



EUROPEAN SUMMER SYMPOSIUM IN ECONOMIC THEORY

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Social Insurance, Information Revelation, and Lack of Commitment

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Liquidity Insurance with Market Information

Luigi Iovino*

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Abstract

Y_r W
 R V V S
 Y_z
 Y_y V W
 V
 W Y_r V
 Y W
 Y

* k Y q $Y Y_z$ x W r V_x ' V
“ x Y_z ff r V_a x V_s y V_r ff V
fi á V_{fi} V_z V
Y
“ z Y

1 Introduction

z

Yff , s V
Y
Y-
Y w V
r s á R s áS
Y “ “ Rj fi S
” — Y V
Y V V
Yz
Yy V
Y Y
V w ff
V
Y á solventV
Ys V w ff
Y V w ff
v Y
w ff
Y á
V Y
Y — — dVcaai V á t
V r ff — R r ff—SV
Y r ff—
Yr V V
V V
Y
Vt u á V Y “

Y_y V
Y_w V
V V
Y V
Y_á V
V Y V
V W
Y
Y_y V
Y_s
Y_t
Y_á Bjj hS i aO
á Yz V
Rz á RabaS t YRabaSSY_w V
V
bi Ygj J e R SWc
V
R SWdgR SWde

V

V

Y_z

Y_w

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Y_y

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Y_s

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Y_z

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Y

d

Related Literature.

Y
Y_z
Vff á Bj hgS
Y á V
Y_r V
ff á Bj hgS Y_w V_z
V
Y
V z r bY Y
Y á
Y_z V_r YRabaS
Y
z V
Y
Y_z
V k
Y_r V
Ys
x RabbS
Y
Y_z s
YRabaS V
V Y- z
R V V Y_r V
SV
after Y V V
W Y
w V
W Rf á Bj i i SVs
RaaSSY á V

$$\begin{array}{c}
Y_w \quad V \\
\\
R \quad x \\
\\
R_{jij}S \quad fl \quad " \quad R_{jj}hS^bY \\
\\
Y_z \quad c_z \quad W \\
\\
Y_z \quad d_z \quad Y\acute{a} \quad e \\
\\
f_z \quad Y_z \quad Y_z \\
V_z \quad W \\
\\
Y\acute{a} \quad g \quad Y_w \quad V \\
\\
r \ b\breve{Y} \quad z \\
Y
\end{array}$$

2 The Model

$$\begin{array}{c}
t = 0, 1, 2 \\
\\
w \quad V \quad A \quad Y \quad Y_v \quad W \\
\\
2Y_r \\
\\
2Y \quad V\theta^H \\
\alpha \quad \theta^L \quad 1 - \alpha V \quad \theta^H > \theta^LY, \quad \tilde{y}(\theta) \\
2 \quad \theta Y_w \quad V_z \quad \tilde{y}(\theta) \\
\\
\tilde{g}(\theta) = \left\{ \begin{array}{l} R \quad YY\theta \\ 0 \quad Y\mathfrak{M} - \theta \end{array} \right. \\
\\
', \quad \Theta \quad V \quad V\Theta = \{\theta^L, \theta^H\}Y \\
\\
2 \quad Y \\
\\
v \quad V \\
\\
Y \quad 2 \\
\\
z \quad L \quad L \\
\\
Y_r \quad V \\
\\
V \quad Y \\
\\
\hline
^b\acute{a} \quad r \quad R_{jj}hSV', \quad R_{j j f}SVt \quad w \quad R_{j j}hSVr \quad YR_{j j j}SV \\
ff \quad R_{aae}SVt \quad YR_{aae}SVx, \quad R_{aa j}SV \quad s \quad YR_{j j j}SY
\end{array}$$

z V I $0V$
 $\rho \sim f_{\theta}(\cdot)$ $1Y$ I
 θ I Y V $2V$
 $\tilde{y}(\theta) IV -$ V
 V $l(\theta) IV$ $l(\theta) < R\theta Y$
 z 1 L LY_z
 $f_{\theta}(\cdot)$ $R^{\epsilon}, -S$ θY
 w V_z $f_{\theta^L}(\rho)/f_{\theta^H}(\rho)$ ρY “ ’ —
 $1Y_z$ V V
 R
 1 SY
 $Y-$
 1
 Y_t Y_r V
 V V
 Y V
 f_{aO} $baaO$ Y
 r ρ $1V$
 V $1Y$ V_{ρ}
 $1Y_z$ ρ
 V V V
 Y_w V
 Y
 z V V_z ρ V
 Y
 w V_z Y_z
 α θ^H $1-\alpha$
 θ^LY YYY $f_{\theta}(\cdot)$
 θY
 Y

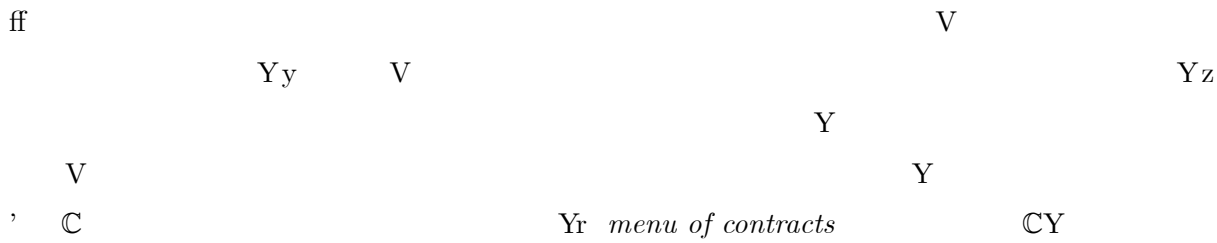
Assumption 1. *Both projects have positive NPV:*

$$\max_{\tilde{\rho}} \left\{ R\theta F_{\theta}(\tilde{\rho}) - 1 - \int_0^{\tilde{\rho}} \rho f_{\theta}(\rho) d\rho \right\} > 0, \quad \theta \in \Theta$$

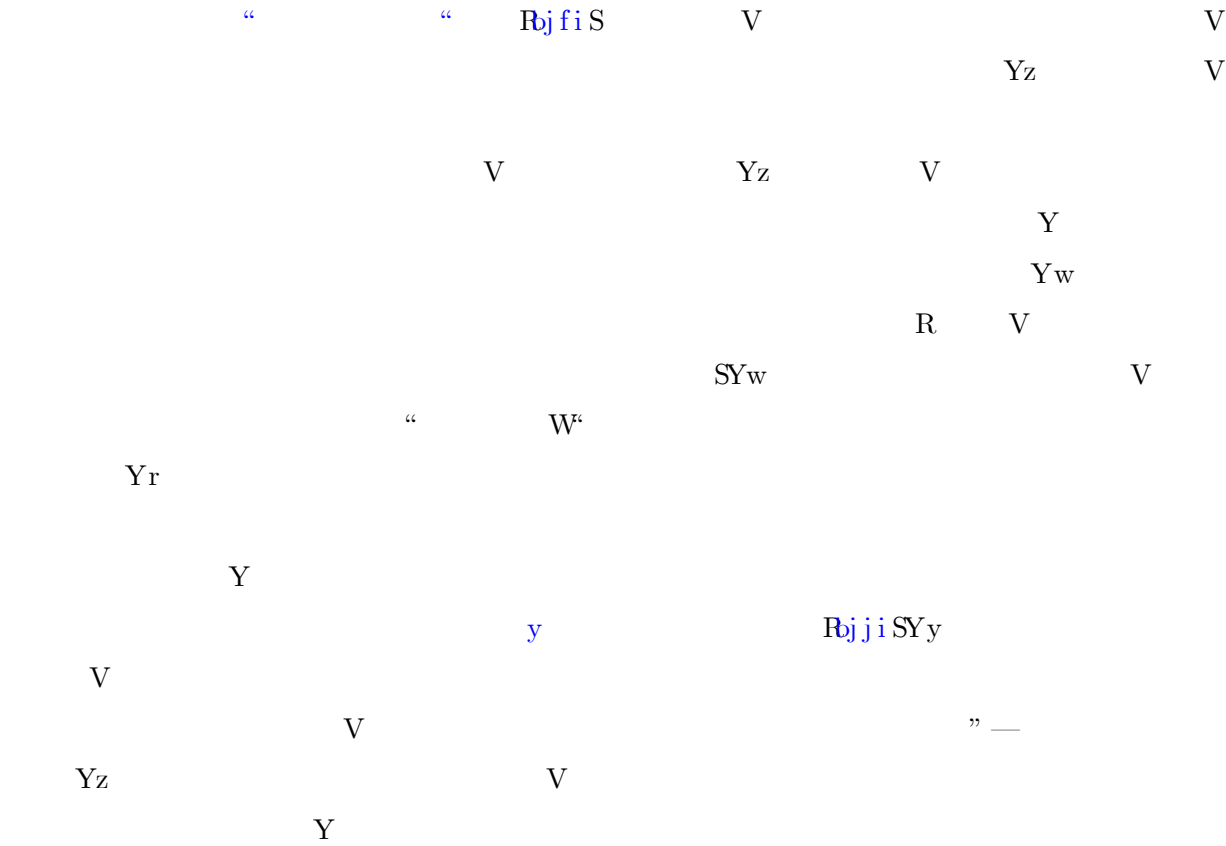
ρV
 V
 SV
 2
 r
 Y
 Y_z
 V
 Y_s
 W
 SY_w
 W
 Y_w
 V_z
 W
 rY
 V
 0
 Y''
 R
 1
 Y_v
 YYY
 Y
 r
 1
 $0Y''$
 V_z
 0
 1
 Y_r
 $1V$
 Y_s
 x
 $R_{j i h} S$
 0
 $1Y$
 $1 R$
 $f i$
 $R_{j i h} S$
 r
 x
 $R_{j j h} S S Y_r$
 V
 z
 $1V$
 V
 Y
 y
 V
 z
 V
 Y
 Y
 h

$$\begin{array}{ccccccc}
r & & V & & & & V \\
& V & & & Y & & \\
& & & & & Y_z & \\
& V & V & & W & Y & V \\
& & & Y_v & & W & \\
& & & Y & & & Y_w \\
V_z & & & & & & \tilde{y}(\theta) \\
l(\theta) & & & Y & & & \\
& & Y_- & V & & & \theta Y \\
& & \tilde{s}_\theta & 1 & & Y & \\
& & \theta & Y & & & \\
& & Y_z & & & & \\
& & & & V & & \\
& & Y & & & & \\
r & V & & & & & V_z \\
& & \tilde{s}_\theta & & & Y_w & V_z \\
& & \tilde{s}_\theta = \theta Y & & & & \\
w & V_z & & V & & V & \\
& & \rho & Y & & & Y \\
& & & & & & \\
& D_i(\theta, p, \rho) & & & iV & & \\
& \theta V & & \rho V & & pY_z & \\
& D_i(\theta, p, \rho) & V & i & & \kappa & \\
& & Y \acute{a} & & & & V \\
& & & & z & i & D_i(\theta, p, \rho)Y \\
& & & & u \sim N(0, \sigma_u^2)Y & & \\
& & & u & & \kappa & \\
& & Y & & & & \\
& & & & Y_z & \phi_P(\cdot|\theta, \rho) & \\
& & & & & & \theta \\
& & \rho & Y_- & V & & \\
& & & & & Y_w & V \\
& \kappa = 1 & & Y & & & \\
& & & & & & \\
& & & & & & i
\end{array}$$

1. payments $\bar{c}^e : \mathbb{R} \times \varrho \times \Omega \rightarrow \mathbb{R}_+$, $\underline{c}^e : \mathbb{R} \times \varrho \times \Omega \rightarrow \mathbb{R}_+$, and $c_L^e : \mathbb{R} \times \varrho \rightarrow \mathbb{R}_+$ to the entrepreneur after observing a price p and a liquidity shock ρ , in the case the project is succesful, completed but unsuccessful, and liquidated, respectively;
2. payments to the investor $\bar{c}^i : \mathbb{R} \times \varrho \times \Omega \rightarrow \mathbb{R}$, $\underline{c}^i : \mathbb{R} \times \varrho \times \Omega \rightarrow \mathbb{R}$, and $c_L^i : \mathbb{R} \times \varrho \rightarrow \mathbb{R}$;
3. an initial investment $I \in \mathbb{R}_+$ in the project;
4. a continuation rule $\chi : \mathbb{R} \times \varrho \rightarrow \{0, 1\}$ which, for each value of p and ρ , is equal to 1 whenever the project is continued.



2.2 Moral Hazard



$\mathbb{W}^{\mathbb{K}}$
 \mathbb{Y}
 \mathbb{Y}
 $\mathbb{Y}^{\mathbb{A}}$
 \mathbb{V}
 y \mathbb{V} \mathbb{V} $f_{\theta}(\rho)$
 $\tilde{y}(\theta)\mathbb{V}$ \mathbb{Y}
 \mathbb{V} \mathbb{Y}
 \mathbb{V} \mathbb{V} \mathbb{Y}
 \mathbb{V} z \mathbb{V}
 $R-\rho_0$ \mathbb{Y} ρ_0
 $\mathbb{Y}_{\mathbb{r}}$ z all
 \mathbb{Y}_y \mathbb{V}
 $l(\theta)I$
 $\mathbb{Y}_{\mathbb{u}}$
 \mathbb{Y}
 s \mathbb{V} $R-\rho_0$
 \mathbb{Y}_z \mathcal{C}
 $(R-\rho_0)I$ \mathbb{V}
 \mathbb{Y} \mathbb{V} \mathcal{C}
 $\forall C \in \mathcal{C}$
 $\bar{c}^e(p,\rho,R) \geq (R-\rho_0)I$ $\mathbb{R}\mathbb{S}$
 \mathbb{V} \mathbb{C}^{MH} \mathbb{C} $\mathbb{R}\mathbb{S}\mathbb{Y}$

2.3 Payoffs

x $\mathcal{C} \subseteq \mathbb{C}\mathbb{V}$ $C \in \mathcal{C}$
 $\theta \mathbb{Y}$ *conditional*
 θ k

$$U\left(C;\theta\right)=\mathbb{E}\left[\theta\chi\left(p,\rho\right)\bar{c}^e\left(p,\rho\right)+\left(1-\theta\right)\chi\left(p,\rho\right)\underline{c}^e\left(p,\rho\right)+\left(1-\chi\left(p,\rho\right)\right)c_L^e\left(p,\rho\right)\left|\theta\right.\right]$$

z C_{θ} \mathcal{C} $y(\theta)\mathbb{Y}_{\mathbb{W}}$
 θ $C_{\theta}\mathbb{Y}$ θ z

$$\begin{array}{c}
\mathcal{C} \text{ before} \\
U(\mathcal{C}) = \alpha U(C_\theta; \theta) + (1 - \alpha) U(C_{\theta'}; \theta') \\
\pi(C; \theta) \\
\mathcal{C} \subseteq \mathbb{C} \qquad \theta \qquad C \in \mathcal{C}_k \\
\pi(C; \theta) = \mathbb{E} \left[\theta \chi(p, \rho) \bar{c}^i(p, \rho) + (1 - \theta) \chi(p, \rho) \underline{c}^i(p, \rho) + (1 - \chi(p, \rho)) c_L^i(p, \rho) \right] - \mathbb{E} [1 + \rho \chi(p, \rho)] I \\
\pi(C; \theta) \qquad pV\rho \qquad y(\theta)Y \\
C \qquad Y \\
k \qquad IV \\
V_X(p, \rho)Y \\
\text{”} \qquad V \qquad \textcolor{blue}{d}V \\
Y \qquad V \\
\pi(C; \theta)V \qquad Y \\
r \qquad V \qquad k \\
\pi(\mathcal{C}) = \alpha \pi(C_\theta; \theta) + (1 - \alpha) \pi(C_{\theta'}; \theta') \\
z \qquad \pi(C; \theta) \qquad Y \\
Y \qquad V \quad \mathcal{C} \qquad V \\
\pi(\mathcal{C}) \geq 0 \qquad \mathbb{R}S \\
W \qquad W \qquad V \qquad V \\
\mathbb{R}S \qquad Y
\end{array}$$

2.4 Equilibrium Definition

$$\begin{array}{c}
z \qquad Y \\
Y_z \qquad \textcolor{blue}{d} \\
\textcolor{blue}{e}Y_r \quad V \qquad V \qquad \textcolor{blue}{r} \textcolor{blue}{b}Y_z \\
Y
\end{array}$$

Definition. An equilibrium for the economy is a menu of contracts $\mathcal{C}^* \subseteq \mathbb{C}^{MH}$, a distribution for the asset price $\Phi_P : \mathbb{R} \times \Theta \times \varrho \rightarrow [0, 1]$, and a demand schedule $D_i : \Theta \times \mathbb{R} \times \varrho \rightarrow \mathbb{R}$ for each trader $i \in [0, 1]$ such that

1. Investors choose $\mathcal{C}^*\subseteq\mathbb{C}^{MH}$ to maximize their profits

$$\mathcal{C}^*\in\arg\max_{\mathcal{C}\subseteq\mathbb{C}^{MH}}\pi\left(\mathcal{C}\right)$$

2. After an entrepreneur learns his type, he selects the contract $C_\theta\in\mathcal{C}^*$ to maximize his expected utility

$$C_\theta\in\arg\max_{C\in\mathcal{C}^*}U\left(C;\theta\right)$$

3. Free-entry among investors drive expected profits to zero

$$\pi\left(\mathcal{C}^*\right)=0$$

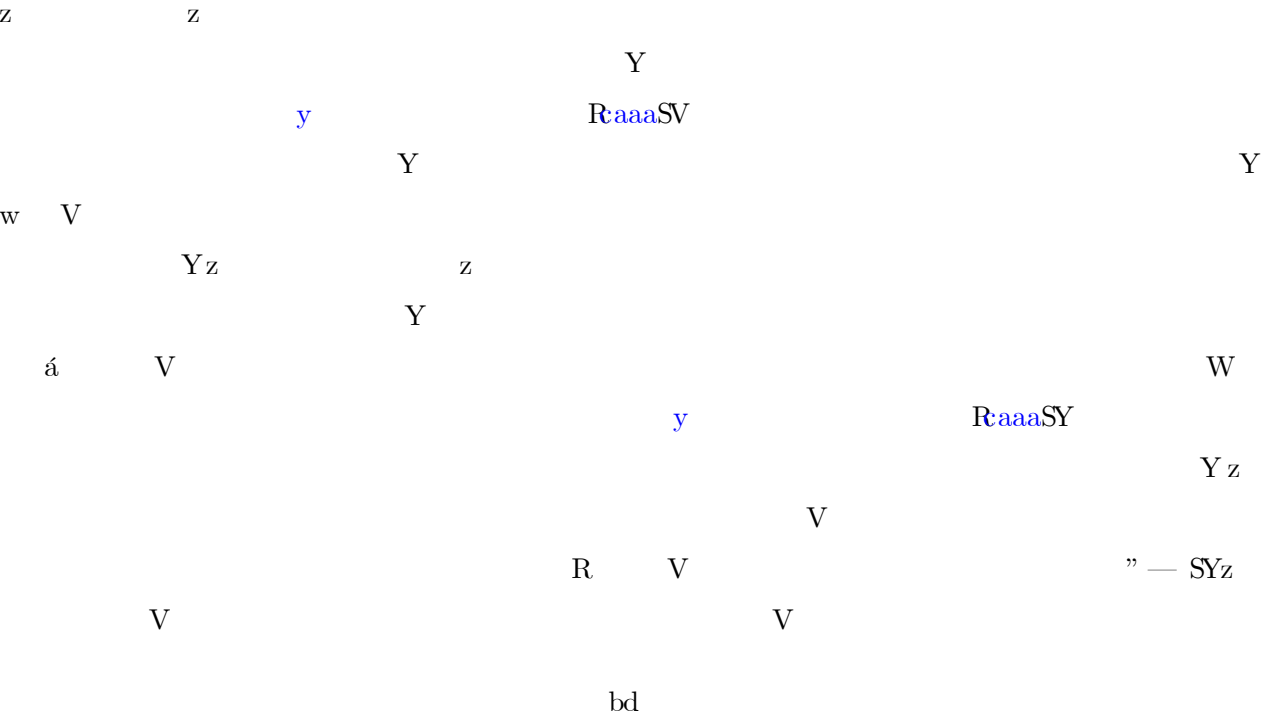
4. For each value of θ , p , and ρ , each trader chooses $D_i\left(\theta,p,\rho\right)$ to maximize expected profits

$$D_i\left(\theta,p,\rho\right)\in\arg\max_x\mathbb{E}\left[\left(\tilde{y}\left(\theta\right)\chi_\theta\left(p,\rho\right)+l\left(\theta\right)\left(1-\chi_\theta\left(p,\rho\right)\right)-p\right)x\middle|\theta,p,\rho\right]-\frac{x^2}{2}+W$$

5. For each value of θ and ρ , $\Phi_P\left(\cdot\middle|\theta,\rho\right)$ is the distribution of prices generated by the equilibrium in the financial market. That is, for each value of θ , ρ , and noisy traders u , $\Phi_P\left(\cdot\middle|\theta,\rho\right)$ is the cdf of the distribution of prices p which solve the equation

$$p=\mathbb{E}\left[\tilde{y}\left(\theta\right)\chi_\theta\left(p,\rho\right)+l\left(\theta\right)\left(1-\chi_\theta\left(p,\rho\right)\right)\middle|\theta,p,\rho\right]-u$$

3 Symmetric Information Benchmark



$$\begin{array}{ccccccc}
& & Y_w & & V & & V \\
& & & & V & & \\
& & Y & & W & & \\
& & & & Y & & \\
" & V & & & & & Y \\
- & V & & & & & RSY_z \\
& & & & & & Y_r \\
& & & & V & & \\
& & & & & & V \\
& & & Y_a & & & \\
& & & Y & & & \\
r & & & V & W & & \\
& & Y_r & V & & & 0V \\
& & & & Y_v & V & 0V \\
& & & & & V & \theta \\
& & Y_w & V & & & \\
& & & Y & & & \\
& & & & W & & Y_w & V \\
\mathcal{C}^{sym} & & & & k & & \\
& & & & \max_{\mathcal{C} \subseteq \mathbb{C}^{MH}} U(\mathcal{C}) & & R-bS \\
& & & & \pi(\mathcal{C}) = 0 & & \\
& & - & V & & & gV \\
R-bSY & & & & & &
\end{array}$$

Proposition 1. *The solution to problem (P1) is given by a menu of contracts \mathcal{C}^{sym} with the following properties*

- 1. The entrepreneur consumes the minimum share of output if the project is successful and nothing otherwise:*

$$\bar{c}_\theta^e(p,\rho) = (R-\rho_0)I_\theta, \; \underline{c}_\theta^e(p,\rho) = 0, \; c_{\theta,L}^e(p,\rho) = 0$$

- 2. The investor receives all the pledgeble income if the project is successful and the liquidation value if the project is liquidated:*

$$\bar{c}_\theta^i(p,\rho) = \rho_0 I_\theta, \; \underline{c}_\theta^i(p,\rho) = 0, \; c_{\theta,L}^i(p,\rho) = l(\theta)I_\theta$$

3. For each θ there exists a threshold ρ_{θ}^* such that $\theta\rho_0 < \rho^*(\theta) < \theta R$ and

$$\chi_{\theta}(p,\rho)=1 \text{ if and only if } \rho \leq \rho_{\theta}^*$$

4. Initial investment is given by:

$$I_{\theta}=\frac{A}{1-l\left(\theta\right)+\int_{\rho_{\theta}^*}^{\infty}\left(\rho-\theta\rho_0+l\left(\theta\right)\right)dF_{\theta}\left(\rho\right)}$$



4 Asymmetric Information without ..nancial markets



$$\pi\left(\mathcal{C}\right)=0$$

$$\mathbb{R}^{\mathbf{t}}\mathbf{S}$$

$$U\left(C_{\theta^i};\theta^i\right)\geq U\left(C_{\theta^j};\theta^i\right)\quad \forall\quad i,j=L,H$$

$$\begin{array}{ccccccc} & & \mathbf{V} & & \mathbf{z} \mathbf{t} & & \\ \mathbf{Y} \mathbf{z} & & & \mathbf{z} \mathbf{t} & & \mathbf{Y} & \mathbf{V} \\ & \textcolor{blue}{\mathbf{g}} \mathbf{V} & & & \textcolor{blue}{-} \mathbf{e} \mathbf{Y} & & \end{array}$$

Proposition 2. *The solution to problem [P2](#) is given by a menu of contracts \mathcal{C}^{asym} with the following properties:*

- The entrepreneur consumes the minimum share of output if the project is successful and nothing otherwise:*

$$c^e_{\theta}(p,\rho)=(R-\rho_0)\,I_{\theta},\,\,c^e_{\theta}(p,\rho)=0,\,\,c^e_{\theta,L}(p,\rho)=0$$

- The investor receives all the pledgeble income if the project is successful and the liquidation value if the project is liquidated:*

$$c^i_{\theta}(p,\rho)=\rho_0I_{\theta},\,\,c^i_{\theta}(p,\rho)=0,\,\,c^i_{\theta,L}(p,\rho)=l\left(\theta\right)I_{\theta}$$

- For each θ there exists a threshold $\rho_{\theta}^{**}(\theta)$ such that*

$$\chi_{\theta}(p,\rho)=1 \text{ if and only if } \rho \leq \rho_{\theta}^{**}$$

where

- the continuation rule for the low type is the same as in the symmetric information benchmark:*

$$\rho_{\theta^L}^{**}=\rho_{\theta^L}^*$$

- the continuation rule for the high type is distorted:*

$$\rho_{\theta^H}^{**}<\rho_{\theta^L}^*.$$

$$\mathrm{bh}$$

4. Initial investment is given by:

$$I_{\theta^H} = \frac{A}{\left(\begin{aligned} &\alpha \left(1 - l \left(\theta^H \right) + \int_{\rho_{\theta^H}^{**}}^{\infty} \left(\rho - \theta^H \rho_0 + l \left(\theta^H \right) \right) dF_{\theta^H} \left(\rho \right) \right) \\ &+ (1 - \alpha) \frac{1 - F_{\theta^L} \left(\rho_{\theta^H}^{**} \right)}{1 - F_{\theta^L} \left(\rho_{\theta^L}^{**} \right)} \left(1 - l \left(\theta^L \right) + \int_{\rho_{\theta^L}^{**}}^{\infty} \left(\rho - \theta^L \rho_0 + l \left(\theta^L \right) \right) dF_{\theta^L} \left(\rho \right) \right) \end{aligned} \right)}$$

and

$$I_{\theta^L} = \frac{1 - F_{\theta^L} \left(\rho_{\theta^H}^{**} \right)}{1 - F_{\theta^L} \left(\rho_{\theta^L}^{**} \right)} I_{\theta^H}$$

ff RbS RbS — c

— bYr V

2 Y V

Y V V Yff RbS — c

Y W

ρ Yy V

V V

Yz V

V

Y

V

V

V

ff RabaS

Y V V Yz ff

RabaS Y

V V

Y Yw Vz V

Y “ Vz

V V Yz V

V V V V

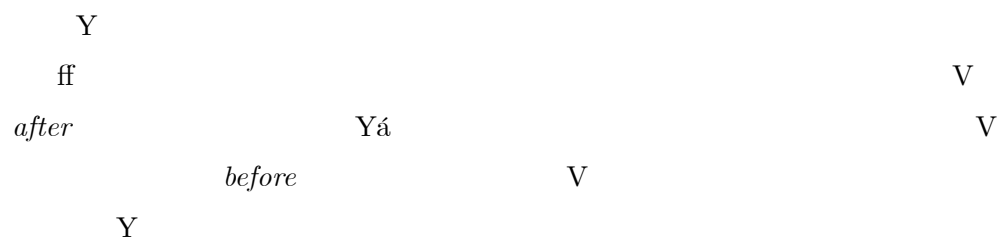
Y“ V V V

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5 Asymmetric Information with Financial markets

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Rbji iSVs

RcaahSS

W

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Yz

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W

Y ff

$\Phi_p\left(\cdot|\theta,\rho\right)$

Y

k

$$\max_{\mathcal{C}\subseteq\mathbb{C}^{MH}}U\left(\mathcal{C};\Phi_p\left(\cdot|\theta,\rho\right)\right)$$

R-dS

k

$$\pi\left(\mathcal{C};\Phi_p\left(\cdot|\theta,\rho\right)\right)=0$$

k

$$U\left(C_{\theta^i};\theta^i,\Phi_p\left(\cdot|\theta^i,\rho\right)\right)\geq U\left(C_{\theta^j};\theta^i,\Phi_p\left(\cdot|\theta^i,\rho\right)\right)\text{ }\forall\text{ }i,j=L,H$$

ca

\mathbb{R}^d \mathbb{S}^2 \mathbb{Y} \mathbb{H}
 \mathbb{C} \mathbb{Y} \mathbb{K}

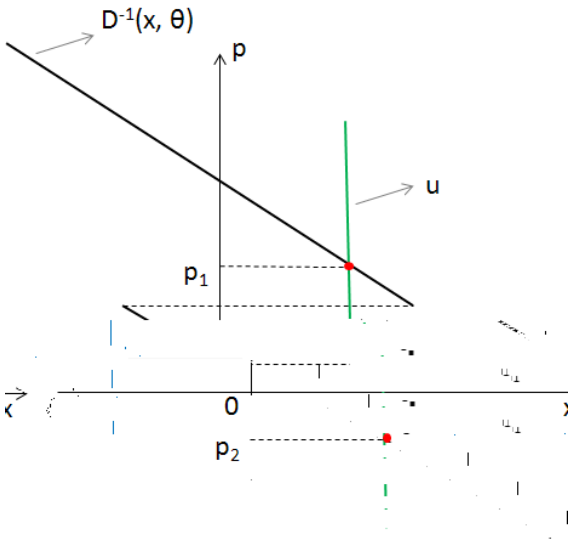
$$p=\mathbb{E}\left[\tilde{y}\left(\theta\right)\chi_{\theta}\left(p,\rho\right)+l\left(\theta\right)\left(1-\chi_{\theta}\left(p,\rho\right)\right)\left|\theta,p,\rho\right]-u\right.$$

$$\mathbb{R}^d \mathbb{S}^2$$

$$\mathbb{E}\left[\tilde{y}\left(\theta\right)\left|\theta\right.\right]=R\theta \mathbb{V} \theta \chi_{\theta}\left(p,\rho\right) \mathbb{Y}^z \left(p,\rho\right) \mathbb{V} l\left(\theta\right) \mathbb{Y} R\theta-p \mathbb{V}$$

$$l\left(\theta\right)-p \mathbb{Y} u \mathbb{Y}$$

$$\theta \rho \mathbb{Y}$$



$$\chi_{\theta}\left(p,\rho\right) \mathbb{Y} u \mathbb{Y}^z \mathbb{V} \mathbb{Y}$$

$$\mathbb{Y}^z \mathbb{V} \mathbb{Y} \mathbb{V} \mathbb{Y} \mathbb{Y}^z \mathbb{Y} \mathbb{Y}^w \mathbb{V}^z \mathbb{Y}$$

$$\mathbb{C} \mathbb{B}$$

$$\begin{array}{c}
\begin{array}{ccccc}
& & Y^{\omega} & & Vz \\
V & & & & \gamma Y \\
& & Yz & & r\ bY\ z & & V \\
R\ S V & & & & \bar{p}_{\theta}(\rho) & & \chi_{\theta}(p,\rho)
\end{array} \\
k \\
\chi_{\theta}(p,\rho) = \begin{cases} 1 & p \geq \bar{p}_{\theta}(\rho) \\ 0 & \end{cases} \quad R\ S \\
pV \\
Y & & & & Y \\
& & \chi_{\theta}(p,\rho)V & & \\
& & k & & \\
\phi_P(p|\theta,\rho) = \begin{cases} \phi_u(R\theta - p) & p \geq R\theta + \bar{p}_{\theta}(\rho) - l(\theta) \\ \gamma\phi_u(R\theta^L - p) & \bar{p}_{\theta}(\rho) \leq p < R\theta + \bar{p}_{\theta}(\rho) - l(\theta) \\ (1-\gamma)\phi_u(l(\theta) - p) & \bar{p}_{\theta}(\rho) - R\theta + l(\theta) \leq p < \bar{p}_{\theta}(\rho) \\ \phi_u(l(\theta) - p) & p < \bar{p}_{\theta}(\rho) - R\theta + l(\theta) \end{cases} \quad R\ S \\
R\ S & & V & & p \\
V & & & & \gamma Y \\
& & Y & & \\
p & & \phi_u & & u \\
& & Yr & & V & & p \\
& & V & & & & Y & & \gamma \\
R\ S & & p \in [\bar{p}_{\theta}(\rho) - R\theta + l(\theta), R\theta + \bar{p}_{\theta}(\rho) - l(\theta)]Y
\end{array}$$

Optimal continuation rules.

$$\begin{array}{c}
R\ S \quad -d \\
& & Yz & & r\ bY\ Vz \\
& & \chi_{\theta}(p,\rho) & & -d \\
& & (p,\rho)Y & & z & & Y^{\omega} & & V \\
& & r\ bY\ Vz & & & & & & \\
V & & V\chi_{\theta^L}(p,\rho) = \chi_{\theta^L}(\rho)V\forall pY & & k
\end{array}$$

$$\begin{array}{ccccccc}
& & Y_y & & V & & V \\
& & Y & & & & \\
& V & & V & & & \\
R & & V & & & & \\
& S & & & W & & Y_{\acute{a}} \\
& & & V & & & \\
& & Y & & & & \\
w & & V & & \bar{\rho}_{\theta^L} & & \bar{p}_{\theta^L}(\rho) = -\infty \quad \rho \leq \bar{\rho}_{\theta^L} \quad \bar{p}_{\theta^L}(\rho) = \infty \\
\rho > \bar{\rho}_{\theta^L} Y_y & & V & & \textcolor{blue}{R}S & & k \\
& & & & \chi_{\theta^L}(\rho) = \begin{cases} 1 & \rho \leq \bar{\rho}_{\theta^L} \\ 0 & \end{cases} & & \\
z & & V & & \bar{\rho}_{\theta^L} & & \\
& \textcolor{blue}{d} & & V & \text{---} & \textcolor{blue}{c}V & \\
& & Y_r & & V & & k \\
& & & & & & \\
Y & & & & & & \\
- & & V & & & & \chi_{\theta^H}(p, \rho) \\
& Y_w & & V & V^* & & \textcolor{blue}{-d}V & V \\
& & & Y_r & V & V_L^* & & \\
& & & & before & & Y & V & z \\
& & \textcolor{blue}{r} \textcolor{blue}{b} Y V & & & & & \bar{p}_{\theta^H}(\rho) Y
\end{array}$$

Lemma 1. The optimal threshold for the high type $\bar{p}_H(\rho)$ in (4) is the solution to:

$$\frac{g_P^L(\bar{p}_{\theta^H}(\rho)) f_{\theta^L}(\rho)}{g_P^H(\bar{p}_{\theta^H}(\rho)) f_{\theta^H}(\rho)} = \frac{\alpha}{1-\alpha} \frac{(\theta^H R - l(\theta^H)) - V^*(\rho - \theta^H \rho_0 + l(\theta^H))}{(\theta^L R - l(\theta^L)) \left(\frac{V^*}{V_L^*} - 1\right)} \tag{R\&S}$$

where

$$g_P^i(\bar{p}_{\theta^H}(\rho)) \equiv \gamma \phi_u(R\theta - \bar{p}_{\theta^H}(\rho)) + (1-\gamma) \phi_u(l(\theta) - \bar{p}_{\theta^H}(\rho)) \, , \quad i = L, H$$

is the pdf of a mixture of two Normal random variables with mean $R\theta$ and $l(\theta)$, respectively, evaluated at the optimal threshold $\bar{p}_{\theta^H}(\rho)$.

$$\begin{array}{ccc}
W & \textcolor{blue}{R\&S} & (\bar{p}_H(\rho), \rho) \\
\bar{p}_{\theta^H}(\rho) Y_w & \text{,} & \textcolor{blue}{b} \quad g_P^i(\bar{p}_{\theta^H}(\rho)) \\
& \gamma \in (0,1) Y & \text{''}
\end{array}$$

$$Y \qquad \qquad \qquad \text{cd}$$

2. The investor receives all the pledgeable income if the project is successful and the liquidation value if the project is liquidated:

$$\bar{c}_{\theta}^i(p,\rho)=\rho_0I_{\theta},\ \underline{c}_{\theta}^i(p,\rho)=0,\ c_{\theta,L}^i(p,\rho)=l(\theta)I_{\theta}$$

3. The low type obtains the same liquidity insurance as in the symmetric information benchmark:

$$\chi_{\theta^L}(\rho)=\begin{cases}1&\text{if }\rho\leq\bar{\rho}_{\theta^L}\\0&\text{otherwise}\end{cases}$$

4. The continuation rule for the high type $\chi_{\theta^H}(p,\rho)$ depends on the asset price. Formally, for each p there exists a threshold $\bar{p}_{\theta^H}(\rho)$ such that

$$\chi_{\theta^H}(p,\rho)=\begin{cases}1&\text{if }p\geq\bar{p}_{\theta^H}(\rho)\\0&\text{otherwise}\end{cases}$$

where $\bar{p}_{\theta^H}(\rho)$ is the unique solution to (6).

5. The initial investment levels are given by (15) and (16) in appendix 1.

6. The optimal contract determines an equilibrium distribution for the asset price $\Phi_P(p|\theta,\rho)$ given by (5).

5.1 Comparative Statics

” z Vz Y z

V

$\sigma_u^2 Y_r$

Y

“ Vz $\sigma_u^2 V$

V

Y– V σ_u^2

Y– V W $\sigma_u^2 V$

W

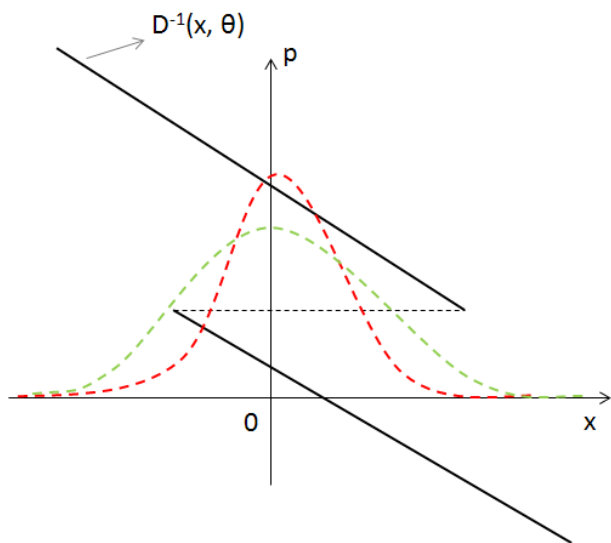
σ_u^2

Y V W

Y V W

Yz V V

Y V V
 Y Y V Y
 V R
 SY
 Y
 r V $'$ \Re a b SY y V_z
 $\sigma_u^2 Y_z$ V V
 Y
 \mathbb{R} SY_z V
 σ_u^2 $\bar{p}_{\theta^H}(\rho)V$ σ_u^2 $\Phi_P(p|\theta,\rho)$
 $Y-$ V $_z$ V V σ_u^2
 V V $\bar{p}_{\theta^H}(\rho)$ Y'
 Y_s V 0
 σ_u^2 V V $\bar{p}_{\theta^H}(\rho)$
 Y 1 V
 $\Phi'_P(p|\theta,\rho)Y$



r W

Y_w $\bar{p}_{\theta^H}(\rho)V$

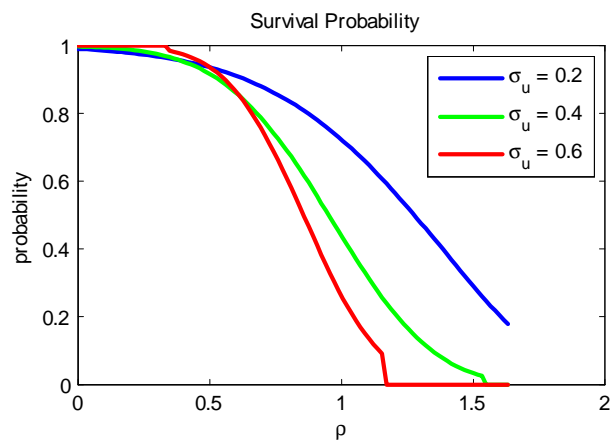
$$\sigma_u^2 \mathbf{V} \qquad \mathbf{V} \qquad \mathbf{V} \qquad \mathbf{Y} \mathbf{r}$$

Proposition 4. *At the optimal contract $\mathcal{C}^{asym p}$, for each ρ the probability of continuation is given by*

$$\Pr \left(p \geq \bar{p}_{\theta^H} \left(\rho \right) \mid \rho \right) = \gamma \Phi \left(R \theta^H - \bar{p}_{\theta^H} \left(\rho \right) \right) + \left(1 - \gamma \right) \Phi \left(l \left(\theta^H \right) - \bar{p}_{\theta^H} \left(\rho \right) \right)$$

Furthermore, there exists a threshold $\tilde{p}_{\theta^H} \left(\gamma \right) > 0$ such that $\Pr \left(p \geq \bar{p}_{\theta^H} \left(\rho \right) \mid \rho \right)$ is decreasing in σ_u^2 for $\bar{p}_{\theta^H} \left(\rho \right) \leq \tilde{p}_{\theta^H} \left(\gamma \right)$ and it is increasing in σ_u^2 for $\bar{p}_{\theta^H} \left(\rho \right) > \tilde{p}_{\theta^H} \left(\gamma \right)$. Finally, $d\tilde{p}_{\theta^H} \left(\gamma \right) / d\gamma < 0$.

$$\begin{array}{ccccccc} \text{---} & \textcolor{blue}{e} & \sigma_u^2 & & & & \\ & \bar{p}_{\theta^H} \left(\rho \right) & & \rho \mathbf{Y} \mathbf{w} & & & \\ & \mathbf{V} & \mathbf{V} & & \mathbf{R} \mathbf{Y} \mathbf{W} \bar{p}_{\theta^H} \left(\rho \right) & \mathbf{S} & \\ & & & \mathbf{Y} & \sigma_u^2 & & \\ & & & & \mathbf{Y} & & \\ \rho \mathbf{Y} & & & & & & \\ & & & & & & \\ & & & & \mathbf{Y} \mathbf{z} & & \\ & \mathbf{Y} \mathbf{r} & & \mathbf{V} & & \mathbf{Y} \mathbf{z} & \\ \mathbf{V} & & & & & & \\ & \textcolor{blue}{e} \mathbf{Y} & & & \textcolor{blue}{R} \textcolor{blue}{S} \mathbf{Y} & & \\ & & \sigma_u^2 \mathbf{V} & & \sigma_u^2 & & \\ g_P^L \left(\bar{p}_{\theta^H} \left(\rho \right) \right) / g_P^H \left(\bar{p}_{\theta^H} \left(\rho \right) \right) \mathbf{Y}'' & \mathbf{V} & & g_P^L \left(\bar{p}_{\theta^H} \left(\rho \right) \right) / g_P^H \left(\bar{p}_{\theta^H} \left(\rho \right) \right) & \sigma_u^2 & & \\ 1 \mathbf{Y} & & & & & & \mathbf{V} \\ & \textcolor{blue}{R} \textcolor{blue}{S} & & & \rho_{\theta^H}^{**} & \text{---} & \textcolor{blue}{c} \mathbf{Y} \\ & & & \mathbf{V} \Pr \left(p \geq \bar{p}_{\theta^H} \left(\rho \right) \mid \rho \right) & & \rho & \\ & & \sigma_u^2 \mathbf{Y} \mathbf{z} & \mathbf{z} & l \left(\theta^H \right) = l \left(\theta^L \right) = 0 & \gamma = 1 \mathbf{Y} & \end{array}$$



Y V
 V
 w V_Z Y
 Y_Z V_Z
 Y

References

- u r w Y p V V
Journal of Political Economy Vbaf RShaj f bVr bj j hY
- r Vr s V — Yu Y
The Quarterly Journal of Economics Vbbe RShdf j bdj hV” bj j j Y
- w r u x Yw V V Y
Journal of Political Economy Vbaf RShf cd egVfi bj j hY
- x W r Vx ’ V r — Ys
kr YcabaY“ z Y
- s s “ x Yr V V Y *American*
Economic Review Vhj RShbe dbV“ bj i j Y
- s á Ys V“ x V á x Y
Y z fi Ys Y “ Y V V *Handbook of Macroe-*
conomics V b *Handbook of Macroeconomics* V cbV bdeb bdj dYv V
— bj j j Y
- á s u x Y— V V Y *New*
Approaches to Monetary Economics V gj ii Vbj i hY
- r s — x Yv Y *Journal*
of Political Economy Vbbe RShci f fbgVfi caagY
- s u á á Yr
Y *American Economic Review* Vi a RShj d bagV“ bj j aY
- s z x Yx Y
fi cabbY á Y
- s Vz x V v á — Y “ W Y
Review of Financial Studies Vcd RShi b i caVw cabaY
- á s fi Y—
Y *Journal of Economic Theory* Vbdf RShf h di bVfi caahY

“ t Vfi ffYx V t ffYy Y
k v Y *Journal of Financial Economics* Vj hRiSeha ei hV
á cabaY

t t á w Yr V V k r
Y *American Economic Review* Vi hRiSkj d j baVu
bjj hY

t Vff “ V ffi Y r
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fl fi t r á Y W
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Review* Vi fRiSkbbag bhVu bji f Y

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dje ebi Vr bji hhY

w “ “ y Y“ Y V V
Y*American Economic Review*VeI RlScgb cjhVbj fi Y

v t — ff “ Y—
Y*Econometrica*VfcRlScb efVfi bjieY

r r Yff Yv V V Y*Journal of Monetary
Economics*VfbRlScff fhVr caaeY

r r Yff áY Y t V V
Y*Journal of Finance*VgfRlScj d cdccVu cabaY

fiYff Yz Y*Econometrica*VehRlScddb dfj Vbj hj Y

“ y Yff u v Y“ Yá Y— W
Y*Journal of Economic Theory*VefRlScbi j bj j Vfi bj ii Y

t ff Y Y*Journal of
Risk & Insurance*Vhi RlScgh ci f Vfi cabbY

“ ff fi v á Yv kr
Y*The Quarterly Journal of Economics*VjaRlScgda ej V

” bj hgY

r ff — á Y x Y
Y*Economic Theory*VdhRlScb c j Vcaai Y

ff ’ á r Ys k u V V
Y*Journal of Money, Credit and Banking*Vcj RlScf bh deV” bj j hY

“ á Y— Y*Journal of Public
Economics*VbaRlScch eehVu bj hi Y

v W Y r V k
Y*Finance Research Letters*VbRlScbb cdV“ caaeY

t Yr Y*Journal of Economic
Theory*VbgRlScgh cahVu bj hhY

A1.1 Proof of results (1) and (2) in Propositions 1, 2, and 3

u

S

V

$$\begin{aligned}
 S(\chi_{\theta^H}, \chi_{\theta^L}, I_{\theta^H}, I_{\theta^L}) &\equiv \alpha \theta^H R I_{\theta^H} \int \int \chi_{\theta^H}(p, \rho) d\Phi_P(p|\theta^H, \rho) dF_{\theta^H}(\rho) \\
 &+ (1 - \alpha) \theta^L R I_{\theta^L} \int \int \chi_{\theta^L}(p, \rho) d\Phi_P(p|\theta^L, \rho) dF_{\theta^L}(\rho) \\
 &+ \alpha l(\theta^H) I_{\theta^H} \int \int (1 - \chi_{\theta^H}(p, \rho)) d\Phi_P(p|\theta^H, \rho) dF_{\theta^H}(\rho) \\
 &+ (1 - \alpha) l(\theta^L) I_{\theta^L} \int \int (1 - \chi_{\theta^L}(p, \rho)) d\Phi_P(p|\theta^L, \rho) dF_{\theta^L}(\rho)
 \end{aligned}$$

$$\mathbb{E}^\theta [\mathbf{c}_{\theta'}^e \chi_{\theta'}]$$

θ

$\theta'V$

V

$$\begin{aligned}
 \mathbb{E}^\theta [\mathbf{c}_{\theta'}^e \chi_{\theta'}] &= \theta \int \int \bar{c}_\theta^e(p, \rho) \chi_\theta(p, \rho) d\Phi_P(p|\theta', \rho) dF_{\theta'}(\rho) \\
 &+ (1 - \theta) \int \int \underline{c}_\theta^e(p, \rho) \chi_\theta(p, \rho) d\Phi_P(p|\theta', \rho) dF_{\theta'}(\rho) \\
 &+ \int \underline{c}_{\theta,L}^e(p, \rho) (1 - \chi_\theta(p, \rho)) d\Phi_P(p|\theta', \rho) dF_{\theta'}(\rho)
 \end{aligned}$$

$$\max_{\{\mathbf{c}^e, \chi_{\theta^H}, \chi_{\theta^L}, I_{\theta^H}, I_{\theta^L}\}} S(\chi_{\theta^H}, \chi_{\theta^L}, I_{\theta^H}, I_{\theta^L})$$

$$\begin{aligned}
 &S(\chi_{\theta^H}, \chi_{\theta^L}, I_{\theta^H}, I_{\theta^L}) - \alpha \mathbb{E}^{\theta^H} [\mathbf{c}_{\theta^H}^e \chi_{\theta^H}] - (1 - \alpha) \mathbb{E}^{\theta^L} [\mathbf{c}_{\theta^L}^e \chi_{\theta^L}] - \alpha I_{\theta^H} - (1 - \alpha) I_{\theta^L} + A \quad \text{Rs} \\
 - &\alpha I_{\theta^H} \int \int \rho \chi_{\theta^H}(p, \rho) d\Phi_P(p|\theta^H, \rho) dF_{\theta^H}(\rho) - (1 - \alpha) I_{\theta^L} \int \int \rho \chi_{\theta^L}(p, \rho) d\Phi_P(p|\theta^L, \rho) dF_{\theta^L}(\rho) \geq 0
 \end{aligned}$$

$$\mathbb{E}^{\theta^L} [\mathbf{c}_{\theta^L}^e \chi_{\theta^L}] \geq \mathbb{E}^{\theta^L} [\mathbf{c}_{\theta^H}^e \chi_{\theta^H}] \quad \text{RS}$$

$$\mathbb{E}^{\theta^H} [\mathbf{c}_{\theta^H}^e \chi_{\theta^H}] \geq \mathbb{E}^{\theta^H} [\mathbf{c}_{\theta^L}^e \chi_{\theta^L}] \quad \text{RS}$$

$$\bar{c}_{\theta^H}^e(p, \rho) \geq (R - \rho_0) I_{\theta^H}, \quad \bar{c}_{\theta^L}^e(p, \rho) \geq (R - \rho_0) I_{\theta^L} \quad \text{RbS}$$

$$\underline{c}_{\theta^H}^e(p, \rho) \geq 0, \quad \underline{c}_{\theta^L}^e(p, \rho) \geq 0 \quad \text{RbS}$$

$$c_{\theta^H,L}^e(p, \rho) \geq 0, \quad c_{\theta^L,L}^e(p, \rho) \geq 0$$

dd

$$\begin{array}{c}
\begin{array}{ccccc}
z & & V & & W \\
& & \textcolor{blue}{R}SV & & \textcolor{blue}{V}S \quad \textcolor{blue}{R}SV \\
& & & & \textcolor{blue}{R}aSV \\
& & \textcolor{blue}{R}bSY & & \\
& & & & V \quad Y \\
V & & z & & zt \quad Y \\
& & & & \\
& & \mu V \nu V \theta^H \gamma_H(p, \rho) \phi_P(p|\theta^H, \rho) f_{\theta^H}(\rho) V \theta^L \gamma_L(p, \rho) \phi_P(p|\theta^L, \rho) f_{\theta^L}(\rho) V \\
(1 - \theta^H) \varphi_H(p, \rho) \phi_P(p|\theta^H, \rho) f_{\theta^H}(\rho) V (1 - \theta^L) \varphi_L(p, \rho) \phi_P(p|\theta^L, \rho) f_{\theta^L}(\rho) & & & & W \\
& & V & & Y \\
& & & & Y Y Y \bar{c}_{\theta^H}^e(p, \rho) V \bar{c}_{\theta^H}^e(p, \rho) V \bar{c}_{\theta^L}^e(p, \rho) V \\
\bar{c}_{\theta^L}^e(p, \rho) & & V & & V
\end{array} \\
\\
\begin{array}{c}
-\mu \alpha \chi_{\theta^H}(p, \rho) - \nu \frac{\theta^L}{\theta^H} \chi_{\theta^H}(p, \rho) + \gamma_H(p, \rho) \geq 0 \\
-\mu \alpha \chi_{\theta^H}(p, \rho) - \nu \frac{1 - \theta^L}{1 - \theta^H} \chi_{\theta^H}(p, \rho) + \varphi_H(p, \rho) \geq 0 \\
-\mu (1 - \alpha) \chi_{\theta^L}(p, \rho) + \nu \chi_{\theta^L}(p, \rho) + \gamma_L(p, \rho) \geq 0 \\
-\mu (1 - \alpha) \chi_{\theta^L}(p, \rho) + \nu \chi_{\theta^L}(p, \rho) + \varphi_L(p, \rho) \geq 0 \\
-\mu \alpha (1 - \chi_{\theta^H}(p, \rho)) - \nu \frac{\theta^L}{\theta^H} (1 - \chi_{\theta^H}(p, \rho)) + \delta_H(p, \rho) \geq 0 \\
-\mu (1 - \alpha) (1 - \chi_{\theta^L}(p, \rho)) + \nu (1 - \chi_{\theta^L}(p, \rho)) + \delta_L(p, \rho) \geq 0
\end{array} \\
\\
\begin{array}{c}
'' \\
\begin{array}{ccc}
\bar{c}_{\theta^H}^e(p, \rho) \geq (R - \rho_0) I_{\theta^H} & & \bar{c}_{\theta^H}^e(p, \rho) \geq 0 \\
(p, \rho) & \textcolor{blue}{R}Y\textcolor{blue}{W} \chi_{\theta^H}(p, \rho) = 1SV & \gamma_H(p, \rho) = 0 \\
\varphi_H(p, \rho) = 0Y & & \\
\bar{c}_{\theta^H}^e(p, \rho) \geq (R - \rho_0) I_{\theta^H} & \bar{c}_{\theta^H}^e(p, \rho) \geq 0 & Y \\
V & &
\end{array}
\end{array} \\
\\
\begin{array}{c}
\bar{c}_{\theta^H}^e(p, \rho) = (R - \rho_0) I_{\theta^H} \\
\\
\bar{c}_{\theta^L}^e(p, \rho) = 0 \\
\\
r \quad V \\
\\
-\mu (1 - \alpha) R \chi_{\theta^L}(p, \rho) + \nu R \chi_{\theta^L}(p, \rho) + \gamma_L(p, \rho) \leq 0 \\
-\mu (1 - \alpha) R \chi_{\theta^L}(p, \rho) + \nu R \chi_{\theta^L}(p, \rho) + \varphi_L(p, \rho) \leq 0 \\
\\
- \quad V \quad \bar{c}_{\theta^L}^e(p, \rho) = \infty \quad \bar{c}_{\theta^L}^e(p, \rho) = \infty \quad \textcolor{blue}{R}SY
\end{array}
\end{array}$$

$$\begin{aligned}
& \text{z} \quad \text{Y} \\
& \text{z} \quad (p, \rho) \quad \chi_{\theta^L}(p, \rho) = 1V \\
& \quad \bar{c}_{\theta^L}^e(p, \rho) \quad \underline{c}_{\theta^L}^e(p, \rho) \quad \bar{c}_{\theta^L}^e(p, \rho) \geq (R - \rho_0) I_{\theta^L} \quad \underline{c}_{\theta^L}^e(p, \rho) \geq 0 \\
& \text{YZ} \quad \text{V} \quad \gamma_L(p, \rho) = 0 \quad \varphi_L(p, \rho) = 0 \quad \text{k} \\
& \quad -\mu(1 - \alpha) + \nu = 0 \\
& \text{s} \quad \text{V} \quad \gamma_L(p, \rho) = 0 \quad \varphi_L(p, \rho) = 0 \quad (p, \rho) \quad \chi_{\theta^L}(p, \rho) = 1Y \\
& \quad \text{V} \quad (p, \rho) \quad \gamma_L(p, \rho) > 0 \quad \varphi_L(p, \rho) > 0V \\
& \bar{c}_{\theta^L}^e(p, \rho) \quad \underline{c}_{\theta^L}^e(p, \rho) \quad \text{Y} \\
& \quad \text{V} \quad -\mu(1 - \alpha) + \nu = 0V \quad (p, \rho) \quad \chi_{\theta^L}(p, \rho) = 0Y \\
& \acute{\text{a}} \quad \gamma_L(p, \rho) > 0 \text{ R} \quad \text{S} \quad \varphi_L(p, \rho) > 0 \text{ R} \\
& \quad \text{SY} \quad \bar{c}_{\theta^L}^e(p, \rho) = \infty V \\
& \underline{c}_{\theta^L}^e(p, \rho) = \infty Y \text{ s} \quad \text{V} \quad \text{V} \\
& \quad \text{Y} \quad \text{V} \quad \gamma_L(p, \rho) = 0 \\
& \varphi_L(p, \rho) = 0 \quad (p, \rho) \quad \chi_{\theta^L}(p, \rho) = 0Y \text{z} \quad \gamma_L(p, \rho) = 0 \\
& \quad (p, \rho) \quad \chi_{\theta^L}(p, \rho) = 1Y \quad \text{V} \gamma_L(p, \rho) = 0 \quad (p, \rho)Y \quad \text{V} \quad \text{V}
\end{aligned}$$

$$\begin{aligned}
& \int \gamma_L(p, \rho) (\bar{c}_{\theta^L}^e(p, \rho) - (R - \rho_0) I_{\theta^L}) = 0 \\
& I_{\theta^L} \quad \text{V} \quad \text{V} \quad I_{\theta^L} = \infty Y \quad \text{R.S} \quad \text{Y} \\
& \text{V}
\end{aligned}$$

$$\bar{c}_{\theta^H}^e(p, \rho) \geq (R - \rho_0) I_{\theta^H}, \quad \bar{c}_{\theta^L}^e(p, \rho) \geq (R - \rho_0) I_{\theta^L}$$

$$\begin{aligned}
& \underline{c}_{\theta^H}^e(p, \rho) \geq 0, \quad \underline{c}_{\theta^L}^e(p, \rho) \geq 0 \\
& c_{\theta^H, L}^e(p, \rho) \geq 0, \quad c_{\theta^L, L}^e(p, \rho) \geq 0
\end{aligned}$$

A1.2 Proof of Lemma 1

$$\begin{aligned}
& \text{t} \quad \text{k} \\
& \max_{\{\chi_{\theta^H}, \chi_{\theta^L}, I_{\theta^H}, I_{\theta^L}\}} S(\chi_{\theta^H}, \chi_{\theta^L}, I_{\theta^H}, I_{\theta^L})
\end{aligned}$$

$$\begin{aligned}
& S(\chi_{\theta^H}, \chi_{\theta^L}, I_{\theta^H}, I_{\theta^L}) - \alpha \mathbb{E}^{\theta^H} [\mathbf{c}_{\theta^H}^e \chi_{\theta^H}] - (1 - \alpha) \mathbb{E}^{\theta^L} [\mathbf{c}_{\theta^L}^e \chi_{\theta^L}] - \alpha I_{\theta^H} - (1 - \alpha) I_{\theta^L} + A \\
& - \alpha I_{\theta^H} \int \int \rho \chi_{\theta^H}(p, \rho) d\Phi_P(p|\theta^H, \rho) dF_{\theta^H}(\rho) - (1 - \alpha) I_{\theta^L} \int \int \rho \chi_{\theta^L}(p, \rho) d\Phi_P(p|\theta^L, \rho) dF_{\theta^L}(\rho) \geq 0
\end{aligned}$$

$$\mathbb{E}^{\theta^L}\left[\mathbf{c}_{\theta^L}^e\chi_{\theta^L}\right]\geq\mathbb{E}^{\theta^L}\left[\mathbf{c}_{\theta^H}^e\chi_{\theta^H}\right]$$

$$\begin{array}{ccccc} \text{s} & & \text{V} & \text{z} & I_{\theta^H} \\ & & & & \text{V} \\ I_{\theta^L} & \text{Yr} & & \mathbf{c}^e & \end{array}$$

$$\mathbf{k}$$

$$V^* = \max_{\{\chi^H,\chi^L\}} A \frac{\tilde{S}\left(\chi_{\theta^H},\chi_{\theta^L}\right)}{\tilde{D}\left(\chi_{\theta^H},\chi_{\theta^L}\right)}$$

$$\begin{aligned}\tilde{S}\left(\chi_{\theta^H},\chi_{\theta^L}\right)\quad\equiv\quad&\alpha l\left(\theta^H\right)+\alpha\left(\theta^HR-l\left(\theta^H\right)\right)\int\int\chi_{\theta^H}\left(p,\rho\right)d\Phi_P\left(p|\theta^H,\rho\right)dF_{\theta^H}\left(\rho\right)\\&+\left(1-\alpha\right)l\left(\theta^L\right)+\left(1-\alpha\right)\left(\theta^LR-l\left(\theta^L\right)\right)\int\int\chi_{\theta^H}\left(p,\rho\right)d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}\left(\rho\right)\end{aligned}$$

$$\begin{aligned}\tilde{D}\left(\chi_{\theta^H},\chi_{\theta^L}\right)\quad\equiv\quad&\alpha\left(1-l\left(\theta^H\right)+\int\int\left(\rho-\theta^H\rho_0+l\left(\theta^H\right)\right)\chi_{\theta^H}\left(p,\rho\right)d\Phi_P\left(p|\theta^H,\rho\right)dF_{\theta^H}\left(\rho\right)\right)\\&+\left(1-\alpha\right)\frac{\int\int\chi_{\theta^H}\left(p,\rho\right)d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}\left(\rho\right)}{\int\int\chi_{\theta^L}\left(p,\rho\right)d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}\left(\rho\right)}\\&\times\left(1-l\left(\theta^L\right)+\int\int\left(\rho-\theta^L\rho_0+l\left(\theta^L\right)\right)\chi_{\theta^L}\left(p,\rho\right)d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}\left(\rho\right)\right)\end{aligned}$$

$$\text{z}\qquad\qquad\chi_{\theta}\left(p,\rho\right)$$

$$\chi_{\theta}\left(p,\rho\right)=\left\{\begin{array}{ll}1&p\geq\bar{p}_{\theta}\left(\rho\right)\\0&\end{array}\right.$$

$$\begin{array}{ccccc} \bar{p}_{\theta}\left(\rho\right) & \text{Yr} & & \text{V} & \bar{p}_{\theta}\left(\rho\right)\text{V} \\ & p & & \mathbf{k} & \end{array}$$

$$\phi_P\left(p|\theta,\rho;\bar{p}_{\theta}\left(\rho\right)\right)=\left\{\begin{array}{ll}\phi_u\left(R\theta-p\right) & p\geq R\theta+\bar{p}_{\theta}\left(\rho\right)-l\left(\theta\right)\\ \gamma\phi_u\left(R\theta^L-p\right) & \bar{p}_{\theta}\left(\rho\right)\leq p<R\theta+\bar{p}_{\theta}\left(\rho\right)-l\left(\theta\right)\\ \left(1-\gamma\right)\phi_u\left(l\left(\theta\right)-p\right) & \bar{p}_{\theta}\left(\rho\right)-R\theta+l\left(\theta\right)\leq p<\bar{p}_{\theta}\left(\rho\right)\\ \phi_u\left(l\left(\theta\right)-p\right) & p<\bar{p}_{\theta}\left(\rho\right)-R\theta+l\left(\theta\right)\end{array}\right.$$

k

$$V^* = \max_{\{\bar{p}_{\theta^H}(\rho), \bar{p}_{\theta^L}(\rho)\}} A \frac{\tilde{S}\left(\bar{p}_{\theta^H}(\rho), \bar{p}_{\theta^L}(\rho)\right)}{\tilde{D}\left(\bar{p}_{\theta^H}(\rho), \bar{p}_{\theta^L}(\rho)\right)}$$

$$\mathrm{dg}$$

$$\begin{aligned}\tilde{S}\left(\bar{p}_{\theta^H}\left(\rho\right),\bar{p}_{\theta^L}\left(\rho\right)\right) &\equiv \alpha l\left(\theta^H\right)+\alpha\left(\theta^H R-l\left(\theta^H\right)\right) \int \int_{\bar{p}_{\theta^H}(\rho)}^{\infty} d \Phi_P\left(p \mid \theta^H, \rho\right) d F_{\theta^H}(\rho) \\ &+ (1-\alpha) l\left(\theta^L\right)+(1-\alpha)\left(\theta^L R-l\left(\theta^L\right)\right) \int \int_{\bar{p}_{\theta^H}(\rho)}^{\infty} d \Phi_P\left(p \mid \theta^L, \rho\right) d F_{\theta^L}(\rho)\end{aligned}$$

$$\begin{aligned}\tilde{D}\left(\bar{p}_{\theta^H}\left(\rho\right),\bar{p}_{\theta^L}\left(\rho\right)\right) &\equiv \alpha\left(1-l\left(\theta^H\right)+\int \int_{\bar{p}_{\theta^H}(\rho)}^{\infty}\left(\rho-\theta^H \rho_0+l\left(\theta^H\right)\right) d \Phi_P\left(p \mid \theta^H, \rho\right) d F_{\theta^H}(\rho)\right) \\ &+ (1-\alpha) \frac{\int \int_{\bar{p}_{\theta^H}(\rho)}^{\infty} d \Phi_P\left(p \mid \theta^L, \rho\right) d F_{\theta^L}(\rho)}{\int \int_{\bar{p}_{\theta^L}(\rho)}^{\infty} d \Phi_P\left(p \mid \theta^L, \rho\right) d F_{\theta^L}(\rho)} \\ &\times\left(1-l\left(\theta^L\right)+\int \int_{\bar{p}_{\theta^L}(\rho)}^{\infty}\left(\rho-\theta^L \rho_0+l\left(\theta^L\right)\right) d \Phi_P\left(p \mid \theta^L, \rho\right) d F_{\theta^L}(\rho)\right)\end{aligned}$$

$$''\hspace{10em}YYY\bar{p}_{\theta^L}\left(\rho\right)$$

$$\max_{\bar{p}_{\theta^L}(\rho)}\frac{(\theta^L R-l\left(\theta^L\right))\int \int_{\bar{p}_{\theta^L}(\rho)}^{\infty} d \Phi_P\left(p \mid \theta^L, \rho\right) d F_{\theta^L}(\rho)}{1-l\left(\theta^L\right)+\int \int_{\bar{p}_{\theta^L}(\rho)}^{\infty}\left(\rho-\theta^L \rho_0+l\left(\theta^L\right)\right) d \Phi_P\left(p \mid \theta^L, \rho\right) d F_{\theta^L}(\rho)}\hspace{1em}\text{RbcS}$$

$$\hspace{10em}\text{RbcS}\hspace{10em}\text{Y}$$

$$\hspace{1em}\text{V}\hspace{10em}\chi_{\theta^L}\hspace{10em}p\text{Yz}\hspace{10em}\text{V}$$

$$\hspace{1em}\rho_L^*$$

$$\chi_{\theta^L}\left(\rho\right)=\left\{\begin{array}{ll}1&\rho\leq\rho_L^*\\0&\end{array}\right.$$

$$\hspace{1em}\text{u}\hspace{10em}V_L^*\hspace{10em}\text{RbcS}\hspace{10em}\chi_{\theta^L}\left(\rho\right)$$

$$\text{R}\hspace{10em}\text{Sk}$$

$$V^*=\max_{\bar{p}_{\theta^H}(\rho)}A\frac{\tilde{S}\left(\bar{p}_{\theta^H}\left(\rho\right),\rho_L^*\right)}{\tilde{D}\left(\bar{p}_{\theta^H}\left(\rho\right),\rho_L^*\right)}\hspace{1em}\text{RbdS}$$

$$\text{t}\hspace{10em}\text{k}$$

$$\begin{aligned}&\int \int_{\bar{p}_{\theta}(\rho)}^{\infty} d \Phi_P\left(p \mid \theta, \rho\right) d F_{\theta}(\rho) \\ = &\int \int_{R \theta+\bar{p}_{\theta^H}(\rho)-l(\theta)}^{\infty} \phi_u\left(R \theta-p\right) f_{\theta}(\rho) d p d \rho+\int \int_{\bar{p}_{\theta^H}(\rho)}^{R \theta+\bar{p}_{\theta^H}(\rho)-l(\theta)} \gamma \phi_u\left(R \theta-p\right) f_{\theta}(\rho) d p d \rho\end{aligned}$$

$$\mathrm{d} \mathbf{h}$$

$$\begin{array}{ccc} & \mathbf{z} & \Phi_P\left(p|\theta,\rho\right)\mathbf{Y} \\ \mathbf{Y}\mathbf{Y}\mathbf{Y}\bar{p}_{\theta^H}\left(\rho\right) & & \mathbf{k} \end{array}$$

$$-\gamma \phi_u\left(R\theta-\bar{p}_{\theta^H}\left(\rho\right)\right)f_{\theta}\left(\rho\right)-\left(1-\gamma\right)\phi_u\left(l\left(\theta\right)-\bar{p}_{\theta^H}\left(\rho\right)\right)f_{\theta}\left(\rho\right)\equiv-g_P^{\theta}\left(\bar{p}_{\theta^H}\left(\rho\right)\right)f_{\theta}\left(\rho\right)$$

$$\mathbf{w-t} \qquad \mathbf{R\textcolor{blue}{b}dS} \qquad \mathbf{k}$$

$$\begin{array}{l} -\alpha\left(\theta^HR-l\left(\theta^H\right)\right)g_P^H\left(\bar{p}_{\theta^H}\left(\rho\right)\right)f_{\theta^H}\left(\rho\right)-\left(1-\alpha\right)\left(\theta^LR-l\left(\theta^L\right)\right)g_P^L\left(\bar{p}_{\theta^H}\left(\rho\right)\right)f_{\theta^L}\left(\rho\right) \\ +\quad V^*\left(\alpha\left(\rho-\theta^H\rho_0+l\left(\theta^H\right)\right)g_P^H\left(\bar{p}_{\theta^H}\left(\rho\right)\right)f_{\theta^H}\left(\rho\right)+\left(1-\alpha\right)\frac{\theta^LR-l\left(\theta^L\right)}{V_L^*}g_P^L\left(\bar{p}_{\theta^H}\left(\rho\right)\right)f_{\theta^L}\left(\rho\right)\right)=0 \end{array}$$

$$\begin{array}{c} \mathbf{V} \qquad \qquad \qquad \bar{p}^*(\rho)\mathbf{V} \\ \\ \frac{g_P^L\left(\bar{p}_{\theta^H}\left(\rho\right)\right)f_{\theta^L}\left(\rho\right)}{g_P^H\left(\bar{p}_{\theta^H}\left(\rho\right)\right)f_{\theta^H}\left(\rho\right)}=\frac{\alpha}{1-\alpha}\frac{\theta^HR-l\left(\theta^H\right)-V^*\left(\rho-\theta^H\rho_0+l\left(\theta^H\right)\right)}{\left(\theta^LR-l\left(\theta^L\right)\right)\left(\frac{V^*}{V_L^*}-1\right)} \end{array}$$

$$\mathbf{R\textcolor{blue}{g}S} \qquad \qquad \mathbf{Y''}$$

$$\mathbf{V^*} \qquad \qquad \mathbf{V_L^*Y}$$

$$\begin{array}{ccccccc} \mathbf{w} & & \mathbf{V_L^*} & & \mathbf{V^*} & \bar{p}_{\theta^H}\left(\rho;V^*,V_L^*\right) & \mathbf{R} \qquad \mathbf{z} \\ & & & & \mathbf{V^*} & \mathbf{V_L^*S} & \end{array}$$

$$\frac{g_P^L\left(\bar{p}_{\theta^H}\left(\rho;V^*,V_L^*\right)\right)f_{\theta^L}\left(\rho\right)}{g_P^H\left(\bar{p}_{\theta^H}\left(\rho;V^*,V_L^*\right)\right)f_{\theta^H}\left(\rho\right)}=\frac{\alpha}{1-\alpha}\frac{\theta^HR-l\left(\theta^H\right)-V^*\left(\rho-\theta^H\rho_0+l\left(\theta^H\right)\right)}{\left(\theta^LR-l\left(\theta^L\right)\right)\left(\frac{V^*}{V_L^*}-1\right)} \qquad \mathbf{R\textcolor{blue}{e}S}$$

$$\begin{array}{ccccccc} \mathbf{w} & & & & \mathbf{V_L^*} & & \mathbf{R\textcolor{blue}{b}cS} \\ \bar{p}_{\theta^H}\left(\rho;V^*,V_L^*\right)\mathbf{Y} & \mathbf{r} & \mathbf{V} & & \mathbf{V_L^*V} & & \mathbf{V^*} \ \mathbf{R\textcolor{blue}{b}eS} \\ \bar{p}_{\theta^H}\left(\rho;V^*,V_L^*\right) & & & & & & \chi_{\theta^H}\left(p,\rho;V^*,V_L^*\right)\mathbf{Y} \end{array}$$

$$,$$

$$\Gamma\left(V^*,V_L^*\right)=A\frac{\tilde{S}\left(\bar{p}_{\theta^H}\left(\rho;V^*,V_L^*\right),\rho_L^*\right)}{\tilde{D}\left(\bar{p}_{\theta^H}\left(\rho;V^*,V_L^*\right),\rho_L^*\right)}$$

$$\begin{array}{ccc} & \mathbf{bd} & \\ \mathbf{V^*} & & \bar{p}_{\theta^H}\left(\rho;V^*,V_L^*\right)\mathbf{V}\rho_L^*\mathbf{V} \qquad \mathbf{V_L^*Y} \end{array}$$

$$\mathbf{V^*}=\Gamma\left(\mathbf{V^*},\mathbf{V_L^*}\right)$$

$$\mathbf{z} \qquad \qquad \mathbf{Y}$$

$$\mathbf{di}$$

w

W

zt

k

$$I_{\theta^H} = \frac{A}{\tilde{D}(\chi_{\theta^H}, \chi_{\theta^L})}$$

RfS

$$I_{\theta^L} = \frac{\int \int \chi_{\theta^H}(p, \rho) d\Phi_P(p|\theta^L, \rho) dF_{\theta^L}(\rho)}{\int \int \chi_{\theta^L}(p, \rho) d\Phi_P(p|\theta^L, \rho) dF_{\theta^L}(\rho)} \frac{A}{\tilde{D}(\chi_{\theta^H}, \chi_{\theta^L})}$$

RgS

w

V

$$\chi_{\theta}(p,\rho)$$

RS

Y_Z

$$R, \theta^H, \theta^L,$$

$$l\left(\theta^H\right), l\left(\theta^L\right)$$

$$\left.\frac{d}{dp}\left(\frac{g_P^L(p)}{g_P^H(p)}\right)\right|_{p=(R\theta^L+l(\theta^H))/2}<0$$

$$g_P^L\left(\bar{p}_{\theta^H}\left(\rho\right)\right)/g_P^H\left(\bar{p}_{\theta^H}\left(\rho\right)\right)\quad \textcolor{blue}{R}\textcolor{blue}{S}$$

R

S

$$\bar{p}_{\theta^H}(\rho) \mathbf{Y}$$

A1.3 Proof of Proposition 4

w

RSV

$$\rho \mathbf{V}$$

$$\Pr\left(p\geq\bar{p}_{\theta^H}\left(\rho\right)|\rho\right)=\gamma\int_{\bar{p}_{\theta^H}\left(\rho\right)}^{R\theta+\bar{p}_{\theta^H}\left(\rho\right)-l\left(\theta\right)}\phi_u\left(R\theta-p\right)dp+\int_{R\theta+\bar{p}_{\theta^H}\left(\rho\right)-l\left(\theta\right)}^{\infty}\phi_u\left(R\theta-p\right)dp$$

$$, \quad u=R\theta^H-p\mathbf{V}$$

$$\begin{aligned} &= \gamma \int_{l(\theta^H)-\bar{p}_{\theta^H}(\rho)}^{R\theta-\bar{p}_{\theta^H}(\rho)} \phi_u(u) du + \int_{-\infty}^{l(\theta)-\bar{p}_{\theta^H}(\rho)} \phi_u(u) dp \\ &= \gamma \Phi(R\theta^H-\bar{p}_{\theta^H}(\rho)) + (1-\gamma) \Phi(l(\theta^H)-\bar{p}_{\theta^H}(\rho)) \end{aligned}$$

$$\mathbf{Y}\mathbf{Y}\mathbf{Y}\sigma_u^2\qquad \bar{p}_{\theta}(\rho)\qquad \mathbf{k}$$

$$\begin{aligned} & -\frac{1}{2}\frac{1}{\sigma_u^2}\left(\gamma\int_{l(\theta^H)-\bar{p}_{\theta^H}(\rho)}^{R\theta-\bar{p}_{\theta^H}(\rho)}\frac{1}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}du+\int_{-\infty}^{l(\theta)-\bar{p}_{\theta^H}(\rho)}\frac{1}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}dp\right) \\ & +\left(\gamma\int_{l(\theta^H)-\bar{p}_{\theta^H}(\rho)}^{R\theta-\bar{p}_{\theta^H}(\rho)}\frac{u^2}{2\left(\sigma_u^2\right)^2\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}du+\int_{-\infty}^{l(\theta)-\bar{p}_{\theta^H}(\rho)}\frac{u^2}{2\left(\sigma_u^2\right)^2\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}dp\right) \\ & =\frac{1}{2\left(\sigma_u^2\right)^2}\left(\gamma\int_{l(\theta^H)-\bar{p}_{\theta^H}(\rho)}^{R\theta-\bar{p}_{\theta^H}(\rho)}\frac{u^2-\sigma_u^2}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}du+\int_{-\infty}^{l(\theta)-\bar{p}_{\theta^H}(\rho)}\frac{u^2-\sigma_u^2}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}dp\right) \\ & =\frac{1}{2\left(\sigma_u^2\right)^2}\left(\gamma\int_{-\infty}^{R\theta-\bar{p}_{\theta^H}(\rho)}\frac{u^2-\sigma_u^2}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}du+(1-\gamma)\int_{-\infty}^{l(\theta)-\bar{p}_{\theta^H}(\rho)}\frac{u^2-\sigma_u^2}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}dp\right) \end{aligned}$$

dj

$$\begin{aligned}
R\theta - \bar{p}_\theta(\rho) &< 0 \\
l(\theta) - \bar{p}_\theta(\rho) &> 0 \\
R\theta - \bar{p}_\theta(\rho) &> 0 > l(\theta) - \bar{p}_\theta(\rho)
\end{aligned}
\qquad \gamma$$

A1.4 Proof of Proposition 5

$$\begin{array}{ccc}
\text{RbS} & \text{RcS} & \\
\text{bVcV} & \text{dY} & \text{---}
\end{array}$$

$$\begin{array}{ccc}
\text{w} & \text{W} & \text{---e} \\
& \text{k} &
\end{array}$$

$$I_\theta^{AS} = \frac{A}{1 - l(\theta) + \int \int (\rho - \theta \rho_0 + l(\theta)) \chi_\theta^{AS}(p, \rho) \, d\Phi_P^{AS}(p|\theta, \rho) \, dF_\theta(\rho)}$$

$$\text{RISY}$$

$$\begin{array}{c}
\text{k} \\
\max_{\{\chi_{\theta^H}^{AS}, \chi_{\theta^L}^{AS}\}} \left(R\theta^H \int \int \chi_{\theta^H}^{AS}(p, \rho) \, d\Phi_P^{AS}(p|\theta^H, \rho) \, dF_{\theta^H}(\rho) - 1 - \int \int \rho \chi_{\theta^H}^{AS}(p, \rho) \, d\Phi_P^{AS}(p|\theta^H, \rho) \, dF_{\theta^H}(\rho) \right) I_{\theta^H}^{AS}
\end{array}$$

$$\begin{array}{cc}
\text{zt} & \text{k}
\end{array}$$

$$\begin{aligned}
&\left(R\theta^L \int \int \chi_{\theta^L}^{AS}(p, \rho) \, d\Phi_P^{AS}(p|\theta^L, \rho) \, dF_{\theta^L}(\rho) - 1 - \int \int \rho \chi_{\theta^L}^{AS}(p, \rho) \, d\Phi_P^{AS}(p|\theta^L, \rho) \, dF_{\theta^L}(\rho) \right) I_{\theta^L}^{AS} \\
&\geq \left(R\theta^L \int \int \chi_{\theta^H}^{AS}(p, \rho) \, d\Phi_P^{AS}(p|\theta^L, \rho) \, dF_{\theta^L}(\rho) - 1 - \int \int \rho \chi_{\theta^H}^{AS}(p, \rho) \, d\Phi_P^{AS}(p|\theta^L, \rho) \, dF_{\theta^L}(\rho) \right) I_{\theta^H}^{AS}
\end{aligned}$$

$$\begin{array}{cc}
\text{r} & \text{V}_Z \\
& \chi_\theta^{AS}(p, \rho)
\end{array}$$

$$\chi_\theta^{AS}(p, \rho) = \begin{cases} 1 & p \geq \bar{p}_\theta^{AS}(\rho) \\ 0 & \end{cases}$$

$$\begin{array}{cc}
\bar{p}_\theta^{AS}(\rho) \text{Y z} & \text{V} \\
\text{RS} & \text{k}
\end{array}$$

$$\phi_P^{AS}(p|\theta, \rho) = \begin{cases} \phi_u(R\theta - p) & p \geq R\theta + \bar{p}_\theta^{AS}(\rho) - l(\theta) \\ \gamma \phi_u(R\theta^L - p) & \bar{p}_\theta^{AS}(\rho) \leq p < R\theta + \bar{p}_\theta^{AS}(\rho) - l(\theta) \\ (1 - \gamma) \phi_u(l(\theta) - p) & \bar{p}_\theta^{AS}(\rho) - R\theta + l(\theta) \leq p < \bar{p}_\theta^{AS}(\rho) \\ \phi_u(l(\theta) - p) & p < \bar{p}_\theta^{AS}(\rho) - R\theta + l(\theta) \end{cases}
\qquad \text{RbhS}$$

$$\begin{array}{c}
\begin{array}{cc}
\mathbf{z} & \mathbf{k} \\
\max_{\{\bar{p}_{\theta^H}^{AS}, \bar{p}_{\theta^L}^{AS}\}} \left(R\theta^H \int_0^\infty \int_{\bar{p}_{\theta^H}^{AS}(\rho)}^\infty d\Phi_P^{AS}(p|\theta^H, \rho) dF_{\theta^H}(\rho) - 1 - \int_0^\infty \int_{\bar{p}_{\theta^H}^{AS}(\rho)}^\infty \rho d\Phi_P^{AS}(p|\theta^H, \rho) dF_{\theta^H}(\rho) \right) I_{\theta^H}^{AS} & \\
\mathbf{zt} & \mathbf{k} \\
\left(R\theta^L \int_0^\infty \int_{\bar{p}_{\theta^L}^{AS}(\rho)}^\infty d\Phi_P^{AS}(p|\theta^L, \rho) dF_{\theta^L}(\rho) - 1 - \int_0^\infty \int_{\bar{p}_{\theta^L}^{AS}(\rho)}^\infty \rho d\Phi_P^{AS}(p|\theta^L, \rho) dF_{\theta^L}(\rho) \right) I_{\theta^L}^{AS} & \\
\geq \left(R\theta^L \int_0^\infty \int_{\bar{p}_{\theta^H}^{AS}(\rho)}^\infty d\Phi_P^{AS}(p|\theta^L, \rho) dF_{\theta^L}(\rho) - 1 - \int_0^\infty \int_{\bar{p}_{\theta^H}^{AS}(\rho)}^\infty \rho d\Phi_P^{AS}(p|\theta^L, \rho) dF_{\theta^L}(\rho) \right) I_{\theta^H}^{AS} & \\
\mathbf{x} & \mathbf{zt} \qquad \qquad \qquad \mathbf{V} \\
I_{\theta^H}^{AS} & \mathbf{Y} \\
, \quad V_H^{AS} & V_L^{AS} \qquad \qquad \qquad \mathbf{V} \\
\mathbf{Y} & \mathbf{Y}_Z \qquad \mathbf{V} \\
\mathbf{YYY}\bar{p}_{\theta^L}^{AS}(\rho) & \mathbf{k} \\
(1 + V_L^{AS})\rho - R\theta^L - V_L^{AS}(\theta\rho_0 + l(\theta)) \geq 0 & \mathbf{RbS} \\
\bar{p}_{\theta^L}^{AS}(\rho)\mathbf{Yr} & \mathbf{V} \qquad \mathbf{V} \\
\mathbf{k} & \\
\chi_{\theta^L}(\rho) = \begin{cases} 1 & \rho \leq \bar{\rho}_L^{AS} \\ 0 & \end{cases} & \\
\bar{\rho}_L^{AS} & \mathbf{biY} \qquad \mathbf{RSY} \\
& \mathbf{V} \qquad \mathbf{zt} \\
I_{\theta^H}^{AS} & \mathbf{k} \\
\max_{\{\chi_{\theta^H}^{AS}, \chi_{\theta^L}^{AS}\}} \left(R\theta^H \int \int \chi_{\theta^H}^{AS}(p, \rho) d\Phi_P^{AS}(p|\theta^H, \rho) dF_{\theta^H}(\rho) - 1 - \int \int \rho \chi_{\theta^H}^{AS}(p, \rho) d\Phi_P^{AS}(p|\theta^H, \rho) dF_{\theta^H}(\rho) \right) I_{\theta^H}^{AS} & \\
V_H^{AS} \equiv \max_{\bar{p}_{\theta^H}^{AS}(\rho)} \frac{R\theta^H \int_0^\infty \int_{\bar{p}_{\theta^H}^{AS}(\rho)}^\infty d\Phi_P^{AS}(p|\theta^H, \rho) dF_{\theta^H}(\rho) - 1 - \int_0^\infty \int_{\bar{p}_{\theta^H}^{AS}(\rho)}^\infty \rho d\Phi_P^{AS}(p|\theta^H, \rho) dF_{\theta^H}(\rho)}{R\theta^L \int_0^\infty \int_{\bar{p}_{\theta^H}^{AS}(\rho)}^\infty d\Phi_P^{AS}(p|\theta^L, \rho) dF_{\theta^L}(\rho) - 1 - \int_0^\infty \int_{\bar{p}_{\theta^H}^{AS}(\rho)}^\infty \rho d\Phi_P^{AS}(p|\theta^L, \rho) dF_{\theta^L}(\rho)} V_L^{AS} & \\
\mathbf{z} & \mathbf{zt} \qquad \qquad \qquad \mathbf{R} \\
V_L^{AS} & \mathbf{SY} \qquad \mathbf{YYY}\bar{p}_{\theta^H}^{AS}(\rho) \qquad \mathbf{k} \\
\frac{g_P^{AS}(\bar{p}_{\theta^H}^{AS}(\rho)|\theta^L, \rho) f_{\theta^L}(\rho)}{g_P^{AS}(\bar{p}_{\theta^H}^{AS}(\rho)|\theta^H, \rho) f_{\theta^H}(\rho)} = \frac{V_L^{AS}(\rho - R\theta^H)}{V_H^{AS}(R\theta^L - \rho)} & \mathbf{RbJS} \\
& \mathbf{eb}
\end{array}
\end{array}$$

$$g_P^{AS}\left(\bar{p}_{\theta^H}^{AS}(\rho)\,|\theta,\rho\right)=\gamma\phi_u\left(R\theta-\bar{p}_{\theta^H}^{AS}(\rho)\right)+\left(1-\gamma\right)\phi_u\left(l\left(\theta\right)-\bar{p}_{\theta^H}^{AS}(\rho)\right)$$

$$\begin{array}{ccccccc} \text{bj} & & \bar{p}_{\theta^H}^{AS}(\rho)Y^- & & V & & - & & \text{dV} & & \text{bj} \\ V_H^{AS} & & & & V_H^{AS} & & & & Y_Z & & VV_H^{AS} \\ & & & & & & & & - & & \text{dY} \\ & & & & & & & & & & \text{RS} \end{array}$$

v kr
 z z
 Yz V
 Yz c z
 Y
 z Vz
 Y
 Yw Vz
 V
 Y- V Vz
 Y-
 V z V W
 Yz V V
 Y- V
 Y
 r ff á Rbj hgSV W
 V
 L L W Y V V
 W Yz V W V ex-ante V
 Y
 V ex-post
 Y
 u V Y
 t V V ff á Rbj hgSY
 ” Y V V
 Y“ V V
 W Y
 Y
 — Yff Rbj hj SV?V ?

Y ff á

Rbj hgSV Y?

ff á Rbj hgS

Y

— Rbj ieS

Y W V W

W W V

W Y— Y

R

SY z

Y— Rbj ieS

V V

Yz V ff W V

”

ff á Rbj hgS Y

z Vs x RaagS

— Rbj ieS

Yr V

ff W

W Y

Y s x RaagS

’ Y V

V

ff W Yy V ff á Raai S

V s x RaagS V

’ Y

x Vz

ff á Rbj hgSYz

W Y

r V W

Y— V W

ff á Rbj hgSV V

Rbj ieSV V R S Y V

ee

[illegible]

ex-post

Y z V W

V W Y

r V V z

ff á Rbj hgS Yz

Y á

z Vz V V V V á Rbj hgSV

Yz V V α L LY

w V W

V k

$$\max_{\mathcal{C} \subseteq \mathbb{C}^{MH}} U\left(C_{\theta^H}; \theta^H, \Phi_p\left(\cdot | \theta^H, \rho\right)\right) \qquad \text{R-eS}$$

k

$$\pi\left(C_{\theta}; \theta, \Phi_p\left(\cdot | \theta, \rho\right)\right) = 0 \forall \theta \in \Theta$$

$$U\left(C_{\theta^i}; \theta^i, \Phi_p\left(\cdot | \theta^i, \rho\right)\right) \geq U\left(C_{\theta^j}; \theta^i, \Phi_p\left(\cdot | \theta^i, \rho\right)\right) \quad \forall \quad i, j = L, H$$

” W R W

SYr V

Y W

Yz Lr áL R á S

Y

Proposition 5. *The solution to [P4](#) (which coincides with the equilibrium of the game when the latter exists) is given by a menu of contracts \mathcal{C}^{AS} with the following properties:*

1. *The entrepreneur consumes the minimum share of output if the project is successful and nothing otherwise:*

$$\bar{c}_{\theta}^{e,AS}(p,\rho) = (R-\rho_0)I_{\theta}^{AS}, \; \underline{c}_{\theta}^{e,AS}(p,\rho) = 0, \; c_{\theta,L}^{e,AS}(p,\rho) = 0$$

2. The investor receives all the pledgeable income if the project is successful and the liquidation value if the project is liquidated:

$$\bar{c}_\theta^{i,AS}(p, \rho) = \rho_0 I_\theta^{AS}, \quad \underline{c}_\theta^{i,AS}(p, \rho) = 0, \quad c_{\theta,L}^{i,AS}(p, \rho) = l(\theta) I_\theta^{AS}$$

3. The low type obtains the same liquidity insurance as in the symmetric information benchmark:

$$\chi_{\theta^L}^{AS}(\rho) = \begin{cases} 1 & \text{if } \rho \leq \bar{\rho}_L^{AS} \\ 0 & \text{otherwise} \end{cases}$$

where $\bar{\rho}_L^{AS}$ is the solution to [18](#) in the appendix.

4. The continuation rule for the high type $\chi_{\theta^H}^{AS}(p, \rho)$ depends on the asset price. Formally, for each ρ there exists a threshold $\bar{p}_{\theta^H}^{AS}(\rho)$ such that

$$\chi_{\theta^H}^{AS}(p, \rho) = \begin{cases} 1 & \text{if } p \geq \bar{p}_{\theta^H}^{AS}(\rho) \\ 0 & \text{otherwise} \end{cases}$$

where $\bar{p}_{\theta^H}^{AS}(\rho)$ is the solution to [19](#) in the appendix.

5. Investment is given by:

$$I_\theta^{AS} = \frac{A}{1 - l(\theta) + \int \int (\rho - \theta \rho_0 + l(\theta)) \chi_\theta^{AS}(p, \rho) d\Phi_P^{AS}(p|\theta, \rho) dF_\theta(\rho)}$$

6. The optimal contract determines an equilibrium distribution for the asset price $\Phi_P^{AS}(p|\theta, \rho)$ given by [\(17\)](#) in the appendix.