Pay-for-luck in CEO compensation: matching and efficient contracting

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Abstract

We develop a stylized model of efficient contracting with matching between firms and managers with state-contingent reservation utility. We show that the optimal contract is designed to retain and insure the manager. The retention motive explains pay-for-luck in executive compensation, while the insurance feature explains asymmetric pay-for-luck. This contract can be implemented with call options based on a single performance measure which generally does not filter out luck. When costs of involuntary managerial turnover differ across firms, and the abilities of different managers are more or less precisely estimated ex-ante, the model can also explain the observed association between pay-for-luck and bad corporate governance.

Keywords: CEO pay; corporate governance; pay-for-luck; stock-options.

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In recent years, CEO pay has attracted considerable attention, both in the popular press and in academic journals. This renewed interest was in part triggered by some puzzling observations such as pay-for-luck, i.e., the evidence that exogenous and contractible shocks to performance do nevertheless have an effect on CEO pay. In response to this and other empirical findings which are inconsistent with a standard version of the principal-agent model, the managerial power or "skimming" hypothesis has been proposed as an alternative paradigm, most notably by Bertrand and Mullainathan (2001) and Bebchuk and Fried (2004). This strand of literature has documented a number of "anomalies", which are suggestive of corporate governance failures. Since then, many papers have shown that some of these apparent "anomalies" can actually be explained in an efficient contracting framework (see the literature review of Edmans and Gabaix (2009)), although some puzzles remain (Frydman and Jenter (2010)).

Bertrand and Mullainathan (2001) acknowledge that the principal-agent model may explain pay-for-luck, but they argue that the facts that pay-for-luck is asymmetric (see also Garvey and Milbourn (2006)) and is stronger in firms with worse corporate governance provides supporting evidence for a skimming model of executive pay. In this paper, we explain these phenomena as the equilibrium outcome of a simple principal-agent model of efficient contracting.

Our baseline model builds on Harris and Holmstrom (1982). We consider a two-period principal-agent relationship between risk-neutral shareholders and a risk averse manager with state-contingent reservation utility. As in Oyer (2004), there is no moral hazard problem. We derive the optimal contract without any restriction on the contracting space. We find that the manager tends to be more rewarded for good luck than he is penalized for bad luck. That is, there is "asymmetric" pay-for-luck. Two assumptions are crucial to obtain this result: the difference in risk preferences between the manager and the shareholders, and one-sided commitment.[1] More precisely, we assume that the firm can commit to employ the manager next period, but the manager cannot commit to work for the firm next period. In this setting, we show that a risk averse manager accepts a lower pay in the first period in exchange for insurance against a

[1] Both assumptions are also in Harris and Holmstrom (1982).

low reservation utility in the second period. This is possible because the firm can commit to pay an endogenously generated "minimum wage" next period. However, in case the reservation utility of the manager is sufficiently high next period, the firm must adjust his pay upward to retain him. This is because the manager cannot commit (ex-ante) to work for the firm next period. Therefore, with an optimal contract, remuneration will be downward rigid *and* sensitive to factors that affect the reservation wage of the manager (including luck) on the upside, so that pay-for-luck will be asymmetric.

Moreover, we show that stock-options provide both the required downward protection and the required upside participation, so that an optimal contract may be implemented with stock-options. This result is all the more interesting that the standard principal-agent model of effort choice does not explain the widespread use of stock-options in managerial compensation (Hall and Murphy (2002), Dittmann and Maug (2007)).[2] The model also predicts an increase in stock-options based compensation as general managerial skills become relatively more important than firm-specific skills. To the extent that this has been the case in the 1980s and the 1990s (as argued in Murphy and Zabojnik (2004) and Frydman (2007)), the model can explain the rise in options-based executive compensation in this period.

Our second main result is that pay-for-luck will be stronger (relative to pay-for-performance) at firms with a high cost of involuntary CEO turnover. Indeed, the pay of the manager in the second period must be state-contingent for two reasons. First, firm performance in the first period provides some information on the manager's ability, which affects his reservation wage in the second period. Second, changes in economic conditions ("luck") also affect the reservation wage of the manager. But firm performance is all the more informative about managerial ability that this ability is not precisely estimated ex-ante – think about young managers, or managers with a short tenure, for example. It follows that state-contingent pay will put a higher weight on firm performance (rather than luck) for managers whose ability is less precisely estimated ex-ante. In addition, if we assume that dismissing a manager and hiring a new one is costly

[2] This being said, postulating loss aversion (Dittmann Maug and Spalt (2010)) or a larger action set for the manager (Dittmann and Yu (2010)) improves its explanatory power.

(Taylor (2010)), and that this cost differs across firms, then we may expect firms with a high firing cost to hire managers whose ability is more precisely estimated ex-ante. Our simple matching model of managers and firms indeed yields this intuitive result. To summarize, firms with high dismissing costs will hire managers whose ability is more precisely estimated ex-ante, and the contingent pay of these managers will consequently involve more pay-for-luck (rather than pay for performance) than the contingent pay of other managers. Thus, our model predicts that the degree of pay-for-luck relative to pay-for-performance will tend to be higher in firms with high dismissing costs. Given that it is efficient to dismiss bad managers in our model, the high cost of dismissal can be interpreted as an indicator of bad corporate governance.[3]

Lastly, as already noted for example in Murphy and Zabojnik (2004), the skimming hypothesis is hard to reconcile with the facts that, in the past decades, CEO pay increased *and* corporate governance improved. Our model can contribute to explain this phenomenon — although many other factors are probably behind the rise in CEO pay (see, e.g., Gabaix and Landier (2008)). Indeed, we show that an improvement in corporate governance, whether across the board or confined to the subset of badly governed firms, has a spillover effect that increases CEO pay in all firms. The intuition is that equilibrium CEO pay is set by the firms with the worst corporate governance, and that firms are willing to pay more a CEO who is less entrenched.

The hypothesis at the core of our model is that retention is an important determinant of CEO compensation. There is empirical support for this notion: Rajgopal, Shevlin and Zamora (2006) present evidence that CEO pay is structured to match the state-contingent outside employment opportunities of managers, while Oyer and Schaefer (2005) emphasize the limitations of the incentives-based explanation for the adoption of broad-based stock-options plans.

Pay-for-luck is not predicted by the standard moral hazard model of efficient contracting with a risk averse agent: if a worker's pay must be variable for incentive purposes, then the informativeness of the performance measure should be maximized (Holmstrom (1979)), so that

[3] The variables used in Bertrand and Mullainathan (2001) – the existence of a large shareholder, CEO tenure, the board size, and the fraction of insider directors – can also be interpreted as proxies for the cost or difficulty of CEO dismissal.

it is optimal to filter out exogenous shocks to performance. The model of Oyer (2004) explains pay-for-luck as the outcome of efficient contracting by invoking the retention motive and renegotiation costs (see also Himmelberg and Hubbard (2000), and Edmans and Gabaix (2009) for additional references), but it does not explain the facts that pay-for-luck is asymmetric and is associated with poor corporate governance. Gopalan, Milbourn and Song (2009) and Feriozzi (2010) propose some alternative hypotheses to explain asymmetric pay-for-luck, related respectively to strategy choice and the implicit incentives emanating from the threat of bankruptcy.

Section 1 presents the model. Section 2 derives the optimal compensation contract, for any given firm-manager match. Section 3 describes the relevant performance measure for managerial compensation, and introduces stock-options. Section 4 describes the matching equilibrium. Section 5 discusses the results. Section 6 concludes. All proofs are in the Appendix.

1 The model

We consider a two-period economy in which firms compete for CEOs.

CEOs are risk averse with utility function $u(\cdot)$ ($u' > 0$, $u'' < 0$). We assume a limited supply of CEOs: there are m firms and n CEOs, with $m > n$. The ability a of a CEO is normally distributed with a mean of $a > 0$ and a variance σ_a^2. A firm that is not run by a CEO is run by a regular management: we model regular management as an unlimited supply of managers whose expected ability is normalized to zero. We also normalize the reservation utility of CEOs and managers who are not employed to 0.

Firms are risk neutral and maximize their expected profits. In every period, the gross profits (before compensation of the manager) of a firm depend on three factors: its CEO's ability a, business conditions $\tilde{L}$[4], and some unobservable idiosyncratic shock $\tilde{\epsilon}_t$. We assume that $\tilde{\epsilon}_t$ is normally distributed with mean zero and variance σ_ϵ^2, and independent from other random variables for any $t \in \{1, 2\}$.[5] We assume that $\tilde{L}$ is a random variable with positive support, and

[4]We will refer to L as "luck" since L is not under the control of the manager.

[5]Assuming that both $\tilde{\epsilon}_t$ and a are normally distributed makes the Bayesian updating of beliefs on a tractable.

which is also independent from other random variables.[6] We will use the notation $\tilde{L}$ to denote the random variable $\tilde{L}$, and the notation L to denote its realization, which is observable and contractible. The luck shock L represents any exogenous contractible shock which is outside the control of the CEO. We assume that the gross profits of the firm write as:

$$\pi_t = (\alpha + s_t a + \tilde{\epsilon}_t)\tilde{L} \tag{1}$$

where $s > 0$.

CEOs are thus, on average, improving firm performance with respect to regular management. The variable s_t represents the accumulated experience and firm-specific skills of the CEO. We assume that $s_t = 1$ if the CEO worked for the firm in period $t-1$, and $s_t = \gamma \in (0,1)$ otherwise[7]. In our model, it introduces a cost to replacing the CEO, which is related to the relative importance of general managerial skills as opposed to firm-specific skills in managerial jobs. For example, if general skills predominate, then s approaches one, in which case managers are more easily replaceable.

For any firm, the net profits are the gross profits net of compensation costs. Both gross profits and net profits are observable and contractible. We assume that firms pay out their net profits to shareholders at the end of the second period. We also assume that there is a market for each firm's shares at the beginning of the second period, following the realization of the luck shock $\tilde{L}$, where stock prices are established. We denote by V the implied market value of a given firm at the end of the first period.[8]

Crucially, we assume that a firm can commit to a long-term contract, but a CEO cannot. While firms can and do propose enforceable long-term contracts to their employees, constraints on involuntary servitude prevent employees from forgoing (ex-ante) the option to quit a job.

[6] We do not need to specify the distribution of $\tilde{L}$.

[7] This modelling choice follows Murphy and Zabojnik (2004).

[8] These two assumptions imply that there exists a performance measure (firm value V) which captures both the past performance of the manager and market conditions. In practice, there are reasons to believe that firm value will indeed reflect these two factors.

This one-sided-commitment assumption is natural and was introduced in Harris and Holmstrom (1982), and Holmstrom and Ricart i Costa (1986). We also assume that a manager can neither save nor borrow, so that he does not transfer income from one period to another.[9]

Two types of contracts are feasible. Any firm can propose a spot contract to a CEO at the beginning of the first period, and a spot contract to a CEO at the beginning of the second period. In this case, the employment of the first period manager terminates at the end of the first period. Alternatively, the firm can propose a long-term contract to a CEO at the beginning of the first period. This contract specifies the wage that the firm commits itself to pay the CEO in the first and second periods. It should be stressed that even a CEO who is dismissed is entitled to this second period wage, as specified in the contract. In this case, the wage can be interpreted as a severance payment.

The timeline is as follows. At the beginning of the first period, firms propose contracts to CEOs. At the end of the first period, the luck shock L and first period profits π_1 are realized. Beliefs about the ability of each CEO are updated accordingly. At the beginning of the second period, any CEO can resign, in which case he forgoes his contractual second period payment, but earns his reservation wage W_2, which we derive below. A firm can dismiss a manager who has not resigned, and hire a new CEO (or regular manager) on the spot market, at an extra cost k, with $k \geq 0$. However, in case the firm hires a new manager, it is committed to pay the CEO's wage under long-term contract. At the end of the second period, second period profits π_2 are realized and the firm is liquidated. The discount factor is zero.

2 The optimal long term contract

Spot contracts, reservation wage and dismissal

We first derive the updated beliefs of firms after they have observed π_1 and L. The updated expected ability $\hat{a}$ of a CEO is calculated using Bayes' rule.

[9] As in Harris and Holmstrom (1982) and Holmstrom and Ricart i Costa (1986), the optimal contract is such that the saving restriction is inconsequential.

Result 1: Updated beliefs about CEO's ability

$$\hat{a} = \frac{1}{\gamma}\frac{\gamma^2\sigma_a^2(\pi_1/L - (\alpha - \gamma a)) + \sigma_\epsilon^2\gamma a}{\gamma^2\sigma_a^2 + \sigma_\epsilon^2} \quad (2)$$

Let's now compute the equilibrium of the spot market in the second period. Firms can choose to hire a manager with zero ability on a spot contract earning a profit of αL. Firms can also compete for CEOs. Consider a CEO with expected ability $\hat{a}$. The firm that employed him in the first period is willing to pay up to $\hat{a}L$ to employ him in the second period. All other firms with vacant positions are only willing to pay up to $\gamma\hat{a}L$ to hire this manager.[10] Competition between firms drives the second period reservation wage of a CEO with expected ability $\hat{a}$ to:

$$W_2(\hat{a}, L) = \gamma\hat{a}L. \quad (3)$$

Indeed, (3) is the maximum compensation that a CEO employed in a given firm can expect from another firm. Any given CEO can earn this wage in the second period, whether he is employed under a spot contract or a long-term contract. It follows that the firm which employed this CEO in the first period only needs to match the compensation in (3) to retain its CEO in the second period. For a given L, the set of reservation wages described in (3) clears the market for CEOs at the beginning of the second period.

Finally, in the first period, any good CEO can earn a wage of $W_1 = \gamma a E[\tilde{L}]$ with a spot contract.

We now derive the dismissal rule at the beginning of the second period.

A firm will dismiss a CEO at the beginning of the second period if and only if it increases its expected net profits in the second period. On the one hand, the expected second period profits

[10] This corresponds to the additional profit coming from the hire of an established CEO over a zero-ability manager.

of a firm which does not dismiss its CEO at the beginning of the second period are

$$(\alpha + \hat{a})L - w_2 \tag{4}$$

where w_2 denotes the (as yet undetermined) compensation of the CEO in the second period under his contract. On the other hand, the expected second period profits of a firm which dismisses its CEO at the beginning of the second period and hires a new CEO on the spot market are

$$\alpha L - k - w_2 \tag{5}$$

Comparing the expressions in (4) and (5) yields the optimal firing rule:

Result 2: Dismissal rule

A firm will dismiss its CEO when $\hat{a} < -\frac{k}{L}$.

Dismissing a CEO under a long-term contract can be advantageous if the expected ability of the incumbent CEO is lower than a threshold. Furthermore, this threshold is decreasing in the cost k of changing the CEO, which is expected, and increasing in the luck shock L. The intuition is that the ability of the manager matters all the more when L is high, since firms profits are multiplicative in a and L.

Optimal long-term contract

The optimal long-term contract is defined by a first period wage $w_1^\star$ and a second period wage $w_2^\star(\hat{a}, L)$, where $w_1^\star$ and $w_2^\star(\hat{a}, L)$ solve the following optimization problem:

$$\min_{\langle w_1, w_2(\hat{a}, L)\rangle} w_1 + E[w_2(\hat{a}, L)] \tag{6}$$

$$w_2(\hat{a}, L) \geq W_2(\hat{a}, L) \qquad \text{for all } \hat{a}, L \tag{7}$$

$$u(w_1) + E[u(w_2(\hat{a}, L))] \geq U \tag{8}$$

where U denotes the outside option of the CEO at the beginning of the first period[11]. For the moment, note that U does not affect the shape of the optimal long-term contract.

The firm sets the pay of the CEO in the first and second period to minimize its compensation costs subject to participation constraints. First, the state-contingent participation constraints in (7) guarantee that the firm matches the reservation wage of the CEO in the second period (these constraints are not binding when the firm dismisses its manager).[12] Second, the constraint in (8) guarantees that the CEO accepts the long-term contract at the beginning of the first period – which is the case if and only if the expected utility associated with the long-term contract exceeds the outside option of the CEO, (e.g. the expected utility achieved with a sequence of spot contracts, or with the long-term contract offered by other firms.).

The first-order conditions with respect to w_1 and $w_2(\hat{a}, L)$ are respectively:

$$\mu u'(w_1) = 1 \tag{9}$$

$$\mu u'(w_2(\hat{a}, L)) = 1 + \lambda(\hat{a}, L) \qquad \text{for all } \hat{a}, L \tag{10}$$

where $\lambda(\hat{a}, L)$ and μ are respectively the (nonnegative) Lagrange multipliers associated with the constraints (7) and (8), where $\lambda(\hat{a}, L) \geq 0$ satisfy the complementary slackness condition:

$$\lambda(\hat{a}, L)(W_2(\hat{a}, L) - w_2(\hat{a}, L)) = 0 \qquad \text{for all } \hat{a}, L \tag{11}$$

where $W_2(\hat{a}, L)$ is as in (3). This immediately yields the form of the optimal long-term contract:

Proposition 1: Optimal Contract

The optimal long-term contract is characterized by a first period wage of w_1^* and a second

[11]In a competitive equilibrium, this outside option corresponds to the highest utility offered by other firms proposing optimal contracts.

[12]The second period participation constraint (7) is not binding when a firm dismisses its CEO. On the one hand, a firm dismisses its CEO if and only if $\hat{a} < -\frac{k}{L}$. On the other hand, (7) is binding if and only if $\hat{a} \geq \frac{w_1^*}{\gamma L}$, which is positive. It follows that, in cases when the firm dismisses its CEO, $\hat{a}$ is negative so that (7) cannot be binding.

period wage of

$$w_2^*(\hat{a}, L) = \max\{w_1^*, \gamma \hat{a} L\} \quad (12)$$

The level of w_1^* notably depends on U, and we assume that parameter values are such that w_1^* is positive. At this stage, we do not specify how this state-contingent pay scheme is implemented.

Comparative static results

A long term contract is fully determined by the first-period wage w_1^*. The second period wage is constant until the participation constraint binds when $W_2(s^*) = w_1^*$. The termination decision depends only on k. The level of the first period wage w_1^* is determined using the first period participation constraint. and thus depends on the outisde option U of the CEO.

Interpretation

The optimal long-term contracts displays three interesting features.

First, as in Harris and Holmstrom (1982), the risk averse manager is insured by the firm in the second period: should his reservation wage fall below w_1^*, the firm will nevertheless pay w_1^*. The cost of this insurance is a first period wage w_1^* lower than what the manager could obtain on the spot market. In the first period, the manager pays an insurance premium to be insured in the second period against adverse realizations of his reservation wage.[13] Otherwise, if the reservation wage of the manager in the second period is above w_1^*, the firm matches this reservation wage. The manager cannot be fully insured by the firm (with a constant wage in the second period) because the manager cannot commit in the first period to work for the firm in the second period. To summarize, the optimal contract features downside protection, with a downward rigid second period wage for insurance purposes, and upside participation for retention purposes. The assumption of one-sided commitment is crucial to obtain this result.

Second, the compensation of the manager in the second period depends on business conditions, or "luck" (L). The reason is the same as in Oyer (2004): compensation adjusts to the

[13] They stem from two sources: updating of beliefs regarding the manager's ability, and the luck shock. In Harris and Holmstrom (1982), the only source of uncertainty relates to the updating of beliefs.

level required to retain the CEO, and this level in turn depends on business conditions.

Third, pay for luck is asymmetric. Indeed, the pay of the manager is sensitive to luck (at the margin) if and only if the luck shock is su ciently positive:

PROPOSITION 1: ASSYMMETRIC PAY FOR LUCK

For a given $\underline{L}$, and for any given value of $a + \epsilon$, $\frac{dW_2}{dL} > 0$ if $L > \underline{L}$, and $\frac{dW_2}{dL} = 0$ if $L < \underline{L}$.

Proof: see Appendix.

This result highlights that, with an optimal contract, managers are \rewarded for good luck", but they are not symmetrically \penalized for bad luck": pay-for-luck is asymmetric. This is due to the insurance against adverse states of the world (including bad luck) provided by the optimal contract whenever managers are risk averse.

As long as the expected termination costs are not too large, a long-term contract dominates a sequence of spot contracts. This is because a long-term contract allows to partly insure the manager in the second period. In the remainder of the paper, we assume that this is the case.[14]

3 Implementation with stock-options

The optimal second period compensation of the manager depends on his expected ability $\hat{a}$ and on the luck shock L. In this section, we show that the optimal long-term contract can be implemented by giving the manager stock-options. We also discuss how the optimal compensation contract depends on the model parameters.

We rst show that the optimal second-period compensation of the manager can be expressed as a function of rm value at the beginning of the second period and the luck shock. To this end, we rst derive rm value, V. In a competitive market for rm shares with risk neutral

[14] This assumption can be microfounded by assuming that σ_a^2 and k are sufficiently low.

shareholders, firm value at the beginning of the second period is:

$$V = \pi_1 - w_1 + E[\pi_2 | \pi_1, L] - w_2(\hat{a}, L) = \pi_1 - w_1^* + (\alpha + \hat{a})L - \max\{w_1^*, \gamma \hat{a} L\} \tag{13}$$

if $\hat{a} > -\frac{k}{L}$. Otherwise, the incumbent manager is dismissed at the beginning of the second period, with a compensating payment of w_1^*, while his replacement receives a fixed wage, so that firm value does not matter for compensation purposes. Recall that,

$$\hat{a} = \frac{1}{\gamma} \frac{\gamma^2 \sigma_a^2 (\pi_1 / L - (\alpha - \gamma a)) + \sigma_\epsilon^2 \gamma a}{\gamma^2 \sigma_a^2 + \sigma_\epsilon^2}.$$

Substituting $\pi_1 = V + w_1^* - (\alpha + \hat{a})L + \max\{w_1^*, \gamma \hat{a} L\}$, and isolating $\hat{a}$, we get that:

$$\begin{aligned} W_2(\hat{a}, L) &= w_1^* \text{ when } \gamma \hat{a} L \le w_1^* \\ W_2(\hat{a}, L) &= \gamma \hat{a} L = \left(\frac{\gamma^2 \sigma_a^2 + \sigma_\epsilon^2}{\gamma^2 \sigma_a^2} + \frac{1 - \gamma}{\gamma} \right)^{-1} \left(V + w_1^* - 2\alpha L + \gamma a L \left(\frac{\sigma_\epsilon^2}{\gamma^2 \sigma_a^2} - 1 \right) \right). \end{aligned}$$

We thus get:

$$W_2(\hat{a}, L) = \max\{w_1^*, \psi w_1^* + \psi V + \psi \eta L\} \tag{14}$$

$$\text{where} \quad \psi \equiv \left(\frac{\gamma^2 \sigma_a^2 + \sigma_\epsilon^2}{\gamma^2 \sigma_a^2} + \frac{1 - \gamma}{\gamma} \right)^{-1} \quad \text{and} \quad \eta \equiv \gamma a \left(\frac{\sigma_\epsilon^2}{\gamma^2 \sigma_a^2} - 1 \right) - 2\alpha \tag{15}$$

Notice that $\psi < 1$.

We now specify how the optimal contract described in (12) and (14) can be implemented with stock-options on a measure which is constructed to incorporate both changes in firm value and luck. Consider the measure $P(V, L)$, constructed as

$$P(V, L) = V + \eta L \tag{16}$$

Then the state-contingent payment $w_2(\hat{a}, L)$ in (12) can be implemented by making payments

to the agent contingent on the measure P:

$$w_2(\hat{a}, L) = \max\{w_1^*, \psi w_1^* + \psi P(V, L)\} \qquad (17)$$

This immediately leads us to this important result:

Proposition 2: Optimal contract and stock options

The state-contingent optimal payment in (17) may be implemented by giving the manager a xed wage w_1^* and ψ stock-options on P with exercise price $\kappa = \frac{w_1^*(1-\psi)}{\psi}$, which vest and expire at the beginning of the second period, after the rm has decided whether to retain or to dismiss its manager.

This notably implies that a CEO who is dismissed will not exercise his options, sine the options are out of the money. Moreover, under the optimal contract, the stock-options are not fully indexed:

Corollary: Under the optimal contract, $\frac{d}{dL}P(V, L) \neq 0$.

This means that the luck shock is not fully ltered out of the measure P. Intuitively, this is because the reservation wage of the manager depends on this luck shock.[15] Furthermore, the degree of pay-for-luck relative to pay-for-performance, which is measured by η, is increasing in σ_ϵ^2, and decreasing in σ_a^2. There will be more pay-for-luck relative to pay-for-performance when V is a noisy measure of a, and when the initial uncertainty on the ability of the executive is low. This suggests that pay-for-luck will be relatively strong for non-CEOs (whose σ_ϵ^2 is high), as well as for old CEOs or CEOs with a long tenure (whose σ_a^2 is low). On the contrary, young

[15] More precisely, firm value net of luck, which may be viewed as a measure of "pure performance", matters for the reservation wage insofar as it leads to updating on the manager's ability. Likewise, pure luck matters for the reservation wage insofar as the latter is sensitive to luck, or "economic conditions".

CEOs or CEOs with a short tenure should be less paid for luck, all else equal. In the limit, as the ratio $\frac{\sigma_\epsilon^2}{\sigma_a^2}$ tends to infinity, η also tends to infinity, and state-contingent remuneration only depends on luck.

The optimal contract depends on the relative importance of general managerial skills, which is measured by γ:

Proposition 3: Under the optimal contract, $\frac{d\eta}{d\gamma} < 0$ and $\frac{d\psi}{d\gamma} > 0$.

First, the degree of pay-for-luck relative to pay-for-ability is decreasing in γ, which measures the relative importance of general managerial skills as opposed to firm-specific skills. As γ approaches zero, there is only pay-for-luck and no pay-for-performance. Indeed, if only firm-specific skills matter, then the manager's ability is irrelevant for other firms, so that his reservation wage does not depend on his ability, but only on luck. On the contrary, our model predicts that a rise in the importance of general managerial skills (as proxied by γ) should be accompanied by a reduction in pay-for-luck relative to pay-for-performance.[16]

Second, the quantity ψ of stock-options given to the CEO is an increasing function of γ. If general skills became progressively more important in the 1980s and the 1990s, as argued by Murphy and Zabojnik (2004) and Frydman (2007), then our model can explain why CEOs received increasing amounts of stock-options throughout this period (Frydman and Jenter (2010), figure 2).

It is noteworthy that the manager is not more exposed to risk with this long-term contract than he would be with a spot contract (he is even less exposed to risk with the long-term contract because of the embedded insurance). Indeed, the manager is only exposed to variations in his reservation wage – on the upside. Firm value V consists in two components: some exogenous shocks which can be filtered out (here represented by L), and the "pure measure of performance", i.e., firm profits once exogenous influences (such as L) have been removed. Since the beliefs on

[16] Murphy and Zabojnik (2004) and Frydman (2007) argue that the relative importance of general managerial skills increased in the past three decades.

the manager's ability are updated based on this \pure measure of performance", any residual noise in this measure will *also* a ect the updated beliefs on the manager's ability, and therefore his reservation wage.[17]

Finally, it should be noted that the rm could simply commit to paying the manager the xed wage w_1^* in the rst period and 2, and adjust his pay at the beginning of the second period depending on $\hat{\alpha}$ and L. While this is certainly possible, this would not be optimal if we were to assume that there exists a renegotiation or transaction cost, no matter how small. Even with an arbitrarily small renegotiation cost, the long-term contract described in this section would strictly dominate the contract described above in this paragraph. By contrast, in Oyer (2004), with an arbitrarily small renegotiation cost, it will almost always[18] be optimal to use spot contracts instead of long-term contracts.[19]

4 Matching equilibrium: corporate governance and and pay-for-luck

The literature on pay-for-luck has documented a correlation between pay-for-luck and corporate governance. This empirical fact is at the root of the skimming theory that states that manangers set their own pay and can more easily do so in rms with bad corporate gorvernance. To incorporate this important dimension in our model, we develop a matching model between

[17]Here our results differ from Oyer's (2004). In Oyer, the tradeoff is between exposing the manager to risk (by indexing his pay on some variable which is imperfectly correlated with his reservation wage) and incurring renegotiation or transactions costs with interim re-contracting. We also differ from standard models of moral hazard, where the optimal contract is the outcome of a tradeoff between exposing the agent to a noisy measure of his effort and providing incentives for effort.

[18]As long as the degree of risk aversion and the variance of the performance measure are strictly positive.

[19]This is not the only potential reason why it is preferable to use an explicit contract rather than an ex-post adjustment to relate the compensation of the manager pay in the second period to firm value at the end of the first period. For example, suppose that the manager can invest (at a cost) in firm-specific skills at the beginning of the first period. There will then typically be a time-inconsistency problem: ex-ante, the firm would like to commit to pay the manager more for increases in firm value, so that the manager invests efficiently in firm-specific skills. However, ex-post, since investment in firm-specific skills does not increase the reservation wage of the manager, it is in the interests of the firm to renege on this promise. In this perspective, an explicit contract may be used as a commitment device.

firms and managers.

We assume that firms differ in terms of their cost of dimissing their CEO and hiring a new one. We will interpret this cost as an index of corporate governance: the cost k that each firm must incur to dismiss a CEO at the beginning of the second period is inversely related to the level of corporate governance. In general, corporate governance measures the extent to which the firm is managed in the interests of the providers of funds (i.e., the shareholders). In the context of our model, the only relevant difference across firms is the ability of the manager. In addition, all managers have the same expected ability ex-ante. Therefore, in our model, corporate governance measures the extent to which a firm can dismiss a manager with low ability and hire a more efficient replacement manager instead. This is captured by the cost k. The assumption that a better corporate governance results in a lower cost of firing the manager is also in Acharya, Gabarro, and Volpin (2011).

We assume that different managers have different σ_a^2, so that managers can be ranked according to the variance of their ability. That is, managers' reputation is more or less established.[20] There are no information asymmetries: for each manager, the value of σ_a^2 is common knowledge, but neither the firms nor the manager observe a.

This corresponds to a matching model with nontransferabilities, as studied in Legros and Newman (2007).[21] An equilibrium is defined by a matching function indicating which type of firm is employing which type of manager, and equilibrium long term contracts. Conditions for equilibrium are that the matching function is consistent, i.e., each manager is matched with only one firm. The second condition is that no firm can break its match and improve its expected profit by proposing a contract to an already matched manager that would prefer that contract. Legros and Newman (2007) derive sufficient conditions on the Pareto frontiers generated by a match that ensure positive or negative assortative matching. In our setup, the equilibrium is characterized by negative assortative matching: a firm with lower dismissal costs

[20] We do not need to be more specific about the distribution of σ_a^2 across managers.

[21] Other matching models between managers and firm in the CEO compensation literature include Gabaix and Landier (2008), and Edmans, Gabaix, and Landier (2009).

will be matched with a relatively riskier manager. Intuitively, it is more likely that the estimated ability of a risky manager (with a high σ_a^2) will fall below the firing threshold of any firm (which is necessarily negative). To minimize the costs of dismissals and the costs of inefficient continuation of managers with low ability, it is more efficient to match a risky manager with a firm with good corporate governance (low k).

Proposition 4: Negative assortative matching

The matching equilibrium is characterized by negative assortative matching. Firms with low dismissal costs are matched with high variance CEOs, while firms with high dismissal costs are matched with low variances CEOs. The firms with highest dismissal costs are matched with reular managers with zero ability and zero variance.

Proof: See appendix.

Heterogeneity among firms and managers and the resulting matching equilibrium that we described yields two important results. First, it can explain that changes in corporate governance in a subset of firm have spillover effects on manager compensation in other firms. We know that in equilibrium, all firms will make nonnegative expected profits. The expected profit made by any firm with a cost of firing of k is equal to the expected profit it would make by giving a manager with the lowest σ_a^2 the same contract as the firm with which this manager is matched. But we know that this manager is matched with the firm with the highest firing cost k, i.e., with the worst corporate governance. This latter firm will therefore make zero expected profits, while all other firms (with a lower k) will make strictly positive expected profits. Their expected profits will be increasing in the maximum value of k among the existing firms. The equilibrium condition that firms' expected profits are nonnegative implies that an increase in the maximum value of k (i.e., a deterioration in the quality of corporate governance at firms which are already badly governed) results in a lower fixed wage for all managers, and more generally in lower managerial compensation.[22] The profits of firms On the contrary, an improvement in

[22] In practice, the fixed portion of pay is not limited to the salary, but it also includes items such as deferred

the quality of corporate governance of badly governed firms triggers an increase in pay in all firms. Our model can explain the fact that CEO pay rose as corporate governance improved (Holmstrom and Kaplan (2001), Murphy and Zabojnik (2004)). This fact cannot be explained by the skimming hypothesis, which would predict the opposite.

Second, our matching model can explain the Bertrand and Mullainathan (2001) finding that firms with bad corporate governance use contracts that display more pay for luck. Indeed, in a matching equilibrium, firms with bad corporate governance are matched with managers whose ability is more precisely estimated, and conversely. This means that firms with bad corporate governance design optimal long-term contracts for different types of managers than those of firms with good corporate governance. Since firms with different levels of corporate governance are matched with different types of managers, it is in principle possible that the observed differences in CEO pay across firms with different types of corporate governance are explained by differences in CEO characteristics.

To take an extreme example, suppose that some managers have a known ability ($\sigma_a^2 = 0$). Since $\hat{a} = a$ with probability one for these managers, only future business conditions, or "luck" (L), make the second period reservation wage of these managers stochastic. On the contrary, the reservation wage of managers with an unknown ability ($\sigma_a^2 > 0$) is stochastic for two reasons: future business conditions, and the updating of beliefs regarding their ability following their performance in the first period. We know that managers with a known ability will be matched to firms with the worse corporate governance, while managers with unknown ability will be matched to firms with better corporate governance. In the former case, all the variability in the second period pay of the manager will be attributable to luck. In the latter case, it will be attributable both to luck and to the updating of beliefs on manager's ability. That is, in firms with bad corporate governance, managers will be exclusively paid for luck, but they will not be paid for performance (since firm performance is not informative about the manager's ability, it is pure noise and should be filtered out of the contract). On the contrary, in firms with better

compensation and pension entitlements.

corporate governance, managers will be paid for luck and for performance.

In a more general case where the ability of different managers is estimated with a different precision, our model relates the degree of pay-for-luck relative to pay-for-performance of a given manager to his σ_a^2:

Proposition 6: $\frac{\frac{dw_2}{dL}}{\frac{dw_2}{dV}}$ is (weakly) decreasing in σ_a^2.

In turn, since managers with high σ_a^2 are matched to firms with bad corporate governance (i.e., firms with a high k), the model predicts that the degree of pay-for-luck relative to pay-for-performance is higher in firms with worse governance:

Corollary: $\frac{\frac{dw_2}{dL}}{\frac{dw_2}{dV}}$ is (weakly) increasing in k.

The higher the cost of dismissal, the more a firm pays its CEO for luck rather than for performance.

In summary, we have shown that firms with bad corporate governance will be matched with managers with a relatively low σ_a^2: firms that are badly governed will optimally select managers with a low variance of ability ("safe managers"). In addition, we have shown in equations (1) and (15) in the previous section that pay-for-luck is decreasing in σ_a^2. The model therefore predicts that pay-for-luck will be stronger in firms with bad corporate governance.

5 Other predictions and empirical implications

Our model can explain the fact that "governance correlates very little with pay for performance, only with pay for luck" (Bertrand and Mullainathan (2001)). In our model with explicit contracts, the sensitivity of pay for performance to σ_a^2 is zero, while the sensitivity of pay for luck to σ_a^2 is proportional to the sensitivity of η to σ_a^2, which is strictly negative. To summarize,

the model predicts that firms with better governance tend to be matched with managers whose ability is more uncertain (i.e., with a high σ_a^2). In addition, the model does not predict any cross-sectional variation in pay-for-performance across these firm-manager matches. However, it predicts that firms with better corporate governance (and the associated high σ_a^2) will tend to have less pay-for-luck.

Even though we do not explicitly derive predictions on this dimension in our simple two-periods model, there are reasons to believe that σ_a^2 will decrease over the tenure of a manager, as his ability becomes more accurately measured. An implicit prediction of our model is therefore that pay-for-luck should increase with CEO tenure. This is all the more interesting that Bertrand and Mullainathan derive a similar prediction with the skimming model: their hypothesis is that CEOs with a longer tenure are more entrenched, so that they can extract more monetary benefits in the form of asymmetric pay-for-luck. By contrast, equations (1) and (15) show that pay-for-luck is decreasing in the variance of the estimate of CEO ability. Since CEO ability is typically estimated more accurately over time, we expect pay-for-luck to increase over time, for a given CEO. In addition, the variance of the CEO ability is not related to pay-for-performance in our model, so that we expect no relationship between tenure and pay-for-performance. As is made clear in Bertrand and Mullainathan, the data suggest that there is indeed a positive relationship between tenure and pay-for-luck, but no relationship between tenure and pay-for-performance.[23] These two predictions, which are empirically validated, are common to the skimming model and the efficient contracting model.

This being said, Bertrand and Mullainathan also find that the positive relationship between tenure and pay-for-luck holds only for firms without a large shareholder on the board (with a large shareholder on the board, the relationship is not statistically significant). The skimming model can explain this, whereas our model of efficient contracting cannot.[24]

[23] Garvey and Milbourn (2003) also find that relative performance evaluation, which consists in filtering out one type of exogenous shock, namely the market index, is stronger for younger managers. They interpret this finding as evidence that firms tend to let older (and more wealthy) CEOs hedge against market fluctuations themselves, since they are better able to do so than young CEOs.

[24] Obviously, this does not mean that no model of efficient contracting can explain this feature of the data.

The model does not give any prediction regarding the frequency of firm-CEO separations depending on a measure of corporate governance. On the one hand, firms with bad corporate governance will tend to hire CEOs with a more precisely estimated ability, which tends to reduce bad surprises and the associated forced turnover. On the other hand, CEOs whose ability are more precisely estimated may be older, and therefore closer to retirement, which tends to increase voluntary turnover. A priori, it is not clear which effect will dominate.[25] In addition, the model predicts that forced turnovers will follow a downward revision of shareholders' beliefs about the CEO ability, but they will be unrelated to luck. In line with this latter prediction, Garvey and Milbourn (2006) do not find any statistically significant relationship between CEO turnover and luck.

6 Conclusion

The observation that CEOs are rewarded for luck has been at the center of the recent debate concerning CEO compensation. The skimming hypothesis of managerial behavior argues that CEOs set themselves their pay and that this explains the pay-for-luck phenomenon and the two other empirical facts linked with pay-for-luck: Pay-for-luck is asymmetric with CEOs being rewarded in good times but not in bad times, and Pay-for-Luck is more present in firms with bad governance,in the sense that managers are entrenched and firms cannot easily dismiss their inefficient CEOs.

In this paper, we developed a model of CEO compensation where all empirical regularities linked with the pay-for-luck phenomenon can be explained within the efficient contracting framework.

For example, the model of CEO discretion of Gromb, Burkart and Panunzi (1997) predicts that a CEO who is less monitored by shareholders (which will typically be the case with a large shareholder on the board) will take more initiatives. On the contrary, a CEO who is closely monitored by shareholders will tend to manage the firm more conservatively. All else equal, this suggests that the updating on CEO ability will tend to be much faster in the former case: the model would thus predict a stronger (positive) relationship between tenure and pay-for-luck for firms with a large shareholder on the board.

[25] Kaplan and Minton (2006) find no statistically significant relationship between CEO turnover and corporate governance.

Our analysis builds on Oyer and stresses the importance of retention motives to explain optimal contract. Firms want need to match the outside option of their CEOs and given that the options are likely to be better in good times, we get the pay-for-luck phenomenon. Asymmetric Pay-for-luck is generated naturally in our model as the consequence of an insurance motive when business conditions are less favorable. Finally, when we embed our model in a matching model between firms and CEOs, we see that firms with high costs of dismissal will be matched with CEOs with little variance, and that the amount of pay-for-luck in the optimal contract becomes more important as the variance of CEO's ability becomes small.

We do not claim that our model integrates all aspects of CEO compensation. In particular, we decided to completely ignore the incentives aspect of the problem. However, we show that the empirical anomalies pointed out by the defenders of the skimming hypothesis can be rationalized in a model of efficient contracting and that more convincing arguments need to be made before dismissing the efficient contracting paradigm.

Another contribution of our paper is to point out the importance of retention motives in the market for CEO and that more work is needed to understand how competition and retention interplay to influence optimal compensation schemes. Finally, our model based mostly on retention motives display an optimal contract that can be implemented using stock options. This finding rationalizes the use of stock options in CEO compensation, something that the literature on contracts designed to provide incentives could not achieve.

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8 Appendix

Proof of Proposition 2: According to (12), $\frac{dw_2}{dL} > 0$ if and only if $W_2(\hat{a}, L) > w_1^*$. According to (14), this is the case if and only if

$$\kappa + nV + (1 - n)bL > w_1^*$$

Or

$$L > \frac{1}{b}\Big(w_1^* - \kappa - \frac{\sigma_a^2}{\sigma_\epsilon^2 + \sigma_a^2}(a + \bar{\epsilon})\Big) \equiv \underline{L}$$

If this condition is not satisfied, then $w_2 = w_1^*$, and $\frac{dw_2}{dL} = 0$.

Proof of the Corollary of Proposition 3:

Comparing (17) and (12), we get, for $\gamma\hat{a}L \geq w_1^*$

$$\gamma\hat{a}L = nw_1^* + nP(V, L) \tag{18}$$

So that

$$\frac{d}{dL}\Big\{\gamma\hat{a}L\Big\} = n\frac{d}{dL}P(V, L) \tag{19}$$

Stock-options are fully indexed if and only if $\frac{d}{dL}P(V, L) = 0$. But this cannot be the case since the left-hand-side of (19) is different from zero, expect on a set of measure zero (for $\hat{a} = 0$).

Proof of Negative Assortative Matching

We will use the condition in proposition1-ii in Legros and Newman (2007) to prove that the matching will satisfy Negative Assortative Matching (NAM), that is firms with better corporate governance match with riskier CEOs.

For that consider two firms with firing costs $k > k'$ and two CEOs indexed by risk $\sigma^2 < \sigma'^2$.

Consider the long term contracts offered by the firm with high costs of firing to both types

of CEOs that lead to the same profit for the firm. To prove NAM, we just need to show that if the firm with low firing cost proposes long term optimal contracts to the CEOs that give them as much as the contracts proposed by the high cost firm, it would make larger profits when it proposes a contract to the risky CEO.

First, note that the expected utility of a CEO for a given long term contract depends only on w_1. This means that the low cost firm would propose the same long term contract as the high cost firm if it needs to ensure the same expected utility to the CEO.

The CEO is fired if the estimated ability of the current CEO is too low compared to the average ability of a new CEO. The dismissal condition is: $\hat{a} \quad k/L = a^k(L)$.

The extra profit of a firm with low dismissal costs that proposes the same contract as a firm with bad corporate governance is thus for a given L : $_\sigma(\quad k/L)\left(k \quad k'\right)$. When $\sigma^2 < \sigma'^2$, we have that $_\sigma\left(a^k(L)\right) > \ _{\sigma'}\left(a^k(L)\right)$ for $a < a$. Given that $a^k(L) < a$, we have that $_{\sigma_a}\left(a^k(L)\right)\left(k \quad k'\right)$ increases as σ^2 increases. Integrating over L, we get the result of negative assortative matching .

Proof of Proposition 6: Either $W_2 < w_1^*$, in which case $\frac{dw_2}{dL} = 0$ and $\frac{dw_2}{dV} = 0$, so that $\frac{\frac{dw_2}{dL}}{\frac{dw_2}{dV}}$ is independent of σ_a^2. Or $W_2 \quad w_1^*$, in which case $\frac{dw_2}{dL} = n(\frac{\sigma_\epsilon^2}{\sigma_a^2} + 1)$ and $\frac{dw_2}{dV} = n$, so that $\frac{\frac{dw_2}{dL}}{\frac{dw_2}{dV}} = \left(\frac{\sigma_\epsilon^2}{\sigma_a^2} + 1\right) \quad 1$, which is positive and decreasing in σ_a^2.