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Asset Markets with Heterogeneous Information

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Abstract

I define a notion of competitive equilibrium for asset markets where assets are heterogeneous and traders have heterogeneous information about them. Markets are defined by a price and a procedure for clearing trades. Any asset can in principle be traded in any market but traders can use their information to impose acceptance rules which specify which goods they are willing to trade in each market. I then apply this notion to a model of distressed sales under asymmetric information and examine whether it can account for fire sales: sharp drops in prices when distressed agents need to sell assets. Standard models of asymmetric information with informed sellers, heterogenous assets and identical uninformed buyers predict the opposite phenomenon, as more distressed sellers on average sell less-adversely-selected pools of assets. With heterogeneity among buyers in their ability to distinguish assets of different qualities, the possibility of fire sales depends on the joint distribution of wealth and ability.

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1 Introduction

[illegible] (1970), [illegible]

[illegible] (1980) [illegible] (1987) [illegible]

[illegible] (1976, [illegible] 1977, [illegible] 1977, [illegible] 1981, [illegible] 2009). [illegible] (1992, 1996, [illegible] 2002, [illegible] 2006) [illegible] 2010, [illegible] 2012, [illegible] 2011) [illegible]

[illegible]

[illegible]

[illegible]

[illegible] In [illegible] ("[illegible]"), [illegible] ("[illegible]") [illegible] In [illegible] i.e. [illegible]

F[illegible] A I [illegible] L[illegible] O[illegible] F[illegible]

O[illegible] W[illegible] fire[illegible] (2010) [illegible] In [illegible] fire[illegible]

2 The Economy

Dates

[illegible] $t = 1$ [illegible] $t = 2$. C[illegible] t [illegible] c_t.

[illegible] It [illegible] "[illegible]" [illegible] "[illegible]" [illegible] "[illegible]" [illegible]

[illegible]

[illegible]

Agents and preferences

A [illegible] B [illegible] $b \in B$. P [illegible]

$$u(c_1; c_2) = c_1 + c_2$$

[illegible]

[illegible] $v \in V$. P [illegible]

$$u(c_1; c_2; v) = c_1 + (v) c_2$$

[illegible] (v) 1. [illegible] V [illegible] $(v) < 1$ [illegible]

[illegible]

L [illegible]

[illegible]

[illegible] F [illegible]

Endowments and Assets

B [illegible] b [illegible] $w(b)$ [illegible] $t = 1$.

[illegible] $i \in I$. A [illegible] i [illegible] $q(i)$ [illegible] $t = 2$.

I [illegible] $q(i)$ [illegible] i. A [illegible] $q(i)$ [illegible]

v [illegible] $e(i; v)$ [illegible] i.

Information

E [illegible] i ([illegible] $q(i)$) [illegible] B [illegible]

[illegible] i [illegible] b [illegible] $x(i; b)$ [illegible]

i. If $x(i; b) \neq x(i'; b)$ [illegible] $q(i) \neq q(i')$ [illegible] b [illegible]

[illegible] O [illegible]

[illegible]

3 Equilibrium

Markets

Seller's problem

[illegible] v [illegible]

$$\max_{c_1;c_2;s} u(c_1;c_2;v) \tag{1}$$

$$s{:}t{:}$$

$$c_1 = \sum_{i \in I} \left[\sum_{M} p(m) \quad (i;m)\, ds(m;i) \right] \tag{2}$$

$$c_2 = \sum_{i \in I} q(i) \left[e(i;v) \qquad {}_{M} \quad (i;m)\, ds(m;i) \right] \tag{3}$$

$$s(m;i) \quad e(i;v) \quad 8i;m \tag{4}$$

$$(i;m)\, ds(m;i) \quad e(i;v) \quad 8i \tag{5}$$

C[illegible] (2) [illegible] $t = 1$ [illegible] F[illegible] i, [illegible] $s(m;i)$ [illegible] m, [illegible] $(i;m)$, [illegible] $p(m)$. A[illegible] (2). C[illegible] (3) [illegible] $t = 2$ [illegible] F[illegible] i [illegible] $e(i;v)$ [illegible] $q(i)$ [illegible] C[illegible] (4) [illegible] $(i;m) < 1$. It [illegible] 100 [illegible] 30 [illegible] 30% [illegible] C[illegible] (5) [illegible] N[illegible] not [illegible]

$$s(M;i) \quad e(i;v) \quad 8i \tag{6}$$

If I [illegible] (6) [illegible] (5), [illegible] G[illegible] (1992, 1996), G[illegible] (2010), G[illegible] (2012) [illegible] C[illegible] (2011) [illegible] (6).

[illegible] $s(m;i)$ [illegible] $i;m$ [illegible] $(i;m) = 0$ [illegible] F[illegible] (1) [illegible]

[illegible] I [illegible]

[illegible] $(i;m)$, [illegible]

[illegible]

Definition 1. A [illegible](1) is [illegible] robust [illegible]

$^n(i;m) > 0$ [illegible] $c_1^n; c_2^n; s^n$ [illegible]

1. $c_1^n; c_2^n; s^n$ [illegible]

$$\max_{c_1;c_2;S^i} u(c_1;c_2;v)$$

$$s{:}t{:}$$

$$c_1 = \sum_{i2I} \left[p(m) \quad {}^n(i;m)\, ds^n(m;i) \right]$$
$$\qquad\qquad\qquad M$$

$$c_2 = \sum_{i2I} \left[q(i)\, e(i;v) \quad {}^n(i;m)\, ds^n(m;i) \right]$$
$$\qquad\qquad\qquad M$$

$$s^n(m;i) \quad e(i;v) \quad 8i;m$$

$$^n(i;m)\, ds^n(m;i) \quad e(i;v) \quad 8i$$

2. $^n(i;m)\ ! \quad (i;m)$

3. $c_1^n\ !\ c_1$, $c_2^n\ !\ c_2$ [illegible] $s^n(M_0;i)\ !\ s(M_0;i)$ [illegible] $M_0 \quad M$.

Buyer's orders

[illegible]

[illegible] A [illegible]

[illegible] "I [illegible] 5 [illegible] i [illegible] $i\ 2\ f\ 1;2;4g$". I

[illegible]

Definition 2. An acceptance rule [illegible] $: I\ !\ f\ 0;1g$.

$(i) = 1$ [illegible] i [illegible] $(i) = 0$ [illegible]

B[illegible]

[illegible]

[illegible] i [illegible] $x(i;b)$. F[illegible]

[illegible]

Definition 3. An [illegible] is feasible for buyer b [illegible]

[illegible] b's [illegible] i.e. [illegible]

$$(i) = (i^0) \; [illegible] \; x(i; b) = x(i^0; b)$$

In [illegible]

[illegible]

[illegible]

I [illegible]

[illegible] b [illegible] $_b$.

Allocation functions

B[illegible] m [illegible]

[illegible] I [illegible], I [illegible]

[illegible]

Definition 4. An allocation function A [illegible] A (;) [illegible] I [illegible]

[illegible] 2 .

A [illegible]

[illegible] E [illegible]

[illegible] I [illegible] m [illegible]

A (; ; m)

A [illegible] m [illegible]

[illegible] A (; ; m) [illegible] A (i; ; m) [illegible] i. [illegible]

m [illegible] A (I; ; m) < 1, [illegible] in

[illegible] m [illegible]

[illegible]

Buyer's problem

B[illegible]

[illegible] F[illegible]

d [illegible]

i	$q(i)$	(i) [illegible] b_1	(i) [illegible] b_2	[illegible]
1	0	0	0	1:5
2	0	1	0	1:5
3	1	1	1	1:5
		$d_{b_1} = 1$	$d_{b_2} = 1$	

[illegible] 1: E[illegible]

B[illegible] b [illegible]

$$\max_{c_1;c_2;d} u(c_1; c_2) \tag{7}$$

$$s{:}t{:}$$

$$c_1 = w(b) \quad p(m)\, A(I; \; ; m\;)\, dd(\; ; m\;) \tag{8}$$

$$c_2 = \quad \sum_{i \in I}^{M} \; q(i)\, A(i; \; ; m\;) \quad dd(\; ; m\;) \; ! \quad M \tag{9}$$

$$d(\;_b; M) = d(\; ; M) \tag{10}$$

$$c_1 \quad 0 \quad c_2 \quad 0 \tag{11}$$

C[illegible] (8) [illegible] $t = 1$ [illegible] I[illegible] m, [illegible] $A(I; \; ; m\;)$ [illegible] $p(m)$ [illegible] H[illegible] C[illegible] (9) [illegible] $t = 2$ [illegible] C[illegible] (10) [illegible]

Clearing algorithms

E[illegible] $p(m)$ [illegible] A [illegible] a[illegible]

[illegible] 1. [illegible] A[illegible] 1 [illegible] 2 [illegible] a [illegible] $t = 2$ [illegible] 3 [illegible] m, [illegible] b_1 [illegible] b_2. [illegible] b_1 [illegible] 2 [illegible] 3 [illegible]

[illegible] 1. [illegible] b_2 [illegible]

[illegible] 1 [illegible] 2 [illegible] 3 [illegible]

[illegible] 3. E[illegible] 1.5

[illegible]

O[illegible] "[illegible] b_1 [illegible]

[illegible] b_2 [illegible]

[illegible]

$$
A(i;\ ) = \begin{cases} \text{if } = \{0;1;1\} & \text{if } = \{0;0;1\} & \\ 0 & 0 & \text{if } i = 1 \\ 0.5 & 0 & \text{if } i = 2 \\ 0.5 & 1 & \text{if } i = 3 \end{cases}; \qquad (i) = \begin{cases} 0 & \text{if } i = 1 \\ \frac{1}{3} & \text{if } i = 2 \\ 1 & \text{if } i = 3 \end{cases} \tag{12}
$$

[illegible] b_1 [illegible] 1. [illegible]

[illegible] 2 [illegible] 3 [illegible] 0.5 [illegible]

A[illegible] b_2 [illegible] He[illegible] 3 [illegible]

[illegible] F[illegible] 3 [illegible] $\frac{1}{3}$ [illegible]

[illegible] 2 [illegible] 1 [illegible]

A[illegible] b_2 [illegible]

[illegible] b_1 [illegible]

$$
A(i;\ ) = \begin{cases} \text{if } = \{0;1;1\} & \text{if } = \{0;0;1\} & \\ 0 & 0 & \text{if } i = 1 \\ 0.75 & 0 & \text{if } i = 2 \\ 0.25 & 1 & \text{if } i = 3 \end{cases}; \qquad (i) = \begin{cases} 0 & \text{if } i = 1 \\ \frac{1}{2} & \text{if } i = 2 \\ \frac{5}{6} & \text{if } i = 3 \end{cases} \tag{13}
$$

A[illegible] b_2 [illegible] 3, [illegible] 0.5 [illegible] 1.5 [illegible]

[illegible] 2. A [illegible] b_1 [illegible] 0.75 [illegible]

[illegible] 2 [illegible] 0.25 [illegible] 3.

C[illegible]

[illegible]

[illegible]

[illegible] I [illegible]

[illegible]

[illegible]

[illegible]

[illegible] E[illegible] o [illegible] a [illegible] T [illegible] $(\cdot)$ [illegible] a [illegible] m [illegible] g. [illegible] m

Definition 5. A clearing algorithm [illegible]

1. a [illegible] K
2. a [illegible] $\omega(\cdot;\cdot)$ [illegible] $\{1;\ldots;K\}$ [illegible] 2 .

In [illegible] 1, [illegible] $K = 3$ [illegible]

$$\omega(k;\ ) = \begin{cases} \text{[illegible]} \ = \{0;1;1\} \ \text{[illegible]} \ = \{0;0;1\} & \text{[illegible]} \\ 1 \quad 0 \quad 0 & \text{[illegible]}\ k = 1 \\ 0 \quad 1 \quad 0 & \text{[illegible]}\ k = 2 \\ 0 \quad 0 \quad 1 & \text{[illegible]}\ k = 3 \end{cases} \tag{14}$$

[illegible] $_1 = \{0;1;1\}$ [illegible] $_2 = \{0;0;1\}$ [illegible] ([illegible]) [illegible]

[illegible] $K = 3$ [illegible]

$$\omega(k;\ ) = \begin{cases} \text{[illegible]} \ = \{0;1;1\} \ \text{[illegible]} \ = \{0;0;1\} & \text{[illegible]} \\ 0 \quad 1 \quad 0 & \text{[illegible]}\ k = 1 \\ 1 \quad 0 \quad 0 & \text{[illegible]}\ k = 2 \\ 0 \quad 0 \quad 1 & \text{[illegible]}\ k = 3 \end{cases} \tag{15}$$

A [illegible] m [illegible] D [illegible] a [illegible] D [illegible] . F [illegible] $_0$ [illegible] $D(\ _0)$ [illegible] $2\ \ _0$. [illegible] $S : I \to R^+$. $S(i)$ [illegible] i-[illegible]

G [illegible] D, [illegible] S [illegible] $\{K;\omega\}$, [illegible] A [illegible] w

1. [illegible] k [illegible] $S^k(i)$. [illegible] kth [illegible]

$$A^k(i;\alpha) = \begin{cases} \frac{\alpha(i)S^k(i)}{\sum_{i\in I}\alpha(i)S^k(i)di} & \text{if } \sum_{i\in I}\alpha(i)S^k(i)di > 0 \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

[illegible] [1]

---

[1] Allocating this amount might not be feasible: For instance, suppose that in the example from Table 1 buyer b_2 demanded 2 units instead of 1 and the clearing algorithm prescribed the measures ω given by (14). In round 2, the algorithm would attempt to assign 2 units of asset 2 to buyer b_2 when only 1 remains.

To complete the description of the algorithm, it is necessary to describe what happens if this arises. Let

$$r(k;i) \equiv \frac{S^k(i)}{\sum_{\alpha\in\Omega} A^k(i;\alpha)\,\omega(k;\alpha)\,D(\alpha)}$$

$$r(k) \equiv \min_i \min\{r(k;i);1\}$$

$r(k;i)$ is the ratio of the residual supply of asset i as of round k to the total demand that the algorithm attempts to allocate during round k. The amount the algorithm actually allocates per unit of demand is given by:

$$A^k(i;\alpha)\,r(k)$$

This means that if in some round there is insufficient supply to meet demand for some asset, the allocation received by all acceptance rules in that round is reduced in the same proportion. This could leave some acceptable assets un-allocated. Therefore if $r(k) < 1$ and there are any assets remaining such that $\alpha(i) = 1$ for an acceptance rule α such that round k is in the support of $\omega(k;\alpha)$, then round k is repeated until this is no longer the case.

For example, suppose the assets and acceptance rules are as in Table 1, buyer b_1 demands 2 units and buyer b_2 demands 8 units. Suppose further that the allocation algorithm is $K = 3$ and the following measures:

$$\omega(k;\alpha) = \begin{cases} \text{if } \alpha = \{0;1;1\} & \text{if } \alpha = \{0;0;1\} & \text{any other} & \\ 0 & 0.25 & 0 & \text{if } k = 1 \\ 1 & 0.75 & 0 & \text{if } k = 2 \\ 0 & 0 & 1 & \text{if } k = 3 \end{cases}$$

In the first round, the algorithm attempts to allocate the following amounts per unit demanded

$$A^k(i;\alpha) = \begin{cases} \text{if } \alpha = \{0;1;1\} & \text{if } \alpha = \{0;0;1\} & \\ 0 & 0 & \text{if } i = 1 \\ 0.5 & 0 & \text{if } i = 2 \\ 0.5 & 1 & \text{if } i = 3 \end{cases}$$

2. [illegible] k is

$$A(i;\ ;k\ ) = \sum_{j=1}^{k} A^j(i;\ )\,!\,(j;\ ) \qquad (17)$$

[illegible] is

$$A(i;\ ) = \sum_{j=1}^{K} A^j(i;\ )\,!\,(j;\ ) \qquad (18)$$

3. It [illegible]

$$\begin{aligned} S^1(i) &= S(i) \\ S^k(i) &= S(i) \quad \sum_{2} A(i;\ ;k \quad 1)\,D(\ ) \quad \text{for} \quad k \quad 2 \end{aligned} \qquad (19)$$

However, this results in

$$r(k=1; i=3) \quad \frac{1{:}5}{0{:}5 \quad 1 \quad 2+1 \quad 0{:}25 \quad 8} = 0{:}5$$

and therefore pro-rated demand is

$$A^k(i;\ )\,r(k) = \begin{cases} \text{if} = f\,0;1;1g & \text{if} = f\,0;0;1g \\ 0 & 0 & \text{if } i = 1 \\ 0{:}25 & 0 & \text{if } i = 2 \\ 0{:}25 & 0{:}5 & \text{if } i = 3 \end{cases}$$

This allocates a total of 0.5 units of each of assets 2 and 3 to buyer b_1 and 1 unit of asset 3 to buyer 2. This exahusts the supply of asset 3 but not of asset 2, which is acceptable to buyer b_1. Therefore the first round is repeated on the remaining supply of $f\,1{:}5;1;0g$ and demands of 1 unit for buyer b_1 and 1 unit ($1 = 8 \quad 0{:}25 \quad 1$) for buyer b_2, resulting in

$$A^k(i;\ ) = \begin{cases} \text{if} = f\,0;1;1g & \text{if} = f\,0;0;1g \\ 0 & 0 & \text{if } i = 1 \\ 1 & 0 & \text{if } i = 2 \\ 0 & 0 & \text{if } i = 3 \end{cases}$$

which requires no further pro-rating. Since the supply of acceptable assets has been exhausted, buyer b_2 receives nothing further in the second round. Overall, the allocation functions that result from this combination of supply, demand and clearing algorithm are:

$$A(i;\ ) = \begin{cases} \text{if} = f\,0;1;1g & \text{if} = f\,0;0;1g \\ 0 & 0 & \text{if } i = 1 \\ 0{:}75 & 0 & \text{if } i = 2 \\ 0{:}25 & 0{:}125 & \text{if } i = 3 \end{cases}$$

4. [illegible] F [illegible] i, [illegible] $_i$ [illegible] 2 : (i) = 1 g [illegible] i. [illegible]

$$(i) = \begin{cases} \displaystyle\sum \frac{A(i;\ )D(\ )}{S(i)} & \text{if } S(i) > 0 \\ 1 & \text{if } S(i) = 0 \text{ and } D(\ _i) > 0 \\ 0 & \text{if } S(i) = 0 \text{ and } D(\ _i) = 0 \end{cases} \qquad (20)$$

[illegible] (20) [illegible] If [illegible] If [illegible] i [illegible] (i) = 1 . [illegible] If [illegible] i [illegible]

[illegible] M [illegible]

Definition of equilibrium

A [illegible]

1. C [illegible] $c_{1;v}; c_{2;v}; s_v$ [illegible]

2. [illegible] $S(\ ;m)$

3. C [illegible] $c_{1;b}; c_{2;b}; d_b$ [illegible]

4. D [illegible] $D(\ ;m)$

5. A [illegible] $A(\ ;\ ;m)$ [illegible]

6. A [illegible] $(\ ;m)$ [illegible]

[illegible]

1. $c_{1;v}; c_{2;v}; s_v$ [illegible] (1) [illegible] v, [illegible] $(\ ;m)$ [illegible]

2. $c_{1;b}; c_{2;b}; d_b$ [illegible] (17) [illegible] b, [illegible] $A(\ ;\ ; m)$ [illegible]

3. F[illegible] m:

(a) [illegible]

$$S(i; m) = \sum_s s(m; i) \quad \forall i$$

(b) [illegible]

$$D(\ _0; m) = \sum_b d_b(\ _0; m) \quad \forall \ _0$$

[illegible]

(c) $A(\ ;\ ; m)$, $(\ ; m)$, $S(\ ; m)$ [illegible] $D(\ ; m)$ [illegible] (18) [illegible] (20)

4 Two examples

[illegible] $I = [0; 1]$, [illegible] $v \in [0; 1]$ [illegible] $b \in [0; 1]$. In [illegible]

$$(v) = \begin{cases} 1 & \text{if } v \\ 0 & \text{if } v < \end{cases}$$

[illegible] $t = 1$ ([illegible] A [illegible] $q(i) = I(i > \ )$ [illegible] $\in (0; 1)$. [illegible] 1 [illegible] ([illegible] $i >$) [illegible] 1 [illegible] $t = 2$ [illegible] ([illegible] i) [illegible] E [illegible] i.e. $e(i; v) = 1$ [illegible] $w(b)$ [illegible]

[illegible] Fig (1).

In [illegible] b [illegible] $x(i; b) = I(i > b\ )$. [illegible] $x(i; b) = 1$. [illegible] $b \frac{i}{}$ [illegible] $x(i; b) = 1$, [illegible] $b > \frac{i}{}$ [illegible] $x(i; b) = 0$ [illegible] b [illegible] b [illegible]

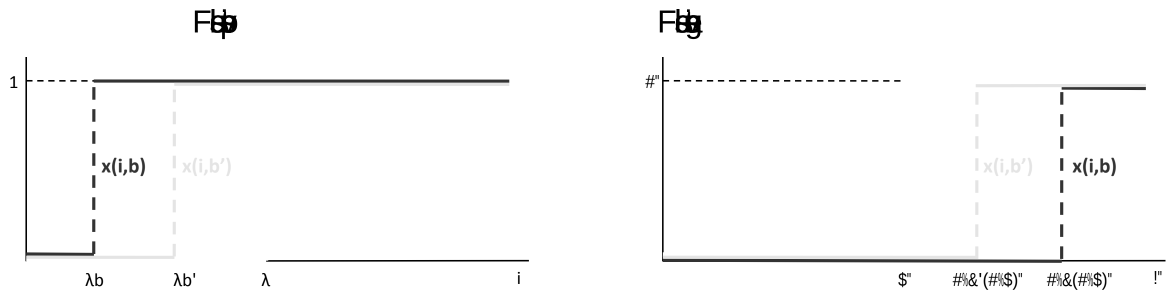


Fig 1: [illegible]

F[illegible] b [illegible] i [illegible] $b^0 > b$.

C[illegible] b [illegible] $x(i;b) = I(i > 1 \quad b(1 \quad))$. [illegible] $x(i;b) = 0$ [illegible] $b \quad \frac{1 \quad i}{1}$ [illegible] $x(i;b) = 1$ [illegible] A[illegible] b [illegible]

I[illegible]

Definition 6.

1. [illegible] less-restrictive- rst [illegible] $[0;1]$ [illegible]

   $$!(K_0) = \begin{cases} 1 & \text{[illegible]} \; g \in K_0 \\ 0 & \text{[illegible]} \end{cases}$$

   [illegible] $(i) = I(i \quad g)$ [illegible]

   $$!(K_0) = \begin{cases} 1 & \text{[illegible]} \; 1 \in K_0 \\ 0 & \text{[illegible]} \end{cases}$$

   [illegible]

2. [illegible] more-restrictive- rst [illegible] $[0;1]$ [illegible]

   $$!(K_0) = \begin{cases} 1 & \text{[illegible]} \; 1 \quad g \in K_0 \\ 0 & \text{[illegible]} \end{cases}$$

[illegible] $(i) = I(i \quad g)$ [illegible]

$$\omega(K_0) = \begin{cases} 1 & \text{if } 1 \in K_0 \\ 0 & \text{otherwise} \end{cases}$$

[illegible]

3. [illegible] non-restrictive-first-then-more-restrictive-first [illegible] $[0; 1]$ [illegible]
   [illegible]

$$\omega(K_0) = \begin{cases} 1 & \text{if } 0 \in K_0 \\ 0 & \text{otherwise} \end{cases}$$

[illegible] $(i) = 1 \ \forall i;$ [illegible]

$$\omega(K_0) = \begin{cases} 1 & \text{if } 1 \quad g \in K_0 \\ 0 & \text{otherwise} \end{cases}$$

[illegible] $(i) = I(i \quad g)$ if $g > 0$, [illegible]

$$\omega(K_0) = \begin{cases} 1 & \text{if } 1 \in K_0 \\ 0 & \text{otherwise} \end{cases}$$

[illegible]

[illegible] C [illegible] [illegible]

False Positives Case

[illegible] A [illegible] m [illegible] p [illegible] D [illegible] $v <$) [illegible] i is [illegible] $i >$ [illegible] 1 [illegible] i . [illegible] b

[illegible] $b < b$ [illegible] $b > b$ [illegible]

[illegible] $x(i;b) = 1$, [illegible]

[illegible] b [illegible] $[b\ ;\ 1]$, [illegible] b [illegible]

[illegible] B[illegible] $b < b$ [illegible]

[illegible] b [illegible] p [illegible]

$$\int_{b}^{1} \frac{1}{(1 \quad b) + \quad (1 \quad)} \frac{w(b)}{p} db = 1 \tag{21}$$

$$p \ = \frac{(1 \quad)}{(1 \quad b\ ) + \quad (1 \quad)} \tag{22}$$

F[illegible]

1. [illegible]

$$s_v(m;i) = \begin{cases} 1 & \text{[illegible]} \begin{cases} i \\ p(m) \quad p \ \text{[illegible]} \quad v < \\ \text{[illegible]} \\ p(m) \quad 1 \end{cases} \\ 0 & \text{[illegible]} \end{cases} \tag{23}$$

[illegible]

$$c_{1;v} = \begin{cases} p \ \int_0^1 (i;m\ )\, di & \text{[illegible]} \ v < \\ p \ \int_0 (i;m\ )\, di & \text{[illegible]} \ v \end{cases}$$

$$c_{2;v} = \begin{cases} 0 & \text{[illegible]} \ v < \\ 1 & \text{[illegible]} \ v \end{cases} \tag{24}$$

2. [illegible]

$$S(i;m) = \begin{cases} 1 & \text{[illegible]} \ i \\ 1 & \text{[illegible]} \ i > \ , \ p(m) \quad 1 \\ & \text{[illegible]} \ i > \ , \ p(m) \ 2 \ [p\ ;1) \\ 0 & \text{[illegible]} \ i > \ , \ p(m) < p \end{cases} \tag{25}$$

3. Ddl

$$d_b(; m) = \begin{cases} \frac{w(b)}{p} & f \; m = m \; , \quad (i) = x(i;b) \; d \quad b > b \\ 0 & bw \end{cases} \quad (26)$$

ggpm

$$c_1(b) = \begin{cases} w(b) & f \; b < b \\ 0 & f \; b \quad b \end{cases}$$

$$c_2(b) = \begin{cases} 0 & f \; b < b \\ \frac{w(b)}{p} \frac{(1 \quad)}{(1 \quad b) + \quad (1 \quad)} & f \; b \quad b \end{cases} \quad (27)$$

4. Dh

$$D(\;\; _0; m) = \begin{cases} \frac{1}{p} \int_{f \, b:[\; (i) = x(i;b)] 2 \; _0 g} w(b) \, db & f \; m = m \\ 0 & bw \end{cases} \quad (28)$$

5. Ab

$$A(I_0; \; ; m) = \begin{cases} \frac{\int_{i 2 I_0} (i) I(i \quad) di}{\int_{i 2 I} (i) I(i \quad) di} & f \; p(m) < p \; d \int_{i 2 I} (i) I(i \quad) di > 0 \\ 0 & f \; p(m) < p \; d \int_{i 2 I} (i) I(i \quad) di = 0 \\ \frac{\int_{i 2 I_0} (i)[I(i \quad) + \quad I(i > \quad)] di}{\int_{i 2 I} (i)[I(i \quad) + \quad I(i > \quad)] di} & f \; p(m) 2 [p \; ; 1) \; d \int_{i 2 I} (i)[I(i \quad) + \quad I(i > \quad)] di > 0 \\ \frac{\int_{i 2 I_0} (i) di}{\int_{i 2 I} (i) di} & f \; p(m) \quad 1 \; d \int_{i 2 I} (i) di > 0 \\ 0 & f \; p(m) \quad p \; d \int_{i 2 I} (i) di = 0 \end{cases} \quad (29)$$

6. Pyg

$$(i; m) = \begin{cases} 1 & f \; m = m \; d \quad i > \\ \int_b^i \frac{1}{(1 \quad b) + \quad (1 \quad)} \frac{w(b)}{p} db & f \; m = m \; d \quad i \; 2 \; [b \; ; \;] \\ 0 & f \; m = m \; d \quad i < b \\ 0 & f \; m \, 6 = m \end{cases} \quad (30)$$

Proposition 1. Equations (21)-(30) describe an equilibrium.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$(b) = \frac{1}{p}\,\frac{(1 \quad)}{(1 \quad b) + (1 \quad)}$$

[illegible]

[illegible]

is (1) [illegible] b ([illegible] b b) [illegible] $\frac{w(b)}{p} \frac{(1 \quad)}{(1 \quad b) + (1 \quad)}$ [illegible]

[illegible] (21). [illegible]

[illegible] m [illegible]

[illegible] p . F[illegible] i 2 (b ;], [illegible] 1 [illegible] b

[illegible] $\frac{w(b)}{p} \frac{1}{(1 \quad b) + (1 \quad)}$ [illegible] b 2 b ; $\bar{i}$; [illegible]

[illegible] i. [illegible] (30). N[illegible]

[illegible] L[illegible] [illegible]

[illegible]

[illegible] b [illegible]

Proposition 2. In any equilibrium, the price and allocations are those of the equilibrium described by equations (21)-(30).

False Negatives Case

In[illegible]

$$s_v(m; i) = \begin{cases} 1 & \text{if } \begin{cases} p(m) \quad p(i) \text{ and } v < i \\ \text{or} \\ p(m) \quad 1 \end{cases} \\ 0 & \text{otherwise} \end{cases} \tag{31}$$

[illegible] (23), [illegible]

[illegible] ([illegible])) [illegible]

[illegible] $p(i)$ [illegible]

As a first [illegible]

[illegible] F[illegible] p, [illegible]

$$E(i; p) \qquad p(j)\, dj \tag{32}$$

$E(i; p)$ [illegible] $t = 1$ [illegible] (; i) [illegible]

[illegible] $p(i)$. A [illegible]

$$W(i) \quad \int_{\hat{b}(i)}^{1} w(b)\,db \tag{33}$$

$$\hat{b}(i) \quad \frac{1 \quad i}{1}$$

$\hat{b}(i)$ [illegible] $x(i;b) = 1$, i.e. [illegible] i [illegible] $W(i)$ [illegible] $t = 1$ [illegible] i [illegible]. If [illegible]

[illegible]

$$E(i;p) = W(i) \tag{34}$$

[illegible] (32) [illegible] (33), [illegible] (34) [illegible]

$$p^E(i) = \frac{1}{(1 \quad)} w \quad \frac{1 \quad i}{1} \tag{35}$$

In [illegible] p^E [illegible] (i) [illegible] (i) [illegible] 1 [illegible] (i) [illegible]

Monotonicity

Let $< i < i^0$. B [illegible] i [illegible] i^0 [illegible] i^0 [illegible] (i.e. [illegible] $x(i;b) = 1$ [illegible] $x(i^0;b) = 1$ [illegible]). It [illegible] i [illegible] i^0 [illegible] i [illegible] i^0 is [illegible] $p(i)$ [illegible]

D[illegible] p^M [illegible] M [illegible]. A [illegible] p^E. [illegible] M, [illegible] Fig. 2, [illegible] p^E [illegible]

1. $E\ i;p^M \quad W(i)$. [illegible] $p^M(i)$, [illegible]
2. p^M [illegible] $(i;i^0)$ [illegible] $E\ i;p^M < W(i)$. [illegible]

[illegible]

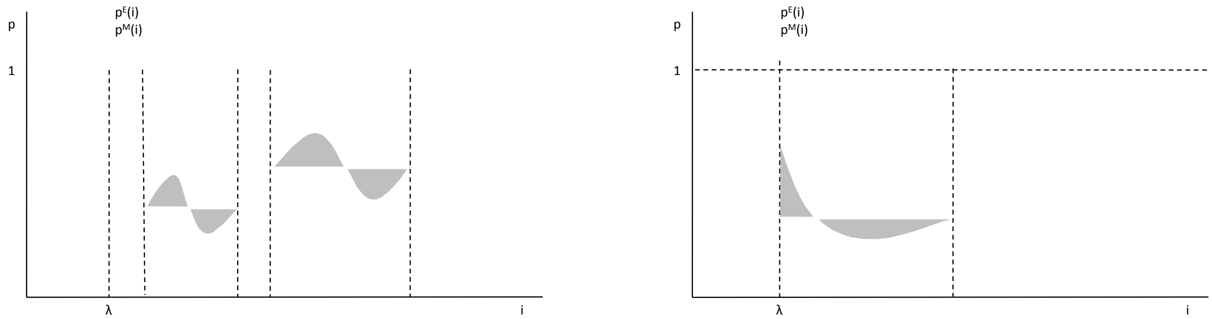


Fig 2: M[illegible] p^E

[illegible]

[illegible]

Upper bound

[illegible] $p^M(i)$ [illegible] 1 [illegible]

D[illegible]

$$p^M(i) \quad \min \; p^M(i); 1$$

[illegible] $p(m) > 1$.

Non-selective buyers

[illegible] i [illegible] m [illegible]

[illegible] A [illegible] $(i) = 1$

(i.e. [illegible] $(i \quad)$

[illegible] ([illegible] i [illegible] $(i; \;))$ [illegible]

[illegible]

$$p^N(i) \quad \frac{(i \quad)}{(i \quad) +} \tag{36}$$

[illegible] $p(m) < p^N(i)$. H[illegible]

[illegible] $t = 1$ [illegible] (a

[illegible] $p^N(i)$ [illegible] D[illegible] i^N [illegible]

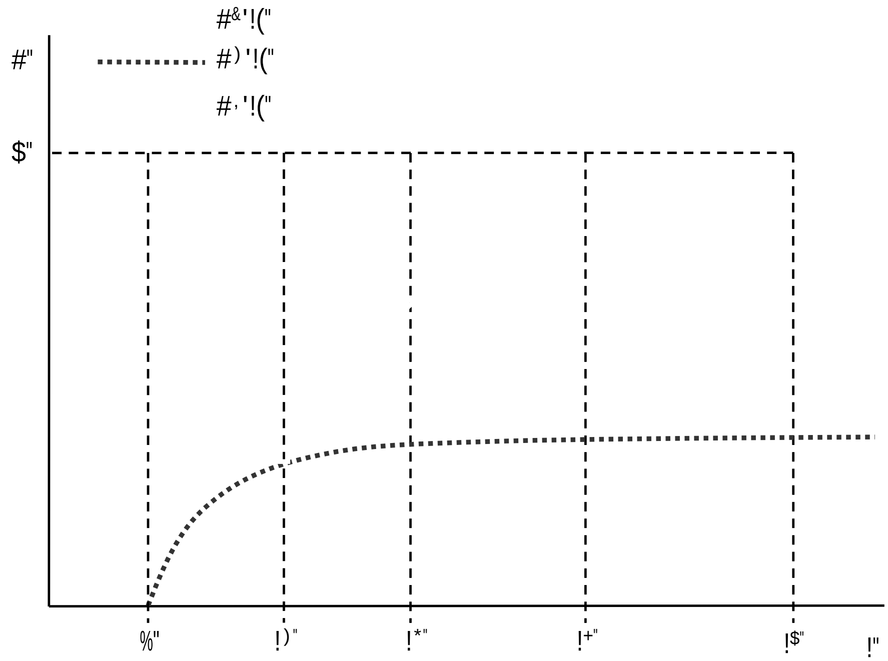


Fig. 3: E

i

$$i^N \quad \max \; i : p^N(i) \quad p^M(i)$$

O p (i)

$$p(i) = \begin{cases} p^N \quad i^N & \text{f} \; i \quad i^N \\ p^M(i) & \end{cases} \tag{37}$$

Fig. 3.

B

1. b $\frac{1 \quad i^N}{1}$.

$; i^N$ in Fig. 3.

d $d = \frac{w(b)}{p(m)}$ $p^N \quad i^N$

(i) =

$I(i > 1 \quad b(1 \quad))$.

2. $b \in \frac{1}{1} \; i^1; \frac{1}{1} \; i^N$, i^1

$$i^1 \quad \left(\begin{array}{ll} \min\{ i : p \; (i) = 1 \} \\ 1 \end{array} \right.$$

1 i.e. $i^N; i^1$ in Fig. 3.

$d = \frac{w(b)}{p(m)}$ $p \; (1 \quad b(1 \quad))$

$(i) = x(i; b)$.

3. $b < \frac{1 \quad i^1}{1}$

1.

(a) $p(m) = 1$ $(i) = I(i > 1 \quad b(1 \quad))$

(b) $p(m) = p^N \quad i^N$

$(i) = 1$

(c)

(a) $b^1; \frac{1 \quad i^1}{1}$, b^1

$$\int_{b^1}^{\frac{1 \quad i^1}{1}} w(b) \, db = 1 \quad i^1$$

$d = w(b)$ $p(m) = 1$

$(i) = x(i; b)$. In

$(i^1; 1)$

(b) $b < b^1$

$$\frac{\left[i^N \quad \frac{W^0}{p(i^N)} \quad i^N \quad + \right]}{(i^N \quad)}$$

$$W_0 \qquad w(b)\,db$$

$$\{b : p\,(1 \quad b(1 \quad)) = p^N (i^N)\}$$

$p(m) = p \quad i^N$

fi ... $(i) = 1$. In ...

$; i^N$

$b \quad \frac{1 \quad i^N}{1}$

F ...

$; i^N \; , \quad i^A; i^B$ ... $[i^1; 1]$ in Fig. 3 ...

more ...

fi ...

H ...

F ...

fi ...

A ...

1. F ... $p(m) < p \quad i^N$:

$$A(I_0; \; ; m) = \begin{cases} \frac{\int_{i \in I_0} (i) I(i \quad) di}{\int_{i \in I} (i) I(i \quad) di} & \text{if} \int_{i \in I} (i) I(i \quad) di > 0 \\ 0 & \text{otherwise} \end{cases}$$

2. F ... $p(m) \in \; p \quad i^N \; ; 1$, ... fi ...

i ... $p\;(i) = p(m)$, ...

$i^A; i^B$ in Fig. 3 ...

(a) F ... $(\;) = I(i \quad g)$ ... $g >$, ...

is

$$A(I_0; \; ; m) = \begin{cases} 1 & \text{if } g \in I_0 \\ 0 & \text{otherwise} \end{cases}$$

(b) F[illegible]

$$A(I_0; ; m) = \begin{cases} \frac{\int_{i \in I_0} (i) I(i \quad) di}{\int_{i \in I} (i) I(i \quad) di} & \text{if } \int_{i \in I} (i) I(i \quad) di > 0 \\ 0 & \text{[illegible]} \end{cases}$$

3. F[illegible]

D[illegible] $i(m)$ [illegible]

$$i(m) \quad \max_i \{ i : p(i) \quad p(m) \}$$

[illegible]

$$A(I_0; ; m) = \begin{cases} \frac{\int_{i \in I_0} (i)[I(i \quad) + I(i > \quad)] I(i \quad i(m)) di}{\int_{i \in I} (i)[I(i \quad) + I(i > \quad)] I(i \quad i(m)) di} & \text{if } \int_{i \in I} (i)[I(i \quad) + I(i > \quad)] I(i \quad i(m)) di > 0 \\ 0 & \text{[illegible]} \end{cases} \tag{38}$$

P[illegible]

1. I[illegible] $p(m) = p(i)$ [illegible] $i > i^N$ [illegible]
   [illegible]
$$(j) = \begin{cases} 1 & \text{if } j \quad i \\ 0 & \text{[illegible]} \end{cases}$$

2. I[illegible] $i > i^N$ [illegible] $p(i) = p(m)$ [illegible]
   [illegible]
$$(j) = \begin{cases} 1 & \text{if } j \quad \min\{ i : p(i) = p(m) \} \\ 0 & \text{[illegible]} \end{cases}$$

3. I[illegible] $p(m) = p \quad i^N$ [illegible]
   [illegible]
$$(j) = \begin{cases} 1 & \text{if } j \\ 1 \quad \frac{W_0}{p(i^N)} \frac{1}{(i^N \quad)} & \text{[illegible]} \end{cases}$$

[illegible] F[illegible]

[illegible]

[illegible]

[illegible]

5 Fire Sales

The "fire sale" [illegible] In [illegible] fire [illegible] p [illegible] , [illegible] From (21) and (22), [illegible]

$$\frac{dp}{d} = \frac{(p)^2}{{}^2(1\quad)}\left[1 \quad b \quad \frac{\frac{(1\quad b)}{{}^2(1\quad)}p + \frac{1}{p}\int_b^1 \frac{(1\quad)w(b)}{[\ (1\quad b)+\ (1\quad)]^2}db}{\frac{}{(1\quad)}p + \frac{1}{p}\frac{w(b)}{(1\quad b)+\ (1\quad)}}\right] \tag{39}$$

[illegible] (39) [illegible] fire [illegible]

Proposition 3. 1. $\lim_{w(b)\to 1} \frac{dp}{d} = \frac{(p)^2}{{}^2(1\quad)}(1 \quad b) > 0$

2. If $w(b) = 0$, then $\frac{dp}{d} = \quad \frac{1}{}\int_b^1 \frac{(1\quad)w(b)}{[\ (1\quad b)+\ (1\quad)]^2}db < 0$

3. If $w(b) = w$ for all b, then $\frac{dp}{d} = 0$

[illegible] On [illegible] (2010). On [illegible] G [illegible] fi [illegible]

[illegible] 3 [illegible] If $w(b)$ [illegible] In [illegible] fire

[illegible] $w(b)$ [illegible]

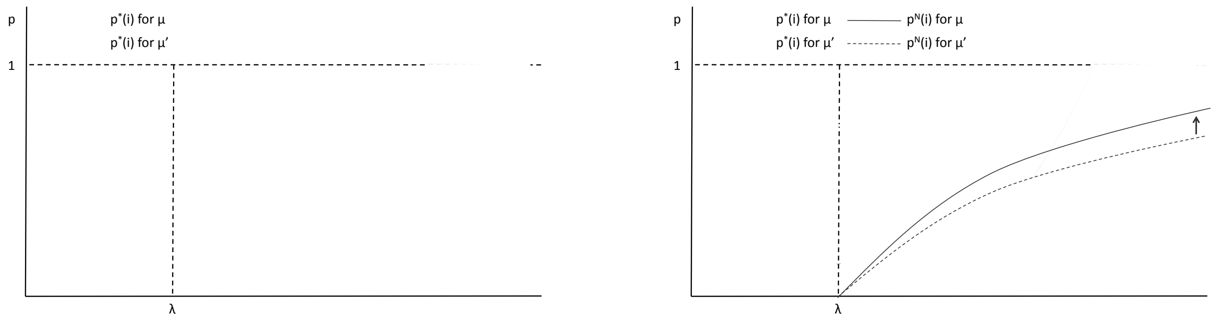
[illegible] (1992, 1997, [illegible] 1997) [illegible]

[illegible] (1998) [illegible]

[illegible] (1994, 1998, [illegible] 2008) [illegible]

[illegible] (35) [illegible] p^E. [illegible] p^N [illegible] Fig. 4. [illegible] (36), [illegible] $p^N(i)$ is [illegible]

E[illegible] 1 E[illegible] 2



F[illegible] 4: E[illegible] [illegible]

b[illegible] p^N [illegible] [illegible] [illegible] [illegible] F[illegible] 4.

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Appendix

[N[illegible]]

Proof of Proposition 1

1. [illegible]

   [illegible] (30) [illegible] $i \in [\ ;1]$ [illegible] $(i; m\ ) < 1$ [illegible] $i \in (b\ ;\ ]$ [illegible] m , [illegible] A [illegible] (1) [illegible] m [illegible] $i \in [b\ ;1]$ [illegible] $i \in [b\ ;\ ]$ [illegible] $i \in (\ ;1]$. I[illegible] (23). [illegible] (24) [illegible]

2. [illegible]

   (25) [illegible] (23) [illegible]

3. B[illegible]

   [illegible] (??) [illegible]

   D[illegible]

$$(m;b) \quad \begin{cases} \frac{q(i)a(i;\ (\ ;m;b);m)di}{p(m)A(I;\ (\ ;m;b);m)} & \text{f} \ A(I;\ (\ ;m;b);m) > 0 \\ 0 & [illegible] \end{cases} \tag{40}$$

[illegible]

$$^{max}(b) \quad \max_{m} \quad (m;b)$$
$$M^{max}(b) \quad \arg\max_{m} \quad (m;b)$$

B[illegible]

(a) [illegible] f $^{max}(b) < 1$, [illegible]

$$d(m;b) = 0 \quad 8m$$

(b) [illegible] f $^{max}(b) > 1$, [illegible]

$$\int_{m 2 M^{max}(b)} \frac{d(m;b)}{p(m)} dm = w(b)$$

[illegible]

$$d(m;b) = 0 \quad 8m \, 2\!\!= M^{max}(b)$$

[illegible] (29),

$$(m;b) = \begin{cases} \frac{1}{p(m)} \frac{(1 \quad)}{(1 \quad b) + (1 \quad)} & \text{f} \;\; p(m) \quad p \\ 0 & [illegible] \end{cases}$$

[illegible]

$$^{max}(b) = \frac{1}{p} \frac{(1 \quad)}{(1 \quad b) + (1 \quad)}$$

[illegible]

$$M^{max}(b) = f\, m \, 2 \, M \quad s{:}t{:} \quad p(m) = p \, g$$

[illegible] (22), [illegible] $b < b$ [illegible] $(b) < 1$ [illegible] $d(m;b) = 0$ [illegible] $b > b$ [illegible] [illegible] $p(m) = p$; [illegible] n [illegible] n m [illegible] (26). [illegible] (27) [illegible] [illegible]

4. D[illegible]

(28) [illegible] (26) [illegible]

5. A[illegible]

In [illegible] m [illegible] (19) [illegible]

$$S^n(i;m) = S^{n-1}(i;m) \quad \forall i;n;m \neq m$$

[illegible] By [illegible]

$$a^n(i; ;m) = a^1(i; ;m) \quad \forall i;n;m \neq m$$

[illegible]

$$a(i; ;m) = a^1(i; ;m) = \begin{cases} \frac{(i)S(i)}{\int (i)S(i)di} & \text{if } \int (i)S(i)di > 0 \\ 0 & \text{[illegible]} \end{cases} \quad \forall i;m \neq m \tag{41}$$

In [illegible] m, [illegible] (21) [illegible] (25) [illegible] (28), [illegible] $!(; 1) = 1 \quad \forall$, [illegible] (41) [illegible] (41) [illegible] (25) [illegible] (29).

6. [illegible]

(30) [illegible] (20).

Proof of Proposition 2

I first [illegible]

Lemma 1. Consider an arbitrary market m and suppose $i < j$. Then in any equilibrium the residual supplies after n rounds of clearing in market m satisfy

$$\frac{S^n(j)}{S^{n-1}(j)} \quad \frac{S^n(i)}{S^{n-1}(i)}$$

Proof. By [illegible] (19) [illegible] (16)

$$\begin{aligned} \frac{S^n(i)}{S^{n-1}(i)} &= 1 - \frac{\int \frac{(i)S^{n-1}(i)}{\int (i)S^{n-1}(i)di} !(;n)dD( )}{S^{n-1}(i)} \\ &= 1 - \int \frac{(i)}{\int (i)S^{n-1}(i)di} !(;n)dD( ) \end{aligned}$$

[illegible]

$$\frac{S^n(j)}{S^{n-1}(j)} - \frac{S^n(i)}{S^{n-1}(i)} = \int \frac{(i) - (j)}{\int (k)S^{n-1}(k)dk} !(;n)dD( ) \tag{42}$$

Gi[illegible]

$$(i) = x(i;b)$$

$$(i) = 1$$

$$(i) = 1 \quad x(i;b)$$

[illegible]

[illegible] $x = 1$. [illegible] $(i) \quad (j)$, [illegible]

[illegible] (42) [illegible] □

L[illegible]

[illegible] In[illegible]

[illegible]

Lemma 2. Let

$$(;m) \begin{cases} \frac{q(i)a(i;\,;m\,)di}{p(m)A(I;\,;m\,)} & \text{if } A(I;\,;m\,) > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$^{max}(;m) \Big(\frac{1}{p(m)} \frac{q(i) \quad (i)S(i;m)di}{} $$

From [illegible] n,

$$A^n(I;\ ;m\ ) = \begin{cases} 1 & \text{if } \quad (i)\,S^{n\ 1}(i;m)\,di > 0 \\ 0 & \text{[illegible]} \end{cases}$$

L[illegible] n [illegible] n [illegible] $A^n(I;\ ;m\ ) = 1$, [illegible] (43) [illegible]

$$(\ ;m\ ) = \frac{1}{p(m)} \quad q(i) \left[\sum_{n=1}^{n} \frac{!\ (\ ;n\ )}{P_{n=1}^{n}\ !\ (\ ;n\ )} a^n(i;\ ;m\ ) \right] di$$

$$= \frac{1}{p(m)} \sum_{n=1}^{n} \left[\frac{!\ (\ ;n\ )}{P_{n=1}^{n}\ !\ (\ ;n\ )} \quad {}^{1}\ a^n(i;\ ;m\ )\,di \right]$$

$$= \frac{1}{p(m)} \sum_{n=1}^{n} \left[\frac{!\ (\ ;n\ )}{P_{n=1}^{n}\ !\ (\ ;n\ )} \frac{{}^{1}\ (i)\,S^{n\ 1}(i;m)\,di}{(i)\,S^{n\ 1}(i;m)\,di} \right]$$

L[illegible] 1 [illegible] n

$$\frac{{}^{1}\ (i)\,S^{n\ 1}(i;m)\,di}{(i)\,S^{n\ 1}(i;m)\,di}$$

i[illegible] n [illegible] $!\ (\ ;\ 1) = 1$,
[illegible] $(\ ;m\ ) = {}^{max}(\ ;m\ )$. □

L[illegible] 2 [illegible] n
[illegible] It [illegible]
[illegible]

Lemma 3. Let

$$(b;m) \qquad \max_{\text{feasible for } b} \quad (b;m)$$

In any equilibrium

$$(b;m) \qquad {}^{max}(\ _b;m)$$

where $_b$ is de ned by equation (??)

Proof. By [illegible] b [illegible]

[illegible]

$$(b; m) = \quad (\; ; m \;)$$
$$\max (\; ; m \;)$$
$$\max (\; _b; m)$$

[illegible] $_b$ [illegible]
$\frac{q(i) \quad (i) S(i; m \;) di}{(i) S(i; m \;) di}$ [illegible] □

K[illegible] m
m [illegible] m^0 [illegible]
[illegible] (b; m), [illegible] b [illegible] m. [illegible]
[illegible]

Lemma 4. In equilibrium there is trade at only one price

Proof. A[illegible] p_H [illegible] p_L. If [illegible]
[illegible] m [illegible] $p = p_L$, [illegible]
[illegible] $i >$ [illegible] p_L. In [illegible] (1), [illegible]
$v <$ [illegible] $i >$, [illegible] $s(i; m; v) = 1$, i.e. [illegible]
[illegible] $i >$ [illegible] $p > p_L$. [illegible] m [illegible]
$p(m) \in (p_L; p_H]$.

$$S(i; m) = \begin{cases} & \text{if } i > \\ 1 & \text{if } i \end{cases}$$

Fix [illegible] b [illegible] m [illegible] $_b$ [illegible]
$p(m) \in (p_L; p_H)$. In [illegible]

$$(\; _b; m) = \max (\; ; m \;)$$
$$> \quad (b; m^0) \text{ [illegible] } m^0 \text{ [illegible] } p(m^0) = p_H$$

[illegible], [illegible]
[illegible] m [illegible] m^0 [illegible] $p(m) < p_H$. [illegible]
b [illegible] p_H. [illegible] b, [illegible]
[illegible] p_H. □

N[illegible] p , [illegible]
[illegible]

Lemma 5. Define

$$(i; p) \qquad (i; m)\, dm$$
$$m : p(m) = p$$

In any equilibrium where there is trade at p, $(i; p) = 1$ for all $i >$.

Proof. A [illegible]

[illegible] $(i; p) < 1$ [illegible] $i >$ [illegible] $(i; p) < 1$ [illegible]

$i >$. By Lemma 4, [illegible]

[illegible]

[illegible] (1), [illegible] $s(i; m; v) = 1$ [illegible] m [illegible] $i >$, [illegible]

$S(i; m) =$ [illegible] $i >$ [illegible] $p(m)$ 1. N[illegible] b [illegible] m^0 [illegible]

$p(m^0) < p$ [illegible] $_b$ [illegible]

[illegible] m^0 [illegible]

$$\begin{aligned}
({}_b; m^0) &= \frac{1}{p(m^0)} \frac{(1 \quad)}{{}_b\ S(i; m^0)\, di + (1 \quad)} \\
&\quad \frac{1}{p(m^0)} \frac{(1 \quad)}{{}_b\ S(i; m)\, di + (1 \quad)} \\
&> \frac{1}{p} \frac{(1 \quad)}{{}_b\ S(i; m)\, di + (1 \quad)} \\
&= {}^{\max}({}_b; m) \\
&\quad (b; m) \quad \text{[illegible]} \quad m \text{ s.t. } p(m) = p
\end{aligned}$$

[illegible] (1) [illegible]

$S(i; m) \quad S(i; m^0)$ [illegible] $p(m) < p(m^0)$, [illegible] i; [illegible] $p(m^0) < p$ [illegible]

Lemma 3. [illegible] b [illegible] m^0 [illegible]

[illegible] $p(m) = p$. [illegible] b, [illegible]

[illegible] p . □

[illegible] Lemmas (1)-(5) P[illegible] 2 [illegible]

Let p and b [illegible] (21) and (22).

[illegible] $p_H > p$. F[illegible]

[illegible] m [illegible] $p = p_H$, [illegible] $S(i; m) = 1$ for i and $S(i; m)$ for $i >$.

[illegible]

$$(b; m) \quad \frac{1}{p_H} \frac{(1 \quad)}{(1 \quad b) + (1 \quad)}$$

[illegible] b [illegible] b_H, [illegible]

$$\frac{1}{p_H}\,\frac{(1\quad)}{(1\quad b_H)+\quad(1\quad)}=1$$

Eq. (22) [illegible] $b_H > b$.

[illegible] b [illegible]

$$\frac{(1\quad)}{(1\quad b)+\quad(1\quad)}\,\frac{w(b)}{p_H}$$

[illegible]

$$\int_{b_H}^{1}\frac{1}{(1\quad b)+\quad(1\quad)}\,\frac{w(b)}{p_H}\,db$$

[illegible] $p_H > p$ [illegible] $b_H > b$, eq. (21) [illegible] 1, [illegible]

Lemma (5).

[illegible] $p_L < p$. F[illegible]

[illegible] m [illegible] $p(m) \in (p_L; 1)$, [illegible] $S(i;m) = 1$ for i [illegible] $S(i;m) =$ for $i >$. [illegible] $_b$ [illegible]

[illegible] $p \in (p_L; 1)$, [illegible] b [illegible]

$$=\frac{1}{p}\,\frac{(1\quad)}{(1\quad b)+\quad(1\quad)}$$

[illegible] p_L, [illegible] $b > b_L$

[illegible]

$$\frac{1}{p_L}\,\frac{(1\quad)}{(1\quad b)+\quad(1\quad)}$$

[illegible] $p = p_L$, [illegible] b_L [illegible]

$$\frac{1}{p_L}\,\frac{(1\quad)}{(1\quad b_L)+\quad(1\quad)}=1$$

F[illegible] $b > b_L$ [illegible]

fi [illegible]

$$\int_{b_L}^{1} \frac{(1 \quad)}{(1 \quad b) + \quad (1 \quad)} \frac{w(b)}{p_L} db$$

[illegible] $p_L < p$ [illegible] $b_L < b$, [illegible] (21) [illegible]
[illegible] $(1 \quad)$, [illegible]
[illegible]

[illegible] $p = p$ [illegible]
[illegible]

Monotonicty transformation

F[illegible] $p(i) : [a;1] \to R^+$, [illegible] M, [illegible]
[illegible] p [illegible] p^M, [illegible]

1. F[illegible] i, [illegible]

$$(\ i) \quad \max_{i^0} \int_i^{i^0} [p(i) \quad p(j)]\, dj$$
$$s.t.: \ i \quad i^0$$

N[illegible]

(a) $(\ i) \quad 0$ [illegible] $i^0 = i$ [illegible]

i f p [illegible] i, [illegible] $(\ i) = 0$

i [illegible]

2. D[illegible]

$$P = \{ i \in [a;1] : (\ i) > 0 \}$$

3. C[illegible] P [illegible] F[illegible]
[illegible] k, [illegible] i_k [illegible]

4. [illegible]

(a) $(i_k) = 0$

(b) $(i_k) > 0$. [illegible] $i_k = a$; [illegible] , i_k [illegible]
[illegible]

5. F[illegible] i_k [illegible]

(a) In[illegible] $(i_k) = 0$:

$$p_k = p(i_k)$$

[illegible] i_k^0 [illegible] i^0 [illegible]

$$\int_{i_k}^{i^0} (p_k \quad p(i))\, di = 0$$

N[illegible] $i^0 = i_k$ [illegible] a[illegible]
[illegible] F[illegible] [illegible]
i_k [illegible] $(i) > 0$ [illegible] i_k [illegible] (i) [illegible]
[illegible]

(b) In[illegible] $(i_k) > 0$:

i. If [illegible] $p_k; i^0$ [illegible]

$$\int_{i_k}^{i^0} (p_k \quad p(i))\, di = 0$$

$$p_k = p(i^0)$$

[illegible] $p_k; i_k^0$ [illegible]

ii. O[illegible] $i_k^0 = 1$ [illegible] p_k [illegible]

$$\int_{i_k}^{1} (p_k \quad p(i))\, di = 0$$

6. p^M [illegible] $p(i)$ [illegible]

$$p^M(i) = p_k \quad \text{[illegible]} \quad i \; 2 \; [i_k; i_k^0]; \quad \text{[illegible]} \quad k$$