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Rational Inattention, Multi-Product Firms and the Neutrality of Money

*Ernesto Pasten (Central Bank of Chile)
Raphael Schoenle (Brandeis University)

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Ernesto Pasten† Raphael Schoenle‡

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Abstract

A quantitative model of rationally inattentive firms no longer generates significant monetary non-neutrality once the multi-product nature of firms is considered. The reason lies in economies of scope in information processing: as firms produce more goods, returns to gathering information on common monetary rather than good-specific shocks increase. Calibrating our model to US CPI data, where stores price a large number of goods, suggests no monetary non-neutrality. Calibrating our model to PPI data, where firms price a much smaller number of goods, leads to only modest non-neutrality, at least according to conventional benchmarks. Only if the multi-product nature of firms is ignored, do both our calibrations lead to significant monetary non-neutrality.

JEL codes: E3, E5, D8

Keywords: rational inattention, multi-production, money neutrality

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†*ernesto.pasten@tse-fr.eu*. Senior Economist, Central Bank of Chile; Research Fellow, Toulouse School of Economics.

‡*schoenle@brandeis.edu*. Assistant Professor, Department of Economics, Brandeis University.

1 Introduction

It is one of the big questions in macroeconomics whether changes in the monetary aggregates can have large real effects on the economy, or whether money is "neutral." From a theoretical point of view, the answer of course crucially depends on the approach taken to modeling price-setting. Models where price-setting is based on the Theory of Rational Inattention, which was put forward in seminal work by Sims (1998, 2003), have concluded that there can indeed be large and persistent real effects. According to this theory, humans must optimally allocate their limited capacity to process information, their "attention," to reduce observation noise of their variables of interest. In the context of the question of monetary non-neutrality, large real effects of money can then easily arise when firms decide to allocate little of their attention to monetary shocks. A small friction is usually enough to lead to such monetary non-neutrality, as argued by Sims (2010). Mackowiak and Wiederholt (2009) show that this conclusion is relevant in a quantitative model of the US economy.

In this paper, we arrive at a substantially different quantitative conclusion when we take into account the multi-product nature of firms.[1] This is because there are economies of scope in information processing implicit in the rational inattention model: Once a firm has used part of its attention to observe with a given precision the realization of a shock, the acquired information can be used at no additional cost in all pricing decisions. Such economies of scope are ignored when firms are assumed to produce a single good only. When firms produce or sell more than

[1]Several pieces of evidence strongly suggest that this is a realistic assumption. While the multi-product nature of firms has been well established in international trade such as in Bernard et al. (2010) or Goldberg et al. (2008), there is also strong evidence for the assumption in pricing: For the retail sector, the Food Marketing Institute (http://www.fmi.org/research-resources/supermarket-facts) reports that its members' stores in the US sold an average of almost 40,000 different products in 2010. The FMI is an industry-based research institution whose members are retailers and wholesalers accounting for 3=4 of all food retail sales in the U.S. For good-producing firms, 98.5% of goods listed in the Production Price Index (PPI) in the U.S. are produced by multi-product firms which produce on average about 4 goods Bhattarai and Schoenle (2011). Even if firms produce multiple goods, one might argue that prices are set by independent decision units. Evidence suggests this is not the case. First, the PPI data defines a firm as a price-setting unit, with the aforementioned average of 4 goods per such unit. Moreover, Zbaracki et al. (2004), who conduct a detailed case study of the pricing process within a firm, report that regular prices for all goods produced by the firm are decided at a centralized level while sales prices for all goods sold in a given geographical area are decided at a local level. At both levels, a single decision unit sets multiple prices.

one good, they find more it attractive to pay attention to shocks common to all their pricing decisions. Monetary and firm-specific shocks have this common-shock property, but not good-specific shocks. We need these three types of shocks to calibrate our model to the micro price data. We then find that monetary non-neutrality in our calibrated model quickly vanishes as firms produce more goods. Uncovering these economies of scope and studying their aggregate implications are the main contributions of this paper.

To this purpose, we augment the model of Mackowiak and Wiederholt (2009) where single-product firms are hit by firm-specific shocks and nominal aggregate demand shocks (in short, "monetary shocks"). We assume that instead multi-product firms are hit by monetary, firm-specific and good-specific shocks. We then present new empirical moments from the datasets used by the Bureau of Labor Statistics (BLS) to compute the Consumer Price Index (CPI) and the Producer Price Index (PPI). We calibrate our model to match these moments and then we conduct a set of experiments with this model. We obtain a number of results that we separate into theoretical, empirical, and quantitative contributions.

Theoretically, we find that, in addition to the economies of scope explained above, there are two other forces that affect the strength of the rational inattention friction. The first is the fact that firms must pay attention to a larger number of good-specific shocks as they produce more goods. We call this the "income effect" since it resembles the situation of a consumer who sees her budget constraint more binding as their consumption basket expands to more goods. Another force is the "aggregation effect" which simply acknowledges that the information-processing capacity of firms may be correlated to the number of goods they produce.[2] While the overall effect of these forces may go in any direction, we show three theoretical results under the assumption of white noise shocks, which allows for closed-form solution: (1) Holding the extent of the friction constant, measured as the cost to firms due to the friction per good produced, attention to monetary

[2] One may presume that the information-processing capacity of firms must depend on their investment in information technologies and the way in which they organize work teams. Since there is no theory to guide us on modeling this link, we do not restrict the aggregation effect except to provide no incentives for firms to merge or delegate their pricing decisions. This condition coincides with a constant cost of the friction per good produced.

shocks increases as firms produce more goods. (2) Strategic complementarity in pricing decisions amplifies the effect on reducing monetary non-neutrality when the attention of firms to monetary shocks increases. (3) If we force the model to yield a constant degree of monetary non-neutrality as firms produce more goods, the cost of firms due to the friction per good produced increases as firms produce more goods.

In the empirical part, we present new moments from the BLS data for the CPI and PPI that are critical to quantifying the extent of monetary non-neutrality in a rational inattention model. PPI data allow us to estimate the number of goods produced by a firm, so we compute statistics after sorting firms in four bins according to the number of goods they produce. This way, we can examine how key statistics change as firms produce more goods. Since CPI data does not provide such an estimate of the number of goods, we compute statistics for the whole CPI sample.

Our main empirical finding is that log price changes only imperfectly co-move within firms. In the CPI data, 60% of the cross-sectional dispersion of log price changes is due to within-firm dispersion. In the PPI data, this ratio is increasing as firms produce more goods, from 20% (for bin 1, where firms produce between 1 and 3 goods) to 55% (for bin 4, where firms produce more than 7 goods). This finding is important because a critical assumption for our theoretical results is that good-specific shocks exist. If there are no good-specific shocks, then the number of goods produced has no effect on firms' attention allocation, and thus on monetary non-neutrality. However, in the absence of good-specific shocks, the model predicts perfect co-movement of prices within firms, which is not what we find in the data. Second, our analysis of the micro data shows that the average absolute size of price changes in the CPI data is 9.6%, which is in line with the findings of Klenow and Kryvtsov (2008). In the PPI data, this statistic is decreasing in the number of goods firms produce, from 8.5% for bin 1 to 6.5% for bin 4. Third, we find that good-level inflation has a negative serial correlation. The median estimate of an AR(1) coefficient is -.29 in the CPI data while in the PPI data it ranges from -.05 for bin 1 to -.03 to bin 4.

Our main quantitative results may be summarized in five points. We start by allowing shocks

to be persistent in the model and set our calibration benchmark as in Mackowiak and Wiederholt (2009), who target the average absolute size of price changes in the CPI data to find large and persistant real effects of money. Our findings are: (1) When we additionally target the serial correlation of per-good inflation in the CPI data, the cumulated effect of a monetary shock is cut by two relative to the benchmark when firms produce a single good. (2) When we additionally target the ratio of within-dispersion of price changes, this cumulated effect is cut by three relative to the benchmark when firms produce two goods. Money is almost neutral when firms produce eight goods or more. Since our most reliable estimate for the number of goods sold by a store is 40,000 (from the FMI's 2010 report), we conclude that rational inattention predicts perfect neutrality of money for CPI data. (3) Firms in the PPI data produce in average 4 goods. To match the three target moments for all bins in this dataset, we must assume different processes of shocks across bins. Once we do so, the cumulated effect of a money shock is cut by ten relative to the benchmark. Money still has sizable real effects on impact, but with almost no persistence. (5) If we force the model for PPI data to yield the same monetary non-neutrality as in the benchmark, the cost of the friction must be nine times higher, 1.9% of steady state revenues. This cost is high relative to the 0.34% of steady state revenues assumed by Midrigan (2011), the 0.23% of revenues for "managerial costs"[3] or even the 1.23% of revenues for the total cost of price changes computed by Zbaracki et al. (2004, pp. 521). Once we set the frictional cost as in Midrigan (2011), monetary non-neutrality is cut by five relative to the benchmark.[4]

Overall, these calibration results suggest that a quantitative rational inattention model which incorporates the multi-product nature of firms is unable to generate large and especially long-lasting aggregate inertia when the friction is small or even when it is similar to standard benchmarks.

Literature review. The economies of scope in information processing highlighted in this paper have not been stressed before in the fast-growing literature linking rational inattention and monetary non-neutrality, as in Sims (2006),Woodford (2009, 2012), Mackowiak and Wiederholt (2009),

[3]These costs are for information gathering, management activities for information processing, and communication.

[4]Our measure of monetary neutrality is difficult to compare with that of Midrigan (2011). In any case, contrasting the rational inattention model with the menu cost model is out of the scope of this paper.

Matejka and McKay (2012) and Paciello and Wiederholt (2011). They are also unexplored in other applications of rational inattention, such as asset pricing (Peng and Xiong, 2006), portfolio choice (Huang and Liu, 2007; Mondria, 2010), rare disasters (Mackowiak and Wiederholt, 2011), consumption dynamics (Luo, 2008), home bias (Mondria and Wu, 2010), the current account (Luo et al., 2012) and foundations for logic models (Matejka and McKay, 2011). Besides, this paper is complementary to the study of multi-production in the menu cost model as in Sheshinski and Weiss (1992), Alvarez and Lippi (2011), Bhattarai and Schoenle (2011) and Midrigan (2011). An important result in this literature is that introducing multi-product firms in menu cost models may increase monetary non-neutrality; in our case it is the opposite. The reason is that in menu cost models there is an extensive margin on price changes which in rational inattention models does not exist. Our empirical results are also novel since most empirical work keeps an eye on menu cost models, e.g. Bils and Klenow (2004), Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008), so they do not provide the statistics necessary to calibrate a rational inattention model. Finally, this paper shares its critical tone with Venkateswaran and Hellwig (2011) who, without explicitly modelling rational inattention, question the assumption in Mackowiak and Wiederholt (2009) that there exist an independent source of information for each type of shocks.

Layout. Section 2 displays the model set up. Section 3 solves the model without frictions. Section 4 assumes white noise shocks to solve the model under rational inattention. Section 5 uses this simplified model to obtain our theoretical results. Section 6 presents our empirical results from both CPI and PPI data in the US. Section 7 calibrates our model with persistent shocks to CPI and PPI data to yield our quantitative results. The appendix collects tables and figures, presents the generalized problem of the firm, and relaxes some simplifying assumptions made in the main text.

2 A model of multi-product, rationally inattentive firms

This section present an augmented version of the rational inattention model of Mackowiak and Wiederholt (2009) that incorporates multi-product firms which face monetary, firm-specific and

good-specific shocks. We then procesd in the subsequent sections to solve the model in a frictionless world and under the assumption of white noise.

Basic ingredients. Consider an economy with a continuum of goods with measure one indexed by $j \in [0, 1]$ and a continuum of monopolistic producers with measure $\frac{1}{N}$ indexed by $i \in \left[0, \frac{1}{N}\right]$ for $N \in \mathbb{N}$. Each firm i produces N goods randomly drawn from the pool of goods without replacement. Denote by $\aleph_i$ the set that collects the identity of the N goods produced by firm i.

Each good j contributes to the profits of its producing firm according to

$$\pi\left(P_{jt}, P_t, Y_t, F_{it}, Z_{jt}\right), \tag{1}$$

where P_{jt} is the fully flexible price of good j, P_t is the aggregate price, Y_t is real aggregate demand, and F_{it} and Z_{jt} are two idiosyncratic, exogenous random variables, the former specific to firm i and the latter specific to good j, all at time t. The function $\pi(\cdot)$ is assumed to be independent of which and how many goods the firm produces, twice continuously differentiable and homogenous of degree zero in the first two arguments. Idiosyncratic variables F_{it} and Z_{jt} satisfy

$$\int_0^{\frac{1}{N}} f_{it} di = 0, \tag{2}$$

$$\int_0^1 z_{jt} dj = 0, \tag{3}$$

where small case variables generically denote log-deviations from steady-state levels. Hence, f_{it} and z_{jt} have a direct interpretation as firm-specific and respectively, good-specific shocks.

Nominal aggregate demand Q_t is assumed to be exogenous and stochastic, satisfying

$$Q_t = P_t Y_t, \tag{4}$$

where aggregate prices are obtained according to

$$p_t = \int_0^1 p_{jt} dj. \tag{5}$$

The problem of the firm. The period profit function of firm i is

$$\sum_{n \in \aleph_i} \pi \left(P_{nt}, P_t, Y_t, F_{it}, Z_{nt} \right),$$

which sums up the contribution to profits of all goods produced by firm i.

The key assumption of rational inattention models is that firms are constrained in the "flow of information" that they can process in each period t,

$$I \left(\left\{ Q_t, F_{it}, \{Z_{nt}\}_{n \in \aleph_i} \right\}, \{s_{it}\} \right) \leq \kappa (N)$$

where $Q_t, F_{it}, \{Z_{nt}\}_{n \in \aleph_i}$ are variables of interest for firm i that are not directly observable, s_{it} is the vector of signals that firm i actually observes, the function $I(\cdot)$ measures the information flow between observed signals and variables of interest, and $\kappa (N)$ is an exogenous limited capacity that without loss of generality is assumed to depend on the number N of goods the firm produces.

The information flow $I(\cdot)$ is a measure of how informative the observation of a signal is with respect to a given variable. This measure has been proposed by Shannon (1948) and has a complicated functional form that, as it becomes apparent below, does not need to be specified here except for computational purposes. Therefore, we relegate it to the appendix. However, to provide intuition, if one denotes by U_t an arbitrary unobservable variable of interest and as O_t an arbitrary observable signal, and assumes that U_t and O_t are Gaussian i.i.d. processes, then the information flow between U_t and O_t is given by

$$I \left(\{U_t\}, \{O_t\} \right) = \frac{1}{2} \log_2 \left(\frac{1}{1 - \rho_{U,O}^2} \right), \tag{6}$$

which is increasing in $|\rho_{U,O}|$, the absolute correlation between U_t and O_t. Hence, a given information flow pins down the precision of signals with respect to the variables of interest.

We also assume that vector of signals s_{it} may be partitioned into $N+1$ subvectors

$$\left\{ s_{it}^{a}, s_{it}^{f}, \{s_{nt}^{z}\}_{n \in \aleph_i} \right\};$$

each subvector is correlated to one target variable such that $\{q_t, s_{it}^a\}$, $\left\{ f_{it}, s_{it}^f \right\}$ and $\{z_{nt}, s_{nt}^z\}_{n \in \aleph_i}$ are independent of each other. All variables are assumed to be Gaussian, jointly stationary, and there exists an initial infinite history of signals:

$$s_i^1 = \{s_{i-\infty}, \ldots, s_{i1}\}.$$

These assumptions imply that the information flow is additively separable according to

$$I\left(\left\{Q_t, F_{it}, \{Z_{nt}\}_{n \in \aleph_i}\right\}, \{s_{it}\}\right) = I\left(\{Q_t\}, \{s_{it}^a\}\right) + I\left(\{F_{it}\}, \left\{s_{it}^f\right\}\right) + \sum_{n \in \aleph_i} I\left(\{Z_{n\tau}\}, s_{nt}^z\right).$$

Hence, the problem of the firm i may be represented as

$$\max_{\{s_{it}\} \in \Gamma} \mathbb{E}_{i0} \left[\sum_{t}^{\infty} \beta^t \left\{ \sum_{n \in \aleph_i} \pi\left(P_{nt}^*, P_t, Y_t, F_{it}, Z_{nt}\right) \right\} \right] \tag{7}$$

where

$$P_{nt}^* = \arg\max_{P_{nt}} \mathbb{E}\left[\pi\left(P_{nt}, P_t, Y_t, F_{it}, Z_{nt}\right) \mid s_{it}\right] \tag{8}$$

subject to

$$I\left(\{P_t, Y_t\}, \{s_{it}^a\}\right) + I\left(\{F_{it}\}, \left\{s_{it}^f\right\}\right) + \sum_{n \in \aleph_i} I\left(\{Z_{nt}\}, \{s_{nt}^z\}\right) \leq \kappa(N) \tag{9}$$

$$\Leftrightarrow \kappa_a + \kappa_f + \sum_{n \in \aleph_i} \kappa_n \leq \kappa(N).$$

where $I\left(\{P_t, Y_t\}, \{s^a_{it}\}\right) = I\left(\{Q_t\}, \{s^a_{it}\}\right)$ in (9) since the only source of aggregate disturbances is Q_t. The absence of nominal rigidities implies that the pricing problem in (8) is static. The firm, however, must consider its whole discounted expected stream of profits to allocate its information flow capacity, its “attention”, among a set of signals. These signals are restricted to satisfy

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model consistent with the exogeneity of N. Hypothetically, if the function $\kappa(N)$ were such that the per-good cost of the friction was decreasing in N, then firms would have an incentive to merge their pricing decisions. We model firms as decision units, not as productive units. Hence, such mergers would look like firms producing more than N goods. The opposite would occur when firms delegated their pricing decisions because their per-good cost of the friction is increasing in N.

Nevertheless, we also explore other alternatives to discipline $\kappa(N)$. In Section 7.6 we show that the model-predicted micro moments do not respond to changes in $\kappa(N)$. At the same time, the responsiveness of prices to monetary shocks are highly sensitive to these changes. We think that this finding invalidates a calibration strategy seeking to set $\kappa(N)$ to match target moments from the data. Instead, we take an agnostic approach: We compute the value of $\kappa(N)$ needed to keep constant the degree of monetary non-neutrality delivered by model as N varies.

To close this section, we define the equilibrium in this economy as follows:

Definition 1 *An equilibrium is a collection of signals $\{s_{it}\}$, prices $\{P_{jt}\}$, the aggregate price level $\{P_t\}$ and real aggregate demand $\{Y_t\}$ such that*

1. *Given $\{P_t\}, \{Y_t\}, \{F_{it}\}_{i\in\left[0,\frac{1}{N}\right]}$ and $\{Z_{jt}\}_{j\in[0,1]}$, all firms $i \in \left[0, \frac{1}{N}\right]$ choose the stochastic process of signals $\{s_{it}\}$ at $t = 0$ and the price of goods they produce, $\{P_{nt}\}_{n\in\aleph_i}$ for $t \geq 1$.*

2. *$\{P_t\}$ and $\{Y_t\}$ are consistent with equations (4) and (5) for $t \geq 1$.*

Extensions. Our setup has simplifying assumptions such as homogeneous number N of goods produced by firms, independent profits $\pi(\cdot)$ across produced by the same firm, and signals providing information only about one type of shocks. In section 5 we augment our model to allow for heterogeneity in N which we calibrate to PPI data in section 7. To show robustness of our analysis, we relax other assumptions in the appendix to show that they yield either counterfactual predictions or lead to substantial changes.

3 A frictionless economy

Here, we solve the model in the case of infinite information processing capacity. This yields the conventional flex-price result of perfect monetary neutrality with no effect of multi-good production.

We assume that $\kappa(N) \to \infty$, so the firm is able to choose infinitely precise signals regarding all variables of interest. Hence, we only need to solve for (8). To do so, denote as Q, $F_i = F\ \forall i$ and $Z_j = Z\ \forall j$ the non-stochastic steady state level of these variables. The properties of $\pi(\cdot)$ imply

$$\pi_1\left(1, 1, Y_t, F, Z\right) = 0,$$

which follows from the optimality of prices. This equation solves for the steady-state level of real aggregate demand Y, and equation (4) for the steady-state aggregate price level $P = Q/Y$.

A second-order approximation of the problem of firm i around the steady-state is

$$\max_{\{p_{nt}\}_{n\in\aleph_i}} \sum_{n\in\aleph_i} \left\{ \begin{array}{c} \widehat{\pi}_1 p_{nt} + \frac{\widehat{\pi}_{11}}{2} p_{nt}^2 + \widehat{\pi}_{12} p_{nt} p_t + \widehat{\pi}_{13} p_{nt} y_t + \widehat{\pi}_{14} p_{nt} f_{it} + \widehat{\pi}_{15} p_{nt} z_{nt} \\ + \, terms\ independent\ of\ p_{nt}. \end{array} \right\}$$

with $\widehat{\pi}_1 = 0$, $\widehat{\pi}_{11} < 0$, $\widehat{\pi}_{12} = -\widehat{\pi}_{11}$ and all parameters identical for all goods and all firms. Hence, the optimal pricing rule for each good $n \in \aleph_i$ for all i is

$$p_{nt}^{\diamondsuit} = p_t + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} y_t + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} f_{it} + \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} z_{nt} \equiv \quad {}_t + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} f_{it} + \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} z_{nt}. \tag{10}$$

Aggregating among goods and using equations (2), (3), $y_t = q_t - p_t$ from (4), and (5), the log-deviation of aggregate prices with respect to the steady state is

$$p_t^{\diamondsuit} = q_t$$

which implies the conventional result of monetary neutrality since aggregate prices react one-to-one with an innovation in nominal aggregate demand. Note that multi-good production plays no role in determining this conventional flex-price result.

4 An analytical version of the model

Here, we solve the model under the assumption that the information capacity $\kappa(N)$ is finite. This implies that the model generates real effects of monetary shocks. We show this by using a simplified version of the model that yields full analytic solution. Specifically, assume that q_t, f_{it} and z_{jt}—the nominal aggregate shock and the two types of idiosyncratic shocks—follow white noise processes, respectively with variances σ_q^2, σ_f^2 for all $i \in \left[0, \frac{1}{N}\right]$ and σ_z^2 for all $j \in [0, 1]$. We relax this assumption in section 6, when a quantitative version of the model is introduced.

From equation (10), the optimal price of good $n \in \aleph_i$ of an arbitrary firm i that solves (8) is

$$p_{nt}^{*} = \mathbb{E}\left[\;_{t} \mid s_{it}^{a}\right] + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\mathbb{E}\left[f_{it} \mid s_{nt}^{f}\right] + \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}\mathbb{E}\left[z_{jt} \mid s_{nt}^{z}\right]. \tag{11}$$

The second-order approximation for the expected loss in profits of good $n \in \aleph_i$ due to the friction is

$$\widetilde{\pi}\left(p_{nt}^{\diamondsuit}, p_t, y_t, f_{it}, z_{nt}\right) - \widetilde{\pi}\left(p_{nt}^{*}, p_t, y_t, f_{it}, z_{nt}\right) = \frac{|\widehat{\pi}_{11}|}{2}\left(p_{nt}^{\diamondsuit} - p_{nt}^{*}\right)^2; \tag{12}$$

this expression is labelled in the subsequent analysis as the"frictional cost."

To solve the attention allocation problem, we guess that $p_t = \alpha q_t$, so that

$$\;_{t} = \left[\alpha + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}(1 - \alpha)\right] q_t. \tag{13}$$

In addition, signals chosen by firm $i \in \left[0, \frac{1}{N}\right]$ are restricted to have the structure

$$
\begin{aligned}
s_{it}^{a} &= {}_t + \varepsilon_{it}, \\
s_{it}^{f} &= f_{it} + e_{it}, \\
s_{nt}^{z} &= z_{nt} + \psi_{nt},
\end{aligned}
$$

where $\sigma_{\varepsilon i}^2$, σ_{ei}^2 and $\sigma_{\psi n}^2$ are the variance of ε_{it}, e_{it} and ψ_{nt}.[5]

The problem of a firm i can be reduced to minimize the discounted sum of frictional costs for all goods produced by the firm,

$$
\min_{\kappa_a, \{\kappa_n\}_{n \in \aleph_i}} \sum_{n \in \aleph_i} \left\{ \sum_{t=1}^{\infty} \beta^t \frac{|\widehat{\pi}_{11}|}{2} \mathbb{E}\left[\left(p_{nt}^{\diamondsuit} - p_{nt}^{*} \right)^2 \right] \right\} \tag{14}
$$

where $p_{nt}^{\diamondsuit}$ is given by (10) and p_{nt}^{*} in (11) is given by

$$
p_{nt}^{*} = \frac{\sigma_{\Delta}^2}{\sigma_{\Delta}^2 + \sigma_{\varepsilon i}^2} s_{it}^{a} + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{ei}^2} s_{it}^{f} + \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} \frac{\sigma_z^2}{\sigma_z^2 + \sigma_{\psi n}^2} s_{nt}^{z}. \tag{15}
$$

Since q_t, f_{it} and z_{jt} are white noise, the functional form of information flow in (6) applies, so the constraint in (9) becomes

$$
\begin{aligned}
\frac{1}{2} \log_2 \left(\frac{\sigma_{\Delta}^2}{\sigma_{\varepsilon i}^2} + 1 \right) + \frac{1}{2} \log_2 \left(\frac{\sigma_f^2}{\sigma_{ei}^2} + 1 \right) + \frac{1}{2} \sum_{n \in \aleph_i} \log_2 \left(\frac{\sigma_z^2}{\sigma_{\psi n}^2} + 1 \right) &\leq \kappa(N) \\
\kappa_a + \kappa_f + \sum_{n \in \aleph_i} \kappa_n &\leq \kappa(N)
\end{aligned}
$$

Using the definitions of κ_a, κ_f and κ_n implicit in the constraint, the problem of the firm is

$$
\min_{\kappa_a, \{\kappa_n\}_{n \in \aleph_i}} \frac{\beta}{1-\beta} \frac{|\widehat{\pi}_{11}|}{2} \left[2^{-2\kappa_a} \sigma_{\Delta}^2 N + \left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}} \right)^2 2^{-2\kappa_f} \sigma_f^2 N + \left(\frac{\widehat{\pi}_{15}}{\widehat{\pi}_{11}} \right)^2 \sum_{n \in \aleph_i} 2^{-2\kappa_n} \sigma_z^2 \right] \tag{16}
$$

[5] Mackowiak and Wiederholt (2009) show that this structure of signals is optimal. This result is not affected by the modifications to their model introduced here.

subject to

$$\kappa_a + \kappa_f + \sum_{n\in\aleph_i} \kappa_n \leq \kappa(N). \tag{17}$$

which delivers the optimality conditions

$$\kappa_a^* = \kappa_f^* + \log_2(x_1), \tag{18}$$

$$\kappa_a^* = \kappa_n^* + \log_2\left(x_2\sqrt{N}\right), \quad \forall n \in \aleph_i \tag{19}$$

for $x_1 \equiv \frac{|\widehat{\pi}_{11}|\sigma}{\widehat{\pi}_{14}\sigma_f}$ and $x_2 \equiv \frac{|\widehat{\pi}_{11}|\sigma}{\widehat{\pi}_{15}\sigma_z}$. The assumptions that all parameters are the same for all firms and goods along with the conditions in (18) and (19) imply that the attention paid to aggregate and firm-specific signals, κ_a^* and κ_f^*, is the same for all firms, and that the attention paid to good-specific signals is the same for all goods within all firms, $\kappa_n^* = \kappa_z^*$ for all $n \in \aleph_i$ and all i.

In addition, the conditions in (18) and (19) along with the constraint imply

$$\kappa_a^* = \begin{cases} \kappa(N) & \text{if } \sqrt[N]{x_1}x_2 > \frac{2^{\left(1+\frac{1}{N}\right)\kappa(N)}}{\sqrt{N}}, \\ \frac{1}{N+2}\left[\kappa(N) + \log_2(x_1) + N\log_2\left(x_2\sqrt{N}\right)\right] & \text{if } \sqrt[N]{x_1}x_2 \in \left(\frac{2^{-\kappa(N)/N}}{\sqrt{N}}, \frac{2^{\left(1+\frac{1}{N}\right)\kappa(N)}}{\sqrt{N}}\right), \\ 0 & \text{if } \sqrt[N]{x_1}x_2 \leq \frac{2^{-\kappa(N)/N}}{\sqrt{N}}. \end{cases} \tag{20}$$

In words, for a given N, the smaller is either capacity $\kappa(N)$ or parameters x_1 and x_2, the smaller is firms' attention to nominal aggregate demand shocks, or equivalently, the larger is the noise of firms' signals correlated to these shocks. A smaller x_1 is obtained when the volatility σ_f of the firm-specific shocks is larger relative to the volatility σ_Δ of the aggregate compound variable in (13) and/or when frictionless prices are more responsive to firm-specific shocks, that is when $\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}$ is larger. Similarly, a smaller x_2 is obtained when $\frac{\sigma_z}{\sigma}$ is larger and/or when $\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}$ is larger.

All firms are identical, so the price of any good $n \in \aleph_i$ for any firm i follows

$$p_{nt}^* = \left(1 - 2^{-2\kappa_a^*}\right)\left(\;_t + \varepsilon_{it}\right) + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\left(1 - 2^{-2\kappa_f^*}\right)\left(f_{it} + e_{it}\right) + \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}\left(1 - 2^{-2\kappa_z^*}\right)\left(z_{nt} + \psi_{nt}\right) \tag{21}$$

where ε_{it}, e_{it} are the realization of the noise of signals observed by firm i of an aggregate demand shock and a shock specific to firm i, while ψ_{nt} is the realization of the noise of signals of a shock specific to good n. Aggregating among all goods and firms, the log-deviation of the aggregate price level with respect to the steady state is

$$p_t^* = \left(1 - 2^{-2\kappa_a^*}\right) \quad {}_t = \left(1 - 2^{-2\kappa_a^*}\right)\left[\alpha + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}(1-\alpha)\right] q_t$$

which confirms the guess $p_t^* = \alpha q_t$ for

$$\alpha = \frac{\left(2^{2\kappa_a^*} - 1\right)\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}}{1 + \left(2^{2\kappa_a^*} - 1\right)\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}}. \tag{22}$$

This is the most important result of the theory of Rational Inattention: $\alpha < 1$ for any finite κ_a^*. When the capacity of firms to process information is limited, prices no longer react one-to-one with an innovation in nominal aggregate demand; hence money becomes non-neutral. The more attention the firm pays to idiosyncratic information—the higher are either κ_f^* or κ_z^*—, the lower is κ_a^*, so monetary non-neutrality is stronger. Moreover, for a given κ_a^*, the stronger is complementarity in pricing decisions among firms—the smaller is $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} > 0$—, the stronger is money non-neutrality.

5 Theoretical results

This section presents five key propositions that relate the choice of attention to the multi-product nature of firms and the existence of good-specific shocks. To this purpose, we conduct a set of comparative statics analyses of the model solved so far. We start by succinctly exposing the three forces by which these ingredients affect the attention allocation of firms. Then, we proceed with the propositions.

The first force affecting the allocation of attention are the economies of scope in information

processing: The more goods a firm produces, the more pricing decisions can benefit from the acquired information that is common to all goods. These economies of scope are capturing in the first order conditions in (18) and (19), which we rewrite as

$$\begin{aligned}\kappa_a^* &= \kappa_z^* + \log_2\left(x_2\sqrt{N}\right), \\ \kappa_f^* &= \kappa_z^* + \log_2\left(\frac{x_2}{x_1}\sqrt{N}\right),\end{aligned}$$

after imposing that $\kappa_n^* = \kappa_z^*$ for all $n \in \aleph_i$ and all i. In words, the difference in attention paid by the firm to aggregate and good-specific shocks is increasing in N since $x_2 > 0$ while the difference in attention paid to firm-specific and good-specific shocks is increasing in N since $x_1, x_2 > 0$.

The second force is the income effect: Firms must pay attention to signals regarding more goods as firms produce more goods. This force is captured by the N in the left hand side of the constraint (17), which we rewrite as

$$\kappa_a + \kappa_f + N\kappa_z \leq \kappa(N)$$

after imposing that $\kappa_n^* = \kappa_z^*$ for all $n \in \aleph_i$ and all i. Just as a consumer whose consumption basket expands with N, if $\kappa(N)$ is kept constant, a firm producing more goods has to distribute its attention among more shocks, so its information capacity becomes more binding. Thus, the income effect reduces the incentives of firms to allocate attention to all shocks.

The third force is the aggregation effect, which is captured by the unspecified functional form of firms' information capacity $\kappa(N)$ in (17). One such choice which we subsequently use is a function such that the burden of the friction per good produced is constant in N. We generally refer to this burden as the "frictional cost" which we define as the per-good loss $C_n(N)$ that rational inattentive firms minimize:

$$C_n(N) \equiv \frac{|\widehat{\pi}_{11}|}{2}\left[2^{-2\kappa_a^*}\sigma_\Delta^2 + \left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}}\right)^2 2^{-2\kappa_f^*}\sigma_f^2 + \left(\frac{\widehat{\pi}_{15}}{\widehat{\pi}_{11}}\right)^2 2^{-2\kappa_z^*}\sigma_z^2\right]. \tag{23}$$

Next we turn to study the interaction of these forces. We start by dropping good-specific shocks:

Proposition 1 *If good-specific shocks do not exist ($\sigma_z = 0$) or are irrelevant for pricing decisions ($\pi_{15} = 0$) then the allocation of attention is invariant to N if $\kappa(N) = \kappa$. Moreover, prices of goods produced by the same firm perfectly comove.*

Proof. When $\sigma_z = 0$ or $\pi_{15} = 0$, firms ignore signals s_t^z regarding firm-specific shocks, so $\kappa_z^* = 0$. Then κ_a^* follows from combining the condition in (18) and the constraint $\kappa_a + \kappa_f = \kappa$:

$$\kappa_a^* = \frac{1}{2}\left[\kappa + \log_2(x_1)\right].$$

which is constant in N. Moreover, the optimal pricing rule in (21) reduces to

$$p_{nt}^* = \left(1 - 2^{-2\kappa_a^*}\right)\left(\quad_t + \varepsilon_{it}\right) + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\left(1 - 2^{-2\kappa_f^*}\right)\left(f_{it} + e_{it}\right)$$

which varies only with aggregate or firm-specific disturbances $\quad_t$, f_{it}, ε_{it} and e_{it}. ■

Intuitively, firms can equally exploit the economies of scope in information processing by paying attention to aggregate or firm-specific shocks for any N. Further, there is no income effect since the number of shocks hitting firms is constant in N. Besides, the assumption that $\kappa(N) = \kappa$ means there is no aggregation effect and constant frictional cost in N. Hence, the constraint is invariant to N which only enters in firms' objective as a scale effect.

Proposition 1 is useful for benchmarking. The rational inattention model of Mackowiak and Wiederholt (2009) considers single-product firms exposed to aggregate and firm-specific shocks. Proposition 1 shows that, when such shocks hit the whole firm, the number of goods produced has no impact on attention allocation and thus on monetary non-neutrality. However, this result has one important caveat. The model in this case predicts perfect correlation of price changes within firms because prices do not respond to any good-specific disturbance. As anticipated in the introduction and documented in section 6, this prediction is strongly counterfactual according to both CPI data and PPI data. Hence, we study a setup with good-specific shocks.[6] We start

[6]The appendix solves for an extension of the model that specifies a common signal for all good-specific shocks that

by assuming $\kappa(N) = \kappa$ and an exogenously constant responsiveness α of aggregate prices to a monetary shock α.

Proposition 2 *If $\kappa(N) = \kappa$ and α in (13) is exogenously constant, attention κ_a^* to monetary shocks is increasing in N for $N > \hat{N}$ and $\kappa_a^* \in [0, \kappa]$, where $\hat{N}$ solves*

$$\log \hat{N} + \frac{1}{2}\hat{N} = \kappa \log 2 - \log(x_2/x_1) - \log(x_2) - 1.$$

Proof. α constant implies that $x_1 \equiv \frac{|\widehat{\pi}_{11}|\sigma}{\widehat{\pi}_{14}\sigma_f}$ and $x_2 \equiv \frac{|\widehat{\pi}_{11}|\sigma}{\widehat{\pi}_{15}\sigma_z}$ are also constant. $\hat{N}$ solves $\frac{\partial \kappa_a^*}{\partial N} = 0$ for the interior solution of (20) after setting $\kappa(N) = \kappa$. ■

A constant α allows to abstract from the feedback between firms' allocation of attention and the responsiveness of aggregate prices to shocks. This feedback is introduced in the next proposition. Moreover, setting $\kappa(N)$ constant shuts down the aggregation effect, so only the economies of scope in information processing and the income effect play a role as firms produce more goods. These two forces go in opposite directions. Proposition 2 states that the former dominates the latter when $N > \hat{N}$. Since $N \in \mathbb{N}$, $\log 2 < 0$ and $x_1, x_2 > 0$, $\hat{N} \geq 1$ only holds if x_2/x_1 and/or x_2 are small enough. Since $x_1 \equiv \frac{|\widehat{\pi}_{11}|\sigma}{\widehat{\pi}_{14}\sigma_f}$ and $x_2 \equiv \frac{|\widehat{\pi}_{11}|\sigma}{\widehat{\pi}_{15}\sigma_z}$, x_2 is small either when $\frac{\sigma}{\sigma_z}$ is small, that is, the volatility of good-specific shocks is high relative to the compound aggregate variable $_t$, or when $\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}$ is large, that is, frictionless prices are highly responsive to good-specific shocks. Similarly, x_2/x_1 is small either when $\frac{\sigma}{\sigma_f}$ is small or when $\frac{\widehat{\pi}_{15}/|\widehat{\pi}_{11}|}{\widehat{\pi}_{14}/|\widehat{\pi}_{11}|}$ is high, that is, frictionless prices are highly responsive to good-specific shocks relative to firm-specific shocks.

If $\kappa(N)$ varies freely with N – so that the aggregation effect is added with restrictions – the same result holds for a slightly different condition for $\hat{N}$. In particular, if $\kappa(N)$ is increasing (decreasing), $\hat{N}$ is smaller (higher) than in Proposition 2. We next introduce the endogeneity of α:

Proposition 3 *Endogeneity of α amplifies the effect of N on $\kappa_a^* \in [0, \kappa(N)]$ obtained in Proposition 2. Further, this amplification is stronger when the complementarity in pricing decisions is*

affect a given firm. We show that Proposition 1 holds. This result gives ground to our assumption that signals provide information about only one type of shocks.

stronger, that is, when $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} \leq 1$ *is smaller.*

Proof. From equation (22), α is increasing in κ_a for $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} \leq 1$, so ${}_t$ in (13) and thus σ_Δ are also increasing in κ_a^*; hence x_1 and x_2 are increasing in α. Besides, the interior solution of κ_a^* in (20) is increasing in x_1 and x_2; hence κ_a^* is increasing in α. As a result, the effect of N on κ_a^* in (20) gets amplified by the endogeneity of α captured in (22). According to (22), α is more increasing in κ_a as $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ is smaller, therefore this amplification effect is stronger. ■

Proposition 3 states that optimal attention to monetary shocks (κ_a^*) and the degree of monetary non-neutrality (α) are jointly determined by a fixed point that solves equations (20) and (22). Figure 1 draws these two equations in the space (α, κ_a). The interior solution of (20) is drawn in red, while (22) is drawn in blue. In addition, $\kappa_a \in [0, \kappa(N)]$ and $\alpha \in [0, 1]$; dashed lines represent $\alpha = 1$ and $\kappa_a = \kappa(N)$. The equilibrium α is denoted as α_1^*.

Equation (22) is invariant to N but the intercept of (20) may decrease or increase while its slope is decreasing in N. The green line in Figure 1 depicts the case of a higher intercept of (20) as N increases, that is $N > \hat{N}$, so equilibrium α is now α_2^*. As a result, the effect of N on κ_a^* studied when α is treated as constant is amplified by an indirect effect of κ_a^* on σ_Δ^2 in the same direction.

A key observation is that (22) is more flattened for intermediate values of α than for high and low α. Hence, an increase in κ_a has a large effect on α when α is in an intermediate level. This function is more flattened when $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} \leq 1$ is smaller, that is, complementarity in pricing decisions is stronger. This result will play a crucial role in our quantitative analysis when a relatively small increase in the attention of firms to monetary shocks translates into a large reduction of monetary non-neutrality. The next result also plays a crucial role in such analysis.

Proposition 4 *When the capacity function* $\kappa(N)$ *is restricted to preserve a constant frictional cost in* N*, and thus to provide no incentives for firms to merge or split, firms' attention* $\kappa_a^*(N)$ *to monetary shocks is unambiguously increasing in* N *and satisfies*

$$\kappa_a^*(N) = \kappa_a^*(1) + \frac{1}{2}\log_2\left(\frac{N+2}{3}\right) + \log_2\left[\frac{\sigma_\Delta\left(\kappa_a^*(N), \sigma_q\right)}{\sigma_\Delta\left(\kappa_a^*(1), \sigma_q\right)}\right].$$

Proof. Using the first order conditions in (18) and (19), the frictional cost in (23) becomes

$$C_n(N) = \frac{|\widehat{\pi}_{11}|}{2} 2^{-2\kappa_a^*(N)} \sigma_\Delta^2 (N+2).$$

From $\;_t$ in (13), σ_Δ is a function of α and σ_q; α in (22) is a function of $\kappa_a^*(N)$, so σ_Δ is a function of $\kappa_a^*(N)$ and σ_q. The equation for $\kappa_a^*(N)$ is obtained from solving $C_n(N) = C_n(1)$. ■

The above proposition present a very important result. After the capacity of firms is disciplined to imply a constant burden of the friction – which gives no incentives for firms to merge or split their pricing decisions – the attention of firms $\kappa_a^*(N)$ to monetary shocks in an economy where firms produce N goods is pinned down by firms' would-be attention $\kappa_a^*(1)$ to monetary shocks in an economy populated by single-product firms, the number of goods N itself, and the volatility of monetary shocks, σ_q. Further, $\kappa_a^*(N)$ is unambiguously increasing in N. In the language used in Proposition 2, the aggregation effect offsets the income effect enough such that the economies of scope in information processing are the dominant force for any $N \geq 1$. The amplification effect due to complementarity in pricing decisions of Proposition 3 is captured in the last term in the right-hand side, which is positive for $N > 1$, since higher attention to monetary shocks leads to higher volatility of the compound aggregate variable $\;_t$.

Proposition 4 is so important because it provides the intuition for our main quantitative results. For a given extent of the friction, the model underestimates firms' attention to monetary shocks—and thus overstates monetary non-neutrality—under the assumption of single-product firms. This result holds even when the parametrization of the model changes with N if the volatility of nominal aggregate shocks σ_q is kept constant.[7] In our subsequent numerical exercises, we pin down this parameter directly from the data. The next result is just the flip side of Proposition 4.

[7] The exception is the degree of strategic complementarity, which is captured by $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ implicit in Δ_t. If one assumes that strategic complementarity among firms' pricing decisions decreases strongly ($\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ increases) as N increases, then the monotonicity of $\;^*_a(N)$ may not hold for some values of N.

Proposition 5 *When firms frictional cost is*

$$C_n(N) = \frac{N+2}{3} C_n(1)$$

firms' attention to aggregate information is constant for any N, $\kappa_a^*(N) = \kappa_a^*(1)$.

Proof. It follows from $C_n(N)$ in Proposition 4 after imposing $\kappa_a^*(N) = \kappa_a^*(1)$. ■

In words, for any parametrization and for an unrestricted aggregation effect (that is, a free $\kappa(N)$), firms' burden of the friction must be unambiguously increasing in N to force the model to yield a constant degree of monetary non-neutrality. This result is important for section 7.6 where we discuss alternative ways to quantitatively calibrate $\kappa(N)$.

Heterogeneous firms. We now study an economy that is populated by multi-product firms that are heterogeneous in the number of goods they produce. Consider G groups of firms such that firms in group $g = 1, ..., G$ produce N_g goods. Each group has measure ω_g satisfying $\sum_{g=1}^{G} \omega_g N_g = 1$. The processes of firm-specific and good-specific shocks are independent for each group, so these shocks still wash out when prices are aggregated. All parameters are the same for all groups.

For a given guess $p_t^* = \alpha q_t$, the solution of κ_a^* is still represented by (20) only replacing N by N_g. We confirm this guess for

$$\alpha = \frac{\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} \sum_{g=1}^{G} \omega_g N_g \left(1 - 2^{-2\kappa_a^*(N_g)}\right)}{1 - \left(1 - \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}\right) \sum_{g=1}^{G} \omega_g N_g \left(1 - 2^{-2\kappa_a^*(N_g)}\right)}$$

which collapses to (22) when $G = 1$. Propositions 1, 2, 3 and 5 hold in this setup. The key condition in Proposition 4 gets modified to

$$\kappa_a^*(N) = \kappa_a^*(1) + \frac{1}{2} \log_2 \left(\frac{N+2}{3}\right).$$

which is still increasing in N although there are two differences with respect to before. First, $\kappa_a^*(1)$

is now the attention paid to monetary shocks by single-product firms in the new economy – before it was the attention in an economy populated only by single-product firms. Second, the last term in the right-hand side of the equation for $\kappa_a^*(N)$ in Proposition 4 is zero since the volatility σ_Δ now is common to all firms. This latter modification implies that the effect of N on $\kappa_a^*(N)$ conditional on $\kappa_a^*(1)$ is now less steep. This does not mean that complementarity in pricing decisions plays no role: $\kappa_a^*(1)$ is higher than in an economy with only single-product firms. This is because $\kappa_a^*(1)$ is increasing in σ_Δ and σ_Δ is higher in an economy where there are multi-product firms.

6 Empirical results

In this section, we report new statistics from the CPI and PPI micro data that can be used to calibrate a quantitative version of our model. Our main empirical finding is that log price changes only imperfectly co-move within firms. Therefore, our assumption of good-specific shocks is a sensible one. For the CPI, we compute statistics for the whole sample. For PPI, we compute statistics for the whole sample and for four 'bins' which firms are assigned to according to the number of goods they produce. Thus we can track the way in which these statistics change as the number of goods that firms produce increases. All statistics, including standard deviations, are reported in table 1.

Data description. We use confidential, monthly transaction-level micro price data collected by the U.S. Bureau Labor Statistics (BLS) to construct the Consumer Price Index (CPI) and the Producer Price Index (PPI).[8]

Our CPI data span the time period from 1988 to 2009 and contains approximately $125,782$ outlets. An outlet usually corresponds to a non-producing retailer or, in colloquial terms, a store. We exclude sales and zero price changes from the data. The exclusion of sales is common in studies of price setting since sales are usually considered practices of firms that not necessarily are

[8]A detailed description of the CPI dataset is in Nakamura and Steinsson (2008) or Bils and Klenow (2004) while of the PPI data in Bhattarai and Schoenle (2011).

related to the business cycles (for instance, Guimaraes and Sheedy (2012)). We exclude zero price changes because we consider that this is more consistent with our model, in which prices are fully flexible. Further, Mackowiak and Wiederholt (2009), which we use as benchmark in section 7, calibrate their model to statistics excluding sales and zero price changes.

The median (mean) number of goods sampled from a single outlet is 1.39 (2.05) with an standard deviation of 2.03 goods.[9] In this data, XX% of outlets have less than 3 goods in the sample. Hence, we consider that the CPI data does not provide enough variation in the number of goods sold to estimate how the number of goods varies with key statistics. For instance, the Food Marketing Institute—which collects information of stores, supermarkets and pharmacies accounting for $2/3$ of the total retail sales in the US—reports that about $40,000$ different goods were sold on average by a single store in the US in 2010. Therefore, we compute our statistics of CPI data for the whole sample without trying to condition on the number of goods sampled per outlet.

In contrast, the sampling methodology in the PPI ensures a monotonic mapping from the number of goods produced by that firm and the number of sampled goods from a given firm, as argued in Bhattarai and Schoenle (2011). In our analysis, we therefore interpret the number of sampled good as the number of produced goods. In particular, we compute statistics for the whole sample and for four 'bins'. These bins contain firms producing between 1 and 3 goods (bin 1), between 3 and 5 goods (bin 2), between 5 and 7 goods (bin 3), and more than 7 goods (bin 4). Table 1 reports the relative measure of these bins according to the number of observations and the employment used by firms in each bin. Looking at these measures, it is clear that PPI data provides much more variation in the number of goods sampled per firm than the CPI data.

Our PPI data span the time period from 1998 to 2005, and contains approximately $28,575$ productive firms. We exclude zero price changes, but we do not control for sales since this practice is sufficiently less common for productive firms to leave results unchanged.[10] We find that the

[9]The median is not integer because for each outlet we compute the mean number of goods sampled for the time period for which the outlet is sampled. This observation also applies to PPI data discussed below.

[10]As work by Nakamura and Steinsson (2008) has shown, sales are not a factor in determining the behavior of PPI prices. We have computed our statistics excluding prices flagged as sales prices, and have found no significantly different results.

median (mean) number of goods per firm is 4 (4.13) with a standard deviation is 2.55 goods. The median number of goods per firm are 2 (bin 1), 4 (bin 2), 6 (bin 3), and 8 (bin 4).

We next describe the statistics we compute and report our results.

Average absolute per-good inflation. As a measure of the magnitude of price changes, we compute the "average size of absolute price changes." We denote this statistics as $\overline{\pi}$. Labelling time as t, firms as i and goods produced by firm i at time t as $n \in \aleph_{it}$,

$$|\overline{\pi}| = \frac{1}{I} \sum_{i=1}^{I} \left[\frac{1}{N_i} \sum_{n \in \aleph_i} \left[\frac{1}{T_n} \sum_{t=1}^{T_n} |\pi_{nt}| \right] \right]$$

where $\pi_{nt} \equiv p_{nt} - p_{nt-1}$ is the non-zero inflation for good n, T_n is the total number of periods for which inflation for good n can be computed, N_{it} is the number of goods produced by firm i in the sample, and I is the total number of firms in the sample. Thus, we first compute for each good the magnitude of price changes, conditional on non-zero price changes. Second, we compute firm-level averages, and finally, take their mean across all firms in the full sample or one of the bins.

This statistics for CPI data, according to Klenow and Kryvtsov (2008), is 9.6%. Our own computation is XX%. For PPI data, the average absolute per-good inflation (standard deviations) for the whole sample is 7.8%, while for bins 1 to 4 the magnitude is as follows: 8.5% (X), 7.9% (X), 6.8% (X) and 6.5% (X). Thus, there is a clear trend: as the number of goods increases, the magnitude of price changes becomes smaller. As a robustness check, we have taken medians in the aggregation step. This leaves the trend unchanged, as also reported in Bhattarai and Schoenle (2011).

Serial correlation of per-good inflation. We denote this statistics as ρ_n for good $n \in \aleph_i$. We obtain this statistic by computing the OLS and median quantile estimation of the parameter of an $AR(1)$ coefficient for π_{nt}. We estimate the following specification:

$$\pi_{n,t} = \rho_n \pi_{n,t-1} + \epsilon_{n,t}$$

where we condition on non-zero price changes. Similarly we compute the median quantile regression by estimating the following:

$$\hat{\rho}_n = argmin_{\rho_n} E[|\pi_{n,t} - \rho_n \pi_{n,t-1}|]$$

We find that the median estimate of the AR(1) coefficient is -0.29 in the CPI, while it ranges from -0.05 in bin 1 to -0.03 in bin 4 for the PPI data. All coefficients are statistically highly significant.

Cross-sectional dispersion of per-good inflation. This statistics is denoted as $\widetilde{\sigma}$ and defined as

$$\widetilde{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\left[\frac{\sum_{i=1}^{I_t}\sum_{n\in\aleph_i}(\pi_{nt}-\overline{\pi}_t)^2}{\sum_{i=1}^{I_t} N_{it} - 1}\right]$$

where $\overline{\pi}_t$ is the average inflation of all goods sampled at time t, N_{it} is the total number of goods sampled for firm i at time t, I_t is the total number of firms at time t, and T is the total number of periods in our data.

Within-firm comovement of per-good inflation. This statistics is denoted by r and is defined as

$$r = \frac{1}{T}\sum_{t=1}^{T}\left[\frac{\sum_{i=1}^{I_t}\sum_{n\in\aleph_i}(\pi_{nt}-\overline{\pi}_{it})^2}{\sum_{i=1}^{I_t}\sum_{n\in\aleph_i}(\pi_{nt}-\overline{\pi}_t)^2}\right]$$

where $\overline{\pi}_{it}$ is the average inflation across all goods sampled for firm i at time t.

Using this statistic, our main empirical finding is that log price changes only imperfectly comove within firms. In the CPI data, 60% of the cross-sectional dispersion of log price changes is due to within-firm dispersion. In the PPI data, this ratio is increasing as firms produce more goods, from 20% (for bin 1, where firms produce between 1 and 3 goods) to 55% (for bin 4, where firms produce more than 7 goods).

This finding is important because a critical assumption for our theoretical results is that good-

specific shocks exist. If there are no good-specific shocks, then the number of goods produced has no effect on firms' attention allocation, and thus on monetary non-neutrality. However, in the absence of good-specific shocks, the model predicts perfect co-movement of prices within firms, which is not what we find in the data. This thus strongly suggests that we need to include good-specific shocks into a rational inattention model of multi-product firms.

7 Quantitative results

This section studies the effects of multi-production in our rational inattention model, allowing shocks q_t, f_{it} and z_{jt} to be persistent. The model now has no analytical solution mainly because the information flow function $I(\cdot)$ has no simple representation. The problem of the firm and its numerical solution algorithm are fully displayed in the appendix.

We seek to provide quantitative answers to two related questions: How sensitive are the aggregate predictions of the rational inattention model as firms produce more goods? And, what is the degree of monetary non-neutrality predicted by a calibrated rational inattention model of multi-product firms?

To answer these questions, we run a number of calibration exercises which take into account our new empirical facts from CPI and PPI data. We close this section by summarizing our main findings.

7.1 Baseline calibration

We start by replicating results of Mackowiak and Wiederholt (2009) for an economy of single-product firms. We nest their calibration in our model by setting

$$N = 1; \kappa(1) = 3; \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} = .15; \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} = 0; \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} = 1.$$

Capacity $\kappa(1) = 3$ implies a so-considered small frictional cost of .21% of firms' steady state revenues. The complementarity in pricing decisions $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} = .15$ is in the lower bound of the range suggested by Woodford (2003) and is exactly what Mackowiak and Wiederholt (2009) assume. Besides, they refer to idiosyncratic shocks as firm-specific, but since their model has single-product firms, idiosyncratic shocks are indistinguishable between our firm-specific and good-specific shocks. To replicate their exercise, we must shut down either of them. We find more appealing to start our analysis assuming these shocks as good-specific, $\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} = 0$. As Mackowiak and Wiederholt (2009), we set idiosyncratic and monetary shocks to be equally important in profits, so $\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} = 1$. This parameter enters in the model through $x_2 \equiv \frac{|\widehat{\pi}_{11}|\sigma}{\widehat{\pi}_{15}\sigma_z}$, so setting $\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} = 1$ implies that $\frac{\sigma}{\sigma_z}$ must be pinned down from the data.

To do so, Mackowiak and Wiederholt (2009) obtain σ_Δ as follows. They estimate an $AR(1)$ process for GNP quarterly data spanning 1959:1–2004:1 to obtain the volatility and persistence of q_t, $\sigma_q = 2.68\%$ and $\rho_q = .95$. Then, for computational simplification, they approximate this process by a $MA(20)$:

$$q_t = \sum_{k=0}^{20} \left(1 - \frac{k}{20}\right) v_{t-k} \tag{24}$$

where $v_t \sim N(0,1)$ and coefficients decrease linearly with the order of lags up to 20 lags. Hence an innovation in nominal aggregate demand dies out after 21 periods. Given the process for q_t, the compound aggregate variable $\;_t$ also follows a $MA(20)$:

$$_t = \sum_{k=0}^{20} \left[\left(1 - \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}\right)\alpha_k + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}\left(1 - \frac{k}{20}\right)\rho_q\right] v_{t-k}$$

where $\{\alpha_k\}$ are the parameters of the guessed process of aggregate prices, which is also $MA(20)$:

$$p_t = \sum_{k=0}^{20} \alpha_k v_{t-k} \tag{25}$$

such that $\{\alpha_k\}$ are found in equilibrium. See a detailed explanation in the appendix.

For idiosyncratic volatility σ_z, they assume that these shocks follow an $MA(20)$ similar to

(24) with an adjusted scale of coefficients to match the 9.6% average absolute per-good inflation reported by Klenow and Kryvtsov (2008) for CPI data in the US. This implies $\sigma_z = 11.8\sigma_q$.

We then replicate exactly Mackowiak and Wiederholt (2009) results: Firms' attention is $\kappa_a^*(1) = .09$ to monetary shocks and $\kappa_z^*(1) = 2.91$ to idiosyncratic shocks, which yield large and long-lasting monetary non-neutrality. Figure 2 depicts the response of aggregate prices after an innovation of 1% in q_t. The black line draws the response of frictionless prices; this response inherits the process assumed for q_t in (24). The blue line draws the response of aggregate prices under rational inattention. Prices absorb on impact only 2.7% of the innovation in q_t, their response remains sluggish relative to the response of frictionless prices for 20 periods (the output deviation is less than 5% of the shock) and the cumulated response of prices is only 22% of the cumulated response of frictionless prices. As anticipated above, the frictional cost is .21% of the firm's steady state quarterly real revenue $\overline{Y}$. This cost is considered small and is interpreted as giving little incentives for firms to increase their information capacity if such decision were endogenous. Importantly, these results are also interpreted as a confirmation of Sims' statement about the ability of the Rational Inattention model to generate large macroeconomic effects with a small friction.

7.2 Multi-product firms

We now assume $N > 1$ under our baseline calibration. Section 6 reports that CPI data does not provide information to calibrate N. Indirect estimates indicate $N \approx 40,000$ (the Food Marketing Institute's 2010 report). However, the effect of multi-production is so strong that it suffices, for illustration, to report results for $N = 2, 4$ and 8. All exercises target the 9.6% of average absolute per-good inflation and to yield a frictional cost per good produced of $.21\%\overline{Y}$.

For $N = 2$, firms' attention is $\kappa_a^*(2) = .36$ and $\kappa_z^*(2) = 2.92$ respectively to monetary and idiosyncratic shocks. Figure 2 draws in red the response of prices to a monetary shock. Prices absorb on impact 15% of the innovation in q_t, their response is almost identical to the frictionless prices response after 7 periods (the output deviation is less than 5% of the shock thereafter), and

their cumulated response is 74% that of frictionless prices. In few words, monetary non-neutrality is cut by three in magnitude and duration. Note that this result holds when firms' attention to monetary shocks is still a small portion of the firm's total capacity. This strong effect is due to complementarity in pricing decisions, as stated by Proposition 3.

For $N = 4$, $\kappa_a^*(4) = .58$ and $\kappa_z^*(4) = 2.9$. Figure 2 draws in green the response of prices to a shock in q_t. Prices absorb 28% of the shock on impact, their response follows closely that of frictionless prices after 4 quarters (almost no real effects thereafter) and their cumulated response is 86% that of frictionless prices. For $N = 8$, in magenta in figure 2, this result is even stronger: $\kappa_a^*(8) = .9$ and $\kappa_z^*(8) = 2.87$, prices absorb 50% of the shock on impact, the output deviation is less than 5% of the shock after 2 quarters and prices cumulated response is 93% that of frictionless prices. Given these results, we find it uninformative to report results for $N = 40,000$: Money is fully neutral.

7.3 Serial correlation of per-good inflation

We now calibrate the persistence of idiosyncratic shocks z_{jt} to match the persistence of per-good inflation in CPI data in the US. So far we have followed Mackowiak and Wiederholt (2009) by assuming that z_{jt} is as persistent as q_t, which implies almost no serial correlation of per-good inflation. Instead, according to {Bils and Klenow (2004), the first-order serial correlation of per-good inflation is $-.05$. Our own computation in table 1 is $-.29$. Both computations are methodologically different,[11] but both suggest that idiosyncratic shocks are less persistent than monetary shocks.

We now set z_{jt} to follow an MA process which, as q_t in (24), its coefficients decrease linearly with the order of lags. However, to match the $-.05$ first-order serial correlation of per-good inflation, z_{jt} must follow a $MA(4)$. We must also set $\sigma_z = 10.85\sigma_q$ to match the average absolute per good inflation. For the $-.29$ first-order serial correlation, z_{jt} must follow a $MA(1)$ with coefficient

[11] Bils and Klenow (2004) compute this statistic by averaging the coefficient of $AR(1)$ regressions for inflation of 123 categories in the CPI data, including sales and zero price changes, between 1995 and 1997. We compute the coefficient from an $AR(1)$ quantile regressions for inflation of each item in the CPI data, excluding sales and zero price changes, between 1989 and 2009. Our computation is consistent with other statistics we report.

.33 and $\sigma_z = 9.95\sigma_q$. We also target on yielding $.21\%\overline{Y}$ of per-good frictional cost.

Results are not significantly different to those above for $N > 1$, so we do not report them. Figure 3 depicts the response of prices to a 1% innovation in q_t for $N = 1$. The black and blue lines draw respectively the response of frictionless prices and prices under rational inattention in our baseline calibration. The red line draws the response of prices under rational inattention calibrated to yield $-.05$ serial correlation: $\kappa_a^*(1) = .18$ and $\kappa_z^*(1) = 2.82$, the response of prices on impact is 6.3% of the shock, the output deviation is less than 5% of the shock after 14 periods, and the cumulated response of prices is 48% that of frictionless prices. The green line draws the response of prices when the model yield $-.29$ serial correlation of per-good inflation. Now $\kappa_a^*(1) = .19$ and $\kappa_z^*(1) = 2.66$, the response of prices on impact is 6.8% of the shock, the output deviation is less than 5% of the shock after 12 periods, and the cumulated response of prices is 51% that of frictionless prices.[12]

We conclude that, when the model is calibrated to match the serial correlation of per-good inflation found in the data, the monetary non-neutrality predicted by the model is substantially reduced for $N = 1$. This is because, when the process of a shock is less persistent, any mistakes of firms when tracking this shock have lower impact on future mistakes tracking this shock; hence firms pay less attention to this shock. For a more detailed argument, see the appendix.

7.4 Co-movement of prices within firms

We now introduce firms-specific shocks to match the imperfect comovement of price changes observed within firms. Specifically, we target the 60% ratio of within-firm variance to total cross-sectional variance of per-good inflation for CPI data reported in table 1. In our exercises above with no firm-specific shocks, this statistic is 50% for $N = 2$, 75% for $N = 4$ and 88% for $N = 8$.

[12] If we would use OLS instead of quantile regressions to estimate an $AR(1)$ process for per-good inflation, the median is $-.22$. The model calibrated to match this statistics yield similar results from from those reported here.

To adjust the relative volatility of firm-specific and good-specific shocks, we assume

$$\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} = 1;$$

that is, firm-specific shocks f_{it} have the same weight in firms' profits than aggregate and good-specific shocks, q_t and z_{nt}.[13] For any N, we must set the process of firm-specific and good-specific shocks to follow an $MA(1)$ with parameter .33 to match $-.29$ serial correlation of per-good inflation. Also for any N, the total volatility of these shocks must be $9.8\sigma_q$ to match 9.6% average absolute per-good inflation. In contrast, to match the 60% ratio of within-firm dispersion, the calibration of σ_f/σ_z is specific to any N. For $N = 2$, we set $\sigma_f = 0$ since the highest within-firm dispersion ratio we can generate is 50%, so results for this case are the same than in section 7.3. We set $\sigma_f = .5\sigma_z$ for $N = 4$ and $\sigma_f = .67\sigma_z$ for $N = 8$. Finally, consistently with our previous exercises, we calibrate $\kappa(N)$ to yield $.21\%\overline{Y}$ of per-good frictional cost.

For $N = 4$, firms' attention is $\kappa_a^*(4) = .67$, and $\kappa_f^*(4) = \kappa_z^*(4) = 2.63$ respectively for monetary, firm-specific and good-specific shocks. Aggregate prices absorb on impact 32% of a monetary shock, the output deviation is less than 5% of the shock after 3 periods, and prices' cumulated response is 88% of the frictionless prices' response. For $N = 8$, $\kappa_a^*(4) = .98$, $\kappa_f^*(4) = 3.35$ and $\kappa_z^*(4) = 2.43$, prices absorption on impact is 53%, the output deviation is less than 5% of the shock after 2 periods, and prices' cumulated response is 94% that of frictionless prices. In a nutshell, monetary non-neutrality still vanishes quickly as N increases—actually, quicker than in our previous exercises.

7.5 PPI moments

We now take the version of our model with heterogeneous number of goods produced by firms from section 5, allow shocks to be persistent, and calibrate it to PPI data. As discussed in section 6, this dataset does provide an estimate for N. Specifically, we assume there are four groups of

[13] Similarly than for x_2, $\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}$ enters in the model's predictions through $x_1 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta}{\widehat{\pi}_{14}\sigma_f}$.

firms which produce $2, 4, 6$ and 8—the median number of goods per firm in each bin. The relative measures of these groups are bins' shares of total employment. We keep our baseline calibration except for the processes of firm-specific and good-specific shocks. As above, we calibrate these processes to match three statistics: the average absolute per-good inflation, the within-firm variation ratio of per-good inflation, and the first-order serial correlation of per-good inflation. Besides, we still target on yielding $.21\%\overline{Y}$ of per-good frictional cost.

We start our analysis by assuming that the processes for firm-specific and good-specific shocks are the same in all bins. These processes are calibrated to match the above mentioned statistics for bin 1. The model-predicted moments are reported in italics in table 2 and contrasted with the moments computed from the data. The model fails to account for these moments for other bins. While the average absolute per-good inflation and the serial correlation of per-good inflation are invariant to N in the model, they are decreasing in the data. Besides, the ratio of within-firm variation is less increasing in N in the model than in the data.

We then calibrate firm-specific and good-specific shocks independently for each group to match the statistics for all bins. Figure 4 draws the response of prices to a shock in q_t. For comparison, we draw in back and blue the response of frictionless prices and aggregate prices in our baseline calibration (section 7.1). The response of prices of each bin are not very different between them. This is because the strategic complementarity in pricing decisions. Aggregate prices absorb on impact 53% of the shock in q_t, output is less than .05% higher than its steady state after 3 periods, and aggregate prices cumulated response is 93% of frictionless prices response. We conclude that monetary non-neutrality is cut by ten relative to Mackowiak and Wiederholt (2009).

7.6 The role of complementarity in pricing

To further highlight the importance of the complementarity in pricing decisions in our analysis, we conduct the following exercises. First, we add a new group with zero measure in our model

calibrated to PPI data of section 7.5.[14] This new group contains firms identical to those in our baseline calibration: They produce a single good and are exposed to highly volatile and persistent idiosyncratic shocks. Section 7.1 reports that the model predicts large monetary non-neutrality when the economy is populated only by these firms. Instead, here prices in this group respond quickly to monetary shocks: They absorb on impact 29% of a shock in q_t, there are almost no real effects after 5 periods, and prices cumulated response is 82% that of frictionless prices.

The second exercise is to reduce the complementarity in pricing decisions, which we do by increasing $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ from .15 to .85. This modification has two effects. On the one hand, an increase in firms' attention to monetary shocks has a milder effect on reducing monetary non-neutrality when $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ is higher. This comes from Proposition 3. On the other hand, for a given level of firms' attention to monetary shocks, monetary non-neutrality is lower when $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ is higher. This comes from equation (22). Our model calibrated to all bins in the PPI data yields aggregate prices absorbing on impact 69% of a shock in q_t, almost no real effects only after 2 periods, and an cumulated response of prices of 96% that of frictionless prices. Hence, after reducing the complementarity, monetary non-neutrality in our model is still low relative to Mackowiak and Wiederholt (2009).

7.7 Calibrating $\kappa(N)$

Thus far our assumption of a constant per-good frictional cost allows us to pin down firms' capacity for $\kappa(N)$ given N. We now depart from this assumption. First, we try to calibrate $\kappa(N)$ directly from the data. For this, we need extra statistics from the data. For this, we use the total cross-sectional correlation of per-good inflation for CPI data. We then take our model for CPI data from section 7.4 for $N = 2$ and solve the model for a grid of $\kappa(2)$. Results are reported in table 3. We find that the model's predictions regarding moments that can be contrasted to the micro data are highly insensitive to changes in $\kappa(N)$, while the model's predictions regarding monetary non-neutrality are highly sensitive to changes in $\kappa(N)$. Since the moments computed to CPI and PPI

[14] Recall single-product firms produce only 1.5% of goods listed in the the PPI data.

data have a sample error and our numerical solution also involves some error, we think that this approach does not allow us to make robust conclusions.[15]

An alternative approach is use alternative criteria to discipline $\kappa(N)$. One informative exercise is to take our calibrated model to PPI data and for the model to yield the same degree of monetary non-neutrality as Mackowiak and Wiederholt (2009). We keep the assumption that the frictional cost per good produced must be equal across bins: otherwise firms would have incentives to merge or split their pricing decisions. In this case, the frictional cost per good produced is 1.9% of steady state revenues. This cost is high relative to the .23% of revenues for "managerial costs" or even the 1.23% of revenues for the total cost of price changes computed by Zbaracki et al. (2004, pp. 521). For another benchmark, Midrigan (2011) calibrates a menu cost model assuming $N = 2$ using the distribution of prices for a given store. There cost of the friction there is .34% of steady state revenues. Once we force our model for PPI data to yield this level of cost, aggregate prices absorb on impact 24% of the shock, the output deviation is less than 5% of the shock after 5 periods, and the cumulated response of aggregate prices is 83% that of frictionless prices.

7.8 Quantitative results: Summary

We now use our quantitative results to answer the questions posed at the beginning of this section:

How sensitive are the aggregate predictions of the rational inattention model as firms produce more goods? Our answer is: very sensitive. Our calibrated model to CPI data in section 7.4 shows that the cumulated real effect of money is cut by three, six and eleven when firms are respectively assumed to produce two, four and eight goods instead of one.

What is the degree of monetary non-neutrality predicted by a calibrated rational inattention

[15] Note also that the model is unable to match this new statistics. This is because the model assumes the distribution of prices changes to be Gaussian, which is not satisfied in the data. Although relaxing the Gaussian assumption in the model is plausible, that would add new parameters to calibrate and would not solve the problem illustrated here that prevents us from calibrating (N) as any other parameter. Besides, we choose to match with the model the properties of time volatility in the data (the average absolute per-good inflation) instead of the cross-section dispersion for comparison with previous work.

model of multi-product firms? Since calibrating capacity from the data leads to too blunt conclusions, we cannot provide a clear cut answer to this question. However, given our answer to the first question, the very large number of goods that stores sell in reality, and the large dispersion of price changes within firms, we think that there is no room to conclude that the rational inattention model could predict large sluggishness of stores prices to a monetary shock. In contrast, there is evidence that productive firms produce a much smaller number of goods than the number of goods that stores sell. Thus we think that, according to the rational inattention model, there is some room for price sluggishness. However, this sluggishness is low for any level of firms' costs due to the friction that are comparable to previous work in rational inattention, what is inferred from quantitative exercises for competing theories, or measures from case studies. Overall, in contrast to the common view, we conclude that a calibrated rational inattention model is unable to generate large aggregate inertia when the friction is assumed small or even when it is assumed at the level of some standard benchmarks.

8 Conclusion

In this paper, we have explored the implications of a model of rational inattention for the real effects of money when firms produce more than one good. We have uncovered the important role of economies of scope in information processing when rational inattentive agents are modeled as taking more than one decision at a time: Processing information is difficult, but once information is acquired, it can be freely used for all decisions it concerns. Monetary information concerns all pricing decisions; good-specific information concerns only a few. We have shown that in a quantitative model with good-specific shocks, attention to monetary shocks increases as firms sell and produce more goods. In particular, a model with these features predicts perfect neutrality of money when calibrated to CPI data in the US; calibrated to PPI data in the US, it predicts little non-neutrality unless a large friction is assumed.

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Appendix A: Tables and Figures

CPI	bin 1	bin 2	bin 3	bin 4	total
# goods, mean					2.045
# goods, median					1.390
average absolute per good inflation					9.6%
serial correlation, median					$-.291$
within variation ratio					60%
PPI					
# goods, mean					4.13
# goods, median	2	4	6	8	4
share of total employment	25.0%	27.7%	16.0%	31.3%	
average absolute per good inflation	8.5%	7.9%	6.8%	6.5%	7.8%
serial correlation, median	$-.050$	$-.057$	$-.033$	$-.032$	$-.043$
within variation ratio	20.0%	32.0%	43.0%	55.0%	38.0%
total cross sectional variance	X	X	X	X	X

	bin 1	bin 2	bin 3	bin 4	total
average absolute per good inflation, data	8.5%	7.9%	6.8%	6.5%	7.8%
average absolute per good inflation, model	8.5%	8.5%	8.5%	8.5%	8.5%
median serial correlation, data	$-.050$	$-.057$	$-.033$	$-.032$	$-.043$
median serial correlation, model	$-.051$	$-.050$	$-.050$	$-.050$	$-.050$
within-firm variation ratio, data	20.0%	32.0%	43.0%	55.0%	38.0%
within-firm variation ratio, model	20.0%	29.7%	32.3%	34.5%	29.3%

Table 2 – PPI moments and model-generated moments

	data (s.d.)	κ = 3.5	κ = 4.5	κ = 6	κ = 7.5	κ = 9
av. absolute per-good inflation	9.6% ()	9.4%	9.5%	9.7%	9.7%	9.8%
median serial correlation	$-.29$ ()	$-.29$	$-.29$	$-.29$	$-.29$	$-.29$
median within-firm var. ratio	60% ()	50%	50%	50%	50%	50%
median cross-sectional var.	X ()	11.7%	12%	12.1%	12.2%	12.2%
$\kappa_a^*(2)$		.075	.179	.414	723	1.21
prices cumulated response (relative to frictionless prices)		25%	48%	77%	89%	97%

Table 3 – CPI moments and model-generated moments

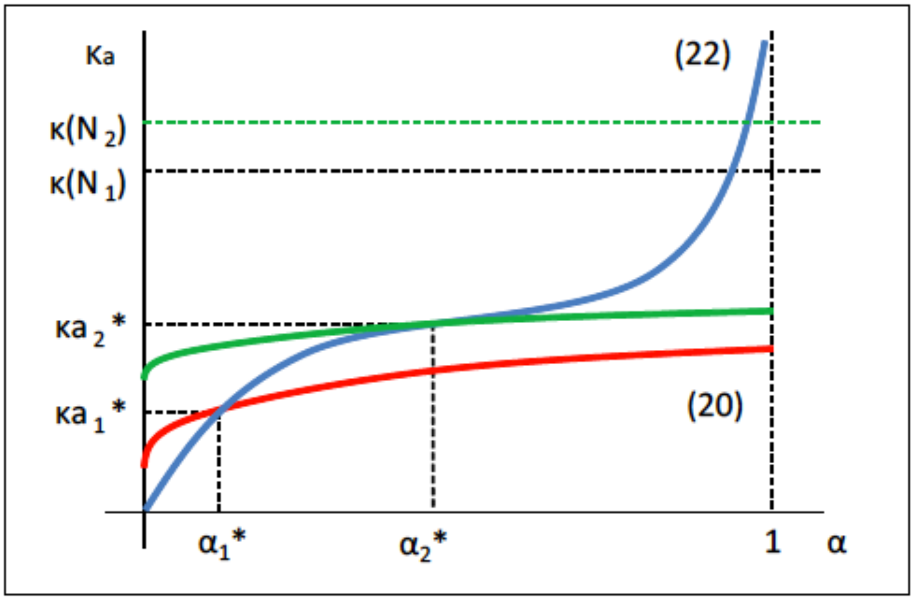

Figure 1 – Equations (20) and (22) in the space (α, κ_a)

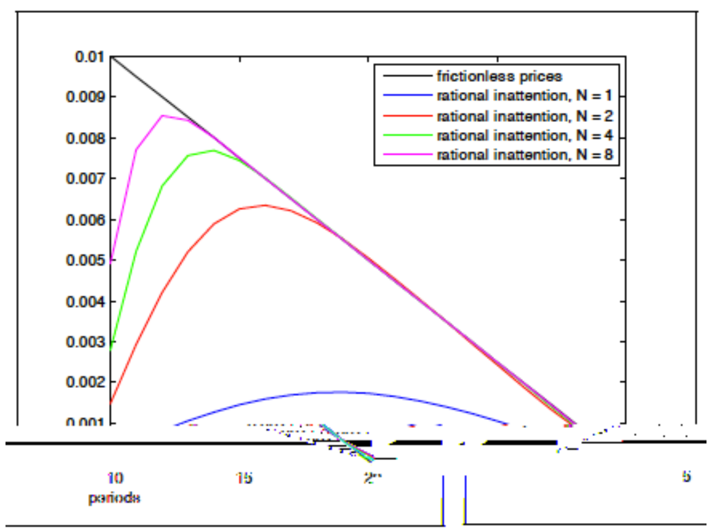

Figure 2 – Response of prices to a 1% impulse in q_t for sections 7.1 and 7.2

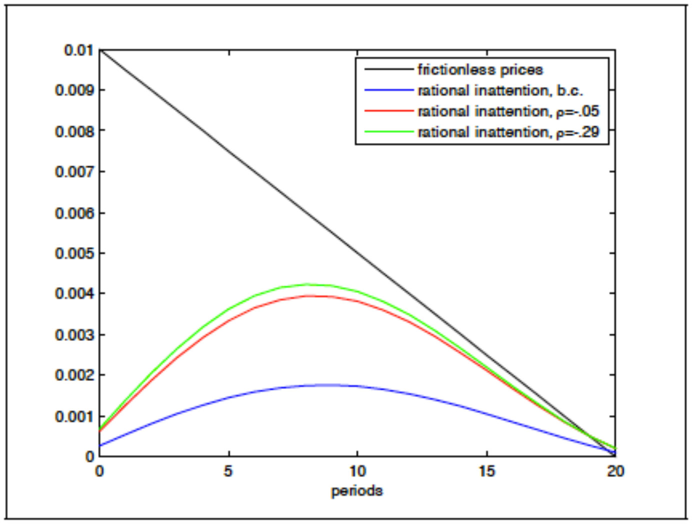

Figure 3 – Response of prices to a 1% impulse in q_t for section 7.3

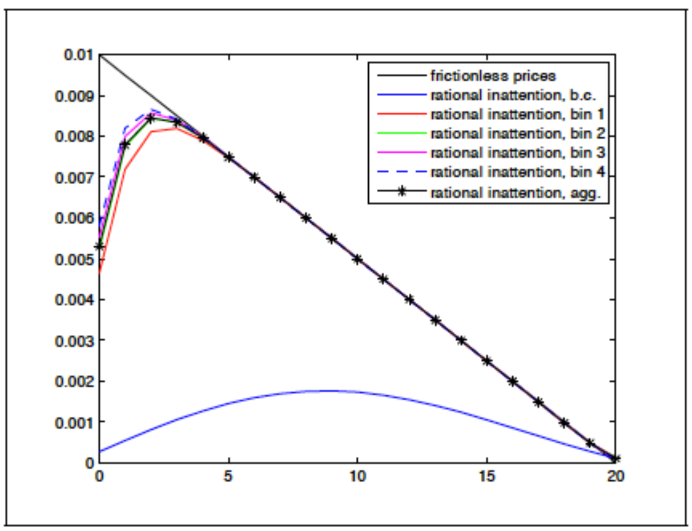

Figure 4 – Response of prices to a 1% impulse in q_t for section 7.5

Appendix B: The firms' problem and its numerical solution

This appendix displays the analytical representation of firms' problem in the setup of section 7 and explains the numerical algorithm applied to solve it. This appendix adapts to our setup a similar presentation by Mackowiak and Wiederholt (2009). Assume that firms are exposed to three types of shocks:

$$q_t = \sum_{l=0}^{\infty} a_l v_{t-l},$$

$$f_{it} = \sum_{l=0}^{\infty} b_l \xi_{t-l},$$

$$z_{jt} = \sum_{l=0}^{\infty} c_l \zeta_{t-l},$$

where q_t is a nominal aggregate demand shock (interpreted as a "monetary" shock), f_{it} is a shock idiosyncratic to each firm $i \in \left[0, \frac{1}{N}\right]$, z_{jt} is a shock idiosyncratic to each good $j \in [0,1]$, and $\{v_{t-l}, \xi_{t-l}, \zeta_{t-l}\}_{l=0}^{\infty}$ are innovations following Gaussian independent processes.

We guess that the log-deviation of aggregate prices follows

$$p_t = \sum_{l=0}^{\infty} \alpha_l v_{t-l}$$

which, given the definition of $\;_t$ in (10) and $y_t = q_t - p_t$, implies

$$\;_t = \left(1 - \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}\right) \sum_{l=0}^{\infty} \alpha_l v_{t-l} + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} \sum_{l=0}^{\infty} a_l v_{t-l} \equiv \sum_{l=0}^{\infty} d_l v_{t-l} \tag{26}$$

The problem of firm $i \in \left[0, \frac{1}{N}\right]$ has two stages. In the first stage, firms must choose conditional expectations for $\;_t$, f_{it} and $\{z_{nt}\}_{n \in \aleph_i}$ to minimize the deviation of prices with respect to frictionless optimal prices subject to the information capacity constraint:

$$\min_{\hat{\Delta}_{it}, \{\hat{z}_{nt}\}_{n \in \aleph_i}} \sum_{n \in \aleph_i} \left\{ \sum_{t=1}^{\infty} \beta^t \frac{|\widehat{\pi}_{11}|}{2} \mathbb{E}\left[\left(p_{nt}^{\diamondsuit} - p_{nt}^{*}\right)^2 \right] \right\}$$

which is equivalent to

$$\min_{\hat{\Delta}_{it},\widehat{f}_{it},\{\hat{z}_{nt}\}_{n\in\aleph_i}} \left\{ \begin{array}{c} \mathbb{E}\left[\left({}_t - \widehat{\ }_{it}\right)^2\right] N + \left(\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\right)^2 \mathbb{E}\left[\left(f_{it} - \widehat{f}_{it}\right)^2\right] N \\ + \left(\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}\right)^2 \sum_{n\in\aleph_i} \mathbb{E}\left[\left(z_{nt} - \widehat{z}_{nt}\right)^2\right] \end{array} \right\}$$

subject to the process of ${}_t$, f_{it} and $\{z_{nt}\}_{n\in\aleph_i}$ and the information capacity constraint

$$I\left({}_t, \widehat{\ }_{it}\right) + I\left(f_{it}, \widehat{f}_{it}\right) + \sum_{n\in\aleph_{we}} I\left(z_{nt}, \widehat{z}_{nt}\right) \leq \kappa\left(N\right).$$

The function $I\left(\cdot\right)$ is the information flow. For instance, this function for ${}_t$ takes the form:

$$I\left({}_t, \widehat{\ }_{it}\right) \equiv -\frac{1}{4\pi}\int_{-\pi}^{\pi} \log_2\left[1 - C_{\Delta_t,\widehat{\Delta}_{it}}\left(\omega\right)\right] d\omega$$

where $C_{\Delta_t,\widehat{\Delta}_{it}}\left(\omega\right)$ is called coherence function, which is defined as follows. Let describe the conditional expectations $\widehat{\ }_{it}$ as

$$\widehat{\ }_{it} = \sum_{l=0}^{\infty} g_l v_{t-l} + \sum_{l=0}^{\infty} h_l \varepsilon_{t-l},$$

then

$$C_{\Delta,\widehat{\Delta}_{we}}\left(\omega\right) \equiv \frac{\frac{G\left(e^{-i\omega}\right)G\left(e^{i\omega}\right)}{H\left(e^{-i\omega}\right)H\left(e^{i\omega}\right)}}{\frac{G\left(e^{-i\omega}\right)G\left(e^{i\omega}\right)}{H\left(e^{-i\omega}\right)H\left(e^{i\omega}\right)} + 1},$$

where $G\left(e^{i\omega}\right) = g_0 + g_1 e^{i\omega} + g_2 e^{i2\omega} + ...$ and $H\left(e^{i\omega}\right) = h_0 + h_1 e^{i\omega} + h_2 e^{i2\omega} + ...$

If the conditional expectations $\widehat{f}_{it}$ and $\{\widehat{z}_{nt}\}_{n\in\aleph_i}$ are described by

$$\begin{aligned} \widehat{f}_{it}^* &= \sum_{l=0}^{\infty} r_l \xi_{t-l} + \sum_{l=0}^{\infty} s_l \varepsilon_{t-l}, \\ \widehat{z}_{nt}^* &= \sum_{l=0}^{\infty} w_{nl} \zeta_{t-l} + \sum_{l=0}^{\infty} x_{nl} e_{nt-l} \text{ for } n \in \aleph_i. \end{aligned}$$

Then the problem may be represented as

$$\min_{g,h,r,s,\{w_n,x_n\}_{n\in\aleph_i}} \left\{ \begin{array}{c} \left[\sum_{l=0}^{\infty}(d_l-g_l)^2+\sum_{l=0}^{\infty}h_l^2\right]N+\left(\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\right)^2 N\left[\sum_{l=0}^{\infty}(b_l-r_l)^2+\sum_{l=0}^{\infty}s_l^2\right] \\ +\left(\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}\right)^2\sum_{n\in\aleph_{we}}\left[\sum_{l=0}^{\infty}(c_l-w_{nl})^2+\sum_{l=0}^{\infty}x_{nl}^2\right] \end{array} \right\}$$

$$s.t.\ I\left(\ _t,\widehat{\ }_{it}\right)+I\left(f_{it},\widehat{f}_{it}\right)+\sum_{n\in\aleph_{we}}I\left(z_{nt},\widehat{z}_{nt}\right)\leq\kappa\left(N\right)$$

where $g,h,r,s,\{w_n,x_n\}_{n\in\aleph_i}$ represent vectors of coefficients. The first order conditions for g and h are

$$\begin{aligned} g_l &: 2\left(d_l^*-g_l^*\right)N=-\frac{\mu_a}{4\pi\log\left(2\right)}\int_{-\pi}^{\pi}\frac{\partial\log\left[1-C_{\Delta,\widehat{\Delta}_{we}^*}\left(\omega\right)\right]}{\partial g_l}d\omega, \\ h_l &: 2h_l^*N=\frac{\mu_a}{4\pi\log\left(2\right)}\int_{-\pi}^{\pi}\frac{\partial\log\left[1-C_{\Delta,\widehat{\Delta}_{we}^*}\left(\omega\right)\right]}{\partial h_l}d\omega \end{aligned}$$

where μ_a is the Lagrangian multiplier. Similar conditions must be satisfied by r^* and s^* and by $\{w_n^*,x_n^*\}_{n\in\aleph_i}$ but without N.

The second stage of the problem is to obtain optimal signals structures that deliver $\widehat{\ }_{it}^{*}=\widehat{\ }_{it}\left(\kappa_a^*\left(N\right),N\right)$ and $\widehat{z}_{nt}^*=\widehat{z}_{nt}\left(\kappa_n\left(N\right),N\right)$. Since we are interested in the aggregate implications of the model, we do not solve this part.

Numerically, we truncate the memory of all processes to 20 lags, which is the same order assumed for the MA process for q_t. Then we start from a guess for α to compute d, we find $g^*,h^*,r^*,s^*,\{w_n,x_n\}_{n\in\aleph_i}$ by using the Levenberg-Marquardt algorithm to solve the system of first-order conditions plus the information flow constraint after imposing symmetry in $\{w_n,x_n\}_{n\in\aleph_i}$. With these vectors, we compute $I\left(\ _t,\widehat{\ }_{it}\right)=\kappa_a^*\left(N\right)$, $I\left(f_{it},\widehat{f}_{it}\right)=\kappa_f^*\left(N\right)$ and $I\left(z_{nt},\widehat{z}_{nt}\right)=\kappa_z^*(N)$ and the vector α. We use this α as guess for a new iteration upon convergence on α.

Appendix C: Extensions

This appendix relaxes some expositional assumptions made in the set up studied in the main text. These extensions either yield counterfactual results or no substantive changes to our conclusions.

Common signals

[. . .]

Persistent shocks

We now solve a simplified version of our model that allows for persistent shocks by keeping at least partial closed-firm solution. Assume that the process of q_t is such that $\ _t$ is $AR(1)$ with persistency ρ_Δ. Idiosyncratic shocks f_{it} and z_{jt} are also $AR(1)$ respectively with persistency ρ_f for all i and ρ_z for all j. The starting guess is now

$$p_t = \sum_{l=0}^{\infty} \alpha_l v_{t-l}, \tag{27}$$

where $\{v_{t-l}\}_{l=0}^{\infty}$ is the history of nominal aggregate demand innovations.

The firms' problem may be cast in two stages. In the first stage, firms choose

$$\begin{aligned} &\min_{\hat{\Delta}_{it},\{\hat{z}_{nt}\}_{n\in\aleph_i}} \sum_{n\in\aleph_i} \left\{ \sum_{t=1}^{\infty} \beta^t \frac{|\widehat{\pi}_{11}|}{2} \mathbb{E}\left[\left(p_{nt}^{\diamondsuit} - p_{nt}^{*}\right)^2 \right] \right\} \\ \rightarrow \quad &\min_{\hat{\Delta}_{it},\{\hat{z}_{nt}\}_{n\in\aleph_i}} \frac{\beta}{1-\beta} \frac{|\widehat{\pi}_{11}|}{2} \left\{ \begin{array}{c} \mathbb{E}\left(\ _t - \hat{\ }_{it}\right)^2 N + \left(\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\right)^2 \mathbb{E}\left(f_{it} - \hat{f}_{it}\right)^2 N \\ + \left(\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}\right)^2 \sum_{n\in\aleph_i} \mathbb{E}\left(z_{nt} - \hat{z}_{nt}\right)^2 \end{array} \right\} \end{aligned}$$

subject to

$$
\begin{aligned}
I\left(\left\{ \quad_t, \hat{\ }_{it}\right\}\right) &\leq \kappa_a, \\
I\left(\left\{ f_{it}, \hat{f}_{it}\right\}\right) &\leq \kappa_f, \\
I\left(\left\{ z_{nt}, \hat{z}_{nt}\right\}\right) &\leq \kappa_n, \quad for\ n \in \aleph_i \\
\kappa_a + \sum_{n \in \aleph_{ie}} \kappa_n &\leq \kappa(N)
\end{aligned}
$$

For the second stage, firms choose the signals that deliver $\hat{\ }^*_{it}, \{\hat{z}^*_{nt}\}_{n \in \aleph_{we}}$. As in the appendix B, we omit this stage. Our representation for the firm's problem follows from a result in Sims (2003): The solution of

$$
\min_{b,c} \mathbb{E}\left(U_t - O_t\right)^2
$$

where U_t is an unobservable and O_t is an observable variable, subject to

$$
\begin{aligned}
U_t &= \rho U_{t-1} + a u_t, \\
O_t &= \sum_{l=0}^{\infty} b_l u_{t-l} + \sum_{l=0}^{\infty} c_l \varepsilon_{t-l}, \\
\kappa &\geq I\left(\{U_t, O_t\}\right)
\end{aligned}
$$

yield

$$
\mathbb{E}\left(U_t - O_t^*\right)^2 = \sigma_T^2 \frac{1-\rho^2}{2^{2\kappa} - \rho^2}.
$$

Therefore, firms' problem may be represented as

$$
\min_{\kappa_a, \kappa_f, \{\kappa_n\}_{n \in \aleph_i}} \frac{\beta}{1-\beta} \frac{|\widehat{\pi}_{11}|}{2} \left[\frac{1-\rho_\Delta^2}{2^{2\kappa_a} - \rho_\Delta^2} N \sigma_\Delta^2 + \left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}}\right)^2 \frac{1-\rho_f^2}{2^{2\kappa_f} - \rho_f^2} N \sigma_f^2 + \left(\frac{\widehat{\pi}_{15}}{\widehat{\pi}_{11}}\right)^2 \sum_{n \in \aleph_i} \frac{1-\rho_z^2}{2^{2\kappa_z} - \rho_z^2} \sigma_z^2 \right]
$$

subject to

$$
\kappa_a + \kappa_f + \sum_{n \in \aleph_i} \kappa_n \leq \kappa(N).
$$

This problem is identical to that solved in section 5 $\rho_\Delta = \rho_z = 0$. Its optimality conditions are

$$\begin{aligned} \kappa_a^* + f\left(\rho_\Delta, \kappa_a^*\right) &= \kappa_f^* + f\left(\rho_f, \kappa_f^*\right) + \log_2 x_1 \\ \kappa_a^* + f\left(\rho_\Delta, \kappa_a^*\right) &= \kappa_z^* + f\left(\rho_z, \kappa_z^*\right) + \log_2 x_2 \sqrt{N} \end{aligned}$$

where $x_1 \equiv \frac{|\widehat{\pi}_{11}|\sigma\ \ (1-\rho\ \ )}{\widehat{\pi}_{14}\sigma_f\left(1-\rho_f\right)}$, $x_2 \equiv \frac{|\widehat{\pi}_{11}|\sigma\ \ (1-\rho\ \ )}{\widehat{\pi}_{14}\sigma_z(1-\rho_z)}$ and $f(\rho, \kappa) = \log_2\left(1 - \rho^2 2^{-2\kappa}\right)$.

The function $f(\rho, \kappa)$ is weakly negative and increasing in κ, so the difference in attention to aggregate and good-specific shocks, $\kappa_a^* - \kappa_z^*$, is still increasing in N. As before, the difference $\kappa_a^* - \kappa_f^*$ remains invariant to N. The function $f(\rho, \kappa)$ is also decreasing in $|\rho|$. Hence, a decrease in persistency of idiosyncratic shocks ρ_f and ρ_z implies an increase of κ_a^* relative to κ_f^* and κ_z^*.

Interconnected profits

We now depart from our assumption that firms are decision units but not production units. Now we assume that firms' production or commercialization processes are integrated such that the pricing decision of one good is not independent of other goods produced by the same firm. We capture this 'interconnection' by assuming that the function $\pi(\cdot)$ in (1) is now

$$\pi\left(P_{nt}, P_t, Y_t, F_{it}, Z_{nt}, \{P_{-nt}\}_{-n \in \aleph_i}\right),$$

for a given good $n \in \aleph_i$ produced by firm i. Our notation remains the same as in the main text for aggregate prices P_t, real aggregate demand Y_t, firm-specific shocks F_{it} and good-specific shocks Z_{nt}. The novelty comes in the last argument, $\{P_{-nt}\}_{-n \in \aleph_i}$, which represents the prices set by firm i for all its produced goods except good n.

Hence, the firm i's objective is now

$$\max_{\{s_{it}\} \in \Gamma} \mathbb{E}_{i0}\left[\sum_t^{\infty} \beta^t \left\{\sum_n \pi\left(P_{nt}^*, P_t, Y_t, F_{it}, Z_{nt}, \left\{P_{-nt}^*\right\}_{-n \in \aleph_i}\right)\right\}\right]$$

where

$$P_{nt}^{*} = \arg\max_{P_{nt}} \mathbb{E}\left[\sum_{n} \pi\left(P_{nt}^{*}, P_t, Y_t, F_{it}, Z_{nt}, \left\{P_{-nt}^{*}\right\}_{-n\in\aleph_i}\right) \mid s_{it}\right]$$

subject to

$$I\left(\{P_t, Y_t\}, \{s_{it}^{a}\}\right) + I\left(\{F_{it}\}, \left\{s_{it}^{f}\right\}\right) + \sum_{n\in\aleph_i} I\left(\{Z_{nt}\}, \{s_{nt}^{z}\}\right) \leq \kappa(N)$$

$$\Leftrightarrow \kappa_a + \kappa_f + \sum_{n\in\aleph_i} \kappa_n \leq \kappa(N).$$

The problem is identical to the one in the main text with the exception that optimal prices must take into account their effect on the contribution to profits of all goods produced by the same firm. In steady state, prices must solve

$$\pi_1\left(1, 1, Y_t, F, Z, \{1\}_{-n\in\aleph_i}\right) + (N-1)\,\pi_6\left(1, 1, Y_t, F, Z, \{1\}_{-n\in\aleph_i}\right) = 0;$$

which implicitly assumes equal marginal effect of the price of any good on other good's profits.

A second order approximation of the total profits function is

$$\begin{aligned}
&(\widehat{\pi}_1 + \widehat{\pi}_6 (N-1))\, p_{nt} + \tfrac{1}{2}(\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1))\, p_{nt}^2 + (\widehat{\pi}_{12} + \widehat{\pi}_{62}(N-1))\, p_{nt} p_t \\
&+ (\widehat{\pi}_{13} + \widehat{\pi}_{63}(N-1))\, p_{nt} y_t + (\widehat{\pi}_{14} + \widehat{\pi}_{64}(N-1))\, p_{nt} f_{it} + \widehat{\pi}_{15} p_{nt} z_{nt} \\
&+ \textstyle\sum_{-n\in\aleph_i} \widehat{\pi}_{65} p_{nt} z_{-nt} + 2\sum_{-n\in\aleph_i} \widehat{\pi}_{16} p_{nt} p_{-nt} \\
&+ \mathit{terms\ independent\ of}\ p_{nt}.
\end{aligned}$$

Hence, the optimal frictionless price solves

$$\begin{aligned}
p_{nt}^{\diamondsuit} \;=\; & \frac{\widehat{\pi}_{12} + \widehat{\pi}_{62}(N-1)}{\left|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)\right|} p_t + \frac{\widehat{\pi}_{13} + \widehat{\pi}_{63}(N-1)}{\left|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)\right|} y_t + \frac{\widehat{\pi}_{14} + \widehat{\pi}_{64}(N-1)}{\left|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)\right|} f_{it} + \\
& \frac{\widehat{\pi}_{15}}{\left|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)\right|} z_{nt} + \sum_{-n\in\aleph_i} \frac{\widehat{\pi}_{65}}{\left|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)\right|} z_{-nt} + 2\sum_{-n\in\aleph_i} \frac{\widehat{\pi}_{16}}{\left|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)\right|} p_{-nt}^{\diamondsuit}.
\end{aligned}$$

The interconnection between profit functions has two implications on optimal frictionless prices. First, frictionless prices respond to all good-specific shocks that hit a given firm. Second, frictionless prices respond to other prices set by the same firm. The solution for optimal frictionless prices may be represented by

$$p_{nt}^{\diamondsuit} = a_0 p_t + a_1 y_t + a_2 f_{it} + a_3 z_{nt} + \sum_{-n \in \aleph_i} a_4 z_{-nt}.$$

For instance, the optimal frictional price for $N = 2$ solves

$$p_{nt}^{\diamondsuit} = \left(\left|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)\right| - 2\widehat{\pi}_{16}\right)^{-1} \left\{ \begin{array}{c} \left[\widehat{\pi}_{12} + \widehat{\pi}_{62}(N-1)\right] p_t + \left[\widehat{\pi}_{13} + \widehat{\pi}_{63}(N-1)\right] y_t \\ + \left[\widehat{\pi}_{14} + \widehat{\pi}_{64}(N-1)\right] f_{it} + \widehat{\pi}_{15} z_{nt} + \widehat{\pi}_{65} z_{-nt} \end{array} \right\}$$

Further, to obtain neutrality of frictionless prices, parameters must satisfy

$$a_0 = 1$$

and to ensure that $a_1 > 1$, parameters must satisfy

$$\left|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)\right| - 2\widehat{\pi}_{16} > 0.$$

Turning to solve for optimal prices under rational inattention, we start by computing the second-order approximation for

$$\sum_{n,-n \in \aleph_i} \left\{ \widetilde{\pi}\left(p_{nt}^{\diamondsuit}, p_t, y_t, f_{it}, z_{nt}, \left\{p_{-nt}^{\diamondsuit}\right\}_{-n \in \aleph_i}\right) - \widetilde{\pi}\left(p_{nt}^{*}, p_t, y_t, f_{it}, z_{nt}, \left\{p_{-nt}^{*}\right\}_{-n \in \aleph_i}\right) \right\}$$

which solves

$$\frac{\left|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)\right|}{2} \sum_{n \in \aleph_i} \left(p_{nt}^{\diamondsuit} - p_{nt}^{*}\right)^2 - \widehat{\pi}_{16} \sum_{n \in \aleph_i} \sum_{-n \in \aleph_i} \left(p_{nt}^{\diamondsuit} - p_{nt}^{*}\right)\left(p_{-nt}^{\diamondsuit} - p_{-nt}^{*}\right).$$

Guessing $p_t = \alpha q_t$, defining $\quad_t \equiv p_t + a_1 y_t$, imposing

$$p^*_{nt} = \frac{\sigma^2_\Delta}{\sigma^2_\Delta + \sigma^2_{\varepsilon i}} s^a_{it} + a_2 \frac{\sigma^2_f}{\sigma^2_f + \sigma^2_{ei}} s^f_{it} + a_3 \frac{\sigma^2_z}{\sigma^2_z + \sigma^2_{\psi n}} s^z_{nt} + a_4 \sum_{-n \in \aleph_i} \frac{\sigma^2_z}{\sigma^2_z + \sigma^2_{\psi n}} s^z_{nt},$$

and using the definitions of κ_a, κ_f and $\{\kappa_n\}_{n \in \aleph_i}$, firms' objective becomes

$$\min_{\kappa_a, \kappa_f, \{\kappa_n\}_{n \in \aleph_i}} \left\{ \begin{array}{c} \frac{|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)|}{2} \left[\left(2^{-2\kappa_a} \sigma^2_\Delta + a_2^2 2^{-2\kappa_f} \sigma^2_f \right) N + \left(a_3^2 + (N-1) a_4^2 \right) \sum_{n \in \aleph_i} 2^{-2\kappa_n} \sigma^2_z \right] \\ -\widehat{\pi}_{16} (N-1) \left[\left(2^{-2\kappa_a} \sigma^2_\Delta + a_2^2 2^{-2\kappa_f} \sigma^2_f \right) N + \left(2 a_3 a_4 + a_4^2 (N-2) \right) \sum_{n \in \aleph_i} 2^{-2\kappa_n} \sigma^2_z \right] \end{array} \right\}$$

which first-order conditions are

$$\begin{aligned} \kappa^*_a &= \kappa^*_f + \log_2 (\widetilde{x}_1), \\ \kappa^*_a &= \kappa^*_n + \log_2 \left(\widetilde{x}_2 (N) \sqrt{N} \right), \quad \forall n \in \aleph_i \end{aligned}$$

where we denote as κ^*_a, κ^*_f and $\{\kappa^*_n\}_{n \in \aleph_i}$ the optimal allocation of attention of a firm in the economy with interconnected profits. The optimality conditions have a similar form than in the main text. The relationship between the attention paid to monetary and firm-specific shocks is still invariant to the number N of goods produced by a firm, while the difference between the attention paid to monetary and good-specific shocks varies with N. The economies of scope in information processing is also still captured by the term $\sqrt{N}$ in the right hand side. However, the interconnection of profits creates another link that is captured by $\widetilde{x}_2 (N)$, which in the main text is a parameter and here is a function of N :

$$\begin{aligned} \widetilde{x}_1 &\equiv \frac{\sigma_\Delta}{a_2 \sigma_f}, \\ \widetilde{x}_2(N) &\equiv \left[\frac{\left(\frac{|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)|}{2} - \widehat{\pi}_{16} (N-1) \right) \frac{\sigma^2}{\sigma^2_z}}{\frac{|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)|}{2} \left(a_3^2 + (N-1) a_4^2 \right) - \widehat{\pi}_{16} (N-1) \left(2 a_3 a_4 + a_4^2 (N-2) \right)} \right]^{\frac{1}{2}} \end{aligned}$$

Hence, after we equalize the per-good frictional cost, we obtain a modified version of Proposi-

tion 4:

$$\kappa_a^*(N) = \kappa_a^*(1) + \frac{1}{2}\log_2\left(\frac{N+2}{3}\right) + \frac{1}{2}\log_2\left(\frac{\sigma_\Delta^2(N)}{\sigma_\Delta^2(1)}\right) + \frac{1}{2}\log_2\left(\frac{|\widehat{\pi}_{11} + \widehat{\pi}_{66}(N-1)| - 2\widehat{\pi}_{16}}{|\widehat{\pi}_{11}|}\right)$$

This expression is identical to the one obtained in Proposition 4 except for the last term on the right hand side, which does not exists such proposition. The positive response of prices to real aggregate demand ensures that this term exists. This term represents the degree of complementarity of the productive or commercialization processes of the goods produced by a single firm. Hence, a reasonable assumption is that it is increasing in N. Otherwise, the firm could exploit better the complementarity in production and commercialization by quitting the production of some goods.