Centre for Economic Policy Research

STUDY CENTER GERZENSEE

# EUROPEAN SUMMER SYMPOSIUM IN ECONOMIC THEORY

**Generously hosted by**
Study Center Gerzensee

Monday 1 – Friday 12 July 2013

# Consumers' Imperfect Information and Price Rigidities

Jean-Paul l'Huillier (EIEF)

EIEF WORKING PAPER SERIES

EIEF

Einaudi Institute for Economics and Finance

EIEF Working Paper 09/12

August 2012

# Consumers' Imperfect Information and Price Rigidities

by

Jean-Paul L'Huillier (EIEF)

# Consumers' Imperfect Information and Price Rigidities

Jean-Paul L'Huillier*

September 2011

**Abstract**

This paper develops a model of price rigidities and information diffusion in decentralized markets with private information. First, I provide a strategic microfoundation for price rigidities, by showing that firms are better off delaying the adjustment of prices when they face a high number of uninformed consumers. Second, in an environment where consumers learn from firms' prices, the diffusion of information follows a Bernoulli differential equation. Therefore, learning follows nonlinear dynamics. Third, the price rigidity produces an informational externality that affects welfare. Fourth, the dynamics of output are hump-shaped due to consumer learning.

**Keywords**: signaling, logistic curve, distortion.

## 1 Introduction

Starting with Lucas (1972), many economists have embraced the idea that dispersed information is a powerful tool for explaining some macroeconomic puzzles, for instance the existence of nominal rigidities and real effects of money. Mankiw and Reis (2002), Reis (2006), and Mackowiak and Wiederholt (2009) are recent examples that have explored related issues – such as the persistence of inflation, or the effects of endogenous attention allocation. The focus of this rapidly

*Einaudi Institute for Economics and Finance. I am highly indebted to my advisors Olivier Blanchard, Michael Golosov, and Guido Lorenzoni for generously providing me with guidance and suggestions that significantly improved the paper. I also thank Daron Acemoglu, Fernando Alvarez, Luca Anderlini, Marios Angeletos, Pierpaolo Benigno, Arnaud Costinot, Giovanni Dell'Ariccia, Tatyana Deryugina, Mathias Dewatripont, Peter Eso, Veronica Guerrieri, Jessica Haugsjaa, Larry Jones, Anton Kolotilin, Pablo Kurlat, Francesco Lippi, Matt Notowidigdo, Mike Powell, Roberto Rigobón, David Romer, Daniele Terlizzese, Nico Trachter, Aleh Tsyvinski, Kenichi Ueda, Iván Werning, Tim Willems, and Muhamet Yildiz, and seminar participants at MIT, the Dallas Fed, the Fed Board, Chicago Booth, USC, EIEF, Bocconi, Universidad Católica de Chile, the Richmond Fed, TSE, the ECB, LUISS, the Riksbank, University of Geneva, Banca d'Italia, the WIEM workshop and ESSIM for useful comments.

growing literature has been to analyze the behavior of imperfectly informed firms, which slowly adjust prices because of an information arrival delay. This paper analyzes a complementary informational structure: the case of imperfectly informed consumers. It finds that this setup largely confirms previous lessons, for instance the existence of nominal rigidities and persistent, real, effects of monetary shocks. However, it delivers a set of other important results, in particular regarding the nature of the rigidity, the dynamics of learning, and the shape of the dynamic responses of output.

I consider a dynamic economy that is subject to exogenous aggregate shocks. For concreteness, I focus on changes in the supply of money. There are two key assumptions. First, firms are better informed than consumers regarding these shocks. Second, the environment is decentralized. Trade of goods and, therefore, learning occurs when consumers are matched with firms. There are no menu costs. My goal is to study how information is transmitted in this economy, and to characterize the behavior of output and prices.

I obtain four main results. First, I show that firms are better off delaying the adjustment of prices. When a firm adjusts its price, it transmits its private information to consumers. Credibly signaling this information requires the firm to incur a cost. In equilibrium, this cost endogenously depends on the proportion of informed consumers. As the proportion of informed consumers increases, the informational asymmetry decreases together with the cost of information transmission. To sum up, there is endogenous price rigidity arising from the informational asymmetry between firms and consumers. Second, I show that aggregate learning dynamics can be characterized by a Bernoulli differential equation, leading to a logistic pattern for learning. The key to this result is a reinforcement between the adjustment of prices and the degree of informational asymmetry, determined by the proportion of informed consumers. The speed of learning is initially slow, and then increases as the degree of informational asymmetry decreases. Third, there is an important dynamic externality in my model. An individual firm does not take into account how its price adjustment affects aggregate information. Learning in the decentralized equilibrium is, therefore, inefficient. Fourth, in the context of monetary economics, my paper generates hump-shaped responses of output as an equilibrium outcome of decentralized learning.

My paper is most closely related to Golosov, Lorenzoni, and Tsyvinski (2009). As in their model, my environment has two key frictions: private information and decentralized trade. Importantly, as in their work, information revelation is strategic. The key contributions compared to that paper are the derivation of a closed-form solution for the dynamic path of learning, and the analysis of its implications for monetary economics. The nonlinear learning dynamics are most closely related to an important recent contribution by Burnside, Eichenbaum, and Rebelo (2011). There, the speed of belief transmission among agents depends on, among other things,

the number of "infected" agents: the more infected agents, the faster the belief transmission. My model generates a reinforcement between price changes, and thus information revelation, and the number of informed agents. Similar to Burnside et al. (2011), this reinforcement effect produces nonlinear learning. The informational externality is related to Amador and Weill (2010), who derive it in an environment wherein the information transmission is non-strategic, since learning occurs instantaneously when agents meet. In my model, the externality arises in a strategic environment where firms face endogenous costs for transmitting information and are better off delaying information revelation. The hump-shaped responses of output are most closely related to Mackowiak and Wiederholt (2010), who derive a similar result in an environment with rationally inattentive firms and consumers[1].

Specifically, the environment is as follows. I consider an economy populated by firms and consumers. Goods markets are decentralized. The economy is composed of islands. On each island there is a price-setting firm. Consumers travel from one island to the other, buying goods sequentially. Importantly, in every period, the only price observable to consumers is the price on that island. After a finite number of periods, all consumers buy a good sold at an exogenous price proportional to the supply of money. This final good is used to capture the idea that in the long run prices are flexible, and do not reflect any strategic concerns between suppliers and buyers. Consumers are heterogeneously informed. Informed consumers know the realization of monetary shocks, uninformed consumers simply hold prior beliefs. By assumption, all firms in the economy are informed[2]. For tractability, I abstract from firms' idiosyncratic shocks. This allows me to obtain simple rules for the adjustment of prices to aggregate shocks, and to solve for a closed form solution for the dynamic path of consumer learning.

The informational asymmetry between firms and uninformed consumers gives rise to a strategic tension. Because firms are informed, uninformed consumers revise their beliefs about money shocks as a function of firms' prices. Firms make higher profits when consumers believe that monetary shocks are high, because they expect to face higher prices in the long run, and therefore they are willing to spend more in the current period. Therefore, firms have a motive to post high prices and make uninformed consumers believe that monetary shocks are high. However, uninformed consumers use Bayes' rule to update their beliefs, and therefore, in equilibrium, they cannot be misled. Thus, the asymmetry of information changes the nature of price increases. If only few consumers are informed, firms need to incur a signaling cost to credibly increase prices.

[1] Duffie and Manso (2007) study information percolation in a finance setup with decentralized trading. They focus on perfect information revelation in every meeting, and also obtain closed-form results in a dynamic setup.

[2] One can motivate this assumption by supposing that firms are able to learn the realization of shocks from the environment, as for instance by observing demand (see, for instance Hellwig and Venkateswaran 2009). As long as firms observe more transactions than (some) consumers, the setup presented here follows. For more details, see the supplementary material posted on my webpage.

This signaling cost is *not* a physical cost, instead, it arises endogenously due to the strategic motives of the firm. It takes the form of a loss of sales coming from a nominal price that is higher than the perfect information price. This price is a credible message because of informed consumers. In fact, these consumers understand that this price is higher than absent informational frictions, and therefore buy less. Yet if monetary shocks were low, they would perceive this price as even higher and would decrease their demand by an even larger amount. The firm would then make even larger losses. This fact is enough to convince uninformed consumers that shocks are in fact high, because otherwise the firm would rather post low prices. When information propagates, the proportion of informed consumers increases. A high proportion of informed consumers allows the firm to adjust prices at a lower signaling cost, because their disciplining behavior is more important. Thus, only when enough consumers are informed can firms adjust prices without incurring significant costs. To sum up, the model generates an endogenous delay in the adjustment of prices, resulting from the lack of information among consumers, and the signaling element of prices due to firms' private information. This holds even if firms' nominal costs increase when there is a positive monetary shock.

This strategic feature of the model appears to fit a type of anecdotal evidence often mentioned in the literature. Indeed, there is a long standing idea in economics that, when costs or demand increase, firms are reluctant to increase prices because it would trigger a disproportionately adverse reaction among consumers. In my model, the price not only defines the terms of a transaction, it is also a message about how much consumers should spend. This signaling component of the price adds a strategic dimension to the relationship between firms and consumers, and – as explained above – can generate a cost to increasing prices. Therefore, my model provides a purely informational explanation for firms' reluctance to increase nominal prices[3].

I make explicit the cases in which the evolution of informed consumers follows a Bernoulli differential equation. The solution of this equation is well-known – the logistic function. Therefore, the price rigidity implies that learning is nonlinear. The reason is an interaction between price adjustment, and therefore learning, and the proportion of informed consumers. The more firms adjust prices, the higher the proportion of consumers who become informed. However, the higher the proportion of informed consumers, the more firms adjust prices. As long as the initial proportion of informed consumers is small, the reinforcement between these two forces generates an increasing speed of learning that implies a nonlinear diffusion of information. I also show that this nonlinearity is not specific to the explicit case of the Bernoulli differential equation, but is

[3]More generally, one can think about the price of a good being a signal of other determinants of consumers' valuation. For instance, consider the case of a developer of a product of unknown quality to consumers. If the developer knows the quality and sets a price, then the price is a signal that determines consumers' valuation. In this macroeconomic application, the price is a signal about monetary shocks and nominal spending.

generic. Moreover, the decentralized structure of the model is abstract enough that it can have other applications, for instance in finance. Thus, the paper provides a novel way of generating an interaction between the adjustment of prices, and the information in the hands of agents, which leads to nonlinear dynamics in learning.

Firms trade off the signaling costs and the benefits of adjusting prices. However, they do not take into account the impact of price adjustment on aggregate consumer learning. Thus, the model features an informational externality, and aggregate learning is inefficient.

The dynamic responses of output are hump-shaped, an important feature of the data according to Christiano, Eichenbaum, and Evans (2005). In the model, money has a procyclical effect as a result of meetings between firms that did not adjust prices and informed consumers. The proportion of informed consumers is increasing over time due to learning. Therefore, as long as only a few consumers are informed right after a monetary shock, the model delivers an increasing procyclical effect of money. This effect fades away in the long run, when all firms change prices, all consumers become informed, and the economy returns to normal. Taken together, these dynamics result in a hump-shaped output response. Importantly, notice that the model generates this type of output responses as a consequence of the same assumptions generating the price rigidity, i.e. that consumers are imperfectly informed and that they learn from firms' prices.

The paper is organized as follows. Section 2 goes over an example to illustrate the price rigidity result. Section 3 presents this result formally. Section 4 presents the dynamic model. It shows how the rigidity interacts with decentralized trading to generate nonlinear learning dynamics. It then analyzes the implications for output dynamics. Section 5 concludes. Most of the proofs are relegated to the Appendix.

# 2 A Static Example with Linear Demand

This section aims to develop intuition regarding the strategic tension arising when an informed monopolist sells to uninformed consumers. Section 3 develops the results fully. Readers acquainted with the theory of signaling games may want to skip this section to avoid redundance.

Consider the problem of a monopolist selling a good $c$ to a unit mass of consumers, indexed by $i$. Demand is a linear function of the real price, $p/P$

$$c_i\left(\frac{p}{P}\right) = 1 - \frac{p}{P} \quad ,$$

where $p$ is the monopolist's price and $P$ is the price level[4]. The price level can take two values,

[4]In the next section I derive this demand function and other shortcuts used here from first principles.

high ($P^h$) or low ($P^l$), $P^h > P^l$, both with equal probability $Pr(P = P^h) = Pr(P = P^l) = 1/2$. This price level is a device for modeling a monetary shock. Throughout the paper I use the terms price level, monetary shock, or aggregate state of the world interchangeably. In this example, the monopolist has zero costs.

Profit maximization yields $p^h \equiv P^h/2$ when the state is high, and $p^l \equiv P^l/2$ when the state is low. Notice that since $P^h > P^l$, $p^h > p^l$. In this example, and throughout the paper, I will use the term "price increase" to the act of posting the (high) price $p^h$, and to a "price decrease" to the act of posting the (low) price $p^l$. This terminology is used having a dynamic model in mind, in which firms increase prices in the long run in proportion to $P^h$ after a positive monetary shock, and decrease prices in proportion to $P^l$ after a negative monetary shock[5].

Suppose that a proportion $1 - \alpha$ of consumers are uninformed about the price level $P$. The complementary proportion $\alpha$ is informed and knows the realization of $P$. Consider an uninformed consumer. Unless this consumer learns $P$, he is unable to compute the real price of $c$, $p/P$, and thus he is uncertain about much to buy from the monopolist. That is, he is unable to evaluate whether a price $p$ is 'expensive' or 'cheap'. As I will show this feature is key for generating the rigidity in the pricing of the monopolist[6].

Uninformed consumers form an expectation about the inverse of $P$, $E_i[1/P]$. This expectation depends on prior beliefs – determined by the prior distribution of $P$, and on the price posted by the monopolist – which can potentially provide information. Thus, the uninformed have the following demand function:

$$c_i\left(pE_i\left[\frac{1}{P}\right]\right) = 1 - p \cdot E_i\left[\frac{1}{P}\right] \quad .$$

The monopolist knows the realization of the price level, and all consumers know that the monopolist is informed. Our goal now is to analyze different pricing strategies and their implications for demand and profits. The monopolist maximizes revenues

$$p\left(\alpha\left(1 - p\frac{1}{P}\right) + (1-\alpha)\left(1 - p \cdot E_i\left[\frac{1}{P}\right]\right)\right) \quad . \tag{1}$$

The monopolist takes into account that uninformed consumers update their beliefs about the price level upon observation of $p$. If, in equilibrium, the monopolist posts different prices as a function of the price level, uninformed consumers can learn the price level. If the monopolist's price is rigid – in the sense that it does not change with the price level – then uninformed

[5]See Section 4 and the supplementary material posted on my webpage for this model.

[6]The uncertainty about the price level is a modeling device for introducing uncertainty about consumers' valuation. It should be obvious that there are other, more direct ways of producing this uncertainty, as assuming for instance that consumers are uncertain about the value of some parameter of their utility function. In this macroeconomic study, prices reveal information about (nominal) valuation. This feature is key for my results.

consumers keep their prior beliefs[7]

$$E_i\left[\frac{1}{P}\right]=\frac{1}{2}\frac{1}{P^h}+\frac{1}{2}\frac{1}{P^l} \quad .$$

This fact gives rise to the following strategic tension. Notice from (1) that the monopolist is better off if uninformed consumers believe that the price level is high. The reason is that they would increase their demand for any $p$, and the monopolist would get higher profits. Thus, the monopolist has a motive to make them believe so. However, uninformed consumers understand the monopolist's strategy and use Bayes' rule when updating their beliefs, and therefore cannot be misled. Therefore, price increases are more difficult to implement than under perfect information. To understand this, suppose no consumer is informed ($\alpha = 0$). Can there be an equilibrium where the monopolist posts the same prices as under perfect information, $p^h$ and $p^l$? The answer in *no*, and the reason is as follows. Suppose that such an equilibrium is possible. When consumers see $p^h$, they understand the price level is high and spend more (in nominal terms). The opposite happens if consumers see $p^l$: they understand the price level is low and spend less. But this implies that the monopolist receives higher nominal profits when it posts $p^h$. Then, when the price level is low, it has a profitable deviation: to post $p^h$. Indeed, in this case consumers think that the price level is high, the monopolist increases nominal (and real) profits. This immediately shows that the alleged equilibrium is in fact not one.

If the proportion of informed is high enough, there exists an equilibrium where the firm posts perfect information prices. The following result establishes this fact.

**Result 1** *If, and only if*

$$\alpha \geq \frac{P^l}{P^h} \quad , \tag{2}$$

*there exists an equilibrium where the firm posts the same prices as under perfect information.*

**Proof (sketch).** Optimal prices are $p^h$ and $p^l$. The Incentive Compatibility (IC) constraint for the firm when the price level is low is

$$p^l\left(1-\frac{p^l}{P^l}\right) \geq p^h\left(\alpha\left(1-\frac{p^h}{P^l}\right)+(1-\alpha)\left(1-\frac{p^h}{P^h}\right)\right) \quad .$$

Solving this inequality for $\alpha$ yields (2).

■

The intuition for this result is that informed consumers discipline the firm by buying less when the state of low and the firms posts $p^h$. If $\alpha$ is high enough, then there are enough

[7] Formally, the monopolist and uninformed consumers play a signaling game.

informed consumers to discipline the firm to the point that there is an equilibrium at $p^h$ and $p^l$. Generally, even if only few consumers are informed, it turns out that equilibria with flexible prices are possible, but in these equilibria the price posted when the state is high is strictly higher than under perfect information (at a price $\overline{p}$ such that $\overline{p} > p^h$). In other words, there is a distortion at the top. This distortion at the top implies that the monopolist gets strictly lower average (real) profits than in the perfect information benchmark. As such, the model endogenously generates a cost to adjusting prices when there is imperfect information among consumers, and the firm is superiorly informed. Here, this loss is necessary for information transmission.

According to some authors, the idea of costly nominal price increases could lie at the root of the existence of nominal price rigidities and real effects of money[8]

provides a rationale for price rigidity, and is stated formally in the next section[9].

# 3 The Static Model

Consider the problem of a monopolist selling a good $c$ to uninformed consumers. In this model the monopolist knows the realization of the state of world, whereas uninformed consumers do not. As I will show, this asymmetry of information leads to price rigidity, in the sense that the price of the monopolist does not react to the state of the economy.

**Consumers.** There is a unit mass of consumers indexed by $i$. Consumer $i$ has the following utility function of consumption:

$$u(c_i) + C_i \quad . \tag{3}$$

I make the following assumptions concerning the utility function $u(c_i)$.

**Assumption 1** *The utility function $u(c_i)$ is twice continuously differentiable on $R_{++}$, strictly increasing, and strictly concave.*

The budget constraint is

$$pc_i + PC_i = \text{Income}_i \quad . \tag{4}$$

Utility (3) is linear in $C_i$ and therefore consumption of good $c$ is independent of income. A possible interpretation is that spending on $c$ is a small proportion of total income. Under this interpretation, I refer to good $c$ as a particular consumption good, and to good $C$ as all other consumption of the individual, and to its price $P$ as a "price level".

Goods $c$ and $C$ are bought sequentially. The consumer first buys good $c$. Then, he buys all other consumption $C$. Consumers buy good $c$ from a monopolist who sets the price $p$. As in the previous section, $P$ is drawn from a binary probability distribution over $\mathfrak{P} = \{P^h, P^l\}$, where $P^h > P^l$. I refer to $P = P^h$ as the high state, and to $P = P^l$ as the low state. I assume that both states are equally likely: $Pr(P = P^h) = Pr(P = P^l) = 1/2$. Income$_i$ is consumer $i$'s income.

**Information.** Informed consumers know the realization of the price level when buying from the monopolist. There is a proportion $\alpha$ of informed consumers. The complementary proportion

[9] Matejka (2010) also analyzes the case of imperfectly informed consumers and perfectly informed firms. He also obtains a price rigidity, but using different tools (rational inattention).

$1-\alpha$ is uninformed and does not know the price level. However, they know the distribution of possible realizations. All consumers know their income when buying from the monopolist. Also, all consumers observe the monopolist's price $p$ when deciding how much to buy.

The monopolist is informed, i.e. he observes the price level before setting his price[10]. To simplify the analysis, it is assumed the monopolist knows the proportion of informed consumers. The monopolist cannot discriminate between informed and uninformed consumers.

Informed consumers maximize (3) subject to (4) under perfect information. These consumers know $P$ and maximize their utility without any uncertainty.

The monopolist and uninformed consumers play the following one-shot game. First, the monopolist observes the realization of the price level $P$. After having observed the price level, the monopolist posts a price $p$. Uninformed consumers observe $p$, form beliefs $\mu$ about $P$, and decide how much to demand from the monopolist[11].

Formally, this sequence of events define a signaling game. The sender of the signaling game is the monopolist. The type of the sender is defined by referring to different possible information sets he can access[12]. Therefore, there are two possible types of monopolist: the "high type" – the monopolist who observed a high realization of the price level, $P^h$, and the "low type" – the monopolist who observed a low realization of the price level, $P^l$. The message of the sender is the price $p$. The receiver is the set of uninformed consumers, whose action is $c_i(\cdot)$, where $i$ belongs to the set of uninformed. This action depends on beliefs $\mu_i$.

**Monopolist's Problem.** To simplify the exposition, here the monopolist produces at zero cost. In Appendix A.4 I show that all results generalize to the presence of marginal costs. The monopolist chooses $p$ to maximize revenues:

$$\max_p pc(p, P, \mu_i(p)) \tag{5}$$

where $c(\cdot)$ is total demand for good $c$, to be derived below. The monopolist sets a price $p$. Consumers observe the price $p$ and submit their demand. Then, production takes place, and the monopolist sells as much as it is demanded. As it will become clear, total demand $c(p, P, \mu_i(p))$ depends on three objects. First, it depends directly on the price $p$. Second, it depends on the price level $P$, because the demand of informed consumers depends on $P$. Third, it depends on beliefs held by the uninformed $\mu_i(p)$, which in turn depend on the monopolist's price $p$.

[10]This assumption will be relaxed in the dynamic model.

[11]A related paper here is by Jones and Manuelli (2002), where other structures of information are considered (symmetric, informed buyer and uninformed seller, uninformed buyer and informed seller, and uninformed buyer and uninformed seller). However, their setup is more restrictive than mine, since it considers only indivisible trade with pure strategies. On the other hand, they consider endogenous information acquisition.

[12]This is the standard definition of "type" in game theory.

**Equilibrium definition.** I now define a perfect Bayesian equilibrium of the game. I first describe the strategy of the monopolist. I focus on pure strategies. A pure strategy for the monopolist $p$ is a mapping

$$p : \mathfrak{P} \longrightarrow R_+ \quad , \tag{6}$$

that assigns a price $p$ to each state of nature $P \in \mathfrak{P}$. Next, I describe beliefs $\mu_i(p)$ of uninformed consumer $i$. I focus on symmetric beliefs. Beliefs are a probability distribution over $\mathfrak{P}$ defined by a mapping

$$\mu_i : R_+ \longrightarrow [0,1] \quad , \tag{7}$$

that assigns a probability $\mu_i(p)$ to the high state of nature $P^h$. Mapping (7) is consistent with Bayes' rule on the path of equilibrium play. Because I focus on pure strategies for the monopolist, the requirement is simply that, for any equilibrium prices (6), denoted $p(P^h)$ and $p(P^l)$ for the high and low states respectively, if $p(P^h) \neq p(P^l)$ (a separating equilibrium), then $\mu_i(p(P^h)) = 1$ and $\mu_i(p(P^l)) = 0$. If instead $p(P^h) = p(P^l)$ (a pooling equilibrium), then $\mu_i(p(P^h)) = \mu_i(p(P^l)) = 1/2$. Beliefs $\mu_i(p)$ are unrestricted for other prices.

I now describe the strategy of uninformed consumers. I focus on symmetric pure strategies. A symmetric pure strategy $c_i$ for a given uninformed consumer $i$ is a mapping

$$c_i : R_+ \times \mathfrak{P} \times [0,1] \longrightarrow R_{++} \quad ,$$

that assigns a demand $c_i$ to each price $p$, each state $P$, and beliefs $\mu_i(p)$. A perfect Bayesian equilibrium requires that both the firm and the uninformed consumers play a best response. Given these definitions, I can now define an equilibrium formally.

**Definition 1** *A Perfect Bayesian Equilibrium (PBE) is a list ($p(P)$,$\mu_i(p)$,$c_i$), for all $i$, such that*

1. *There is no profitable deviation from posting p, given consumers' play,*
2. *$\mu_i(p)$ is derived using Bayes' rule on the equilibrium path,*
3. *consumption decisions $c_i$ maximize utility (3), given the budget constraint (4), beliefs $\mu_i(p)$ and firm's play.*

Having defined an equilibrium of the game, it is now useful to present consumers' optimality conditions for good $c$ because they provide intuition for the strategic tension between the monopolist and uninformed consumers.

**Consumers' Optimality Conditions for good $c$.** In the case of informed consumers, marginal utility of $c_i$ is equated to the relative price of the goods:

$$u'(c_i) = \frac{p}{P} \quad . \tag{8}$$

This equation pins down the demand for good $c$ by consumer $i$:

$$c_i\left(p\frac{1}{P}\right) \quad . \tag{9}$$

Notice that because of the quasilinearity of preferences, the demand (9) does not depend on income.

In the case of uninformed consumers, marginal utility of $c_i$ is equated to the expected relative price of the goods[13]:

$$u'(c_i) = E_{\mu_i(p)}\left[\frac{p}{P}\right] \quad , \tag{10}$$

where $E_{\mu_i(p)}\left[\frac{p}{P}\right]$ is simply the expectation of relative prices using beliefs $\mu_i(p)$, that is

$$E_{\mu_i(p)}\left[\frac{p}{P}\right] = \mu_i(p)\frac{p}{P^h} + (1-\mu_i(p))\frac{p}{P^l} \quad .$$

Because the expectation $E_{\mu_i(p)}\left[\frac{p}{P}\right]$ is conditional on $p$ (consumer $i$ observes it), the price $p$ can be taken out of the expectation operator, to obtain the demand function

$$c_i\left(pE_{\mu_i(p)}\left[\frac{1}{P}\right]\right) \quad . \tag{11}$$

At this point, it is important to notice that both the demand of the informed (9) and the demand of the uninformed (11) depend on the price chosen by the monopolist $p$ times (the inverse of) a deflator. In the case of the informed, this deflator is equal to the inverse of the price level $1/P$. In the case of the uninformed, this deflator is equal to a belief about the inverse of the price level $E_{\mu_i(p)}[1/P]$. Since I focus on symmetric strategies for uninformed consumers, I write total demand as

$$c(p, P, \mu_i(p)) = \alpha c_i\left(p\frac{1}{P}\right) + (1-\alpha)c_i\left(pE_{\mu_i(p)}\left[\frac{1}{P}\right]\right) \quad . \tag{12}$$

An interesting feature of (12) is that is it increasing in $\mu_i(p)$. This fact has a key implication for the strategic motives of the monopolist: it prefers uninformed consumers to believe that the price level is high, because in that case the deflator $E_{\mu(p)}\left[\frac{1}{P}\right]$ is low. Notice that this fact is

[13] To get this expression, substitute Income$_i$ from (4) into (3) and then take the first order condition with respect to $c_i$.

independent of the state of the world.

## 3.1 Perfect Information Benchmark

To develop intuition, here I consider a perfect information benchmark where all consumers are informed, i.e. $\alpha = 1$. In this case, total demand is

$$c(p, P, \mu_i(p)) = c_i\left(p\frac{1}{P}\right) \quad . \tag{13}$$

Plugging (13) into (5), the monopolist's problem is

$$\max_p pc_i\left(p\frac{1}{P}\right) \quad . \tag{14}$$

**Lemma 1** *When all consumers know the value of the price level, the monopolist's price is proportional to the price level and demand is the same in both states of nature.*

**Proof.** Taking the first order condition for the problem (14) and rearranging, get

$$c_i\left(p\frac{1}{P}\right) + p\frac{1}{P}c_i'\left(p\frac{1}{P}\right) = 0 \quad . \tag{15}$$

From condition (15) one can conclude that the monopolist's optimal price is proportional to the price level. Since total demand depends on the monopolist's price divided by the realization of the price level, this demand is the same in both states of nature.

■

## 3.2 Heterogeneous Information

In this case, there is a proportion of consumers that do not know the realization of the price level $P$, and therefore $\alpha < 1$. Here I will analyze the equilibria of the game between the firm and uninformed consumers[14]. I will show that, for low enough values of $\alpha$ this game admits equilibria with price rigidity, i.e. pooling equilibria in which the firm posts the same price in both states

[14] An interesting, non-strategic, feature of this model without marginal costs is worth highlighting. Suppose no consumer is informed ($\alpha = 0$), and fix consumers' beliefs $E_{\mu_i(p)}\left[\frac{1}{P}\right] \equiv \Xi$. Then, to maximize its revenue, the monopolist posts a price $p$ inversely proportional to consumers' beliefs. To see this, take the first order condition

$$c_i(p\Xi) + p\Xi c_i'(p\Xi) = 0 \quad ,$$

which implies that $p$ is inversely proportional to $\Xi$. The optimality result of pooling equilibria for few informed consumers presented later in this section hinges on this fact, but is valid even in the presence of marginal costs that are proportional to the price level $P$.

of the world. Moreover, I will also show that, for low enough values of $\alpha$, these equilibria deliver higher (ex-ante) profits than separating equilibria.

The following assumption is necessary to make this game tractable.

**Assumption 2** *The revenue function $pc_i(p/P)$ has strict increasing differences in $(p,P)$ and is single-peaked.*

By this assumption (together with Assumption 1) this signaling game belongs to the well-known class of monotonic signaling games (Cho and Sobel 1990, the proof is in Appendix A.1). This has two implications. First, the firm is better off if uninformed customers believe that the state of the world is high, independently of the actual realization of the state. This is because total demand and profits are decreasing in uninformed consumers' beliefs. Second, the game has the single-crossing property. This means that the high type is more at ease in posting high prices than the low type. The reason is that informed consumers know that he is the high type, and revenues on that share of the market increase faster with price increases than revenues on informed consumers of the low type. Together, these two properties make this game tractable.

As it is usually the case among signaling games, this game has many equilibria. I first characterize separating equilibria. The following proposition characterizes a benchmark separating equilibrium, the one where both types get the highest (ex-post) profits possible, also called the "Least Cost Separating Equilibrium".

**Proposition 1 (Best Separating Equilibrium)** *The following is the Best Separating Equilibrium. Define $\underline{\alpha}$ by*

$$p^l c_i\left(p^l \frac{1}{P^l}\right) = p^h\left(\underline{\alpha} c_i\left(p^h \frac{1}{P^l}\right) + (1-\underline{\alpha}) c_i\left(p^h \frac{1}{P^h}\right)\right) \quad , \tag{16}$$

*where*

$$p^h = \arg\max_p pc_i\left(p\frac{1}{P^h}\right) \quad , \tag{17}$$

$$p^l = \arg\max_p pc_i\left(p\frac{1}{P^l}\right) \quad . \tag{18}$$

*Then, $\underline{\alpha} < 1$ and,*

- *if $\alpha \geq \underline{\alpha}$:*
  - *The firm posts the same prices as in the perfect information benchmark, $p^h$ and $p^l$. Moreover, for a given equilibrium set of prices $p(P)$, define ex-ante real profits as*

$$\Pi(p(P)) = \frac{1}{2}\frac{1}{P^h}\pi(P^h) + \frac{1}{2}\frac{1}{P^l}\pi(P^l) \quad . \tag{19}$$

*where* $\pi(P) = pc(p, P, \mu_i(p))$*. In this case, ex-ante real profits* $\Pi(p(P))$ *are equal to ex-ante real profits in the perfect information benchmark:*

$$\Pi^* = \frac{1}{2}\frac{1}{P^h}\pi(P^h) + \frac{1}{2}\frac{1}{P^l}\pi(P^l) \quad , \tag{20}$$

*where* $\pi(P^h) = \max_p pc_i\left(p \cdot 1/P^h\right)$ *and* $\pi(P^l) = \max_p pc_i\left(p \cdot 1/P^l\right)$*.*

- *If* $\alpha < \underline{\alpha}$*:*
  - *The firm posts* $p^l$ *and* $\overline{p} > p_h$ *such that*

$$p^l c_i\left(p^l \frac{1}{P^l}\right) = \overline{p}\left(\alpha c_i\left(\overline{p}\frac{1}{P^l}\right) + (1-\alpha)c_i\left(\overline{p}\frac{1}{P^h}\right)\right) \quad . \tag{21}$$

*In this case,* $\overline{p}$ *is strictly decreasing and* $\Pi(p(P))$ *is strictly increasing in* $\alpha$*.*

The proof is in the appendix. This proposition shows that when the proportion of informed consumers is high enough, the high type can separate from the low type by posting the perfect information prices $p^h$ and $p^l$. The reason is that, in this case, the proportion of informed consumers is high enough to discourage the low type from imitating him: if the low type posts $p^h$, the informed know that his price is too high and they reduce their demand, thereby decreasing the low type's profits. In other words, informed consumers discipline the monopolist. Formally this is expressed by the IC constraint for the low type (16). When the proportion of informed is lower, the only way a separating equilibrium is possible is by having the high type post a price strictly higher than $p^h$, so that the low type does not imitate. Figure 1 is a graphical illustration of this proposition. On the right panel I plot real average ex-ante profits in this equilibrium. The plot shows that ex-ante profits are increasing[15] in $\alpha$, and reach $\Pi^*$ when $\alpha \geq \underline{\alpha}$.

Having characterized a benchmark separating equilibrium, I will now characterize a benchmark pooling equilibrium. Pooling equilibria are interesting for the study of nominal rigidities since in these equilibria the firm sets the same price independently of the state of the world. Pooling equilibria exist when the proportion of informed is low. The following proposition characterizes a benchmark pooling equilibrium. This is the pooling equilibrium at the price corresponding to profit maximization when $\alpha$ is equal to zero (no consumer knows the state of the world[16]). I also show that when $\alpha$ is equal to zero, this equilibrium reaches the perfect

[15] In a related paper, Benabou and Gertner (1993) analyzes a search market that features a similar informational asymmetry and shows how price increases trigger search.

[16] In a dynamic cash in advance model, this equilibrium corresponds to keeping the price unchanged after a monetary shock, as explained in detail in the supplementary material posted on my webpage.

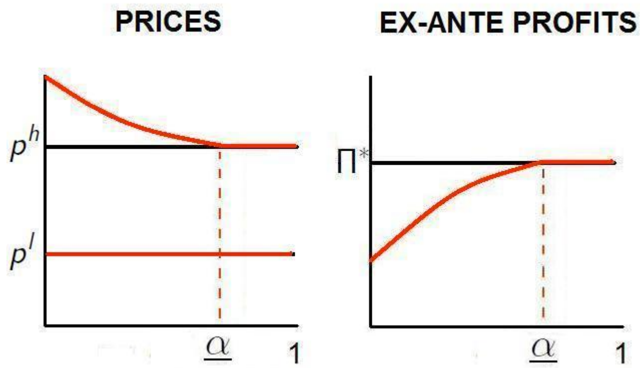


Figure 1: Best Separating Equilibrium (Proposition 1).

information level of ex-ante profits $\Pi^*$.

**Proposition 2 ($p^*$-Pooling Equilibrium)** *Consider $p^*$ such that*

$$p^* = \arg\max pc_i \left( p \left[ \frac{1}{2} \cdot \frac{1}{P^h} + \frac{1}{2} \cdot \frac{1}{P^l} \right] \right) \quad , \tag{22}$$

*and consider the lowest $\overline{\alpha}$ such that*

$$p^* \left( \overline{\alpha} c_i \left( p^* \frac{1}{P^h} \right) + (1-\overline{\alpha}) c_i \left( p^* \left[ \frac{1}{2} \cdot \frac{1}{P^h} + \frac{1}{2} \cdot \frac{1}{P^l} \right] \right) \right) \geq \max_p \left\{ \overline{\alpha} pc_i \left( p \frac{1}{P^h} \right) + (1-\overline{\alpha}) pc_i \left( p \frac{1}{P^l} \right) \right\} \tag{23}$$

*and*

$$p^* \left( \overline{\alpha} c_i \left( p^* \frac{1}{P^l} \right) + (1-\overline{\alpha}) c_i \left( p^* \left[ \frac{1}{2} \cdot \frac{1}{P^h} + \frac{1}{2} \cdot \frac{1}{P^l} \right] \right) \right) \geq p^l c_i \left( p^l \frac{1}{P^l} \right) \quad . \tag{24}$$

*For all $\alpha \leq \overline{\alpha}$, there exists a pooling equilibrium at $p^*$. If $\alpha = 0$, ex-ante profits reach $\Pi^*$. Moreover, ex-ante profits $\Pi(p^*)$ are strictly decreasing in $\alpha$.*

The proof is in the appendix. When the proportion of informed consumers is low this equilibrium exists because, according to (23) and (24), both types do not want to deviate. Figure 2 is a graphical illustration of this proposition. In the right panel ex-ante profits are decreasing in $\alpha$, and there is a unique $\alpha^*$ where the ex-ante profit functions cross[17].

A comparison of the Best Separating Equilibrium and the $p^*$-Pooling Equilibrium in terms of ex-ante real profits delivers that the latter ex-ante dominates the former when the proportion

[17]This results generalizes to the presence of marginal costs proportional to the price level $P$, see the supplementary material posted on my webpage.

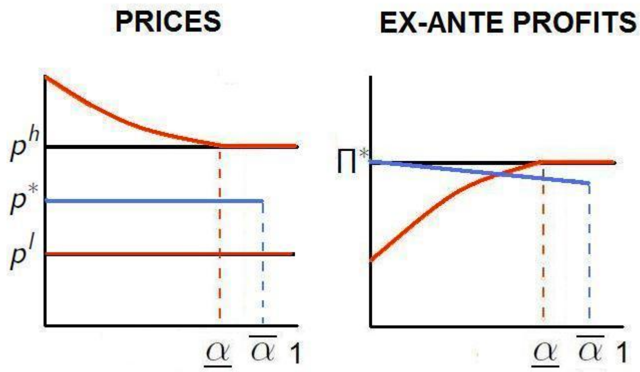


Figure 2: Best Separating Equilibrium and $p^*$-Pooling Equilibrium (Propositions 1 and 2).

of informed consumers is low enough. The next proposition develops this result in the case of any utility function satisfying Assumptions 1 and 2.

**Proposition 3 (Price Rigidity)** *There is $\alpha^P > 0$ s.t., if $\alpha \leq \alpha^P$, the $p^*$-Pooling Equilibrium delivers higher ex-ante profits than any separating equilibrium.*

The proof is in the appendix. The intuition for this result is the following. There is an ex-ante trade-off between two possible distortions. The first distortion arises in separating equilibria: in any separating equilibrium, when the proportion is informed consumers is low enough, there is a distortion at the top, because the firm needs to post a higher price than under perfect information to be able to signal the state of the world to uninformed consumers. This distortion hurts ex-ante profits. On the other hand, another type of distortion arises in pooling equilibria: in any pooling equilibrium the price posted does not correspond to beliefs of the informed in all states of nature, making them buy a different quantity than under perfect information, creating a distortion that hurts ex-ante profits. The first type of distortion is bigger the lower the proportion of informed consumers. The opposite happens in the second type of distortion, which is bigger the higher the proportion of informed consumers. Thus, each of these distortions varies monotonically with $\alpha$, but in opposite directions. As shown in the appendix, this holds even in the presence of marginal costs proportional to the price level $P$.

A symmetric result follows when the proportion of informed consumers is high enough.

**Proposition 4** *There is $\alpha^S > 0$ s.t., if $\alpha \geq \alpha^S$, the Best Separating Equilibrium delivers higher ex-ante profits than any pooling equilibrium.*

The proof is in the appendix. The intuition for this result is the same as for Proposition 3. There is an ex-ante trade-off between two types of distortions. The distortion arising in the

Best Separating Equilibrium is small when the proportion of informed consumers $\alpha$ is high, and therefore this equilibrium ex-ante dominates any pooling equilibrium[18]. The following lemma establishes that under some equilibria existence conditions there is a unique $\alpha^*$ that balances out this ex-ante trade-off.

**Lemma 2** *Consider* $\alpha^*$ *such that*

$$\alpha^* \left( \frac{1}{2} \frac{1}{P^h} p^* c_i \left( p^* \frac{1}{P^h} \right) + \frac{1}{2} \frac{1}{P^l} p^* c_i \left( p^* \frac{1}{P^l} \right) \right) + (1-\alpha^*)\Pi^* = \frac{1}{2} \frac{1}{P^h} \left( \overline{p} c_i \left( \overline{p} \frac{1}{P^h} \right) \right) + \frac{1}{2} \frac{1}{P^l} \left( p^l c_i \left( p^l \frac{1}{P^h} \right) \right) \tag{25}$$

*If* $\overline{\alpha} \geq \alpha^*$*, then both the Best Separating Equilibrium and the* $p^*$*-Pooling Equilibrium exist at* $\alpha^*$*. Then,* $\alpha^*$ *is a unique cutoff such that, if* $\alpha \geq \alpha^*$*, the Best Separating equilibrium delivers higher ex-ante profits, and if* $\alpha < \alpha^*$*, the* $p^*$*-Pooling Equilibrium delivers higher ex-ante profits.*

**Proof (sketch).** The LHS of (25) are ex-ante profits in the $p^*$-Pooling Equilibrium, and the RHS are ex-ante profits in the Best Separating Equilibrium. This equation defines a unique $\alpha^*$ where both are equal. If at $\alpha^*$ both equilibria exist, the result follows from both continuity and strict monotonicity of these functions. ■

In order to apply these results in a macroeconomic framework, an issue that I need to confront is equilibrium selection. In this application I am interested both in pooling and separating equilibria. Pooling equilibria are useful because they feature prices that do not react to the aggregate state and demand of the informed that is procyclical – high when the state is high and low when the state is low. Separating equilibria are useful because their outcomes provide the counterpart, i.e. prices that adjust and – as long as $\alpha$ is high enough – quantities that mimick perfect information. In a dynamic economy, these features are particularly attractive in the "long run". With these purposes in mind, a possible selection criterion is based on the idea of ex-ante real profits used in Propositions 3 and 4 and Lemma 2. The following definition outlines my procedure in detail.

**Definition 2 (Criterion** $S$**)** *If, for a given* $\alpha$*, the* $p^*$*-Pooling Equilibrium does not exist, pick the Best Separating Equilibrium. If both the Best Separating Equilibrium and the* $p^*$*-Pooling Equilibrium exist, compute ex-ante profits in both of them, and pick the one that delivers higher ex-ante profits.*

[18] One may wonder why, in the pooling equilibrium, informed consumers do not transmit their information to uninformed consumers. Taken literally, my wording of the problem implicitly assumes that such communication is not possible. However, one may instead imagine that the firm faces a unit mass of either informed or uninformed consumers, with probabilities $\alpha$ and $1 - \alpha$ respectively. In this case this problem of information transmission among consumers does not arise.

Notice that Criterion $S$ selects a unique equilibrium for all parameter values. For simplicity, criterion $S$ focuses on equilibria I characterized in Propositions 3 and 4, but considering also all other PBE would deliver similar qualitative results. The reason is that, as characterized in Proposition 3, for $\alpha$ low enough the $p^*$-Pooling Equilibrium dominates *all* separating equilibria. Considering other pooling or semi-separating equilibria would, if anything, obtain even more price rigidity, and more delays in the way the dynamic economy reacts to monetary shocks. To simplify my analysis, I omit the consideration of a larger set of PBE.

Selection criterion $S$ can be justified by imagining that the firm has a commitment device that allows to choose, before the state is realized, a pricing plan. These pricing plans have to be 'credible', i.e. the consumers need reasons to believe that, ex-post, the firm will keep its promise. In other words, the firm can only commit to prices that satisfy a PBE and, when the state of the world is low, the firm will not be tempted to fake that the state of the world is high. A formal way of justifying this criterion is to modify the game using a mechanism design approach similar to Maskin and Tirole (1992)[19].

However, it is important to emphasize that the above proposed criterion is not crucial for any of the dynamic results of Section 4, including the nonlinearity of learning, the informational externality and the general shape of the responses to a monetary shock. As long as a non-zero fraction of firms play a pooling equilibrium, similar results would be obtained. What is crucial is that, for low proportion of informed $\alpha$, the pooling equilibrium exists, and that for high values of $\alpha$ *only* the Best Separating Equilibrium exists, so that when enough consumers become informed all firms change prices and the economy returns to steady state. Thus, any selection criterion that allows to have a non-zero fraction of firms play pooling equilibria when they exist would work[20].

Finally, a remark is needed regarding the type of nominal rigidity obtained here. This model, as it is the case in most informational theories of nominal rigidity, starting with Lucas (1972), features a nominal rigidity in which firms price changes do not perfectly *correlate* with the aggregate state. That is, in separating equilibria prices are 'flexible' in the sense that there is a one-to-one mapping between the state and firms' prices, and in pooling equilibria prices are

[19] In the literature, there is no consensus on how to select equilibria in signaling games. The criterion I use here is most closely related to the notion of 'undefeated equilibria' put forth by Mailath, Okuno-Fujiwara, and Postlewaite (1993). A popular criterion is the Intuitive Criterion. However, in a richer model with more states of the world, clearly a relevant extension of this model for the analysis of monetary policy, this criterion looses its bite: it fails to select a unique equilibrium. Therefore, it would not be useful (Cho and Kreps 1987, p. 212). As I show in the supplementary material posted on my webpage, the procedure I propose selects a unique equilibrium for more than two types.

[20] For instance, if one were to Pareto rank equilibria, this would also favor the $p^*$-Pooling Equilibrium for low values of $\alpha$ and the Best Separating Equilibrium for high values of $\alpha$. The reason is that the $p^*$-Pooling Equilibrium is less distortionary in the former case – and thus provides higher welfare, and the Best Separating Equilibrium in the latter. See Proposition 5 for more details regarding welfare.

'rigid' in the sense that firms' prices are the same in different states. But notice that this notion of 'rigidity' is different from the one used in other models, as for instance in a model with menu costs. In richer versions of this model firms' prices could change as a function of other parameters (marginal costs, demand parameters, etc.), but as long as these changes do not correlate with the aggregate state, according to the notion used in this paper, they are 'rigid'[21].

## 3.3 Comparative Statics in the Presence of Marginal Costs

In a more general model, all the cutoffs presented above should depend on firm specific characteristics. To illustrate this point, let me consider the case where the monopolist has a linear marginal cost of production $kP$. For tractability, I assume $k$ is known by both the firm and consumers. I analyze which equilibrium among the Best Separating vs. the $p^*$-Pooling Equilibrium is ex-ante optimal. The following numerical result follows[22].

**Result 3 (Comparative Statics of $\alpha^*$)** *Assume $u(c_i) = c_i - \frac{1}{2}c_i^2$, and consider the Best Separating Equilibrium and the $p^*$-Pooling Equilibrium. For $k \leq \hat{k}$, there is $\alpha \leq \hat{\alpha}$ where both equilibria exist. In this region there is $\alpha^*(k)$ such that:*

- *for $\alpha > \alpha^*(k)$, ex-ante profits are higher in the Best Separating Equilibrium,*
- *for $\alpha < \alpha^*(k)$, ex-ante profits are higher in the $p^*$-Pooling Equilibrium.*

*Moreover, $\alpha^*(k)$ is decreasing with $k$.*

As this result shows, which equilibrium delivers higher ex-ante profits depends on firms' marginal cost $kP$. The higher $k$, the lower the critical value $\alpha^*$. Figure 3 plots this cutoff as a function of $k$ and shows that it is decreasing. The region below the curve is where the $p^*$-Pooling Equilibrium delivers higher ex-ante profits, then region above the curve is where the Best Separating Equilibrium delivers higher ex-ante profits.

This result has an interesting application in a macroeconomic model. Indeed, one can write a model where firms are heterogeneous and thus have different cutoffs for adjusting prices. The presence of firms playing separating equilibria allows for the possibility of consumer learning. This, in turn, has implications for the proportion of firms playing the Best Separating Equilibrium. Thus, it seems that a dynamic model can deliver interesting feedback effects between

[21] This type of rigidity has also been used in the literature on other setups with asymmetric information. An early example is provided by Wilson (2002), where it is shown that bargaining problems in which prices are negotiated in nominal terms, and one party has superior information about real terms, have equilibrium outcomes insensitive to this information. Another more recent example is by Kennan (2010).

[22] Unfortunately, these comparative statics are not tractable analytically. The reason is that in the presence of marginal costs, analyzing (the equivalent to) equation (25) is involved due to the fact that prices $p^*$ and $\overline{p}$ vary with $k$.

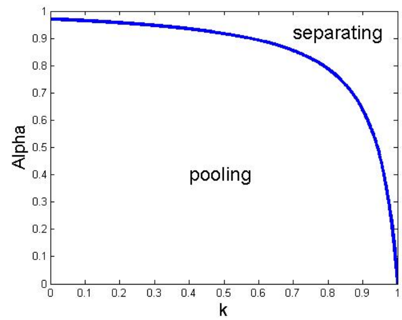


Figure 3: Cutoff $\alpha^*(k)$, and regions where $p^*$-Pooling/Best Separating Equilibrium delivers highest ex-ante profits ($P^h/P^l = 1.03$).

consumer learning and the proportion of firms playing separating equilibria. I explore these themes in the next section.

# 4 The Dynamic Model

This section analyzes a dynamic economy where consumers are imperfectly informed and learn from prices in a decentralized market for goods. Prices are temporarily rigid for the reasons explained previously. I focus on two aspects of the problem. First, I study how information about the aggregate state of the economy is diffused among consumers. Second, I analyze the dynamic implications of consumer learning for the effects of monetary shocks on output.

Essentially, this dynamic model consists in repeating the framework of last section, keeping track of the evolution of the proportion of informed consumers. I divide a macroeconomy in a unit mass of islands. Consumers go randomly form one island to the other, buying goods sequentially. This allows learning about the aggregate state. For tractability purposes, I focus on an environment with two basic features. First, the only state variable of the problem is the proportion of informed consumers. Second, the event that a given consumer is paired with the same firm twice is a zero probability event, and therefore each time firms and consumers play the static game presented in the previous section. Moreover, firms are heterogeneous and adjust prices at different stages, which allows for a smooth evolution of learning.

## 4.1 Dynamic Setup

For presentational purposes here I present a simplified setup with the essential ingredients to understand the dynamics of learning and output. Everything can also be presented in a full-blown cash-in-advance general equilibrium environment, in order to show that these dynamics

can be obtained in a standard macroeconomic framework. However, such a framework is involved and has many elements that are tangential to the essence of the problem. It is therefore relegated to the Appendix.

Time is continuous and indexed by $t$. It runs from $t = 0$ to a (large) terminal date $T$. There is a unit mass of consumers, indexed by $i$.

The economy is geographically divided into a unit mass of islands, indexed by $j$. On every island there is a firm, also indexed by $j$. This firm is a price-setting monopolist.

Firm $j$ has a cost function $k(c; \theta_j)$, where $c$ is the quantity produced and $\theta_j$ is a firm specific parameter. Firms are heterogeneous. Heterogeneity is modeled by imposing that $\theta_j$ is a continuous random variable with pdf

$$\theta_j \sim F(\theta_j) \quad ,$$

and support $\mathfrak{S} = [0, \overline{\theta}]$. As in the static model of the previous section, the aggregate state of the economy is defined by the realization of the price of a final good that consumers buy at $T$. I call this price the "long-run price level" $P_T$. I use this long-run price level as a way of modeling a monetary shock, using the idea that in the long run prices are flexible and proportional to money supply[23]. $P_T$ is drawn from the same binary distribution over $\mathfrak{P} = \{P^h, P^l\}$ used in the previous section.

**Information.** As above, by assumption firms are informed[24]. At every instant $t$, there is a proportion $\alpha_t$ of consumers that are informed and know the realization of $P_T$, and thus the complement $1 - \alpha_t$ of consumers is uninformed and has prior beliefs about $P_T$. For simplicity, $\theta_j$, for all $j$, is known by both firms and consumers.

At every instant $t$ consumers are randomly paired with islands for consumption at a rate $r > 0$ per unit of time. More specifically, the population of consumers is divided into a unit mass of representative groups of unit mass. Each of these groups is sent to an island. Thus, by the law of large numbers, every island receives a representative sample composed by a proportion $\alpha_t$ of informed consumers and a proportion $1 - \alpha_t$ of uninformed consumers. On each island (as in the previous Section) the monopolist and uninformed consumers play a signaling game where the monopolist posts a price, uninformed observe the price and update their beliefs about the aggregate state, and then decide how much to buy. This shopping pairing between islands and consumers is repeated over time.

[23]The cash-in-advance framework uses a process for money supply to generate these changes in the long-run price level. Therefore, in that model the state of the world is determined by actual monetary shocks.

[24]In the full-blown model presented in the supplementary material posted on my webpage, I relax this assumption and let firms learn the realization of the state from demand.

## 4.2 Diffusion of Information

The following definition is necessary in order to derive the dynamics of learning.

**Definition 3 (Proportion of Firms Adjusting Prices)** *According to Criterion S, the mass of firms playing the Best Separating Equilibrium (BSE) is a function $\beta(\alpha)$ defined over $[0,1]$:*

$$\beta(\alpha) = \int_{Play\ BSE\ for\ \alpha} f(\theta_j) d\theta_j \quad .$$

Having defined this proportion, the evolution of the proportion of informed consumers is given by the differential equation

$$\frac{d\alpha_t}{dt} = r(1-\alpha_t)\beta(\alpha_t) \quad . \tag{26}$$

According to (26), the amount of learning at time $t$ is equal to the proportion of uninformed consumers $1-\alpha_t$ that are paired, at a rate $r$ per unit of time, with firms playing a separating equilibrium $\beta(\alpha_t)$. Unless $\beta(\alpha)$ is a constant, this is a nonlinear differential equation. This type of equations are usually difficult to solve in closed form. However, it is possible to analyze the properties of the solution by directly studying (26). In a case made explicit below $\beta$ is such that this equation admits a well-known closed-form solution[25].

The following Lemma establishes some important properties of $\beta$.

**Lemma 3** *$\beta(\alpha)$ is a (weakly) increasing function, and it is bounded above by 1.*

**Proof.** Consider $\alpha'$ and $\alpha$ such that $\alpha' \geq \alpha$. Consider a firm $j$. It follows from Lemma 2 and Criterion $S$ that if firm $j$ plays the Best Separating Equilibrium at $\alpha$, it also plays this equilibrium for $\alpha'$. If firm $j$ plays the $p^*$-Pooling Equilibrium, then it either plays the Best Separating Equilibrium at $\alpha'$, or it plays the $p^*$-Pooling Equilibrium. Thus, $\beta(\alpha)$ is a (weakly) increasing function. Because the total mass of firms is 1, $\beta(\alpha)$ is bounded above by 1. ■

[25] This equation is derived from the discrete time model presented in the supplementary material posted on my webpage as follows. For a given $\alpha_t$, the proportion of informed consumers at $t+1$ is

$$\alpha_{t+1} = \alpha_t + r(1-\alpha_t)\beta(\alpha_t) \quad .$$

If the unit of time is $h$, then this equation is

$$\alpha_{t+h} = \alpha_t + r(1-\alpha_t)\beta(\alpha_t)h \quad ,$$

which is equivalent to

$$\frac{\alpha_{t+h} - \alpha_t}{h} = r(1-\alpha_t)\beta(\alpha_t) \quad .$$

Taking the limit as $h$ approaches zero yields (26).

**Closed form solution.** Consider a firm $j$. For this firms $j$, the cutoff $\alpha^*$ is a function of the (idiosyncratic) parameter $\theta_j$, and therefore it can be written $\alpha^*(\theta_j)$. If this function satisfies the following assumption, it is easy to derive a closed form solution for (26).

**Assumption 3 (Monotonicity Over $\mathfrak{S}$)** *The function $\alpha^*(\theta_j)$ is continuously differentiable and strictly monotonic.*

An example where this assumption holds is provided in p. 20, where it is shown that in the case of linear marginal costs $k$, the cutoff $\alpha^*(k)$ is strictly decreasing in $k$. However, more generally, it is only needed that this cutoff is strictly monotonic in some idiosyncratic parameter of the firm, together with the fact that some firms play the Best Separating Equilibrium for arbitrarily low $\alpha_0$. The following assumption guarantees this.

**Assumption 4 (Learning Starts Off for Arbitrary Low $\alpha_0$)** *If $\alpha^*(\theta_j)$ is increasing, then $\alpha(0) = 0$. If $\alpha^*(\theta_j)$ is decreasing, then $\alpha^*(\overline{\theta}) = 0$.*

The following proposition establishes that under these assumptions it is possible to find a distribution of firms such that (26) has an explicit analytical solution.

**Proposition 5 (Closed Form Learning Dynamics)** *Under Assumptions 3 and 4, there is a pdf $f(\theta_j)$ such that, for $t < T$, $\alpha_t$ follows*

$$\frac{d\alpha_t}{dt} = \begin{cases} r(1-\alpha_t)\cdot b\alpha_t & \text{if} \quad b\alpha_t \leq 1 \\ r(1-\alpha_t) & \text{if} \quad b\alpha_t > 1 \end{cases} \quad , \tag{27}$$

*where $b > 1$. The solution has the form*

$$\alpha_t = \begin{cases} \frac{1}{1+C_1 e^{-rbt}} & \text{if} \quad b\alpha_t \leq 1 \\ 1 - C_2 e^{-rt} & \text{if} \quad b\alpha_t > 1 \end{cases} \quad . \tag{28}$$

Also, $\alpha_T = 1$.

**Proof.** Since $\alpha^*(\theta_j)$ is strictly monotonic, it is invertible:

$$\alpha^{*-1}(\theta_j) = \theta_j^*(\alpha) \quad .$$

If $\theta_j^*(\cdot)$ is strictly increasing, then

$$\beta(\alpha_t) = \int_0^{\theta_j^*(\alpha_t)} f(\theta_j)d\theta_j = F(\theta_j^*(\alpha_t)) \quad .$$

Now solve

$$F(\theta_j^*(\alpha_t)) = b\alpha_t$$

by substituting $\alpha_t$ by $\alpha^*(\theta_j)$, obtaining

$$F(\theta_j) = b\alpha^*(\theta_j) \quad ,$$

which gives the expression for $F$. To obtain $b$, solve

$$b = \frac{1}{\lim_{\theta_j \to \overline{\theta}} \alpha^*(\theta_j)} \quad .$$

$F$ is defined over the support of $\theta_j$, it is strictly increasing, and bounded above by 1. Thus, it is a cumulative distribution function. Then, $f(\theta_j) = \frac{d}{d\theta_j} F(\theta_j)$. Since $\overline{\alpha} < 1$ (see Proposition 1), $\alpha^* < 1$ and $b > 1$. The case of a strictly decreasing $\theta_j^*(\cdot)$ is similar.

By differentiating (28) one finds that this is a solution of (27).

■

For $b\alpha_t \leq 1$, equation (27) is called a Bernoulli differential equation, and its solution is given by the logistic function (28). For $b\alpha_t > 1$ (all firms have changed prices), learning follows a negative exponential function. The constant $C_1$ is a function of the exogenous initial condition $\alpha_0$:

$$C_1 = \frac{1}{\alpha_0} - 1 \quad ,$$

and

$$C_2 = \left(\frac{1}{b}\right)^{\frac{1}{b}} \left(1 - \frac{1}{b}\right)^{1-\frac{1}{b}} C_1^{\frac{1}{b}} \quad .$$

The Bernoulli differential equation, together with its solution the logistic curve, have a wide range of applications in the natural sciences. For instance, they are used in medicine to model spread of diseases and in ecology to model the diffusion of species. Similar nonlinear patterns of diffusion have been used recently by Burnside, Eichenbaum, and Rebelo (2011) to explain movements in housing markets. There is a tight link between these applications and the modeling of information diffusion among consumers, that I explain after plotting this solution.

Figure 4 shows the solution (28) for $b = 2$, $r = .15$, and $\alpha_0 = .01$. As it is well-known, this curve is S-shaped. Learning is initially slow, it accelerates and then slows down in the long-run. The reason for the nonlinear behavior of learning is due to a reinforcement effect between $\alpha_t$, the proportion of informed consumers, and $\beta(\alpha_t)$, the proportion of firms changing prices (i.e. playing the Best Separating Equilibrium): the higher $\alpha_t$, the higher $\beta(\alpha_t)$. But also other

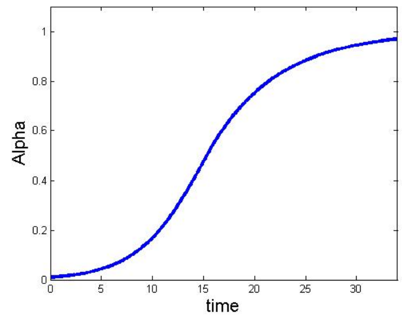


Figure 4: Learning in the Case of the Bernoulli Equation ($b = 2$, $r = .15$, and $\alpha_0 = .01$)

things equal, the higher $\beta(\alpha_t)$, the faster $\alpha_t$ increases. Initially, as long as $\alpha_0$ is small, few firms change prices and therefore few consumers learn. Over time $\alpha_t$ increases, triggering further price changes and allowing for faster learning among consumers. In the long-run learning slows down because the proportion of uninformed consumers has declined.

This type of reinforcement effect is also present in most population dynamics models. Generally, the reinforcement implies that there is a range where the rate at which a population grows is increasing in the size of the population. For instance, in the model by Burnside et al. (2011), agents transmit beliefs about different long-run fundamentals of housing by bilateral meetings. The diffusion of beliefs resembles the spread of a disease. Agents with tighter priors are more likely to convert others to their beliefs. In their model, the higher the number of infected agents, the higher the probability that a non-infected agent becomes infected in a bilateral meeting. But also the higher is the probability of infection per meeting, the higher is the number of infected agents. Thus, the model features a reinforcement between the number of infected and the probability of infection per meeting. In my model, the reinforcement effect is tightly linked to Proposition 4 and Criterion $S$, according to which the Best Separating Equilibrium is played for large proportions of informed consumers. Notice however that Criterion $S$ is here not crucial. As explained in p. 19, what is crucial is that for large values of $\alpha_t$ pooling equilibria do not exist, and therefore only separating equilibria, which involve learning, are possible. More formally, there is a reinforcement effect whenever the proportion of firms changing prices is an increasing function of the proportion of informed consumers, as established in Lemma 3. In this case, there is a scope for an acceleration of learning. This reasoning suggests that this feature of the model does not only appear in the logistic solution, but it is more general. Indeed, the following proposition establishes that for a large class of functional forms for $\beta$, if $\alpha_0$ is small enough, there is a region where the speed of learning is increasing[26].

[26] In an important paper investigating the geographical properties of female labor participation in the U.S., Fogli and

**Proposition 6 (Nonlinear Learning)** *Suppose that $\beta(\cdot)$ is continuously differentiable over $(0,1)$, and that*

$$\lim_{\alpha \to 0} \beta(\alpha) = 0 \quad ,$$

*and*

$$\lim_{\alpha \to 0} \beta'(\alpha) > 0 \quad .$$

*Then, there exists $\alpha_0 > 0$ and $t$ such that*

$$\frac{d}{dt}\left(\frac{d\alpha_t}{dt}\right) \geq 0 \quad .$$

The proof is in the Appendix. The key to this result is that for low values of $\alpha_0$ few firms change prices, and therefore learning is slow. However, the proportion of firms adjusting prices is increasing for all $\alpha$, and therefore there is a range where the speed of learning is increasing.

It is important to notice that the existence of this reinforcement effect relies here on the existence of an informational externality among firms. Indeed, when a firm adjusts its price or not, it does not take into consideration the impact on aggregate information, and by implication on other firms. It is easiest to understand the effects of this externality by assuming that firms are homogeneous. As the next lemma shows, there are cases where the economy can get stuck in a no-learning situation.

**F-28 77 3(4 77 4 (In(formational)- 3(Eexternalt)328(y)1 ()) 0 g 0 G F53 10 9091 f24113242**

$$\int \left( \frac{1}{2} \left\{ \int u(c_{it})\ dt + C_{iT} \right\} \Big|_{P=P^h} + \frac{1}{2} \left\{ \int u(c_{it})\ dt + C_{iT} \right\} \Big|_{P=P^h} \right) di$$

*than the market allocation by imposing that the Best Separating Equilibrium is played from* $t = 0$ *until* $\overline{T} < T$*, where* $\alpha_{\overline{T}}$ *is such that there are no more distortions, i.e. at* $\overline{T}$*:*

$$c(p, P, \mu_i(p)) = c_i \left( p^h \frac{1}{P^h} \right) \quad .$$

The proof is in the Appendix. The intuition for this result is as follows. When firms play the $p^*$-Pooling Equilibrium forever, informed consumers' bundle of good $c$ varies across states. If their demand is linear they consume on average the same quantity, but by risk aversion they suffer an average welfare loss. The planner can improve on this allocation. The policy consists on imposing that firms adjust prices until the proportion of informed reaches a threshold where all firms post perfect information prices and there are no distortions. If $T$ is large enough, the policy increases total average welfare.

The loss of welfare associated with the externality stands in stark contrast with the nature of the rigidity at the individual level. Indeed, there is a sense in which this rigidity is optimal in the local economy. The rigidity resulting from criterion $S$ can be thought as an optimal arrangement between firms and consumers in order to react to aggregate shocks in the presence of asymmetric information (for instance, as in Maskin and Tirole 1992). However, in the aggregate, this rigidity implies an externality that creates a welfare loss[27].

The positive implications of nonlinear learning for the dynamics of the economy are clear: given that information spreads slowly initially, nonlinear learning implies that – as long as $\alpha_0$ is small – there is a significant delay and persistence in the aggregate responses a monetary shock. In the remainder of this section I develop a more detailed analysis of the dynamic response of output.

## 4.3 Dynamics of output

Due to the presence of pooling equilibria, the model is able to generate a procyclical effect of monetary shocks on output. Every time an informed consumer is paired with a firm playing the $p^*$-Pooling Equilibrium, he buys a different quantity than under perfect information. If the state is high, this consumer buys more than under perfect information. If the state is low, he buys less than under perfect information.

Figure 5 shows percentage deviations from steady state (or perfect information benchmark) in the dynamics of output in the high and low states, for different values of the relative ratio of

[27] In a related paper, Miccoli (2011) also studies (in a different economy) regulation under informational externalities.

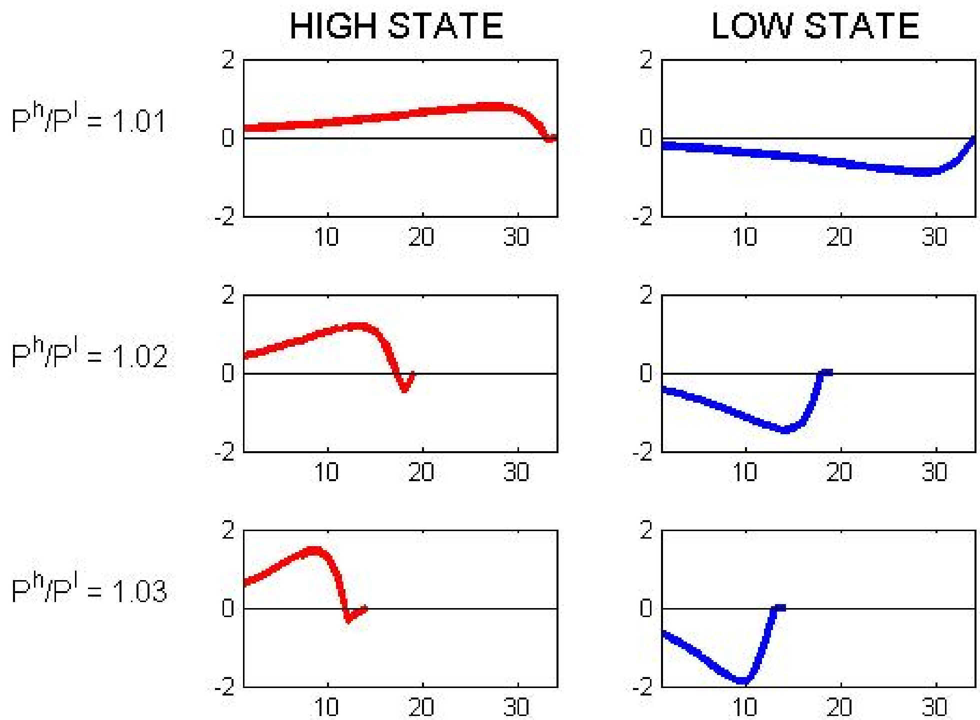


Figure 5: Dynamics of Output for Different Values of $P^h/P^l$ (y-axis units are percentage points).

the price level across states $P^h/P^l$. Output at $t$ is given by aggregate consumption of good $c$ at time $t$. In this simulation preferences are quadratic ($u(c_{it}) = c_{it} - \frac{1}{2}c_{it}^2$), marginal costs are linear ($k(c;\theta_j) = \theta_j c$), the distribution of $\theta_j$ is uniform ($\theta_j \sim U[0,1]$), and the initial proportion of informed consumers is $\alpha_0 = .15$. The steady state is defined as the level of consumption under perfect information[28].

Several features of these plots are worth highlighting. First, the dynamics of output are hump-shaped, which is an important feature of the data (Christiano, Eichenbaum, and Evans 2005). The next proposition establishes analytically, for the case of a low state, that the dynamics of output are hump-shaped as long as $\alpha_0$ is small enough.

**Proposition 7** *If the state is low and Assumption 3 holds, there is $\alpha^H > 0$ s.t., if $\alpha_0 \leq \alpha^H$, the response of output is hump-shaped.*

The proof is in the Appendix. When the state is high it is difficult to show this result for

[28]The supplementary material posted on my webpage shows that in general equilibrium the level of consumption of good $c$ at time $t$ does not affect the consumption level of the final good at $T$.

general functional forms. This is due to the presence of the distortion at the top in the Best Separating Equilibrium[29]. This distortion enters the expression for the evolution of the dynamic effects of money, and without more assumptions it is not possible to sign it for low values of $\alpha_0$. However, Figure 5 shows numerically that with quadratic preferences the response of output in this case is also hump-shaped. This is indeed the case for a large set of parameter values.

The logic behind this result is quite simple. To get intuition, consider a economy composed by only one firm and a unit mass of consumers. Suppose that the state is low and that there are three periods. Suppose that the proportion of informed consumers grows exogenously and is given by three values $\alpha_1$, $\alpha_2$ and $\alpha_3$, where $\alpha_1 < \alpha_2 < \alpha_3$. Suppose also that this firm has a cutoff $\alpha^*$ as in Section 3, and that

$$\alpha_1, \alpha_2 < \alpha^*$$

and

$$\alpha_3 > \alpha^* \quad .$$

Figure 6 plots demand and output for this firm, as deviations from the perfect information level of demand. At period 1 there is a negative deviation. At period 2 the deviation is strictly bigger (in absolute value), since the proportion of informed is strictly bigger and the firm is still posting a rigid price. Therefore, between periods 1 and 2, the procyclical effect is increasing in absolute value. In period 3 the firm adjusts the price, consumers learn, and the level of demand goes back to normal. Altogether, these dynamics imply a hump in the response of output. This example shows that as long as there is a range where the effect is increasing, the shape of the response will be hump-shaped, given that in the long run the economy goes back to normal. For this it is only needed that the initial proportion of informed consumers is small enough[30]. Notice, then, that the model generates hump-shaped responses as an outcome of the same set of assumptions generating the rigidity itself, i.e. consumers are imperfectly informed and they learn from firms' prices[31].

The second feature to notice from the dynamics of output in Figure 5 is that their shape changes with the size of the shock (i.e. with the ratio between the long-run price level in the two states $P^h/P^l$). When shocks are large, firms change prices faster and this allows for faster

[29] See Proposition 1 and Figure 3.2.

[30] Of course, in the model, the proportion of firms playing the $p^*$-Pooling Equilibrium is decreasing over time, potentially leading to a monotonically decreasing response of output. However, the proof of Proposition 7 shows that for low enough values of $\alpha_0$ this effect is negligible.

[31] Using rational inattention, Mackowiak and Wiederholt (2010) also obtain hump-shaped dynamics due to endogenous learning among consumers.

Figure 6: Intuition for the hump-shaped dynamics of output.

learning among consumers. The economy returns to steady state faster. In this simulation, the larger the shocks, the larger the procyclical effect of money.

A third feature is the asymmetry of the responses: the effect on output is more pronounced in the low than in the high state. The reason is the distortion at the top generated by the Best Separating Equilibrium. This distortion implies that, in the aggregate, prices increase faster than they decrease, the distortion at the top dampening[32] the procyclical effect of money in the high state[33].

To summarize, the model delivers rich dynamics of output. First, they are hump-shaped because consumers learn about the state of the world. Second, their shape varies endogenously depending on the distribution of states. Third, their shape is asymmetric: it varies depending on whether the shock leads to a positive or negative effect on output.

# 5 Conclusion

This paper presented a strategic microfoundation for price rigidities. The microfoundation is based on an informational asymmetry between firms and consumers. The basic assumption is that some consumers are less informed than firms about the aggregate state of the economy. As shown, this implies that, in equilibrium, there is an endogenous cost to adjusting prices. The level of the cost is a monotone function of the number of informed consumers. When few

[32]The reason this effect is small and only leads to a slightly dampened but positive effect of money, instead of a negative effect, is that only a few firms cause it. This is precisely because, according to Criterion $S$, firms avoid this distortion.

[33]These features seem consistent with the data. For instance, Cover (1992) documents evidence of negative monetary shocks having larger output effects. Peltzman (2000) documents that prices conditionally increase faster than they fall. Kim and Ruge-Murcia (2009) document recent evidence of asymmetric rigidity in the case of wages. However, these are only ‘hints’ and more research is needed to put the different pieces together.

consumers are informed, firms are better off not adjusting prices.

As noted in the introduction, the form of rigidity obtained here has nothing to do with nominal quantities per se. For concreteness, here I focused on a monetary economy and therefore the rigidity here is nominal. However, it is clear that similar ideas can be applied to obtain rigidities in real prices.

In the dynamic, decentralized markets environment, consumers learn from prices. There is an interesting interaction between price adjustment, and the amount of information in hand of consumers. Firms wait until enough consumers are informed to adjust prices. This generates a "reinforcement effect": the higher the proportion of informed consumers, the more firms adjust prices. However, the more firms adjust prices, the higher the proportion of informed consumers. Therefore, firms' pricing generates a nonlinearity in learning dynamics. The implication for the dynamics of aggregate adjustment is that, as long as initially few consumers are informed, adjustment is initially slow, and accelerates later.

A priori, these learning dynamics are not particular to goods markets, nor to aggregate macroeconomic shocks. A similar informational and market structure can potentially be used to generate nonlinear dynamics in other markets, as asset markets, where the diffusion of information is assumed to be of first order importance. The basic ingredients needed are that agents learn from prices, and that there is an increase in the likelihood of price adjustment for large amounts of information among agents.

Finally, I studied the implications of consumer learning for the dynamics of output. The model has rich implications: the dynamics are hump-shaped, have a different speed of adjustment for different size of shocks, and are asymmetric: different for positive and negative shocks.

# A Appendix

## A.1 Characterization of the Game

First, it is necessary to define the following well-known property for a function of two variables.

**Definition 4 (Increasing Differences Property)** *A function $f(x, y)$ has strict increasing differences in $(x, y)$ if, for $x' > x$ and $y' > y$,*

$$f(x, y') - f(x, y) < f(x', y') - f(x', y) \quad . \tag{29}$$

The following lemma shows that under some conditions this game is a standard monotonic signaling game (Cho and Sobel 1990).

**Lemma 6 (Characterization of the Game)** *If $\alpha > 0$ and $pc_i(p \cdot 1/P)$ has strict increasing differences in $(p, P)$, this is a monotonic signaling game. It satisfies:*

1. *Monotonicity.*

   *Let $\mu_i'(p)$ and $\mu_i(p)$ be two possible beliefs of the uninformed. If $\mu_i'(p) > \mu_i(p)$, then, for all $p$, $pc(p, P, \mu_i'(p)) > pc(p, P, \mu_i(p))$.*

2. *Single-crossing. For any $p' > p$, we have that, for arbitrary demand of the uninformed, $p'c(p', P^l, \mu_i(p)) \geq pc(p, P^l, \mu_i(p)) \implies p'c(p', P^h, \mu_i(p)) > pc(p, P^h, \mu_i(p))$*

**Proof.** I first prove monotonicity, and then single-crossing.

1. Monotonicity. By Assumption 1 $u'(c_i)$ is a strictly decreasing function. Thus the demand of the uninformed $c_i(pE_{\mu_i(p)}[1/P])$ is strictly increasing in $\mu_i(p)$. Therefore, for any $\mu_i'(p) > \mu_i(p)$, $pc(p, P, \mu_i'(p)) > pc(p, P, \mu_i(p))$.

2. Single-crossing.

   Consider $p$, $p'$, such that $p < p'$, and assume

   $$p'c(p', P^l, \mu_i(p)) \geq pc(p, P^l, \mu_i(p))$$

   This is equivalent to

   $$p'c(p', P^l, \mu_i(p)) - pc(p, P^l, \mu_i(p)) \geq 0$$

   Since $c(p, P, \mu_i(p))$ has strict increasing differences in $(p, P)$,

$$p'c(p', P^h, \mu_i(p)) - pc(p, P^h, \mu_i'(p)) > p'c(p', P^l, \mu_i(p)) - pc(p, P^l, \mu_i'(p)) \geq 0$$

and therefore

$$p'c(p', P^h, \mu_i(p)) > pc(p, P^h, \mu_i(p))$$

■

## A.2 Main Proofs

For supplementary proofs, see the supplementary material posted on my webpage.

### A.2.1 Proof of Proposition 2

Off equilibrium path beliefs are $\mu_i(p) = 0$ (pessimistic). Given these beliefs, the cutoff $\overline{\alpha}$ is the lowest $\alpha$ for which both the IC constraint of the high and low types ((23) and (24)) hold and thus this is an equilibrium.

I now show that if $\alpha = 0$, $\Pi(p^*) = \Pi^*$. For $\alpha = 0$,

$$\Pi(p^*) = \frac{1}{2}\frac{1}{P^h}p^* c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right) + \frac{1}{2}\frac{1}{P^l}p^* c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right) \quad . \tag{30}$$

From FOCs (22), (17) and (18), notice that

$$p^*\left[\frac{1}{2}\cdot\frac{1}{P^h} + \frac{1}{2}\cdot\frac{1}{P^l}\right] = p^h \cdot \frac{1}{P^h} = p^l \cdot \frac{1}{P^l} \quad . \tag{31}$$

Also,

$$c_i\left(p^*\left[\frac{1}{2}\cdot\frac{1}{P^h} + \frac{1}{2}\cdot\frac{1}{P^l}\right]\right) = c_i\left(p^h\frac{1}{P^h}\right) = c_i\left(p^l\frac{1}{P^l}\right) \equiv c^{ss} \quad . \tag{32}$$

Together, (31) and (32) imply that the right hand side of (30) is equal to $\Pi^*$.

More generally,

$$\Pi(p^*) = \alpha\left(\frac{1}{2}\frac{1}{P^h}p^* c_i\left(p^*\frac{1}{P^h}\right) + \frac{1}{2}\frac{1}{P^l}p^* c_i\left(p^*\frac{1}{P^l}\right)\right) + (1-\alpha)\Pi^* \quad . \tag{33}$$

Since the revenue function is single-peaked, this is strictly decreasing in $\alpha$ .

■

### A.2.2 Proof of Proposition 3

Since the Best Separating Equilibrium is the least cost separating equilibrium, in all other separating equilibria the price posted by the high type is greater than $\overline{p}$. Thus, in all other separating equilibrium, ex-ante profits $\Pi(p(P))$ are lower than in the Best Separating Equilibrium. All other separating equilibria are therefore ex-ante dominated.

In the Best Separating Equilibrium, $\Pi(p(P))$ is continuous and strictly increasing in $\alpha$. From (33), $\Pi(p^*)$ is continuous at $\alpha = 0$, reaches $\Pi^*$ at $\alpha = 0$, and it is strictly decreasing thereafter. Thus, there is a boundary $[0, \alpha^*]$ away from $\alpha = 0$ where $\Pi(p(P))$ is strictly higher in the $p^*$-Pooling Equilibrium than in the Best Separating Equilibrium. ■

### A.2.3 Proof of Proposition 4

Pick $\alpha^S = \underline{\alpha}$. In the Best Separating Equilibrium $\Pi(p(P)) = \Pi^*$ for $\alpha \geq \alpha^S$. Because the revenue function is single-peaked, in any potential pooling equilibrium at $\tilde{p}$, if $\alpha \geq \underline{\alpha}$, $\Pi(\tilde{p}) < \Pi^*$. ■

### A.2.4 Proof of Proposition 6

Notice that

$$\frac{d}{dt}\left(\frac{d\alpha_t}{dt}\right) = \frac{d}{d\alpha_t}\left(\frac{d\alpha_t}{dt}\right) \cdot \frac{d\alpha_t}{dt} \quad ,$$

and therefore $Sign\left(\frac{d}{dt}\left(\frac{d\alpha_t}{dt}\right)\right) = Sign\left(\frac{d}{d\alpha_t}\left(\frac{d\alpha_t}{dt}\right)\right)$. Now,

$$\frac{d}{d\alpha_t}\left(\frac{d\alpha_t}{dt}\right) = (1-\alpha_t)\beta'(\alpha_t) - \beta(\alpha_t) \quad .$$

Since

$$\lim_{\alpha \to 0} \beta(\alpha) = 0 \quad ,$$

and

$$\lim_{\alpha \to 0} \beta'(\alpha) > 0 \quad ,$$

there exists $\alpha_0 > 0$ such that

$$\frac{d}{d\alpha_t}\left(\frac{d\alpha_t}{dt}\right) \geq 0 \quad .$$

■

### A.2.5 Proof of Proposition 7

Suppose that $\alpha^*(k_j)$ is strictly increasing. Deviations from the steady state level of output are

$$\begin{aligned} D(\alpha_t) &= \int_{k^*(\alpha_t)}^{\infty} \alpha_t \left[ c_i\left(p^* \frac{1}{P^l}\right) - c_i\left(p^l \frac{1}{P^l}\right)\right] f(k)dk \\ &+ \int_{k^*(\alpha_t)}^{\infty} (1-\alpha_t) \left[ c_i\left(p^* \left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right) - c_i\left(p^l \frac{1}{P^l}\right)\right] f(k)dk \\ &+ \int_{0}^{k^*(\alpha_t)} \left[ c_i\left(p^l \frac{1}{P^l}\right) - c_i\left(p^l \frac{1}{P^l}\right)\right] f(k)dk \\ &= \int_{k^*(\alpha_t)}^{\infty} \alpha_t \left[ c_i\left(p^* \frac{1}{P^l}\right) - c_i\left(p^l \frac{1}{P^l}\right)\right] f(k)dk \quad , \end{aligned} \tag{34}$$

where I used that $c_i\left(p^l \frac{1}{P^l}\right) = c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right)$. By differentiating this expression with respect to $\alpha_t$ one gets

$$\frac{d}{d\alpha_t} D(\alpha_t) = -\alpha_t \left[ c_i\left(p^* \frac{1}{P^l}\right) - c_i\left(p^l \frac{1}{P^l}\right)\right] f(k^*(\alpha_t)) \frac{d}{d\alpha_t} k^*(\alpha_t)$$

$$+ \int_{k^*(\alpha_t)}^{\infty} \left\{ c_i\left(p^* \frac{1}{P^l}\right) - c_i\left(p^l \frac{1}{P^l}\right)\right.$$

$$\left. +\alpha_t \left[ c_i'\left(p^* \frac{1}{P^l}\right)\frac{1}{P^l} - c_i'\left(p^*\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right)\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right] \frac{d}{dk} p^*(k^*(\alpha)) \frac{d}{d\alpha_t} k^*(\alpha_t) \right\} f(k)dk \quad , \tag{35}$$

Now,

$$\frac{d}{dt} D(\alpha_t) = \frac{d}{d\alpha_t} D(\alpha_t) \frac{d\alpha_t}{dt}$$

and $\frac{d\alpha_t}{dt} \geq 0$, therefore $Sign\left(\frac{d}{dt} D(\alpha_t)\right) = Sign\left(\frac{d}{d\alpha_t} D(\alpha_t)\right)$.
Now

$$\lim_{\alpha_0 \to 0} k^*(\alpha_0) = 0 \quad ,$$

and therefore, since

$$c_i\left(p^* \frac{1}{P^l}\right) - c_i\left(p^l \frac{1}{P^l}\right) \leq 0 \quad ,$$

we have that

$$\lim_{\alpha_0 \to 0} \frac{d}{d\alpha_t} D(\alpha_0) \leq 0$$

and thus there is $\alpha^H$ such that, $\forall \alpha_0 \leq \alpha_H$, $\frac{d}{d\alpha_t} D(\alpha_0) \leq 0$. Therefore, the response of output is increasing (in absolute value) for low enough $\alpha_0$. It converges to zero in the long run. It has then a hump-shaped form.

The proof in the case of strictly decreasing $\alpha^*(\cdot)$ is similar.

■

## References


Amador, M. and P.-O. Weill (2010). Learning from prices: public communication and welfare. *Journal of Political Economy 118*, 866–907.

Banerjee, A. V. (1992). A simple model of herd behavior. *Quarterly Journal of Economics 57*(3), 797–817.

Benabou, R. and R. Gertner (1993). Search with learning from prices: does increased inflationary uncertainty lead to higher markups. *Review of Economic Studies 60*(1), 69–93.

Blinder, A., E. Canetti, D. Lebow, and J. Rudd (1998). *Asking About Prices: A New Approach to Understanding Price Stickiness*. New York. Russell Sage Foundation.

Burnside, C., M. Eichenbaum, and S. Rebelo (2011). Understanding booms and busts in housing markets. *Working Paper*.

Cho, I.-K. and D. M. Kreps (1987). Signaling games and stable equilibria. *Quarterly Journal of Economics 102*(2), 179–221.

Cho, I.-K. and J. Sobel (1990). Strategy stability and uniqueness in signaling games. *Journal of Economic Theory 50*, 381–413.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Cover, J. P. (1992). Asymmetric effects of positive and negative money-supply shocks. *Quarterly Journal of Economics 107*(4), 1261–1282.

Duffie, D. and G. Manso (2007). Information percolation in large markets. *American Economic Review Papers and Proceedings 97*, 203–209.

Fehr, E., G. Kirchsteiger, and A. Riedl (1993). Does fairness prevent market clearing? *Quarterly Journal of Economics 108*.

Fogli, A. and L. Veldkamp (2011). Nature or nurture? Learning and the geography of female labor force participation. *Econometrica 79*(4), 1103–1138.

Golosov, M., G. Lorenzoni, and A. Tsyvinski (2009). Decentralized trading with private information. *Mimeo, MIT*.

Greenwald, B. and J. E. Stiglitz (1989). Toward a theory of rigidities. *American Economic Review Papers and Proceedings 79*(2), 364–369.

Hall, R. and C. Hitch (1939). Price theory and business behavior. *Oxford Economic Papers 2*, 12–45.

Heidhues, P. and B. Köszegi (2005). The impact of consumer loss aversion on pricing. *Mimeo, University of Bonn and University of California, Berkeley*.

Hellwig, C. and V. Venkateswaran (2009). Setting the right prices for the wrong reasons. *Journal of Monetary Economics 56*, 57–77.

Jones, L. and R. E. Manuelli (2002). Volatile policy and private information: The case of monetary shocks. *Journal of Economic Theory 99*, 265–296.

Kahneman, D., J. L. Knetsch, and R. H. Thaler (1986). Fairness as a constraint on profit seeking: Entitlements in the market. *American Economic Review 76*(4), 728–741.

Kennan, J. (2010). Private information, wage bargaining and employment fluctuations. *Review of Economic Dynamics 77*, 633–664.

Kim, J. and F. J. Ruge-Murcia (2009). How much inflation is necessary to grease the wheels? *Journal of Monetary Economics 56*, 365–377.

Lagos, R. and R. Wright (1995). A unified framework for monetary theory and policy analysis. *Journal of Political Economy 113*(31), 463–484.

Lucas, R. E. (1972). Expectations and the neutrality of money. *Journal of Economic Theory 4*, 103–124.

Lucas, R. E. and N. L. Stokey (1987). Money and interest in a cash-in-advance economy. *Econometrica 55*(3), 491–513.

Mackowiak, B. and M. Wiederholt (2009). Optimal sticky prices under rational inattention. *American Economic Review 99*(3), 769–803.

Mackowiak, B. and M. Wiederholt (2010). Business cycle dynamics under rational inattention. *mimeo*.

Mailath, G. J., M. Okuno-Fujiwara, and A. Postlewaite (1993). Belief-based refinements in signaling games. *Journal of Economic Theory 60*.

Mankiw, G. and R. Reis (2002). Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve. *Quarterly Journal of Economics 117*(4), 1295–1328.

Maskin, E. and J. Tirole (1992). The principal-agent relationship with an informed principal, II: Common values. *Econometrica 60*(1), 1–42.

Matejka, F. (2010). Rigid pricing and rationally inattentive consumer. *Working Paper, Princeton*.

Miccoli, M. (2011). Optimal dynamic public communication. *Working paper, Stanford*.

Nakamura, E. and J. Steinsson. Price setting in a forward-looking customer markets. *Journal of Monetary Economics (forthcoming)*.

Okun, A. M. (1981). *Prices and Quantities: A Macroeconomic Analysis.* Brookings Institution.

Peltzman, S. (2000). Prices rise faster than they fall. *Journal of Political Economy 108*(3), 466–502.

Reis, R. (2006). Inattentive producers. *Review of Economic Studies 73*, 793–821.

Rotemberg, J. J. (2005). Customer anger at price increases, changes in the frequency of prices changes and monetary policy. *Journal of Monetary Economics* (52), 829–852.

Wilson, R. (2002). A bargaining model of monetary nonneutrality. *Mimeo, Stanford*.

# Supplementary Material for “Consumers’ Imperfect Information and Price Rigidities”

Jean-Paul L’Huillier
September 2011

**Preamble:** For printing convenience, I have split the original 55-pages paper into the main document and this one. This document contains additional results: supplementary proofs, the model with marginal costs, a model with three types, and the cash-in-advance general equilibrium framework.

## A.3 Supplementary Proofs

### A.3.1 Proof of Proposition 1

The cutoff $\underline{\alpha}$ is obtained using the Incentive Compatibility (IC) constraint for the low type (16). This inequality states that if the low type imitates the high type, the $1-\alpha$ proportion of uninformed consumers believe that he is the high type (and have beliefs $\mu_i(p)=1$). However, informed consumers know that he is the low type (and their beliefs are fixed at $\mu_i=0$).

Because of Assumptions (1) and (2), the game has the single crossing property (strictly) and therefore $\underline{\alpha}<1$. Indeed, in equation (16) we have that

$$p^h c_i\left(p^h \frac{1}{P^h}\right) > p^h c_i\left(p^h \frac{1}{P^l}\right)$$

and therefore $\underline{\alpha}<1$.

Once this cutoff obtained, there are two cases:

- $\alpha \geq \underline{\alpha}$.

  In this case, (16) is satisfied at $p^h$ and $p^l$ defined by (17) and (18). Firms optimization in each state yields $p^h$ and $p^l$. Therefore, this is the Best Separating Equilibrium, and if off equilibrium path beliefs are $\mu_i(p)=0$ (pessimistic), there are no deviations for either type. Since consumers uses Bayes’ rule and have beliefs $\mu_i(p)=1$ when facing the high type and $\mu_i=0$ when facing the low type, ex-ante profits are $\Pi^*$.

- $\alpha < \underline{\alpha}$.

  In this case, the IC constraint for the low type is satisfied for a price $\overline{p}$ defined by (21). Because the game has the single-crossing property $\overline{p}$ always exists and is s.t. $\overline{p}>p^h$. The

low type posts $p^l$ and gets the highest profits possible. The high type posts $\overline{p}$ and gets the highest profits possible ensuring he is not imitated by the low type. If off equilibrium path beliefs are $\mu_i(p) = 0$ (pessimistic), then there are no profitable deviations for either type. The low type does not deviate from its perfect information optimal price. For the high type, write the optimal deviation

$$\tilde{p} = \arg\max\left\{p\left(\alpha c_i\left(p\frac{1}{P^h}\right) + (1-\alpha)c_i\left(p\frac{1}{P^l}\right)\right)\right\} \quad .$$

We need to check that

$$\overline{p}c_i\left(\overline{p}\frac{1}{P^h}\right) \geq \tilde{p}\left(\alpha c_i\left(\tilde{p}\frac{1}{P^h}\right) + (1-\alpha)c_i\left(\tilde{p}\frac{1}{P^l}\right)\right) \quad . \tag{36}$$

The LHS of (36) can be written

$$\overline{p}c_i\left(\overline{p}\frac{1}{P^h}\right) = (1-\alpha)\overline{p}c_i\left(\overline{p}\frac{1}{P^h}\right) + \alpha\overline{p}c_i\left(\overline{p}\frac{1}{P^h}\right) \pm \alpha\overline{p}c_i\left(\overline{p}\frac{1}{P^l}\right). \tag{37}$$

From (21) we know that

$$\overline{p}\left(\alpha c_i\left(\overline{p}\frac{1}{P^l}\right) + (1-\alpha)c_i\left(\overline{p}\frac{1}{P^h}\right)\right) = p^l c_i\left(p^l\frac{1}{P^l}\right) \quad .$$

and thus (37) is

$$= \alpha\left[\overline{p}c_i\left(\overline{p}\frac{1}{P^h}\right) - \overline{p}c_i\left(\overline{p}\frac{1}{P^l}\right)\right] + p^l c_i\left(p^l\frac{1}{P^l}\right)$$

that by single-crossing revenue function and strict increasing differences

$$> \alpha\left[\tilde{p}c_i\left(\tilde{p}\frac{1}{P^h}\right) - \tilde{p}\left(\tilde{p}\frac{1}{P^l}\right)\right] + \tilde{p}c_i\left(\tilde{p}\frac{1}{P^l}\right) \quad ,$$

showing that (36) holds.

From (16), $\overline{p}$ is strictly decreasing in $\alpha$, and therefore $\Pi(p(P))$ is strictly increasing in $\alpha$, as claimed.

This completes the proof.

■

### A.3.2 Proof of Lemma 5

Under laissez-faire, firms always play the $p^*$-Pooling Equilibrium. Given that $\alpha_0 > 0$, there is a distortion in the consumption bundle of informed at every instant $t \in [0, T]$. Define the perfect

information consumption of good $c$

$$c^{ss} = c_i\left(p^h \frac{1}{P^h}\right) \quad .$$

Because of the distortion, informed consumers instantaneous ex-ante utility

$$\frac{1}{2}u\left(c_{it}\left(p^* \frac{1}{P^h}\right)\right) + \frac{1}{2}u\left(c_{it}\left(p^* \frac{1}{P^l}\right)\right) <$$

$$u\left(\frac{1}{2}c_{it}\left(p^* \frac{1}{P^h}\right) + \frac{1}{2}c_{it}\left(p^* \frac{1}{P^l}\right)\right) = c^{ss} \quad ,$$

where in the last step I used the linearity of $c_i(\cdot)$.

Under regulation, after $\overline{T}$ instantaneous utility of all consumers is $u(c^{ss})$. Thus, if $T - \overline{T}$ is large enough, welfare under regulation is strictly higher than welfare under laissez-faire.

■

## A.4 Results in the Presence of Marginal Costs

As argued in the body of the text, all results of Section 3 can be extended to the case of marginal costs proportional to the price level $P$. In this appendix I prove this claim.

Suppose the monopolist's cost function is of the form $k(c_i(p/P)) \cdot P$. Notice that this cost function is proportional to the price level $P$. For tractability, I assume the function $k(\cdot)$ is known by both the firm and consumers. I make the following assumption about this function and the implied profit function.

**Assumption 5** *The profit function $\pi(p, P) = pc_i(p/P) - k(c_i(p/P))P$ is twice continuously differentiable on $R_{++}$, single-peaked at a maximum, and has strict increasing differences in $(p, P)$.*

The following lemma states that, under perfect information, the optimal price of the monopolist is proportional to the price level $P$.

**Lemma 7** *When all consumers know the value of the price level, the monopolist's price is proportional to the price level.*

**Proof.** Under perfect information the monopolist's problem is

$$\max_p \left\{ pc_i\left(p\frac{1}{P}\right) - k\left(c_i\left(p\frac{1}{P}\right)\right)P \right\} \quad .$$

Taking the first order condition delivers

$$c\left(p\frac{1}{P}\right)+p\frac{1}{P}c_i'\left(p\frac{1}{P}\right)-k'\left(c_i\left(p\frac{1}{P}\right)\right)c_i'\left(p\frac{1}{P}\right)=0 \quad .$$

From this equation it is clear that $p$ is proportional to $P$. ■

The following proposition characterizes the Best Separating Equilibrium and ex-ante real profits in this equilibrium. It generalizes Proposition 1.

**Proposition 8 (Best Separating Equilibrium)** *The following is the Best Separating Equilibrium. Define $\underline{\alpha}$ by*

$$\begin{aligned} p^l c_i\left(p^l\frac{1}{P^l}\right)-k\left(c_i\left(p^l\frac{1}{P^l}\right)\right)P^l \quad &= \quad \underline{\alpha}\left[p^h c_i\left(p^h\frac{1}{P^l}\right)-k\left(c_i\left(p^h\frac{1}{P^l}\right)\right)P^l\right] \\ &\quad +(1-\underline{\alpha})\left[p^h c_i\left(p^h\frac{1}{P^h}\right)-k\left(c_i\left(p^h\frac{1}{P^h}\right)\right)P^l\right] \quad , \end{aligned} \tag{38}$$

*where*

$$p^h=\arg\max_p\left\{pc_i\left(p\frac{1}{P^h}\right)-k\left(c_i\left(p\frac{1}{P^h}\right)\right)P^h\right\} \quad , \tag{39}$$

$$p^l=\arg\max_p\left\{pc_i\left(p\frac{1}{P^l}\right)-k\left(c_i\left(p\frac{1}{P^l}\right)\right)P^l\right\} \quad . \tag{40}$$

*Then, $\alpha<1$ and,*

- *if $\alpha\geq\underline{\alpha}$:*
  - *The firm posts the same prices as in the perfect information benchmark, $p^h$ and $p^l$. Moreover, for a given equilibrium set of prices $p(P)$, define ex-ante real profits as*

    $$\Pi(p(P))=\frac{1}{2}\frac{1}{P^h}\pi(P^h)+\frac{1}{2}\frac{1}{P^l}\pi(P^l) \quad ,$$

    *where $\pi(P)=pc(p,P,\mu_i(p))$: in this case, ex-ante real profits $\Pi(p(P))$ are equal to ex-ante real profits in the perfect information benchmark:*

    $$\Pi^*=\frac{1}{2}\frac{1}{P^h}\pi^*(P^h)+\frac{1}{2}\frac{1}{P^l}\pi^*(P^l) \quad ,$$

    *where $\pi^*(P_h)=\max_p pc(p/P^h)-k(c(p/P^h))P^h$ and $\pi^*(P_h)=\max_p pc(p/P^l)-k(c(p/P^l))P^l$.*
- *If $\alpha<\underline{\alpha}$:*
  - *The firm posts $p^l$ and $\overline{p}>p^h$ such that*

$$
\begin{aligned}
p^l c_i\left(p^l \frac{1}{P^l}\right) - k\left(c_i\left(p^l \frac{1}{P^l}\right)\right) P^l \quad &= \quad \alpha\left[\overline{p} c_i\left(\overline{p}\frac{1}{P^l}\right) - k\left(c_i\left(\overline{p}\frac{1}{P^l}\right)\right) P^l\right] \\
&\quad +(1-\alpha)\left[\overline{p} c_i\left(\overline{p}\frac{1}{P^h}\right) - k\left(c_i\left(\overline{p}\frac{1}{P^h}\right)\right) P^l\right] \quad .
\end{aligned}
$$

*In this case, $\overline{p}$ is strictly decreasing and $\Pi(p(P))$ is strictly increasing in $\alpha$.*

**Proof (sketch).** Given Assumptions 1 and 5, the objective function of the monopolist is single-peaked and satisfies the single-crossing property. Then, the proof is identical to the proof of Proposition 1. ■

**Proposition 9 ($p^*$-Pooling Equilibrium)** *Consider $p^*$ such that*

$$
p^* = \arg\max_p \left\{ p c_i\left(p\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right) - k\left(p\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right)\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]^{-1} \right\} \quad ,
$$

*For given $P^h$ and $P^l$, suppose that there is $\overline{\alpha}$ such that*

$$
\begin{aligned}
&\overline{\alpha}\left[p^* c_i\left(p^*\frac{1}{P^h}\right) - k\left(c_i\left(p^*\frac{1}{P^h}\right)\right) P^h\right] \\
&+(1-\overline{\alpha})\left[p^* c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right) - k\left(c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right)\right) P^h\right] \\
&\geq \max_p \left\{\overline{\alpha}\left[c_i\left(p^*\frac{1}{P^h}\right) - k\left(c_i\left(p^*\frac{1}{P^h}\right)\right) P^h\right]\right. \\
&\left. + (1-\overline{\alpha})\left[p^* c_i\left(p^*\frac{1}{P^l}\right) - k\left(c_i\left(p^*\frac{1}{P^l}\right)\right) P^h\right]\right\}
\end{aligned}
\tag{41}
$$

*and*

$$
\begin{aligned}
&\overline{\alpha}\left[p^* c_i\left(p^*\frac{1}{P^l}\right) - k\left(c_i\left(p^*\frac{1}{P^l}\right)\right) P^l\right] \\
&+(1-\overline{\alpha})\left[p^* c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right) - k\left(c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h} + \frac{1}{2}\frac{1}{P^l}\right]\right)\right) P^l\right] \\
&\geq p^l c_i\left(p^l\frac{1}{P^l}\right) - k\left(c_i\left(p^l\frac{1}{P^l}\right)\right) P^l \quad .
\end{aligned}
$$

*Consider the lowest possible $\overline{\alpha}$. Then, for all $\alpha \leq \overline{\alpha}$, there exists a pooling equilibrium at $p^*$.*

*If $\alpha = 0$, this equilibrium reaches ex-ante profits. Moreover, ex-ante profits $\Pi(p^*)$ are strictly decreasing in $\alpha$.*

**Proof.** Off equilibrium path beliefs are $\mu_i(p) = 0$. Given these beliefs, for all $\alpha \leq \overline{\alpha}$ the IC constraints for both the high and low types ((41) and (42)) are satisfied and thus this is an equilibrium.

I now show that if $\alpha = 0$, $\Pi(p^*) = \Pi^*$. For $\alpha = 0$,

$$\begin{aligned} \Pi(p^*) &= \frac{1}{2}\frac{1}{P^h}\left[p^* c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h}+\frac{1}{2}\frac{1}{P^l}\right]\right) - k\left(c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h}+\frac{1}{2}\frac{1}{P^l}\right]\right)\right)P^h\right] \\ &+ \frac{1}{2}\frac{1}{P^l}\left[p^* c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h}+\frac{1}{2}\frac{1}{P^l}\right]\right) - k\left(c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h}+\frac{1}{2}\frac{1}{P^l}\right]\right)\right)P^l\right] \quad . \end{aligned} \tag{42}$$

Similar to Proposition 2,

$$c_i\left(p^*\left[\frac{1}{2}\frac{1}{P^h}+\frac{1}{2}\frac{1}{P^l}\right]\right) = p^h c_i\left(p^h\frac{1}{P^h}\right) = p^l c_i\left(p^l\frac{1}{P^l}\right) \equiv c^{ss} \quad ,$$

where $p^h$ and $p^l$ are defined by (39) and (40). Thus,

$$\Pi^*(p^*) = \left[\frac{1}{2}\frac{1}{P^h}+\frac{1}{2}\frac{1}{P^l}\right][p^* c^{ss}] - \frac{1}{2}\frac{1}{P^h}\left[k\left(c^{ss}\right)P^h\right] - \frac{1}{2}\frac{1}{P^l}\left[k(c^{ss})P^l\right] = \Pi^* \quad .$$

More generally,

$$\begin{aligned} \Pi(p^*) &= \alpha\left(\frac{1}{2}\frac{1}{P^h}\left[p^* c_i\left(p^*\frac{1}{P^h}\right) - k\left(c_i\left(p^*\frac{1}{P^h}\right)\right)P^h\right]\right. \\ &+ \left.\frac{1}{2}\frac{1}{P^l}\left[p^* c_i\left(p^*\frac{1}{P^l}\right) - k\left(c_i\left(p^*\frac{1}{P^l}\right)\right)P^l\right]\right) + (1-\alpha)\Pi^* \quad . \end{aligned} \tag{43}$$

Since the profit function is single-peaked, this is strictly decreasing in $\alpha$.

■

As shown in this proof the key to the result that, when $\alpha = 0$, $p^*$ is ex-ante optimal relies on the fact that ex-ante real costs are the same as under perfect information.

Under Assumption 5, and having established Propositions 8 and 9, it is straightforward to extent Propositions 3 and 4 to the presence of marginal costs.

## A.5 The Model with Three Types

In this Appendix I show how the number of types can be augmented. This is an easy task due to three basic properties of the model: the monotonicity of the game, the single-crossing property, and the independence of demand from income. Together, these three properties ensure that a) the Best Separating Equilibrium is similar to the one presented in Proposition 1, b) the $p^*$-Pooling Equilibrium is also similar to the one in Proposition 2, and c) the results on ex-ante profits (Propositions 3 and 4, and Lemma 2) and Criterion $S$ follow through. Here I characterize equilibria with three types.

**The Game with Three Types.** The price level $P$ is now drawn over $\mathfrak{P} = \{P^H, P^M, P^L\}$, where $P^H > P^M > P^L$ and $Pr(P = P^H) = Pr(P = P^M) = Pr(P = P^L) = 1/3$. I call "low type" to the firm that knows that the state is $P^L$, "medium type" to the firm that knows that the state is $P^M$, and "high type" to the firm that knows that the state is $P^H$. Uninformed consumers' beliefs are a probability distribution over $\mathfrak{P}$ defined by two mappings

$$\mu_i^H : R_+ \longrightarrow [0,1] \quad ,$$

and

$$\mu_i^M : R_+ \longrightarrow [0,1] \quad ,$$

that assign probabilities to the high and medium states.

All other definitions of the problem remain the same.

**Proposition 10 (Best Separating Equilibrium)** *The following is the Best Separating Equilibrium. For a given $\alpha$, the low type posts $p(P^L) = p^L$*

*then the medium type posts* $p(P^M) = p^M$. *Otherwise, the medium type posts* $p(P^M) = \overline{p}^M$ *such that*

$$p^L c_i\left(p^L \frac{1}{P^L}\right) = \overline{p}^M\left(\alpha c_i\left(\overline{p}^M \frac{1}{P^L}\right) + (1-\alpha)c_i\left(\overline{p}^M \frac{1}{P^M}\right)\right) \quad . \tag{47}$$

*Consider* $p^H$ *such that*

$$p^H = \arg\max_p pc_i\left(p\frac{1}{P^H}\right) \quad . \tag{48}$$

*If*

$$p(P^M)c_i\left(p(P^M)\frac{1}{P^M}\right) > p^H\left(\alpha c_i\left(p^H \frac{1}{P^M}\right) + (1-\alpha)c_i\left(p^H \frac{1}{P^H}\right)\right) \quad , \tag{49}$$

*then the high type posts* $p(P^H) = p^H$. *Otherwise, the high type posts* $p(P^H) = \overline{p}^H$ *such that*

$$p(P^M)c_i\left(p(P^M)\frac{1}{P^M}\right) = \overline{p}^H\left(\alpha c_i\left(\overline{p}^H \frac{1}{P^M}\right) + (1-\alpha)c_i\left(\overline{p}^H \frac{1}{P^H}\right)\right) \quad . \tag{50}$$

*Define ex-ante real profits by*

$$\Pi(p(P)) = \frac{1}{3}\frac{1}{P^L}\pi(P^L) + \frac{1}{3}\frac{1}{P^M}\pi(P^M) + \frac{1}{3}\frac{1}{P^H}\pi(P^H) \quad , \tag{51}$$

*where* $\pi(P) = pc(p, P, \mu_i(P))$. *Then,* $\Pi(p(P))$ *is (weakly) increasing in* $\alpha$.

**Proof.** The low type posts $p^L$ and, if off-equilibrium path beliefs are $\mu_i^H(p) = 0$ and $\mu_i^M(p) = 0$ (pessimistic), he finds no profitable deviation. (46) (or (47)) ensures that the low type does not imitate the medium type. A fortiori, by monotonicity, he does not imitate the high type. Using the same steps as in p. 42 one can proof that there are no profitable deviations for the medium type. A similar reasoning shows that this is an equilibrium for the high type as well.

I now show that (51) is weakly increasing in $\alpha$. Consider $\alpha' > \alpha$. If types post $p^H$, $p^M$ and $p^L$, then there are no distortions and $\Pi(\alpha') = \Pi(\alpha)$. If for $\alpha$ either $p(P^M) \neq P^M$ or $p(P^H) \neq P^H$, then:

- If $p(P^H) \neq P^H$, $p(P^H)$ is strictly decreasing in $\alpha$, and therefore $\Pi(\alpha)$ is strictly increasing,
- Similarly, if $p(P^M) \neq P^M$, $p(P^M)$ is strictly decreasing in $\alpha$, and therefore $\Pi(\alpha)$ is strictly increasing.

■

**Proposition 11 ($p^*$-Pooling Equilibrium)** *Consider* $p^*$ *such that*

$$p^* = \arg\max pc_i\left(p \cdot B\right) \quad , \tag{52}$$

*where* $B = \left[\frac{1}{3} \cdot \frac{1}{P^H} + \frac{1}{3} \cdot \frac{1}{P^M} + \frac{1}{3} \cdot \frac{1}{P^L}\right]$, *and consider the highest* $\overline{\alpha}$ *such that*

$$p^* \left( \overline{\alpha} c_i \left( p^* \frac{1}{P^H} \right) + (1-\overline{\alpha}) c_i \left( p^* \cdot B \right) \right) \geq \max_p \left\{ \overline{\alpha} p c_i \left( p \frac{1}{P^H} \right) + (1-\overline{\alpha}) p c_i \left( p \frac{1}{P^L} \right) \right\} \quad , \tag{53}$$

*and*

$$p^* \left( \overline{\alpha} c_i \left( p^* \frac{1}{P^M} \right) + (1-\overline{\alpha}) c_i \left( p^* \cdot B \right) \right) \geq \max_p \left\{ \overline{\alpha} p c_i \left( p \frac{1}{P^M} \right) + (1-\overline{\alpha}) p c_i \left( p \frac{1}{P^L} \right) \right\} \quad , \tag{54}$$

*and*

$$p^* \left( \overline{\alpha} c_i \left( p^* \frac{1}{P^L} \right) + (1-\overline{\alpha}) c_i \left( p^* \cdot B \right) \right) \geq p^L c_i \left( p^L \frac{1}{P^L} \right) \quad . \tag{55}$$

*For all* $\alpha \leq \overline{\alpha}$*, there exists a pooling equilibrium at* $p^*$*. If* $\alpha = 0$*, ex-ante profits reach* $\Pi^*$*. Moreover, ex-ante profits* $\Pi(p^*)$ *are strictly decreasing in* $\alpha$*.*

The proof is similar to the proof of Proposition 2. Given that ex-ante profits are increasing in the Best Separating Equilibrium, and decreasing the the $p^*$-Pooling Equilibrium, all results concerning ex-ante profits follow through. Moreover, because of single-crossing the Best Separating Equilibrium always exists. Therefore, a criterion similar to Criterion $S$ can be used to apply this game in a monetary framework.

## A.6 The Cash in Advance General Equilibrium Framework

The goal of this Section is to show that the simple dynamic model of Section 4 is compatible with a cash in advance general equilibrium framework.

In this model money has an explicit role due to a cash in advance constraint (Lucas and Stokey 1987). In each island there is a price-setting firm. Firms are heterogeneous and differ in productivity levels.

**Time.** To simplify the exposition, time is discreet and every period indexed by $\tau = 0, \ldots$. Every period is subdivided into $N + 2$ subperiods, indexed by $t = 0, 1, \ldots, N + 1$. For the analysis of the dynamics after a monetary transfer, I am interested in the dynamics across subperiods. (The model of Section 4 is obtained by letting $N \to \infty$.)

**Geography.** There is a unit mass of islands indexed by $j \in [0, 1]$. Every island is populated by a unit mass of households, indexed by $i$. Therefore, a household in this economy is identified by a double index $ij$. On every island, households own a firm that produces a consumption good $c$. The firm is a monopolist on the island. Each one of these firms produce the same good.

**Shoppers and Workers.** Every household is divided into a shopper and a worker.

**Consumption.** In the first $N+1$ subperiods of every period $\tau$ shoppers are randomly sent to islands for consumption of good $c_t$. Therefore, in every subperiod $t \leq N+1$, every monopolist receives a new mass of shoppers that demand good $c_t$. Good $c_t$ is sold by monopolists on credit. In the rest of the paper I will refer to these goods as credit goods.

In the last subperiod $N+1$ of every period $\tau$, shoppers buy a cash good $C$ from a centralized, representative competitive firm. Goods bought in decentralized markets – credit goods – are denoted in lowercase, and the good bought in the centralized markets – the cash good – are denoted in uppercase. As it will become clear, goods bought in centralized markets will have prices that will be proportional to the money supply and will all be denoted in uppercase. (Goods bought in decentralized markets need not have prices proportional to the money supply, and will be denoted in lowercase.)

**Goods and Labor Markets.** Goods $c_t$ are sold on every island by a price-setting monopolist. Good $C$ is sold in a centralized competitive market by a representative firm. There is a centralized competitive labor market where all workers supply labor $L$. Labor is supplied in a centralized market and therefore it is denoted in uppercase. All firms in the economy hire labor from this market.

**Timing and Information Assumptions.** At the beginning of every period $\tau$ there is a monetary transfer $\nu_\tau$. I will refer to $\nu_\tau$ as the "monetary transfer" or the "monetary shock" interchangeably. This monetary transfer is not immediately revealed to households.

I know describe a sequence of events, to be repeated every subperiod.

At the first subperiod $t = 0$, every island receives a representative sample of shoppers. An exogenous proportion of shoppers $\alpha_{0\tau}$ knows the monetary transfer $\nu_\tau$ when buying credit good $c_0$. The complementary proportion $1 - \alpha_{0\tau}$ does not know the transfer when buying. In this period, firms are uninformed. Since firms sell to a representative sample of consumers, they learn the state by observing demand.

At every subperiod $t = 1, \ldots, N$, every island receives a representative sample of shoppers. In the first subperiod $t = 1$, the proportion of $\alpha_{1\tau}$ knows the monetary transfer $\nu_\tau$ when buying credit good $c_1$. The complementary proportion $1 - \alpha_{1\tau}$ does not know this transfer. At this

point firms are informed about the monetary shock, and therefore, on islands with flexible prices it is possible to learn the monetary transfer from prices. Therefore, the proportion of shoppers that know the monetary shock grows endogenously: $\alpha_{2\tau} \geq \alpha_{1\tau}$, and more generally, $\alpha_{t+1\tau} \geq \alpha_{t\tau}$, $t = 1, \ldots, N$.

Meanwhile, workers supply labor in the centralized labor market. In equilibrium, the wage $W_\tau$ is flexible and proportional to the money supply. For this reason, the wage immediately reveals the transfer to workers. Shoppers do not observe the wage.

At the last subperiod $t = N + 1$, each shopper receives the monetary transfer $\nu_\tau$ and buys the cash good $C$. At the end of the period, the worker goes back home bringing labor income, and the household pays the consumption of credit goods. Profits from firms are received, and financial markets open. At this point, all agents in the economy know the monetary transfer $\nu_\tau$. This implies that all agents know the money supply.

**Games Between Monopolists and Uninformed Shoppers.** Every time a particular informed firm and a sample of shoppers are matched, the firm and the proportion of uninformed shoppers play the following signalling game. The firm's type is determined by the monetary shock $\nu_\tau$. After observing the monetary shock, the firm posts a price $p_{jt\tau}$. Uninformed shoppers observe this price, form beliefs about the monetary shock $\mu(p_{jt\tau})$, and decide how much to demand.

**Stochastic Process for Money.** Money supply evolves as:

$$\log M_\tau = \log M_{\tau-1} + \nu_\tau \quad , \tag{56}$$

where $\nu_\tau$ is shock that can take two values, $\nu_h > 0$ or $\nu_l < 0$, both outcomes with equal probability. I further impose that

$$E\left[e^{-\nu_\tau}\right] = 1 \quad . \tag{57}$$

This centering assumption implies that the the inverse of the money supply, i.e. the real value of a 1 dollar bill, is a martingale:

$$E\left[\frac{1}{M_\tau}\right] = E\left[\frac{e^{-\nu_\tau}}{M_{\tau-1}}\right] = \frac{1}{M_{\tau-1}} \quad . \tag{58}$$

This assumption will deliver that the $p^*$-Pooling Equilibrium corresponds to posting, at period $\tau$, the perfect information price of previous period $\tau - 1$. In other words, under this assumption, the rigid price corresponds to a non price adjustment.

**Households' Problem.** Household $i$ of island $j$ faces the problem

$$\max E_{ijt\tau}\left[\sum_{\tau=0}^{\infty}\beta^{\tau}\left(\sum_{t=0}^{N}u(c_{ijt\tau})+v(C_{ij\tau})-L_{ij\tau}\right)\right] \quad , \tag{59}$$

where $c_{ijt\tau}$ is consumption of good $c$ at subperiod $t$ time $\tau$, produced by a randomly matched firm $\hat{j}$ of island $\hat{j}$, $C_{ij\tau}$ is consumption of the credit good, and $L_{ij\tau}$ is labor supplied by the worker. $\hat{j}(i,j,t,\tau)$ is a function that designates firm $\hat{j}$, that sells to household $ij$ at subperiod $t$ time $\tau$. $E_{ijt\tau}$ is an expectation taken at different stages of each period, and using the relevant decision maker's (shopper or worker) specific information set. This maximization is subject to the budget constraint

$$\sum_{t=1}^{N}p_{\hat{j}t\tau}c_{ijt\tau}+P_{\tau}C_{ij\tau}+M_{ij\tau}+B_{ij\tau}=(1+R_{\tau})B_{ij\tau-1}+M_{ij\tau-1}+\tilde{\nu}_{\tau}+W_{\tau}L_{ij\tau}+s_{ij}\Pi_{j\tau} \quad , \tag{60}$$

where $\tilde{\tau_{\tau}}$ is a lump-sum transfer consistent with the process of money[34].

The cash-in-advance constraint for consumption of good $y$ is

$$P_{\tau}C_{ij\tau}\leq M_{ij\tau-1}+\tilde{\nu}_{\tau} \quad . \tag{61}$$

At this point it is important to emphasize that households' preferences are quasilinear. By eliminating wealth effects on household choices, this assumption simplifies the analysis in three ways. First, similar to Lagos and Wright (1995), it allows to handle the heterogeneity in households' information. This heterogeneity implies different choices for credit goods $c_t$ for different shoppers. However, the absence of wealth effects implies that all other choices are in fact the same, and therefore the cross-sectional distribution of money and bond holdings collapses into a degenerate distribution. For this reason, I do not need to keep track of it. A second implication of the absence of wealth effects is that there is no option value before consuming credit goods, which simplifies the games between firms and shoppers enormously. Third, linearity in labor supply implies that every set of equilibria of these games is consistent with a general equilibrium in which the labor market clears.

**Monopolists.** On every island $j$ there is a monopolist that produces good $c_t$ using technology

$$c_{jt\tau}=A_jL_{jt\tau} \quad , \tag{62}$$

[34]More specifically, $\tilde{\nu}_{\tau}$ is such that $\tilde{\nu}_{\tau}=M_{\tau}-M_{\tau-1}$. I will show that in equilibrium all agents have the same money holdings, and therefore this is a correct way of writing the transfer is possible.

where $L_{jt\tau}$ is labor hired by firm $j$ at subperiod $t$ time $\tau$. Write the firm's real marginal costs as

$$k_j = \frac{\frac{W_\tau}{M_\tau}}{A_j} \quad , \tag{63}$$

where $W_\tau$ is the nominal wage.

**Representative Firm Producing $C$.** There is a representative firm that hires labor and produces good $C$ according to a simple linear technology. The labor productivity if this firm is normalized to 1:

$$C = L \quad .$$

**Definition of Equilibrium.** A general equilibrium of this economy is given by allocations $\{c_{ijt\tau}, C_{ij\tau}\}$, beliefs $\mu_{i\hat{j}t\tau}(p_{jt\tau})$, labor supply decisions $\{L_{ij\tau}\}$, prices $\{p_{jt\tau}, P_\tau\}$, nominal wages, $\{W_\tau\}$, interest rates $\{1 + R_\tau\}$, for all $i$, $j$, $t$, $\tau$, s.t.

1. Households' conditions for optimality and corresponding constraints are satisfied;
2. equilibrium strategies for the games played between monopolists and shoppers satisfy Bayesian Perfection:
   - monopolists post prices to maximize profits, given consumers' play,
   - uninformed shoppers use Bayes' rule on the path of equilibrium play,
   - shoppers' demand satisfies the condition for optimality;
3. the representative firm maximizes profits taking the price as given;
4. goods, labor, bonds, and money markets clear.

### A.6.1 General Equilibrium.

**Household $ij$'s Optimality Conditions.** The conditions for optimality are computed as follows. Each time the shopper of household $ij$ is matched with a monopolist, he computes the first order condition:

$$\beta^\tau u'(c_{ijt\tau}) = p_{\hat{j}t\tau} E_{\mu_{ijt\tau}}[\lambda_{ij\tau}] \quad . \tag{64}$$

$E_{ij\tau t}[\cdot]$ is an expectation taken using the shopper's information set. This information set contains information previously collected plus the information revealed by the contemporaneous price of the monopolist.

When the shopper finally arrives to buy the cash good, he computes a first order condition for consumption of this cash good after observing the price. This good is sold by a representative competitive firm in a centralized market, and therefore its price reveals the realization of the monetary shock to the shopper, in case he did not know it already. Therefore, at this point the shopper does not face any uncertainty, and the first order condition is:

$$\beta^\tau v'(C_{ij\tau}) = P_\tau(\lambda_{ij\tau} + \psi_{ij\tau}) \quad . \tag{65}$$

The worker arrives at the market for labor and computes a first order condition for labor supply after observing the equilibrium wage. This is a centralized and competitive market, and therefore this wage reveals the realization of the monetary shock to the worker. Therefore, the worker does not face any uncertainty, and the first order condition is:

$$\beta^\tau = W_\tau \lambda_{ij\tau} \quad . \tag{66}$$

The first order condition for money holdings is computed at a financial market at the end of the period, and therefore under perfect information:

$$\lambda_{ij\tau} = E_\tau[\lambda_{ij\tau+1} + \psi_{ij\tau+1}] \quad . \tag{67}$$

The first order condition for bond holdings is also computed under perfect information:

$$\lambda_{ij\tau} = (1 + R_{\tau+1})E_\tau[\lambda_{ij\tau+1}] \quad . \tag{68}$$

**Cash Good and Labor Markets.** It is possible to show that every set of PBE of the games played between monopolists and shoppers is consistent with a general equilibrium. The reason is the linearity of the disutility of labor, which implies that individual labor choices are always consistent with the resource constraint of the economy.

In equilibrium, the price of the cash good $C$ is pinned down by the cash in advance constraint, and therefore is proportional to the money supply. Optimality of production for the representative firm immediately implies that the wage $W_\tau$ is also proportional to money supply $M_\tau$. After a normalization, we have that all of these three quantities are equal:

$$P_\tau = W_\tau = M_\tau \quad . \tag{69}$$

The nominal interest rate is determined in expectation of next period's shock, and therefore is equal to a constant, for all $\tau$. Consumption of the credit good is also constant every period.

**Demand for Credit Good $c_{t\tau}$ by Shopper $ij$.** Substituting (69) and (66) into (64):

$$u'(c_{ijt\tau}) = p_{\hat{j}t\tau} E_{\mu_{ijt\tau}} \left[ \frac{1}{M_\tau} \right] \quad . \tag{70}$$

From this equation I get the demand:

$$c_{ijt\tau} \left( p_{\hat{j}t\tau} E_{\mu_{ijt\tau}} \left[ \frac{1}{M_\tau} \right] \right) \quad . \tag{71}$$

As in the partial equilibrium model of section 3, the demand for credit goods depends on a deflated version of the price posted by firms. Here, the (inverse of the) deflator is a belief about the inverse of the real value of a unit of money, and therefore the consumer transforms the nominal price posted by the monopolist into a real price. If this estimate is low (corresponding to a belief that the supply of money is high), a given nominal price is deflated into a low real price, increasing the shopper's demand.

**Aggregate Demand for $c_{t\tau}$.** At every subperiod $t\tau$ a proportion $\alpha_{t\tau}$ of shoppers know the monetary aggregate. Therefore, aggregate demand is

$$\int_{\mathcal{M}_{\hat{j}t\tau}} c_{ijt\tau}(p, M_\tau, \mu_{ijt\tau}) didj = \alpha_{t\tau} c_{ijt\tau} \left( p_{\hat{j}t\tau} \cdot \frac{1}{M_\tau} \right) + (1-\alpha_{t\tau}) c_{ijt\tau} \left( p_{\hat{j}t\tau} \cdot E_{\mu_{ijt\tau}}(p_{\hat{j}t\tau}) \left[ \frac{1}{M_\tau} \right] \right) \quad , \tag{72}$$

where $\mathcal{M}_{\hat{j}t\tau} \in [0,1] \times [0,1]$ is the subset of consumers matched with firm $\hat{j}$ at subperiod $t\tau$.

At this point, notice that the total demand (72) that every firm faces is the same as (12) in Section 3. Also, firms' production functions are linear, which implies that profit functions satisfy Assumption 5. Firms meet every consumer only once and therefore play the one-shot game described in Section 3. Thus, all results of Sections 3 and 4 follow through. Therefore, as claimed, it is possible to write a cash in advance general equilibrium framework compatible with all the results of the paper.