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## **The Ratchet Effect Re-examined: A Learning Perspective**

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# The Ratchet Effect Re-examined: A Learning Perspective

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## Abstract

We examine the ratchet effect in a situation where both principal and agent are uncertain about the difficulty of the job, and must learn this over time. Since the agent can always increase his future continuation value by shirking, this must be deterred by higher powered incentives today. However, with a continuum of effort levels, high powered incentives provide an incentive for the agent to over-work. This implies an impossibility result: *no interior effort level is implementable* today. We explore the role of rent sharing in solving the problem. We also explain why the problem of implementability is less severe either with full commitment or in models of limited liability.

Keywords: ratchet effect, moral hazard, learning. JEL codes: D83, D86.

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# 1 Introduction

The ratchet effect is one of earliest problems noted by modern incentive theory, being prominent in discussions of Soviet planning (Berliner, 1954; Bergson, 1976). If the factory met or exceeded its plan target, the target for subsequent years was increased, thereby reducing current effort incentives for the factory manager (Weitzman, 1980). The problem also arises in capitalist firms, as Milgrom and Roberts (1990) note. When a firm installs new equipment, both firms and workers have to learn what is the appropriate work standard. It is efficient to use future information to adjust the standard. But this reduces work incentives today. Time and motion studies may reduce the degree of uncertainty regarding the technology, and ameliorate the effect, but their role is limited in contexts where a worker's performance improves with experience. Mathewson (1931), Roy (1952) and Edwards (1979) are workplace studies that document the importance of "output restriction". The ratchet effect is also prominent in the marketing literature. Salesmen are often paid bonuses that depend on the excess of sales over a quota, that is usually adjusted based on past performance. It also arises in a regulatory context, where both the regulator are uncertain about the effects of new technology (see Meyer and Vickers, 1997).<sup>1</sup>

Theoretical work on optimal contracts in the presence of the ratchet effect usually assumes that agent already has private information. It studies the dynamic mechanism design problem without commitment, on how to induce the agent to reveal her private information. This work includes Lazear (1986), Gibbons (1986), Freixas, Rochet & Tirole (1985), and Lafont & Tirole (1986). Lazear (1986) argues that high powered incentives are able to overcome the ratchet effect, and without any efficiency loss, assuming that the worker is risk neutral. Gibbons (1986) shows that Lazear's result depends upon an implicit assumption of long term commitment; in its absence, one cannot induce efficient effort provision by the more productive type. Lafont and Tirole (1986) prove a general result, that one cannot induce full separation given a continuum of types. Kanemoto and Macleod (1992) show that if the private information pertains to worker ability rather than job characteristics, then the ratchet effect is does not apply. Ickes and Samuelson (1987) explore the role of job transfers in mitigating the ratchet effect.

This paper studies the ratchet effect in a situation where both employer and the

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<sup>1</sup>See also Meyer (1995) and Meyer et al. (1996).

worker are learning about the technology, with ex ante symmetric information, a situation which has not been, to our knowledge, formally analyzed. Indeed, in Bolton and Dewatripont (2005) dynamic information revelation without commitment is seen as synonymous with the ratchet effect. This identification is, in our view, overstated. When uncertainty pertains not to the worker's innate characteristic, but rather to the nature of the job or match specific productivity — as, for example, when new machinery is introduced — learning becomes important. If the worker does acquire private information, this takes time, and one must allow for contractual remedies that could address this at the outset. Milgrom and Roberts (1990) present an illuminating (albeit somewhat informal) discussion of this problem. They assume a linear technology and normally distributed shocks, and argue that the ratchet effect implies that incentives are more high powered at the beginning, and the effort induced increases over time. While long term commitments alleviate the problem, it may be hard to stick to these commitments since they are likely to be inefficient ex post.<sup>2</sup> While their discussion is extremely insightful, they do not make explicit the assumptions underlying their analysis.

Our analysis can be seen as a part of the emerging literature on dynamic moral hazard with learning, or experimentation in a principal agent context that includes Bergemann and Hege (1998, 2005), Horner and Samuelson (2009), Kwon (2011) and Bhaskar and Mailath (2013). It also relates to Holmstrom's (1982) career concern model, and the literature that develops this further. We discuss this relation in detail in section 5.

Our key findings are that the ratchet effect gives rise to a very serious incentive problem, more serious than that which arises in the dynamic information revelation context. Indeed, in the absence of rent sharing, the problem can be very severe indeed, and lead to a complete breakdown of incentive contracts.

We study optimal contracting where the firm (the principal) and the agent (the worker) are ex ante symmetrically informed and uncertain about the difficulty of the job, and learn about this over time. To focus on the ratchet effect, we assume that the principal cannot commit to long term contracts, but chooses short-term contracts optimally. We allow for a rather general information structure, and study the dynamic

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<sup>2</sup>They discuss at length the case of Lincoln Electric, which developed a reputation for never revising the piece rate downwards in the light of information — see also Harvard Business Case study.

moral hazard problem that arises in this learning environment. We consider first a situation without rent-sharing, where the agent need not be paid any more than his outside option. Since the uncertainty pertains to the nature of the job, the outside option does not depend upon what is learnt regarding the job.

In our context, the ratchet effect arises from the possibility that the agent can manipulate the beliefs of the principal, by shirking. Suppose that both principal and agent have a common prior  $\lambda$  that the job is good or easy, and bad with complementary probability. Consider a contract where the principal seeks to induce some non zero effort level  $e^*$ . If the agent chooses  $e^*$ , and some output  $y^k$  is realized, both principal and agent will update their beliefs regarding the job to  $\mu_{e^*}^k$ , and the second period contract minimizes expected wage payments subject the incentive and individual rationality constraints defined by  $\mu_{e^*}^k$ . Suppose now that the agent deviates to  $e < e^*$ . Since effort is privately chosen, the principal's belief is still  $\mu_{e^*}^k$  after  $y^k$ , while the agent's belief is  $\mu_e^k$ . Under fairly general conditions, the agent's continuation value *strictly increases* if he shirks a little relative to  $e^*$ . The intuition comes from the fact that the individual rationality constraint always binds given belief  $\mu_{e^*}^k$ . If the agent is more pessimistic about the job (i.e.  $\mu_e^k < \mu_{e^*}^k$ ), then the constraint is violated, while if the agent is more optimistic (i.e.  $\mu_e^k > \mu_{e^*}^k$ ), then the individual rationality constraint holds strictly, and the agent makes a surplus above his reservation utility. Now, when the IR constraint is violated, the agent will simply refuse the contract and earn his reservation utility, and therefore suffers no loss. Since the agent accepts the payoff gains but can refuse the payoff losses, he will benefit as long as there is some signal  $y^k$  such that  $\mu_e^k > \mu_{e^*}^k$ , i.e. where he is more optimistic regarding the job than the principal thinks that he is.

We show that there always exists some signal  $y^k$  such that  $\mu_e^k > \mu_{e^*}^k$  as long as  $e < e^*$ . This follows from the martingale property of beliefs. The expectation of the agent's posterior, over all signal realizations, must equal his prior,  $\lambda$ , regardless of whether the agent performs the experiment  $e^*$  or the experiment  $e$ . Since good signals have higher probability under  $e^*$  than under  $e$ , this equality of expectations can only be satisfied if there are some signals such that  $\mu_e^k > \mu_{e^*}^k$ .

Since the agent's second period continuation value is higher when he deviates to low effort in period one, as compared to the case where he does not deviate, this implies that the incentive constraint in the first period must be modified. That is, the principal must provide greater incentives for high effort than she would need to

do in a static context, where there was no second period. This argument is quite general, and holds as long as the signal structure satisfies the property that a signal that is informative of high effort is also informative of the job being easy. Thus current period incentives have to be sufficiently high powered so as to deter all such downward deviations. This is the point made by Milgrom and Roberts (1990) – our contribution, so far, is to show that this argument generalizes with rather mild assumptions on the information structure. In particular, it is possible that the agent is more pessimistic than the principal after some realizations of the output – the argument still applies.

The main contribution of the paper is to show that by deterring downward deviations, the principal makes upward deviations profitable. Let  $W(e, e^*)$  denote the agent's continuation value at  $t = 2$ , net of his outside option, when the principal seeks to induce  $e^*$ , and he chooses  $e < e^*$ . The preceding arguments imply that  $W(e, e^*) > 0$  for  $e < e^*$ .  $W(e^*, e^*) = 0$  since the agent gets his reservation utility on the equilibrium path. Also,  $W(e, e^*) \geq 0$  for  $e > e^*$  since the agent cannot get less than his outside option. We see therefore that there is fundamental convexity of the continuation value function – take any convex combination  $\lambda e + (1 - \lambda)e' = e^*$ , and we note that  $\lambda W(e, e^*) + (1 - \lambda)W(e', e^*) > 0 = W(e^*, e^*)$ . Since the continuation value function is strictly convex at  $e^*$ , the first order condition for implementing  $e^*$  can never be satisfied, as long as  $e^* \in (0, 1)$ .

One solution to this problem is to the leaving of rents to the agent. This makes the continuation value smooth at  $e^*$ . If the rents are large enough, and the cost of effort function is sufficiently convex, this alleviates the implementability problem. We also explore limited liability models, and show that the ratchet effect is a less robust phenomenon in such a context.

## 2 The model

Our model combines moral hazard with uncertainty regarding job difficulty. Specifically, the job is either good (easy) or bad (hard), i.e. the job type is  $\alpha \in \{G, B\}$ . Let  $\lambda \in (0, 1)$  denote the common prior that  $\alpha = G$ . The agent chooses effort  $e \in \{0, 1\}$ . Let  $y \in Y = \{y_1, y_2, \dots, y_n\}$  denote the signal that is realized following effort choice. This depends, stochastically, on both the type and the effort chosen. Let  $p_{e\alpha}^k$  be the probability of signal  $y^k$  given effort  $e$  and type  $\alpha \in \{G, B\}$ . Thus for each signal  $y^k$ ,

we have a 4-tuple  $(p_{0B}^k, p_{1B}^k, p_{0G}^k, p_{1G}^k)$ . With a slight abuse of notation, we may also define  $p_{1\mu}^k$  (resp.  $p_{0\mu}^k$ ) to be the probability of signal  $k$  when effort level 1 (resp. 0) is chosen, given that  $\mu$  is the probability that the agent is type  $G$ .

We shall distinguish two types of likelihood ratio, the likelihood ratio on efforts for a given type (or belief over types) and the likelihood ratio over types for a given effort choice. The former is relevant for providing effort incentives, while the latter determines Bayesian learning. Let  $\ell_\alpha^k = \frac{p_{1\alpha}^k}{p_{0\alpha}^k}$  be the likelihood ratio for signal  $k$  for type  $\alpha$ . Generalizing this,  $\ell_\mu^k = \frac{\mu p_{1G}^k + (1-\mu)p_{1B}^k}{\mu p_{0G}^k + (1-\mu)p_{0B}^k}$  denote the likelihood ratio for signal  $k$  when  $\mu$  is the probability that the agent is type  $G$ . Let  $\ell_e^k = \frac{p_{eG}^k}{p_{eB}^k}$  be the likelihood ratio for signal  $k$  for effort level  $e$ .

Our main assumption, that is maintained throughout this paper, is as follows:

**A1** All probabilities belong to  $(0, 1)$ . For some  $y^k$ ,  $p_{1G}^k \neq p_{0B}^k$  i.e. there exists some informative signal. For any informative signal  $y^k$ ,  $p_{1B}^k$  and  $p_{0G}^k$  lie in the interior of the interval spanned by  $p_{1G}^k$  and  $p_{0B}^k$ , i.e.  $p_{1B}^k, p_{0G}^k \in (\min\{p_{1G}^k, p_{0B}^k\}, \max\{p_{1G}^k, p_{0B}^k\})$ .

To provide some intuition for this assumption, let  $Y^H$  be the set of high signals, where  $p_{1G}^k > p_{0B}^k$ . Then this assumption implies that if  $y^k \in Y^H$ ,  $\ell_\alpha^k > 1$  for  $\alpha \in \{G, B\}$  and  $\ell_e^k > 1$  for  $e \in \{0, 1\}$ . That is, if a signal is more likely when a given type of agent chooses high effort, it is also more likely for a given effort level when the job is the good type. This implies that signals that are indicative of high effort are also indicative of the agent being the good type. Similarly, let  $Y^L$  be the set of low signals, where  $p_{1G}^k < p_{0B}^k$ . The assumption implies that if  $y^k \in Y^L$ ,  $\ell_\alpha^k < 1$  for  $\alpha \in \{G, B\}$  and  $\ell_e^k < 1$  for  $e \in \{0, 1\}$ , so that a low signal indicates low ability as well as low effort. Finally, we may have some uninformative signals when  $p_{1G}^k = p_{0B}^k$ , where all likelihood ratios are one, but since there is at least one informative signals, both  $Y^H$  and  $Y^L$  are non-empty. Let  $Y^U$  denote the set of uninformative signals, and let  $\Pr(Y^U)$  denote the probability that an uninformative signal is realized – this does not depend upon effort choice or ability.

We shall assume that the agent's payoff in any period is given by  $u(w) - c(e)$  where  $u(\cdot)$  is strictly concave, and unbounded, while  $c(\cdot)$  is increasing.

We begin our analysis by focusing on the principal's cost minimization problem. That is, we assume that the principal seeks to induce high effort in every period, and solve for the sequentially optimal dynamic contract that minimizes expected wage costs. Specifically, we study the dynamic game induced by this contracting problem, and solve for perfect Bayesian equilibria that satisfy sequential rationality, with beliefs

given by Bayes rule. We do not have to deal with out of equilibrium beliefs, since there are no observable deviations. Since effort choice by the agent is private and public signals have full support, the principal does not see an out of equilibrium action, except when the game ends by the agent refusing the contract (at which point, beliefs are moot).

### 3 Binary effort

The focus of our paper is on the case where the principal may make commitments within the period, but cannot commit to future contracts. We shall also focus on the case the principal seeks to induce high effort with probability one. There is an initial common prior probability  $\lambda \in (0, 1)$  that the job is good. Suppose that the principal and agent interact for two periods. The agent discounts future payoffs at rate  $\delta \in (0, 1]$ , while principal discounts at rate  $\beta \in [0, 1]$ . We shall assume that neither the principal nor the agent can commit in period one regarding the contract in period two. One interpretation of the model is that there are two short term principals, one arriving in period one and the second arriving in period two, after consumption has taken place in period one. The principal in period two observes the public signal (output) in period one. This implies that wages paid have to satisfy incentive compatibility and individual rationality period by period.

Suppose that the principal wants to induce  $e = 1$  in both periods. Period 2 contracts are straightforward. Given that  $e = 1$  is chosen, the principal's beliefs about the agent's beliefs are degenerate, and are given by  $\mu_1^k$  after signal  $y^k$ . Thus the period two contract after signal  $y^k$  is given by  $w(\mu_1^k) \in \mathbb{R}^{|Y|}$ . Let  $w_j(\mu_1^k)$  denote the wage paid under the optimal second period contract after second period signal realization  $y_j$  given that the principal has belief  $\mu_1^k$ . We now show that the agent can always increase his continuation value by deviating to  $e = 0$  in the first period. Suppose that the agent deviates in period one and chooses  $e = 0$ . His period two beliefs are now different from the principal's beliefs about the agent's beliefs. In particular, there is at least one signal realization such that he becomes more optimistic about his ability. Since the agent suffers no penalty when he becomes more pessimistic – he quits and gets his outside utility, which is the same as he would get when he chooses high effort, this implies that his continuation value increases.

The fact that the agent always becomes more optimistic at some signal realization



after deviating is a consequence of the martingale property of beliefs. For any effort level  $e$  that the agent chooses, the expectation of his posterior must equal his prior,  $\lambda$ . Thus his expected beliefs under  $e = 0$  must equal his expected beliefs under  $e = 1$ . Since  $e = 1$  makes signals in  $Y^H$  more likely than when  $e = 0$  is chosen the equality of expectations can only be satisfied if there is some signal realization  $y$  such that  $\mu_0^k > \mu_1^k$ , where  $\mu_e^k$  is the posterior probability that the agent is the good type given signal realization  $y^k$  and effort choice  $e$ .

**Lemma 1** *There exists some  $k$  such that  $\mu_0^k > \mu_1^k$ .*

**Proof.** From the martingale property of beliefs,  $\mathbf{E}(\mu_1^k | e = 1) = \mathbf{E}(\mu_0^k | e = 0) = \lambda$ , i.e.

$$\sum_{k=1}^n p_{0\mu}^k \mu_0^k = \sum_{k=1}^n p_{1\mu}^k \mu_1^k.$$

This can be written as

$$\sum_{k=1}^n$$

The following examples illustrate our arguments. Let output be binary, so that  $y \in \{y^H, y^L\}$ . Tables 1-3 give examples of information structures that satisfy our assumptions, where the entries show the probability of the signal  $y^H$ , and  $0 < q < p < 1$ . In all these examples, high output is most likely when  $\alpha = G$  and  $e = 1$ , and least likely when  $\alpha = B$  and  $e = 0$ . Let  $\lambda = 0.5$ , although any interior value will suffice.

	$e = 1$	$e = 0$
G	$p$	$p$
B	$p$	$q$

Example 1

Consider example 1, where high effort only makes a difference to output when the job is bad. Thus  $e = 0$  is a more informative experiment than  $e = 1$ . Indeed,  $e = 1$  is uninformative about the realization of  $\alpha$ , and if the principal induces high effort at  $t = 1$ , his posterior will equal the prior after either signal realization. Since  $e = 0$  is informative, the agent becomes more optimistic after a success and more pessimistic after a failure. That is  $\mu_0^H = \frac{p}{p+q} > \mu_1^H = \frac{1}{2}$  and  $\mu_0^L = \frac{q}{p+q} < \mu_1^L = \frac{1}{2}$ . The agent will quit after observing  $y^H$ . After  $y^H$ , he stays on the job and earns a surplus. In this case, he chooses  $e = 0$ , since his greater optimism implies that the incentive constraint is violated.

	$e = 1$	$e = 0$
G	$p$	$q$
B	$q$	$q$

Example 2

Example 2 is the polar opposite of the first example, since effort only makes a difference when the job is good, and  $e = 0$  is uninformative. If he deviates to  $e = 0$ , he becomes more pessimistic than the principal after a success, and more optimistic after a failure. That is  $\mu_1^H = \frac{p}{p+q} > \mu_0^H = \frac{1}{2}$  and  $\mu_1^L = \frac{q}{p+q} < \mu_0^L = \frac{1}{2}$ . Thus he quits

after a success and earns a surplus after a failure. With greater optimism, he has more incentives to exert effort, and his incentive constraint is slack.<sup>3</sup>

	$e = 1$	$e = 0$
G	$p$	$\frac{p+q}{2}$
B	$\frac{p+q}{2}$	$q$

Example 3

In example 3 high effort raises the probability of success by the same magnitude, regardless of the nature of the job. In this case, it may be verified that  $\mu_0^H > \mu_1^H$  and  $\mu_0^L > \mu_1^L$ , so that the agent is more optimistic than the principal after *either* output realization. Thus the agent always earns a surplus after deviating to  $e = 0$ . Since his incentive constraint is satisfied under his beliefs, it is optimal for him to exert high effort at  $t = 2$ . This example seems slightly paradoxical, since  $\mu_0^k > \mu_1^k$  for all output realizations  $y^k$ . We label this phenomenon *uniform optimism* – as we shall see, many of our subsequent characterization results depend upon whether or not uniform optimism holds. Notice that uniform optimism does not violate the martingale property of beliefs – in our example,  $y^H$  has lower probability under  $e = 0$  than under  $e = 1$ , and thus one can have the equality of  $\mathbf{E}(\mu_1^k | e = 1)$  and  $\mathbf{E}(\mu_0^k | e = 0)$ , even though  $\mu_0^k > \mu_1^k$  for every value of  $k$ .

Turning to period 1 contract, this must satisfy IR with the prior beliefs  $\lambda$  and also a modified IC given these beliefs. We turn to deriving this modified IC. The agent's surplus, i.e. his continuation utility relative to  $\bar{u}$ , after signal  $y^k$  and deviation  $e = 0$  in the event that he stays on the job is denoted by  $V(\mu_0^k, \mu_1^k)$  – it depends upon his belief  $\mu_0^k$ , and also upon  $\mu_1^k$ , since second period wages depend upon  $\mu_1^k$ . If  $\mu_0^k < \mu_1^k$ , the agent quits and  $V(\mu_0^k, \mu_1^k) = 0$ . If  $\mu_0^k \geq \mu_1^k$ , he stays on the job and chooses whichever effort level gives him a higher payoff. Thus  $V(\mu_0^k, \mu_1^k)$  is given by

$$V(\mu_0^k, \mu_1^k) = [\mu_0^k - \mu_1^k] \max \left[ \sum_j (p_{1G}^j - p_{1B}^j) u(w_j(\mu_1^k)), \sum_j (p_{0G}^j - p_{0B}^j) u(w_j(\mu_1^k)) \right], \quad (1)$$

<sup>3</sup>In examples 1 and 2, a weaker version of A1 holds since  $p_{1B}^H$  and  $p_{0G}^H$  are not necessarily strictly less than  $p_{1G}^H$  or strictly greater than  $p_{0B}^H$ . As remark 8 makes clear, the agent necessarily gets a surplus when he is more optimistic, but he could get his reservation utility when he is more pessimistic, e.g. by choosing  $e = 1$  in example 1, and  $e = 0$  in example 2.

if  $\mu_1^k \geq \mu_0^k$ , and  $V(\mu_0^k, \mu_1^k) = 0$  if  $\mu_0^k < \mu_1^k$ .

Therefore, the agent's expected increase in continuation utility from choosing  $e = 0$  in the first period is given by  $W(0, 1)$ , where the first argument denotes the agent's actual action, and the second denotes the action the principal seeks to induce:

$$W(0, 1) = \sum_k V(\mu_0^k, \mu_1^k)(\lambda p_{0G}^k + (1 - \lambda)p_{0B}^k).$$

The modified IC for the first period is given by

$$\mathbf{E}(w^1|e = 1, \lambda) - \mathbf{E}(w^1|e = 0, \lambda) \geq c(1) - c(0) + \delta W(0, 1). \quad (2)$$

$$\lambda \sum_k (p_{1G}^k - p_{0G}^k) u(w_k^1) + (1 - \lambda) \sum_k (p_{1B}^k - p_{0B}^k) u(w_k^1) \geq c(1) - c(0) + \delta W(0, 1). \quad (3)$$

The IR constraint is unaffected and is given by

$$\mathbf{E}(w^1|e = 1, \lambda) - c(1) \geq \bar{u}. \quad (4)$$

Given an equilibrium where the agent chooses  $e = 1$  at  $t = 1$ , the second period contract does not directly depend upon the wages chosen in the first period. This follows from the fact that the second period contract after any signal realization depends only on the beliefs,  $\mu_1^k$ , and not upon first period wages. This implies that  $\mathbf{E}(\hat{V}(\lambda))$  is given, and also does not depend upon first period wages. Since first period wages do not affect the principal's continuation payoff, the optimal first period contract minimizes expected wages subject to the IR constraint and the modified incentive constraint. Thus it is the standard static optimal contract as in Holmstrom (1979), but with a more stringent incentive constraint. The characterization of the optimal contract in this case is standard (i.e. both constraints hold, and the marginal utility at any signal realization is related to the likelihood ratio). Inspecting the constraints (2) and (4), we see that it is as though the agent has a lower cost of low effort, i.e.  $c(0)$  is reduced by an amount equal to  $\delta W(0, 1)$ , while the cost of high effort is unaffected. Thus the incentive scheme needs to be more high powered, in order to dissuade the agent from increasing his continuation value by deviating to low effort. The agent is subjected to greater risk, and in consequence, there is a *dynamic*

*agency cost* that magnifies the static agency cost. Since the individual rationality constraint binds, the agent does not benefit in any way from this dynamic moral hazard. We summarize our results in the following proposition.

**Proposition 2** *The optimal dynamic contract that induces  $e = 1$  in both periods is follows: i) in period 1, the contract wages minimize expected wage payments given the modified IC (2) and the IR (4), which hold with equality. ii) in period 2, the contract after signal realization  $y^k$  is given by the static contract  $\mathbb{W}(\mu_1^k)$ , corresponding to common beliefs  $\mu_1^k$ . Since the incentive constraint is more stringent in the dynamic contract, the first period contract has more high powered incentives and the principal incurs a dynamic agency cost since the agent must bear more risk.*

One question that arises is, can the principal not do better by screening at the beginning of the second period? The answer is no; in equilibrium, the probability that the agent chooses  $e = 0$  is zero, and thus the screening problem is one where the probability assigned by the principal to the agent having a belief  $\mu_0^k$  equals zero. The solution to this screening problem is to offer a contract that is only acceptable to type  $\mu_1^k$ , and which leaves him no rents.

The dynamic agency cost that arises in period one is due to the fact that the agent can increase his continuation value by, choosing low effort. We turn now to some of the factors that influence the size of the dynamic agency cost.

One immediate observation is that the dynamic agency cost increases if the agent becomes more patient, i.e. if  $\delta$  is larger. That is the shadow of the future looms larger if the agent values it more. Notice that the discount factor of the principal plays no role in the analysis, in the absence of commitment.

## 4 A continuum of possible effort levels

We now assume that effort  $e$  must be chosen from  $[0, 1]$ . For simplicity, we focus on the two period case — extension to multiple periods is straight-forward. With a continuum of effort levels, we need to employ the first-order approach to solve for the optimal contract, even in the static case. We therefore assume the Hart-Holmstrom (1987) sufficient conditions for the validity of this approach. Assume that cost of

effort,  $c(e)$  is strictly increasing, strictly convex and differentiable. Assume that the probability of signal  $y^k$ , conditional on type  $\alpha \in \{G, B\}$  and  $e \in [0, 1]$  is linear in effort, and equal to<sup>4</sup>

$$p_{e\alpha}^k = ep_{1\alpha}^k + (1 - e)p_{0\alpha}^k.$$

The revenue from inducing  $e$  at belief  $\mu$ ,

$$R(e, \mu) = \sum_k p_{e\mu}^k y^k,$$

is linear in  $e$ . We assume it is strictly increasing in  $e$ , since otherwise, inducing  $e = 0$  is optimal. Note that the cross-partial  $\frac{\partial R(e, \mu)}{\partial e \partial \mu}$  maybe positive or negative, so that it may be less profitable or more profitable to induce higher effort at optimistic beliefs, as in examples 2 and 3 respectively.

**Definition 3** *Effort  $\hat{e}$  is implementable if there exists some contract  $(w^k)_{k=1}^{|Y|}$  such that  $\hat{e}$  is optimal for the agent. A contract  $(\hat{e}, (\hat{w}^k)_{k=1}^{|Y|})$  is optimal if it maximizes the principal's profits over all  $(e, (w^k)_{k=1}^{|Y|})$  such that  $e$  is implementable.*

**Claim 4** *In the final period, for any public belief  $\mu$ , every effort  $e \in [0, 1]$  is implementable. The profit maximizing level of  $e$ ,  $\hat{e}(\mu)$  and the corresponding optimal wages,  $\hat{w}_j(\mu)$ , satisfy the first order conditions for the principal's maximization problem. Let  $(w^k)_{k=1}^{|Y|}$  be an arbitrary contract at  $t = 2$ . There is a unique effort level that maximizes the agent's utility given this contract.*

We omit the proof the first part of this claim, since it is straightforward, and almost identical to the argument in Hart and Holmstrom (1987). To prove the second part, note that the agent's payoff from choosing  $e$  equals  $\mathbf{E}(u(w|e) - c(e))$ . Since the  $p_{e\mu}^k$  is linear in  $e$ , the first term is linear in  $e$  and  $c(e)$  is strictly concave, so there is a unique solution to the agent's maximization problem.

Turning to the first period, our main result is an impossibility theorem.

**Theorem 5** *Assume that optimal effort at  $t = 2$  is not zero, for all relevant beliefs. If  $e \in (0, 1)$ , then  $e$  is **not** implementable at  $t = 1$ . The set of implementable efforts at  $t = 1$  is  $\{0, 1\}$ .*

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<sup>4</sup>Hart and Holmstrom (1987) assume a linear cost of effort and that the probability of  $y^k$  is a convex combination of two distributions, a "good" one and a "bad". They assume that the weight on the good distribution is an increasing and concave function of effort. To see that that our parameterization is equivalent to theirs, define a new effort variable,  $c(e)$ . This gives linear costs and a concave weighting function.

We now provide the intuition underlying the impossibility result. Let  $e^* \in (0, 1)$  be the effort level that the principal seeks to induce at  $t = 1$ . Fig. 1 graphs the expected continuation value function of the agent, as a function of his effort level  $e$ ,  $W(e, e^*)$ .  $W(e^*, e^*) = 0$ , since the agent has no private information when he chooses the equilibrium level of effort. Since the principal induces positive effort at  $t = 2$ , he must provide incentives for effort, i.e. the agent must be rewarded for outputs that are informative of high effort. This implies that  $W(e, e^*) > 0$  if  $e < e^*$ . Indeed, the appendix shows that the left hand derivative of  $W(e, e^*)$ , evaluated at  $e = e^*$ , is negative. On the other hand, for  $e > e^*$ ,  $W(e, e^*) \geq 0$  since the agent cannot get below his outside option. Thus at  $e^*$ , there is a kink the continuation value function, with the left-hand derivative being strictly negative and the right hand derivative non-negative. If  $Y^+$  is empty, so that the signal structure satisfies uniform optimism, then the right hand derivative is zero. If  $Y^+$  is non-empty, it can be shown that the right hand derivative is strictly positive, using the same arguments as above.

Let  $(w^k)_{k=1}^{|Y|}$  denote an arbitrary contract that seeks to implement  $e^*$ . Let  $\mathbf{E}(u(w)|e, \lambda)$  denote the expected utility of wages, where this expectation is taken with respect to the probability distribution over  $Y$  induced by  $e$ , and given the prior beliefs  $\lambda$ . The probability distribution over  $Y$  given  $e$  and  $\lambda$  is a differentiable function of  $e$ , and so is  $c(e)$ . Thus the first order conditions for  $e^*$  to be optimal for the agent are:

$$\left. \frac{\partial \mathbf{E}(u(w)|e, \lambda)}{\partial e} \right|_{e^*} - c'(e)|_{e^*} + \delta \left. \frac{\partial W^-(e, e^*)}{\partial e} \right|_{e=e^*} \geq 0.$$

$$\left. \frac{\partial \mathbf{E}(u(w)|e, \lambda)}{\partial e} \right|_{e^*} - c'(e)|_{e^*} + \delta \left. \frac{\partial W^+(e, e^*)}{\partial e} \right|_{e=e^*} \leq 0.$$

Since  $\left. \frac{\partial W^-}{\partial e} \right|_{e=e^*} < \left. \frac{\partial W^+}{\partial e} \right|_{e=e^*}$ , the two conditions cannot be simultaneously satisfied.

Our result is quite striking: no interior effort level can be implemented in the first period. That is, the ratchet effect is totally destructive of incentives. The ratchet effect implies that the agent can raise his continuation value by shirking a little relative to  $e^*$ . To overcome this, incentives today must be high powered, so that a little shirking reduces the agent's current payoff. However, this implies that the agent can also increase his current payoff by over-working relative to  $e^*$  – this follows from the fact that current costs and benefits are smooth functions of effort. But over-

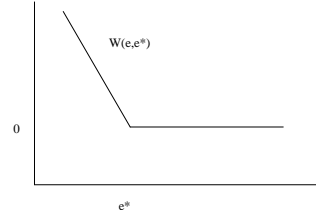


Fig. 1a.  $W(e, e^*)$  under uniform optimism

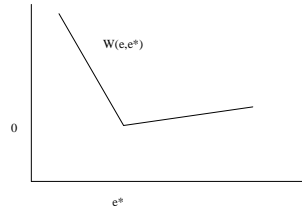


Fig. 1b.  $W(e, e^*)$  without uniform optimism

working cannot *reduce* the agent's continuation value relative to  $e^*$ , since the agent can always quit. In other words, the principal can deter downward deviations, but this makes upward deviations profitable. This point has not been recognized in literature on ratchet effect, which has assumed that high powered incentives can solve the problem.

One may ask if interior effort levels are implementable with positive probability. That is, can the principal design a contract where the agent randomizes over effort levels, using some mixed strategy  $\sigma$ , such that the maximal effort level is some  $e \in (0, 1)$ . Let  $S(\sigma)$  denote the support of  $\sigma$ . The following proposition gives a partial negative answer.

**Proposition 6** *Suppose that there is uniform optimism.  $e = 0$  is implementable. Let  $\sigma$  be a probability distribution over effort levels where  $\sup S(\sigma) \in (0, 1)$ . Then  $\sigma$  is not implementable.*

The appendix provides the proof of this proposition. The underlying intuition is rather similar to that for our main result. If  $\sigma$  is optimal, then  $e^* = \sup S(\sigma)$  must be optimal (even if  $e^*$  does not itself belong to  $S(\sigma)$ ). Under uniform optimism,  $W(e^*, e^*) = \bar{u}$ , since the agent has the most pessimistic beliefs when he chooses  $e^*$ . Since  $W(e, e^*) > \bar{u}$  when  $e < e^*$ , current wage payments must make it unprofitable



for the agent to reduce effort below  $e^*$ . But this means that the agent can increase his current payoff by raising effort above  $e^*$ , and thus  $\sigma$  cannot be optimal.

## 4.1 Rent sharing

We have assumed so far that the firm holds the worker down to her outside option. However, the literature on worker compensation argues that firms often feel that they have to treat their workers fairly (e.g., Bewley, 1999), and this involves a degree of rent sharing. Leaving the worker some rent often improves incentive problems – the efficiency wage model is an example. This is the case in our context as well. Rent sharing may also arise from the worker having some bargaining power, whereby he gets a utility strictly higher than his outside option/

Rent sharing may make the non-implementability problem less severe. Suppose that the firm has a policy of offering a contract where it offers the worker an expected overall payoff of  $\bar{u} + \Delta$ , rather than her reservation utility  $\bar{u}$ . Thus the second period contract,  $\hat{w}(\mu)$ , ensures an expected payoff  $\bar{u} + \Delta$  after every belief  $\mu$ , on the equilibrium path.

Now suppose that the worker deviates in the first period to  $e > e^*$ . If he becomes more pessimistic after signal  $y^k$ , and his payoff is between  $\bar{u}$  and  $\bar{u} + \Delta$ , he will stay on the job. When  $\nu$  is slightly below  $\mu$ ,  $V(\nu, \mu)$  is given by

$$V(\nu, \mu) = \mathbf{E}(u(\hat{w}(\mu)) | \tilde{e}(\nu), \nu) - c(\tilde{e}(\nu)) - \bar{u}, \quad (5)$$

as long as this is positive ( $\tilde{e}(\nu)$  denotes the payoff maximizing effort choice at belief  $\nu$ ). Since the agent will no longer quit if his private belief  $\nu$  is just a little below the public belief  $\mu$ ,  $W(e, e^*)$  decreases as  $e$  is increased beyond  $e^*$ , and is smooth at  $e^*$ . Fig 2a illustrates this for the case of uniform optimism. Now the non differentiability is now at some  $\tilde{e} \gg e^*$ . In the case where there is not uniform optimism, there is an additional effect, since downward deviations below  $e^*$  are also less profitable. There are some signal realizations where the agent becomes more pessimistic after shirking, and since he stays on the job he incurs the cost of earning a rent lower than  $\Delta$ . Thus rent sharing also makes  $W(e, e^*)$  less steep around  $e = e^*$ . Fig 2.b illustrates the case without uniform optimism.

In either case,  $W(e, e^*)$  is decreasing and differentiable at  $e = e^*$ . This implies that the first order conditions for implementing  $e^*$  are satisfied if

$$\left. \frac{\partial \mathbf{E}(u(w)|e, \lambda)}{\partial e} \right|_{e^*} - c'(e)|_{e^*} + \delta \left. \frac{\partial W(e, e^*)}{\partial e} \right|_{e=e^*} = 0.$$

Now the non differentiability is now at some  $\tilde{e} \gg e^*$ . Indeed the value function  $W(e, e^*)$  is necessarily convex as the diagrams illustrate. One has to therefore verify that choosing  $e^*$  is globally optimal. Large upward deviations may be unprofitable if the cost of effort function  $c(e)$  is sufficiently convex.

## 4.2 Risk Neutrality

Our results do not depend upon the agent being risk averse, but they do depend upon the absence of long term commitments. Suppose that the agent is risk neutral, but that contracts are only for one period. In the final period, suppose that the belief is  $\mu$ . Then the principal can make the agent the residual claimant of the project, by charging a fixed rental,  $F(\mu)$ . This must satisfy the individual rationality constraint:

$$\max_e [\mathbf{E}(y|e, \mu) - c(e)] - F(\mu) \geq \bar{u}.$$

The optimal contract maximizes  $F(\mu)$  subject to this constraint, so that

$$F(\mu) = \max_e [\mathbf{E}(y|e, \mu) - c(e)] - \bar{u}.$$

Let  $\hat{e}(\mu)$  denote the value of  $e$  that maximizes  $[\mathbf{E}(y|e, \mu) - c(e)]$ . Thus the principal charges the agent a fee  $F(\mu)$ , that is increasing in  $\mu$  under our assumptions.

Suppose that the agent is offered a contract  $F(\mu)$ , but has belief  $\nu > \mu$ . Then he will accept the contract and his payoff will be

$$\mathbf{E}(y|\hat{e}(\nu), \nu) - c(\hat{e}(\nu)) - F(\mu) = F(\nu) - F(\mu) + \bar{u}.$$

Thus

$$V(\nu, \mu) = F(\nu) - F(\mu) > 0 \text{ if } \nu > \mu.$$

In particular, the derivative is given by

$$\left. \frac{dV(\nu, \mu)}{d\nu} \right|_{\nu=\mu} = \sum_j (p_{\hat{e}(\mu)G}^j - p_{\hat{e}(\mu)B}^j) y^k.$$

Now consider the first period problem. Suppose that the principal wants to implement effort level  $e^*$ . The second period continuation value of the agent when he deviates to  $e < e^*$  is given by  $W(e, e^*) > 0$ . The left hand derivative evaluated at  $e = e^*$  is strictly negative, since  $\left. \frac{dV(\nu, \mu)}{d\nu} \right|_{\nu=\mu} > 0$ . Thus, in order to prevent downward deviations, the agent must be offered more high powered incentives than residual claimancy – his wage payments have to *more variable* than  $y$ . However, this implies that the agent has earned more than  $\bar{u}$  today by increasing his effort level beyond  $e^*$ , and quitting the job tomorrow, when signals in  $Y^-$  are realized. Thus no interior effort level is implementable even when the agent is risk neutral.

This problem can be solved if the agent can sign a long term contract, whereby he commits to buying the project for both periods. The total return from this project is

$$\max \left\{ \sum_k p_{e\lambda}^k \left\{ y^k + \delta \left[ \mathbf{E}(y | \hat{e}(\mu_e^k), \mu_e^k) - c(\hat{e}(\mu_e^k)) \right] \right\} \right\}.$$

Thus the agent would be willing to buy this project for this sum, minus  $(1 + \delta)\bar{u}$ .

### 4.3 Limited Liability & Full commitment

Non implementability is a less of a problem with limited liability – in which case the agent gets a rent. It is also not a problem under full commitment. (To be completed).

## 5 Related Literature

In its most general setting, we have focused on an agent and a principal who are both learning about the state of the world. The state is chosen by nature once and for all. The agent chooses an action, which may be thought of as effort or an experiment. The agent's action and the state jointly determine a public signal, but the agent's action is private, and not observed by principal. Phrased in these terms, this problem is being addressed by a growing literature on private experimentation with public signals in an agency setting, and by the career concerns model.

The first paper to deal with this general setting is Holmstrom's (1999) career concerns model. This early paper set out a model where the non-observability of effort gives rise to possible private information between employers and firms. By assuming a linear technology and normally distributed noise, Holmstrom was able to finesse

many of the difficulties that arise due to the privateness of effort choice. In particular, optimal effort only depends upon calendar time, and not upon previous outputs or previously chosen efforts. Thus the agent's optimal continuation strategy does not depend upon his private information. There is a substantial literature that has developed on Holmstrom's career concerns model, retaining the key assumptions of a linear technology and normally distributed noise. Gibbons and Murphy (1992) allow for linear contracts in the context of Holmstrom's model, and show that explicit incentives become more important over time, as uncertainty is reduced. Meyer and Vickers (1997) study the interaction between implicit incentives arising from career concerns and explicit incentives. Dewatripont et al. (1999a) analyze the implications of alternative information structures and technologies, including a multiplicative technology. Their companion paper (1999b) considers career concerns in public organizations. Prat and Jovanovic (2011) analyze long term contracts with full commitment in a setting similar to Holmstrom's. Their main finding is that as information accumulates, the contracting problem becomes easier. De Marzo and Sannikov (2011) analyze a continuous time contracting problem where the state follows a Brownian motion.

More recently, a series of papers on venture capital consider information structures and technologies that are different from Holmstrom's. Bergemann and Hege (1998, 2005) consider the problem of venture capital financing, where output is binary and depends upon the quality of the project as well as the effort of the agent. Horner and Samuelson (2009) analyze the same question under different economic assumptions (where the project rather than capital is the scarce factor). Manso (2011) considers the problem of motivating innovation, in a context where the outcome variable is binary. Kwon (2011) analyzes a limited liability moral hazard problem, where the probability of success depends upon an unobserved state variable that is partially persistent. Bhaskar and Mailath (2013) examine the case of binary effort and many periods, and show that the incentive problem becomes increasingly harder with the time horizon.

## 6 Appendix

### 6.1 The static model

Suppose that the principal wants to induce  $e = 1$ . The principal's optimal contract depends upon second-order beliefs, i.e. his beliefs regarding the agent's beliefs regarding his own type. Let us suppose that the principal assigns probability one to the agent assigning probability  $\mu$  to being the good type. Let  $w_k$  denote the wage paid in the event that signal  $y^k$  is realized. The incentive constraint corresponding to this belief is given by

$$\mu \sum_k (p_{1G}^k - p_{0G}^k) u(w_k) + (1 - \mu) \sum_k (p_{1B}^k - p_{0B}^k) u(w_k) \geq c(1) - c(0). \quad (6)$$

The individual rationality constraint given this belief is given by

$$\mu \sum_k p_{1G}^k u(w_k) + (1 - \mu) \sum_k p_{1B}^k u(w_k) - c(1) \geq \bar{u}. \quad (7)$$

The optimal contract that induces  $e = 1$  minimizes expected wage payments subject to these constraints, and is standard, as in Holmstrom (1979) or Mirrlees (1999). Let  $\mathbb{W}(\mu) = (w_k(\mu))_{k=1}^n$  denote the profile of wages corresponding to this optimal contract. The important thing that matters for our purpose is that wages are increasing in  $\tilde{\ell}_\mu^k$ , the likelihood ratio corresponding to belief  $\mu$ . In particular, if we compare two signals  $y^l \in Y^L$  and  $y^h \in Y^H$ , then  $w_l(\mu) < w_h(\mu)$  for any belief  $\mu$ .

Our first results concern the optimal strategy and utility of an agent who is offered contract  $\mathbb{W}(\mu)$ , but who in fact has belief  $\nu$ . If the agent accepts the contract, then he will choose effort optimally. Let  $\Delta(\nu|\mu)$  denote the payoff difference between choosing  $e = 1$  and  $e = 0$  given belief  $\nu$ :

$$\Delta(\nu|\mu) = \nu \sum_k (p_{1G}^k - p_{0G}^k) u(w_k) + (1 - \nu) \sum_k (p_{1B}^k - p_{0B}^k) u(w_k) - c(1) + c(0). \quad (8)$$

Using the fact that the incentive

The individual rationality constraint is given by

$$\mathbf{E}(u(\delta w^{jk})|\hat{e}_1, \hat{e}_{ej}) - c(\hat{e}_1) - \delta \mathbf{E}(c(\hat{e}_{2j})|\hat{e}_1) \geq (1 + \delta)\bar{u}.$$

The second period incentive constraints (on path) after choosing  $\hat{e}_1$  and observing signal  $y^k$  is given by

constraint (6) binds at belief  $\mu$ , this can be re-written as

$$\Delta(\nu|\mu) = (\nu - \mu) \sum_k [(p_{1G}^k - p_{0G}^k) - (p_{1B}^k - p_{0B}^k)] u(w_k(\mu)). \quad (9)$$

Thus the agent's optimal effort choice is  $e = 1$  if  $\Delta(\nu|\mu) \geq 0$  and  $e = 0$  if  $\Delta(\nu|\mu) < 0$ . Since  $\Delta(\nu|\mu)$  is linear in  $(\mu - \nu)$ , there are three possibilities. If the term under the summation sign in (9) is zero, then  $\Delta(\nu|\mu) = 0$  for all  $\nu$  and the IC holds. Otherwise, either the IC binds strictly for all  $\nu > \mu$  and is violated for all  $\nu < \mu$  or vice versa.

*Lemma 7 If  $\nu > \mu$ , the agent gets utility that is strictly greater than  $\bar{u}$ ; he accepts the contract and chooses high effort if  $\Delta(\nu|\mu) \geq 0$  and low effort if  $\Delta(\nu|\mu) < 0$ . If  $\nu < \mu$ , the agent rejects the contract since he gets a utility that is strictly lower than  $\bar{u}$ , regardless of his effort choice.*

The agent's optimal strategy if he accepts the contract, as a function of  $\Delta(\nu|\mu)$ , has already been derived. Since the agent's IR constraint binds at belief  $\mu$ , his utility is the same regardless of whether he chooses  $e = 1$  or  $e = 0$ , i.e.  $U(e = 1, \mu) = U(e = 0, \mu)$ .

$$U(e = 1, \nu) - U(e = 1, \mu) = (\nu - \mu) \sum_k (p_{1G}^k - p_{1B}^k) u(w_k(\mu)). \quad (10)$$

Now,  $p_{1G}^k - p_{1B}^k > 0$  if  $y^k \in Y^H$ , and  $p_{1G}^k - p_{1B}^k < 0$  if  $y^k \in Y^L$ . Since wages are uniformly higher for signals in  $Y^H$  than for signals in  $Y^L$ , (10) has the same sign as  $(\nu - \mu)$ . Similarly,

$$U(e = 0, \nu) - U(e = 1, \mu) = (\nu - \mu) \sum_k (p_{0G}^k - p_{0B}^k) u(w_k(\mu)). \quad (11)$$

Now,  $p_{0G}^k - p_{0B}^k > 0$  if  $y^k \in Y^H$ , and  $p_{0G}^k - p_{0B}^k < 0$  if  $y^k \in Y^L$ . Thus (11) has the same sign as  $(\mu - \nu)$ . We conclude that the agent's payoff from accepting the contract is strictly greater than  $\bar{u}$  as long as  $(\nu - \mu) > 0$ , and strictly less if  $(\nu - \mu) < 0$ .

*Remark 8 If assumption A1 is weakened, so that for any informative signal  $y^k$ ,  $p_{1G}^k$  and  $p_{0G}^k$  lie in the closed interval spanned by  $p_{1B}^k$  and  $p_{0B}^k$ , (rather than the open interval), this would imply that either (10) or (11) has the same sign as  $(\nu - \mu)$ . In this case, the agent would get a surplus when he is more optimistic, as in the*

proposition, but may get a payoff equal to his reservation utility by choosing effort optimally when  $(\nu - \mu) < 0$ .

**Lemma 9** *If  $\hat{e}(\mu) \neq 0$ , then  $V(\nu, \mu) > 0$  if  $\nu > \mu$ .*

To see this, suppose that , so that the optimal contract in the ..nal period,  $\hat{w}_j(\mu)$ , must satisfy the ..rst order conditions for  $\hat{e}(\mu)$  to be optimal, i.e.<sup>5</sup>

$$\left. \frac{\partial \mathbf{E}(u(\hat{w}(\mu))|e, \mu)}{\partial e} \right|_{\hat{e}(\mu)} = c'(e)|_{\hat{e}(\mu)} > 0. \quad (12)$$

This can be re-written as

$$\sum_j (p_{1\mu}^j - p_{0\mu}^j) u(\hat{w}_j(\mu)) = c'(e)|_{\hat{e}(\mu)} > 0. \quad (13)$$

But this implies that

$$\sum_j (p_{\hat{e}(\mu)G}^j - p_{\hat{e}(\mu)B}^j) u(\hat{w}_j(\mu)) > 0.$$

Thus,

$$V(\nu, \mu) = (\nu - \mu) \sum_j (p_{\hat{e}(\mu)G}^j - p_{\hat{e}(\mu)B}^j) u(\hat{w}_j(\mu)) > 0.$$

We now derive the conditions that a period one contract must satisfy if some effort level  $e^* \in (0, 1)$  is to be implementable. Since  $e^*$  is in the interior, we must ensure that the agent does not want to deviate either downwards or upwards.

Let  $Y^-$  denote the set of signal realizations such that  $\mu_0^k > \mu_1^k$ , i.e.  $Y^- = \{y^k \in Y : \ell_0^k > \ell_1^k\}$ . Lemma 1 has established that  $Y^-$  is non-empty. Let  $Y^+$  denote the set of signal realizations such that  $\mu_0^k < \mu_1^k$ , i.e.  $Y^+ = \{y^k \in Y : \ell_0^k < \ell_1^k\}$ .

**Lemma 10** *Let  $e, \tilde{e} \in [0, 1]$ , and  $e < \tilde{e}$ . Then  $\mu_e^k > \mu_{\tilde{e}}^k$  if and only if  $y^k \in Y^-$ .  $\mu_e^k < \mu_{\tilde{e}}^k$  if and only if  $y^k \in Y^+$ .*

Suppose that the principal seeks to implement effort level  $e^*$  at  $t = 1$ . This lemma implies that if the agent deviates downwards, to  $e < e^*$ , then he becomes more

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<sup>5</sup> If  $e^*(\mu) = 1$ , then equation (12) applies to the left hand derivative of  $c(e)$  at 1.

optimistic than the principal after signals in  $Y^-$ . If he deviates upwards, so that  $e > e^*$ , then he becomes more optimistic than the principals after signals in  $Y^+$ . Of course, under uniform optimism,  $Y^+$  is empty.

Let  $W(e, e^*)$  denote the second continuation value of the agent (over and above his reservation utility  $\bar{u}$ ) when the principal seeks to implement  $e^*$ , and when the agent chooses  $e$  instead. If signal  $y^k$  is realized, then the agent's continuation value conditional on the signal is  $V(\mu_e^k, \mu_{e^*}^k)$  as long as  $\mu_e^k \geq \mu_{e^*}^k$ . If  $\mu_e^k < \mu_{e^*}^k$ , then his continuation value is zero since he can always quit the job. Thus  $W(e, e^*)$  is bounded below by zero. Clearly,  $W(e^*, e^*) = 0$  since  $V(\mu_{e^*}^k, \mu_{e^*}^k) = W(e, e^*)$  can be written as

$$W(e, e^*) = \begin{cases} \sum_{y^k \in \bar{Y}} p_{e\lambda}^k V(\mu_e^k, \mu_{e^*}^k) & \text{if } e < e^* \\ \sum_{y^k \in Y^+} p_{e\lambda}^k V(\mu_e^k, \mu_{e^*}^k) & \text{if } e > e^*. \end{cases}$$

Our purpose here is to evaluate left-hand and right hand derivatives of  $W(e, e^*)$  at  $e = e^*$ . These are required for the first order conditions for the implementability of  $e^*$ .

The left hand derivative of  $W(e, e^*)$  with respect to  $e$ , evaluated at  $e = e^*$ , is given by

$$\left. \frac{\partial W^-(e, e^*)}{\partial e} \right|_{e=e^*} = \sum_{y^k \in Y^-} \frac{\partial p_{e\lambda}^k}{\partial e} V(\mu_{e^*}^k, \mu_{e^*}^k) + \sum_{y^k \in Y^-} p_{e^*\lambda}^k \frac{dV(\mu_e^k, \mu_{e^*}^k)}{de}.$$

Since  $V(\mu_{e^*}^k, \mu_{e^*}^k) = 0$ , this simplifies to

$$\left. \frac{\partial W^-(e, e^*)}{\partial e} \right|_{e=e^*} = \sum_{y^k \in \bar{Y}} p_{e^*\lambda}^k \left. \frac{\partial V(\mu_e^k, \mu_{e^*}^k)}{\partial \mu_e^k} \right|_{\mu_e^k = \mu_{e^*}^k} \left. \frac{\partial \mu_e^k}{\partial e} \right|_{e=e^*} \quad (14)$$

We now show that each term under the summation sign is strictly negative. The first term,  $p_{e^*\lambda}^k$ , is positive. To evaluate the second term, let us consider the derivative of  $V(\nu, \mu)$  with respect to  $\nu$ . When  $\nu > \mu$ ,  $V(\nu, \mu)$  is given by

$$V(\nu, \mu) = \mathbf{E}(u(\hat{w}(\mu)) | \tilde{e}(\nu), \nu) - c(\tilde{e}(\nu)) - \bar{u}, \quad (15)$$

where  $\tilde{e}(\nu)$  denotes the payoff maximizing effort choice at belief  $\nu$ . The derivative of this expression with respect to  $\nu$  equals



$$\frac{dV(\nu, \mu)}{d\nu} = \frac{\partial \mathbf{E}(u(\hat{w}(\mu))|\tilde{e}(\nu), \nu)}{\partial \nu} + \left[ \frac{\partial \mathbf{E}(u(\hat{w}(\mu))|\tilde{e}(\nu), \nu)}{\partial e} - c'(\tilde{e}(\nu)) \right] \frac{\partial \tilde{e}(\nu)}{\partial \nu}. \quad (16)$$

By the envelope theorem, the second term is zero, and so

$$\frac{dV(\nu, \mu)}{d\nu} = \frac{\partial \mathbf{E}(u(\hat{w}(\mu))|\tilde{e}(\nu), \nu)}{\partial \nu}. \quad (17)$$

Evaluating this derivative at  $\nu = \mu$ , we get

$$\left. \frac{dV(\nu, \mu)}{d\nu} \right|_{\nu=\mu} = \left. \frac{\partial \mathbf{E}(u(\hat{w}(\mu))|\hat{e}(\mu), \nu)}{\partial \nu} \right|_{\nu=\mu} = \sum_j (p_{\hat{e}(\mu)G}^j - p_{\hat{e}(\mu)B}^j) u(\hat{w}_j(\mu)).$$

This implies

$$\left. \frac{dV(\nu, \mu)}{d\nu} \right|_{\nu=\mu} = \left. \frac{\partial \mathbf{E}(u(\hat{w}(\mu))|\hat{e}(\mu), \nu)}{\partial \nu} \right|_{\nu=\mu} = \sum_j (p_{\hat{e}(\mu)G}^j - p_{\hat{e}(\mu)B}^j) u(\hat{w}_j(\mu)) > 0.$$

Consider now the second term under the summation size in 14, the derivative of beliefs:

$$\frac{\partial \mu_e^k}{\partial e} = \frac{\lambda(1-\lambda) [p_{1G}^k p_{0B}^k - p_{1B}^k p_{0G}^k]}{[\lambda p_{eG}^k + (1-\lambda) p_{eB}^k]^2}.$$

The numerator is negative if  $p_{1G}^k p_{0B}^k < p_{1B}^k p_{0G}^k$ , i.e. if  $\ell_1^k < \ell_0^k$ . Thus the second term is negative, since  $y^k \in Y^-$ .

Since the left hand derivative of  $W(e, e^*)$  at  $e = e^*$  in equation (14) is the sum strictly negative terms, we conclude that the left hand derivative is strictly less than zero.

**Proof of proposition 6:** Let  $\bar{w}$  solve  $u(\bar{w}) - c(0) = \bar{u}$  and set  $w^k = \bar{w}$  for all  $k$ . By choosing  $e = 0$ , the agent ensures himself  $\bar{u}$  and a continuation value of zero. If he chooses  $e > 0$ , his current payoff is reduced by  $c(e) - c(0)$ . Since there is uniform optimism,  $\mu_e^k \leq \mu_0^k \forall k$ , and his continuation value is also zero. Thus this contract implements  $e = 0$ , and is the cost minimizing contract for doing so.

**Proof.** Let  $(w^k)_{k=1}^{|Y|}$  be an arbitrary first period contract that induces  $e \in (0, 1)$  with

positive probability, and let  $\sigma$  denote the agent's local behavior strategy under this contract, i.e. his randomization over effort levels, and let  $\tilde{U}$  denote the total payoff of the agent under this strategy. Let  $\tilde{e} = \sup S(\sigma)$  and assume  $\tilde{e} \in (0, 1)$ . Consider a sequence  $e_n \uparrow \tilde{e}, e_n \in \sup S(\sigma)$ . Since both the current payoff and  $W(e)$  are continuous in  $e$ , the total payoff of choosing  $\tilde{e}$  is also  $\tilde{U}$ , i.e. choosing  $\tilde{e}$  is optimal under the contract. If the agent chooses  $\tilde{e}$ ,  $W(\tilde{e}) = 0$  since we have uniform optimism. Furthermore, the left hand derivative of  $W(e)$ , evaluated at  $\tilde{e}$ , is strictly negative, as we have already established. Thus the derivative of the current payoff at  $\tilde{e}$ ,  $\left. \frac{\partial [\mathbf{E}(w|e, \lambda) - c(e)]}{\partial e} \right|_{\tilde{e}}$ , is strictly positive. ■

Claim 11 *If  $e \in S(\sigma)$ , then  $e \leq \hat{e}$ .*

Proof. Since there is uniform optimism,  $W(e) \leq 0$  for  $e > \hat{e}$ . Since  $\hat{e}$  yields a higher current payoff than any  $e > \hat{e}$ ,  $e$  cannot be optimal. ■

Claim 12  $\sup S(\sigma) = \hat{e}$ .

Proof. Suppose  $\sup S(\sigma) = \tilde{e} < \hat{e}$ . Then  $W(\tilde{e}) = 0$ , since there is uniform optimism. But then  $\hat{e}$  yields a higher total payoff than  $\tilde{e}$ . ■

Suppose that  $\tilde{e}, \hat{e} \in S(\sigma)$ . Since  $W(\hat{e}) = 0$ , indifference requires

$$W(\tilde{e}) = [\mathbf{E}(y|\hat{e}, \lambda) - c(\hat{e})] - [\mathbf{E}(y|\tilde{e}, \lambda) - c(\tilde{e})].$$

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