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Great Moderation or Great Mistake: Can rising confidence in low macro-risk explain the boom in asset prices?

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Great Moderation and Great Leverage: Financial trade and asset prices when investors disagree about risk

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Abstract

Whether the "Great Moderation" in macroeconomic volatility observed since the mid-1980s was an outlier of "good luck" or an expression of more structural change was first debated in the late 1990s during times of rising leverage and asset prices. We show how disagreement about economic volatility gives rise to perceived gains from trade that can increase leverage and asset prices. This is because investors with dispersed posteriors value more highly the convexity of profits implied by leveraged asset purchases, whose returns increase with high payoffs but are insensitive to a deterioration of bad realisations that lead to bankruptcy of investors. We analyse a simple general equilibrium economy where investors temporarily disagree on the dispersion of payoffs as they update their posteriors at different speeds in reaction to a long-lived fall in observed volatility. Despite constant and common expected payoffs and risk-neutrality, the economy experiences a temporary rise in leverage, price volatility and average asset prices.

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1 Introduction

By the mid-1990s, both investors and academic economists had recognised the fall in macroeconomic volatility that many developed economies had experienced about a decade earlier. Disagreement persisted, however, about the origins and implications of this "Great Moderation" in volatility of GDP and other macroeconomic aggregates. Over the same time period, financial markets in many of the same countries saw both a strong rise in asset prices, and a significant increase in the leverage of households and financial firms. This paper looks at the behaviour of asset prices and investor leverage during a period when an observed fall in the dispersion of past output realisations leads to temporary disagreement about the outlook for aggregate risk in the near future. We show how risk-neutral investors with a relatively dispersed posterior distribution of payoffs purchase assets through leveraged investments from those with more concentrated posteriors even though both groups share the same fundamental valuation of assets. This is because the possibility of bankruptcy by leveraged investors leads to perceived gains from trade from disagreement about risk: limited liability increases the value of leveraged investments to agents with dispersed posteriors, who gain from the upside but go bankrupt when payoffs are low. Investors with concentrated posteriors perceive the risk of bankruptcy to be lower, implying gains from trade that would remain unrealised in the absence of leverage.

We show first theoretically how disagreement about the volatility of asset payoffs can lead to an increase in asset trade and leverage as well as a rise in asset prices in the general equilibrium of a static economy where two groups of investors share the same belief about mean asset payoffs but have different views about their volatility. In a simplified, dynamic version of the model, prices fluctuate in a range strictly above the common fundamental value of the asset. This is because leveraged investors occasionally go out of business but the remaining investors are happy to pay a speculative premium in anticipation of higher prices once leveraged investors re-enter the market in the future. Finally, we use a quantitative version of this simple model to analyse a scenario that captures the experience of the US economy during the Great Moderation period. When some investors update their posteriors about the dispersion of aggregate payoffs less quickly than others, temporary disagreement about volatility leads to a period of increased leverage and price volatility as well as a moderate boom in average asset prices.

The literature on the consequences of heteregenous investor beliefs has focused on dis-

agreement about mean payoffs.[1] Particularly, Miller (1977)'s seminal article shows how asset prices rise when investors disagree about future mean payoffs and the absence of short-selling makes the marginal investor become more optimistic. Harrison and Kreps (1978) analyse a dynamic version of the model and show that even in periods where optimists are not in the market, pessimists are happy to pay a speculative premium above their fundamental asset valuation in anticipation of rising prices when optimists return to the market in the future. Geanakoplos (2001) introduces leverage into this framework, whereby investors can issue debt collateralised by the assets they want to buy in order to increase the amount they can invest. This allows optimists to increase their asset purchase and thus makes the marginal investor more optimistic, increasing prices further. At the same time it leads to fluctuations in asset prices in response to changes in investor balance sheets. Simsek (2013) uses a similar model with two groups of investors to show how leveraged investment dampens the effect of belief disagreements on prices when optimism is concentrated on the downside, i.e. when optimists have relatively positive views on the distribution of low shock realisations. If optimists are particularly positive about the upside potential of the asset, however, leveraged investments amplify the effect of belief disagreements.

Interestingly, Miller (1977)'s original article associates higher payoff risk with stronger disagreement about mean payoffs. Neither his article, nor the literature that it preceeds, however, has analysed disagreement in beliefs about risk per se.[2] Although the latter trivially leaves prices unaffected in Miller (1977)'s original frame-work with risk-neutral investors and no leverage, we show how leverage introduces a convexity in the profit function that makes expected profits rise with the dispersion of asset payoffs. In other words, investors with a relatively more dispersed posterior feature both Simsek (2013)'s upside optimism and downside pessimism, leading to perceived gains from trade and a boom in equilibrium asset prices.

The effect of the Great Moderation on asset prices has been studied previously in environments with learning about the volatility of payoffs under homogeneous priors (Lettau et al. (2008), Broer and Kero (2011)). Even when investors share the same prior mean and variance of output growth, however, differences in the "strength" of their priors, or in the speed at which their posterior reacts to new information, will lead to temporary

[1] See (Xiong, 2013) for a survey.

[2] In an early reaction to (Miller, 1977), (Jarrow, 1980) has pointed out the importance of the variance-covariance structure of asset returns for the effect of short-selling constraints on asset prices. His focus is very different to the one in this paper, however.

heterogeneity in posterior beliefs when moments of observed data exhibit a sudden change such as the fall in GDP volatility during the Great Moderation. Particularly, when some agents adjust their posteriors more quickly than others, a permanent fall in volatility leads to a gradual fall in the average of volatility estimates across investors, but also to a temporary rise in their dispersion as posteriors of \quick updaters" react more strongly initially, but those of \latecomers" eventually catch up and both converge to the new data moment. In line with this intuition, both the academic literature and investment research during the second half of the 1990s discussed a variety of potential mechanisms behind the observed fall in volatility. Speci cally, some participants saw the period of low volatility as a mere outlier of \good luck" in an unchanged stochastic environment, while others interpreted it as the result of fundamental changes in the economy such as central bank independence, logistical innovation, globalisation, nancial innovation, etc.

We see this debate, generally, as an expression of increased heterogeneity in posterior beliefs about the structure of the economy. More speci cally, in this paper, we operationalise this heterogeneity as an increase in the dispersion of beliefs about the volatility in future aggregate output and investment payo s. Due to the convexity of pro ts from leveraged investment, which are constant and zero below a bankruptcy threshold but increase linearly with asset payo s above it, this belief dispersion leads to gains from trade as investors with more volatile posteriors perceive both more upside risk and downside risk, but the latter does not a ect their expected pro ts. We see our contribution in drawing attention to this previously overlooked mechanism and formalising it in a simple general equilibrium economy. Speci cally, we rst analyse a general equilibrium version of the static environment in Simsek (2013). When investors are risk-neutral and have prior payo distributions with a common mean, there are no gains from trading the asset on simple spot markets without leverage. However, investors with more dispersed posteriors can be seen as \upside optimists" and \downside pessimists", in Simsek (2013)'s language. The result that leveraged investments amplify upside optimism but dampen downside optimism then implies that disagreement about second moments of payo s necessarily leads to scope for asset trade and a rise in prices in a static economy. We then and present a simple dynamic version of this economy, to show that general equilibrium prices uctuate in a range strictly above their fundamental valuation that is shared by both optimists and pessimists. Thus, despite constant and common fundamental valuations of the asset, disagreement about the second moments introduces both price volatility, as low payo realisations make leveraged investors go bankrupt and temporarilly leave the market, and

a boom in prices. That equilibrium prices exceed the fundamental value of the asset even when only investors with concentrated posteriors are in the market is due to a speculative premium, as in Harrison and Kreps (1978), that makes them pay a higher price today in anticipation of rising prices upon return of the natural buyers in the future. Note that the convexity of payo s that drives most results is also behind the potential for risk-seeking behaviour in nancial markets with limited liability (as in, for example, Murdock et al. (2000)). The environment of this paper, however, lacks any potential for moral hazard as actions are perfectly observed and the buyers of collateralised debt agree with leveraged investors on the functional relationship between exogenous asset payo s and bankruptcy. The only source of disagreement, and the origin of perceived gains from trade between the two, is the second moment of the distribution of asset payo s.

2 Disagreement about payoff dispersion in a static economy

2.1 The General Environment

We study an economy that exists for two periods $t \in \{1, 2\}$. There are two types of agents, 0 and 1, both of unit-mass, who receive an endowment $n_i, i = 0, 1$ of the unique consumption good in period 1, and who are endowed with $\overline{a}_i, i = 1, 2$ units of an asset (a \tree") that pays a stochastic amount $s \in S = [s_{min}, s_{max}]$, $s_{min} > 0$ in period 2. Agents are assumed to be risk-neutral, and thus trade in order to maximise the present discounted sum of consumption $U_i = c_i + \frac{1}{R}c_i'$.

We assume that types di er in their beliefs about the distribution of random payo s s, summarised by two distribution functions $f_i : S \longrightarrow R^+, i = 0, 1$. We assume that both types expect payo s to be the same on average, but that type 0 believes them to be more tightly distributed:

$$\mathbf{A1} : E_1(s) = E_0(s) \equiv E_s, \ f_0 \succ^2 f_1 \tag{1}$$

where $\succ^2$ denotes second-order stochastic dominance

2.2 Asset markets

Agents trade in 2 asset markets: In t=1, agent i purchases $a_i - \overline{a}_i$ units of the physical asset at price p. In addition, agents can trade collateralised loan contracts. These contracts are characterised by a promised face value that agents pay unless they declare bankruptcy, in which case all their assets are sold to honour loan contracts. We normalise contracts to 1 unit of collateral. Thus collateralised loan contracts have unit-payo s equal to $\phi_i(s) = min\{s, \overline{s}\}$, where $\overline{s}$ is the promised face value, or riskyness of the loan. In t=1, agents trade these contracts at price q.

2.3 Optimal Behaviour at given prices

2.3.1 Profits

Buyers of collateralised loans pay a sum q to their counterparty today, for a promise of $E_i[min\{s, \overline{s}\}]$. So for a quantity of loans a_i^l, net pro ts are

$$ {}^l_i = a_i^l[\frac{E_i[min\{s, \overline{s}\}]}{R} - q] \tag{2}$$

A2: Loan issuers have all bargaining power, so $q = \frac{E_i[min\{s,\overline{s}\}]}{R}$

Other than buying assets outright using consumption goods as payment, agents can engage in leveraged asset purchases by issuing loans using their wealth, including the newly purchased assets, as collateral. Expected net pro ts from buying an additional $a_i - \overline{a}_i$ units of the asset using a collateralised loan are

$$ {}^a_i = a_i \frac{[E_i(s) - E_i(min(s, \bar{(s)}))]}{R} - m \tag{3}$$

$$ with\ a_i p = m + \overline{a}_i + q a_i = m + \overline{a}_i + a_i \frac{E_j\{min\{s, \overline{s}\}\}}{R} \tag{4}$$

for a cash payment of m. Note that, due to our bankruptcy assumption, all assets of agent i serve as collateral, not just his net purchases.

Result 1:

$$ {}^l_1 <= {}^l_0\ \forall\ \overline{s} \in (s_{min}, s_{max}), \forall p, q, R \tag{5}$$

Result 2:

$$ {}^{a}_{1} > {}^{a}_{0} \ \forall\ \overline{s} \in (s_{min}, s_{max}), \forall p, q, R \tag{6}$$

Proof: The result follows from 2nd order stochastic dominance and the strict concavity of $\frac{E_i[min\{s,\overline{s}\}]}{R}$ in s when $\overline{s} \in (s_{min}, s_{max})$.

2.3.2 Participation

Result 3:

If equilibrium price p is such that

C1 $\frac{E_s}{R} = \underline{p} < p < \overline{p} = \frac{E_s + E_0(min(s,\overline{s})) - E_1(min\{s,\overline{s}\})}{R}$

all type 1 agents invest their endowments in leveraged assets, while type 0 agents hold all collateralised loans (tie-breaking).

Proof:

The first inequality in C1 ensures that type 0 agents prefer to consume their ressources in t = 1 rather than buying risky assets. Moreover, they are indifferent between consuming today and buying collateralised loans.

The second inequality implies that type 1 agents prefer leveraged asset purchases over consumption today.

$$ {}^{a}_{1} > 0 \tag{7}$$

$$\Leftrightarrow E({}^{a}_{1}(m_1)) = \frac{m_1[E_s - E_1(min\{s,\overline{s}\})]}{[pR - E_0(min\{s,\overline{s}\})]} > U(m_1) = m_1 \tag{8}$$

where the second row substitutes the definition of profits and the budget constraint for leveraged investments.

2.3.3 Type 1's problem

Since type 1 is the agent that engages in leveraged asset purchases, she solves the following problem

$$\begin{aligned}
& max_{C_1,m_1,a_1,\overline{s}} \quad C_1 + \frac{a_1[E_s - E_1(min\{s,\overline{s}\})]}{R} \\
& s.t. \quad a_1 p = m_1 + a_1 \frac{E_0[min\{s,\overline{s}\}]}{R} \\
& C_1 = n + \overline{a}_1 - m_1 \\
\Leftrightarrow \quad & max_{m_1,\overline{s}} \;\; (n_1 + \overline{a}_1 p - m_1) + \; m_1 R_1^a \\
& = (n_1 + \overline{a}_1 p - m_1) + m_1 \frac{[E_s - E_1(min\{s,\overline{s}\})]}{Rp - E_0\{min\{s,\overline{s}\}\}} \qquad (9)
\end{aligned}$$

If $R_1^a = \frac{[E_s - E_1(min\{s,\overline{s}\})]}{Rp - E_0\{min\{s,\overline{s}\}\}} \geq R$ for any $\overline{s}$, it is optimal to invest the whole endowment, so $m_1 = n_1$. The rst order condition for $\overline{s}$ can be written as

$$\frac{(n_1 + \overline{a}_1 p)}{Rp - E_0\{min\{s,\overline{s}\}\}}[(1 - F_1(\overline{s})) - R_1^l(1 - F_0(\overline{s}))] = 0 \qquad (10)$$

Result 4: If p is such that C1 holds for some $\overline{s} \in (s_{min}, s_{max})$, $R_1^a(p,\overline{s})$ has an interior maximum at some $\overline{s}^\star \in (s_{min}, s_{max})$.

Proof: Note that$R_1^a = 0$ at $\overline{s} = s_{max}$. Also, if $p > \frac{E_s}{R}$, $R_1^a(s_{min}) = \underline{R_1^a} < 1$. But if at some $\overline{s}'$ $p < \frac{E_s + E_0(min\{s,\overline{s}\}) - E_1(min(s,\overline{s}'))}{R}$, then $R_1^a(\overline{s}') > 1$. The statement then follows from continuity of R_1^a.

Corollary 5:

There is $p \geq \frac{E_s}{R}$ such that type 1 agents invest in leveraged assets given p.

Result 6:

If type 1 agents invest in leveraged assets, then she chooses $\overline{s}$ such that $1 - F_0(\overline{s}) \leq 1 - F_1(\overline{s})$.

Proof: Follows from the rst order condition (10) and the condition for investment in leveraged assets $R_1^a \geq R$.

2.4 General Equilibrium

Definition: An equilibrium is a set of quantities and prices $c_i, c_i', a_i, b_i, p, q$, such that agents act optimally given their beliefs and markets clear.

The equilibrium depends on the size of type 1's endowment:

1. If $n_1 = 0, a_1 = 0$, there is no trade in collateralised loans, so $p = \frac{E_s}{R}$.

2. If $n_1 > 0$ or $a_1 > 0$, p and $\overline{s}$ are given by the following equations

$$PS^1 : \mathbb{C} = m_1(1 - F_0) - (1 - F1)(Rp - E_0[min\{s, \overline{s}\}]) = 0 \tag{11}$$

$$p = max\{\overline{p}, p^{\star}\} \tag{12}$$

$$PS^2 : (1 - \overline{a}_1)p^{\star} - n_1 - \frac{E_0[min\{s, \overline{s}(p^{\star})\}]}{R} = 0 \tag{13}$$

3. Particularly, if $n_1 \geq \overline{n} = (1 - \overline{a})\frac{E_s - E_0[min\{s,\overline{s}\}] - E_1[min\{s,\overline{s}\}}{R} - \frac{E_0[min\{s,\overline{s}\}]}{R}$, type 1's endowment is large enough to buy type 0's assets at the maximum price $\overline{p}$ that ensures her participation $\overline{p} = E_s + E_0[min\{s, \overline{s}\}] - E_1[min\{s, \overline{s}\}$. Only at this price, which implies an expected return of R on leveraged investments, is she willing to consume the resources that remain after issuing the optimal collateralised loan and purchasing all assets.

4. If $0 < n_1 \leq \overline{n}$, we have $p < \overline{p}$, which implies $R_1^a > R$ and $c_1 = 0$. Type 1 agents expect a return on leveraged investments larger than R, so invest all their funds to buy type 0's assets.

Result 7:
PS^2 is upward-sloping. PS^1 is downward-sloping.
Proof: The second part of the statement follows from

$$\frac{dp}{d\overline{s}}|_{PS^1} = -\frac{\frac{\mathbb{C}}{d\overline{s}}}{\frac{\mathbb{C}}{d\overline{p}}} \tag{14}$$

Concavity of R_1^a at the optimum implies that the numerator is negative. Since $\frac{(C)}{d\overline{p}} < 0, \forall P, \overline{s}$ the result follows.

2.5 Comparative statics

This section looks at the e ect of \belief-divergence", in the sense of a further mean-preserving contraction to f_0. For this we assume that the distribution function f_0 is parameterised by a variable v such that

1. f_0 is continuous in v for all s

2. $E_{0,v}(s) = E_s, \forall v$

3. $f_0(v_1)$ second order dominates $f_0(v_2)$ whenever $v_1 > v_2$

4. $F_0(v2, s) - F_0(v1, s)$ is downward sloping in s whenever $v_1 > v_2$ and crosses the zero line once at $s^\star$.

Result 8: $\frac{\delta p}{\delta v} > 0$ if either $n_1 > \overline{n}$ or $\frac{\delta PS^1}{\delta \overline{v}} > 0$. The latter is necessarily the case when equilibrium riskyness s is below $s^\star$.
If $n_1 > \overline{n}$, $F_0 = F_1 = \frac{1}{2}$ from PS_1 and $R_1^a = 1$. So $\overline{s}$ does not change in response to v, but $\overline{p}$ rises.
If $0 < n_1 < \overline{n}$, we can use PS^1 and PS^2 to get from the implicit function theorem the partial derivative of the price with respect to v as

$$\frac{\delta p}{\delta v} = \frac{\frac{\frac{\delta PS^1}{\delta \overline{s}}}{\frac{\delta PS^2}{\delta \overline{s}}}[\frac{\delta PS^2}{\delta v}] - \frac{\delta PS^1}{\delta v}}{\frac{\delta PS^1}{\delta p} - \frac{\delta PS^2}{\delta p}} \tag{15}$$

We know:

1. $\frac{\delta PS^1}{\delta \overline{s}} < 0$ (from optimality of $\overline{s}$)
2. $\frac{\delta PS^2}{\delta \overline{s}} = -(1 - F_0(\overline{s})) < 0$
3. $\frac{\delta PS^2}{\delta \overline{v}} = -\frac{\delta E_0(min\{s,\overline{s}\})}{\delta \overline{v}} < 0$
4. $\frac{\delta PS^1}{\delta \overline{p}} - \frac{\delta PS^2}{\delta \overline{p}} = -R(1 - F_1(\overline{s})) - 1 < 0$
5. $\frac{\delta PS^1}{\delta \overline{v}} = -[E_s - E_1(min\{s, \overline{s}\})]\frac{\delta F_0}{\delta \overline{v}} + (1 - F_1)\frac{\delta E_0(min\{s,\overline{s}\})}{\delta \overline{v}}$

Thus $\frac{\delta p}{\delta v} > 0$ if $\frac{\delta PS^1}{\delta \overline{v}} > 0$. This is the case for any $\overline{s} < s^\star$, since $\frac{\delta F_0}{\delta \overline{v}} < 0$.

□

Discussion
A mean-preserving contraction in f_0 is equivalent to lenders updating their beliefs to a lower level of risk. This increases the expected payo from a collateralised loan of given riskyness, and thus increases the amount they are willing to lend to investors for leveraged investment. For investors, this always increases expected pro ts at a given price and level of riskyness. However, it has an ambiguous e ect on marginal pro ts and thus

the optimal value of riskyness $\overline{s}$. Speci cally, while a rise in v increases the return at any given riskyness, it can increase or decrease $1 - F_0$, the marginal e ect of a change in $\overline{s}$ on pro ts at given returns. Only when the marginal bene t of a change in riskyness rises with v ($\frac{\delta PS^1}{\delta \overline{v}} > 0$), however, is the e ect on asset demand, and thus the general equilibrium e ect on prices unambiguously positive.

3 A Dynamic model

In an environment like that of the previous section, where assets are claims to a single stochastic payo in the future, limited liability of leveraged investments naturally implies the convex pro ts of leveraged investors the are behind most of the results. When assets are claims on a sequence of payo s, however, investor wealth at any point in time equals the sum of the realised payo and the price of the asset. The curvature of pro ts thus depends on the function that links general equilibrium asset prices to current payo s and other state variables in the economy. This section circumvents this problem by assuming that payo s have a discrete support, as opposed to the continuous support in the previous section. Speci cally, we aim to present the simplest example of such an economy. To introduce mean-preserving spreads, the analysis requires distributions with at least 3 points of strictly positive mass. As it turns out, however, payo realisations close to the value of promised net payments by leveraged investors to creditors introduce a potential for multiple equilibria: if prices are high these payo s are consistent with credit repayments as promised and continued leveraged investment that justi es high prices. If prices, on the other hand, are low, leveraged investors cannot pay back their creditors, go bankrupt and leave the market for one period, warranting the fall in prices. While this multiplicity is interesting and potentially also arises in other models of collateralised investments, we choose an environment where it does not arise because payo realisations are always su ciently \far away" from the value of promised net payments to be consistent with either bankruptcy or continued investment, but not both. This requires at least 4 support points of payo s, which is the case we focus on.

3.1 The environment

We look at a version of the previous model where time is in nite $t = 0, 1, 2, \ldots$. Every period physical assets pay a random amount of the consumption good $s_t,\ \forall t$ that is inde-

pendent across periods. Agents of both types are in nitely lived and receive consumption endowment $n_i, \forall t$. So the resources available to agent i at the beginning of the period equal her investment payo plus her endowment. Agents maximise the present discounted value of consumption through decisions on consumption and asset purchases every period. As before they trade physical assets and collateralised loans whose face value for next period $\overline{s}_{t+1}$ is agreed on in t. For simplicity I normalise total assets to 1 and assume that type 0 agents own all asset in $t = 0$.
Agents' liability is limited to their total period t assets plus their period t endowment. The budget constraint for leveraged investors is

$$a_{it+1}(p_t - \frac{E_j[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]}{R}) + c_{it} \leq a_{it}(max\{n_i + p_t + s_t - \overline{s}, 0\}) \tag{16}$$

For providers of collateralised loans we have

$$a_{jt+1}\frac{E_i[min\{n_j + p_{t+1} + s_{t+1}, \overline{s}\}]}{R} + c_{it} \leq n_i + a_{jt}(min\{n_j + p_t + s_t, \overline{s}\} \tag{17}$$

3.2 3 dimensions of dynamic behaviour

There are three dimensions of dynamic bevaviour:

1. The endogenous evolution of relative wealth
2. Learning about the distribution of s_t
3. Endogenous price uctuations, with potentially di erent beliefs about the price process p_t by type 1 and 2 agents

This section focusses on dimensions 1 and 3. We thus abstract from 2. by assuming that belief disagreements are constant, but introduce learning explicitly in the quantitative analysis of the next section.

3.3 Equilibrium definition

1. Sequences of prices and quantities as functions of the state of the economy $(s_t, \overline{s}_t, a_{it}, a_{jt})$ s.t.
2. agents optimise given belief $f_{it} = f_i$

3. markets for consumption and assets clear

Discussion: Note how we specify an equilibrium where agents agree on the price and policy functions of the underlying state of the economy, but disagree on the distribution of exogenous shocks.

3.4 Equilibrium Description and Problem

Type i agents invest in leveraged assets if

$$R_i^a = \frac{E_i[p_{t+1} + s_{t+1} - min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]}{p_t - \frac{E_j[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]}{R}} \tag{18}$$

$$= \frac{E_i[p_{t+1}] + E_s - E[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]}{p_t - \frac{E_j[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]}{R}} > R \tag{19}$$

Asset market clearing requires $R_i^a \geq R \geq R_j^a$: only 1 type can expect to make strictly positive pro ts from leveraged investments. However, asset prices are not necessarily weakly convex in s_t, partly because they are bounded below and above respectively by the minimum and maximum of their expected discounted value to either agent with or without leverage. So $E_1[p_{t+1}]$ may be greater or smaller than $E_0[p_{t+1}]$.

3.5 A discrete process for s_t

This section shows an example of an economy where disagreement in beliefs about the volatility of asset payo s have the following implications:

1. Prices are strictly above the discounted expected value of payo s.

2. Prices uctuate over time although the expected payo is constant.

For this, we make the following assumption:

A3: The perceived distribution of s_t $f_{i,0}$ is symmetric around E_s with mass points at $\{s_1, s_2, s_3, s_4\} = \{E_s - 2n, E_s - n, E_s + n, E_s + 2n\}$. Particularly, $f_{1,0}(s_i) = \frac{1}{4}, i = 1, 2, 3, 4,$ while $f_{0,0}(s_2) = f_{0,0}(s_3) = \frac{1-\epsilon}{2}$ and $f_{0,0}(s_3) = f_{0,0}(s_4) = \frac{\epsilon}{2}$ where ϵ is an in nitesimally small number.

3.5.1 Equilibrium Dynamics

Proposition 1
There is an equilibrium with the following properties:

- $\overline{s}_t = \underline{p} + E_s, \forall t$
- If type 1 invested in $t-1$, she goes bankrupt in the two low-income states where $s_t < E_s$. If bankrupt, $c_1 = a_1 = 0$. In case she did not invest in period $t-1$, or if she did and $s_t > E_s$, she buys all assets and consumes a positive amount $a_1 = 1, c_1 > 0$.
- The price process has only two support points:
    1. If agent 1 is not bankrupt, $p_t(s_t) = \overline{p} = \frac{E_s + \frac{R-\frac{1}{2}}{R} \;\; E[\varphi(s)]}{R-1}$
    2. If agent 1 is bankrupt, $p_t(s_t) = \underline{p} = \frac{E_s + \frac{1}{2} \;\; E[\varphi(s)]}{R-1}$

Proof:
The proof is by guess and verify: guess that $P(s_1) = p(s_2) = \underline{p} < E_s < p(s_3) = p(s_4) = \overline{p}, \forall t$ and de ne $p = \overline{p} - \underline{p} > 0$. Guess also that agent 1 buys as much leveraged claims as she can and sets $\overline{s} = E_s$. To verify that this is an equilibrium, rst calculate the price in periods where agent 1 is bankrupt as

$$\underline{p} = \frac{E_0[p_{t+1}] + E_s}{R} = \frac{E_s + \frac{1}{2} \;\; p}{R-1} \tag{20}$$

The expected return to agent i from a leveraged investment is

$$R_i^a = \frac{E_i[p_{t+1}] + E_s - E_i[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]}{p_t - \frac{E_j[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]}{R}} \tag{21}$$

Since $E_1[p_{t+1}] = E_0[p_{t+1}]$ and $E_1[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}] < E_0[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]$, $R_1^a > R_0^a$. So whenever agent 1 has funds to invest, she will spend them on leveraged asset puchases in equilibrium. The maximum value of $\overline{p}$ at which she is willing to do so is that which implies $R_1^a = 1$. If at this value agent 1 can a ord to buy all assets whenever she has positive resources, this is the only equilibrium price outside bankruptcy periods: outside bankruptcy agent 1 is \too rich" for asset purchases to exhaust her budget even at the maximum price. So asset market clearing requires that she has strictly positive consumption. For this to be an equilibrium, she needs to be indi erent between investing

and consuming, implying an equilibrium price consistent with $R_i^a = 1$. To see this, first calculate $\overline{p}$ from $R_1^a = 1$ as

$$\overline{p} = \frac{E_1[p] + E_s + \quad E[\varphi(s_{t+1})]}{R} \tag{22}$$

$$= \underline{p} + \frac{E[\varphi(s_{t+1})] - \quad p}{R - 1} \tag{23}$$

$$= \frac{E_s + \frac{R - \frac{1}{2}}{R} \quad E[\varphi(s_{t+1})]}{R - 1} \tag{24}$$

where the last line uses the definition of $\underline{p}$. This also yields

$$p = \frac{E[\varphi(s)]}{R} = \frac{E[\varphi(s)]}{R} = \frac{\frac{1}{4}n}{R} \tag{25}$$

To show that at this price, agent 1 can indeed always buy all assets in states s_3, s_4, calculate the excess ressources (multiplied by R for convenience) without initial asset holdings ($a_{1,t} = 0$) in state s_3, the non-bankruptcy state with lowest income, after buying all assets ($a_{1,t+1} = 1$) at the maximum price

$$Rn + E_0[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]) - R\overline{p} \tag{26}$$

$$= Rn + E_0[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]) - E_1[p] - E_s - \quad E[\varphi(s_{t+1})] \tag{27}$$

$$= Rn + E_1[min\{n_i + p_{t+1} + s_{t+1}, \overline{s}\}]) - E_1[p] - E_s \tag{28}$$

$$= Rn + \frac{1}{4}[\underline{p} + E_s - n] + \frac{1}{4}[\underline{p} + E_s] + \frac{1}{2}[\underline{p} + E_s] + \frac{1}{4}[\underline{p} + E_s] - E_1[p] - E_s \tag{29}$$

$$= Rn - \frac{1}{2} \quad p - \frac{1}{4}n \tag{30}$$

$$= Rn - (\frac{1}{8R} + \frac{1}{4})n > 0 \tag{31}$$

Since in all other non-bankrupt states agent 1 has funds that are larger, she can always buy all assets at the maximum price. So $\overline{p}$ is the only equilibrium price outside bankruptcy periods.[3]

To show that $\overline{s} = \underline{p} + E_s$ is an optimal choice for loan riskyness, note that agent 1's total funds in state s_i before paying creditors equal $p(s_i) + s_i + n$. Thus, for $\overline{s} : \underline{p} + E_s > \overline{s} > \underline{p} + E_s - n$ she expects to go bankrupt with probability $\frac{1}{4}$, while agent 0 expects her to go bankrupt with probability 0. Thus, in this range, agent 1 expects to make

[3] In a similar fashion, it is easy to show that the agent 0 always consumes a positive amount after buying collateralised loans worth $\frac{E_0[min f n_i + p_{t+1} + s_{t+1}, \overline{s} g]}{R}$.

pro ts from issuing claims that she perceives as risky, but agent 0 does not. Similarly, for $\overline{s} : \overline{p} + E_s + 3n > \overline{s} > \overline{p} + E_s + 2n$ agent 1 expects to go bankrupt with probability $\frac{3}{4}$, while agent 0 expects her to go bankrupt with certainty. So agent 1 will not be able to increase the funds available for asset purchase by increasing $\overline{s}$ beyond $E_s + 2n$. In other words

$$\frac{dR_1^a}{d\overline{s}}|\underline{p} + E_s - n < \overline{s} < \underline{p} + E_s = R_1^a \times 1 - \frac{3}{4} >$$

$$\frac{dR_1^a}{d\overline{s}}|\underline{p} + E_s \le \overline{s} \le \overline{p} + E_s + 2n = R_1^a \times \frac{1}{2} - \frac{1}{2} = 0 \frac{dR_1^a}{d\overline{s}}|\overline{p} + E_s + 2n < \overline{s} \le \overline{p} + E_s + 3n = R_1^a \times 0 - \frac{1}{4} <$$

Since $R_1^a = 1$ in equilibrium outside bankruptcy, agent 1 is indi erent between any $\overline{s} : \underline{p} + E_s \le \overline{s} < \overline{p} + E_s + 2n$.

Conjecture 1

The equilibrium prices are unique. Multiplicity in consumption follows from that of loan riskyness which can take values $\overline{s} : \underline{p} + E_s \le \overline{s} < \overline{p} + E_s + 2n$ in equilibrium.

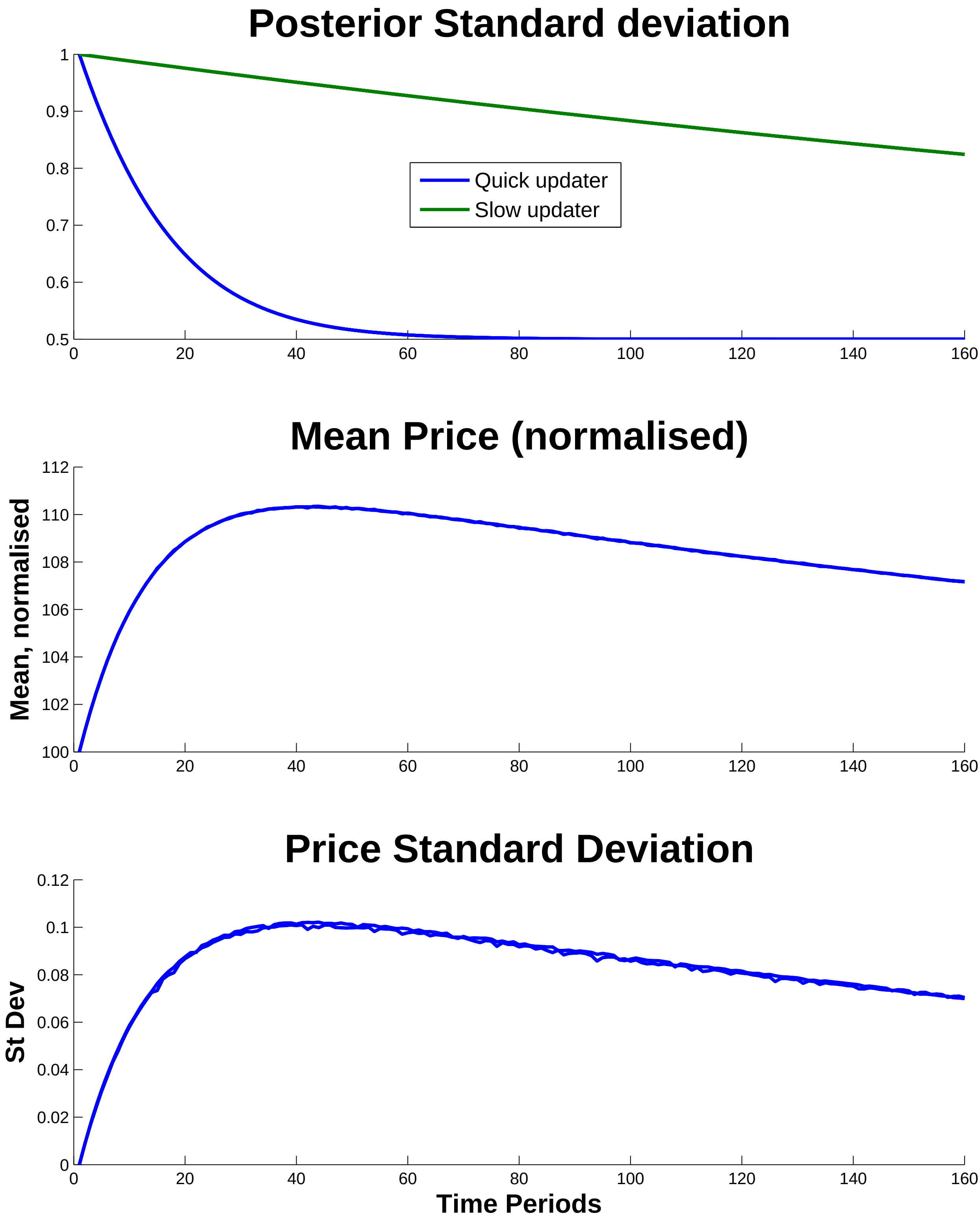
4 The Great Moderation, belief disagreement and asset prices

When investors are risk-neutral, a fall in volatility such as during the Great Moderation should leave their fundamental valuation unchanged. The previous sections showed, however, that with leveraged asset purchases, equilibrium prices nevertheless rise. This section uses the simple dynamic model from the previous section to illustrate the e ect of the Great Moderation on asset prices when the convexity-e ect of heterogeneous beliefs about second moments is the only channel through which a fall in volatility a ects asset prices.

The general environment is the same as in the previous section. Particularly, payo s are distributed on 4 support points. In order to introduce learning in this environment in the simplest possible way, we make the following assumptions: The Great Moderation takes the form of a contraction in the support of a binomial distribution of payo s. Thus, before the Great Moderation, payo s take values in $\{s_1, s_4\}$ only, while during the Great Moderation they uctuate between s_2 and s_3. Investors' prior before the Great Moderation coincides with the true distribution. Once they observe realisations of payo s in $\{s_2, s_3\}$

they therefore notice a change in the environment. To update their prior for the payo distribution in the following periods $\widehat{f}_{it+s}$

Figure 1


Posterior Standard deviation
1
0.9
0.8
0.7
0.6
0.5
0
20
40
60
80
100
120
140
160
Quick updater
Slow updater
Mean Price (normalised)
Mean, normalised
112
110
108
106
104
102
100
0
20
40
60
80
100
120
140
160
Price Standard Deviation
St Dev
0.12
0.1
0.08
0.06
0.04
0.02
0
0
20
40
60
80
100
120
140
160
Time Periods

5.2 Franchise Value

To be added.

6 Conclusion

To be added.

References

Broer, Tobias and Afroditi Kero. 2011. *Great moderation or great mistake: Can rising confidence in low macro-risk explain the boom in asset prices?*.

Geanakoplos, John. 2001. *Liquidity, default and crashes: Endogenous contracts in general equilibrium.*

Harrison, J Michael and David M Kreps. 1978. *Speculative investor behavior in a stock market with heterogeneous expectations*, The Quarterly Journal of Economics **92**, no. 2, 323–36.

Jarrow, Robert A. 1980. *Heterogeneous expectations, restrictions on short sales, and equilibrium asset prices*, Journal of Finance **35**, no. 5, 1105–13.

Lettau, Martin, Sydney C. Ludvigson, and Jessica A. Wachter. 2008. *The declining equity premium: What role does macroeconomic risk play?*, Review of Financial Studies **21**, no. 4, 1653–1687.

Miller, Edward M. 1977. *Risk, uncertainty, and divergence of opinion*, Journal of Finance **32**, no. 4, 1151–68.

Murdock, Kevin C., Thomas F. Hellmann, and Joseph E. Stiglitz. 2000. *Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough?*, American Economic Review **90**, no. 1, 147–165.

Simsek, Alp. 2013. *Belief disagreements and collateral constraints*, Econometrica **81**, no. 1, 1–53.

Xiong, Wei. 2013. *Bubbles, crises, and heterogeneous beliefs.*