



EUROPEAN SUMMER SYMPOSIUM IN ECONOMIC THEORY

Generously hosted by
Study Center Gerzensee

Monday 1 – Friday 12 July 2013

On absolute auctions and secret reserve prices

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Abstract

From a theory viewpoint, the use of auctions with zero public reserve prices also called absolute auctions, or the use of auctions with secret reserve prices is somehow puzzling despite being common. By allowing that buyers differ in their processing of past data regarding how the participation rate varies with the auction format and how reserve prices are distributed when secret, we show in a competitive environment that these auction formats may endogenously emerge. We also analyze how buyers with various sophistications and sellers with various costs sort into the different formats, thereby offering a range of testable predictions. Alternative approaches are reviewed.

Keywords: competing auctions, absolute auctions, secret reserve prices, endogenous entry, rational expectations, analogy-based expectations.

JEL classification : D03, D44

1 Introduction

Over the past decades, eBay has provided a wonderful large-scale laboratory for the analysis of the kind of auction formats and instruments that are used by real sellers to sell their goods.¹ One observation that can be made is that auctions with no or very low reserve prices are frequently used even in cases in which it would seem that the reservation value of the seller is above the chosen reserve price.² Another is that secret reserve prices

We thank participants in various seminars and workshops including CERGE- mechanism design workshop, UCL, X-CREST, the workshop on online search and search advertising organized by TSE and Microsoft and also Jozsef Sakovics who discussed a preliminary version of our paper at EARIE 2012 (Roma).

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¹Recently, auctions have become less popular on eBay to the advantage of fixed price mechanisms. This shift is largely due to the change of fees in 2008 that has become less favorable to auctions. Our analysis below probably fits less well with the recent trend on eBay (due to the shift toward fixed price mechanisms) but it should be relevant for other auction platforms and not necessarily electronic ones.

²See the survey in Hasker and Sickles (2010).

are also often used, especially for items of high quality (Bajari and Hortaçsu, 2003³). Furthermore, field experiments have shown that those formats may be profitable at least for some types of goods⁴.

Auctions with no reserve price are sometimes referred to as absolute auctions. An informal argument proposed in their favor is that not posting a reserve price is one way of attracting more participants in the auction, which is advantageous to sellers. That more participation is to be expected with lower reserve prices seems obviously right, and this is actually confirmed in the empirical literature.⁵ However, greater participation is not the seller's objective per se, and if the good then happens to be sold at too low a price (below the seller's valuation), the seller would have been better off keeping the object. Theoretical models of auctions with endogenous participation (Levin and Smith 1994, McAfee 1993) all conclude that in scenarios with rational buyers, endogenous participation should lead rational sellers to post reserve prices set at their own valuation level. These theoretical models do not therefore explain why absolute auctions (or more generally, reserve prices below the seller's valuation) are used.

An informal argument sometimes offered in favor of secret reserve prices is that if one would like to post a high reserve price then it is better to keep this reserve price secret so as not to discourage participation. However, if bidders are fully rational, they should anticipate this, thereby mitigating the positive effect of secrecy. More precisely, the sellers who offer the most attractive (i.e. lowest) reserve prices amongst the secret prices would prefer to disclose this reserve price publicly, as these sellers gain nothing from being pooled with less attractive sellers. But, this in turn would lead to an unraveling argument and no seller would choose a secret reserve price in equilibrium.⁷ An extra disadvantage of secret

³ For all auctions listed on eBay in 2009, Einav et al. (2011) report that only one percent of these have a secret reserve. However, the use of a secret reserve price entails significant extra fees (at least 2% nowadays) so that this instrument is not valuable for goods of low value. In the data set of Hossain (2008) which involves golf drivers, a quarter of the auctions have a secret reserve price.

⁴ For collectible trading cards, Reiley (2006) finds that absolute auctions raise 25% higher revenue than auctions with a reserve approximately equal to the book value of the card in the case of low-quality cards. This difference is small and insignificant for cards of higher quality. The evidence on the profitability of the use of secret reserve prices is mixed (see the discussion in Bajari and Hortaçsu, 2004). It should be mentioned that field experiments analyzing the effect of secret reserves (see e.g. Katkar and Reiley, 2006) suffer from a major concern in that they compare a public reserve to a secret reserve set at the same level, whereas the optimal secret and public reserves differ in models with endogenous entry (see the analysis below).

⁵ We abstract from signalling and shill bidding issues that may invalidate this argument. These will be discussed later.

⁶ These consider respectively the case where potential entrants learn their valuation after and before their participation decision. The respective terminologies are auctions with entry and auctions with participation costs. See also Peter's (2011) survey on competing mechanisms.

⁷ Two previous attempts to solve the secret reserves 'puzzle' rely on the possibility of committing ex ante to stick to secret reserves, i.e. before the seller learns her valuation. Li and Tan's (2000) argument is

reserve prices is related to commitment concerns: Even if sellers are homogeneous, a seller posting a secret reserve price in a private value environment has an incentive to raise her reserve price strictly above her valuation (as can be inferred from Myerson, 1981), but this would lead to suboptimal participation and thus be detrimental to the seller (as can be inferred from Levin and Smith 1994 and McAfee 1993).

The first main contribution of this paper is to provide a rationale for the use of absolute auctions and secret reserve prices in a competing auction environment in which while some buyers have rational expectations, some other buyers mistakenly miss the relation of the participation rate to the reserve price and yet some other buyers miss that the distribution of reserve prices when secret need not coincide with the entire distribution of reserve prices. Specifically, the first set of mistaken buyers forms a view about the aggregate participation rate simply by averaging the number of bidders per auction without keeping track of the level of the reserve price when a specific number of bidders showed up. The second set of mistaken buyers looks at all reserve prices previously used and reasons as if the reserve price when secret was drawn from this (empirical) distribution. Such mistakes can be interpreted by saying that some explanatory variables are being dropped when assessing the participation rate (for the first set of buyers) or when assessing the distribution of secret reserve prices (for the second set of buyers). It should be noted that dropping some explanatory variables represents a common practice (and possibly a mistake) for econometricians, especially in the face of data sets of limited size. Thus, we do not necessarily have the view that those buyers who do not have rational expectations suffer from cognitive limitations, as such buyers can be thought of as being less experienced, thereby meaning they make their decisions based on data sets of more limited size.

Specifically, we consider a competing auction setting with rational (or experienced) sellers and more or less inexperienced buyers, which we model by assuming that there are three (cognitive) types of buyers: those who are fully rational (FR), those referred to as fully coarse (FC) who miss the relation of the participation rate to the auction format, and those referred to as partially coarse (PC) who see correctly the effect of the auction format on the participation rate but miss that the reserve price is not distributed in the

based on risk aversion and works for first-price auctions but not for second-price or English auctions. The example developed by Vincent (1995) relies on interdependent valuations. In both cases, if the reserve price policy were chosen after the seller learns her valuation, the unravelling argument would apply.

⁸In the spirit of the scarce dataset story, one might consider that this creates an extra source of heterogeneity in buyers' expectations (because the samples considered by different buyers need not be the same). We abstract from this randomness (which is not needed for our point) by assuming that each individual buyer has access to a sample of arbitrarily large size (and yet consider more or less explanatory variables depending of his level of experience).

same way whether it is set publicly or privately. In our economy, sellers have heterogeneous valuations for the goods they sell, which can be interpreted as allowing for heterogeneity in the opportunity costs of sellers of having their object (momentarily) unsold. Buyers -whatever their (cognitive) type- choose which auction to participate to depending on the observable characteristics of the auction format, and learn their valuation after their participation decision. The rule of the auctions is that of the second (or ascending) price auction possibly with a reserve price, public or private. The relative mass of sellers to buyers as well as the shares of the various cognitive types are extra parameters of the model together with the distribution of valuations of sellers and buyers.

We consider competitive equilibria with a continuum of agents in which each seller chooses the format that maximizes her expected payoff taking into account how the format affects participation, and each buyer chooses formats that maximize his perceived expected payoff as defined by his cognitive type. It follows the spirit of competitive equilibrium due to our consideration of a continuum of agents so that when an individual seller contemplates the choice of the various auction formats, she takes the perceived utility of the various types of buyers as given (and not as being influenced by her choice of format).

We show that a competitive equilibrium exists. In addition to establishing that absolute auctions and secret reserve prices are used in equilibrium, our second main contribution is to characterize how the various buyers and sellers sort into the various auction formats. In any competitive equilibrium, sellers with low (yet strictly positive) valuations use absolute auctions, sellers with high valuations use secret reserve prices (set above their valuation in accordance with Myerson, 1981), and sellers with intermediate valuations use public reserve price auctions in which the reserve price is set at their valuation. Fully coarse buyers all choose the objects which are sold through absolute auctions. Fully rational buyers all choose the objects which are sold through a strictly positive public reserve price, and partially coarse buyers choose objects which are either sold through a secret reserve price or through strictly positive public reserve prices.

To get an intuition for our results, observe that from the viewpoint of a fully coarse buyer, absolute auctions are those that look most attractive given that any other format presents the disadvantage of having a less favorable reserve price and fully coarse buyers do not appreciate that participation may decrease with the reserve price. If there were only fully coarse buyers, competition between sellers would take a form similar to that of a Bertrand competition on the choice of reserve price, thereby leading all active sellers whatever their valuation to offer absolute auctions. We also observe that if an auction

format is to attract some fully rational buyers, it must have a public reserve price set at the seller's valuation. This is a result analogous to the one obtained in Levin and Smith (1994) or McAfee (1993): Given that participation adjusts to make fully rational buyers equally well off, the objective of a seller coincides with the welfare net of the opportunity cost of participating buyers, which leads the seller to set a reserve price that optimizes welfare, hence set at her valuation. Finally, the presence of partially coarse buyers invalidates the unravelling argument used to rule out secret reserve prices because now the seller offering the lowest secret reserve price is also pooled (in the perception of partially coarse buyers) with sellers offering (lower) public reserve prices.

The rough intuition provided so far explains why absolute auctions, secret reserve prices and public reserves set at the seller's valuation would emerge. What our characterization result establishes is the further (sorting) property that sellers with low valuations choose absolute auctions, sellers with high valuations choose secret reserve prices and those with intermediary valuations opt for public and positive reserve prices: The intuition comes from the fact that sellers with lower valuations are those who suffer less from not having a reserve price while sellers with higher valuations are those who benefit more from having a higher reserve price.

In Section 2, we describe the model and the competitive equilibrium (noting the similarity of the approach with Jehiel's (2005) analogy-based expectation equilibrium). In Section 3, we characterize the competitive equilibrium and derive its main properties. In Section 4, we review the empirical literature, discussing how it fits with our main theoretical predictions. In Section 5, we discuss alternative and complementary explanations, including risk aversion, inattention, shill bidding, auction fever, QRE and level-k approaches. We argue that by contrast to our theory, none of the competing approaches provides a set of predictions that accommodates the set of empirical observations surveyed in Section 4. Section 6 concludes.

2 The model

We consider an economy with a continuum of sellers and buyers, the ratio of the mass of buyers to the mass of sellers being denoted by $\lambda > 0$. Each individual seller sells one good through an auction, and she can decide to put a reserve price $r \in \mathbb{R}_+$ and whether or not to disclose it to participants $d \in \{\text{public}; \text{secret}\}$. We let $s = (r; d)$ and we denote the set of available reserve price policies by \mathcal{S} . Other than the reserve price, the rule of

the auction for any object is that of the second-price auction. The winner (if any) is the bidder with highest bid if this bid is above the reserve price, and he pays the maximum of the second highest bid (if any) and the reserve price. We note that such a specification of auction formats fits well with the working of most auction sites including eBay.

Sellers simultaneously choose their auction formats. At the time a seller chooses her auction format, she knows how much she would value keeping the object. We assume that sellers are heterogeneous in that respect, some sellers having larger inventory costs than others, say. The cumulative distribution of sellers' valuations is denoted by $G(\cdot)$ with support $[\underline{v}; \bar{v}]$ and $0 < \underline{v} < \bar{v}$.

Regarding buyers, we assume that they do not know ex ante how much they would value the various goods for sale, and that they need to spend some time inspecting a good in order to assess how valuable it is to them. We assume that each individual buyer inspects just one good, and thus is able to participate to only one auction⁹. Thus, an individual buyer makes his decision on which auction to participate in, based on what he sees from the auction format (the reserve price if public and the announcement that the reserve price is secret otherwise). Upon participating in an auction, the buyer learns his valuation for the corresponding object. The cumulative distribution of buyers' valuations for each object is denoted by $F(\cdot)$ with support $[0; 1]$. That is, we assume objects are ex ante symmetrically valuable for buyers. To simplify the analysis of secret reserve prices, we assume that $F(\cdot)$ is regular, that is, the function $x \mapsto \frac{1 - F(x)}{f(x)}$ is assumed to be increasing in x .

For later use, it is convenient to let s refer to what buyers see from the auction format $s = (r; d)$. It can be described as $s = r$ if $d = \text{public}$ and $s = \text{secret}$ if $d = \text{secret}$. We let S denote the set of s .

A key (innovative) feature of our model is that we consider buyers who are heterogeneous in their understanding of how participation rate relates to the auction format, and how reserve prices are distributed when secret. All buyers are otherwise assumed to know the distribution $F(\cdot)$ from which their valuation is drawn. Specifically, we consider three groups of buyers:

The fully rational buyers FR who are perfectly rational,

⁹This is obviously a simplifying assumption, as in reality the search process may lead buyers to inspect more than one object (and sellers not to choose their formats simultaneously). Yet, the main insights to be developed next would carry over more generally, as long as the various agents have no market power so that they take as given their (perceived) expected payoffs in equilibrium.

The fully coarse buyers FC who only know the aggregate participation rate over all auction formats, but miss the correlation between the auction format and the participation rate (due to coarse processing of past participation rates in the available data), and

The partially coarse buyers PC who have a correct understanding of the participation rate but when the reserve price is secret consider it is distributed according to the aggregate distribution of reserve prices whether secret or public (which corresponds to not keeping track in the record of past data whether reserve prices were set secretly or not).

We let $i = FR; FC; PC$ and $\alpha_i > 0$ denote the share of buyers of type i in the population.

Sellers are assumed to be perfectly rational, as usual.

Before we define our notion of equilibrium, some preliminaries as well as extra piece of notation are helpful.

Preliminaries.

In equilibrium, according to their type, buyers distribute themselves across the various auctions. This results in a random number of participants in each individual auction. Specifically, we adopt the view that if a mass x of buyers distribute themselves uniformly over a mass y of sellers, each individual seller receives a random number of buyers that is distributed according to a Poisson distribution with mean $\lambda = \frac{x}{y}$. That is,

$$\Pr(\text{number of participants} = n) = e^{-\lambda} \frac{\lambda^n}{n!}:$$

This formulation corresponds to the limit distribution obtained in a setting with n_B buyers and n_S sellers, each buyer choosing (independently) a given seller with probability $\frac{1}{n_S}$, where n_B and n_S would tend to infinity while the ratio $\frac{n_B}{n_S}$ would be kept asymptotically close to λ . Thus, our formulation combines the idea that individual decisions to participate are made independently and symmetrically across buyers of the same type and the idea of large market leading us to work directly with the limit (Poisson) distribution. It should be noted that the use of the Poisson distribution to model participation in large economies is common in the search literature, especially for labor markets (see for example Rogerson et al. 2005). Obviously, if the participation rate of buyers of type i is α_i for $i = FR; PC; FC$

then the overall participation rate is $\lambda = \lambda^{FR} + \lambda^{PC} + \lambda^{FC}$, since the sum of independent Poisson distributions with various means is a Poisson distribution with mean equal to the sum of individual means.¹⁰

Notation

The following notation will be used in the sequel.

$V_n(r)$ is the expected (interim) utility of a buyer participating in an auction with n other participants and a reserve price r .

$u_n(r; v)$ is the expected (interim) utility of a seller with valuation v offering an auction with reserve price r in which n buyers participate.

$u_{\text{sell}}(\lambda; r; v) = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} u_n(r; v)$ is the expected (ex ante) utility of a seller with valuation v using a reserve price r in which participation follows a Poisson distribution with mean λ .

$u_b(\lambda; r) = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} V_n(r)$ is the expected (ex ante) utility of a buyer participating in an auction with reserve price r in which participation follows a Poisson distribution with mean λ .

$\mathbf{u}_b(\lambda; f) = \int_0^R u_b(\lambda; r) f(r) dr$ is the expected (ex ante) utility of a buyer participating in an auction in which participation follows a Poisson distribution with mean λ and in which the reserve price is distributed according to the density f .

Competitive equilibrium

Since we consider an economy with a continuum of sellers and buyers, it seems legitimate to assume that a deviation by a single seller would not affect the perceived equilibrium utilities of the various types of buyers. Accordingly, the participation rate of the various groups of buyers adjust so that the perceived expected payoff of any buyer in any format he participates to [resp. does not participate to] corresponds to [resp. is below] the equilibrium one. Sellers maximize their expected payoff taking into account how the aggregate participation rate depends on the auction format.

¹⁰ Because of this additivity property, the implicit symmetry assumption behind the Poisson distribution is actually much weaker than it might appear at first glance insofar as we could split buyers of the same type into several subgroups each characterized by a different Poisson participation distribution so that the (underlying) symmetry assumption applies only to the subgroups. We elaborate more on the foundations of equilibrium models with the Poisson distributions in Jehiel and Lamy (2013). Nevertheless, we emphasize that our model precludes the possibility that buyers of the same group coordinate their participation decisions, thereby resulting in different expected utilities for buyers of the same group, as it is, for example, the case in Engelbrecht-Wiggans' (1993) model of sequential entry.

To define the competitive equilibrium formally, we introduce for each v , the strategy of sellers with reservation value v that we denote by σ_v . This is a measure over possible auction formats S (possibly concentrated on just ones $\in S$). We introduce for each $i \in \{FC; PC; FR\}$ a Poisson parameter function $\lambda^i : S \rightarrow \mathbb{R}_+$, where $\lambda^i(s)$ characterizes the distribution of participation of buyers of type i in an auction with observable characteristic s .

Definition 1 A competitive equilibrium is defined as a pair $(\sigma_v)_{v \in [\underline{v}, \bar{v}]}$ and $(\lambda^i)_{i \in \{FC; PC; FR\}}$, where σ_v stands for the strategy of a seller with valuation v , and $\lambda^i : S \rightarrow \mathbb{R}_+$ describes the distributions of participation of buyers of type i in the various auction formats where

1. (Utility maximization for sellers) for any $v \in [\underline{v}, \bar{v}]$,

$$\text{Supp}(\sigma_v) \subseteq \text{Arg} \max_{s \in (r, d) \in S} u_{\text{sell}}(s; r; v)$$

$$\text{where } u(s) = u^{FC}(s) + u^{PC}(s) + u^{FR}(s),$$

2. (Utility maximization for buyers) for $i = FC; PC; FR$, there exists $V^i \geq 0$ such that for each $s \in S$,

$$\lambda^i(s) > 0 \Rightarrow b_b^i(s) = V^i; \quad (\text{resp: } =) \quad (1)$$

3. (Matching conditions) for $i = FC; PC; FR$,

$$\int_{\underline{v}}^{\bar{v}} \int_S \lambda^i(s) v(s) ds dG(v) = \int b_b^i:$$

where $b_b^i(s)$ corresponds to the perceived expected payoff of a buyer of type i when choosing formats :

$$b_b^i(s = r) = \begin{cases} u_b(r; r) & \text{for } i = FR; PC \\ u_b(-; r) & \text{for } i = FC \end{cases} \quad (2)$$

and

$$b_b^i(s = \text{secret}) = \begin{cases} u_b(\text{secret}; \text{secret}) & \text{for } i = FR \\ u_b(\text{secret}; -) & \text{for } i = PC \\ u_b(-; -) & \text{for } i = FC \end{cases} \quad (3)$$

with

$$\bar{\pi} := \frac{R_v}{R} \frac{R_s}{R} \frac{\int_0^s v(s) ds}{\int_0^s v(s) ds} \frac{dG(v)}{dG(v)}; \quad (4)$$

$$\pi_{secret}(r) = \frac{R_v}{R} \frac{R_1}{R} \frac{\int_0^r v(r; secret) dG(v)}{\int_0^r v(r; secret) dr^0} \frac{dG(v)}{dG(v)};^{11} \quad (5)$$

and

$$\bar{\pi}(r) := \frac{R_v}{R} \frac{R_s}{R} \frac{(\int_0^r v(r; public) + \int_0^r v(r; secret)) dG(v)}{\int_0^r v(s) ds} \frac{dG(v)}{dG(v)}; \quad (6)$$

Part 1 of the definition implies that a seller with reservation value v is required to pick a format which maximizes her expected payoff given the participation rate $\pi(s)$ attached to any format $s = (r; d)$ with observable characteristics s . Part 3 reflects the constraint that buyers must participate in one and only one auction and the aggregate ratio of buyers of type i to sellers is π^i . In part 2, the constants V^i , $i \in \{FC; PC; FRg\}$, correspond to the perceived expected payoff that buyers of type i expect in the auctions they participate. Condition (1) implies that whatever the format, either the participation rate of i buyers is positive and the format is perceived to deliver an expected utility to buyers of V^i , or the participation rate is zero and the perceived expected payoff of a buyer is lower than V^i .

The perceived expected payoff coincides with the true expected payoff for FR buyers as well as for PC buyers who contemplate non-secret reserve price auctions $s = (r; public)$. By contrast, the perceived expected payoff involves an aggregate participation rate for FC buyers and it involves an aggregate distribution of reserve prices for PC buyers who contemplate secret reserve prices. A simple interpretation of our competitive equilibrium is that the various types of buyers correspond to different ways of processing data from past auctions in order to assess the participation rate and the distribution of reserve prices when secret.¹² Whereas FR buyers make the best use of the data, FC buyers only assess the aggregate participation rate (simply looking at the mean of participation number and assuming the true participation follows a Poisson distribution with mean equal to the empirical aggregate mean as described in (4)), PC buyers assess the distribution of secret reserve prices by looking at all previous reserve prices (whether public or private) and reasoning as if the distribution of secret reserve price coincides with that aggregate distribution as described in (6) which contrasts with the true distribution described in (5).

¹¹ If the denominator is null, then we can take any distribution on R_+ for $\pi_{secret}(\cdot)$.

¹² We also have the view that the data set available to the buyers is large so that all empirical distributions match the true empirical distributions.

Comments: 1) It should be noted that the participation rates $\pi^i(s)$, $i \in \{FC; PC; FRg\}$, are defined irrespective of whether a format $\pi = (r; d)$ is offered in equilibrium. It is determined to ensure that a buyer who participates in such an auction would obtain his perceived equilibrium utility V^i . This specification of the participation rates (covering also non-chosen formats) is a simple way to capture the trembling hand refinements that rule out non-meaningful equilibria. 2) The competitive equilibrium just defined is in the spirit of the analogy-based expectation equilibrium (Jehiel, 2005) developed for games whereby players bundle various decision nodes or states in order to form their expectations about others' behaviors. From this perspective, our fully-coarse buyers bundle all participation decisions of buyers at all auction formats into the same analogy class, and partially-coarse buyers put all reserve price decisions of sellers into the same analogy class.¹³ 3) Depending on the feedback regarding previous auctions that prevails in a particular environment where auctions with secret reserves can be used, we could also consider variants where aggregation is over all public reserves or over all auctions but with a weight reflecting the frequency with which the reserve is publicly disclosed. Our insights would not qualitatively change in these variants.

3 The competitive equilibrium

In this Section, we characterize the main features of the competitive equilibrium as defined above. Three auction formats play a key role in the analysis: absolute auctions for which the reserve price is null and public, secret reserve price auctions for which the reserve price is not disclosed, and open reserve auctions for which the reserve price is strictly positive and public. The corresponding acronyms are AA, SR and OR. Among OR auctions, those in which the reserve price is set at the seller's valuation will play a key role: These are referred to as truthful-open auctions, and TO is the corresponding acronym. Among SR auctions, those in which the (secret) reserve price is set at $r^M(v)$ with

$$r^M(v) = \frac{1 - F(r^M(v))}{f(r^M(v))} = v; \quad (7)$$

where v is the seller's valuation, play a key role. Observe that $r^M(v)$ is uniquely defined thanks to our regularity assumption.

¹³ To make this fit the framework of the analogy-based expectation equilibrium, we have to decompose into two decision nodes the choice of disclosure policy d and reserve price r , and in addition decompose the participation decisions in the various auction formats.

3.1 Characterization

In preparing our characterization result, we make a number of preliminary observations.

FR buyers participate only in TO auctions

We first observe that when a public reserve price is chosen and when it attracts some buyers who correctly assess their payoffs (in open reserve price auctions), then the seller posts a reserve price equal to her valuation. The detailed argument appears in Appendix A, but the intuition is simple enough to be spelled out in the main text. Given that sellers are utility-takers, their objective coincides with the welfare generated in their auction net of the utility of the participating buyers. From Condition (1) in the definition of competitive equilibria, in auctions that attract some participants who correctly assess their payoffs, the expected (ex ante) utility of all participants is fixed to V^{FR} so that the analysis match the one with only rational buyers. In a TO auction, buyers receive their marginal contribution to the welfare in the auction so that buyers' objective when they choose to participate or not in a given auction is aligned with the welfare when those buyers who participate have correct expectations. Consequently, the participation rate turns out to be the one that maximizes the welfare net of the opportunity costs of FR buyers. Thus the TO auction does not solely maximize the welfare ex-post (i.e. for any given participation rate), but also ex-ante insofar as it induces the optimal participation rate, thereby explaining why with rational buyers, sellers cannot do better than posting a reserve price equal to their valuation.

Lemma 3.1 If $(r; \text{public}) \in \text{Supp}(\underline{v})$ and $^{FR}(r) + ^{PC}(r) > 0$, then $r = v$.

Turning to SR auctions, observe that FR buyers do not participate to them. Indeed, if $^{FR}(\text{secret}) > 0$, we raise a contradiction by following a standard unraveling argument: Among those sellers who are proposing SR auctions, those who choose the lowest reserves would strictly prefer to make it public, since it would raise the participation rate. Thus, we have:

Lemma 3.2 If $\int_{\underline{v}}^{\underline{R}_1} \int_0^{\underline{R}_1} v(r; \text{secret}) dr dG(v) \neq 0$, then $^{FR}(\text{secret}) = 0$.

Combined with Lemma 3.1, we obtain that almost all FR buyers should participate to TO auctions in equilibrium. Thus, if there were only FR buyers, neither AA nor SR auctions would emerge in equilibrium.

FC buyers participate only in AA auctions

We next note that in equilibrium FC buyers opt for absolute auctions. This follows as these buyers do not perceive that participation is affected by the auction format, so the auction with lowest public reserve price is obviously the format that looks most attractive to FC buyers. This induces a kind of Bertrand competition among sellers to look most attractive to FC buyers, which, given that the share of FC buyers is positive, leads some sellers, in equilibrium, to propose auction formats with minimum reserve price, that is, AA auctions. (If no AA were proposed, a deviation to an AA auction would attract infinitely many FC participants and would thus be profitable.) To sum up,

Lemma 3.3 Absolute auctions are proposed in equilibrium, i.e. $\int_{\underline{v}}^{\bar{v}} v(0; \text{public}) dG(v) > 0$. Moreover, fully-coarse buyers select only absolute auctions, i.e. if $F^C(s) > 0$ then $s = (0; \text{public})$. Only fully-coarse buyers participate in absolute auctions, i.e. $(0) = F^C(0) = \frac{R_{\bar{v}}}{\int_{\underline{v}}^{\bar{v}} v(0; \text{public}) dG(v)} \cdot \frac{F^C(b)}{dG(v)}$.

PC buyers participate in SR auctions and possibly also in TO auctions

A simple corollary of Myerson's (1981) analysis is that if a seller with valuation v opts for a SR auction and expects some participation, she will pick a reserve price set at $r^M(v)$ as defined in (7). This is because the participation rate in a SR auction is independent of r (since r is not observed) and $r^M(v)$ maximizes the seller's expected payoff with n participants $\pi_n(r; v)$, irrespective of n .

We next observe that a positive mass of PC buyers participate to SR auctions in equilibrium. If it were not the case, we would raise a contradiction by establishing that the sellers who are proposing the highest public reserves would prefer to keep it secret because the perception of PC buyers regarding the expected reserve price would be lower yielding thus a strictly higher participation rate and thus an higher expected payoff to the seller. The positive participation in SR auctions also implies that a seller, no matter what her valuation v is, can guarantee a payoff strictly above her valuation, thereby ensuring that participation must be positive in all formats chosen in equilibrium. These observations are gathered in the following lemma.

Lemma 3.4 $\int_{\underline{v}}^{\bar{v}} \int_0^{R_1} v(r; \text{secret}) dr dG(v) \neq 0$, $(\text{secret}) = P^C(\text{secret}) > 0$, and $r = r^M(v)$ if $(r; \text{secret}) \in \text{Supp}(\underline{v})$; Moreover, $(r) > 0$ if $(r; \text{public}) \in \text{Supp}(\underline{v})$.

The sorting properties

The previous lemmas imply that in equilibrium sellers propose either an AA auction expecting only FC buyers to participate, or a TO auction expecting FR and possibly some PC buyers to participate, or a SR auction with reserve price $r^M(v)$ expecting only PC buyers to participate. This is summarized by:

Corollary 3.5 Consider $s = (r; d) \in \text{Supp}(\nu)$. If $d = \text{public}$, then either $r = 0$ and $(s) = F^C(0) > 0$ or $r = v > 0$ and $(s) = P^C(v) + F^R(v) > 0$. If $d = \text{secret}$, then $r = r^M(v)$ and $(s) = P^C(\text{secret}) > 0$.

What remains to be determined is how sellers with various valuations sort into these three possible auction formats. Intuitively, the higher the valuation of the seller, the more this seller benefits from a higher reserve price, thereby leading sellers with low valuations to prefer AA, sellers with high valuations to prefer SR and sellers with medium range valuations to prefer TO. This intuition turns out to be correct, as we now detail.

For $k = \text{AA}; \text{TO}; \text{SR}$ and $v > 0$, we let $u_{\text{sell}}^k(v)$ denote the seller's expected payoff $u_{\text{sell}}(k; r^k(v); v)$ where the participation rate is defined by $\frac{AA}{v} := (0)$, $\frac{TO}{v} := (v)$ for $v > 0$ and $\frac{SR}{v} := (\text{secret})$ and the reserve price is defined by $r^{AA}(v) := 0$, $r^{TO}(v) := v$ and $r^{SR}(v) := r^M(v)$, which corresponds to the three possible equilibrium choices made by a seller with valuation v , as shown above. A key observation is the following lemma:

Lemma 3.6 The functions $u_{\text{sell}}^{TO}(v)$, $u_{\text{sell}}^{AA}(v)$ and $u_{\text{sell}}^{SR}(v)$ are quasimonotone increasing on $(0; 1)$ where a real function is quasimonotone increasing on R if for any pair $(x; x^0) \in I^2$ with $x > x^0$ we have that $(x) \geq 0$ implies that $(x^0) < 0$.

Lemma 3.6 can be viewed as establishing a form of single-crossing condition between the functions $u_{\text{sell}}^{TO}(v)$, $u_{\text{sell}}^{AA}(v)$ and $u_{\text{sell}}^{SR}(v)$.

To see why Lemma 3.6 holds, observe that for a given distribution of participants and for a given reserve price, the derivative of the seller's expected utility with respect to her valuation is equal to the probability that the good remains unsold. For the Poisson distribution with mean λ , this particularizes into: $\frac{\partial u_{\text{sell}}(r; v)}{\partial v} = e^{-(1-F(r))}$.

Let $r > r^0$ and suppose (as it turns out to be the case) that $(r) < 0 = (r^0)$. If $u_{\text{sell}}(r; v) \geq u_{\text{sell}}(r^0; v)$ then for all $v^0 < v$, we would have that $u_{\text{sell}}(r; v^0) < u_{\text{sell}}(r^0; v^0)$, thereby explaining why the monotonicity of (\cdot) implies some form of single-crossing. The single-crossing required for Lemma 3.6 is a little more involved though, since

the reserve price in TO and SR auctions depends on \bar{v} . However, since the chosen reserve price is the result of optimization, an application of the envelope theorem allows us to conclude as it is detailed in Appendix E.

Corollary 3.5 and Lemma 3.6 together imply that for any candidate equilibrium, we can define in a unique manner three parameters, two thresholds \underline{v} and \bar{v} with $\underline{v} < \bar{v} < v < \bar{v}$ and a share $\alpha \in [0; 1]$,¹⁴ such that: 1) FC buyers choose AA; 2) FR buyers and a share α of PC buyers choose TO; 3) A share $1 - \alpha$ of PC buyers choose SR;¹⁵ 4) Sellers with $v < \underline{v}$ propose AA; 5) Sellers with $v \in [\underline{v}; \bar{v}]$ propose TO; and 6) Sellers with $v > \bar{v}$ propose SR.

The first threshold \underline{v} is defined so that a seller with valuation \underline{v} is indifferent between AA and TO, and the second threshold is defined so that a seller with valuation \bar{v} is indifferent between TO and SR. That is,

$$u_{\text{sell}}^{\text{TO}}(\underline{v}) = u_{\text{sell}}^{\text{AA}}(\underline{v}) \text{ and } u_{\text{sell}}^{\text{TO}}(\bar{v}) = u_{\text{sell}}^{\text{SR}}(\bar{v}): \quad (8)$$

The quasi-monotonicity of $u_{\text{sell}}^{\text{TO}}(v)$, $u_{\text{sell}}^{\text{AA}}(v)$ and $u_{\text{sell}}^{\text{SR}}(v)$ ensures that the sorting of sellers is as described above. Finally, it should also be the case that PC buyers find their choice of auction best given their perception. That is, $\frac{\partial u_{\text{buy}}^k}{\partial v} = 0$ implies that¹⁶ (resp: $>$)

$$b_b^{\text{PC}}(\text{secret}) \leq b_b^{\text{PC}}(v) \quad (\text{resp: } =) \quad (9)$$

Conversely, we show in Appendix F that any triple $(\underline{v}; \bar{v}; \alpha) \in T$ where $T := \{(\underline{v}; \bar{v}; \alpha) \in [\underline{v}; \bar{v}]^2 \times [0; 1] \mid \bar{v} > \underline{v} > v_1\}$ which satisfies (8) and (9) induces a competitive equilibrium. The Appendix also establishes the existence of such a triple. Our discussion is summarized in the following proposition which is illustrated in Figure 1.

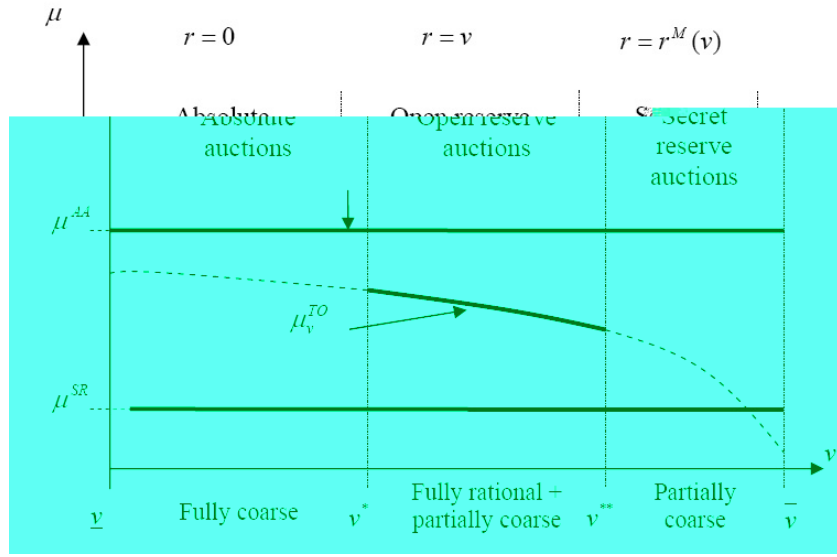
Proposition 3.7 There exists a competitive equilibrium. Any competitive equilibrium is characterized by a triple $(\underline{v}; \bar{v}; \alpha) \in T$ with $\underline{v} < \bar{v} < v < \bar{v}$, $\alpha < 1$ such that: 1) (8) and (9) jointly hold; 2) Sellers with reservation values in $[\underline{v}; \bar{v}]$ propose absolute

¹⁴ Since there is a strictly positive measure of buyers participating in either AA, TO or SR auctions, each of those formats should be proposed with positive probability. Then for any $k \in \{\text{AA}; \text{TO}; \text{SR}\}$, $k \in \text{Arg max}_{k' \in \{\text{AA}; \text{TO}; \text{SR}\}} u_{\text{sell}}^{k'}(v)$ on a positive measure of reservation values. From Lemma 3.6, we have equivalently $\underline{v} < \bar{v} < v < \bar{v}$.

¹⁵ From PC buyers' matching condition and Lemma 3.4, we have $1 - \alpha = \frac{(b_b^{\text{PC}}(\text{secret}) - \int_{\underline{v}}^{\bar{v}} \int_0^{\infty} v(r, \text{secret}) dr) dG(v)}{b_b^{\text{PC}}(\text{secret})} > 0$.

¹⁶ Note that the aggregate distribution of reserve prices $\bar{r}(\cdot)$ which is used to compute $b_b^{\text{PC}}(\text{secret})$ itself depends on \underline{v} and \bar{v} . The distribution $\bar{r}(\cdot)$ in (6) is defined as: $\bar{r}(r) = G(v) \cdot \delta(r - v)$ (where $\delta(\cdot)$ denotes the Dirac distribution); $\bar{r}(r) = g(r)$ if $r \in (\underline{v}; \bar{v})$; $\bar{r}(r) = g(v)$ for $r = r^M(v)$ with $v \in (\underline{v}; \bar{v}]$ and $\bar{r}(r) = 0$ almost everywhere else.

Figure 1: Equilibrium form



auctions; 3) Sellers with reservation values in $(v^* ; v^{\bar{v}})$ propose open reserve price auctions with the reserve price set at v ; 4) Sellers with reservation values in $(v^{\bar{v}} ; \bar{v}]$ propose secret reserve price auctions with the reserve price set at $r^M(v)$; 5) Fully-coarse buyers select absolute auctions; 6) Fully-rational buyers select open reserve auctions; and 7) Partially-coarse buyers may mix between the open reserve auctions (with probability μ_v^{TO}) and the secret reserve auctions (with probability $1 - \mu_v^{TO}$). Equilibrium participation rates satisfy the following: $\mu^{AA} = \mu^{FC}(0) = \frac{\mu^{FC} b}{G(v^*)}$, μ_v^{TO} is decreasing in v , and $\mu^{SR} = \mu^{PC}(\text{secret}) = \frac{(1 - \mu^{FC}) \mu^{PC} b}{1 - G(v^{\bar{v}})}$.

It should be noted that our equilibrium construction did not make use of the endogenous (though mistaken) perception of FC buyers as described by (see equations (2) and (3) in the definition of competitive equilibrium). More precisely, our analysis remains unchanged, as long as FR buyers do not perceive that the participation rate varies with the format. Regarding PC buyers, the situation is somehow different: The equilibrium is sensitive to the details of how their expectations are specified, as their perceived payoff is used to determine whether a share of them (and which) would go for TO auctions. A variant that deserves to be considered is the one in which PC buyers base their beliefs exclusively on the auctions where the reserve price has been disclosed ex-post. Alone, this kind of selection bias is not sufficient to provide a rationale for the emergence of secret reserves: it would distort bidders' beliefs toward auctions with low reserve prices

(those where the reserve price has been disclosed with higher probability), but not beyond the support of the true distribution of secret reserves so that the unraveling argument still prevails. By contrast, if PC buyers form their belief about the distribution of secret reserve prices by aggregating all the reserve prices that are disclosed, then compared to our current model the attractiveness of SR auctions to PC buyers is reinforced, thereby explaining that our results would not change qualitatively.¹⁷ In relation to this, it should be mentioned that if one were to endogenize the mass of PC buyers, the perceived payoff of FC (as well as PC) buyers would matter.

3.2 Main properties

We consider three main questions in this Subsection: 1) How do the reserve prices in SR auctions relate to the ones chosen in OR auctions? 2) How do the participation rates vary in the various auction formats? 3) How disappointed buyers are depending on their cognitive type, i.e. how do real and perceived expected payoffs differ?

We first observe that in equilibrium, all reserve prices set secretly lie above the reserve prices set publicly. This follows from Proposition 3.7 because SR auctions are chosen by higher valuation sellers and secret reserve prices $r^M(v)$ lie strictly above v (which corresponds to the reserve price chosen in a TO auction).

Proposition 3.8 Pick r, r^0, v and v^0 such that $(r; \text{secret}) \in \text{Supp}(v)$ and $(r^0; \text{public}) \in \text{Supp}(v)$, then $r > r^0$.

We then consider how participation rates compare to each other in AA, TO and SR auctions.

Proposition 3.9 Equilibrium participation rates satisfy the following: $\alpha(r) < \alpha^{AA}$ for any $r > 0$ with $\lim_{r \downarrow 0^+} \alpha(r) < \alpha^{AA}$, $\alpha^{AA} > \alpha^{SR}$ and there is a threshold $\bar{v} > v$ such that $\alpha(r) > \alpha^{SR}$ if and only if $r < \bar{v}$.

The discontinuity in terms of participation when we switch from positive to null reserves (i.e., $\lim_{r \downarrow 0^+} \alpha(r) < \alpha^{AA}$) implies that public and very low (yet non-zero) reserve prices are not used in equilibrium, since sellers strictly prefer AA auctions to them. Moreover, $\alpha(v) < \alpha^{SR}$ as otherwise a seller with valuation v would strictly prefer a SR

¹⁷ Secret reserve prices are typically not disclosed ex-post when the reserve price is not reached during the course of the auction so that auctions with low secret reserve prices are overrepresented compared to the ones with high secret reserves.

auction to a TO auction, thereby explaining why $e > v$. Thus, combining these observations, on the equilibrium path, the participation rate is larger in (any) AA auctions than in (any) OR auctions and larger in (any) OR auctions than in (any) SR auctions.

We next consider how costly it is to participate in an AA or an SR auction. To this end, for $k = AA; TO; SR$, we let $V_k(v)$ denote the true expected payoff of a buyer participating in an auction of type k when the seller's valuation equals v . Note that $V_{AA}(v)$ is independent of v (it is $u_b(AA; 0)$), and it can thus be more simply denoted by V_{AA} .

For auctions in which FR buyers participate, $V_k(v)$ corresponds to V^{FR} since perceived and true expected payoffs coincide for FR buyers. This is also true regarding PC buyers in auctions with public reserves. In particular, we have $V_{TO}(v) = V^{FR}$ for $v \geq (v; \bar{v})$. It is also straightforward to check that for any proposed format the expected payoff of a given participant cannot be strictly larger than V^{FR} , since otherwise it would imply that FR buyers could get strictly more than what they get in equilibrium. The next proposition establishes the stronger result that FR buyers would be strictly worse off by participating in AA and SR auctions. This follows as the only reason why some sellers propose AA and SR auctions (as opposed to TO auctions) is that this allows them to induce more participation than the corresponding reserve price policies would produce were buyers to be fully rational, but then this implies that FR buyers are not tempted by such auctions.

Proposition 3.10 $V_{AA} < V^{FR}$ and $V_{SR}(v) < V^{FR}$ for $v \geq [v; \bar{v}]$.

A related but different issue regarding coarse buyers is how their perceived expected payoff compares to the true expected payoff they derive in the auctions in which they participate. We say that a buyer experiences disappointment if his perceived expected payoff is strictly smaller than his true expected payoff. The measure of disappointment is interesting to the extent that it can be related to the frequency of complaints observed on auction sites (see Section 4 for a review of this). From the equilibrium condition (9), we have that PC buyers weakly prefer the SR auctions proposed in equilibrium to TO auctions, or equivalently that $V^{PC} \geq V^{FR}$ and from Proposition 3.10 we have $V^{FR} > V_{SR}(v)$. Together, this implies that:

Proposition 3.11 Partially-coarse buyers experience disappointment in any secret reserve price auction in which they participate. That is, $V_{SR}(v) < V^{PC}$ for any $v \geq [v; \bar{v}]$.

Turning to FC buyers, Proposition 3.9 guarantees that $(s) < V_{AA}$ for any $s \geq$

Since $\bar{b} > 0$. From the matching conditions, this implies that $\bar{b} = b < \bar{b}^{AA}$, i.e. that FC buyers underestimate the participation rate, which further implies that $V^{FC} = u_b(\bar{b}; 0) > u_b(\bar{b}^{AA}; 0) = V_{AA}$.

Proposition 3.12 Fully-coarse buyers experience disappointment in any auction in which they participate: $V_{AA} < V^{FC}$.

Finally, we touch on the question whether or not sellers benefit from being in a population with less sophisticated buyers. It is unfortunately not so easy to derive general comparative statics results regarding how the shares of the various types of buyers affect sellers' and buyers' payoffs in a competitive equilibrium. Yet, one can easily illustrate that sellers do not necessarily benefit from having coarser buyers, as the presence of less sophisticated buyers may make competition tougher. To illustrate this, suppose that all buyers are FC buyers, then our analysis trivially implies that in a competitive equilibrium, all sellers propose an AA auction and the participation rate for all sellers is thus $\bar{b} = b$.¹⁸ By contrast, if all buyers are FR buyers, then all sellers use a TO auction. Clearly, buyers are better off in the case in which all buyers are FC buyers and it is not difficult to show that all sellers whatever their valuation are worse off when all buyers are FC buyers (if a seller proposes an AA auction with only FR buyers, she will attract an average number of entrants that is strictly above \bar{b} (due to a comparative advantage w.r.t. to the other auction format proposed in equilibrium) and thus raises a strictly larger revenue than in the equilibrium with only FC buyers).

One can also illustrate with a simple example how sellers can benefit from having more PC buyers. Start from an initial situation with only FR buyers and consider then that a small share $\epsilon > 0$ of the buyers switch from FR to PC: In equilibrium, a positive share of sellers will then switch from TO to SR auctions and share between themselves the full mass of PC buyers (this follows from the fact that SR auctions offer an extra rent due to larger reserve prices). The total mass of sellers who still propose TO auctions receive more participation on the whole and so does each seller individually. The sellers who have switched to SR auctions receive a higher payoff than with the TO auctions and are thus better off compared to their initial equilibrium payoff with only FR buyers.

¹⁸ More precisely, we consider implicitly in our discussion that $u_{sell}(\bar{b}; 0; \bar{v}) \geq \bar{v}$ so that all sellers prefer to post an AA auction rather posting no auction at all or equivalently an auction that receives no entrants. If $u_{sell}(\bar{b}; 0; \bar{v}) < \bar{v}$, the discussion should be nuanced since some sellers would somehow quit the market which is beneficial to the other sellers and detrimental to the buyers.

4 Discussion of the empirical/experimental literature

This Section summarizes the main empirical findings on competitive auctions and relates them to our theoretical results. In a previous working paper version (Jehiel and Lamy, 2011), we considered a slightly different version of the model in which the heterogeneity between sellers reflected not solely differences in terms of reservation values but also in terms of the distribution of buyers' valuations. In that model, from the viewpoint of buyers, the goods for sale were not fully homogenous but were differentiated according to a parameter capturing the quality of the good. This previous work leads roughly to similar insights if the role of the seller's valuation is replaced by the quality of the good, which corresponds to what is typically observed in the empirical literature. In particular, in Jehiel and Lamy (2011) we establish that sellers of highest quality goods opt for SR auctions, sellers with lowest quality goods opt for AA auctions and sellers with intermediate quality goods opt for TO auctions. In the rest of the Section, we present the main empirical/experimental findings and the corresponding theoretical results in our model appear next in bracket (up to the quality/seller's reservation value twist).

As far as SR auctions are concerned, Bajari and Hortaçsu (2003) and Hossain (2008) note that secret reserve prices are more often associated with goods of higher quality [Proposition 3.7]. Furthermore, according to their counterfactual estimates, Bajari and Hortaçsu (2003) find that the expected revenue difference between SR and OR auctions

Another important puzzle is the use of absolute auctions as emphasized by Hasker and Sickles (2010). Low reserves have often been perceived as supporting the theory of endogenous as opposed to exogenous entry à la Myerson, where sellers should set reserves that are strictly above their reservation values. However, in many cases we observe reserve prices that are much lower than any reasonable reservation value of the seller. In the millions of auctions analyzed by Einav et al. (2011), a quarter of the auctions have a reserve price which is less than three percent of the final price. This comes out naturally in our setup with heterogeneous data processing from past auctions.

The results in the field experiment of Reiley (2006) (see footnote 4) are consistent with our work insofar as the expected revenue difference between AA and OR auctions falls with quality [Lemma 3.6]: This revenue raises from 2.70\$ to 3.40\$, and drops from 10.05\$ to 9.93\$, for respectively low- and medium-value cards when we move from an OR to an AA auction. A number of field experiments suggest that AA auctions may maximize the seller's revenue (Walley and Fortin 2005, Barrymore and Raviv 2009). For homogenous goods, Simonsohn and Ariely (2008) find nearly identical revenues for AA and OR auctions.²⁰ Taking advantage of a natural experiment with a local discontinuity in the reserve price policy for second-hand car auctions in U.K., Choi et al. (2010) observe a discontinuity in participation according to experience: For (almost) identical items, more experienced buyers participate more in auctions with higher reserves, an observation that appears also in Simonsohn and Ariely (2008). This is consistent with our result saying that FR buyers participate in auctions with higher reserve prices than FC buyers [Proposition 3.7] if we have in mind that FR buyers are more experienced than FC buyers, which is in line with the econometric interpretation sketched in Introduction.

Without any structural model, Simonsohn and Ariely (2008) find that buyers' expected payoff is higher in OR than in AA [Proposition 3.10], since they observe that buyers in AA are less likely to win and also pay more on average when they do win, which is hard to reconcile with a model with fully rational buyers but is compatible with our model. As expected, these authors also find that participation is greater in AA as compared to the OR auctions [Proposition 3.9]. Exploiting the experiments made by some sellers to auction identical goods, Einav et al. (2011) find that the upper tail of auction prices is heavier when sellers use a very low public reserve price rather than a moderately high

²⁰In the case of homogenous goods (i.e. $\nabla = \underline{v}$), Proposition 3.7 should be changed so that in any competitive equilibrium sellers would mix between AA, SR and TO auctions, which is consistent with Simonsohn and Ariely (2008).

one. This feature arises already in Levin and Smith's (1994) model with private values since the upper tail of the price distribution increases with the number of participants. However, the presence of fully coarse buyers makes this feature even sharper in our model, since it induces a discontinuity in the participation rate [Proposition 3.9].²¹

Bajari and Hortaçsu (2003) confirm the theory of Levin and Smith (1994) by finding that the OR auctions that maximizes the expected payoff of the seller is the TO auction, and this holds independently of the seller's reservation value. This is also consistent with our results: In any of our competitive equilibria, the optimal OR auction is the TO, as the bidders entering OR auctions behave like rational buyers so that some parts of our analysis match closely the standard model with only rational buyers.

Jehiel and Lamy's (2011) analysis yields a richer set of prediction. In particular the participation rate in SR auctions is shown to increase with quality while a negative effect of quality on participation is (typically) obtained for TO auctions. The idea is that PC buyers find that SR auctions with higher qualities are unambiguously more attractive because they fail to anticipate that they are associated with higher reserves. This result is somehow consistent with Bajari and Hortaçsu (2003) who found that there is a significantly positive effect of quality on participation for SR auctions, while the effect is much smaller (and not statistically significant) for the full sample.

We have reviewed above most of the empirical findings available on Internet auctions, and we have checked how they relate to our theoretical results. Our analysis suggests extra properties that it would be nice to investigate empirically. For example, it would be interesting to test whether sellers choosing AA auctions are receiving more negative feedbacks as Proposition 3.12 suggests, and it would be interesting to explore whether buyers choosing SR auctions are less experienced than buyers choosing OR auctions (with non-negligible reserve price), as should be inferred from Proposition 3.7.

We conclude this Section by warning that some of the results should not be taken to the letter, as it may be possible to amend our definition of cognitive types to obtain more nuanced insights. For example, if positive reserve prices clearly below the seller's valuation

²¹ Simonsohn and Ariely (2008) find that, conditional on the current price, auctions that start at a lower minimum bid surprisingly receive more new bids later in the auction dynamic, and advance informal herding or escalating commitment explanations. However, those 'new' bids include additional bids from bidders who had already entered the auction, thereby making the herding phenomenon less clear (many bidders place more than one bid on eBay). In a similar behavioral vein, Hossain (2008) develops a model (with exogenous entry) where one bidder does not understand how much he's ready to pay for the good but can only evaluate whether he is prepared to pay more than the current price. He obtains that the seller's expected payoff is highest with a secret reserve price. See also our discussion about 'auction fever' in Section 5.3.

(say by inspecting the resale opportunities) are observed, it would be inconsistent with our basic model, but it may be accounted for by having buyers who identify the effect of the reserve price on participation only coarsely, say distinguishing participation rate only according to whether the reserve price is below or above a threshold. More work is required though to analyze fully such extensions of the basic model.

5 Complementary and/or competing explanations

We view the main contribution of our paper as providing a simple extension of a standard model that is able to explain most (if not all) of the empirical regularities reported in Section 4. The basic innovative ingredient of our model is that some buyers process past data in a coarse way so as to assess the participation rate in the various auction formats and the distribution of reserve prices when secret. It should be mentioned that our model is parsimonious insofar as it requires only two new parameters (\bar{p}^C and \bar{p}^C) as compared with the standard model. Besides, the idea that some agents of the economy would bundle data from different contexts (which is the central theme of the analogy-based expectation equilibrium to which our theory offers a competitive equilibrium counterpart) is we believe a fruitful way to think of anomalies in various contexts (see also Cooper and Kagel (2009), Huck et al. (2011) and Grimm and Mengel (2012) for various effects of learning across games in the lab). In the rest of this section, we present alternative approaches and check each time the fit with the empirical regularities reported in Section 4.

5.1 Rational buyers

The evidence that OR auctions are uniformly more profitable than AA auctions from buyers' perspective is hard to reconcile with full rationality, which provides a first reason why some departure from rational expectation is needed to explain the data. We next investigate from a more theoretical viewpoint whether a departure is required to explain the emergence of AA and SR auctions. Clearly, within the model developed above such a departure is needed, but could it be that modifications of the basic model provide some rationale without requiring the presence of cognitive limitations? We review here several modifications and indicate why they would not do the job.

Heterogeneous buyers

Allowing for heterogeneous buyers coming from different groups of buyers with different valuation distributions F_j would not alter the analysis of the competitive equilibrium in a world with fully rational agents: In any competitive equilibrium, sellers would all use TO auctions and thus neither SR nor AA auctions would emerge. We elaborate on this in Jehiel and Lamy (2013).

Risk-aversion

Risk aversion is sometimes proposed as an explanation to reconcile theory with experimental findings (especially on first-price auctions, see, e.g., Bajari and Hortaçsu, 2005). In our model of second-price auctions with endogenous participation, it is clear that risk-aversion cannot play in favor of secret reserves (risk-averse buyers would not like the less transparent formats, and the unraveling argument mentioned in the introduction would certainly destabilize the presence of SR auctions in a world with only fully-rational buyers). The effect of risk aversion on absolute auctions is less straightforward, but the attractiveness of an AA (for a buyer) relative to a TO auction in a world with only rational buyers seems to result mainly from the small probability of being the sole participant and thus winning the good for free. Intuition suggests that buyers' payoffs are more uncertain in an AA than a TO auction, thereby making it unlikely that risk aversion alone can explain the emergence of AA.

Interdependence

Assuming individual buyers do not have enough time to assess how they would value the good sold at auctions they participate to, some interdependence may be expected to the extent that the information held by the various bidders may combine to assess more fully the quality of the good for sale. In such a setting, sellers' maximization program is still equivalent to total welfare maximization, but buyers' payoffs in a second-price auction are no longer aligned with total welfare, and thus the equilibrium participation rate in TO auctions does not correspond anymore to the welfare-maximizing one, which further implies that there may be an incentive for sellers to distort their reserve price (as compared with the ex post efficient TO format). A general investigation of this issue goes beyond the scope of this paper, but by making buyers bidding more cautiously, the winner's curse phenomenon is a force suggesting that buyers' expected payoff is larger than their contribution to the welfare (as it is the case in Levin and Smith's (1994) model

when valuations are interdependent). Because of this, one would expect reserve prices to be set above the seller's valuation so as to better align participation incentives with the welfare criterion. If so, it is unlikely that interdependence alone would lead to the emergence of AA auctions²²

5.2 Auction fees

Our model considers no costs associated to the choice of the various formats. In auction sites such as eBay, there are fees and these may depend on the format. To the extent that extra fees are imposed to SR auctions (it can be up to 50\$ on eBay), SR auctions are even less attractive to sellers than assumed in our main model²³. As far as public reserve prices are concerned, the fee is sometimes increasing in the reserve price for some sellers, which may provide a more direct rationale for the use of AA formats. Nevertheless, starting from the 2008 reform on the fee structure, the first 50 auction listing per month of a given seller are free on eBay so that the fee explanation could have some bite only for professional sellers (and mainly for low value items since the insertion fee can never be larger than 2\$).

So, obviously, some fee structures may give rise to the use of AA formats, but it seems that the widespread use of AA and SR auctions on eBay and beyond cannot be explained simply by the fee structure.²⁴

5.3 Auction fever

An informal argument sometimes given in favor of auctions that start with a very low price (i.e. both absolute auctions and auctions with a secret reserve and no minimum bid) is that those formats induce herding or bidding frenzies (see, e.g. Simonsohn and Ariely (2008)). Starting low is perceived as a way to create auction fever : Due to some psychological mechanism, some auction participants may then bid above their true valuation as considered by Malmendier and Szeidl (2008). At first glance, it would seem auction fever would make AA and SR auctions attractive to sellers. This would indeed be true

²² Concerning SR auctions, the unravelling argument still prevails with interdependent valuation in cases in which participation is exogenous (see Lamy 2009). Nevertheless, the unravelling argument is less clear with endogenous participation given that it is no longer true under interdependent valuations that more participants makes the seller better off.

²³ In the early days of eBay (in particular for the period covered by Bajari and Hortaçsu 2003), this option was entirely free and SR auctions were much more popular (14% for their dataset).

²⁴ Auction fees do not help explain the above mentioned results from field experiments about the profitability of AA and SR auctions (Walley and Fortin 2005, Katkar and Reiley 2006, Reiley 2006, Barrymore and Raviv 2009), since those results do not take into account any fees (whether they exist or not in the corresponding auction platforms).

if participation were exogenous, given that auction fever reduces the buyers' rents, which is beneficial to the seller. However with endogenous participation, if buyers are rational at the participation stage (because say they are in a cold mode), then buyers should participate more cautiously in AA or SR auctions so that all the extra rents captured by the seller *ceteris paribus* would be lost through reduced participation. As a result, AA and SR auctions would be suboptimal for the seller compared to TO auctions (not to mention the extra allocative inefficiencies that are typically associated to auction fever), and sellers would in turn choose TO auctions. Thus, auction fever *per se* does not explain the emergence of AA nor of SR auctions in a competitive environment with endogenous participation.

5.4 Inattention

We now consider the possibility that some share $\alpha \in (0, 1)$ of buyers would not pay attention to the format when making their participation decision. This is different from our modelling of FC buyers, since FC buyers do observe the format (at least the reserve price if public) but somehow erroneously believe participation is not responsive to the format. The participation decision of inattentive buyers by contrast does not respond to the format, since inattentive buyers make their participation decision as if they did not observe the format. Clearly, if there were only inattentive buyers, a seller with valuation v would pick a reserve price (whether public or secret) set at $r^M(v)$, since participation would be exogenously given (uniform splitting of buyers into the various auctions). Henceforth, we refer to an auction format with $r = r^M(v)$ as an MO auction.

With a mix of (attentive) FR buyers and inattentive IN buyers, sellers either choose a TO auction as it is the case when facing only FR buyers or choose an MO auction. This conclusion follows the same logic as the one developed in the main model: As long as FR buyers find it profitable to participate, it is best to offer them a TO auction, and when FR buyers do not participate, it is best to offer a MO auction given that participation is not responsive to the choice of format. What remains to be determined is how sellers with various v sort into TO and MO auctions, and it is readily verified that sellers with higher valuations benefit more from having a higher reserve price than sellers with lower valuations, thereby implying that there exists a threshold $\bar{v} > \underline{v}$ such that sellers with valuations $v < \bar{v}$ opt for a TO auction and sellers with valuations $v > \bar{v}$ opt for an MO auction.

Thus, the presence of inattentive buyers does not allow to explain the emergence of AA auctions and it provides only a weak rationale for the use of SR auctions, since when sellers find it optimal to choose MO auctions they are indifferent between making their reserve price public or not, as long as

essentially similar to a secret reserve price in this setting²⁷. Suppose next that because shill bidding is illegal, not every seller resorts to it, and only a share α of fraudulent sellers consider it while the remaining share consists of honest sellers. Assuming that buyers are fully rational, there are a priori many equilibria if deviations from the equilibrium path are interpreted as meaning that there is a greater chance that the seller be a shill bidder. However, reasonable restrictions on these off-path interpretations²⁸ yield the prediction that no matter how small α is, honest sellers select Myerson's reserve price $r^M(v)$ in any equilibrium, while fraudulent sellers will eventually set a reserve price at Myerson's level $r^M(v)$ (but possibly through some shill bidding activity). Thus, even a small share of shill bidding activity may have a big impact on the competitive equilibrium when all buyers are fully rational.

Consider now the case with a mix of FR and FC buyers, assume that a share of sellers can resort to shill bidding, and consider as in our main model that sellers may have heterogeneous valuations. A natural specification for the FC buyers' beliefs is that they ignore shill bidding (where shill bids are perceived as regular bids). In such a situation, to the extent that α is not too big, fraudulent sellers will pick an AA auction expecting to attract more (FC) buyers and they will have incentive to do so irrespective of their valuation. So AA auctions will be picked both by honest sellers with low valuations and by fraudulent sellers with arbitrary valuations. So when FR and FC buyers coexist, the presence of a small share of fraudulent sellers does not induce a big efficiency loss on the working of the auction house unlike in the case with only FR buyers, thereby suggesting a potentially stabilizing role of FC buyers.

To sum up, shill bidding alone (i.e., assuming that buyers are fully rational) would not explain the emergence of AA nor of SR auctions. Besides, to the extent that one expects a small share of shill bidding activity to have a small impact on the working of the competitive equilibrium, it is inconsistent with full rationality but not with our formulation of cognitive limitations.

²⁷ With heterogeneous sellers, the equilibrium analysis is not clear since sellers with valuation v may announce a public reserve strictly above $r^M(v)$.

²⁸ The required selection idea is that if a seller proposes an off-the-path reserve price, then she is perceived to use shill bidding with a probability of at most α (which can be rationalized on the grounds that fraudulent sellers, who are likely to be more active/experienced players, are less likely to tremble).

5.6 Other models of bounded rationality

The quantal response equilibrium (QRE) is a popular model used to explain anomalous behaviors, especially in experimental settings, and may be combined with risk aversion in the context of auctions (see Goeree et al. 2002, for first-price auctions). We briefly discuss what might be expected if we consider that participation decisions are taken according to QRE.²⁹ Our intuition is that the equilibrium participation rate as a function of the public reserve should be flatter under QRE than Nash equilibrium,³⁰ which suggests that the optimal reserve price should be larger in QRE than in Nash equilibrium (where it is set at the seller's valuation). Note that in the limit where the QRE error parameter goes to infinity, participation becomes irresponsive to the announced format, in which case the reserve price should be set at Myerson's optimal level. Thus, QRE does not seem to give rise to AA or SR auctions.

The level-k framework is another popular approach used to explain anomalies (see for example Crawford and Iriberri, 2007 in the context of auctions). Applied to our participation game, it would see level-0 buyers as using uniformly random participation over all auction formats, and thus level-1 buyers would behave as our FC buyers, thereby giving rise to AA auctions in a competitive equilibrium. More problematic would be the implication of level-2 buyers who would opt for the auction with second lowest reserve price (as they would expect all other level-1 buyers to participate only in the auctions with lowest reserve price). The presence of level-2 buyers would in turn lead to the emergence of auctions with very low but positive reserve prices. Considering level-k buyers with $k > 2$ would only give rise to AA auctions or to auctions with arbitrarily small reserve prices. Thus, this approach would not be able to explain the use of reserve prices significantly away from 0, which does not fit with the empirical observation.³¹ Overall, it seems to us, the level-k approach is not so well suited for our problem.

²⁹ On the contrary, we maintain that sellers maximize their expected payoffs while buyers bid their valuations at the bidding stage. Note that in the second-price auction with private values, QRE predicts that buyers should bid both above and below their valuations.

³⁰ This is analogous to flatter responses to asymmetric payoffs within QRE in matching pennies games (Goeree et al., 2003).

³¹ It is also not easy to apply the level-k approach to the analysis of SR auctions given that the range of possible reserve price is not a priori known (so it is not clear how to define level-0 expectation for the distribution of secret reserve prices). Brown et al. (2012) apply the level-k approach to get around the unravelling argument in the film review industry. In their case, they suggest that there is a pre-conceived range of possible qualities, thereby indicating a way to formalize the level-0 belief.

6 Conclusion

We have shown how the presence of buyers who do not have rational expectations about the behavior of the other players, but whose expectations are however consistent with some empirical feedback, may explain the emergence of absolute auctions and secret reserve price auctions in competitive environments with rational sellers. We have also reviewed the empirical literature on Internet auctions and checked that the most salient findings there can all be explained within our framework which leads to a segmentation of the market not solely according to preferences (namely sellers' reservation values) but also according to buyers' cognitive types. Moreover, we have reviewed the main competing theories and suggested that they failed to capture the same properties shared by our model and the empirical findings. The recent change in eBay fee policy³² would provide the opportunity of further tests of our theory. From a policy perspective and similarly to what eBay has done for auctions with secret reserve prices, our analysis suggests that eBay should not limit herself to flat fees (i.e. insertion fees that do not depend on the reserve price) but should introduce some extra fees for auctions with too low a reserve price in order to make sellers internalize the disappointment they induce.

While we have focused on the widespread use of absolute auctions and secret reserve price auctions for which there is an impressive collection of empirical evidence, we believe that approaches similar to ours can be used to shed light on other puzzles. For example, Einav et al. (2012) observe on eBay auctions that setting null shipping fees may be profitable, which could be rationalized with buyers similar to our FC buyers who would not see the impact of the shipping fee on the participation rate.³³

In another application, it is a stylized fact in certification/grade-disclosure environments that a non-negligible proportion of subjects with bad signals prefer to hold them back. The standard explanation for the absence of complete unraveling (which standard theory predicts) is the fact that certification may be costly, so that those who receive the worst grades do not pay for it. However, this type of argument is less compelling in environments in which the grades are available to the subjects for free. The estimates in Conlin and Dickert-Conlin (2010) reveal that colleges underestimate the relationship

³² From spring 2013 on, eBay has suppressed the insertion fees for all kinds of sellers. The model is now the one of sellers who rent a 'virtual shop' of a given size that allows them to auction a fixed number of goods of a given category.

³³ A puzzling pattern emerges from the field experiments led by Hossain and Morgan (2006): the exact splitting between the reserve price and the shipping fees, which should not matter for rational buyers, do [resp. do not] matter in the low [resp. high] reserve price treatment. Note that this is consistent with our view that buyers who participate to auctions with higher reserve are more experienced.

between an applicant's action in submitting (or not) his SATI score and the actual score. In auctions for baseball cards, Jin and Kato (2006) provide empirical evidence of buyers' naïveté: Some buyers overestimate the quality of the card when sellers do not pay to be graded by a professional certifier, especially if the seller also claims that the quality is high. For example, sellers claiming top qualities instead of nothing raise their revenue by 50%. Jin and Kato (2006) also note that the average winner of a graded-card auction is more experienced than winners of ungraded-card auctions, thereby suggesting a sorting similar to ours with a segment of the market framed to capitalize on buyers' coarseness. These findings can be related to our analysis of SR auctions and how PC buyers form their beliefs over the distribution of reserve prices when secret.

In yet another application, it has long been observed that people do not react rationally to the characteristics of lotteries/contests. In particular, participants seem to under-react to an increase in the number of other contestants (see Lim et al. (2009) and the references therein). This feature remains in the lab once we remove charity motives and the small probabilities/large prize effects that are often inherent to lotteries. Such a puzzle can be rationalized by considering subjects who would not adjust their expectation about others' effort to the number of participants, somehow similarly to our modeling of FC buyers. The analysis in greater detail of these applications is a clear subject for further research.

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Appendix

A Proof of Lemma 3.1

For any $r \geq R_+$ and $V^{FR} \geq 0$, we define $\psi(r; V^{FR})$ as the solution (in β) to $u_b(\beta; r) = V^{FR}$ if a solution exists³⁴ while we let $\psi(r; V^{FR}) := 0$ otherwise. In other words, the

³⁴ If a solution exists then it is necessarily unique since $\frac{\partial u_b(\beta; r)}{\partial \beta} = \sum_{n=0}^N \frac{1}{n!} e^{-\frac{n}{n!} [V_{n+1}(r) - V_n(r)]} < 0$ because $V_n(r) = \int_r^{R_+} F^n(x) dx - F(x)$ is decreasing in n .

participation intensity $\beta(r; V^{FR})$ corresponds to the one that would prevail in a setup with only FR buyers having the expected payoff V^{FR} . From condition (1), note that we have $\beta(r) = \beta(r; V^{FR})$ with $\beta(r) = \beta(r; V^{FR})$ if $F^{FR}(r) > 0$.

As a small roundabout, let us consider the maximization with respect to r of the function $u_{\text{sell}}(\beta(r; V^{FR}); r; v)$. Let $W_n(r; v)$ denote the expected total welfare (sum of all agents' payoffs) in an auction with n bidders, a reserve price r and a valuation v of the seller, i.e. $W_n(r; v) = \beta_n(r; v) + nV_{n-1}(r)$.

From the definition of $\beta(\cdot; \cdot)$, we have that $u_{\text{sell}}(\beta(r; V^{FR}); r; v) = TW(\beta(r; V^{FR}); r; v)$ where $TW(\cdot; r; v) := \sum_{n=0}^{\infty} \frac{1}{n!} W_n(r; v) - V^{FR}$ corresponds to the total expected welfare net of the expected opportunity cost V^{FR} incurred by FR buyers who could have got V^{FR} (in expectation) by not participating in the auction (and there are β of them in expectation).

A key well known property of the second-price auction is that the social contribution to the welfare of a new participant coincides with his expected payoff when the reserve price is set at the seller's valuation (this corresponds to the pivotal mechanism or the Vickrey auction). Specifically, for all n , we have:

$$W_{n+1}(v; v) - W_n(v; v) = V_n(v): \quad (10)$$

As buyers obtain the incremental surplus they generate in TO auctions, we have that $\frac{\partial TW(\cdot; v; v)}{\partial \beta} = \sum_{n=0}^{\infty} \frac{1}{n!} V_n(v) - V^{FR}$. Since $V_n(v)$ is decreasing in n , we obtain that

$$\text{Arg max}_0 TW(\cdot; v; v) = f(v; V^{FR})g: \quad (11)$$

Furthermore, it is readily verified that, for a fixed $n \geq 1$, maximizing expected welfare $W_n(r; v)$ with respect to r requires setting the reserve price at the seller's valuation $r = v$. If $\beta(v; V^{FR}) > 0$ [resp. $\beta(v; V^{FR}) = 0$], then it follows that the maximization program $\max_{0 \leq r \leq v} TW(\cdot; r; v)$ has a unique solution given by $\beta = \beta(v; V^{FR})$ and $r = v$ [resp. has a unique solution for which is given by $\beta = 0$ while r can take any value]. On the whole, we obtain that the set of solutions of the maximization program

$$\max_{r \geq 0} u_{\text{sell}}(\beta(r; V^{FR}); r; v) \quad (12)$$

corresponds to the singleton $r = v$ if $\beta(v; V^{FR}) > 0$ and to the set $\{r : \beta(r; V^{FR}) = 0\}$ otherwise (which includes also $r = v$). This is the argument why TO auctions should emerge with fully rational buyers: It confirms earlier findings, in particular by Levin and

Smith (1994) and Peters and Severinov (1997)³⁵.

Coming back to our setup with a mix of different types of buyers and considering a seller with reservation value v choosing in equilibrium a public reserve price $\bar{r} \notin v$ such that $F^R(r) > 0$, then the seller's expected payoff equals $u_{\text{sell}}(\bar{r}; V^{FR}); r; v$. Let us show that this seller could have raised a strictly larger revenue if she had chosen the TO auction. If $F^R(v) > 0$, the seller's expected payoff in the TO auction equals $u_{\text{sell}}(v; V^{FR}); v; v$ and we conclude from our resolution of the maximization program (12). If $F^R(v) = 0$, then the TO auction yields a (weakly) higher participation rate than $(v; V^{FR})$ while this latter participation rate yields the expected payoff $u_{\text{sell}}(v; V^{FR}); v; v$ which is strictly larger than $u_{\text{sell}}(\bar{r}; V^{FR}); r; v$ as it comes from our resolution of the maximization program (12) and since $(\bar{r}; V^{FR}) = (r) > 0$ (because $F^R(r) > 0$). We conclude after noting that the seller's revenue increases with participation in TO auctions. On the whole, we have shown that the only auctions with public reserves that are proposed in equilibrium and in which FR buyers participate are TO auctions. Since PC buyers have the same (correct) beliefs as FR buyers w.r.t. auctions with public reserves, by a similar argument we observe that if a PC buyer selects an auction with a public reserve price, it must be a TO auction, which concludes the proof.

B Proof of Lemma 3.2

Suppose that $\int_{\underline{v}}^{\bar{v}} \int_0^1 v(r; \text{secret}) dr dG(v) \notin 0$ and $F^R(\text{secret}) > 0$. Since this implies that $(\text{secret}) > 0$, we obtain then that for any $v \in [\underline{v}; \bar{v}]$, $(r; \text{secret}) \in \text{Supp}(\nu)$ implies that $r = r^M(v)$.

Let $\theta := \inf \{v \in [\underline{v}; \bar{v}] : (r^M(v); \text{secret}) \in \text{Supp}(\nu)\}$. Note that since G has no atom, then the measure $\int_{\underline{v}}^{\bar{v}} \int_0^1 v(r; \text{secret}) dG(v)$ contains no atom too and is thus not a Dirac measure $\text{at}^M(\theta)$. Since $V_n(r)$ is strictly decreasing in r , we obtain that $u_b(\cdot; r)$ is strictly decreasing in r . We have thus $u_b(\text{secret}; \text{secret}) < u_b(\text{secret}; r^M(\theta))$. By continuity, the previous inequality remains true in the neighborhood of $r^M(\theta)$ so that there exists v such that $(r^M(v); \text{secret}) \in \text{Supp}(\nu)$ and $u_b(\text{secret}; \theta) < u_b(\text{secret}; r^M(v))$.

From (1), we must have $u_b(r^M(v); r^M(v)) = V^{FR}$. Since $F^R(\text{secret}) > 0$, we have also $u_b(\text{secret}; \text{secret}) = V^{FR}$. On the whole we obtain that $u_b(r^M(v); r^M(v)) < u_b(\text{secret}; r^M(v))$. Since the function $r \mapsto u_b(r; r)$ is strictly decreasing for any r , we obtain that $(r^M(v)) > (\text{secret})$. Since the seller's revenue strictly increases with entry

³⁵ In their section devoted to competing auctions with entry, Peters and Severinov (1997) provide a series of conditions for competitive equilibria. Although their formal analysis is correct, they wrongly conclude in their comments that the equilibrium reserve price lies strictly above the seller's reservation value (see also the corrigendum by Albrecht et al. 2011).

are either AA or TO auctions. From the preliminary remark, we have that any auction that is proposed on the equilibrium path receive some entry. This means in particular that $\bar{\nu}(r) := \frac{R_{\bar{\nu}} - R_1}{\int_0^{\bar{\nu}} \frac{r(r; \text{public})}{\nu(r; \text{public})} d\nu(r)} g(r)$ if $r > 0$ and $\bar{\nu}(0) := \frac{R_{\bar{\nu}} - R_1}{\int_0^{\bar{\nu}} \frac{\nu(0; \text{public})}{\nu(r; \text{public})} d\nu(r)} dG(\nu)$.

Since we have assumed that there is no mass of buyers participating to SR auctions, Lemmas 3.2 and 3.3 imply that $\frac{R_{\bar{\nu}}}{\int_0^{\bar{\nu}} [\text{FR}(\nu) + \text{PC}(\nu)] \nu(\nu; \text{public}) dG(\nu)} = \frac{R_{\bar{\nu}}}{\int_0^{\bar{\nu}} [\text{FR}(s) + \text{PC}(s)] \nu(s) ds} dG(\nu)$. From the matching conditions for FR and PC buyers, we obtain then $\frac{R_{\bar{\nu}}}{\int_0^{\bar{\nu}} \nu(\nu; \text{public}) dG(\nu)} = (\text{FR} + \text{PC}) b > 0$ which further implies that $\frac{R_{\bar{\nu}}}{\int_0^{\bar{\nu}} \bar{\nu}(r) dG(r)} > 0$.

Since FR and PC buyers have the same beliefs w.r.t. auctions with public reserves and we have assumed that there is no mass of buyers participating to SR auctions, we obtain that $V^{\text{FR}} = V^{\text{PC}}$.

Let $\bar{\nu} := \sup \{ \nu \in [\underline{\nu}, \bar{\nu}] : (\nu; \text{public}) \in \text{Supp}(\bar{\nu}) \text{ and } \bar{\nu}(\nu) > 0 \}$. As noted above the measure $\bar{\nu}(\cdot)$ is not a Dirac measure. For any ν and any $r < \bar{\nu}$ (and thus for almost any r in the support of $\bar{\nu}$), we have then $V_n(r) > V_n(\bar{\nu})$ since $V_n(r)$ is strictly decreasing in r . We have thus $b_b^{\text{PC}}(\text{secret}) > u_b(\text{secret}; \bar{\nu})$. By continuity, the previous inequality remains true in the neighborhood of $\bar{\nu}$ so that there exists ν such that $(r^M(\nu); \text{secret}) \in \text{Supp}(\bar{\nu})$, $\bar{\nu}(\nu) > 0$ and $b_b^{\text{PC}}(\text{secret}) > u_b(\text{secret}; \nu)$.

From (1), we must have $b_b^{\text{PC}}(\text{secret}) \leq V^{\text{PC}}$ and thus $u_b(\text{secret}; \nu) < V^{\text{PC}}$. Since $\bar{\nu}(\nu) > 0$, which implies that either $\text{FR}(\nu) > 0$ or $\text{PC}(\nu) > 0$, we have also $u_b(\nu; \nu) = V^{\text{PC}}$. Since the function $\nu \mapsto u_b(\nu; \nu)$ is strictly decreasing, we obtain that $\bar{\nu}(\text{secret}) > \bar{\nu}(\nu)$. Since the seller's revenue strictly increases with entry for any reserve price above its valuation, we obtain that a seller with valuation ν has a strict larger payoff in the TO auction than in the SR auction with the reserve ν which raises a contradiction with $(\nu; \text{secret}) \in \text{Supp}(\bar{\nu})$.

E Proof of Lemma 3.6

Straightforward calculation leads to: $u_{\text{sell}}^{\text{AA}}(\nu) = \nu e^{-\text{AA}} + \sum_{n=1}^P \frac{1}{n!} e^{-\text{AA}} \frac{[\text{AA}]^n}{n!} \int_0^{\bar{\nu}} u d[F^{(2:n)}(u)]$. We have then :

$$\frac{du_{\text{sell}}^{\text{AA}}(\nu)}{d\nu} = e^{-\text{AA}} : \quad (14)$$

From the way we have solved the maximization program (12) and in particular (11), we obtain:

$$u_{\text{sell}}^{\text{TO}}(v) = \max_0 \sum_{n=0}^{\infty} \frac{v^n}{n!} e^{-v} (F^{(1:n)}(v) + \sum_{x=v}^1 x d[F^{(1:n)}(x)]) v^{FR} \quad (15)$$

We emphasize that this is true for any $v \in (0; 1)$ since buyers that enter OR auctions have rational expectations.

From the envelope theorem, the differentiation w.r.t. v leads to

$$\frac{du_{\text{sell}}^{\text{TO}}(v)}{dv} = e^{-v} (1 - F(v)) \quad (16)$$

Since $u_{\text{sell}}^{\text{SR}}(v) = \sum_{n=0}^{\infty} \frac{v^n}{n!} e^{-v} (r^M(v); v)$ and recalling that $\frac{\partial}{\partial r} (r^M(v); v) = 0$, we obtain

$$\frac{du_{\text{sell}}^{\text{SR}}(v)}{dv} = e^{-v} (1 - F(r^M(v))) \quad (17)$$

Lemma E.1 formalizes the tradeoff between a larger reserve price and enhancing participation.

Lemma E.1 In equilibrium, for any $v \in (0; 1)$, we have $u_{\text{sell}}^{\text{AA}}(v) > u_{\text{sell}}^{\text{TO}}(v)$ and $u_{\text{sell}}^{\text{TO}}(v) > u_{\text{sell}}^{\text{SR}}(v)$.

Proof For any $v \in (0; 1)$, we have $u_{\text{sell}}(0; v) < u_{\text{sell}}(v; v) < u_{\text{sell}}(r^M(v); v)$ and the inequalities are strict for $n \geq 1$. We obtain then that $u_{\text{sell}}(\gamma; 0; v) < u_{\text{sell}}(\gamma; v; v) < u_{\text{sell}}(\gamma; r^M(v); v)$ for any $\gamma > 0$. When we apply the first inequality to $\gamma = \text{AA}$, we obtain that $u_{\text{sell}}^{\text{AA}}(v) > u_{\text{sell}}^{\text{TO}}(v)$ implies $u_{\text{sell}}(\text{AA}; v; v) > u_{\text{sell}}(\frac{\text{TO}}{v}; v; v)$ and finally $\text{AA} > \frac{\text{TO}}{v}$ for any $v \in (0; 1)$.

Since $\text{SR} > 0$, we have $u_{\text{sell}}^{\text{SR}}(v) > v$ and then $u_{\text{sell}}^{\text{TO}}(v) > u_{\text{sell}}^{\text{SR}}(v)$ implies that $\frac{\text{TO}}{v} > \text{SR}$. With an analogous argument but with the second inequality, we obtain that $u_{\text{sell}}^{\text{TO}}(v) > u_{\text{sell}}^{\text{SR}}(v)$ imply $\frac{\text{TO}}{v} > \text{SR}$ for any $v \in (0; 1)$. Q.E.D.

In order to show that a given differentiable function is quasimonotone increasing, it is sufficient to show that its derivative is strictly positive at any point where the function is null. Consider v such that $u_{\text{sell}}^{\text{TO}}(v) = u_{\text{sell}}^{\text{AA}}(v)$. From (14) and (16), we have $\frac{d[u_{\text{sell}}^{\text{TO}}(v) - u_{\text{sell}}^{\text{AA}}(v)]}{dv} = e^{-v} (1 - F(v)) - e^{-v} \text{AA}$, which is positive since $\text{AA} > \frac{\text{TO}}{v}$ (Lemma E.1). Consider now v such that $u_{\text{sell}}^{\text{SR}}(v) = u_{\text{sell}}^{\text{TO}}(v)$. From (16) and (17), we have $\frac{d[u_{\text{sell}}^{\text{SR}}(v) - u_{\text{sell}}^{\text{TO}}(v)]}{dv} = e^{-v} (1 - F(r^M(v))) - e^{-v} \frac{\text{TO}}{v}$, which is positive since $\frac{\text{TO}}{v} > \text{SR}$ (Lemma E.1) and $r^M(v) > v$.

F Proof of Proposition 3.7

We have already shown that, for any competitive equilibrium, we have a triple $(v_1; v_2;) \in T$ so that: 1) $\text{Supp}(v) = (0; \text{public})$ if $v < v_1$; 2) $\text{Supp}(v) = (v; \text{public})$ if $v \in (v_1; v_2)$; 3) $\text{Supp}(v) = (r^M(v); \text{secret})$ if $v > v_2$; 4) a share γ [resp. $(1 - \gamma)$] of the PC buyers participate in TO [resp. SR] auctions; 5) FC buyers participate only in AA; and 6) FR buyers participate only in TO. From profit maximization for sellers, we obtain (8) and (9). From profit maximization for buyers and the matching equilibrium conditions, the equilibrium participation rate $\gamma(s) = \frac{P}{k \cdot f_{FC;PC;FR} g} \gamma(s)$ in the various formats AA, TO and SR necessarily have the following form:

$$\gamma(0) = \gamma^{FC}(0) = \frac{F_C b}{G(v_1)}$$

On the interval $(0; 1)$, we have $\gamma(r) = \max \{ \gamma[v_1; v_2;](r); 0 \}$ where the function $\gamma[v_1; v_2;](\cdot)$ is uniquely characterized as the unique continuous function $\gamma : \mathbb{R}_+ \rightarrow \mathbb{R}$ which is a solution of the differential equation

$$\gamma'(r) = - \frac{R_1}{r} \frac{(1 - F(r)) e^{-\gamma(r)(1 - F(r))}}{(1 - F(x))^2 e^{-\gamma(r)(1 - F(x))}} < 0; \quad (18)$$

(this guarantees the indifference of FR and PC buyers regarding the various OR auctions) with the matching condition:³⁶

$$\int_{v_1}^{v_2} \gamma(r) dG(v) = (\gamma^{FR} + \gamma^{PC}) b; \quad (19)$$

For any $z > 0$, the participation rate $\gamma(\cdot; z)$ satisfies also the differential equation (18) at a reserver where $\gamma(r; z) > 0$. Furthermore, we also know that we have $\gamma(s) = \gamma(s; V^{FR})$ if $\gamma^{FR}(s) > 0$. Since there exists some OR auctions such that $\gamma^{FR}(s) > 0$, we obtain that γ and $\gamma(\cdot; z)$ should coincide.

$$\gamma(\text{secret}) = \gamma^{PC}(\text{secret}) = \frac{(1 - \gamma)}{1} \frac{P_C b}{G(v_2)}.$$

We have also $\gamma^{FC}(s) = 0$ for any $s \in S$ if 0 . Furthermore, we have also already shown that $\underline{v} < v_1 < v_2 < \bar{v}$ and $\gamma < 1$.

We are now left with equilibrium existence. The proof contains now three steps. In a first step ('construction'), we build a full strategy profile

³⁶ The Cauchy-Lipschitz Theorem establishes the existence and uniqueness of differential equations when the initial condition has the form $\gamma(r) = \gamma_0$. See the proof of Proposition 3.2 in Jehiel and Lamy (2011) for a formal proof when the usual condition is replaced by our matching condition.

$(b_v[v_1; v_2; \cdot])_{v_2 \in [\underline{v}, \bar{v}]}; (b^i[v_1; v_2; \cdot])_{i \in \{FC; PC; FR\}}$ for any triple $(v_1; v_2; \cdot) \in T$, which should be viewed as an equilibrium candidate. In a second step ('verification'), we show that if a triple $(v; v; \cdot) \in T$ and the corresponding strategy profile satisfy (8) and (9), then the given strategy profile is an equilibrium. In a third step ('existence'), we show that there exists a triple $(v; v; \cdot) \in T$ satisfying (8) and (9) by applying Kakutani fixed point theorem. To lighten the notation we let $t := (v_1; v_2; \cdot)$. Let also $b[t](s) = \prod_{i \in \{FC; PC; FR\}} b^i[t](s)$. The functions $u_{\text{sell}}^k[t](\cdot)$, $k = AA; TO; SR$, are then defined analogously. E.g. $u_{\text{sell}}^{TO}[t](v) = u_{\text{sell}}(b[t](v); v; v)$ for any $v > 0$.

1/ Construction For a given $t \in T$, we build the equilibrium candidate $(b_v[t])_{v_2 \in [\underline{v}, \bar{v}]}; (b^i[t])_{i \in \{FC; PC; FR\}}$ in the following way. Next we adopt the convention that $\frac{x}{0} = 1$ for any $x \in \mathbb{R}$ and we allow $b^i[t](s) = +1$ (we could extend our definition straightforwardly that allow that. In any cases, it would never occur in equilibrium).

$$\text{Supp}(b_v[t]) := \begin{cases} (0; \text{public}) & v < v_1 \\ (v; \text{public}) & \text{if } v \in [v_1; v_2] : \\ (r^M(v); \text{secret}) & v > v_2 \end{cases}$$

Let $b^{FC}[t](0) := \frac{FC}{G(v_1)} b$ and $b^{FC}[t](s) := 0$ for any $s \in S$ if $0 < s$.

For $r \in (0; 1)$, let $b^{FR}[t](r) := \frac{FR}{FR + PC} b[t](r)$, where $b[t](\cdot)$ is characterized by the differential equation (18) once $b[t](r) > 0$ and the condition (19). We also let $b^{FR}[t](0) := 0$ and $b^{FR}[t](\text{secret}) := 0$.

For $r \in (0; 1)$, let $b^{PC}[t](r) := \frac{PC}{FR + PC} b[t](r)$. We also let $b^{PC}[t](0) := 0$ and $b^{PC}[t](\text{secret}) := \frac{(1-r) PC b}{1 - G(v_2)}$.

If $v_2 < \bar{v}$, then we can define $b_{\text{secret}}[t](\cdot)$ as in (5). We also define the distribution $b[t](\cdot)$ as in (6).

2/ Verification

Consider a triple $(v; v; \cdot) \in T$ and the corresponding strategy profile $(b_v[t])_{v_2 \in [\underline{v}, \bar{v}]}, (b^i[t])_{i \in \{FC; PC; FR\}}$ such that (8) and (9) jointly hold. Note that (8) implies that $\underline{v} < v_1 < v_2 < \bar{v}$.

We first check that sellers' profit maximization conditions are satisfied. Combined with Eq. (8), the following lemma allows us to conclude.

Lemma F.1 The functions $u_{\text{sell}}^{TO}[t](v)$, $u_{\text{sell}}^{AA}[t](v)$ and $u_{\text{sell}}^{SR}[t](v)$, $u_{\text{sell}}^{TO}[t](v)$ are quasi-monotone increasing on $(0; 1)$.

Proof The proof follows that in Lemma 3.6: all the properties of an equilibrium that are used to establish Lemma 3.6 are satisfied by our equilibrium candidate $(b_v[t])_{v \in [\underline{v}, \bar{v}]}$, $(b^i[t])_{i \in \{FC, PC, FR\}}$. Q.E.D.

Second we check that buyers' profit-maximization conditions are satisfied. This is straightforward for FC buyers. From the definition of $b_v[t](\cdot)$ we obtain that the equilibrium condition (1) for FR buyers is satisfied for any $s \in (0, 1)$. If AA auctions were strictly profitable for FR buyers, then it would imply that $b[t](0) < b_v[t](0)$. For $v < v_1$ so that the seller's most preferred auction is the AA auction which guarantees that $u_{sell}^{AA}[t](v) = v$. As an application of Lemma I.1, we have then $u_{sell}^{AA}[t](v) < u_{sell}(b_v[t](0); v; v)$. From our resolution of the maximization program (12), we have also $u_{sell}(b_v[t](0); 0; v) = u_{sell}(b_v[t](v); v; v) = u_{sell}^{TO}[t](v)$. On the whole we have shown that $u_{sell}^{AA}[t](v) < u_{sell}^{TO}[t](v)$ which raises a contradiction with the sellers' profit maximization conditions established above. If SR auctions were strictly profitable for FR buyers, then it would imply that $b[t](secret) < b_v[t](secret)$ where $b_v[t](secret)$ is characterized by $u_b(b_v[t](secret); b_{secret}[t]) = u_b(b_v[t](v_1); v_1)$ (note that $u_b(b_v[t](v_1); v_1)$ corresponds to buyers' expected payoff in OR auctions). Then we would have $u_{sell}^{SR}[t](v_2) = u_{sell}(b_v[t](secret); r^M(v_2); v_2) < u_{sell}(b_v[t](secret); r^M(v_2); v_2) = u_{sell}(b_v[t](r^M(v_2)); r^M(v_2); v_2)$ where the last inequality comes from the fact that $u_b(b_v[t](secret); b_{secret}[t]) = u_b(b_v[t](r^M(v_2)); r^M(v_2))$ for any $r^M(v_2) \in R_+$ (since $r^M(v_2)$ is the lower bound of the distribution of secret reserves $b_{secret}[t](\cdot)$) which implies that $b_v[t](secret) = b_v[t](r^M(v_2))$ and thus raises a contradiction. We have thus finished with the analysis of FR buyers. Since PC buyers have the same payoff expectation as FR buyers in auctions with public reserves, we obtain that PC buyers are indifferent between all OR auctions and strictly prefer OR auctions to AA auctions. Since we have assumed that (9) holds, we conclude that PC buyers' maximization conditions hold.

3/ Existence

Lemma F.2 $\frac{\partial b[t](0)}{\partial v} \geq 0$ and $\frac{\partial b[t](r)}{\partial v} \geq 0$ for any $v_1 \in (\underline{v}, v_2)$ and $r > 0$; $\frac{\partial b[t](r)}{\partial v} \geq 0$ and $\frac{\partial b[t](secret)}{\partial v} \geq 0$ for any $v_2 \in (v_1, \bar{v})$ and $r > 0$.

Proof The first and the fourth inequalities are straightforward from our construction (e.g., $\frac{\partial b[t](0)}{\partial v} = \frac{FC \cdot bg(v_1)}{[G(v_1)]^2}$).

Consider $\underline{v} < v_1 < v_1^0 < v_2$. Suppose that $b[v_1; v_2; \cdot](r) > b[v_1^0; v_2; \cdot](r)$ for a given $r > 0$. The differential equation (18) implies then that $b[v_1; v_2; \cdot](r) > b[v_1^0; v_2; \cdot](r) = 0$ for any $r > 0$ while the inequality is strict once $b[v_1; v_2; \cdot](r) > 0$. We thus obtain that $\int_{v_1^0}^{v_2} b[v_1; v_2; \cdot](u) dG(u) > \int_{v_1^0}^{v_2} b[v_1^0; v_2; \cdot](u) dG(u)$ and finally that $\int_{v_1^0}^{v_2} b[v_1; v_2; \cdot](u)$

$dG(u) > (\frac{FR}{G} + \frac{PC}{G}) b$, which raises a contradiction with the matching condition (19).

We then obtain that $\frac{\partial b[t](r)}{\partial v} \geq 0$. The proof is similar for the third inequality. Q.E.D.

For $t \geq T$, we let $\bar{u}_{sell}^{AA}(t) := u_{sell}^{AA}[t](v_1)$ and $\bar{u}_{sell}^{TO;1}(t) := u_{sell}^{TO}[t](v_1)$.

Lemma F.3 The function $v_1 \mapsto \bar{u}_{sell}^{TO;1}(v_1; v_2; \cdot) - \bar{u}_{sell}^{AA}(v_1; v_2; \cdot)$ is quasimonotone increasing on $(\underline{v}; \bar{v})$ for any $v_2 \in (\underline{v}; \bar{v})$ and $\cdot \in [0; 1]$.

Proof $\frac{d(\bar{u}_{sell}^{TO;1}(t) - \bar{u}_{sell}^{AA}(t))}{dv_1} = \frac{d(u_{sell}^{TO}[t](r) - u_{sell}^{AA}[t](r))}{dr} \Big|_{r=v_1} + \frac{\partial b[t](r)}{\partial v} \Big|_{r=v_1} - \frac{\partial u_{sell}(b[t](v_1); v_1; v_1)}{\partial v}$
 $\frac{\partial b[t](0)}{\partial v} - \frac{\partial u_{sell}(b[t](0); 0; v_1)}{\partial v}$. From the way we have proved Lemma 3.6, we obtain that the first term is strictly positive at any point where $\bar{u}_{sell}^{TO;1}(t) = \bar{u}_{sell}^{AA}(t)$. From Lemma F.2, the second term is always positive since we also have $\frac{\partial u_{sell}(\cdot; v_1; v_1)}{\partial v} \geq 0$ for any \cdot . From Lemma F.2, we are done with the third term if we show that $\frac{\partial u_{sell}(b[t](0); 0; v_1)}{\partial v} \geq 0$ once $\bar{u}_{sell}^{TO;1}(t) = \bar{u}_{sell}^{AA}(t) = v_1$ which comes from Lemma I.1 since we have also $b[t](0) > 0$ when $v_1 > \underline{v}$. Q.E.D.

If $v_2 > \underline{v}$, $\lim_{v_1 \downarrow \underline{v}} b[t](0) = +1$, $\lim_{v_1 \downarrow \underline{v}} b[t](v_1) < +1$, $\lim_{v_1 \downarrow v_2} b[t](0) = \frac{FC}{G(v_2)} b$ and $\lim_{v_1 \downarrow v_2} b[t](v_1) = +1$. We obtain then that $\bar{u}_{sell}^{TO;1}(v_1; v_2; \cdot) - \bar{u}_{sell}^{AA}(v_1; v_2; \cdot)$ goes to 1 as v_1 goes to \underline{v} and goes to $+1$ as v_1 goes to v_2 . As a corollary of lemma F.3, there is a unique solution $v_1 \in (\underline{v}; v_2)$ to the equation

$$\bar{u}_{sell}^{TO;1}(v_1; v_2; \cdot) = \bar{u}_{sell}^{AA}(v_1; v_2; \cdot) \quad (20)$$

for any $v_2 > \underline{v}$ and any \cdot . Let $F_1(v_1; v_2; \cdot)$ denote this solution and let $F_1(\underline{v}; \underline{v}; \cdot) = \underline{v}$. Note that $F_1(\cdot)$ (where $F_1(t)$ depends solely on v_2 and \cdot) is a continuous function on T since the expected profit functions \bar{u}_{sell}^{AA} and $\bar{u}_{sell}^{TO;1}$ are continuous.

For $t \geq T$, we let $\bar{u}_{sell}^{TO;2}(t) := u_{sell}^{TO}[t](v_2)$ and $\bar{u}_{sell}^{SR}(t) := u_{sell}^{SR}[t](v_2)$.

Lemma F.4 The function $v_2 \mapsto \bar{u}_{sell}^{SR}(v_1; v_2; \cdot) - \bar{u}_{sell}^{TO;2}(v_1; v_2; \cdot)$ is quasimonotone increasing on $(v_1; \bar{v})$ for any $v_1 \in [\underline{v}; \bar{v})$ and $\cdot \in [0; 1]$.

Proof $\frac{d(\bar{u}_{sell}^{SR}(t) - \bar{u}_{sell}^{TO;2}(t))}{dv_2} = \frac{d(u_{sell}^{SR}[t](r) - u_{sell}^{TO}[t](r))}{dr} \Big|_{r=v_2} + \frac{\partial b[t](secret)}{\partial v} - \frac{\partial u_{sell}(b[t](secret); r^M(v_2); v_2)}{\partial v}$
 $\frac{\partial b[t](r)}{\partial v} \Big|_{r=v_2} - \frac{\partial u_{sell}(b[t](v_2); v_2; v_2)}{\partial v}$. From the way we have proved Lemma 3.6, we obtain that the first term is strictly positive at any point where $\bar{u}_{sell}^{SR}(t) = \bar{u}_{sell}^{TO;2}(t)$. From Lemma F.2, the second and the third terms are always positive since we also have $\frac{\partial u_{sell}(\cdot; r^M(v_2); v_2)}{\partial v} \geq 0$ and $\frac{\partial u_{sell}(\cdot; v_2; v_2)}{\partial v} \geq 0$ for any \cdot . Q.E.D.

If $\epsilon \in [0; 1]$ and $v_1 < \bar{v}$, $\lim_{v_2 \downarrow v_1} b[t](v_2) = +1$, $\lim_{v_2 \downarrow v_1} b[t](secret) = \frac{(1-\epsilon)PC}{1-G(v_1)} b$, $\lim_{v_2 \downarrow \bar{v}} b[t](v_2) = \frac{(\frac{FR}{G} + \frac{PC}{G})b}{G(v_2)G(v_1)}$ and $\lim_{v_2 \downarrow \bar{v}} b[t](secret) = +1$. We obtain then that $\bar{u}_{sell}^{SR}(v_1; v_2; \cdot) - \bar{u}_{sell}^{TO;2}(v_1; v_2; \cdot)$ goes to 1 as v_2 goes to v_1 and goes to $+1$ once v_2 goes

to \bar{v} . As a corollary of lemma F.4, there is a unique solution $v_2 \in (v_1; \bar{v})$ to the equation

$$u_{\text{sell}}^{\text{SR}}(v_1; v_2; \cdot) = u_{\text{sell}}^{\text{TO}; 2}(v_1; v_2; \cdot) \quad (21)$$

for any $v_1 < \bar{v}$ and \cdot . Let $F_2(v_1; v_2; \cdot)$ denote this solution. To complete the definition, we let $F_2(\bar{v}; \bar{v}; \cdot) := \bar{v}$ and $F_2(v_1; v_2; 1) := \bar{v}$ for any $v_1; v_2$. Note that the function $F_2(\cdot)$ is continuous on T .

Take $v_1 < \bar{v}$, $v_2 \in (v_1; \bar{v})$. We note first that $\frac{b[t](\text{secret})}{d} < 0$ while we can show in the same way as in Lemma F.2 that $\frac{a[b[v_1; v_2; \cdot]](r)}{a} > 0$. There is thus a unique solution to the program (note that $b[t]$ does not depend on \cdot):

$$e = 0 \quad \text{(resp: } > \text{)} \quad a_b(b[v_1; v_2; e](\text{secret}); b[v_1; v_2; e]) \quad \text{(resp: } = \text{)} \quad u_b(b[v_1; v_2; e](v_2; v_2); \cdot) \quad (22)$$

Let then $F_3(v_1; v_2; \cdot) := f \circ g$. If $v_2 = v_1$, let $F_3(v_1; v_2; \cdot) = f \circ g$. If $v_1 < v_2 = \bar{v}$, let $F_3(v_1; v_2; \cdot) = f \circ g$. Finally, we also let $F_3(\bar{v}; \bar{v}; \cdot) = [0; 1]$. We can easily check that the correspondence $F_3(\cdot; \cdot; \cdot)$ is upper hemicontinuous on T .

We now have all of the elements required to apply a fixed point Theorem. Consider the correspondence F such that $F(v_1; v_2; \cdot) = (F_1(v_1; v_2; \cdot); F_2(v_1; v_2; \cdot); F_3(v_1; v_2; \cdot))$. The correspondence F is an upper hemicontinuous function from T , which is a convex compact subset of the Euclidian space, to itself. From the Kakutani fixed point Theorem the correspondence F has a fixed point. Note first that for any fixed point $t := (v; v; \cdot)$, we must have $\underline{v} < v < \bar{v}$ and $\cdot < 1$. We conclude the proof by noting that the equations (20-22) guarantee that any fixed point $t := (v; v; \cdot)$ of F satisfies (8) and (9).

G Proof of Proposition 3.9

From Lemma 3.6, we obtain that $u_{\text{sell}}^{\text{AA}}(v) = u_{\text{sell}}^{\text{TO}}(v)$ for any $v \in (0; \bar{v}]$. From Lemma E.1, this implies that $\frac{\text{AA}}{v} > \frac{\text{TO}}{v}(r)$ for any $r \in (0; \bar{v}]$. Since (\cdot) is decreasing, we obtain that $\frac{\text{AA}}{v} > \frac{\text{TO}}{v}(r)$ for any $r > 0$. Suppose that $\lim_{r \downarrow 0^+} \frac{\text{TO}}{v}(r) = \frac{\text{AA}}{v}$. Note from Proposition 3.7 that $\frac{\text{TO}}{v}(r) = \frac{\text{TO}}{v}(r; V_{\text{FR}})$ for any $r > 0$ such that we have also $\lim_{r \downarrow 0^+} \frac{\text{TO}}{v}(r) = \frac{\text{TO}}{v}(0; V_{\text{FR}})$. By continuity, we have then $\lim_{r \downarrow 0^+} (u_{\text{sell}}^{\text{AA}}(r) - u_{\text{sell}}^{\text{TO}}(r)) = 0$. In the same way as in Lemma 3.6 but now at the boundary $r = 0$, this implies that $u_{\text{sell}}^{\text{AA}}(r) < u_{\text{sell}}^{\text{TO}}(r)$ for any $r > 0$ which raises a contradiction. On the whole we obtain that $\lim_{r \downarrow 0^+} \frac{\text{TO}}{v}(r) < \frac{\text{AA}}{v}$.

From Lemma 3.6, we obtain that $u_{\text{sell}}^{\text{TO}}(v) = u_{\text{sell}}^{\text{SR}}(v)$ for any $v \in (0; \bar{v}]$. From Lemma E.1, this implies that $\frac{\text{TO}}{v} > \frac{\text{SR}}{v} > 0$ for any $v \in (0; \bar{v}]$. From (18), we have $\frac{d}{dr} \frac{\text{TO}}{v}(r)$

$$\int_0^1 \frac{(1 - F(v))}{(1 - F(x))^2} dx < 0 \text{ for any}$$