea of this extensive form is two fo d - rst it ma es the competing mechanism ogic trivia This ought to ma e it much easier to understand how various contracting restrictions wor because the game theoretic ogic is minima It i ustrates the set of of outcome functions that can be supported in some competing mechanism game However most important the set of outcome functions that can be supported coincides exact y with the set of outcome functions that can be supported in some competing mechanism game e turn to the argument for the genera case

3 Incomplete Information Games and Mechanism Design

The basic approach in what fo ows is to add the contracting game on top of a basic game of incomp ete information In this basic game there are n p ayers Each p ayer has a nite action set A_i and a nite type set T_i In standard notation A A_{-i} represent cross product spaces representing a p ayers actions and the actions of a the p ayers other than i respective y Simi ar y de ne $T = \prod_i T_i$ and $T_{-i} = \prod_{j\neq i} T_j$ Types are joint y distributed on T according to some common prior

Let q be a mixture over the set of action pro es A The notation Q is used to represent the set of a such mixtures For any action pro e a we write q_a to be the probabi ity of a under q and $q_{a_i} = \sum_{a_{-i}} q_{a_i,a_{-i}}$ e use notation q_{A_i} to represent the margina distribution over A_i and $q_{A_{-i}}$ to be the margina distribution over A_{-i} e assume that p ayers have expected uti ity preferences over otteries Then p ayers preferences are given by $u_i \quad Q \times T \to \mathbb{R}$ where u_i is inear in q An *outcome*

[3] See (Peters and Troncoso-Valverde 2009), who develop the idea from (A.T Kalai and Samet 2010).

function is a mapping $\omega : T \to Q$. So player i's payoff from this outcome function is $\mathbb{E}\{u_i(\omega(t), t)|t_i\}$.

One way to resolve this game is to have a mechanism designer collect information from the players, then tell each player what action to take. We think of the mechanism designer as an enforcer here, not just a coordinator - players have to carry out the action the mechanism designer tells them to whether they want to or not.

Obviously, the mechanism designer can only implement an outcome function ω if it is *incentive compatible.* Specifically, for every i, t_i and t'_i,

$$(3.1) \qquad \mathbb{E}\{u_i(\omega(t), t)|t_i\} \geq \mathbb{E}\{u_i(\omega(t'_i, t_{-i}), t)|t_i\}.$$

Incentive compatibility as defined above is completely standard so there is no need to discuss it further. On the other hand, a player cannot be coerced into participating in this mechanism - he has to agree at the interim stage to be bound by the mechanism designer's ex post recommendation. What makes this problem somewhat complex is the fact that the payoff to a player who chooses not to participate is not exogenous since he can still choose whatever action he wants. In that event, we allow the mechanism designer to implement a punishment that includes a recommendation to the non-participating player about which action he should take. Whatever this punishment is, the players should still want to report their types truthfully, while the non-participating player should want to carry out the recommendation of the mechanism designer after updating his beliefs conditional on receiving the recommendation.

Let $\rho_i : T \to Q$ be an outcome function that is implemented when player i chooses not to participate in the mechanism that implements ω. We refer to this outcome function as a *punishment.* The outcome function ω is *individually rational* if there is a collection of punishments $\{\rho_i\}_{i=1,n}$ such that for every player i,

$$\mathbb{E}\{u_j(\omega(t), t)|t_i\} \geq$$

$$\mathbb{E}\{u_i(\rho_i(t), t)|t_i\} \geq$$

$$(3.2) \qquad \max_{t'_i} \sum_{\tilde{a}_i} \left\{ \max_{a_i} \mathbb{E}\left\{u_i\left(a_i, \rho_{A_{-i}}(t'_i, t_{i-i}), t\right)|t_i, \tilde{a}_i, t'\right\} \right\} \mathbb{E}\{\rho_{\tilde{a}_i}(t'_i, t_{-i})|t_i\}$$

and for each player other than i, there is some belief about distribution of player types such that

$$(3.3) \qquad \mathbb{E}\{u_j(\rho_i(t_j, t_{-j}), t)|t_j\} \geq \mathbb{E}\left\{u_j\left(\rho_i\left(t'_j, t_{-j}\right), t\right)|t_j\right\}.$$

What follows shows that an outcome function ω is supportable as a weak perfect Bayesian equilibrium in *some* competing mechanism game if and only if there is a collection of punishments and beliefs such that (3.1), (3.2) and (3.3) hold. There are two parts to this. First, beginning with an outcome function that satisfies these constraints, we need to construct a competing mechanism game which can be used to support the outcome function as an equilibrium. We have already described this game, it is the reciprocal contracting game described above. Notice that the reciprocal contracting game plays the role of a 'direct mechanism' in the usual revelation principle. General competing mechanism games are analogous to indirect mechanisms.

Most of the work in the rest of the paper is devoted to explaining exactly how the reciprocal contracting game accomplishes this. This borrows a number of methods that are probably unfamiliar, so we break them up a bit in the discussion that follows to explain heuristically how they work. They are combined in the proof of the main theorem which appears in the appendix.

The other part of the proof is to show why competing mechanism games have enough structure to ensure that the constraints given above are all satisfied in equilibrium. We defer this discussion until later in the paper.

3.1. **Reciprocal Mechanisms.** The reciprocal contracting game mimics the classic competing auction game in which players simultaneously announce commitments to direct mechanisms, then report their types to each other. However, the commitment phase of the game may not fully determine the actions of the players. In the competing auction game, for example, buyers are still free to choose which seller to visit. More important here, a deviating player might deliberately want to delay his action choice. To deal with this we add a final stage in which players whose actions are not fully committed play a cheap talk game which determines the final outcome. The basic logic is that players attempt to come to a reciprocal agreement about how to play the game. The cheap talk game determines the outcome when they cannot find such an agreement. To capture all this, the reciprocal contracting game involves three parts - commitments, type reports and the cheap talk game.

To make commitments, the players simultaneous make proposals. The proposal is taken from a common message space $\triangle$ for each player. We describe these proposals and the commitments that are attached to them momentarily, though they are simply a generalization of the proposals described above.

In the reporting part of the game, each player i privately sends a signal in the set $[T_i \times [0,1]]$ to each of the other players. The first element of this signal is a type report, the second is a correlating message used to support randomization. We refer to this message as a 'type report', though the player reports more than just type. In particular, for reasons that will be apparent momentarily, we focus in what follows on situations in which the player chooses his correlating message randomly using a uniform distribution on $[0,1]$. To keep things conceptually simple, we say that a player 'reports his type truthfully' if he announces his true type *and* chooses a correlating message uniformly.

In the cheap talk game, players tell each other what messages they heard in the reporting stage, then based on what they hear from the others, they recommend an action to any player whose actions are not yet fully committed. We refer to the reports about other players type reports in the reporting stage as *recollections.* Of course, the advice that players give other players are just called recommendations. Formally, the messages available to player i in the cheap talk stage are taken from a finite set $M_i = T_{-i} \times A_{-i}$ - just the product of his recollections and his recommendations. Once the cheap talk messages have all been sent, any player who has not yet fully committed himself chooses an action. Since i sends a message in M_i to every other player in the cheap talk stage, the action space for every 'uncommitted' player in the cheap talk game is $\triangle \times [T_i \times [0,1]]^{n-1} \times M_i^{n-1} \times A_i$.

We need first to define the proposals in $\triangle$. Define $\mathcal{T}_i \equiv [T_i \times [0,1]]^{n-1} \times \prod_{j \neq i} [T_j \times [0,1]]$. This is the set of type reports that player i observes at the end of the reporting stage. This is all the type reports i sent to other players, as well as all the type reports he received from other players. Let *Divvt*

measurable mappings $d_i : T \to A_i$. We refer to each of these mappings as a *direct mechanism.*

There are two differences between these mechanisms and more conventional direct mechanisms. First, a fundamental part of mechanism design with informed principals is Myerson's inscrutability principle (Myerson 1983) in which a principal is allowed to choose a mechanism (from a set specified beforehand) at the same instant that his agents send their type reports. This allows the principal to hide his type from agents at the point where they submit their type reports to him. We are going to give players the same option here. In the competing mechanism environment, however, players may want to hide the mechanism they are using from some other players. We let them do this formally by allowing them to condition their actions on the messages they *send* to other players. In particular, if a direct mechanism commits to an arbitrary action unless the message the player sends to each of the other player is the same, then we refer to the mechanism as a *homogeneous* direct mechanism. A direct mechanism that commits to send the same message only to some subset of the other players will be referred to as a partially homogeneous direct mechanism. Observe that committing to send the same message to a group of players is not the same thing as making the messages public, precisely because players can limit the set of the opponents who see the same message.[4]

The second sense in which these mechanism differ from standard direct mechanisms is that they allow players to send one another (correlating) messages. The role of such messages was first pointed out by Peck (1995). Apart from the fact that such messages can be used to support correlation, they will also make it possible to implement what are effectively random mechanisms while still writing mechanisms as mappings from messages into actions (rather than mixtures over actions).

A *punishment mechanism* p_i^j for player i to use against player j is a partially homogeneous direct mechanism in which i's action is independent of player j's type, and in which i commits to an arbitrary action unless the signals he sends to all players other than j coincide.

We can now describe the set of commitment messages $\triangle$ along with the commitments associated with each of them. Let $\delta_i = \left\{ d_i, \left\{ p_i^j \right\}_{j \neq i} \right\}$ be a list consisting of a homogeneous direct mechanism along with $n-1$ punishment mechanisms for player i to use against each of the other players. Heuristically, the first mechanism in this list is a 'cooperative' mechanism that player i will use when the other players are cooperative. The punishment mechanisms represents what i will do when each of the other players unilaterally refuses to cooperate. The notation $\delta = \{\delta_1, \ldots, \delta_n\}$ represents a list of such lists, one for each of the n players. Δ is the set of all such δ.

The function $\lambda_i : \Delta^n \to D_i$ describes the commitment associated with each array of signals. In what follows, the functions d_i and p_i use second round signals as their arguments. In that case, we can abuse notation slightly and write the commitments

[4]Myerson's inscrutable mechanisms implicitly assumes that the principal 'sends' the same message about his type to every agent. What is different here is that agents can also make commitments that are allowed to depend on this common message. It is possible to design mechanisms that check that a principal has sent the same message about his type to each of the other players. The way this is done is to have each of the players tell each other what messages they heard from the other players, and to commit themselves to arbitrary actions unless these reports agree. The details are in (Peters and Troncoso-Valverde 2009).

associated with each array of rst round signa s as

$$3 \qquad \lambda_i \; \delta_i, \delta_{-i} \; = \begin{cases} d_i & \exists \delta^* \;\; \delta_j = \delta^* \forall j \\ p_i^j & \exists \delta^* \;\; \exists \, j \;\; \delta_j \neq \delta^* \wedge j \neq i \\ a_i \in A_i & \text{otherwise} \end{cases}$$

In this notation p_j^i and d_i represent the corresponding e ements of δ_i and the notation ∃ means there exists a unique

By 3 , each array of reciproca mechanisms announced by the p ayers in the rst stage pub ic y commits that p ayer to either a homogeneous direct mechanism a partia y homogeneous punishment mechanism or a ows the p ayer to choose whatever action he i es after istening to cheap ta   messages If the commitments a   agree a   the p ayers can do according to the rst ine of 3 , is to send type reports to the other p ayers These reports fu y determine a   the actions so the cheap ta   game at the end is irre evant If there is a sing e dissenting p ayer second ine on the right hand side of 3 , that p ayer simp y istens to the cheap ta   messages then ta es an action The non-dissenting p ayers send one another type reports send their reco ections to each other then ma e recommendations to uncommitted p ayers In every other history p ayers p ay a cheap ta   game in the third and fourth rounds A wea Perfect Bayesian equi ibrium of this Bayesian game is de ned in the usua way

This forma ism now ma es it possib e to state the rst theorem

Theorem 1. *If there are four or more players and ω is an outcome function satisfying (3.1), (3.2) and (3.3), then there is a weak Perfect Bayesian equilibrium in the reciprocal contracting game that supports ω. Furthermore, along the equilibrium path of this game, all players announce a common signal δ^* in the first stage, report their types truthfully, and choose a correlating message uniformly from the interval* [, 1] *in the second stage.*

The fu proof is contained in Section 1 The important part and more di cu t part of this theorem is the part that shows how to support outcome functions ω that satisfy the restrictions given by 3 1 , 3   and 3 3 as equi ibria The essentia ogic is straightforward - if a   the other p ayers match a commitment δ then a   of them are ob iged to carry it out If some p ayer decides to deviate then the others imp ement a punishment The outcome function ω a ong with the punishments $\{\rho_i\}$ and inequa ities provided by the centra ized mechanism provide the basic bui ding b oc s that are used to construct the strategies in the reciproca contracting game

There are two basic comp ications The rst stems from the fact that the outcome function ω t invo ves a joint randomization possib y invo ving corre ation over the actions of a   the p ayers whi e any commitment δ by p ayer i on y commits him to a randomization over his own actions This is where the corre ating messages are used   e adopt a method from A T Ka ai and Samet   1   and a s ight genera ization of it in Peters and Troncoso- a verde   which converts the private corre ating messages into something that wor s i e a pub ic randomizing device that the p ayers cannot manipu ate Once we have created this device detai s are in the proof , it is straightforward to construct the contracts that imp ement ω and its various punishments ρ_j

The second comp ication stems from the fact that when the mechanism designer punishes a p ayer who refuses to participate he might need to send the p ayer an

informative recommendation. Since there is no centralized mechanism designer in the reciprocal contracting game, we need to find a way to induce the punishing players to send the right recommendations on their own. These recommendations have to depend on the types of all the players. At the same time, there cannot be any incentive for the players who are communicating these recommendations to manipulate them. We accomplish this by having the deviating ignore recommendations from the others unless they agree. This is where the assumption that there are four or more players is used. The deviator will hear at least three recommendations, and will follow them provided at least two of them agree. At the point where the punishing players send their recommendations, they think that they know the types of all the other punishing players. The reason they don't manipulate their recommendation is that they anticipate a very specific type contingent recommendation from the other punishing players and believe their recommendation will be ignored if it doesn't match. Then continuation play supports an outcome in which the punishing players send the same recommendations because each of them expects the others to send that recommendation.

The theorem is stated here for weak Perfect Bayesian equilibrium. It is possible to modify the story in a fairly straightforward was so that beliefs off the equilibrium path are consistent in the sense of sequential equilibrium. We restrict to arbitrary off path beliefs here because the reciprocal contracting game isn't finite, so we are able to avoid a significant technical discussion about how to define sequential equilibrium in continuous games.[5]

3. **Competing Mechanism Games.** The reciprocal contracting game is but one example of a large set of potential games. It has a number of properties that seem unusual relative to the standard literature on common agency and competing auctions. For example, all players are allowed to communicate and to commit. In a common agency, for example, principals can't communicate with each other, and the agent has no commitment ability. In a competing auction game, bidders aren't allowed to send correlating messages. In a problem with collusion, the grand mechanism designer is competing with a group of colluding players who often go to great pains to hide the method they are using to organize. Papers like Yamashita or Peters and Troncoso-Valverde show how the set of supportable outcomes explodes when players are allowed to communicate messages more complex than their payoff types. Sequential commitments support different outcomes than simultaneous commitments (Pavan and Calzolari).

This vast array of modeling choices makes it difficult to come to specific conclusions about the impact of competition. This is one reason that characterizing the set of outcomes supportable as equilibrium is useful, since it is possible to sidestep the complicated game theoretic details. Of course, restricting players' ability to contract will impose restrictions on what can be supported in equilibrium. We return to this issue below. At this point, what we want to do is to show is that

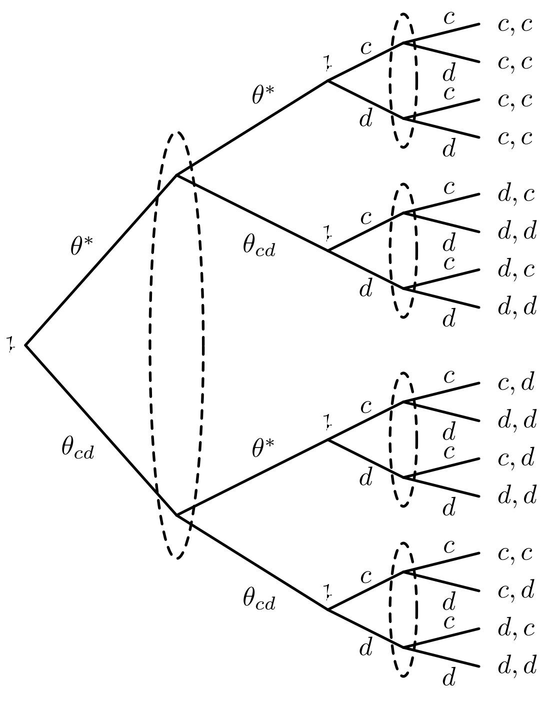
[5]The only part of reciprocal contracts that isn't finite is the correlating messages. However, if mechanisms are Lebesque measurable functions of correlating messages, then the set of mechanisms is actually fairly simple. The functions map into a finite set of actions. If the functions are required to be Lebesque measurable, then they can all be characterized by a finite set of number in $[0, 1]$ where the functions jump from one action profile to another.

whatever restrictions seems appropriate in a particu ar app ication, the equi ibrium outcome functions supported in the app ication can a so be supported with reciproca contracts

The main comp ication in doing this is to try to describe in some fair y genera way what a competing mechanism game is This is hard because there are so many di erent ways to approach competing mechanisms The best nown variants of this come from the competing auction iterature for examp e, Epstein and Peters , Yamashita or Peters and Troncoso- a verde or the iterature on common agency Pavan and Ca zo ari or Martimort and Sto e or Bernheim and hinston 86 in which mechanism designers simu taneous y o er mechanisms which ma e commitments based on a speci c group of p ayers ca ed agents However a usefu description shou d a so capture mode s in which mechanisms are o ered sequentia y as in Pavan and Ca zo ari or private y as in Sega and hinston 3

Rather than trying to deve op this tedious forma ism, we ta e a s ight y di erent approach here e interpret a competing mechanism game an extensive form game of incomp ete information e interpret the nodes of this game as opportunities for p ayers to send messages A path through the game is an ordered sequence of messages Some of these messages convey commitments, some type information, whi e some are just cheap ta In order to interpret the messages, we use an outcome function λ which assigns a pro e of actions to each path through the game tree The pro e of actions indirect y determines each p ayer s pay o

The picture that fo ows shows the reciproca contracting version of a simp e prisoner s di emma game with the cheap ta part eft out to ma e it simp er



In this game, p ayers announce pub ic messages representing commitments over two rounds In the rst round, each p ayer can o er a contract that conditions

directly on the other player's contract. This is the contract θ^* in the picture. The alternative is a contract θ_{cd} that allows the player to defer his choice until the second round. The outcome function λ is displayed on the far right of the picture. Notice that if player 1 announces the message θ^* in his first information set, then the outcome function forces him to use action c in every history in which player 2 uses message θ^*, and to use action d in every other history following that choice. So from the outcome function λ the interpretation of the message θ^* is that it is a reciprocal contract that commits player 1 to use action c if player to sends signal θ^*, and to use d otherwise.

Any set of behavioral strategies specify a possibly random path consisting of a sequence of messages. The outcome function λ converts every sequence of messages into a profile of actions for the players. Player i's payoff in the history in which players send the sequence of messages m is given by $u_i(\lambda(m), t)$. In the extensive form version of the reciprocal contracting game pictured above, the history of messages $\{\theta^*, \theta^*, d, d\}$ supports the profile of actions $\{c, c\}$.

Let $\{\sigma_i, b_i\}_{i=1,\ldots n}$ be behavioral strategies and beliefs for the players specifying mixtures over messages available to players in each of their information sets, and beliefs about the history of play prior to the information set. Let ι be an information set for player i. The continuation game associated with ι is the extensive form game of incomplete information in which each player's type is his payoff type from the original game along with his information about the history of play prior to ι. Beliefs for player i in this continuation game are given by $b_i(\iota)$. For every other player j, the player's type t_j in the continuation game describes (among other things) the most recent information set ι_j in which he sent a message. So player j's belief in the continuation game are his beliefs in the information set ι_j. Associated with each history $h \in \iota$, there is an outcome function that describes type contingent mixtures over action profiles when all players use the continuation strategies associated with $\{\sigma_i, \sigma_{-i}\}$ from the information set ι onward. Using i's beliefs in the information set ι, we write $\rho(t_i, t_{-i}|\sigma_i, \sigma_{-i}, \iota)$ as the outcome function conditional on attaining this information set when players are using the continuation strategies associated with (σ_i, σ_{-i}).

Given an array of behavioral strategies $\{\sigma_i, \sigma_{-i}\}$, a collection of information sets $\mathcal{I}$ is *attainable with probability* π by player i in the continuation game associated with ι if there is a continuation strategy for i at ι such that an information set in $\mathcal{I}$ is reached with probability at least π given i's beliefs $b_i(\iota)$ and the continuation strategies σ_{-i}.

An information set ι for player i has the *no-commitment* property if (i) the outcome function $\rho_{A_{-i}}(t_i, t_{-i}|\sigma_i', \sigma_{-i}, \iota)$ is independent of σ_i' and (ii) for each $a_i \in A_i$, there is a strategy σ_i' such that $\rho_{A_i}(t_i, t_{-i}|\sigma_i', \sigma_{-i})$ assigns probability 1 to the action a_i. In words, a no-commitment information set is one in which i can carry out any action he likes without changing the behavior of the other players. We say that a player i is uncommitted in information set ι if he has a continuation strategy that attains an information set having the no-commitment property with probability 1.

Definition 2. A contracting game is said to be *regular* if for every profile σ of strategies, each player i has a strategy σ_i' that attains some no-commitment information set with probability 1.[6]

[6]This definition is inspired by a similar assumption in (Peters and Szentes 2008).

This restriction is imposed because we are interested in adding contracts that enhance players' strategy sets, not in contracting games that impose arbitrary restrictions on what players can do. For example, consider the complete information game of matching pennies (with payoffs 1 and −1). This game has a unique Nash equilibrium in which both players' payoff is zero, which makes it pretty predictive by game theoretic standards. We already know we could change the outcome of this game by removing actions, or changing timing. We want to know whether contracts that both players would want to use might change the set of equilibrium outcomes for the game.

Suppose we specify the following contracting game: player 2 is allowed to choose one of two contracts. The first commits him to tails, the second to heads. We now allow player 1, still moving simultaneously with player 2, to commit himself in a manner that depends on the commitment made by player 2.[7] The only equilibrium would then have player 1 committing to match (or mismatch) the commitment of player 2. Payoffs would then be 1 for player 1 and [illegible] for player 2.

In this example, we would say equilibrium strategies are not regular for player 2. If player 1 is using his equilibrium strategy, then there are no strategies available to player 2 that allow him to change actions without simultaneously changing player 1's response. The contracting game we build on top of the matching pennies is simply depriving player 2 of the ability to select his action simultaneously with player 1.

It is possible to use the methods we describe below to analyze irregular games. For example, in the matching pennies example, if the asymmetric contract structure seems the right one for some reason, then we could analyze it by changing the original game from matching pennies to sequential matching pennies. The contracting game would then be regular with respect to this sequential game.

The Equivalence of Competing Mechanisms and Reciprocal Contracting

We can now state the main theorem.

Theorem 3. *Suppose the outcome function ω can be supported as a weak Perfect Bayesian equilibrium in a regular contracting game. Then there is a collection of punishments $\{\rho_i\}_{i=1,\dots,n}$ such that (3.1), (3.2) and (3.3) hold.*

Proof. Let $\omega(t)$ be the outcome function supported by some equilibrium of a regular competing mechanism game in which strategies are σ^*. It satisfies (3.1) by the usual revelation principle.

The game is regular, so i has a behavioral strategy σ_i' that attains a no-commitment information set with probability 1. Write

$$\rho(t|\sigma_i') \equiv \mathbb{E}_\iota\{\rho(t|\sigma^*,\iota(\cdot|t,\sigma_i'))\}. \tag{4.1}$$

In words, ρ is the outcome function that prevails when player i uses the behavioral strategy σ_i' then reverts to σ_i^* once a no-commitment information set is attained.

The payoff associated with σ_i' is

$$\mathbb{E}\{u_i(\rho(t|\sigma_i'),t)|t_i\}.$$

[7] This is how the meet the competition argument works.

A p ayer with type t_i can mimic the behavior of a p ayer of type t_i' by adopting the same mixture over feasib e messages in each of his information sets as the type t_i' p ayer does in each of his corresponding information sets Modifying the behaviora strategy in this way provides a new behaviora strategy that attains a no-commitment information set with probabi ity The pay to p ayer a p ayer of type t who does this is

$$\mathbb{E}\{u_i \ \rho \ \ t_i', t_{-i} \ |\sigma_i' \ , \ t_i, t_{-i} \ \ |t_i\}.$$

Furthermore, once this new behaviora strategy reaches a no-commitment information set, we can modify the strategy again by having the p ayer with type t_i adopt his origina strategy σ^* from that information set on So if i has a behaviora strategy that attains a no-commitment information set with probabi ity , then there must be a strategy such that

$$\mathbb{E}\{u_i \ \rho \ t|\sigma_i' \ ,t \ |t_i\} =$$

$$\sum_{A_i} \mathbb{E}\left\{u_i\left(a_i, \rho_{A_{-i}} \ t|\sigma_i' \ ,t\right)|t_i, a_i\right\} \mathbb{E}\{\rho_{a_i} \ t|\sigma_i' \ |t_i\} \geq$$

$$\max_{t_i \in T_i} \sum_{A_i} \max_{a_{i'} \in A_i} \mathbb{E}\left\{u_i\left(a_i', \rho_{A_{-i}} \ \ t_i', t_{-i} \ |\sigma_i' \ ,t\right)|t_i, a_i\right\} \mathbb{E}\{\rho_{a_i} \ \ t_i', t_{-i} \ |\sigma_i' \ |t_i\}.$$

The equa ity fo ows from the aw of iterated expectations and the fact that the joint distribution of actions of the other p ayers is independent of a_i in every no-commitment information set The inequa ity fo ows from the fact that σ_i' attains a no-commitment information set with probabi ity This veri es that the punishment $\rho \ \cdot|\sigma_i'$ satis es 3

The punishment $\rho \ \cdot|\sigma_i'$ satis es 3 3 for the p ayers other than i because of the fact that each such p ayer is using behaviora strategy σ^*, which is sequentia y rationa as part of a wea perfect Bayesian equi ibrium □

Combining this theorem with Theorem gives the fo owing coro ary

Theorem 4. *If an outcome function ω is supportable as an equilibrium in a regular competing mechanism game with four or more players, then it is supportable as an equilibrium in the reciprocal contracting game.*

The basic ogic of reciproca mechanisms is quite simp e - competing mechanisms are comp ex, but u timate y it is possib e to understand quite a bit about them by using we understood ogic that oo s much i e the ogic in repeated games As with the iterature on repeated games, this means that many things can be supported as equi ibrium outcomes It is important to understand that there are two distinct reasons for mu tip icity here As a ways, any particu ar competing mechanism game can have many equi ibrium outcomes For examp e, the reciproca contracting game we described above has a arge number of equi ibrium outcomes

However, there are a so many di erent ways to mode competing mechanisms Each mode can have many equi ibrium outcomes The reciproca contracting game described above can be used to understand a these outcomes This is ana ogous to the fact that there are many di erent incentive compatib

properties that cou d be used to identify co usion Furthermore the reciproca contracting game provides a convenient contracting game ana ogous to a direct mechanism in the usua reve ation story that can be used to thin about strategic issues

One examp e of a regu ar contracting game is any cheap ta extension of the basic Bayesian game described in Section 3 that does not a ow p ayers any additiona commitment abi ity In any such game p ayers are uncommitted in every information set The set of outcome functions supportab e as equi ibrium in such a game is just the set of communications equi ibrium Forges 1 86 of the origina game and can be described forma y by setting $\rho_i = \omega$ for each p ayer in 3 and 3 3 Any communications equi ibrium outcome is supported as an equi ibrium in the reciproca contracting game in the obvious way since $\rho_i = \omega$ for each i

Constraints on Contracting

As we have mentioned most competing mechanism mode s ma e very speci c assumptions about what can and can t be contracted on As we have shown the outcome functions supportab e as wea perfect Bayesian equi ibrium in such game must be contained in the set of outcome functions supportab e as equi ibrium in a reciproca contracting game This suggests that constraints on contracting can be trans ated into constraints on the set of supportab e outcomes

To put it another way no indirect contracting game can support a bigger set of outcomes than the reciproca contracting game un ess it somehow modi es the strategic position of some p ayer in the origina game In this sense the reciproca contracting game is ana ogous to a comp ete mar ets mode where in the contracting sense everything is wor ing as it shou d If it is possib e to articu ate a constraint on contracting the natura approach is to impose that constraint on the reciproca contracting game in order to iso ate its impact from other imp icit restrictions imposed in the indirect contracting game This ma es it possib e to get a better idea of how the constraint wor s

Any ind of comp ete ana ysis of this issue wou d go we beyond the scope of this paper so we on y s etch the way this might wor for a specia case A very simp e constraint on contracting wou d be to assume that some p ayers simp y cou dn t commit at a This is simp e because this constraint on contracting is i e y to oo the same in every contracting game Ca a p ayer a *no-commitment player* if he is uncommitted in the sense of De nition 1 in every one of his information sets That is to say that no matter where he nds himse f in the game he a ways has a behaviora strategy that attains a no-commitment information set with probabi ity 1 [8]

Suppose that the rst m p ayers in a contracting game are no-commitment p ayers The other $n - m$ p ayers wi be referred to as commitment p ayers Suppose there are at east 3 commitment p ayers If we simp y add the restriction that p ayers 1 through m ma e no proposa s in the reciproca contracting game then it is straightforward to show that equi ibrium wi impose a constraint on the outcome function simi ar to the second part of 3 for each of the no-commitment p ayers In other words whatever action the no-commitment p ayers are supposed to ta e in a supportab e outcome it had better be the case that that action is a best rep y

[8] An example would be an agent in an intrinsic common agency who can supply effort to any subset of principals that he wants.

for them conditiona on nowing they are supposed to ta e that action A simi ar restriction has to be added to the punishment

To see what is earned from this suppose we had instead mode ed an indirect contracting game with no commitment p ayers and managed to characterize its equi ibrium There are two possibi ities to consider - there is an outcome in the indirect game that can t be supported in the restricted reciproca contracting game and converse y In the rst case the outcome function can be supported in the unrestricted reciproca contracting game by the theorems above so there must be something about the indirect game that is ma ing it possib e to by pass the contracting restriction hatever it is might be interesting but not because of the contracting restriction Simi ar y if there is an outcome function that is supportab e in the restricted reciproca contracting game but not in the indirect game the conc usion is that the indirect game is imp icit y imposing more restrictions on contracts

The point is simp y to i ustrate that the reciproca contracting approach can be used to study the imp ications of restrictions on contracting even though the game itse f supports a arge number of equi ibria

5 Literature

Epstein and Peters 1 provides a type space and set of mechanisms which a ows agents to convey mar et information a ong with information about their pay- type They show that every mechanism that is - ered in the equi ibrium of a principa -agent type competing mechanism game coincides with a mechanism in *universal set of mechanisms* in which agents report types that convey a their mar et information. The set of mechanisms that is feasib e in a particu ar game maps into a sma subset of the universa set of mechanism Nonethe ess they were ab e to show that provided mechanism were not restricted in how they dea t with pay- types pure strategy equi ibria are typica y robust to expansion of the set of feasib e mechanisms Thus pure strategy equi ibrium in naive direct mechanisms for examp e the equi ibrium in competing direct mechanisms described by McAfee 1 3 can be supported as equi ibrium re ative to the universa set of mechanisms The di cu ty with naive direct mechanisms is that they cannot be used to characterize some of the outcomes that can be supported as equi ibrium re ative to the universa set of mechanisms

The iterature on common agency many competing principa s but on y a sing e agent tried to remedy this by abandoning the reve ation princip e and simp y as - ing for some set of indirect mechanisms that cou d be used to support a outcomes that might qua ity as common agency equi ibrium Martimort and Sto e and Peters 1 show that every robust equi ibrium re ative to any set of indirect mechanisms in common agency is an equi ibrium re ative to the set of menus Pavan and Ca zo ari show a simi ar resu t for common agency using what they ca the set of extended direct mechanisms A robust pure equi ibrium in common agency are equi ibrium re ative to the set of extended direct mechanisms

As usefu as the common agency too s are they have two shortcomings First common agency is specia since there can on y be one agent and principa s can t communicate Second though the set of mechanisms menus that this iterature

offers is considerably simpler than the universal set of mechanisms, they are not sufficiently structured to allow a characterization of supportable outcomes.[9]

Yamashita has recently suggested a way to extend the common agency logic to problems in which each principal has many agents. As in common agency, principals simply ask agents what to do, and commit themselves to carry out the recommendation as long as the majority of the recommendations agree. A characterization theorem for this case is given by Peters and Troncoso-Valverde for competing mechanism games with at least four players and at least one principal.

One consequence of these theorems is that the set of allocations that can be supported as equilibrium with competing mechanisms is large. This fact has been observed before. Starting with the large literature on delegation games (Fershtman and Judd, Fershtman and Kalai), a number of papers have shown large equilibrium sets for special cases (Katz, Tennenholtz, Yamashita, Peters and Troncoso-Valverde). Our paper differs from these in two ways. First we impose no restrictions on the environment. (Katz, Tennenholtz, for example, assume complete information. Yamashita assumes that players who offer contracts have no private information.)

Secondly, like the papers by A.T. Kalai and Samet[10] and Peters and Szentes, we provide a complete characterization of supportable equilibrium outcomes rather than simply illustrating that a large number of equilibrium outcomes can be supported. However we do not assume, as do A.T. Kalai and Samet, that players have complete information.

In this sense, the paper closest to this one is Peters and Troncoso-Valverde which provides a characterization of outcome functions supportable as equilibrium using Yamashita's consensus mechanisms. The difference between the two is the equivalence result given here for reciprocal contracting games and regular competing mechanism games. Generally, the set of outcome function supportable by consensus mechanisms is strictly contained in the set of outcome functions supportable as equilibrium with reciprocal contracting. The reason for the difference is that the reciprocal contracting game allows players who are punishing a deviator to send information to the deviator during the cheap talk part of the game.[11]

Finally, this paper in many way relies on the arguments developed in Peters and Szentes. They show that the simple 'cooperate or be punished' logic that is used to construct reciprocal contracts can be used to understand equilibrium in competing mechanism games no matter how rich the space of contracts is. There are three differences between the two papers. First, the space of feasible contracts in Peters and Szentes is as large as it could be. In other words, the set of feasible contracts in that paper is much larger than the set considered here. The critical result in that paper is that equilibrium outcomes must satisfy (3.1) and (3.) even when the space of contracts is very large. This makes it possible in this paper to turn the result around and show that equilibrium in a reciprocal contracting game can be used to support the same outcomes as can equilibrium with definable contracts. Absent the theorem in their paper there is no way to know

[9]Characterizations of outcomes for special environments have been given by (Peters and Troncoso-Valverde 2009). Though it might not be apparent why yet, we would also include (Tennenholtz 2004) and (A.T Kalai and Samet 2010) in this category.

[10]We borrowed the randomizing trick in (7.1) from this paper.

[11]To see why this might make a difference see (Celik and Peters 2008) or (Dequiedt 2006).

whether the equilibrium outcomes described here are robust to the introduction of new contracts.[12]

The second difference is that Peters and Szentes restrict players ability to communicate once contracts are announced. Here we allow players to continue to communicate, all the while making commitments about how this communication will be handled. This is what is used to expand the set of equilibrium outcomes supportable with reciprocal contracts to include all outcome functions satisfying (3.1) and (3.2) rather than just a subset, as was the case in Peters and Szentes.

Finally and least important, Peters and Szentes restrict attention to pure strategy equilibrium. As a consequence, their characterization does not capture the randomization and correlation that are possible in competing mechanism games. As a consequence, their approach cannot be immediately adapted for a revelation principle.

6. Conclusion

We have shown that all equilibria of competing mechanism games can be understood using reciprocal mechanisms. The advantage of this is that reciprocal mechanisms are conceptually no more difficult to work with than ordinary direct mechanisms. So reciprocal mechanisms provide a useful analytic approach for problems in which a broad class of mechanisms is feasible.

Like direct mechanisms, reciprocal mechanisms make it possible to understand equilibrium outcomes with competition without worrying about the intricacies of particular indirect mechanisms that are used in practice. Apart from the standard logic of incentive constraints, reciprocal mechanisms simply add the logic that if everyone else wants to do something, it is simple to write a contract that commits you to do it too.

Appendix: Proofs

1. Proof of Theorem 3.

Theorem. *If the reciprocal contracting game has four or more players, there is a weak Perfect Bayesian equilibrium that supports the outcome function ω if and only if ω satisfies (3.1), (3.2) and (3.3). Furthermore, along the equilibrium path of this game, all players announce a common signal δ^* in the first stage, report their types truthfully, and choose a correlating message uniformly from the interval $[0,1]$ in the second stage.*

Proof. We begin by showing that any outcome function ω satisfying (3.1), (3.2) and (3.3) can be supported as a weak Perfect Bayesian equilibrium in the reciprocal contracting game. The reciprocal contracts that are announced along the equilibrium path of the game require a direct mechanism for each player and a list of punishment mechanisms. We begin by describing these mechanisms, and the corresponding equilibrium path reciprocal mechanisms.

Index the action profiles in A in some arbitrary way. Let $\omega^k(t)$ be the probability assigned to action profile a^k by the outcome function ω when player types are given by the vector t. The notation a_i^k means the action taken by player i in action profile

[12] Also note that the definition of regularity of competing mechanism games is adapted from their paper. They show that games in which players can use definable contracts are regular.

a^k For $t, x \in T_i$ et t_{ij} and x_{ij} be the type and corre ating message that p ayer i sends to p ayer j The variab es t_j and x_j are the corresponding messages that i receives from $j \neq i$ The vector of messages that i receives from the other p ayers are t_{-i} and x_{-i} Now de ne a homogeneous direct mechanism such that

$$d_i^{\omega} \quad t, x \quad = \begin{cases} a_i^k \quad k = \min_{k'} \quad \sum_{\tau=1}^{k'} \\ \end{cases}$$

The ogic of the proof is then straightforward If a p ayer ma es commitment δ^{ω}, he shou d anticipate the outcome function ω If he deviates, he shou d anticipate p ayers to imp ement a punishment During the punishment he wi hear cheap ta messages that provide him the same information that he wou d have received from the mechanism designer, and so he wi anticipate the same punishment outcome he wou d have faced against the mechanism designer Since the construction of the outcome function ma es this unpro tab e, he won t want to deviate

The comp ication in proving a this is to specify sequentia y rationa continuation p ay that supports the outcome

Continuation Play when all players make the same commitment: If a p ayers ma e the same commitment δ^{ω}, each p ayer shou d choose a corre ating message using a uniform distribution on [, 1], then send that corre ating message and their true type to every other p ayer

The cheap ta game is irre evant in this case, so strategies for the continuation can be de ned arbitrari y As we exp ained above, if p ayers report truthfu y and choose their corre ating messages uniform y, these mechanisms wi joint y imp ement ω Since ω is incentive compatib e by 3 1 and the corre ating messages are non-manipu ab e, these strategies are immediate y sequentia y rationa

e need to specify strategies for information sets in which p ayers ma e a common commitment δ which does not coincide with δ^{ω} In this case, we assume a p ayers report corre ating messages using the uniform distribution as above This reporting strategy is non-manipu ab e, so no p ayer has an incentive to deviate from this reporting strategy no matter type reports are being submitted Conditiona on this, p ayers choose messages from a nite set, and each array of messages determines a pay- Since this de nes a nite Bayesian game, there exists at east one set of strategies that constitute a Bayesian equi ibrium For each common commitment, et these Bayesian equi ibrium strategies constitute the continuation strategies of the p ayers They wi evident y be sequentia y rationa given p ayers interim be iefs Since these information sets p ay no ro e in supporting equi ibrium, we wi not discuss them again

Continuation Play when there is a unilateral deviation in commitments: If there is a uni atera deviation to an a ternative commitment by some p ayer j, then each of the p ayers other than j is committed to the partia y homogeneous punishment mechanism $p_i^{\rho_j}$ Their actions are fu y committed, however they need to send reports and possib y reco ections and recommendations during the cheap ta part of the game If the common commitment is - erent from δ^{ω}, then we proceed as above an use any Bayesian continuation equi ibrium to de ne strategies So we focus on the case in which the common mechanism is δ^{ω}. For the rest of the process, the sequentia rationa ity of the various strategy ru es depend on what p ayers expect to occur ater in the game, so we specify the strategy ru es, then return to provide be iefs that ma e these strategy ru es sequentia y rationa

Reporting Strategies (A): If a p ayers except p ayer j ma e commitment δ^{ω}, then every p ayer, inc uding j, shou d report his type truthfu y to each of the p ayers other than j, we ta e it for granted here and from now on that this inc udes choosing the corre ating message uniform y etc The p ayers other than j shou d babb e[14] when they are reporting to p ayer j

[14]Babble, means to send every feasible message with the same probability.

hen j is a uni atera deviator, choose as part of the equi ibrium three arbitrary p ayers other than j who wi communicate with j during the cheap ta game e refer to these three p ayers as j s *communication partners.*

Recollections (B): If a p ayers except j ma e commitment δ^ω, then each of j s communication partners shou d truthfu y reco ect to each other the type report they heard from j

Recommendation strategies are more comp ex, since there are some - path issues to dea with o dea with on path recommendations, observe that in the centra ized imp ementation described in 3 , 3 and 3 3 , when p ayer j refuses to participate, he nonethe ess sends a type report to the mechanism designer, who then acts in the ro e of a mediator, and sends a recommendation to him that depends on the type t_j he reported, and on the reports t_{-j} the mechanism designer has received from the other p ayers he probabi ity distribution over this recommendation is the margina distribution $\rho_{A_j}\ \ t_j, t_{-j}$ since j is supposed to want to carry out this recommendation Let $\rho_{A_j}^{a_j}\ \ t_j, t_{-j}$ be the probabi ity assigned to action a_j by this margina distribution

Recommendations (C): For each communication partner i, if at east one of the reco ections of the other two communications partners coincides with the report $\tilde{t}_j$ that i received from j, then i shou d send j the recommendation

$$a_j \quad j = \min_{k'} \sum_{\tau=1}^{k'} \rho_{A_j}^{a_{k'}} \left(\tilde{t}_j, t_{-j}\right) \geq \lfloor \sum_{i' \neq j} x_{i'} \rfloor, \tag{3}$$

where t_{-j} and $x_{i'}$ are i s reco ection of the type reports from the p ayers other than j Otherwise, each of the communication partners shou d babb e to j Each of the p ayers who is not a communication partner with j shou d babb e to j

Note that, as above, if p ayers corre ating messages are chosen uniform y and the reco ections of a three p ayers coincide, then from j s perspective, each of the three p ayers wi send him the same recommendation, and each such recommendation a_j wi occur with the same probabi ity as it wou d have if the centra ized mechanism designer were ma ing the recommendation after hearing reports $\tilde{t}_j$ and t_{-j}

he strategy of the uni atera deviator depends on the recommendations he receives o de ne strategies, choose some action that maximizes his expected pay - given his interim be iefs and given that he expects the other p ayers to report their types truthfu y to one another in other words, he shou d anticipate the punishment $\rho_{A_{-j}}\ \ t_{-j}$ e wi refer to this action as p ayer j s interim best rep y

P ayer j now expects to hear the same recommendation from exact y three p ayers Furthermore, this recommendation shou d have positive probabi ity given the strategy de ned by 3 and his interim be iefs he other p ayers are babb ing in our construction, so j shou d ignore any recommendations that they ma e he information sets in which Bayes ru e does not admit a we de ned posterior be ief are those in which the deviator j sends a common type report to at east two of his three communication partners but does not get a common recommendation bac A ternative y, Bayes ru e does not give a posterior be ief in information sets in which the three communication partners ma e a common recommendations, but this recommendation does not ie in the support of 3 e refer to these as *off-path* information sets

Information sets in which Bayes rule is well defined are those in which j sent a common type report to at least two of his communication partners and receives a common recommendation, say a_j, in the support of 3 back from each of them. What j does in these information sets should depend on the common type report. If this common type report was t_j, then he should update his beliefs about t_{-j} given the recommendation a_j using 3 with $t_j = t_j$. He should then choose a best reply to $\rho_{A_{-j}}$ t_{-j}. Whichever action this happens to be will be referred to as his best reply given the report t_j and recommendation a_j. We refer to these information sets as *on-path information sets with type report t_j and recommendation a_j.*

Actions (D): If every player but j commits to δ^{ω}, in any on-path information set with type t_j and report a_j, j should use the action that is a best reply to t_j and a_j. In particular if t_j is js true type, then he should obey the recommendation a_j of his communication parters. In any off-path information set j should use his interim best reply.

Finally, in all the histories for which there are three or more distinct commitments, players simply play a cheap talk continuation game using their interim beliefs. Any strategy rules for the players that are part of a Perfect Bayesian equilibrium of this cheap talk game will act as appropriate sequentially rational continuation strategies. Such an array of strategy rules will exist because the game is finite. Again, since these information sets arise after unilateral deviations, we don't need to discuss them further.

This completes the description of the strategy rules. We can now verify their sequential rationality. We already explained why strategies are sequentially rational when all players make the same commitment, and why they are sequentially rational when players make 3 or more different commitments. So we begin with reporting strategies after a unilateral deviation (Case A above). In this case, each non-deviating player expects js communication partners to confirm each others recollections (by B) then make (by C) the same recommendation to j that the centralized mechanism designer would have made. By (D) above, j is expected to carry out this recommendation. So players anticipate the outcome function ρ_j. It is then sequentially rational for players other than j to report their types to one another truthfully by 3 3.

The deviating player on the other hand, is not committed, so he can send different reports to each of the non-deviators. If he sends the same report t_j to at least two of his communication partners, then by (B) and (C), he will face the punishment ρ_j, anticipating by 3 to receive the same distribution of recommendations as what he would have received from the mechanism designer. If he sends three different reports, then by the second part of (C) he will receive an uninformative report. By 3, it is sequentially rational then for him to report the same type to all players.[15]

The sequential rationality of the non-deviators reports to the deviator are immediate since the others don't pay any attention to them.

By (B) and the fact that j reports truthfully, if i is one of js communication partners, he expects each of the others to hear a recollection that coincides with the type report they heard from j which they expect to coincide with the type report they heard. As a result, they believe that their own recollection will be ignored.

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[15]Observe the role that the recollections player here, since they prevent the deviating player from sending different messages to each of his partners in order to try to get more information out of them.

It is therefore sequentia y rationa for i to reco ect j s type truthfu y no matter what he be ieves

A simi ar reasoning is used for recommendations though there are a coup e of o path possibi ities Again et i be one of j s communication partners who heard j report t_j If i hears at east one reco ection that coincides with t_j and be ieves on path that j reported truthfu y to each of his communication partners and that if there is a reco ection from one of the communication partners that doesn t coincide with t_j that this report was the resu t of a tremb e then he shou d be ieve that both his communication partners have reported t_j to each other He wi then expect both of them to ma e the same recommendation determined by 3 to j no matter what recommendation he sends By D , any recommendation he ma es wi be ignored in this case This estab ishes that it is sequentia rationa for him to send a report determined by 3

On the other hand if both of the reco ections he receives di er from t_j then i shou d be ieve that j tremb ed from truthfu reporting to babb ing independent y to each p ayer By D , this means that j is going to use his interim best rep y no matter what recommendations he receives nder these circumstances it is sequentia y rationa for i to babb e when ma ing his recommendation

Fina y if j sends three di erent reports to his communication partners he shou d expect them to truthfu y reco ect these to each other By C this wou d induce them to babb e when ma ing recommendations Thus it is sequentia y rationa for j to use his interim best rep y If he ma es two or more common reports t_j and receives a common recommendation a_j as a resu t then it is sequentia y rationa for him to use his best rep y to t_j and a_j by de nition If he ma es two or common reports but doesn t get a common recommendation then any sequentia y rationa choice of action wi do

Fo owing a of this 3 guarantees that it does not pay any p ayer to deviate from the commitment message δ^ω

To prove the other direction begin with an outcome function ω that is supportab e as a wea perfect Bayesian equi ibrium in reciproca mechanisms P ayer i s equi ibrium pay o is

$$\mathbb{E}\{u \quad \omega \quad t \quad , t \quad |t_i\}.$$

Let δ' made by some p ayer i that ensures that with probabi ity 1, i chooses his action without commitment Let ρ' t be the outcome function associated with a deviation to δ' by p ayer i The various conditions 3 1 , 3 and 3 3 then fo ow immediate y from the fact that ρ' is continuation p ay associated with a wea perfect Bayesian equi ibrium and from the fact that i cannot bene t by deviating to δ' $\square$

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