Switching Costs in Two-sided Markets*

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Abstract

This paper studies a dynamic two-sided market in which consumers face switching costs between competing products. In addition, they can be naïve or sophisticated as well as have fixed ("loyal") or independent preferences. I first show that if both sides of consumers single-home, then externality and naivete make it more likely that the first-period equilibrium price decreases in switching costs. I prove that this result is true even in a multi-homing regime. I then consider the situation where a proportion of the population on each side is naive, and the others behave rationally. I find that (i) platforms lower prices of one side when this side has more myopic consumers, (ii) platforms raise prices of one side when the other side has more myopic consumers, and (iii) greater firm patience intensifies competition.

Keywords: switching costs, two-sided markets, network externality, naivete, sophistication

JEL Classification: D4, L1

1 Introduction

In two-sided markets, platforms (e.g. Google's Android operating system and Apple's iOS) facilitate the interaction between two groups of agents, such that the participation of one group (e.g. apps developers) raises the value of participating for the other group (e.g. smartphone users). Very often, consumers in these markets also face switching costs between different platforms. These may arise, for instance, from the fact that smartphone users, who have learnt to use Android phones or iPhones, have strong incentives to continue to purchase products from the same firm. Switching costs may also be psychological in nature, meaning that people have their own preferences over brands and tend to tip in favor of the products that they have previously chosen.[1] This begs the

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[1]Klemperer (1995) gives many examples of different kinds of switching costs, and UK Office of Fair Trading documented some useful case studies.

question as to how switching costs affect the pricing strategies and profits of firms competing in two-sided markets. This question is also relevant for other two-sided markets

discriminate between old and new consumers. When firms cannot commit to future prices and customers are of one type, they will then compete fiercely at the earlier stage in order to capture customers whom they will exploit later. This "bargains-then-ripoff" pattern is the main result of the first-generation switching-cost models (see for instance Klemperer (1987a, b)). A second group of works allows for price discrimination, so firms can charge a price to its old customers and a different price to new ones. Chen (1997) analyzes a two-period duopoly with homogeneous goods. Under duopoly, consumers who leave their current supplier have only one firm to switch to. Since there is no competition for switchers, this permits the duopolists to earn positive profits in equilibrium. Taylor (2003) extends Chen's model to many periods and many firms. With three or more firms, there are at least two firms vying business for switchers, and if products are undifferentiated, these firms will compete away their expected profits from serving their current customers in order to attract them, and thus earning zero economic profit. More recent contributions include Biglaiser, Crémer and Dobos (2012), which studies the consequence of heterogeneity of switching costs in an infinite horizon model with free entry. They show that even low switching cost customers are valuable for the incumbent. Related articles that discuss behavior-based price discrimination include Fudenberg and Tirole (2000), and Villas-Boas (1999), but these models do not have explicit switching costs.

The design of pricing strategies to induce agents on both sides to participate has occupied a central place in the research on two-sided markets. The pioneering work is Caillaud and Jullien (2003), who analyze a model of imperfect price competition between undifferentiated intermediaries. In the case where all agents must single-home, the only equilibrium involves one platform attracting all agents and the platform making zero profit. In contrast, when agents can multi-home, the pricing strategy is of a "divide-and-conquer" nature: the single-homing side is subsidized (divide), while the multi-homing side has all its surplus extracted (conquer). Armstrong (2006) advances the analysis by putting forward a model of competition between differentiated platforms by using the Hotelling specification. He finds that the determinants of equilibrium prices are (i) the size of cross-group externalities, (ii) whether charges are levied on a lump-sum or per-transaction basis, and (iii) whether agents single-home or multi-home. His approach is the closest to mine. However, he focuses on a static two-sided markets model, while here with switching costs and different degrees of sophistication the problem becomes a dynamic one. As we shall see later, it is possible to reproduce Armstrong's result for extreme parameter values. Another closely related article is Rochet and Tirole (2006), who build a model of two-sided markets combining usage and membership externalities (as opposed to the pure-usage-externality model of Rochet and Tirole (2003), and the pure-membership-externality model of Armstrong (2006)), and derive the optimal pricing formulas. But they focus on the analysis of a monopoly platform only.

Substantial studies have been conducted in these two areas, switching costs and two-sided markets, separately. Yet only rarely are the analysis approached from a unified perspective. This article seeks to fill the gap. Besides this study, there is currently one

of consumer switching costs.

theoretical article on bridging the gap between the switching-cost and two-sided markets literature. Su and Zeng (2008) analyze a two period model of two-sided competing platforms. Their focus is on the optimal pricing strategy when only one group of agents have switching costs and their preferences are independent, while the current article studies a richer setting in which both sides bear switching costs, and consumers are heterogeneous in terms of loyalty and naivete. Therefore, one can view Su and Zeng (2008) as a special case of this paper.

2 Model

Consider a two-sided market with two periods. There are two groups of consumers, denoted by A and B. Assume that for some exogenous reasons in each period consumers choose to single-home.[3] Section 4.2 will extend the analysis to cover the multi-homing case. Both sides of consumers have switching costs: side-i (A or B) consumers have to incur switching cost $s_i > 0$ if they switch platform in the second period. On each side, consumers are heterogeneous in two dimensions. First, with probability $_i$ consumers' preferences are fixed ("loyal"), and with probability $1 - {}_i$ their preferences are re-distributed on the unit interval in the second period (independent preferences). Second, they can be naive or rational. Naive consumers, who are a fraction $_i$ of the population on side i, maximize their expected payoff in the first period without regard to how their purchasing decision will affect the offers they receive in the following period; while rational consumers, who form a fraction $1 - {}_i$ of side i's population, realize that their choices in the first period will influence the offers they receive in the second period. Naivete is modeled by setting the discount factor of side-i consumers $_i = 0$, while rational consumers have $_i > 0$. Moreover, I distinguish the firm's discount factor, denoted by $_F$, from the consumer's discount factor $_i$. There are two competing platforms, denoted 0 and 1, which enable the two groups to interact. Consider a simple Hotelling model, where consumers on each side are assumed to be uniformly located along a unit interval with the two platforms located at the two endpoints. Throughout the paper, we assume that platforms cannot price discriminate among his previous customers and customers who have bought the rival's product in the previous period.

The utility of a side-i consumer is

$$v_i + e_i n^j_{k,t} - |x - k| - p^i_{k,t};$$

where $i; j \in \{A; B\}; i \neq j$ since the two sides are symmetric. v_i is the intrinsic value of side-i consumers for using either platform. Assume that v_i is sufficiently large such that the market is fully covered. e_i is the benefit that consumer from side i enjoys from interacting with each agent on the other side (for brevity, I ignore the possibility that consumers also care about the number of people in the same group who join the platform). Suppose that each side is of mass 1, so that $n^i_{k,t}$ is the number of agents from side i (A or B) who are attached to platform k (0 or 1) in period t (1 or 2), while

[3] If both sides universally multi-home, switching costs play no role.

the number of agents from the same side in the same time period who are attached to the other platform is denoted by $1 - n^i_{k,t}$. Thus, $e_i n^j_{k,t}$ is the total external benefit from interacting with the other group. x is the location of the consumer. To keep things simple, I assume unit transport cost. Thus, $|x - k|$ is the transport cost when the consumer purchases from platform k. Platform charges are levied on a lump-sum basis: each agent from side i incurs a cost of $p^i_{k,t}$ when he joins platform k at time t.

Platform k's profit at time t is

$$\pi_{k,t} = p^A_{k,t} n^A_{k,t} + p^B_{k,t} n^B_{k,t}.$$

For simplicity, assume that there is no marginal cost of production. Assume also that $e_i < 1$ such that the profit function is well-defined. The solution concept for the game is subgame perfect equilibrium (SPE).

2.1 Second Period: the mature market

We now work backward from the second period, where each platform has already established a customer base. Given the first-period market shares $n^A_{0,1}$ and $n^B_{0,1}$, a side-i consumer, located at x on the unit interval, belonging to platform 0 in the beginning of the second period is indifferent between continuing to buy from platform 0 and switching to platform 1 if

$$v_i + e_i n^j_{0,2} - x - p^i_{0,2} = v_i + e_i(1 - n^j_{0,2}) - (1 - x) - p^i_{1,2} - s_i.$$

The indifferent consumer is given by $x = \theta^i_{0|0}$,[4] where the subscript "0|0" means that the agent buys from platform 0 this period conditional on the fact that he has previously bought from platform 0,

$$\theta^i_{0|0} = \frac{1}{2} + \frac{1}{2}[e_i(2n^j_{0,2} - 1) + p^i_{1,2} - p^i_{0,2} + s_i].$$

Another consumer from side i, positioned at x, previously purchased from platform 1 is indifferent between switching to platform 0 and continuing to purchase from platform 1 if

$$v_i + e_i n^j_{0,2} - x - p^i_{0,2} - s_i = v_i + e_i(1 - n^j_{0,2}) - (1 - x) - p^i_{1,2}.$$

The indifferent consumer $x = \theta^i_{0|1}$ satisfies

$$\theta^i_{0|1} = \frac{1}{2} + \frac{1}{2}[e_i(2n^j_{0,2} - 1) + p^i_{1,2} - p^i_{0,2} - s_i].$$

Taken together,

$$n^i_{0,2} = \lambda_i n^i_{0,1} + (1 - \lambda_i) n^i_{0,1} \theta^i_{0|0} + (1 - \lambda_i)(1 - n^i_{0,1}) \theta^i_{0|1}.$$

[4] With a slight abuse of notation, we will drop the second period subscript in $\theta^i_{0|0,2}$ since customers in the first period do not have any purchase history.

This equation shows that the number of agents from side i belonging to platform 0 in the second period is the sum of three terms. The first element represents with probability ${}_i$ the old customers are loyal. The second and third terms represent with probability $1 - {}_i$ the old customers are switchers (whose preferences are unrelated in the two periods): they can either be switchers who did not switch away from platform 0 or switchers who actually switched from platform 1 to platform 0.

Then solving for $n_{0,2}^A$ and $n_{0,2}^B$ simultaneously, we obtain the second-period market shares of groups A and B as follows:

$$n_{0,2}^i = \frac{+ \ {}_i + (1 - \ {}_i)(p_{1,2}^i - p_{0,2}^i) + e_i(1 - \ {}_i)(1 - \ {}_j)(p_{1,2}^j - p_{0,2}^j)}{2};$$

where

$$\begin{aligned} &= 1 - (1 - \ {}_A)(1 - \ {}_B)e_A e_B; \\ {}_i &= (2n_{0,1}^i - 1)(\ {}_i + (1 - \ {}_i)s_i) + (2n_{0,1}^j - 1)(1 - \ {}_i)e_i(\ {}_j + (1 - \ {}_j)s_j): \end{aligned}$$

Since I have assumed at the beginning that $e_i < 1$, so $\ > 0$.

The respective second-period profit of platforms 0 and 1 can be written as:

$$\begin{aligned} {}_{0,2} &= p_{0,2}^A n_{0,2}^A + p_{0,2}^B n_{0,2}^B; \\ {}_{1,2} &= p_{1,2}^A(1 - n_{0,2}^A) + p_{1,2}^B(1 - n_{0,2}^B): \end{aligned}$$

Substitute $n_{0,2}^A(p_{0,2}^A; p_{1,2}^A; p_{0,2}^B; p_{1,2}^B)$ and $n_{0,2}^B(p_{0,2}^A; p_{1,2}^A; p_{0,2}^B; p_{1,2}^B)$ into these two profit functions. Then, by differentiating ${}_{0,2}$ with respect to $p_{0,2}^A$ and $p_{0,2}^B$, and differentiating ${}_{1,2}$ with respect to $p_{1,2}^A$ and $p_{1,2}^B$, we obtain 4 equations and 4 unknowns.

$$\begin{aligned} \frac{@ \ {}_{0,2}}{@p_{0,2}^i} &= n_{0,2}^i - \frac{p_{0,2}^i}{2}(1 - \ {}_i) - \frac{p_{0,2}^j}{2}e_j(1 - \ {}_i)(1 - \ {}_j); \\ \frac{@ \ {}_{1,2}}{@p_{1,2}^i} &= 1 - n_{0,2}^i - \frac{p_{1,2}^i}{2}(1 - \ {}_i) - \frac{p_{1,2}^j}{2}e_j(1 - \ {}_i)(1 - \ {}_j): \end{aligned}$$

Solving the system of first-order conditions, one finds the following four second-period equilibrium prices.

$$p_{0,2}^i = \frac{1 - e_j(1 - \ {}_i)}{1 - \ {}_i} + \frac{{}_i \ {}_i + \ {}_i \ {}_j}{(1 - \ {}_i)\Delta}; \tag{1}$$

$$p_{1,2}^i = \frac{1 - e_j(1 - \ {}_i)}{1 - \ {}_i} - \frac{{}_i \ {}_i + \ {}_i \ {}_j}{(1 - \ {}_i)\Delta}: \tag{2}$$

where

$$\begin{aligned} \Delta &= 9 - (1 - \ {}_A)(1 - \ {}_B)(e_A + 2e_B)(e_B + 2e_A) > 0; \\ {}_i &= (2n_{0,1}^i - 1)(\ {}_i + (1 - \ {}_i)s_i); \\ {}_i &= 3 - e_j(e_j + 2e_i)(1 - \ {}_i)(1 - \ {}_j) > 0; \\ {}_i &= (1 - \ {}_i)(e_i - e_j): \end{aligned}$$

Note that if each side exerts the same external benefit on the other side ($e_i = e_j = e$ such that $\;_i = 0$), then the second-period equilibrium prices of one side are only influenced by own side's parameters such as loyalty, switching costs, market share, and externality.

2.1.1 E ect of switching costs on second-period pricing

Proposition 1. *In the mature market, the platform with a larger market share of a particular group can increase the price paid by this group of consumers as switching costs of them increase; whereas the other platform with a smaller market share decreases the price paid by this group as switching costs increase.*

Proof.

$$sign\frac{\partial p^i_{0,2}}{\partial s_i} = sign(n^i_{0,1} - \frac{1}{2});$$
$$\frac{\partial p^i_{0,2}}{\partial s_i} = -\frac{\partial p^i_{1,2}}{\partial s_i}.$$

□

The intuition underlying Proposition 1 is simple. When the market share of one group of consumers between the two platforms is asymmetric, the increase in switching costs of this group gives the platform with a larger customer base higher market power, whereas for the platform with a smaller customer base, it has to offer a lower price in order to attract this group of consumers. This represents the "ripoff" price of the "bargains-then-ripoff" pattern, which is fairly standard in the switching-cost literature. Notice that if the market share is equal between platforms, then switching cost has no effect on the second-period price, which is indeed the case when we solve the full equilibrium by taking into consideration the first-period results. The intuitive reason is that the second period is also the last period, and therefore switching cost should play no role.

Proposition 2. *In the mature market, whether the price paid by side-i consumers is increasing in the switching costs of side-j consumers depends on the platform's market share of side j and the relative strength of cross-group externalities.*

Proof.

$$sign\frac{\partial p^i_{0,2}}{\partial s_j} = sign(e_i - e_j)(n^j_{0,1} - \frac{1}{2});$$
$$\frac{\partial p^i_{0,2}}{\partial s_j} = -\frac{\partial p^i_{1,2}}{\partial s_j}.$$

□

The following corollary provides an easily interpretable version of Proposition 2 for the special case of $n^B_{0,1} > \frac{1}{2}$ and $e_B > e_A$.

Corollary 1. *In the mature market, if platform* 0 *has a larger market share of side-*B *consumers than platform* 1*, and the externality of side-*A *consumers on side-*B *consumers is stronger than otherwise (i.e.* $e_B > e_A$*), then the price charged to side-*A *consumers is decreasing in the switching costs of group-*B *consumers.*

The intuition behind Corollary 1 runs as follows. Since platform 0 has a larger side-B customer base, according to Proposition 1 it is able to charge higher prices to exploit these customers. It will then use this positive margin to attract side-A consumers, who are valuable customers to the platform as they exert strong externality on side-B consumers. Note that what platform 1 will do is just the opposite of platform 0 because of the asymmetric market shares.

Likewise, using Propositions 1 and 2 we can easily analyze how switching costs of consumers on each side affect the prices of the two platforms in other scenarios. But the central message is that in a one-sided market with switching costs a platform's current market share is an important determinant of its pricing strategy and profits (see Klemperer (1995)); in a multi-sided market it is crucial to also take into consideration the network externality. Relying on a one-sided logic may overestimate potential anti-competitive effects: according to Proposition 1 prices tend to increase in switching cost on the side that the platform has a larger market share; but this does not necessarily imply anti-competitive motives in two-sided markets, since according to Propositions 2 larger margin on one side is translated into smaller or even negative margin on the other side depending on the magnitude of externalities.

2.1.2 E ect of switching costs on second-period pro t

To compute the second-period equilibrium profit of each platform, we substitute the second-period equilibrium prices $p^i_{0,2}; p^i_{1,2}$ into $n^i_{0,2}$.

$$n^i_{0,2} = \frac{1}{2} + \frac{3 \; {}_i + (1 - \; {}_i)(e_i + 2e_j) \; {}_j}{2\Delta}:$$

Finally, put them into ${}_{0,2}$, we will obtain the second-period equilibrium profit that each platform makes. By differentiating it with respect to s_i, we obtain the following proposition:

Proposition 3. *In the mature market, if one platform has a larger customer base of a particular group, its second-period equilibrium profits are increasing in the switching costs of this group when the following holds true: the platform's first-period market shares of the two sides are not too small, and the cross-group externalities are not too different.*

See Appendix A for the proof.

Proposition 3 suggests that market share and cross-group externalities are important determinants as to whether the second-period profit of the platform increases in switching costs or not.

Corollary 2. *In the mature market, the equilibrium profits of the two platforms can be a decreasing function of switching costs under the following conditions:*

$$\frac{@\ \ {}_{0,2}}{@s_i} < 0$$

when $\ {}_i$ *is small,* $\ {}_j$ *is large,* $e_i > e_j$*, and* $n^i_{0,1} > \frac{1}{2}$*.*

Corollary 2 is a special case of Proposition 3, which shows that the second-period equilibrium profits can be decreasing in the switching costs for some values of loyalty, network externality, and market shares.

2.2 First Period: the new market

I now turn to examine the equilibrium outcomes in the first period where consumers are not attached to any platform. Recall that on side i, a proportion $\ {}_i$ of the consumers are naive (N). Since they care only about today, they have $\ {}_i = 0$. On the contrary, a proportion $1 - \ {}_i$ of side-i's population is rational (R). Since they fully anticipate the influence of their first-period decisions on the second period, they have $\ {}_i > 0$.

A naive consumer of side i is indifferent between buying from platform 0 and platform 1 if

$$v_i + e_i n^j_{0,1} - x - p^i_{0,1} = v_i + e_i(1 - n^j_{0,1}) - (1 - x) - p^i_{1,1};$$

which can be simplified to

$$x = \frac{1}{2} + \frac{1}{2}[e_i(2n^j_{0,1} - 1) + p^i_{1,1} - p^i_{0,1}] = \ {}^i_N:$$

A sophisticated consumer of side i is indifferent between purchasing from platform 0 and platform 1 if

$$v_i + e_i n^j_{0,1} - x - p^i_{0,1} + \ {}_i U^i_{0,2} = v_i + e_i(1 - n^j_{0,1}) - (1 - x) - p^i_{1,1} + \ {}_i U^i_{1,2}:$$

After some rearrangement, this gives

$$x = \frac{1}{2} + \frac{1}{2}[e_i(2n^j_{0,1} - 1) + p^i_{1,1} - p^i_{0,1} + \ {}_i(U^i_{0,2} - U^i_{1,2})] = \ {}^i_R;$$

where $U^i_{0,2}$ and $U^i_{1,2}$ are the respective expected utilities that a side-i agent obtains from joining platforms 0 and 1 in the second period, and are defined as follows.

$$\begin{aligned} U^i_{0,2} = &\ {}_i(v_i + e_i n^j_{0,2} - x - p^i_{0,2}) + (1 - \ {}_i)\ {}^i_{0|0}(v_i + e_i n^j_{0,2} - x - p^i_{0,2}) \\ &+ (1 - \ {}_i)(1 - \ {}^i_{0|0})(v_i + e_i(1 - n^j_{0,2}) - (1 - x) - p^i_{1,2} - s_i): \end{aligned}$$

$U^i_{0,2}$ is the sum of three terms. It shows that with probability $\ {}_i$ the consumer is loyal and chooses to join platform 0 in both periods, with probability $(1 - \ {}_i)\ {}^i_{0|0}$ he has

independent preferences but still chooses to stay with platform 0, and with probability $(1- \ )(1- \ ^i_{0|0})$ he has independent preferences and he switches to platform 1. Similarly,

$$\begin{aligned} U^i_{1,2} = & \ _i(v_i + e_i(1 - n^j_{0,2}) - (1 - x) - p^i_{1,2}) \\ & + (1 - \ _i)(1 - \ ^i_{0|1})(v_i + e_i(1 - n^j_{0,2}) - (1 - x) - p^i_{1,2}) \\ & + (1 - \ _i) \ ^i_{0|1}(v_i + e_i n^j_{0,2} - x - p^i_{0,2} - s_i). \end{aligned}$$

Using $U^i_{0,2}$ and $U^i_{1,2}$, we can obtain a more explicit expression for

$$\ ^i_R = \frac{1}{2} + \frac{e_i(2n^j_{0,1} - 1) + p^i_{1,1} - p^i_{0,1} + \ _i(\ _i + 2(1- \ _i)s_i)\frac{[(1-\mu_i)(e_i+2e_j)\theta_j+(3-\Delta)\lambda_i]}{(1-\mu_i)\Delta}}{2[1 + \ _i(\ _i + (1 - \ _i)s_i)]}.$$

The first-period market share of side-i is represented by

$$n^i_{0,1} = \ _i \ ^i_N + (1 - \ _i) \ ^i_R.$$

Then, we substitute $\ ^i_N$ and $\ ^i_R$ into this, and solve simultaneously for the first-period market shares of sides A and B:

$$n^i_{0,1} = \frac{1}{2} + \frac{e_i(1 - \ _j)(p^i_{1,1} - p^i_{0,1}) + \ _j(e_i \ _i + \ _i)(p^j_{1,1} - p^j_{0,1})}{2[(1 - \ _i)(1 - \ _j) - (e_i \ _i + \ _i)(e_j \ _j + \ _j)]},$$

where

$$\begin{aligned} \ _i = & \ _i + \frac{1 - \ _i}{1 + \ _i(\ _i + (1 - \ _i)s_i)}, \\ \ _i = & \frac{\ _i(\ _i + 2(1 - \ _i)s_i)(3 - \Delta)(1 - \ _i)(\ _i + (1 - \ _i)s_i)}{(1 - \ _i)\Delta[1 + \ _i(\ _i + (1 - \ _i)s_i)]}, \\ \ _i = & \frac{\ _i(\ _i + 2(1 - \ _i)s_i)(e_i + 2e_j)(1 - \ _i)(\ _j + (1 - \ _j)s_j)}{\Delta[1 + \ _i(\ _i + (1 - \ _i)s_i)]}. \end{aligned}$$

The expected profit of platform 0 is

$$\ _0 = p^A_{0,1}n^A_{0,1} + p^B_{0,1}n^B_{0,1} + \ _F \ _{0,2}.$$

The first-order conditions for maximizing $\ _0$ with respect to $p^A_{0,1}$ and $p^B_{0,1}$ are given as follows.

$$\frac{@ \ _0}{@p^i_{0,1}} = n^i_{0,1} - p^i_{0,1}\frac{\ _i(1 - \ _j)}{2 \, '} - p^j_{0,1}\frac{\ _i(e_j \ _j + \ _j)}{2 \, '} + \ _F \left[\frac{@ \ _{0,2}}{@n^i_{0,1}}\frac{@n^i_{0,1}}{@p^i_{0,1}} + \frac{@ \ _{0,2}}{@n^j_{0,1}}\frac{@n^j_{0,1}}{@p^i_{0,1}} \right]$$

where

$$\begin{aligned} ' = & (1 - \ _i)(1 - \ _j) - (e_i \ _i + \ _i)(e_j \ _j + \ _j); \\ \frac{@ \ _{0,2}}{@n^i_{0,1}} = & \frac{6}{(1 - \ _i)\Delta} + \frac{(e_i - e_j) - (e_i + e_j)(e_j + 2e_i)(1 - \ _j)}{\Delta} \quad (\ _i + (1 - \ _i)s_i) = \ _i. \end{aligned}$$

Consider the symmetric equilibrium such that $p^A_{0,1} = p^A_{1,1}$ and $p^B_{0,1} = p^B_{1,1}$, so that $n^A_{0,1} = n^B_{0,1} = \frac{1}{2}$ (hence $\;_i = 0$). Then, we simplify the first-order conditions and solve simultaneously for $p^A_{0,1}$ and $p^B_{0,1}$.

$$p^i_{0,1} = \frac{1 - \;_i}{\;_i} - \frac{\;_j}{\;_j} - e_j - \;_F\;_i.$$

The discussion is summarized in the following proposition.

Proposition 4. *The two-period two-sided single-homing duopoly model, where on each side a proportion* $\;_i$ *of the consumers are naive while the remaining consumers are sophisticated, and a fraction* $\;_i$ *of the consumers have fixed preferences while the others have independent preference, has a unique symmetric equilibrium. The first-period equilibrium prices for group* A *and group* B *are given respectively by*

$$p^A_{0,1} = \frac{1 - \;_A}{\;_A} - \frac{\;_B}{\;_B} - e_B - \;_F\;_A; \quad p^B_{0,1} = \frac{1 - \;_B}{\;_B} - \frac{\;_A}{\;_A} - e_A - \;_F\;_B; \tag{3}$$

and in the second period, each platform's equilibrium pricing strategies are given by

$$p^A_{0,2} = \frac{1 - e_B(1 - \;_A)}{1 - \;_A}; \quad p^B_{0,2} = \frac{1 - e_A(1 - \;_B)}{1 - \;_B}.$$

In equilibrium, the second-period pricing strategies is computed by substituting symmetric market shares into Equations (1) and (2).

3 Discussion

The analysis of the effect of switching costs on first-period prices is complicated as $p^A_{0,1}$ and $p^B_{0,1}$ are long expressions with many variables. An easier way to interpret the results is to start the discussion from pure switching-cost model (à la Klemperer) and pure two-sided model (à la Armstrong), and then extend to mixed models with different ingredients.

3.1 Pure Switching-cost Model

In the simplest two-period model of switching costs with two symmetric firms, zero marginal costs, and undifferentiated competition, the firms knowing that they can exercise its ex post market power in the second period over those consumers who are locked-in ($p_2 = s$), they are willing to price below cost ($p_1 = -\;s$) in the first period to acquire these valuable customers. Consumers foreseeing the possibility of being exploited in the future will become more indifferent between the two competing firms, and the only thing that matters to them is today's price. This implies that even rational consumers behave in a naive way, and this is equivalent to setting $\;_i; \;_i = 0$ in the current model.[5]

[5]Note that one can interpret $\delta_i(1 - \alpha_i)$ as a measure of rationality in the current analysis.

Moreover, $_i, e_i = 0$ because in this simple switching-cost model consumer's loyalty and network externality do not matter.

Together, Equation (3) is simplified to

$$p^i_{0,1} = 1 - \frac{2}{3} \; _F s_i$$

This implies that

$$\frac{\partial p^i_{0,1}}{\partial s_i} < 0.$$

This pattern of attractive introductory offers followed by higher prices to exploit locked-in customers (proved in Proposition 1) is well-known from the switching-cost literature.

3.2 Pure Two-sided Model

In a simple model of two-sided markets, there is only one period so that $_F, \; _i, \; _i = 0$, and loyalty and switching costs are irrelevant ($s_i, \; _i = 0$). Then, $_i = 1$, $_i, \; _i, \; _i = 0$, and Equation (3) is reduced to

$$p^i = 1 - e_j,$$

which is exactly the unique symmetric equilibrium in the two-sided single-homing model in Proposition 2 of Armstrong (2006).[6] The interpretation of this condition is that the side of the market which exerts a larger external benefit on the other side tends to face lower prices.

3.3 Switching Costs, Two-sided Markets and Naivete

Suppose that all consumers are myopic such that $_i, \; _i = 0$. For conciseness, assume preferences in the two periods are unrelated ($_i = 0$). Then, we are left with $s_i, e_i, \; _F > 0$. Equation (3) becomes

$$p^i_{0,1} = 1 - \; _F \; \frac{6}{\Delta} + \frac{e_i - e_j - (e_i + e_j)(e_j + 2e_i)}{\Delta} \; s_i - e_j.$$

Since the term in the square brackets (denoted as henceforth) is positive, this gives

$$\frac{\partial p^i_{0,1}}{\partial s_i} < 0.$$

[6] Since Armstrong's model is of one period only, we can replicate his result by setting $s_i, \mu_i = 0$ in the second-period equilibrium prices or with first-period equilibrium prices by taking away the effect of first period on the second period.

3.4 Switching Costs, One-sided Markets and Rationality

Suppose that all consumers and the two platforms are rational such that $\;_i, \;_F > 0$ and $\;_i = 0$. $e_i = 0$ since there is no two-sidedness. For simplicity, assume that consumers draw new preferences in each period ($\;_i = 0$). Equation (3) can be simplified to

$$p^i_{0,1} = 1 + \;_i s_i + \frac{4}{3}\;_i s_i^2 - \frac{2}{3}\;_F s_i.$$

Differentiating $p^i_{0,1}$ with respect to s_i, we obtain

Lemma 1. *In the two-period single-homing duopoly model, where network externality is absent, all consumers are rational, and their preferences are independent over time,*

i. If consumers are relatively less patient than the platforms, then the relationship between first-period equilibrium prices and switching costs is U-shaped.

ii. If consumers are relatively more patient than the platforms, then first-period equilibrium prices always increase in switching costs.

See Appendix B for the proof.

3.5 Switching Costs, Two-sided Markets and Rationality

The preliminaries are the same as the previous case, $\;_i, \;_F, s_i > 0$ and $\;_i, \;_i = 0$, except that the market is two-sided now ($e_i > 0$). In this case, Equation (3) can be rewritten as

$$p^i_{0,1} = 1 + \;_i s_i - \frac{2(3-\Delta)}{\Delta}\;_i s_i^2 - \frac{2\;_j}{\Delta}(e_j + 2e_i)s_j s_i - \;_F\; s_i - e_j.$$

Differentiating $p^i_{0,1}$ with respect to s_i, we obtain

Lemma 2. *In the two-period two-sided single-homing duopoly model, where all consumers are rational, both groups of consumers have switching costs, each side exerts the same external benefit on the other side, and their preferences are independent over time,*

i. For sufficiently weak externality,

- *If consumers are relatively less patient than the platforms, then the relationship between first-period equilibrium prices and switching costs is U-shaped.*
- *If consumers are relatively more patient than the platforms, then first-period equilibrium prices always increase in switching costs.*

ii. For sufficiently strong externality,

- *If consumers are relatively less patient than the platforms, then first-period equilibrium prices always decrease in switching costs.*

- *If consumers are relatively more patient than the platforms, then the relationship between first-period equilibrium prices and switching costs is inverted U-shaped.*

See Appendix C for the proof.
Collecting Lemmas 1 and 2 we obtain the following proposition.

Proposition 5. *In the two-period two-sided single-homing duopoly model, where all consumers are either naive or sophisticated, and their preferences are independent over time,*

i. If all consumers are sufficiently naive, then first-period equilibrium prices are decreasing in switching costs.

ii. The presence of strong network externality makes it more likely that switching costs will lower the first-period equilibrium prices.

Proof. Immediate from Lemmas 1 and 2. □

The intuition underlying this proposition is as follows. (i) When consumers are

3.6 Heterogeneous Consumers

I now turn to discuss, rather than having all consumers being rational or being naive, the consequences of having heterogeneous consumers on each side, where a proportion $_i$ of side-i consumers are myopic, while $1 - {}_i$ of them are forward-looking. Differentiating Equation (3) with respect to $_i;$ $_j$ and $_F$, we obtain the following:

Proposition 6. *In the two-period two-sided single-homing duopoly model, where on each side a fraction* $_i$ *of the consumers are naive, while* $1 - {}_i$ *of them are rational;* $_i$ *consumers are loyal, while the remaining ones have independent preferences,*

i. $p^i_{0,1}$ *is decreasing in* $_i$ *if* $_i$ *is close to 1.*

ii. $p^i_{0,1}$ *is increasing in* $_j$*.*

iii. $p^i_{0,1}$ *is decreasing in* $_F$*.*

See Appendix D for the proof.

The intuition behind this proposition is as follows. (i) shows that having more naive consumers on one side lowers the price of this side if this side contains many loyal consumers. The reason for this is that when all consumers of this side are loyal ($_i = 1$), they know that once they have purchased they will be locked-in with the same platform forever, and thus have to be offered a bigger carrot in the first period. Moreover, naive consumers, who care only about today, are more attracted by a current price cut. Therefore, an increase in loyal and naive consumers provides more incentives for platforms to compete aggressively. (ii) shows that having more naive consumers on one side will increase the price of the other side. Intuitively, smaller margins to convince the myopic consumers on one side to join the platform must be recouped through larger margins on the other side. This type of unbalanced price structure is common in two-sided markets. (iii) indicates that equilibrium prices are lower when the platforms are more patient. The intuition is that firms compete harder on prices because they foresee the advantage of having a large customer base in the next period.

3.7 Effect of switching costs on first-period profit

In equilibrium,

$$_0 = \frac{1}{2}p^A_{0,1} + \frac{1}{2}p^B_{0,1} + {}_{0,2}.$$

Thus,

$$\frac{@ {}_0}{@s_i} = \frac{1}{2}\frac{@p^A_{0,1}}{@s_i} + \frac{1}{2}\frac{@p^B_{0,1}}{@s_i}$$

because $_{0,2}$ are not affected by s_i in equilibrium.

As is well-known from the switching-cost literature, switching costs raise platforms' profits in the second period as prices are usually higher. However, the presence of market power over the locked-in consumers intensifies competition in the first period, and this

may result in a decrease in overall profit as switching costs increase.[8] More interesting is that here identifies additional channels which may make overall profit decreases in switching costs. In particular, network externality and naivete make it more likely that switching costs lower first-period equilibrium prices, and hence reducing overall profits.

4 Extensions

The analysis so far is based on a single-homing model, but this is not the only market configuration in reality. There are various ways to extend the model, for instance, one may consider the case where one group single-homes while the other group may join either one platform or both (commonly termed as "competitive bottlenecks" because each platform has exclusive market power over the multi-homing consumers), the two groups of consumers bear different switching costs, and the two platforms are asymmetric. I will sketch these extensions in turn, but leave the full-scale analysis of them for a separate study.

4.1 Competitive Bottlenecks

Suppose that side A continues to single-home, while side B may multi-home (or not).[9] Competitive bottleneck framework is typical in, for instance, the computer operating system market (users use a single OS, Windows, Mac or Linux, while engineers develop software for different OS), and online air ticket or hotel bookings (consumers use one comparison site such as skyscanner.com, lastminute.com or booking.com, but airlines and hotels join multiple platforms in order to gain access to each comparison site's consumers). Since side B can choose both platforms, switching costs and loyalty on this side are not relevant $(s_B;\ {}_B = 0)$.[10] The main difference from the single-homing model lies in the market share of side-B consumers, which can be described as follows. A consumer from this group is indifferent between buying and not buying from platform 0 if

$$v_B + e_B n_{0,2}^A - x - p_{0,2}^B = 0;$$

which can be simplified to

$$n_{0,2}^B = x = v_B + e_B n_{0,2}^A - p_{0,2}^B:$$

[8]See for instance Klemperer (1987a).

[9]Side B does not necessarily universally multi-home.

[10]Note that the concept of multi-homing is not compatible with switching costs in the current framework. As an example, think of the smartphone market. If the option to multi-home means consumers are able to use both iPhone and Android systems, then it is not reasonable to impose an additional learning cost on them if they switch platform. Another example is the media market. If multi-homing means that advertisers are free to put ads on platform A, B or both, then it does not make sense to impose a switching cost on advertisers who buy ads on one platform only. Even if we distinguish between learning switching costs (incurred only at a switch to a new supplier) and transactional switching costs (incurred at every switch), as in Nilssen (1992), switching cost is still not relevant on the multi-homing side because transaction costs and learning costs are equivalent in a two-period model, where consumers switch only once. This also explains why it is not useful to consider the case where both sides multi-home within this framework.

Similarly, a side-B consumer is indifferent between buying and not buying from platform 1 if

$$v_B + e_B(1 - n_{0,2}^A) - (1 - x) - p_{1,2}^B = 0;$$

which can be reduced to

$$n_{1,2}^B = 1 - x = v_B + e_B(1 - n_{0,2}^A) - p_{1,2}^B:$$

We solve the game by backward induction as before. To simplify the exposition, we focus on the special case where $e_A = e_B = e$, ${}_A = 0$, and ${}_A = {}_B = {}_F =$. Consider the symmetric equilibrium. We find the following first-period equilibrium prices:

$$\begin{aligned} p_{0,1}^A =& 1 + \frac{s_A}{3} - e^2 - \frac{v_B}{2} - \frac{2 \;\; s_A^2(3e^2 - 2)}{3(1 - e^2)}; \\ p_{0,1}^B =& \frac{v_B}{2}: \end{aligned}$$

Differentiating $p_{0,1}^A$ with respect to s_A, we obtain

Proposition 7. *In the two-period two-sided multi-homing duopoly model, where one side of the consumers multi-homes, while the other side single-homes, each side exerts the same external benefit on the other side, all consumers' preferences are independent over time, and they are equally patient as the platforms,*

i. For the group of consumers who bear switching costs,

- *If network externality is sufficiently weak, then the first-period equilibrium price paid by them always increases in switching costs.*
- *If network externality is sufficiently strong, then the relationship between the first-period equilibrium price paid by them and switching costs is inverted U-shaped.*

ii. If the market is fully covered, then prices tend to be higher on the side that multi-homes, and lower on the side that single-homes.

See Appendix E for the proof.

(i) implies that strong externality makes it more likely that first-period equilibrium prices decrease in switching costs, which is consistent with Proposition 5. However, (ii) is different from the single-homing model. Since side B multi-homes, there is no competition between the two platforms to attract this group. The high prices faced by the multi-homing side is a consequence of each platform having monopoly power over this side, and the large revenues are used in the form of lower prices to convince the single-homing side to join the platform.

Before, in the single-homing model welfare analysis is not meaningful: welfare is always the same because all consumers buy one unit of good, the size of the two groups is fixed, and the whole market is served. It ignores the possible demand-expansion and

demand-reduction effects of switching costs as the total demand is fixed. However, in the multi-homing regime welfare can be affected through participation, which is in turn determined by the price. In the second period, switching cost has no effect on price because this is the final period. Therefore, if switching cost reduces first-period price (see Proposition 7), then switching cost tends to increase welfare.[11] This is because lower price induces more consumers to multi-home. The more consumers multi-home, the better it is for the single-homing consumers as it increases the reach to them.

4.2 Asymmetric Sides

Suppose the two groups are asymmetric such that side-B consumers do not incur any switching costs in the second period ($s_B = 0$), and their preferences are independent ($_B = 0$). For simplicity, assume that all consumers are rational ($_i = 0$), and they have the same discount factor as the firm ($_A = \ _B = \ _F = \ $). Then, $p^B_{0,1}$ in Equation (3) becomes

$$p^B_{0,1} = 1 - e_A.$$

We can easily observe that s_A only influences $p^A_{0,1}$ but not $p^B_{0,1}$, which leads us to the next proposition.

Proposition 8. *In the two-period two-sided single-homing duopoly model, where only one side of the consumers bear switching costs, switching costs will only affect the price for the group of consumers with switching costs, but not the price for the other group.*

The intuitive reason is that s_A affects how side-A consumers will behave tomorrow, and this should in turn influence side-B consumers' choice tomorrow through externality. However, since preferences of side-B consumers in the two periods are unrelated ($_B = 0$), and they do not have switching costs ($s_B = 0$), every period's choice is independent for side-B consumers. Therefore, s_A does not influence the price charged to side B.[12]

4.3 Asymmetric Platforms

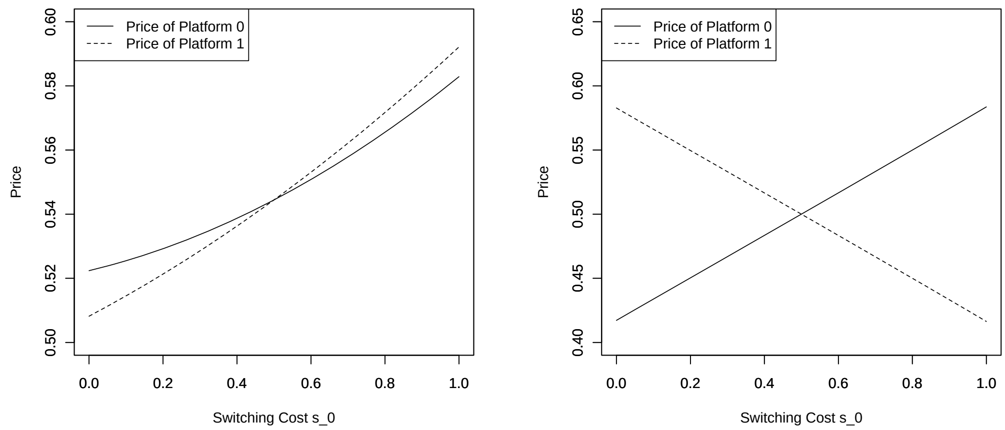
Finally, let us consider an asymmetric market in which the cost of switching from platform 0 to 1, denoted s_0, is different from the cost of switching from platform 1 to 0, denoted s_1. As an example, "iPhones are more expensive than most Samsung smartphones."[13] Can we attribute the difference in the pricing of devices between Apple and Samsung to the fact that Apple has successfully built an ecosystem that makes users hard to switch? To address this question, consider two groups of consumers who are asymmetric in the sense that only consumers from side A incur switching costs in the second period. For simplicity, all consumers here single-home. Following the same procedures,

[11] If there is quality choice as in Anderson et al. (2013), then welfare effects are less clear-cut: platform's investment in quality may change depending on whether multi-purchasing is allowed.

[12] This special case has been studied recently in Su and Zeng (2008). However, they did not emphasize this result, and hence did not provide any intuition.

[13] NBC News, "Apple is biggest US phone seller for first time," 1 February 2013, by Peter Svensson. http://www.nbcnews.com/technology/apple-biggest-us-phone-seller-first-time-1B8210244

and focusing on the special case where $e_A = e_B = e$, $\ _A = 0$, and $\ _A = \ _B = \ _F = \ $, we obtain long expressions of first-period equilibrium prices, which are difficult to generalize. In order to get explicit results, I confine attention to a particular numerical example in which $e = 0{:}5$, $\ = 0{:}1$, $s_1 = 0{:}5$, and $s_0 \in [0; 1]$.



(a) First-period Prices (b) Second-period Prices

Fig. 1. Equilibrium Pricing: the case with asymmetric platforms, asymmetric sides, and single-homing consumers.

The results are illustrated in Figure 1. Panel (a) presents the first-period pricing, and panel (b) reports the second-period pricing. Pricing of platform 0 is shown with a solid line, and that of platform 1 is drawn as a dotted line. It is shown that if $s_0 < s_1$, platform 1 will price lower than platform 0 in the first period, but price higher in the second period. The intuitive reason is that since platform 1 is relatively more expensive to switch away from in the second period, it is willing to price lower in the first period to acquire more consumers whom it can exploit later. On the contrary, if $s_0 > s_1$, platform 1, knowing that consumers will easily switch away tomorrow, prefers to raise its price today. While this result holds for relatively small externality, it needs to be confirmed for larger externality.

5 Conclusion

This article characterized the equilibrium pricing strategies of competing platforms in two-sided markets with switching costs. The main contribution is that it provided a useful model for generalizing arguments already used in the switching-cost and the two-sided markets literature, and for extending beyond the traditional results. Consistent with earlier research, there are some conditions under which switching costs reduce first-period prices but increase second-period prices (à la Klemperer); and prices tend to be lower on the side that exerts a stronger externality (à la Armstrong). However, the current model develops the idea further by proving that in a dynamic two-sided market

- as opposed to a merely static one - externality and naivete make it more likely that the first-period equilibrium price decreases in switching costs. Moreover, sophistication of consumers on each side as well as time preference of the platforms are shown to be crucial to understanding the pricing strategies. Returning to the iOS/Android example written at the opening of this paper, the results here indicates that if Apple has many loyal customers, increasing the sophistication of smartphone users will raise the price paid by this group of consumers; increasing the sophistication of apps developers will, on the other hand, lower the price paid by smartphone users; and prices are even lower when platforms become more patient.

Noteworthy is the fact that loyalty, naivete, network externality and switching costs have similar effects *ex post*: they all provide more incentives for the platforms to compete fiercely in the first period. But they are different *ex ante* in that disloyal consumers who are not attached to any brand may behave loyally because of switching costs, rational consumers anticipating the possibility of being exploited in the future because of switching cost may behave in a naive way today, and consumers may incur "collective" switching costs in the presence of network externality because they meet different people on different platforms.

The results should have important consequences on regulation. In a one-sided market with switching costs, penetration pricing to induce early adoption may call for consumer protection in later periods, for example, through policies that address compatibility issues (increasing compatibility can lower switching costs). In a two-sided market, the situation is subtler than it appears, since asymmetric price structures are common in these markets because they help increasing the participation of different groups of consumers. Therefore, policies should not rely on a one-sided approach as protecting one group of consumers may have broader repercussions on the other groups.

In future work, while the discussion of multi-homing consumers, asymmetric sides and asymmetric platforms in the Extension Section is explorative, a more complete analysis of them could constitute an interesting follow up. Another promising avenue is to study the consequence of introducing price discrimination based on the consumers' purchase history. For instance, one might consider the situation where platforms can set different prices for their own past customers and new customers who have bought the rival's product in the previous period on each side, but solving eight prices together might be a challenging task.

Appendices

A Proof of Proposition 3

The second-period equilibrium profit of platform 0 is

$$
\begin{aligned}
{}_{0,2} &= p^A_{0,2} n^A_{0,2} + p^B_{0,2} n^B_{0,2} \\
&= \frac{1 - e_B(1 - {}_A)}{1 - {}_A} + \frac{{}_A\ {}_A + {}_A\ {}_B}{(1 - {}_A)\Delta} \quad \frac{1}{2} + \frac{3\ {}_A + (1 - {}_A)(e_A + 2e_B)\ {}_B}{2\Delta} \\
&\quad + \frac{1 - e_A(1 - {}_B)}{1 - {}_B} + \frac{{}_B\ {}_B + {}_B\ {}_A}{(1 - {}_B)\Delta} \quad \frac{1}{2} + \frac{3\ {}_B + (1 - {}_B)(e_B + 2e_A)\ {}_A}{2\Delta} \quad :
\end{aligned}
$$

The first-order conditions with respect to s_A and s_B are

$$
\frac{@\ {}_{0,2}}{@s_i} = \frac{@\ {}_{0,2}}{@\ {}_i}(2n^i_{0,1} - 1)(1 - {}_i);
$$

where

$$
\begin{aligned}
\frac{@\ {}_{0,2}}{@\ {}_i} =& \frac{{}_i}{(1 - {}_i)\Delta} \quad \frac{1}{2} + \frac{3\ {}_i + (1 - {}_i)(e_i + 2e_j)\ {}_j}{2\Delta} \\
&+ \frac{3}{2\Delta} \quad \frac{1 - e_j(1 - {}_i)}{1 - {}_i} + \frac{{}_i\ {}_i + {}_i\ {}_j}{(1 - {}_i)\Delta} \\
&+ \frac{{}_j}{(1 - {}_j)\Delta} \quad \frac{1}{2} + \frac{3\ {}_j + (1 - {}_j)(e_j + 2e_i)\ {}_i}{2\Delta} \\
&+ \frac{(1 - {}_j)(e_j + 2e_i)}{2\Delta} \quad \frac{1 - e_i(1 - {}_j)}{1 - {}_j} + \frac{{}_j\ {}_j + {}_j\ {}_i}{(1 - {}_j)\Delta} \quad :
\end{aligned}
$$

Therefore,

$$
sign\frac{@\ {}_{0,2}}{@s_i} = sign(n^i_{0,1} - \frac{1}{2})
$$

if

$$
n^i_{0,1} > \frac{1}{2} \text{ and } \frac{@\ {}_{0,2}}{@\ {}_i} > 0
$$

or

$$
n^i_{0,1} < \frac{1}{2} \text{ and } \frac{@\ {}_{0,2}}{@\ {}_i} > 0:
$$

Together, they imply the conditions that $n^A_{0,1}; n^B_{0,1}$ are not too close to zero, as well as e_A and e_B are not too different.

Since platforms 0 and 1 are symmetric (i.e. differentiated in the same way), the qualitative result is the same for both platforms.

B Proof of Lemma 1

$$\frac{\partial p^i_{0,1}}{\partial s_i} = \frac{8}{3}\,{}_i s_i + {}_i - \frac{2}{3}\,{}_F;$$

$$\frac{\partial^2 p^i_{0,1}}{\partial s_i^2} = \frac{8}{3}\,{}_i > 0;$$

$$\frac{\partial p^i_{0,1}}{\partial s_i}\Big|_{s_i=0} < 0 \text{ if } {}_i < \frac{2}{3}\,{}_F.$$

C Proof of Lemma 2

$$\frac{\partial p^i_{0,1}}{\partial s_i} = {}_i - \frac{4\,{}_i s_i(3-\Delta)}{\Delta} - \frac{2\,{}_j(e_j+2e_i)s_j}{\Delta} - {}_F\ .$$

Given that $e_A = e_B = e;\ {}_A = {}_B = \ ;\ {}_i = 0$,

$$\frac{\partial p^i_{0,1}}{\partial s_i} = 1 - \frac{4s_i(3-\Delta)}{\Delta} - \frac{6es_j}{\Delta} - \frac{2}{3}\,{}_F;$$

$$\frac{\partial^2 p^i_{0,1}}{\partial s_i^2} = -\frac{4\,{}_i(3-\Delta)}{\Delta} \quad \begin{array}{ll} > 0 & \text{if } e \text{ is small,} \\ < 0 & \text{if } e \text{ is large,} \end{array}$$

$$\frac{\partial p^i_{0,1}}{\partial s_i}\Big|_{s_i=0} < 0 \text{ if } \quad 1 - \frac{6es_j}{\Delta} < \frac{2}{3}\,{}_F.$$

D Proof of Proposition 6

Differentiating Equation (3) with respect to ${}_i;\ {}_j$ and ${}_F$, we obtain the following:

$$\frac{\partial p^i_{0,1}}{\partial\ {}_i} \quad \begin{array}{ll} \leq 0; & \text{if } {}_i \to 1 \text{ or } e_i; e_j \text{ small,} \\ > 0; & \text{if } {}_i \to 0 \text{ and } e_i; e_j \text{ large,} \end{array}$$

since

$$\frac{\partial p^i_{0,1}}{\partial\ {}_i} > 0 \text{ if } \frac{{}_i + 2(1-\ {}_i)s_i}{1+2(1-\ {}_i)s_i} > \frac{\Delta}{3}.$$

$$\frac{\partial p^i_{0,1}}{\partial\ {}_j} \geq 0.$$

$$\frac{\partial p^i_{0,1}}{\partial\ {}_F} = -\ {}_i \leq 0.$$

E Proof of Proposition 7

For part (i),

$$\begin{aligned}
\frac{\partial p_{0,1}^A}{\partial s_A} &= \frac{}{3} - \frac{4\ \ s_A(3e^2-2)}{3(1-e^2)}; \\
\frac{\partial^2 p_{0,1}^A}{\partial s_A^2} &= -\frac{4\ \ (3e^2-2)}{3(1-e^2)} \quad \begin{cases} > 0 & \text{if } e \text{ is small,} \\ < 0 & \text{if } e \text{ is large,} \end{cases} \\
\frac{\partial p_{0,1}^A}{\partial s_A}\Big|_{s_A=0} &= \frac{}{3} > 0:
\end{aligned}$$

For part (ii), we compare the multi-homing model with the single-homing model with asymmetric switching costs. On the side with switching costs,

$$p_{mh}^A < p_{sh}^A \ \text{ if } e + \tfrac{v_B}{2} > 1;$$

where mh denotes the multi-homing framework, and sh denotes the single-homing framework. On the side without switching costs,

$$p_{mh}^B > p_{sh}^B \ \text{ if } e + \tfrac{v_B}{2} > 1:$$

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