



# EUROPEAN SUMMER SYMPOSIUM IN ECONOMIC THEORY

**Generously hosted by**  
Study Center Gerzensee

Monday 1 – Friday 12 July 2013

## **Pledgeability and Liquidity**

\*Venky Venkateswaran (Stern School of Business, New York University)  
Randall Wright (Wisconsin)

*The views expressed in this paper are those of the author(s) and not those of the funding organisation(s), nor of CEPR, which takes no institutional policy positions.*

# PLEDGABILITY AND LIQUIDITY: A NEW MONETARIST MODEL OF FINANCIAL AND MACROECONOMIC ACTIVITY\*

Venky Venkateswaran<sup>†</sup>      Randall Wright<sup>‡</sup>

January 28, 2013

## Abstract

When limited commitment hinders unsecured credit, assets help by serving as collateral. We study models where assets differ in *pledgability* – the extent to which they can be used to secure loans – and hence *liquidity*. Although many previous analyses of imperfect credit focus on producers, we emphasize consumers. Household debt limits are determined by the cost households incur when assets are seized in the event of default. The framework, which nests standard growth and asset-pricing theory, is calibrated to analyze the effects of monetary policy and financial innovation. We show that inflation can raise output, employment and investment, plus improve housing and stock markets. For the baseline calibration, optimal inflation is positive. Increases in pledgability can generate booms and busts in economic activity, but may still be good for welfare.

---

\*We thank Guido Menzio, Guillaume Rocheteau, Neil Wallace, Yu Zhu and Chao He for their input, as well as participants in seminars or conference presentations at Wisconsin, Columbia, USC, the 2012 SED Meetings in Cyprus, and the 2012 FRB Chicago Conference on Money, Banking and Payments. Wright thanks the National Science Foundation and the Ray Zemon Chair in Liquid Assets at the Wisconsin School of Business for research support. We also thank the Toulouse School of Economics, where we began this project, for their hospitality. The usual disclaimers apply.

<sup>†</sup>Pennsylvania State University (vuv10@psu.edu).

<sup>‡</sup>University of Wisconsin-Madison, FRB Minneapolis, FRB Chicago and NBER (rwright@bus.wisc.edu).

Collateral is, after all, only good if a creditor can get his hands on it. Niall Ferguson, *The Ascent of Money*.

## 1 Introduction

This project develops a theory of the role of assets in the exchange process and uses it to study a variety of issues in macro, monetary and financial economics, both analytically and quantitatively. Our approach begins with the premise that the intertemporal allocation of resources is hindered by limited commitment. Interacted with some notion of imperfect monitoring or record keeping, as stressed in monetary economics by, e.g., Kocherlakota (1998) and Wallace (2010), limited commitment implies that assets have a role in facilitating credit transactions. In our view, desiderata for a theory that tries to take this seriously are: (1) it must use a general equilibrium approach in the sense of working within a *complete and internally consistent* description of an economic environment; (2) it must go beyond classical equilibrium analysis by modeling agents as *trading with each other*, not simply against their budget constraints. Only when one has such a theory can one reasonably ask *how* agents trade: Is exchange bilateral or multilateral? Are the terms of trade taken parametrically or set strategically? Do they use barter, money or credit? If they use credit, how is repayment enforced? It is from this vantage that we study financial and macroeconomic activity.

By way of example, suppose that you want something, either a consumption or a production good, from someone now, but you have no good that they want at the moment, so you cannot barter directly. If you will have something at a later date that they want – maybe cash, maybe goods or claims to goods, or general purchasing power – you can promise that if they give you what you want now you will reciprocate by transferring something of value to them in the future. But they worry you may renege (that is what a lack-of-commitment friction means). What mechanism can provide incentives that encourage you to honor your obligations? Theories like Kehoe and Levine (1993,

2001) and Alvarez and Jermann (2000) punish those who default by taking away their access to future credit. That can be difficult, however, when there is imperfect monitoring or record keeping, including the extreme situation where agents are anonymous. With limited ability to punish those who renege, unsecured credit does not work well.

In this situation there emerges a role for assets in the facilitation of intertemporal exchange. There are two ways this can work. First, if you want something and have assets at hand, you can turn them over to a counterparty now and finalize the transaction. In this case assets serve as a means of payment, or medium of exchange, as in Kiyotaki and Wright (1989, 1993). Second, you can assign to the seller the right to seize some of your assets in the event that you renege on your promised payment. In this case the assets serve as collateral, as in Kiyotaki and Moore (1997, 2005). Collateral is useful in the presence of commitment issues because it helps ensure compliance: if you fail to honor an obligation you lose the collateral, and, to the extent that you value it, this helps deter opportunistic misbehavior (notice that for this to work it is not at all necessary that the counterparty values the collateral; it is enough that you do). While these two ways in which assets may facilitate intertemporal exchange – serving as a means of payment or as collateral – look different on the surface, they are often in fact equivalent.<sup>1</sup>

This essay proceeds with the interpretation that assets serve as collateral. In this situation, what matters is the fraction of one’s assets that can be seized in the event of default. In the language of Holmstrom and Tirole (2011), what matters is *pledgability*, which is related to *liquidity*. We formalize this in a framework that nests standard growth and asset-pricing theory as special cases, and can be viewed as an extension of the New Monetarist models recently surveyed by Williamson and Wright (2010) and Nosal and Rocheteau (2011): most of the ingredients are standard, but some applications are novel.

---

<sup>1</sup>Suppose at date  $t$  you have assets that will be worth  $\phi_s$  at date  $s > t$ , and you use them to secure a loan between  $t$  and  $s$ . If no punishment is available except forfeiture of collateral, clearly your debt limit is  $\phi_s$  because you will honor an obligation if and only if it is less than the value of the collateral. It is equally clear that, instead of using the assets as collateral, you can turn them over and finalize the transaction at  $t$ . At least, this is the case without some reason to prefer either immediate or deferred settlement. We talk more below about why one may have such a preference.

Since we can price currency as well as capital, equity, real estate etc., we can analyze the effects of monetary policy on investment, stock markets, housing markets etc. Classic results by Fisher, Mundell, Tobin et al. emerge as special cases, clarifying how inflation affects asset returns. It also affects output and employment, and the model can generate a stable, exploitable, long-run Phillips curve. One can also analyze open-market operations, and other policies where the public and private sectors swap assets. One can also study the impact of financial development.

The framework is tractable enough that many results can be derived analytically, but we also calibrate the model to study the aggregate effects of monetary policy and financial innovation quantitatively. In the baseline calibration, higher inflation rates over some range increase output, employment, investment, the price and quantity of housing, and the value of the stock market. This is driven mainly by a Mundell-Tobin effect that makes agents want to substitute out of real balances, and into other pledgeable assets, when inflation rises. The nominal returns on illiquid assets go up one-for-one with inflation, à la Fisher, but the nominal returns on partially-liquid assets go up by less. Hence inflation reduces the real returns on bonds, capital and housing. To our surprise, in the baseline calibration welfare is increasing in inflation over a reasonable range. Again, this is due to the Mundell-Tobin effect, combined with the fact that capital accumulation tends to be too low due to the taxation of asset income – without such taxes, or without a Mundell-Tobin effect, the Friedman rule is optimal.

For the baseline calibration the optimal inflation rate is very close to the mean in the data. One has to be careful, however, because the optimal policy is somewhat sensitive to parameter values. In an alternative calibration that looks similar to the benchmark along most dimensions, the optimal inflation rate is negative, although still above the Friedman rule. In terms of the financial variables, increases in the pledgability of home equity (the loan-to-value ratio) initially lead to a boom in house prices, construction, investment and employment; further increases in pledgability eventually lead to a bust.

This nonmonotonicity can generate a housing-fueled expansion followed by a recession, in principle, although it is hard account for all of the boom and bust behavior since 2000. Increases in the pledgability of other assets have similar effects. Still, financial innovation can increase welfare, even if it might look bad for some macro variables. We think these kinds of computational exercises constitute a step in the right direction for research that tries to model the microfoundations of the exchange process.<sup>2</sup>

The rest of the paper is organized as follows. Section 2 lays out the basic assumptions. This includes a discussion of debt limits, since they are the heart of the model, and of pricing mechanisms, since the theory allows different approaches to determining the terms of trade. Section 3 defines equilibrium and describes three possible outcomes: liquidity may be so plentiful that the economy gets by without using money; liquidity may be less plentiful but money does not help; and liquidity may be sufficiently scarce that money becomes essential. For each case we derive analytic predictions about asset markets and macroeconomic activity. Section 4 briefly discusses extensions. Section 5 presents the quantitative analysis. The model is calibrated and used to study the effects of inflation on allocations, asset prices and welfare, and to study the effects of financial innovation. Section 6 concludes.<sup>3</sup>

---

<sup>2</sup>Early work in this vein was not meant to be quantitative; the goal was rather to elucidate the roles of various frictions, including spatial or temporal separation, limited commitment and imperfect information, on transactions patterns. As methods and models advance, it becomes increasingly possible to incorporate key elements of the theory into fully-articulated macro models. Although the literature studying similar models quantitatively is not huge, ours is obviously not the first attempt, but rather than listing individual contributions, in the interests of space, we refer readers to Aruoba et al. (2011) for citations.

<sup>3</sup>A related paper is Lester et al. (2012), where differential liquidity is modeled using information frictions: some traders are unable to recognize asset quality. In that paper, agents who do not recognize quality reject assets outright, which avoids bargaining under asymmetric information, but then liquidity differs only on the extensive margin (acceptance by more or fewer counterparties). One can tackle bargaining under asymmetric information in the model, as in Rocheteau (2011), Li and Rocheteau (2011), and Li et al. (2012), and also get liquidity differentials on the intensive margin (acceptance of assets up to endogenous limits), but that is complicated, and often relies on special protocols like take-it-or-leave-it offers. Our approach is based on commitment rather than information frictions – i.e., on pledgability rather than recognizability – which is much easier. This allows us to go well beyond those papers in terms of applications and quantitative analysis. There is much more work on the microfoundations of monetary economics that is related, some of which is discussed below, but there are too many papers to list individually. Therefore we refer readers to the above-mentioned surveys on New Monetarist economics, and to Gertler and Kiyotaki (2010) for a survey of related work from a somewhat different perspective.

## 2 Environment

This Section describes preferences, technology etc., then discusses credit frictions and mechanisms for determining the terms of trade.

### 2.1 Fundamentals

There is a  $[0, 1]$  continuum of infinitely-lived households. Each period in discrete time has two distinct markets that meet sequentially. One is a frictionless centralized market, called AD for Arrow-Debreu, where agents trade assets, labor and certain consumption goods. The other is a market where they trade different goods subject to various frictions that impede credit, as detailed below, called KM for Kiyotaki-Moore. We assume KM convenes before AD, but little depends on this. All agents always participate in AD, while only a measure  $2\sigma \leq 1$ , chosen at random each period, participate in KM. By not participating, we mean some households neither derive utility from, nor have an endowment of, KM goods that period. Of the measure  $\sigma$  that participate, they all have an endowment  $\bar{q}$ , while  $\sigma/2$  have utility function  $u_b(Q)$  and  $\sigma/2$  have utility function  $u_s(Q)$ , where  $u'_b(Q) > u'_s(Q)$  for all  $Q$ , and the subscripts signify buyer and seller. Buyers and sellers meet in KM, and potentially trade because the former have higher marginal utility.

In terms of modeling strategy, this alternating market structure is meant to capture in a tractable way the obviously correct notion that in reality not all economic activity takes place in frictionless settings, nor does it all take place in settings with search, limited commitment or other frictions. The theory is qualitatively robust to changing the details. Thus, one can instead assume that households in the DM have the same utility but different endowments. Or that  $q$  is a factor of production and agents realize different investment-opportunity shocks, as in as in Kiyotaki and Moore (1997, 2005). Or, as in much related work, gains from trade can arise from random matching between households, with  $\sigma$  interpreted as the probability of meeting someone who can produce something you want. And instead of alternating the two markets, one can have them both always open, as in

Williamson (2006), with agents transiting randomly between them. Also, instead of having households trade with each other in KM they could trade with producers or retailers. All these alternatives may be more or less appropriate in given applications, but they do not change the basic insights.<sup>4</sup>

If a seller in KM gives  $q \leq \bar{q}$  of his endowment to a buyer, the cost to the former is  $c(q) \equiv u_s(\bar{q}) - u_s(\bar{q} - q)$ , while the gain for the latter is  $u(q) \equiv u_b(\bar{q} + q) - u_b(\bar{q})$ . Strictly speaking,  $q$  is a transfer, while  $Q_b = \bar{q} + q$  and  $Q_s = \bar{q} - q$  are net consumption for buyers and sellers, but we sometimes we refer to  $q$  as KM consumption. Given  $u_b$  and  $u_s$  satisfy the usual monotonicity and curvature assumptions, so do  $u$  and  $c$ . This notation looks very much like the setup in models where there is random matching and sellers are households that produce (e.g., Lagos and Wright (2005)). The interpretation here, making KM a pure-exchange market, implies that all production occurs in AD, which is not at all crucial but is convenient for some purposes discussed below, like measuring employment. In any case, for a seller to hand over  $q$ , it is obvious that he must get something in return. While many models of this type adopt the interpretation of assets as a means of payment, as we said in the Introduction, here they are used as collateral to secure promises of payment in the next AD market.

This specification captures in an abstract way the notion that households sometimes want to make certain purchases – including some surprise needs, like household or automobile repairs and medical treatment – for which they need loans. They need loans in the model, formally, because in between two AD markets they have no current receipt of labor, asset or other income. These loans require collateral, formally, because limited commitment means agents are free to renege on payment promises. If no punishments are available beyond seizing collateral, sellers will only accept pledges of future payments up

---

<sup>4</sup>More details, including an explicit description of retailers in a similar setting, are contained in Aruoba et al. (2012). The quantitative work in Section 5 interprets KM as a retail market and calibrates some parameters to match retail markup observations. Also, in Section 5, driven by the data we relax the assumption  $\sigma \leq 1/2$ ; this can be interpreted as having some sellers producing for multiple buyers. We do not do this in the benchmark model because we think it might be a distraction, especially to those familiar with standard random-matching models.



to some limit that depends on the value of one's assets. In reality, unsecured credit is not impossible, of course, and some expenditure on home improvement, medical treatment etc. can be put on one's credit card, but as long as there are limits one may sometimes require collateral. We allow unsecured debt up to a limit; beyond this, assets must be used to secure loans.

Moving to the AD market, as in the standard growth model, there is a numeraire good  $x$  that can be used for consumption or investment, produced by firms using capital and labor according to a technology  $f(k, \ell)$ . We assume that  $f$  is strictly increasing and concave. Usually,  $f$  displays CRS (constant returns to scale), but for some results this is not necessary, and it suffices to assume inputs are complements, in the sense  $f_{k\ell} > 0$ . The profit-maximization conditions are

$$\omega = f_{\ell}(k, \ell) \text{ and } \rho = f_k(k, \ell), \quad (1)$$

where  $\omega$  is the wage and  $\rho$  the rental rate on capital in terms of numeraire.<sup>5</sup> Firms are owned by households, and if there are profits they are dispersed as dividends. As always, CRS implies that profits are 0 in equilibrium.

In addition to market capital  $k$ , households own home capital, or housing,  $h$ . Let  $\delta_k$  and  $\delta_h$  be the depreciation rates on  $k$  and  $h$ . Housing can be in fixed supply  $H$ , in which case  $\delta_h = 0$ , or it can be produced endogenously, as discussed below. There is also equity  $e$  in a Lucas (1978) tree paying dividend  $\gamma$  in each AD market. This asset is always in fixed supply, normalized to 1. There is also fiat money  $m$ . Its supply  $M$  evolves over time depending on policy, so that  $M' = (1 + \pi) M$ . There is also a real bond  $b$  that can be purchased in one AD market and redeemed in the next. Its supply  $B$  also depends on policy. A portfolio  $\mathbf{a} = (b, e, h, k, m)$  lists these assets (as a mnemonic device, in alphabetical order). Let  $\phi = (\phi_b, \phi_e, \phi_h, \phi_k, \phi_m)$  be the asset price vector, with  $\phi_k = 1$  because  $k$  and  $x$  are the same physical object. Notice  $\mathbf{a}$  includes some reproducible assets,

---

<sup>5</sup>Recognizing that the structure described below is complicated, we impose method as much as we can on notation. Thus, quantities are represented by Roman and prices or parameters by Greek letters.

like  $k$ , some in fixed supply, like  $e$ , and some that can be either, like  $h$ . It includes  $m$  and  $b$  so one can discuss traditional monetary policy, and one can add other assets (e.g., foreign currency) as one likes. It is also worth mentioning that, in this model,  $m$  does not need to be literally cash; it can include bank deposits. This avoids a common disconnect between theory and measurement emphasized by, e.g., Lucas (2000).

Households have period utility over AD consumption, housing and labor  $\mathcal{U}(x, h, \ell) = U(x, h) - \zeta \ell$ , where for now  $\zeta = 1$ . As in Lagos and Wright (2005), quasi-linear utility simplifies some analytic results, but there should be no presumption that the basic economic insights hinge on it. Chiu and Molico (2010, 2011) study related models with more general preferences numerically, and derive results quantitatively similar to those in the quasi-linear versions. Also, Wong (2012) shows very similar analytic results obtain under weaker conditions.<sup>6</sup> And as discussed further below, Rocheteau et al. (2008) show exactly the same results obtain without quasi-linear utility if we assume indivisible labor and lotteries, à la Rogerson (1988). In any case, in addition to usual regularity conditions like monotonicity and concavity, for a few results we assume  $x$  and  $h$  are normal goods. Also, households have discount factor  $\beta \in (0, 1)$  between the AD market and the next KM market, but without loss of generality there is no discounting between KM and AD.

A household in AD with portfolio  $\mathbf{a}$  has net worth in terms of numeraire

$$y = y(\mathbf{a}) = b + (\gamma + \phi_e) e + (1 - \delta_h) \phi_h h + (\rho + 1 - \delta_k) k + \phi_m m - d + I, \quad (2)$$

where  $d$  is debt (which could be negative) from the previous KM market and  $I$  denotes other income, including dividends and lump-sum transfers minus taxes. All KM debt is settled each period in AD; given the preference structure, this is without loss in generality as long as  $|d|$  is not so big that we get a corner solution. Also, (2) presumes there is no default, which is true in equilibrium; if one were to default,  $d$  would vanish from the RHS,

---

<sup>6</sup>Ignoring housing for a moment, Wong (2012) shows that in order to get the key result (that  $\hat{\mathbf{a}}$  is independent of  $\mathbf{a}$ ; see below), one can replace  $U(x) - \ell$  with any  $U(x, 1 - \ell)$  satisfying  $|U| = 0$ , where  $|U| = U_{11}U_{22} - U_{12}^2$ . This allows any  $U$  that is homogeneous of degree 1, including  $U = x^\eta (1 - \ell)^{1-\eta}$  or  $U = [x^\eta + (1 - \ell)^\eta]^{1/\eta}$ .

and any assets that were pledged would be subtracted. The individual state variable is  $(y, h)$ , since by assumption one has to own housing at the start of a period to enjoy its service flow.

If  $W(y, h)$  denotes the value function of a household in AD then

$$W(y, h) = \max_{x, \ell, \hat{\mathbf{a}}} \{U(x, h) - \ell + \beta V(\hat{\mathbf{a}})\} \text{ s.t. } x = y + \omega\ell - \phi\hat{\mathbf{a}},$$

where  $V(\hat{\mathbf{a}})$  is the continuation value at market closing, generally depending on the composition of the portfolio  $\hat{\mathbf{a}}$

## 2.2 Debt Limits

While most of the literature following Kiyotaki and Moore (1997, 2005) emphasizes limited credit for firms, the focus here is on households. A household's debt position is

$$d = d(\mathbf{d}) = d_b + (\gamma + \phi_e) d_e + (1 - \delta_h) \phi_h d_h + (\rho + 1 - \delta_k) d_k + \phi_m d_m + d_u, \quad (5)$$

where  $\mathbf{d} = (d_b, d_e, d_h, d_k, d_m, d_u)$  can be interpreted as a vector of asset pledges, plus unsecured debt  $d_u$ . In (5), bond pledges are evaluated at face value, as are money and unsecured debt; pledges of equity are evaluated cum dividend; pledges of capital are evaluated before factor markets convene; and home equity pledges are evaluated at market prices after depreciation. As regards housing, this reflects a timing assumption, that a creditor can seize  $h$  if a debtor defaults, but foreclosure occurs at the end of the period, after the current utility flow and depreciation.

A more important point is that we do *not* have in mind borrowers making promises that oblige them to deliver particular quantities of individual assets. Rather, they only pledge to deliver specified amounts of general purchasing power (numeraire). Since AD is a centralized market, neither borrowers nor lenders care about the instrument of settlement, and the pledges  $d_j$  are only interesting off the equilibrium path in the event of default. If you owe  $d$  and renege, the creditor – or maybe the court, or some other abstract institution – seizes an amount  $D_j(a_j) \leq a_j$  of your holdings of  $a_j$ . Note that  $D_j(a_j)$  is not the gain realized by the creditor from asset seizure, but the loss to the debtor. Sellers give up goods in KM because they want general purchasing power, not specific assets, and they believe you will deliver it, up to a point, because otherwise you will be punished. Seizure in this economy is a punishment device that dissuades opportunistic misbehavior.

Clearly, it is a best response to renege if the loss from forfeiture is less than the value of one's obligations. Of course, there can be additional punishments, as it used to be standard in practice to incarcerate defaulters, and it is now standard in theory to take away their future credit, although this punishment is not without potential problems. One

such problem is that after defaulting on a particular creditor, even leaving renegotiation issues aside, it is not always clear why one cannot go to a different creditor (in terms of formal assumptions, this is where anonymity or lack of record keeping is relevant). Given this, we impose the following *pledgability restrictions*

$$d_j \leq D_j(a_j) \text{ for } j = b, e, h, k, m, \text{ and } d_u \leq D_u \quad (6)$$

where  $D_j(0) = 0$ ,  $D_j(a_j) \leq a_j$  and  $\partial D_j / \partial a_j \geq 0$ , in general, with  $D_m(m) = m$  for money. The upper bound on debt comes from pledging oneself to the hilt,

$$\bar{D}(\mathbf{a}) \equiv D_b(b) + (\gamma + \phi_e) D_e(e) + (1 - \delta_h) \phi_h D_h(h) + (\rho + 1 - \delta_k) D_k(k) + \phi_m m + D_u. \quad (7)$$

The RHS of (7) is the most you can be punished, plus  $D_u$ , all expressed in numeraire. Notice  $D_j(a_j)/a_j$  is the loan-to-value ratio. If  $D_j(a_j) = \mu_j a_j$ , this ratio is constant.<sup>7</sup>

Although debtors honor obligations in terms of general purchasing power here, one might imagine situations where they pledge specific assets. In this case, however, absent additional assumptions, buyers may as well hand the assets over at the time of sale and finalize the transaction. One might say this sounds more like Kiyotaki-Wright than Kiyotaki-Moore, and that would be right, but it does not change the equations. It may be reasonable to think some assets, like cash, are more naturally used as media of exchange, while others, like home equity, are more naturally used as collateral. To be precise about this, one ought to explicitly incorporate assumptions about asset attributes, including portability, divisibility, recognizability etc. (see Nosal and Rocheteau (2011) for a modern take). While this may be interesting for some purposes, the distinction between a medium of exchange and collateral does not matter too much here, so we do not dwell on it too much in what follows.

---

<sup>7</sup>There are alternative ways used in the literature to rationalize debt limits, and in particular  $\mu_j < 1$ . Sometimes there is appeal to diversion: creditors can seize only a fraction of your assets, while you abscond with the rest. Sometimes there is appeal to resources getting used up by seizure, including litigation costs (e.g., Iacoviello (2005)), but for us that is irrelevant – compliance here could be encouraged by the threat of *burning* your assets. Holmstrom and Tirole (2011) take pledgability as a primitive, but provide several ways to motivate it. There are also formalizations based on private information, as discussed fn.3. See Gertler and Kiyotaki (2010) for more discussion and references.

## 2.3 Mechanisms

We now determine the KM terms of trade using an abstract mechanism. We understand that to some readers this discussion may seem strange, but one of the key innovations in modern monetary economics involves exploring various options for pricing. We see no need to be wed to Walrasian pricing, sticky or otherwise, especially since one way to motivate credit market imperfections starts with a search-based approach. So, although one can use Walrasian pricing, and as discussed later it is actually a special case, it is not our preferred benchmark.

Starting with an example, suppose KM trade is bilateral, and buyers make take-it-or-leave-it offers. If one asks for  $q$ , one must compensate a seller for his cost,  $d = \omega c(q) / \zeta$ , subject to  $d \leq \bar{D} = \bar{D}(\mathbf{a})$ . Notice  $c(q)$  is measured in utils, so dividing by  $\zeta$  converts it to time, and multiplying by  $\omega$  converts it to numeraire. Given we normalize  $\zeta = 1$ , for now, the best take-it-or-leave-it offer is described as follows: Let  $u'(q^*) = c'(q^*)$ , so  $d^* = \omega c(q^*)$  is the promise one has to make to get  $q^*$ . If  $d^* \leq \bar{D}$  then a buyer asks for  $q^*$  and promises  $d^*$ ; but if  $d^* > \bar{D}$  he cannot credibly promise  $d^*$ , so he offers  $d = \bar{D}$  and gets  $q = c^{-1}(\bar{D}/\omega) < q^*$ . A generalization of this is the Kalai (1977) proportional bargaining solution, which in this context is described as follows: Let

$$z(q) = \theta c(q) + (1 - \theta) u(q), \quad (8)$$

where  $\theta$  is the buyer's bargaining power, and let  $d^* = \omega z(q^*)$ . Then if  $d^* \leq \bar{D}$  the buyer gets  $q = q^*$  and promises  $d = d^*$ ; but if  $d^* > \bar{D}$  he promises  $d = \bar{D}$  and gets  $q = z^{-1}(\bar{D}/\omega) < q^*$ . The take-it-or-leave-it case is  $\theta = 1$ .

Some readers may be more familiar with generalized Nash bargaining.<sup>8</sup> Kalai bargaining is more tractable than Nash, and has several other advantages: for a class of models

---

<sup>8</sup>In our model, as in Lagos and Wright (2005), generalized Nash is similar except

$$z(q) = \frac{\theta u'(q) c(q)}{\theta u'(q) + (1 - \theta) c'(q)} + \frac{(1 - \theta) c'(q) u(q)}{\theta u'(q) + (1 - \theta) c'(q)}.$$

This is the same as (8) when  $u(q) = c(q) = q$ , or when  $\theta = 1$ . Otherwise, they give the same outcome if and only if  $d \leq \bar{D}$  does not bind, but it may well bind in equilibrium.

including this one, Aruoba et al. (2007) show that Kalai guarantees the trading surpluses are increasing in  $\bar{D}$ , that  $V$  is concave, and that buyers have no incentive to conceal their asset; generalized Nash does not guarantee these results except in special cases like  $\theta = 1$ . Hence, Kalai bargaining is used in much recent monetary economics, and we follow that trend in the quantitative work. For now, however, all we need is:

**Assumption 1** *There is some function  $z$ , continuously differentiable on  $(0, q^*)$ , with  $z'(q) > 0$ ,  $z(0) = 0$  and  $\omega z(q^*) = d^*$ , such that: if  $d^* \leq \bar{D}(\mathbf{a})$  then  $q = q^*$  and  $d = d^*$ ; and if  $\omega z(q^*) > \bar{D}(\mathbf{a})$  then  $d = \bar{D}(\mathbf{a})$  and  $q$  solves  $\bar{D}(\mathbf{a}) = \omega z(q)$ .*

Several approaches used in related models are consistent with Assumption 1. These include price posting with either directed or undirected search, as in Lagos and Rocheteau (2005) or Head et al. (2012); abstract mechanism design, as in Hu et al. (2009); and auctions as in Galenianos and Kircher (2008) or Dutu et al. (2009). Some of these, like auctions, or price posting along the lines of Burdett and Judd (1983), are more interesting and easier to motivate once one departs from bilateral trade. If we have multilateral trade, it also makes sense to consider Walrasian pricing, as discussed by Rocheteau and Wright (2005) in this kind of model (in the context of labor-search models, think of switching from Mortensen and Pissarides (1994) to Lucas and Prescott (1974)). To use Walrasian pricing, simply set  $z(q) = Pq$ , where individuals take  $P$  as given, then set  $P = c'(q)$  in equilibrium. If  $c(q) = q$  this is the same as bargaining with  $\theta = 1$ . For the quantitative work we prefer bargaining, with  $\theta < 1$  calibrated to match markup data, but Walrasian pricing satisfies the same equilibrium conditions when  $\theta = 1$ .

In any case, all we need for now is Assumption 1: there is some  $z(q)$  describing the terms of trade when  $d \leq \bar{D}$  binds. We mention in passing that Assumption 1 can also be derived from deeper principles, using an axiomatic approach, instead of building up to it by way of examples (Gu et al. (2012)). But the important economic point is that none of our theoretical results depend on a particular way of splitting the gains from trade.

### 3 Equilibrium

To solve the portfolio problem (the choice of  $\hat{\mathbf{a}}$ ) in (3), form the Lagrangian:

$$\begin{aligned} \mathcal{L} = & -\phi\hat{\mathbf{a}}/\omega + \beta W[y(\hat{\mathbf{a}}), \hat{h}] + \beta\sigma[u(q) - z(q)] + \Sigma_j \lambda_j [D_j(\hat{a}_j) - d_j] + \lambda_u (D_u - d_u) \\ & + \lambda_q \left[ d_b + (\gamma + \phi'_e)d_e + (1 - \delta_h)\phi'_h d_h + (\rho' + 1 - \delta_k)d_k + \phi'_m d_m + p_d - \omega' z(q) \right]. \end{aligned}$$

The constraints with multipliers  $\lambda_u$  and  $\lambda_j$ ,  $j = b, e, h, k, m$ , say that unsecured pledges are limited by  $D_u$  and pledges secured by  $\hat{a}_j$  are limited by  $D_j(\hat{a}_j)$ . The constraint with multiplier  $\lambda_q$  says KM trade must respect the mechanism,  $d = \omega' z(q)$  if the debt limit is binding with  $d = d(\mathbf{d})$  given by (5). Note that  $\omega'$  and  $\phi'$  are prices next period, since that is when the relevant KM trades occurs.

The FOC's are

$$\hat{a}_j : -\frac{\phi_j}{\omega} + \beta \frac{\partial W[y(\hat{\mathbf{a}}), \hat{h}]}{\partial \hat{a}_j} + \lambda_j \frac{\partial D_j(\hat{a}_j)}{\partial \hat{a}_j} \leq 0, \quad = \text{ if } \hat{a}_j > 0 \quad (9)$$

$$d_j : -\lambda_j + \lambda_q \frac{\partial d(\mathbf{d})}{\partial d_j} \leq 0, \quad = \text{ if } d_j > 0 \quad (10)$$

$$q : \beta\sigma \left[ \frac{\partial u(q)}{\partial q} - \frac{\partial z(q)}{\partial q} \right] - \lambda_q \omega' \frac{\partial z(q)}{\partial q} \leq 0, \quad = \text{ if } q > 0. \quad (11)$$

In (10),  $\partial d(\mathbf{d})/\partial d_j$  is the marginal value of a  $d_j$  pledge – e.g.,  $\partial d(\mathbf{d})/\partial d_e = \gamma + \phi_e$  is how much being able to pledge more  $e$  buys you, since each unit is worth  $\gamma + \phi_e$  in the AD market. A solution to the household's problem is given by (9)-(11), plus  $\omega U_x(x, h) = 1$ , which determines  $x$ , and the budget equation, which determines  $\ell$ . The nature of the results depends on which of three possible situations obtains: (1) liquidity is not scarce, in which case  $m$  cannot be valued; (2) liquidity is somewhat scarce, but  $m$  cannot help; and (3) liquidity is more scarce, and  $m$  is essential. We study these in turn.

#### 3.1 Liquid Nonmonetary Equilibrium

If households have sufficient pledgability to acquire  $q^*$  in KM, liquidity is not scarce. In this case, as is standard, fiat money cannot be valued and  $\phi_m = 0$ . One does not have to take this literally – it could be that there are some buyers in the economy that can't get



credit, or some sellers that don't give credit, under any terms, for whatever reason, and they might always use cash. Still, the nonmonetary economy analyzed here is interesting at least as a benchmark. In this case, with  $q = q^*$ , the pledgability constraints are slack, and so  $\lambda_j = 0$ . Then the FOC (9), which holds at equality in equilibrium, becomes  $\phi_j = \omega\beta\partial W/\partial\hat{a}_j$ . Deriving  $\partial W/\partial\hat{a}_j$  and simplifying, we get the asset-pricing conditions:

$$\phi_b = \frac{\beta\omega}{\omega'} \quad (12)$$

$$\phi_e = \frac{\beta\omega}{\omega'} (\gamma + \phi'_e) \quad (13)$$

$$\phi_h = \frac{\beta\omega}{\omega'} [(1 - \delta_h) \phi'_h + \omega' U_h(x', h')] \quad (14)$$

$$1 = \frac{\beta\omega}{\omega'} (\rho' + 1 - \delta_k). \quad (15)$$

Since  $\omega = 1/U_x(x, h)$ , (12) says the bond price equals the MRS,  $\beta U_x(x', h')/U_x(x, h)$ . Similarly, (13)-(14) set the prices of  $e$  and  $h$  to the MRS times their payoffs. And (15) is the usual capital Euler equation, which in steady state is  $\rho = r + \delta_k$  with  $r = (1 - \beta)/\beta$ . The *accounting return*  $r_j$  on asset  $j$  is next period's payoff over the current price,

$$1 + r_b = 1/\phi_b \quad (16)$$

$$1 + r_e = (\gamma + \phi'_e)/\phi_e \quad (17)$$

$$1 + r_h = (1 - \delta_h) \phi'_h/\phi_h \quad (18)$$

$$1 + r_k = \rho' + 1 - \delta_k. \quad (19)$$

Of course, for housing, which is a consumption good as well as an asset, the true return is  $[(1 - \delta_h) \phi'_h + \omega U_h(x', h')]/\phi_h$ , not only the capital gain. From (12)-(15), the true return on all assets is  $(1 + r)\omega'/\omega$  when liquid is plentiful; this will not be so when it is scarce.

The above results follow directly from the household problem. The next step is to discuss macroeconomic equilibrium, in two versions of the model, one with a fixed stock of housing  $H$  and the other with an endogenous supply. In the first version, given the initial stocks of  $k$  and  $h$ , equilibrium consists of time paths for: (1) AD consumption, capital

investment, housing investment, and employment  $(x, k', h', \ell)$  satisfying

$$1 = f_\ell(k, \ell) U_x(x, H) \quad (20)$$

$$U_x(x, H) = \beta U_x(x', H) [f_k(k', \ell') + 1 - \delta_k] \quad (21)$$

$$h' = H \quad (22)$$

$$x = \gamma + f(k, \ell) - [k' - (1 - \delta_k)k]; \quad (23)$$

(2) KM consumption and debt  $q = q^*$  and  $d = f_\ell(k, \ell) z(q^*)$ ; and (3) asset prices as described above.<sup>9</sup> A steady state satisfies stationary versions of these conditions.

It seems worth spending a little time on steady state in this case, where liquidity is plentiful and money is not valued, before considering more complicated scenarios. To begin, impose stationarity in (20)-(23) and derive

$$\begin{bmatrix} f_{kk} & f_{k\ell} & 0 \\ f_k - \delta_k & f_\ell & -1 \\ U_x f_{\ell k} & U_x f_{\ell \ell} & f_\ell U_{xx} \end{bmatrix} \begin{bmatrix} dk \\ d\ell \\ dx \end{bmatrix} = \begin{bmatrix} dr + d\delta_k \\ kd\delta_k - d\gamma \\ -f_\ell U_{xh} dh \end{bmatrix}.$$

Let the square matrix be  $\mathbf{C}_1$ . Then  $\Delta_1 = \det(\mathbf{C}_1) = f_\ell [f_\ell f_{kk} - (f_k - \delta_k) f_{k\ell}] U_{xx} + |f| U_x > 0$ , where  $|f| = f_{kk} f_{\ell \ell} - f_{k\ell}^2 \geq 0$ , with equality if  $f$  displays CRS. It is now routine to compute the effects of parameters on the allocation and prices, including factor prices and the rental rate on housing,  $R_h = (r + \delta_h) \phi_h$ . These are summarized in Table 1.<sup>10</sup>

Generally, most results are either unambiguous or ambiguous for good economic reasons. Consider those related to housing,

$$\Delta_1 \partial k / \partial H = f_\ell f_{k\ell} U_{xh} \approx U_{xh}$$

$$\Delta_1 \partial \ell / \partial H = -f_\ell f_{kk} U_{xh} \approx U_{xh}$$

$$\Delta_1 \partial x / \partial H = f_\ell [(f_k - \delta_k) f_{k\ell} - f_\ell f_{kk}] U_{xh} \approx U_{xh}$$

---

<sup>9</sup>In case it is not obvious, (20) and (21) are the FOC's for  $x$  and  $\hat{k}$ , after inserting factor prices  $\omega$  and  $\rho$  from (1); (22) clears the housing market, with  $\phi_h$  adjusting to make that happen; and (23) clears the AD goods market. Note that  $\ell$  denotes aggregate labor in these equations. Individual household labor depends on net wealth in AD, which generally differs across households depending on debt from the previous KM market, but we only need aggregate  $\ell$  to define macro equilibrium.

<sup>10</sup>Here we omit the effects on  $q$ , as well as the effects of  $\sigma$  and  $D_j$ , because they are all 0 in this case.

	$k$	$\ell$	$x$	$\omega$	$\rho$	$R_h$
$\gamma$	—	—	+/0	+/0	0	$+N_h/0$
$r$	—	?	—	—	+	$-N_h$
$\delta_k$	?	?	—	—	+	$-N_h$
$H$	$+U_{xh}$	$+U_{xh}$	$+U_{xh}$	$-U_{xh}/0$	0	$-N_x$

Table 1: Results in Case 1 with  $k$  endogenous,  $h$  exogenous.  $+U_{xh}$  means the same sign as  $U_{xh}$  and similarly for  $-U_{xh}$ ;  $+/0$  means  $=0$  for concave  $f$  and  $=0$  for CRS; and  $-N_j$  means  $=0$  if good  $j$  is normal.

where  $A \approx B$  indicate that  $A$  and  $B$  take the same sign. Naturally these depend on whether  $x$  and  $h$  are complements or substitutes in the sense  $U_{xh} \gtrless 0$ , as in many home production models. However,  $\partial\phi_h/\partial H < 0$  is unambiguous, at least if  $x$  is normal, which we interpret as a downward-sloping long-run demand for housing.

For the model with endogenous  $h$ , we introduce into the AD market competitive home builders with convex cost  $g(\cdot)$ , so in equilibrium  $\phi_h = g'[h' - (1 - \delta_h)h]$ . Combining this with the FOC for  $h'$ , we get the housing Euler equation

$$U_x(x, h)g'[h' - (1 - \delta_h)h] = \beta U_x(x', H) \{ (1 - \delta_h)g'[h'' - (1 - \delta_h)h'] + U_h(x', h') \}.$$

Equilibrium with endogenous  $h$  uses this instead of (22), and uses

$$x = \gamma + f(k, \ell) - [k' - (1 - \delta_k)k] - g[h' - (1 - \delta_h)h]$$

instead of (23). In steady state the home builders' FOC is  $g'(\delta_h h) = \phi_h$ , an upward-sloping long-run supply curve. Hence there is a unique  $(h, \phi_h)$  clearing the housing market. Symmetric to Table 1, Table 2 gives parameter effects with  $h$  endogenous but  $k$  fixed, instead of  $k$  endogenous but  $h$  fixed.<sup>11</sup>

Housing is included in the model not only because it is topical, but because it allows us to make several substantive points. First, when  $h$  is endogenous, supply and demand jointly determining  $(h, \phi_h)$ , while when  $h = H$  is fixed demand simply pins down price.

<sup>11</sup>For a few of these results we assume that  $r$  is not too big.

	$h$	$\ell$	$x$	$\omega$	$\rho$	$R_h$
$\gamma$	$+N_h$	$-$	$+N_x$	$+$	$-$	$+N_h$
$r$	$-$	$-N_h$	$?$	$+N_h$	$-N_h$	$+N_{h,x}$
$\delta_h$	$-N_h$	$?$	$?$	$?$	$?$	$+N_{h,x}$
$K$	$+N_h$	$?$	$+N_x$	$?$	$?$	$+N_h$

Table 2: Results in Case 1 with  $h$  endogenous,  $k$  exogenous.  $+U_{xh}$  means the same sign as  $U_{xh}$  and similarly for  $-U_{xh}$ ;  $+/-0$  means  $=0$  for concave  $f$  and  $=0$  for CRS; and  $-N_j$  means  $=0$  if good  $j$  is normal.

We could make a similar point by comparing  $k$  and  $e$ , but that is less interesting because we always have  $\phi_k = 1$ , which is not true for  $\phi_h$ . Second, different from other assets,  $h$  affects utility directly and not only via the budget equation, which has some interesting implications. Suppose, e.g., that  $U(x, h) = \tilde{U}(x) + h^{1-\xi}/(1-\sigma)$ . Then when  $H$  increases, home equity  $\phi_h H$  can go up or down, so liquidity can become less or more scarce, depending on  $\xi \gtrless 1$ . Because of this, welfare can also go up or down as  $H$  increases. By contrast, increases in the supply of  $e$ , which enters the budget equation but not  $U$ , always raises liquidity and welfare in the model. Third,  $h$  provides an plausible situation where it is not equivalent to use an asset as collateral or as a medium of exchange: even if it were possible to hand over part of your house in a KM transaction, our assumptions imply that then it cannot be used that period, so you would prefer deferred settlement secured by  $h$  to finalizing the deal by transferring assets.

We close this case with three other observations. First, we still have to ask, under what conditions do we get equilibrium where liquidity is plentiful? Focusing on steady state, this obtains if and only if  $\bar{D}(\mathbf{a}) > f_\ell(k, \ell)z(q^*)$ , where  $\bar{D}(\mathbf{a})$  is given by (7). Second, in this equilibrium the form of payment is indeterminate, and one can use  $b$ ,  $e$ ... or any combination. Third, when liquidity is plentiful, the model *dichotomizes*: the AD allocation  $(x, k', h', \ell)$  is independent of  $q^*$ . It is known that one can break this dichotomy by interacting  $q$  with  $(x, k', h', \ell)$  in preferences or technology; in what follows we break it by assuming liquidity is scarce, so AD and KM interact via financial considerations.

### 3.2 Illiquid Nonmonetary Equilibrium

Consider next a nonmonetary equilibrium where  $\bar{D}(\mathbf{a})$  is such that buyers cannot get  $q^*$ .

The FOC for  $q$  implies  $\lambda_q = \beta \sigma L(q) / \omega' > 0$ , where

$$L(q) = \frac{u'(q) - z'(q)}{z'(q)}. \quad (24)$$

Although not strictly necessary, to ease the presentation, assume  $L'(q) < 0 \forall q$ .<sup>12</sup> Then for  $j \neq m$  the FOC for  $d_j$  implies  $\lambda_j = \lambda_q \partial d / \partial d_j > 0$ . Hence, KM buyers borrow to the limit  $\bar{D}(\hat{\mathbf{a}})$ , and  $q$  solves  $z(q) \omega' = \bar{D}(\hat{\mathbf{a}})$ . In terms of asset prices, we have

$$\phi_b = \frac{\beta \omega}{\omega'} [1 + D'_b(B) \sigma L(q)] \quad (25)$$

$$\phi_e = \frac{\beta \omega}{\omega'} (\gamma + \phi'_e) [1 + D'_e(1) \sigma L(q)] \quad (26)$$

$$\phi_h = \beta \omega U_h(x', h') + \frac{\beta \omega}{\omega'} (1 - \delta_h) \phi'_h [1 + D'_h(h') \sigma L(q)] \quad (27)$$

$$1 = \frac{\beta \omega}{\omega'} (\rho' + 1 - \delta_k) [1 + D'_k(k') \sigma L(q)]. \quad (28)$$

Compared to (12)-(15), the *liquidity premium*  $D'_j(\hat{a}_j) \sigma L(q)$  now appears on the RHS, because as long as  $D'_j(a_j) > 0$ , having more  $a_j$  relaxes debt limits.

Suppose  $h = H$  is fixed (endogenous  $h$  can be handled as above). An illiquid nonmonetary equilibrium consists of paths for: (1)  $(x, k', h', \ell)$  satisfying

$$1 = f_\ell(k, \ell) U_x(x, H) \quad (29)$$

$$U_x(x, H) = \beta U_x(x', H) [f_k(k', \ell') + 1 - \delta_k] [1 + D'_j(k) \sigma L(q)] \quad (30)$$

$$h' = H \quad (31)$$

$$x = \gamma + f(k, \ell) + (1 - \delta_k) k - k'; \quad (32)$$

(2)  $(q, d)$  satisfying  $d = \bar{D}(\hat{\mathbf{a}})$  and  $z(q) = d / \omega'$ ; and (3) asset prices as described above.

Compared to the previous case, (30) has  $1 + D'_j(k) \sigma L(q)$  multiplying the RHS because

---

<sup>12</sup>This condition holds automatically for many standard mechanisms, including Kalai bargaining and Walrasian pricing, but not generalized Nash bargaining. One can prove the same results without assuming  $L'(q) < 0 \forall q$ , as in Wright (2010), but we prefer to avoid these technicalities.

liquidity considerations now affect investment in productive capital. Steady state satisfies stationary versions of these conditions, including

$$\bar{D}(\hat{\mathbf{a}}) = D_b(B) + D_e(1)\phi_e + D_h(H)\phi_h + [f_k(k, \ell) + 1 - \delta_k]D_k(k) + D_u \quad (33)$$

which makes  $q$  depend on asset prices (for  $b$ ,  $e$  and  $h$ ) and quantities (for  $k$  and for  $h$  when it is endogenous).

For illustration, let  $D_j(\hat{a}_j)/\hat{a}_j = \mu_j$  and assume  $\mu_b = \mu_e = \mu_h = 0 < \mu_k$ , so that for now  $k$  and only  $k$  serves as collateral. Then steady state is summarized by an allocation  $(x, k, \ell, q)$  satisfying:

$$1 = f_\ell(k, \ell)U_x(x) \quad (34)$$

$$r + \delta_k = f_k(k, \ell) + [f_k(k, \ell) + 1 - \delta_k]\sigma\mu_k L(q) \quad (35)$$

$$x = \gamma + f(k, \ell) - \delta_k k \quad (36)$$

$$f_\ell(k, \ell)z(q) = [f_k(k, \ell) + 1 - \delta_k]\mu_k k + D_u \quad (37)$$

From these one derives

$$\mathbf{C}_2 \begin{bmatrix} dq \\ dk \\ d\ell \\ dx \end{bmatrix} = \begin{bmatrix} dD_u + (f_k + 1 - \delta_k)kd\mu_k - \mu_k k d\delta_k \\ dr + (1 + \sigma\mu_k L)d\delta_k - (f_k + 1 - \delta_k)L(\sigma d\mu_k + \mu_k d\sigma) \\ kd\delta_k - d\gamma \\ -f_\ell U_{xh}dH \end{bmatrix}$$

where

$$\mathbf{C}_2 = \begin{bmatrix} f_\ell z' & z f_{\ell k} - \mu_k(f_k + 1 - \delta_k) - \mu_k f_{kk} & z f_{\ell \ell} - \mu_k k f_{k\ell} & 0 \\ (f_k + 1 - \delta_k)\mu_k \sigma L' & (1 + \mu_k \sigma L)f_{kk} & (1 + \mu_k \sigma L)f_{k\ell} & 0 \\ 0 & f_k - \delta & f_\ell & -1 \\ 0 & f_{k\ell} U_x & f_{\ell \ell} U_x & f_{\ell} U_{xx} \end{bmatrix}$$

In general,  $\Delta_2 = \det(\mathbf{C}_2)$  is complicated, but one can show  $\Delta_2 > 0$  at least if  $\sigma\mu_k$  is not too big.

The effects of parameter changes are in Table 3, which has some new twists, compared to Tables 1-2, because now  $q < q^*$  and financial conditions matter. First, an increase in  $r$  lowers  $k$ , but at least when  $\mu_k$  is not too big this increases  $q$ . This is because  $\rho = f_k$  is

	$q$	$k$	$\ell$	$x$	$\omega$	$\rho$
$\gamma$	$-$	$-^*$	$-^*/0$	$+^*$	$+^*/0$	$-$
$r$	$+^*$	$-$	$?$	$-$	$-$	$+$
$H$	$+U_{xh}$	$+U_{xh}^*$	$+U_{xh}^*$	$+U_{xh}^*$	$-U_{xh}^*$	$+U_{xh}$
$D_u$	$+$	$-$	$?$	$+$	$-$	$+$
$\mu_k$	$?$	$+^*$	$-^*$	$+^*$	$-^*$	$-^*$
$\sigma$	$-^*$	$+$	$?$	$+$	$+$	$-$

Table 3: Results in Case 2 with  $k$  endogenous,  $h$  exogenous. Notes:  $+U_{xh}$  means the same sign as  $U_{xh}$  and similarly for  $-U_{xh}$ ;  $+^*$  means the result is ambiguous in general but  $> 0$  if  $k$  is small, and similarly for  $-^*$ ; and  $+ / 0$  means  $= 0$  for concave  $f$  and  $= 0$  for CRS, and similarly for  $- / 0$ .

higher when there is less  $k$ , and on net credit constraints can be relaxed. Second, supposing  $h$  and  $x$  are complements, if  $H$  increases then  $x$  and  $k$  do, too, so credit constraints ease with higher  $H$  even though  $\mu_h = 0$ . The loan-to-value ratio  $\mu_k$  has an ambiguous effect on  $q$ , because  $k$  rises but  $\rho$  falls, and on net debt limits can fall. Similarly, an increase in  $\sigma$  makes agents put more weight on liquidity, which increases  $k$  and hence  $x$ , but has an ambiguous effect on  $q$ . Indeed,  $\partial q / \partial \sigma < 0$  is unambiguous when  $\mu_k$  is small. One might call this a *paradox of liquidity*: individuals can try to relax borrowing constraints by investing in pledgeable  $k$ , but if everyone does so,  $\rho$  can fall enough to tighten credit conditions.

Finally, as in the previous case, we have to ask when this (illiquid nonmonetary) equilibrium exists. The answer is  $f_\ell(k, \ell) z(q^*) > \bar{D}(\hat{\mathbf{a}}) \geq f_\ell(k, \ell) z(q^i)$ . The first inequality says agents cannot borrow enough to get  $q^*$ , with  $\bar{D}(\hat{\mathbf{a}})$  given in (7). The second says they can borrow enough to get at least what they would get in monetary equilibrium, as described next.

### 3.3 Monetary Equilibrium

In monetary equilibrium, the constraints bind, so  $\lambda_q = \beta \sigma L(q) > 0$  and  $\lambda_j = D'_j(p_j) \lambda_q > 0$ . Again, buyers go to the limit  $\bar{D}(\hat{\mathbf{a}})$ , but now this includes real balances. The equations for  $(\phi_b, \phi_e, \phi_h, \phi_k)$  are the same as above, (25)-(28), and now there is a new condition

for pricing currency,

$$\phi_m = \frac{\beta\omega}{\omega'} \phi'_m [1 + \sigma L(q)]. \quad (38)$$

Equilibrium satisfies the relevant conditions for AD, KM and asset prices, as above, but in a monetary economy the determination of  $q$  is very different.

To see this, rearrange (38) as  $\omega' \phi_m / \beta \omega \phi'_m = 1 + \sigma L(q)$ . The LHS is the inflation rate  $1 + \pi = \phi_m / \phi'_m$  times the real interest rate  $1 + r = \omega' / \beta \omega$  on an illiquid bond – i.e., one that is not pledgeable at all – which we can always price even if it does not trade in equilibrium. Therefore, by the Fisher equation, the LHS is the  $1 + i$ , where  $i$  is the return on an illiquid nominal bond. Hence (38) can be rewritten succinctly as

$$i = \sigma L(q), \quad (39)$$

Monetary policy can set inflation or the growth rate of  $M$  (both equal  $\pi$  in steady state). Or, it can peg the interest rate on illiquid nominal bonds  $i$ , then let  $\pi$  and  $M$  evolve endogenously. For concreteness let's say policy pegs  $i$ . Then (39) pins down  $q = q^i$ , with  $\partial q / \partial i = 1 / \sigma L'(q) < 0$ . As usual, increasing  $i$  raises the cost of carrying real balances, and this lowers purchases of  $q$ .

A monetary steady state can be summarized by an allocation satisfying similar conditions to the previous case, except we replace  $\omega z(q) = \bar{D}(\hat{\mathbf{a}})$  with  $i = \sigma L(q)$ . For monetary equilibrium to exist we need  $\bar{D}(\hat{\mathbf{a}}) < f_\ell(k, \ell) z(q^i)$ , which says the  $q^i$  that solves (39) exceeds what one could get using only credit. Obviously, this is more likely to be true when  $i$  is lower. In steady state,  $(x, k, \ell, q)$  satisfies

$$1 = f_\ell(k, \ell) U_x(x) \quad (40)$$

$$r + \delta_k = f_k(k, \ell) + [f_k(k, \ell) + 1 - \delta_k] \sigma D'_k(k) L(q) \quad (41)$$

$$x = \gamma + f(k, \ell) - \delta_k k \quad (42)$$

$$i = \sigma L(q). \quad (43)$$

At the *Friedman rule*  $i = 0$ , we have  $L(q) = 0$ , and (40)-(42) reduce to the equilibrium



conditions for  $(x, k, \ell)$  in Section 3.1. As in many monetary models,  $i = 0$  delivers an efficient AD allocation.

In our model, there is still a question of whether  $i = 0$  also delivers KM efficiency,  $q = q^*$ . The answer depends on the mechanism: it is not hard to verify that  $i = 0$  implies  $q = q^*$  with Walrasian pricing or Kalai bargaining, but not necessarily with generalized Nash bargaining unless  $\theta = 1$ . For Nash with  $\theta < 1$ ,  $i = 0$  is still optimal but it does not achieve the first best. In this case, it would be desirable to set  $i < 0$ , in principle, but there is no equilibrium with  $i < 0$ . This is the New Monetarist version of the New Keynesian *zero lower bound* problem. Here,  $i < 0$  would be desirable, if only it were feasible, because it would correct a problem with Nash bargaining. Still, with any of these pricing mechanisms  $i = 0$  is the optimal policy in this version of the model, but once we introduce other distortions, that may no longer be the case. In Section 5, we introduce capital-income taxation, and find that  $i > 0$  may be optimal.

In any case, setting  $D(a_j) = \mu_j a_j$  and combining (39) and (41), we get

$$f_k(k, \ell) = \frac{r + \delta_k - (1 - \delta_k) \mu_k i}{1 + \mu_k i}.$$

While  $\mu_k$  affects this condition, it cannot affect  $q$ , which is determined by (39). How can pledgability not affect KM trade? The answer is that in monetary equilibrium  $i$  pins down  $q^i$  and then  $q^i$  pins down the debt limit  $\bar{D}(\hat{\mathbf{a}}) = f_\ell(k, \ell) z(q^i)$ , not vice-versa. Heuristically, when the pledgability of a  $k$  increases, other forms of liquidity are crowded out, leaving

$$\bar{D} = \mu_b B + \frac{(1+r) \gamma \mu_e}{r - \mu_e i} + \frac{(1 - \delta_h) (U_h/U_x) \mu_h H}{r + \delta_h - (1 - \delta_h) \mu_h i} + (f_k + 1 - \delta_k) \mu_k K + D_u + \phi_m M$$

the same. Relatedly, increasing  $B$  has no effect on the allocation, as real balances get completely crowded out. This suggests open market operations, or more generally, *quantitative easing*, might not have the impact one expects.

To characterize these and other effects in more detail, one can derive

$$\mathbf{C}_3 \begin{bmatrix} dq \\ dk \\ d\ell \\ dx \end{bmatrix} = \begin{bmatrix} Ld\sigma - di \\ dr + (1 + \mu_k \sigma L) d\delta_k - (f_k + 1 - \delta_k) L(\sigma d\mu_k + \mu_k d\sigma) \\ kd\delta_k - d\gamma \\ -f_\ell U_{xh} dH \end{bmatrix}$$

where

$$\mathbf{C}_3 = \begin{bmatrix} -\sigma L' & 0 & 0 & 0 \\ (f_k + 1 - \delta_k) \mu_k \sigma L' & (1 + \mu_k \sigma L) f_{kk} & (1 + \mu_k \sigma L) f_{k\ell} & 0 \\ 0 & f_k - \delta_k & f_\ell & -1 \\ 0 & f_{\ell k} U_x & f_{\ell \ell} U_x & f_{\ell} U_{xx} \end{bmatrix}$$

and  $\Delta_3 = \det(\mathbf{C}_3) = -\sigma L' (1 + \mu_k \sigma L) \{U_x |f| + f_\ell [f_\ell f_{kk} - f_{k\ell} (f_k - \delta_k)] U_{xx}\} > 0$ . The (extremely sharp) effects of parameters are shown in Table 4, including what is usually called the *Tobin effect*,

$$\Delta_3 \partial k / \partial i = (f_k + 1 - \delta_k) \mu_k \sigma L' (f_\ell^2 U_{xx} + f_{\ell \ell} U_x) > 0 \text{ if } \mu_k > 0.$$

Intuitively, higher inflation gives households the incentive to substitute  $k$  for  $m$  in their portfolios, which is relevant for several of the findings to follow.

	$q$	$k$	$\ell$	$x$	$\omega$	$\rho$	$\phi_e$
$i$	−	+	?	+	+	−	+
$\sigma$	+	0	0	0	0	0	0
$\gamma$	0	−	−	+	0	0	0
$\mu_k$	0	+	?	+	+	−	0
$\mu_e$	0	0	0	0	0	0	+
$r$	0	−	?	−	−	+	0
$\delta_k$	0	?	?	−	−	+	0

Table 4: Results in Case 3 with  $k$  endogenous,  $h$  exogenous.

In particular, since monetary policy affects investment in capital it also affects the labor market. In general, one cannot sign

$$\Delta_3 \partial \ell / \partial i = -(f_k + 1 - \delta_k) \mu_k \sigma L' [f_{k\ell} U_x + (f_k - \delta_k) f_\ell U_{xx}]$$

and so the *Phillips curve* can go either way, but if  $U_{xx}$  is small then we know for sure

that  $\partial\ell/\partial i > 0$ .<sup>13</sup> Here movements in  $\ell$  come along the intensive margin, as changes in hours worked by the representative individual, but as mentioned above, one can assume indivisible labor and use lotteries to generate unemployment. The same equations hold in that model, but now movements in  $\ell$  come along the extensive margin, as changes in the number of agents employed. Thus, the model can generate a long-run exploitable trade-off between inflation and unemployment. Of course, just because it is feasible to reduce unemployment by increasing inflation, that does not mean it is a good idea. As we said,  $i = 0$  is optimal here, although it may not be once one introduce other distortions.

In terms of financial parameters, higher  $\mu_k$  makes  $k$  a better payment instrument, and so it increases investment, but again  $q$  does not change – restricting attention, obviously, to parameter changes that do not take us out of monetary equilibrium. Similarly, increasing  $\mu_e$  raises the price and lowers the return on  $e$  but has no effect on  $q$ . In terms of policy's impact on returns, increasing  $i$  lowers the demand for money and raises the demand for other assets, which we call a *Mundell effect*. This helps clarify the nature of *Fisher's theory* that nominal interest rates increase one-for-one with inflation, leaving real returns the same. One version of this theory, the Fisher equation, says the real return on a illiquid (nonpledgable) bond is pinned down in steady state by  $1 + r = 1/\beta$ , independent of policy. But the critical qualification there is that the asset is illiquid.

Fisher's theory cannot hold for all assets, since  $m$  is an asset, with a 0 nominal return, and so its real return must fall with inflation. How about partially-liquid assets? The real return on equity is

$$r_e = \frac{r - \mu_e i}{1 + \mu_e i} > 0,$$

assuming  $i$  is sufficiently low that monetary equilibrium exists. If  $\mu_e > 0$  then  $\partial r_e / \partial i < 0$ .

---

<sup>13</sup>If  $U(x) = x^{1-\eta}/(1-\eta)$  and  $f(k, \ell) = k^\alpha \ell^{1-\alpha}$  then one can show  $\partial\ell/\partial i > 0$  when  $\eta < \hat{\eta}$ , where  $\hat{\eta} > 1$ . Also, by employment we mean AD labor, since  $q$  is not produced. We set it up this way precisely to identify employment by  $\ell$ , but alternatively, if  $q$  is produced there is an effect going the other way (see Section 5). And we abstract from the effect in Rocheteau et al. (2007) and Dong (2011), where  $x$  and  $q$  interact in utility, making  $\partial\ell/\partial i \gtrless 0$  depend on whether they are substitutes or complements. Some people seem to think employment increases with inflation in the data, although our reading is the opposite (see Berentsen et al. (2011) or Figure 5 below). So perhaps it is good the framework is flexible in its prediction for  $\partial\ell/\partial i$ .

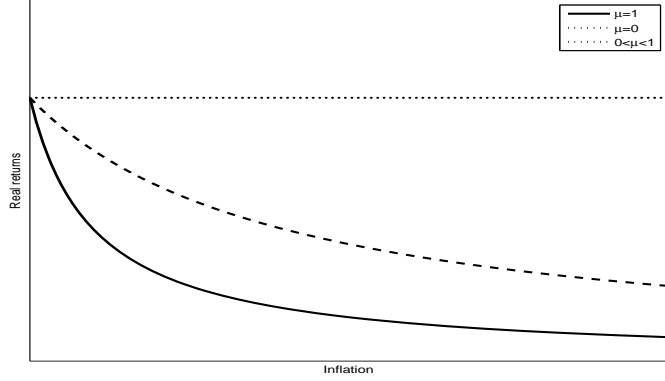


Figure 1: Inflation and asset returns

More generally, for any asset  $j$ ,  $\partial r_j / \partial i \leq 0$  with equality if and only if  $\mu_j = 0$  (i.e., if and only if the asset is not pledgeable at all). In Figure 1, the dotted line is the steady state return on an illiquid asset,  $1 + r = 1/\beta$ , the solid curve is the real return on cash,  $1 + r_m = \phi' / \phi$ , and the dashed curve is the real return on an asset with  $0 < \mu_j < 1$ . Hopefully, this clarifies Fisher's theory: real returns are independent of inflation for illiquid but not liquid assets.

## 4 Extensions

Before getting into the quantitative work, we briefly mention how the framework can be extended in various directions.<sup>14</sup> First, in a stochastic economy liquidity risk is also a source of variation in asset returns. Intuitively, assets that provide liquidity in states where it is most needed command a higher premium. Suppose the dividend on the Lucas tree  $\gamma$  is an i.i.d. random variable with mean  $\bar{\gamma}$ , and a realization that is known at the point of KM trade. In monetary equilibrium, the average return on equity is

$$1 + r_e = \frac{\phi_e + \bar{\gamma}}{\phi_e} = \frac{1 + r}{1 + \mu_e i} - \frac{\mu_e \text{cov}[L(q), \gamma / \phi_e]}{1 + \mu_e i}.$$

The first term on the RHS is the same as before; the second is an adjustment for risk, making  $r_e$  lower if the dividend  $\gamma$  is big in states when  $\lambda_q$  is high.

<sup>14</sup>It is possible to skip this Section without loss of continuity.

Lagos (2010, 2011) assumes  $b$  is riskless while  $e$  has a random dividend. Then the excess return on equity has two parts, one coming from the liquidity differential and one from risk:

$$r_e - r_b = \frac{(1+r)(\mu_b - \mu_e)i}{(1+\mu_e i)(1+\mu_b i)} - \frac{\mu_e \text{cov}[L(q), \gamma/\phi_e]}{1+\mu_e i}.$$

Notice this premium depends on the policy variable  $i$ . Although this can be explored in more detail, we focus below on the pure liquidity premium. But to be clear, there are two parts to Lagos' approach. One is that differences in  $\mu_j$  help explain  $r_e - r_b$ . The other is to notice that even if  $\mu_e = \mu_b$  liquidity affects how one interprets the data. Heuristically, suppose  $r_e = 6\%$  and  $r_b = 1\%$ . A factor of 6 looks like a lot to explain based on risk. But suppose the liquidity value of both assets is worth 4%. Then the "corrected" return on  $e$  is 10% and on  $b$  is 5%, only a factor of 2.

Next, we mention that it is easy to allow match-specific pledgability limits: the loan-to-value ratio  $\mu_j$  for asset  $j$  can depend on who one trades with in the KM market. It is worth pursuing this, in general, but here we only use the idea to demonstrate one way of making pledgability endogenous, following Lester et al. (2012). Suppose there are two assets,  $e$  and  $m$ . Every seller takes  $m$  at face value, while there are two technologies for enforcing debt secured by  $e$ . The first is free, and allows creditors to seize  $\mu_1 e$  in the event of a default. The second has a fixed cost  $\kappa$ , and enables sellers to seize  $\mu_2 e$ , with  $\mu_2 > \mu_1$ .<sup>15</sup> Let us assume  $\kappa_s$  has to be paid each period in the AD market, although it is also interesting to consider once-and-for-all investments. The cost is specific to the individual: agent  $s \in [0, 1]$  must invest  $\kappa_s \in [\underline{\kappa}, \bar{\kappa}]$  to access the superior technology.

Labeling agents so that  $\kappa_s$  is increasing in  $s$ , let  $H(\kappa)$  be the corresponding CDF with support  $[\underline{\kappa}, \bar{\kappa}]$ . Let  $\chi$  be the endogenous fraction of agents that invest in the better technology. Conditional on being a KM seller, one's only relevant characteristic is whether one has made the investment. Conditional on being a buyer,  $\chi$  is the fraction of meetings

---

<sup>15</sup>One can imagine that sellers do not always know the quality of equity, say, and seizing a lemon or counterfeit asset entails no cost for a defaulter. A seller that invests in the requisite information to discern asset quality can accept more assets as collateral. See Lester et al. (2012) for more discussion.

where  $\mu_e = \mu_2$ . The benefit for a seller from having the better technology, depending on the measure of others who have it, is

$$\Omega = \Omega(\chi) \equiv z(q_2) - c(q_2) - z(q_1) + c(q_1),$$

where  $q_1$  and  $q_2$  are KM output in the two types of meetings, which depend on  $\chi$ .

The investment decision of agent  $s$  is obvious: invest if  $\Omega(\chi)$  exceeds  $\kappa_s$ . Defining  $T(\chi) = H[\max\{s : \kappa_s \leq \beta\sigma\Omega(\chi)\}]$ , an equilibrium with endogenous pledgability is a fixed point  $\chi = T(\chi)$ . While  $T(\chi)$  is not necessarily continuous, Lester et al. (2012) show assuming Kalai bargaining that it is increasing. Heuristically, if more sellers have the  $\mu_2$  technology, it is easier to use  $e$  as collateral, which increases the demand for  $e$ . This increases the price  $\phi_e$  and makes agents more keen on being able to trade using  $e$  as collateral, so  $\chi$  goes up. Hence, by Tarski's fixed point theorem an equilibrium exists. There can easily be multiple equilibria, and it is possible that no one invests in the superior technology  $\chi = 0$ , where everyone does  $\chi = 1$ , and where a fraction does  $\chi \in (0, 1)$ . Also, pledgability is not invariant to changes in policy in this economy: the ability to get a loan secured by  $e$  changes when  $i$  changes.

One can also make pledgability endogenous with a simple moral-hazard model. Again consider two assets,  $e$  and  $m$ , and suppose the former requires maintenance to yield the full dividend  $\gamma$ . Agents with  $e$  units of equity choose a maintenance level  $n \in [0, 1]$ , after trading, at a utility cost  $\varepsilon ne$ . Given  $n$  the dividend per unit of equity is  $n\gamma e$ . In principle, a debtor can choose to not maintain an asset that has been pledged to a creditor, renege on his obligation, and forfeit  $e$ . One can show (details available on request) the inability of debtors to commit to  $n$  leads to an endogenous pledgability limit  $D_e(e) = \mu_e e$  with  $\mu = 1 - \varepsilon\omega' / (\gamma + \phi_e)$ . That arises because he will maintain the asset to keep the return to his share (after forfeiture) high. Also, when all agents face the same maintenance cost  $\varepsilon$ , then it is equivalent to use the asset as a medium of exchange or as collateral. But this is no longer true when  $\varepsilon$  differs across agents.

Suppose there are two types, with costs  $\varepsilon_1$  and  $\varepsilon_2 > \varepsilon_1$ , and the fraction of type-1 agents is common knowledge. We consider two cases: ex-ante heterogeneity, where type is perfectly observed in the KM market; and ex post heterogeneity, where type is realized after KM trade and only privately observed. In the first case, collateralized borrowing can be strictly preferred in some meetings, and transfers in others. The intuition is simple: if one agent is known to be better at maintaining the value of  $e$ , then he should hold it. But if differences emerge after KM trade, and again are privately observed, immediate transfers may better facilitate transactions. Private information limits the extent to which ex post liquidity can be pledged. Thus, assets characterized by observable differences in the ability to maintain value are more likely to be used as collateral. We only touch on this idea briefly, but think there is potential for future work. See Holmstrom and Tirole (2011) for more discussion of these kinds of issues. Our point is that it is possible to endogenize pledgability, in various ways, and it is surely interesting, but for the rest of this paper the debt limits are exogenous.

## 5 Quantitative Analysis

We now ask what the theory has to say quantitatively, focusing on the effects of monetary policy and financial innovation. We emphasize that this is *not* a business cycle analysis; the interest is on longer-run phenomena. In terms of the model, this means looking at differences across steady states, or deterministic transitions between steady states, although one can simulate the model with shocks to monetary, financial and other variables. In terms of the data, this means we are mainly interested in observations after filtering to remove higher-frequency effects. This is partly because we want the empirical analysis to correspond to the analytic results in Section 3, and partly because we think longer-run changes in monetary and financial variables are more interesting. Also, to hopefully avoid confusion, we are not claiming that inflation or financial innovation were the only driving forces in the data. The exercises are in the nature of controlled experiments, or numer-

ical partial derivatives, when we ask, counterfactually, what would the model predict if something were the sole driving force? While this is not the only possible approach, it has much precedent, including the practice in macro following Kydland and Prescott (1982) of asking what a model would predict if technology shocks were the sole impulse.

By way of preview, here are some findings:

- liquidity effects can generate a large return premium for assets that are less pledgeable;
- the model can account quite well for the effects of changes in inflation on standard macro aggregates in the data;
- the calibrated parameters imply that reducing inflation to the Friedman rule leads to a *decrease* in welfare;
- the optimal inflation rate is close to the average rate observed in the data, although this is somewhat sensitive to changes in specification;
- financial innovation can generate significant movements in asset prices and economic activity, including expansions and recessions fueled by housing markets.

Before proceeding, we make three minor changes to the model. First, we set  $D_u = 0$ , and ignore Lucas trees, leaving four assets to facilitate intertemporal exchange:  $\mathbf{a} = (b, h, k, m)$ . Second, we add proportional taxes on labor income and asset income (the returns to  $b$  and  $k$ ), denoted  $\tau_\ell$  and  $\tau_a$ , and assume pledgability limits are post-tax:  $d_b \leq [1 - (1 - \phi_b)\tau_a]\mu_b b$  and  $d_k \leq [1 + (\rho - \delta_k)(1 - \tau_a)]\mu_k k$ . While taxes would have been a distraction for the analytic results, they are critical for calibration. Third, the probability that a household needs a KM loan each period is now  $\sigma \in [0, 1]$ . When households were interpreted as trading bilaterally in the KM market, previously, we have  $\sigma \in [0, 1/2]$  since not more than a fraction  $\sigma = 1/2$  can be buyers if each one needs a seller. However, to match some observations, it helps to allow  $\sigma > 1/2$ . This is no problem for



the theory, as one can simply assume some sellers produce for multiple buyers in the KM market, without changing any of the relevant equations discussed above.

Also, we address a measurement issue. With the KM sector modeled as a pure-exchange market, total output in the model economy is well defined according to standard accounting practice – AD output plus KM output all measured in numeraire – and total employment comes only from AD hours. But there are alternative approaches. What if not all KM activity is recorded in the data? At least part of this activity involves cash transactions, some of which may not show up in the official accounts.<sup>16</sup> Moreover, instead of making KM a pure exchange market, one can assume that  $q$  is produced and add KM labor to total employment. Usually we stick to the benchmark interpretation, where  $q$  is counted in GDP while only  $\ell$  is counted as employment, but we also discuss some implications of changing this interpretation.

## 5.1 Calibration

Functional forms are standard. The AD utility and production functions are  $\mathcal{U}(x, h, \ell) = \log x + \psi \log h - \zeta \ell$  and  $f(k, l) = k^\alpha l^{1-\alpha}$ . The KM utility and cost functions are  $u(q) = vq^{1-\eta}/(1-\eta)$  and  $c(q) = q$ . The housing cost function is  $g(I) = I^{1+\xi}/(1+\xi)$ , where  $I = H' - (1 - \delta_h)H$  is net residential investment. As discussed above, the KM terms of trade are determined by Kalai bargaining,  $z(q) = (1 - \theta)u(q) + \theta c(q)$ , although Walrasian pricing emerges as a special case when  $\theta = 1$ . As in much of the literature,  $\alpha$  in the AD production function is set to 0.33 to match labor's and capital's shares in the income accounts. We let the time period be a year, and set  $\beta = 0.95$  so the annual real interest rate on a illiquid bond is 5.26%. However, interest rates on liquid assets can be considerably lower, and 5.26% should be interpreted as the return one would require if an asset could not be used as collateral at all. Estimates of marginal tax rates, especially  $\tau_a$ , vary widely, but we use  $\tau_a = 0.4$  and  $\tau_\ell = 0.3$ , in the range of previous studies.

---

<sup>16</sup>In Wallace (2010) and Aruoba (2010), the analog of our KM market is interpreted as the underground economy. That would be going to far for our purposes of this paper, but we wanted to mention it.

The remaining parameters are calibrated to match some observations for the U.S. over the period 1954-2000 (unless otherwise noted). There are two reasons for stopping at 2000. First, we do not claim to have a great theory of the great contraction, although we believe models that take financial considerations seriously may be on the right track. Second, even before the crisis, developments in financial and housing markets make the 2000’s “special.” As Holmstrom and Tirole (2011) say, “In the runup to the subprime crisis, securitization of mortgages played a major role ... by making nontradable mortgages tradable, [and this] led to a dramatic growth in the US volume of mortgages, home equity loans, and mortgage-backed securities in 2000 to 2008.” As Ferguson (2008) or Reinhart and Rogoff (2009) put it, this allowed consumers to start treating their houses as “ATM machines” (see also Mian and Sufi (2011)). To be clear, the recent period is not a “problem” for the theory, which in fact puts front and center issues relating to credit and pledgability. But our strategy is to calibrate to more “normal” times, then see what the model says about events since the turn of the millennium after adjusting the financial parameters.

The Appendix contains details of the data sources, although housing is the only series that merits discussion. For this, the value of structures plus land is taken from Davis and Heathcote (2007), which has the disadvantage of starting only in 1975, but has the advantage of reliability, mainly because it tries to measure land values accurately. To this we add consumer durables since many of these (e.g., home appliances) are part of household capital. To be consistent, the value of housing services is removed from the GDP data, and durable consumption is added to residential investment. Annual depreciation rates are set to  $\delta_h = \delta_k = 0.10$  to approximately match investment rates in  $h$  and  $k$ . These numbers are slightly higher than those seen in some other studies, at least in part due to the way we adjust our housing and output measures. Inflation averages 3.7% in our sample. Given this, and an average pre-tax real return on 3-month T-bills of 1.8%, we can pin down  $\mu_b = 0.45$  from the optimality condition for  $b$ , which with taxes is  $1 + r_b = [1 + r - (1 + \mu_b i) \tau_a] / (1 + \mu_b i) (1 - \tau_a)$ .

This leaves 10 parameters,  $(\eta, v, \sigma, \theta, \psi, \xi, \mu_k, \mu_h, \zeta)$ , which are calibrated as follows. The KM parameters – the curvature and level of utility  $\eta$  and  $v$ , plus the fraction of agents needing a loan  $\sigma$  – are set to fit: (1) the empirical relationship between inflation and  $M/PY$  (money demand or inverse velocity), defined using sweep-adjusted  $M1$  data, consistent with the idea that in this model  $m$  can well represent money in the bank; and (2) the empirical relationship between inflation and real output. While (1) is standard procedure, (2) is less so and deserves discussion, but we defer that until we see some results. The remaining 7 parameters are set to match the following targets:<sup>17</sup>

1. Housing capital over GDP: 1.92.
2. Housing investment over GDP: 0.15.
3. Market capital over GDP: 1.25.
4. T-bills outstanding over GDP: 0.10.
5. Hours worked over discretionary time: 0.33.
6. The KM markup, price  $z(q)/q$  over marginal cost  $c'(q)$ : 1.30.
7. Home equity loans over housing wealth: 0.03.

Table 5 summarizes the baseline calibration, as well as an alternative calibration discussed below. Although the parameters are set jointly to match the targets, heuristically one can think of subsets of parameters being identified by subsets of the targets, as described in the Table. Already from these numbers one can ask how the model does in

---

<sup>17</sup>These targets are mostly standard, although a couple of comments may be warranted. As in Aruoba et al. (2011), the markup comes from the Annual Retail Trade Survey. In these data, at the low end, Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations have markups between 1.17 and 1.21; at the high end, Specialty Foods, Clothing, Footware, and Furniture have markups between 1.42 and 1.44; so we target 1.3. Also, for  $\mu_h$ , we need to adjust for two factors. First, around 30% of home capital is used to collateralize mortgages, and is therefore not available to secure household consumption loans. Thus, home equity is given by  $0.7(1 - \delta_h)\phi_h h = 0.63\phi_h h$ . Second, in the model, only a fraction  $\sigma$  of households need loans in any period. So home equity loans over housing wealth is  $0.63\sigma\mu_h$ . Given a target for this ratio and a choice for  $\sigma$ , we pin down  $\mu_h$ .

Parameter	Description	Baseline	Alternate	Identified by
$\beta$	Discount factor	0.95	0.95	Set directly
$\alpha$	Capital's Share	0.33	0.33	"
$\pi$	Inflation	0.037	0.037	"
$\delta_h$	Depreciation of housing	0.10	0.10	"
$\delta_k$	Depreciation of capital	0.10	0.10	"
$\tau_\ell$	Labor-income tax	0.30	0.30	"
$\tau_a$	Asset-income tax	0.40	0.40	"
$\psi$	Coefficient on $h$ in $U$	0.86	0.73	Housing/GDP
$\zeta$	Coefficient on $\ell$ in $U$	0.38	0.41	Hours worked
$\xi$	Curvature of $h$ cost fn	0.30	0.30	Housing investment/GDP
$\mu_b$	Pledgability of bonds	0.45	0.45	Real return on bonds
$\mu_k$	Pledgability of capital	0.17	0.06	Capital/GDP
$\mu_h$	Pledgability of housing	0.06	0.04	Home equity loans/housing
$B$	Bond supply	0.08	0.07	T-bills/GDP
$\theta$	Bargaining power	0.86	0.68	Retail markup
$\eta$	Curvature of $u(q)$	0.63	0.39	$M/PY$ and $Y$ vs inflation
$\nu$	Constant in $u(q)$	1.39	1.11	"
$\sigma$	Fraction of KM buyers	0.67	0.75	"

Table 5: Calibration

terms of some macro-finance issues, including the equity premium. From the calibrated  $\mu_b$  and  $\mu_k$ ,  $k$  earns a higher return than  $b$ , with the difference  $r_b - r_k$  approximately  $i(\mu_k - \mu_b)/(1 - \tau) = 0.04$ . This is remarkably close to the empirical equity premium, as measured by the difference in average returns to stocks and bonds over the last 100 years. Mehra and Prescott (1985) highlight the inability of standard models to explain this difference based on risk aversion, and a large body of work has attempted to resolve the issue by modifying standard models in various ways. It turns out that liquidity goes a long way.

As we said earlier, Lagos (2010, 2011) makes a similar point when he demonstrates that models where liquidity is incorporated explicitly can explain a substantial part of the equity premium. Our approach is somewhat different, in that he only considers differences in liquidity on the extensive margin – assets either are or are not accepted in decentralized trade – while the model presented here captures liquidity differences along the intensive

margin – assets are accepted only up to certain limits. The point is not that these two approaches are so very different; rather, we want to emphasize how models of this general class have considerable potential for studying macro-finance issues in various ways. The message is that these models are potentially useful for understanding many observations that are difficult to capture with models that ignore liquidity considerations.

## 5.2 Main Results

To begin, Figure 2 shows steady state values for the standard macro aggregates as a function of inflation  $\pi$ , from a high of 10% to a low of  $-0.05$ , the value associated with the zero lower bound on  $i$ , or the Friedman rule. Inflation is bad for KM consumption  $q$ , as we knew from the analytic results, but now we see the effect is sizable: reducing  $\pi$  from 10% to the Friedman rule increases  $q$  from around 1.1 to 1.7, and even reducing  $\pi$  from 10% to 0 increases  $q$  to almost 1.5. Offsetting this is the higher AD consumption  $x$ , which results mainly from  $k$  increasing due to the Tobin effect. The increase in  $k$  also generates an increase in employment, although the scale on the vertical axis indicates that this effect is not very big (we return to this below). Although this is not true for all parameter values, for the calibrated values, total output increases with  $\pi$ , as the increase in AD production more than offsets the fall in KM trade.

The value of the housing stock  $\phi_h h$  also increases with inflation, consistent with the facts documented in various data sources for the U.S. and other countries by Aruoba et al. (2012), but the explanation is totally different. The suggestion there is that agents substitute out of market and into household production as  $\pi$  increases, leading to higher demand for inputs into household production, including houses. Here the demand for  $h$  increases with  $\pi$  as agents try to substitute other pledgeable assets for cash, similar to the effect on  $k$ . An advantage of our explanation is that it has  $h$  and  $k$  both rising with  $\pi$  in the model, as in the data (see below); the other story tends to have them move in opposite directions. Something similar applies to bonds, which we emphasize because T-bills are

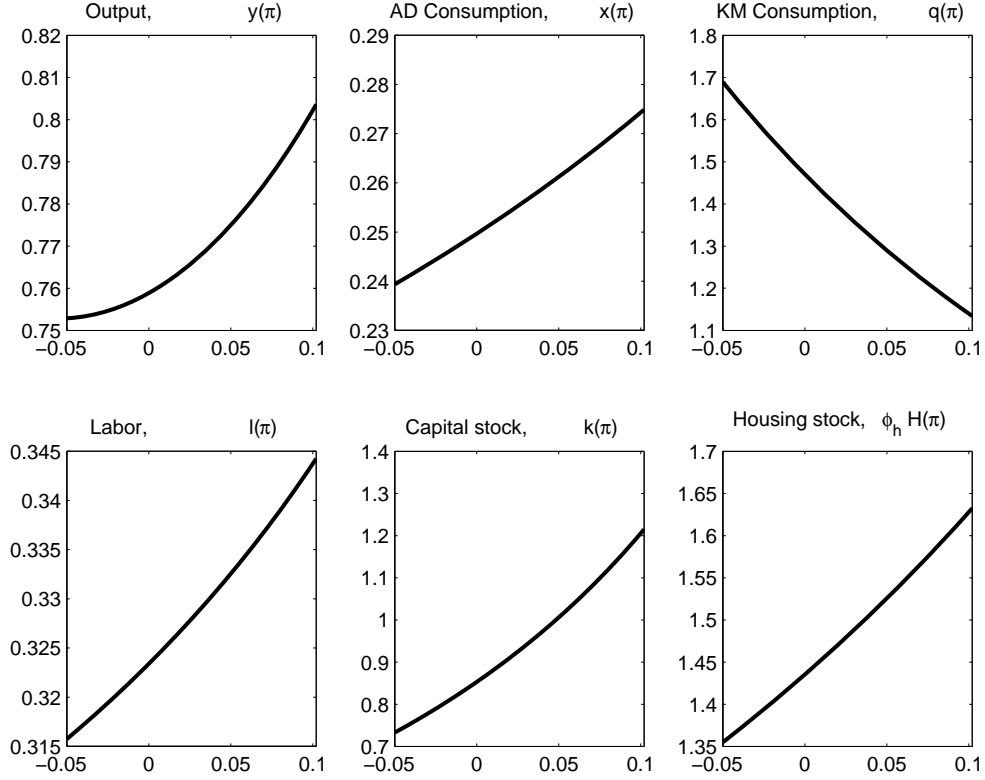


Figure 2: Inflation and aggregate variables (baseline calibration).

the subject of a recent empirical study by Krishnamurthy and Vissing-Jorgensen (2012). Figure 3 shows how  $\pi$  reduces the returns to  $b$ ,  $k$  and  $h$ , which we knew from the analytic results, but here we quantify the effects, with the fall in  $r_b$  larger than in  $r_h$  and  $r_k$ , since the calibration implies  $b$  is more pledgeable. The lower left panel of Figure 3 shows the fall in real balances with  $\pi$ , while the next panel shows the increase in liquidity from other assets,  $\bar{D} - \phi_m M$ . On net, total liquidity falls – or credit tightens – with  $\pi$  since  $q$  falls.<sup>18</sup>

The last panel of Figure 3 shows the cost of inflation, computed as the change in AD consumption that gives the same increase in utility as switching to the Friedman rule, not across steady states, but taking into account the transition as  $k$  and  $h$  adjust. In the calibrated model this cost is *negative* – i.e., switching to the Friedman rule is equivalent

<sup>18</sup>Part of the fall in  $q$  is due to an increase in the wage  $\omega$ , since  $z(q)\omega/\zeta = \bar{D}$  means that at higher wages KM buyers get less for their money.

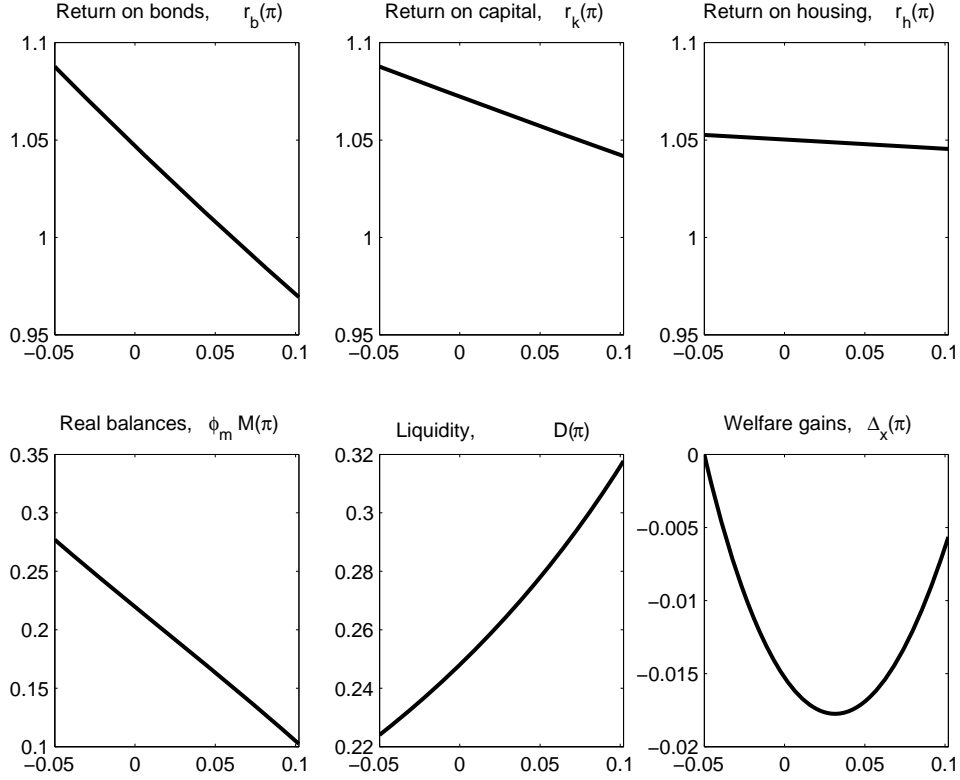
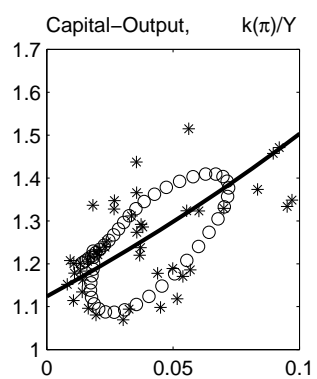
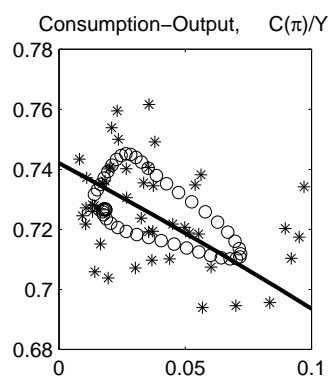


Figure 3: Inflation and aggregate variables (baseline calibration).

to a *reduction* in  $x$ . In terms of magnitude, going from the average  $\pi$  in the sample to the Friedman rule is like reducing  $x$  about 1.5%. This is due to the tax  $\tau_a$ , which tends to make  $k$  too low. Inflation is beneficial because the Tobin effect partially offsets the tax effect on  $k$ , more than compensating for the effect on  $q$ . If  $\pi$  did not affect  $k$  then going from  $\pi = 0.1$  to the Friedman rule would be like an increase in  $x$  of nearly 5%, purely due to the impact on  $q$ . This is much higher than what one finds in reduced-form models like Lucas (2000), but similar to Lagos and Wright (2005). Even with this big impact on  $q$ , the beneficial impact on  $k$ , absent in Lucas (2000) or Lagos and Wright (2005), is the dominating effect. Without taxes the Friedman rule is optimal; with taxes, the calibration implies the optimal inflation rate is 3.5%, which is surprisingly close to the average inflation rate in the data.





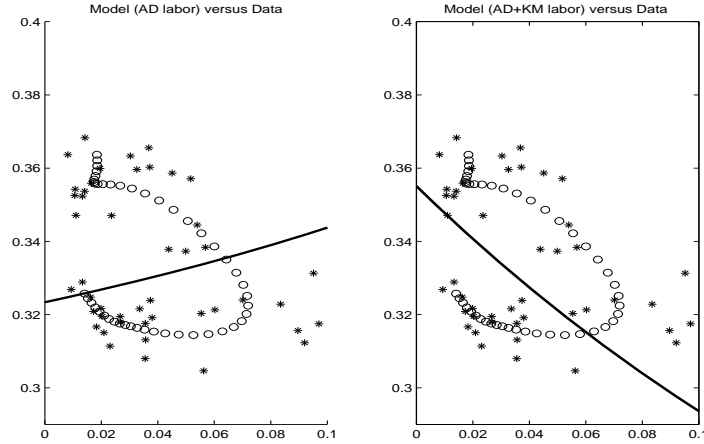


Figure 5: Phillips curves.

The one dimension along which the model does less well is the labor market: it generates a fairly flat but slightly upward-sloping relationship, somewhat at odds with the data. There are two points to make about this. First, one can argue that the scatter plot of  $\pi$  versus  $\ell$  is basically a “cloud,” as Christiano and Eichenbaum (1992) argue for the relationship between productivity and  $\ell$ , and hence, as they conclude, one is not going to match the data with any one forcing variable. Second, as we said above, one can interpret employment differently in the model by having production in the KM market. Figure 5 compares the model-implied Phillips curve to the data under the two interpretations of labor. The left panel is reproduced from the baseline results, which only count AD labor. The right panel adds  $\ell$  and  $q$ , assuming the KM technology produces output one-for-one with hours, and rescaling the sum so that the model is still consistent with the calibration target 0.33. Under this interpretation, the model lines up reasonably well with the facts.

While we do not want to oversell any match between model and data, we feel obliged to defend its relevance against the obvious critique that it is generated assuming no changes in other driving forces, including taxes, technology shocks etc. As explained above, it is standard in macro to ask, counterfactually, what might have happened in the economy if the only driving process were changes in some variable  $z$ . Here  $z$  is given by medium-run

changes in inflation, and the answer is that the main variables of interest would have looked a lot like what was observed in the data. This should shift one's prior toward thinking that maybe inflation is interesting and that the model is reasonable. One still ought to think about robustness, however, particularly with respect to the choice of the targets generating the parameter values.

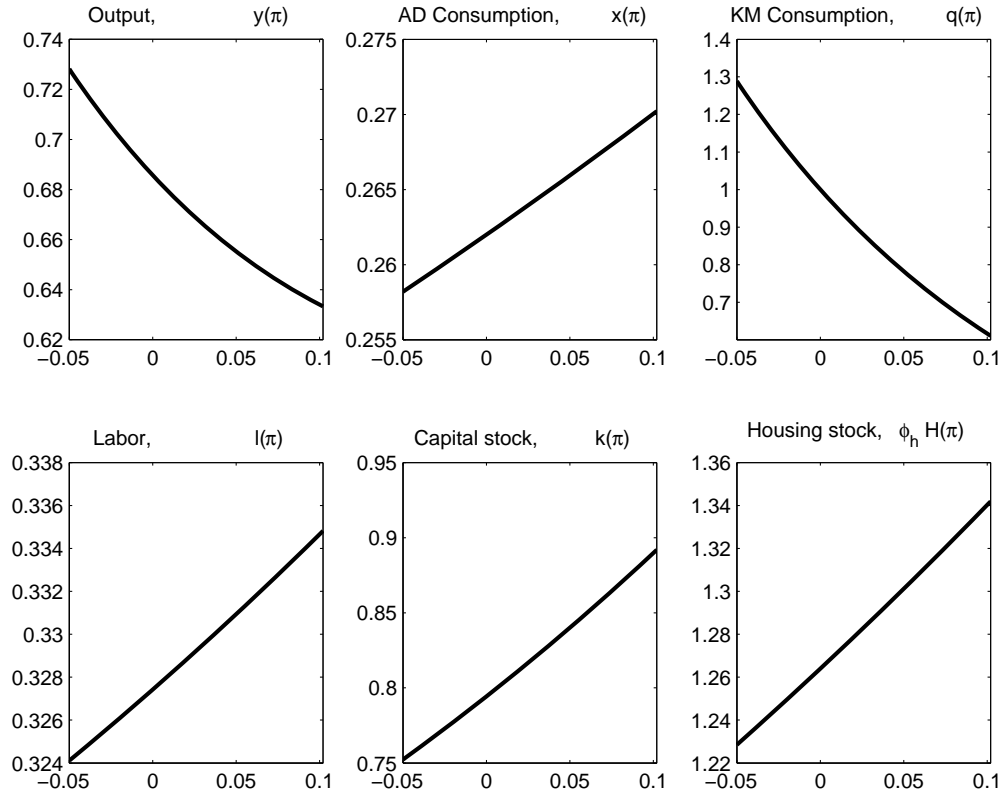


Figure 6: Inflation and aggregate variables (alternative calibration).

Much of our calibration strategy follows well-worn paths, except perhaps matching the relationship between filtered output and inflation. One can try to set the KM parameters  $(\eta, v, \sigma)$  to match only the empirical money demand relationship, although identification is fairly weak – i.e., there are diff

show the results for this calibration, while Figure 8 overlays the relevant data. Comparing these to Figures 2-4, for most variables the results are very similar, and the predictions for the ratios  $k/y$ ,  $c/y$  etc. are still in line with the data. This implies that many predictions are robust along this dimensions, but there are two exceptions.

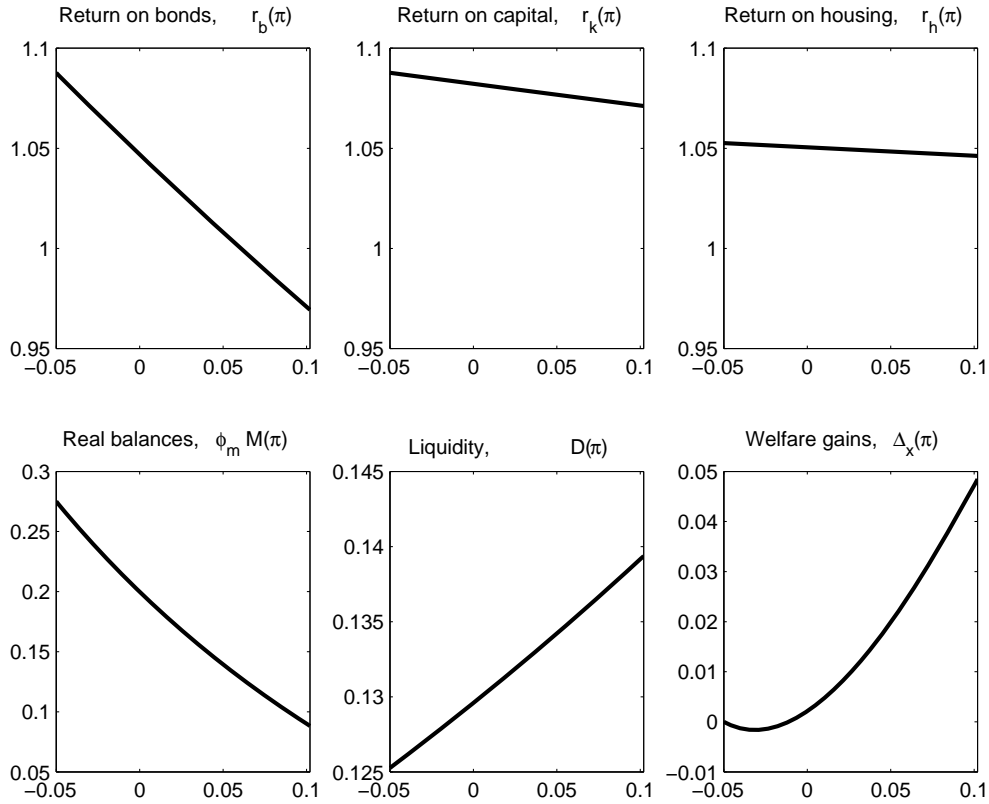


Figure 7: Inflation and aggregate variables (alternative calibration).

First, the baseline model predicts that output increases with  $\pi$ , while the alternative predicts that it falls. This is driven by the difference in the pledgability of capital. Specifically, the baseline calibration selects values for  $(\eta, \sigma, v)$  that require a higher  $\mu_k$  to match the target capital-output ratio, and this makes the Tobin effect stronger. Second, there is a big difference in the implications for welfare. Welfare rises with  $\pi$  over a broad range in the baseline model, since ameliorating the tax-related investment distortion more than compensates for the decline in KM activity. This is not true for the alternative

calibration, where the KM effect dominates, and there are substantial welfare losses from positive inflation. In particular, reducing  $\pi$  from 10% to the Friedman rule is worth almost 5% of  $x$ , and the optimal inflation rate is now negative, although above the Friedman rule. So, while many of the positive implications of the theory are robust, the normative implications are sensitive to the parameterization.<sup>20</sup>

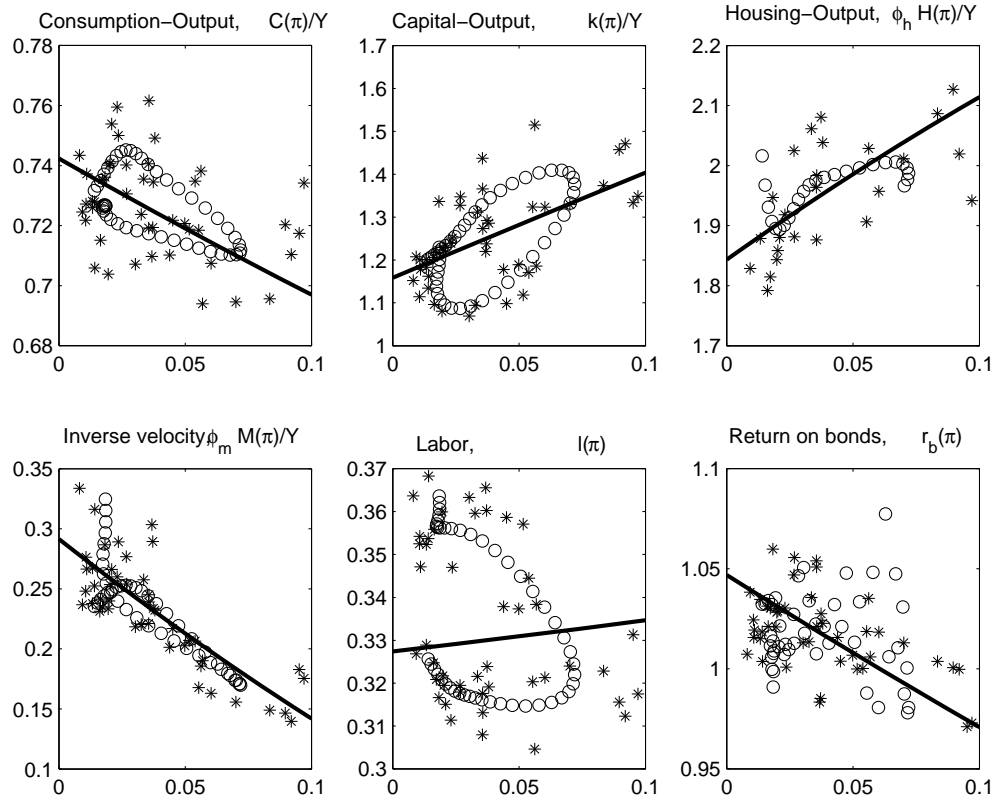


Figure 8: Inflation and ratios (alternative calibration): \* = data; o = HP trends.

In terms of other robustness issues, we recalibrated the model after setting  $\theta = 1$ , which can be interpreted as Walrasian pricing. The results (not shown) are similar to the benchmark, although  $q$  is higher, the loss from going to the Friedman rule is slightly lower, and there are minor differences in the effects of  $\pi$  on  $k$  and  $h$ . Where  $\theta = 1$  misses, of course, it is that it gives a KM markup of 0 rather than 0.3. We also checked robustness by

<sup>20</sup>We discuss this more, in the context of the empirical literature on the effects of inflation, in the Conclusion.

shutting down taxes. The results (not shown) are very different from the benchmark, which is no surprise, since it is well known that taxes have big effects on capital accumulation (e.g., McGrattan et al. (1997)). Consider the cost of inflation. With  $\tau_a = \tau_\ell = 0$ , going from  $\pi = 0.1$  to the Friedman rule, which is now optimal, is worth nearly 18% of  $x$ , with about half coming from KM and half from AD. Even going from  $\pi = 0.04$  to the Friedman rule is worth nearly 5% of  $x$ . We do not think these numbers are too meaningful, however, since taxation is a fact of life, making  $\tau_a = \tau_\ell = 0$  not a compelling calibration. What we take away from this is that  $\pi$  has a big impact on consumption, but this is offset by partially correcting tax distortions on investment.

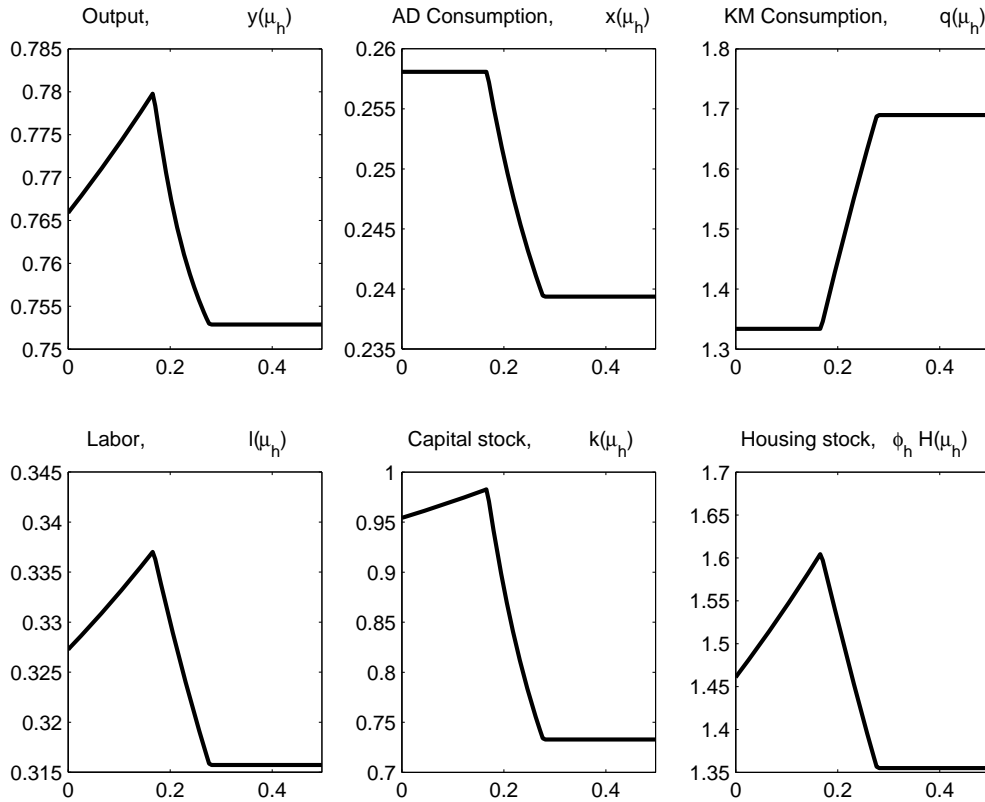


Figure 9: Housing pledgability and aggregate variables.

One can also use the model to quantify the effects of financial innovation, interpreted as changes in the pledgability parameters. Figure 9 shows the effect of raising  $\mu_h$ . When

$\mu_h$  is small, an increase leads to a housing boom, combined with higher output, investment and employment; as the innovation continues, however, it ultimately leads to a bust, with declines in output, investment, employment and the housing market. The reason is that as liquidity becomes more abundant, its marginal value decreases, reducing liquidity premia and asset prices. In particular, once liquidity is sufficiently abundant that buyers can get at least  $q^i$  on credit, money is no longer valued. The kinks in the graphs indicate points where we switch across the different regimes (types of equilibria) described in Section 3, but recall that one does not have to take the nonmonetary regime literally, since some agents in the real world may always use cash.

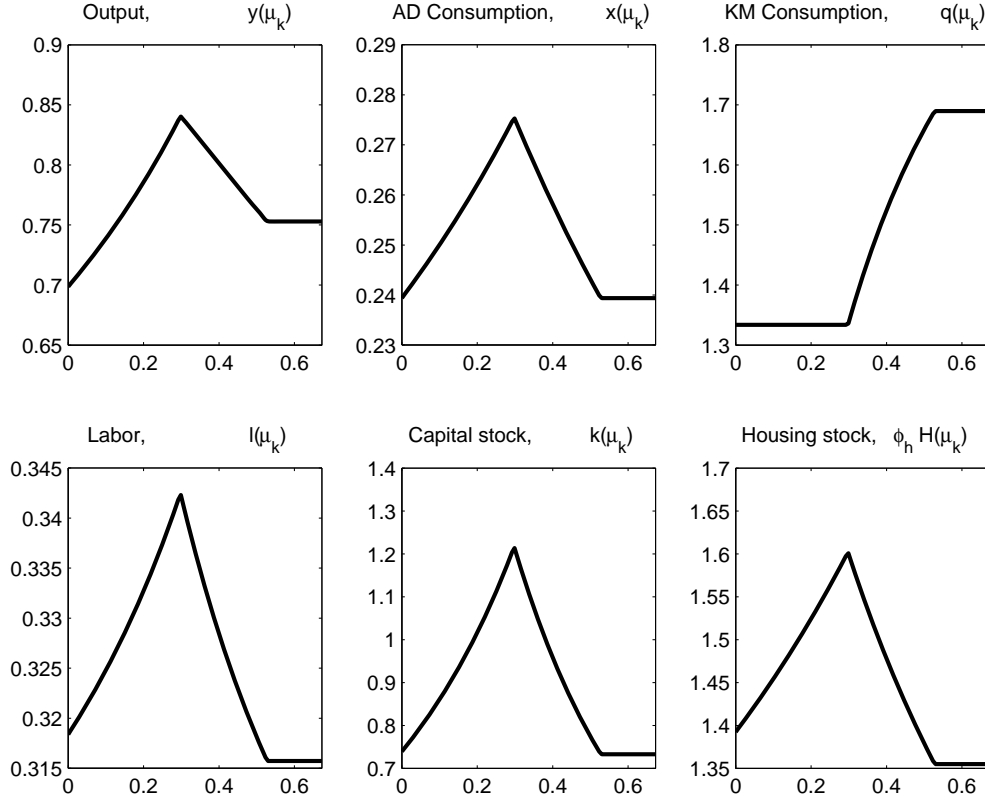


Figure 10: Capital pledgability and aggregate variables.

He et al. (2012) use a related but simpler model to evaluate the possibility that financial innovation spurred the U.S. experience since 2000, along with alternatives, like bubbles as

self-fulfilling prophecies. The findings are similar: while a housing-fueled expansion and contraction can be explained qualitatively by developments in home-equity loan markets, it is difficult to match the facts quantitatively. The ratio of the value of the housing stock (the last panel of Figure 9) to GDP (the first panel) peaks at just over 2, while in the data it reached approximately 2.76 at the height of the boom. Hence, while financial developments may have been significant, there is room for other forces in accounting for the facts. For comparison, Figure 10 repeats the analysis for an increase in  $\mu_k$ . As with  $\mu_h$ , the impact on output, employment, capital and housing is first positive and then negative, for the same reason. What is not reported in these figures is that increases in pledgability typically improve welfare. The general rule is that improvements in credit conditions can be good, even if it looks bad for some macro aggregates.

## 6 Conclusion

We began by thinking about the role of assets in facilitating intertemporal exchange with limited commitment, and other frictions, as stressed by Kocherlakota (1998) and Wallace (2010). A key ingredient in the logic was the asynchronization of receipts and expenditure: sometimes – in the model, in the KM market – agents have demands but no current income, and therefore they need loans. Credit was modeled after Kiyotaki and Moore (1997, 2005), except we concentrate on consumers rather than producers, in the spirit of Kehoe and Levine (1993, 2001), Alvarez and Jermann (2000) and incomplete-market models generally going back at least to Bewley (1977). Debt limits are endogenized by a limited ability to punish defaulters by seizing assets, guided by work on the microfoundations of money going back to Kiyotaki and Wright (1989, 1993), updated using recent advances in the field. A difference from most monetary economics is that agents do not use some asset as a medium of exchange; rather, they have a portfolio, each element of which can be used to a greater or lesser extent to collateralize loans. Motivated by Holmstrom and Tirole (2011), the focus is on pledgability.

Although it nests standard growth and asset-pricing theories as special cases, the framework is still very tractable. To some readers some ingredients might be unfamiliar, like the use of abstract pricing mechanisms in the credit market, but these are standard tools in modern monetary economics, and they make a difference for quantitative work as well as theory. In terms of substantive questions, we used the model to study several issues in macro and financial economics. We analyzed how inflation affects the returns to assets as a function of their pledgability, and how it affects output and employment. We showed there is crowding out of liquidity in monetary economies, e.g., higher inflation reduces real balances, but as this impinges on the demand for alternative assets, total liquidity changes by less than real balances. Relatedly, increases in one asset's pledgability crowds out other forms of liquidity. Although one can derive many of these results analytically, we also presented a quantitative analysis, calibrating the model using standard practice and using it to experiment with monetary policy and financial innovation.

The quantitative results imply that over a relevant range inflation increases output, employment, investment and some but not all consumption, due to a Mundell-Tobin effect. While the nominal returns on illiquid assets go up one-for-one with inflation, nominal returns on liquid assets go up significantly less. This is especially true for bonds, which are very liquid according to the calibration, and hence earn a lower return than capital, consistent with the observed equity premium. The price and quantity of housing increase with inflation, consistent with recent explorations of the data, although our explanation is different. The benchmark model also implies welfare is increasing in inflation over a relevant range, due to the Mundell-Tobin effect combined with investment being too low due to taxes on asset income. For the baseline calibration, the optimal inflation rate was just about the mean in the data, but this is somewhat sensitive to the parameterization. In an alternative calibration, the predictions for most of the variables are similar, except inflation decreases output and the optimal inflation rate is negative.

It is therefore crucial to know whether inflation increases output in the data. There is



a big literature on this, but we can discuss it only briefly here. Bullard and Keating (1995) find that permanent changes in inflation do not increase in real output for most countries, but do for some (low-inflation) countries. Ahmed and Rogers (2000) conclude from a century of U.S. data that “the long-run effects of inflation on consumption, investment, and output are positive ... [and] ratios like the consumption and investment rates are not independent of inflation, which we interpret in terms of the Fisher effect.” Therefore, they say “models generating long-term negative effects of inflation on output and consumption seem to be at odds with data.” On the other hand, for OECD economies Madsen (2003) finds investment is negatively related to inflation. In terms of returns, King and Watson (1997) find nominal interest rates do not adjust one-for-one with permanent inflation, for many plausible identification schemes.<sup>21</sup>

After surveying much literature, Bullard (1999) concludes: “While the overall evidence on these questions is mixed, considering only lower inflation countries leads to the conclusion that permanently higher money growth or inflation is associated with permanently higher output and permanently lower real interest rates.” He also says “this result is inconsistent with many – almost all? – current quantitative business cycle models, which generally predict that permanently higher inflation permanently lowers consumption and output. There is little support for such a prediction in the studies surveyed here. This is an important empirical puzzle that stands as a challenge for future research.” Our model *is* consistent with these findings. Future work might try to better understand the facts, to look at higher-frequency implications of the model, and to apply it to additional policy issues. That is left to future work.

---

<sup>21</sup>Other papers focus more on growth rates, not levels. Gillman and Kejak (2011) find growth, investment and real interest rates tend to be negatively affected by inflation. Gillman and Nakov (2003) find evidence for the US and UK consistent with a Tobin effect, while the growth rate of output is reduced by inflation. Ericsson et al. (2001) find in cross-country regressions that output growth is often negatively related to inflation, but that result is not robust, and correcting for several problems leads to a positive effect for most countries. Benhabib and Spiegel (2009) find nonmonotone effects: when inflation is high it is negatively correlated with growth, while a positive relationship exists for negative to moderate inflation. López-Villavicencio and Mignon (2011) also find nonlinear effects: above a threshold inflation decreases growth, and below it inflation increases growth.

## References

- Ahmed, S. and J. H. Rogers (2000). Inflation and the great ratios: Long term evidence from the u.s. *Journal of Monetary Economics* 45(1), 3–35.
- Alvarez, F. and U. J. Jermann (2000). Efficiency, equilibrium, and asset pricing with risk of default. *Econometrica* 68(4), 775–797.
- Aruoba, S. (2010). Informal sector, government policy and institutions. In *2010 Meeting Papers*, Number 324. Society for Economic Dynamics.
- Aruoba, S. B., M. A. Davis, and R. Wright (2012). Homework in monetary economics: Inflation, home production, and the production of homes. Mimeo, University of Wisconsin-Madison.
- Aruoba, S. B., G. Rocheteau, and C. Waller (2007). Bargaining and the value of money. *Journal of Monetary Economics* 54(8), 2636–2655.
- Aruoba, S. B., C. J. Waller, and R. Wright (2011). Money and capital. *Journal of Monetary Economics* 58(2), 98–116.
- Benhabib, J. and M. M. Spiegel (2009). Moderate inflation and the deflation-depression link. *Journal of Money, Credit and Banking* 41(4), 787–798.
- Berentsen, A., G. Menzio, and R. Wright (2011). Inflation and unemployment in the long run. *American Economic Review* 101(1), 371–398.
- Bewley, T. (1977). The permanent income hypothesis: A theoretical formulation. *Journal of Economic Theory* 16(2), 252–292.
- Bullard, J. (1999). Testing long-run monetary neutrality propositions: Lessons from the recent research. *Federal Reserve Bank of St. Louis Review* (Nov), 57–77.
- Bullard, J. and J. Keating (1995). The long-run relationship between inflation and output in postwar economies. *Journal of Monetary Economics* 36(3), 477–496.
- Burdett, K. and K. L. Judd (1983). Equilibrium price dispersion. *Econometrica* 51(4), 955–969.
- Chiu, J. and M. Molico (2010). Liquidity, redistribution, and the welfare cost of inflation. *Journal of Monetary Economics* 57(4), 428–438.
- Chiu, J. and M. Molico (2011). Uncertainty, inflation, and welfare. *Journal of Money, Credit and Banking* 43, 487–512.
- Christiano, L. and M. Eichenbaum (1992). Current real-business-cycle theories and aggregate labor-market fluctuations. *American Economic Review* 82(3), 430–450.

- Cynamon, B. Z., D. H. Dutkowsky, and B. E. Jones (2006). Redefining the monetary aggregates: A clean sweep. *Eastern Economic Journal* 32(4), 661–672.
- Davis, M. A. and J. Heathcote (2007). The price and quantity of residential land in the united states. *Journal of Monetary Economics* 54(8), 2595–2620.
- Dong, M. (2011, 8). Inflation and unemployment in competitive search equilibrium. *Macroeconomic Dynamics* 15.
- Dutu, R., B. Julien, and I. King (2009). Liquidity constrained competing auctions. Research Paper Number 1068, University of Melbourne.
- Ericsson, N. R., J. S. Irons, and R. W. Tryon (2001). Output and inflation in the long run. *Journal of Applied Econometrics* 16(3), 241–253.
- Ferguson, N. (2008). *The ascent of money: A financial history of the world*. Penguin Press HC.
- Galenianos, M. and P. Kircher (2008). A model of money with multilateral matching. *Journal of Monetary Economics* 55(6), 1054–1066.
- Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis.
- Gillman, M. and M. Kejak (2011). Inflation, investment and growth: a money and banking approach. *Economica* 78(310), 260–282.
- Gillman, M. and A. Nakov (2003). A revised tobin effect from inflation: Relative input price and capital ratio realignments, usa and uk, 1959–1999. *Economica* 70(279), 439–450.
- Gu, C., F. Mattesini, and R. Wright (2012). Money and credit redux. Mimeo, University of Wisconsin-Madison.
- He, C., R. Wright, and Y. Zhu (2012). Housing and liquidity. Mimeo, University of Wisconsin.
- Head, A., L. Q. Liu, G. Menzio, and R. Wright (2012). Sticky prices: A new monetarist approach. *Journal of the European Economic Association* 10(5), 939–973.
- Holmstrom, B. and J. Tirole (2011). *Inside and Outside Liquidity*. The MIT Press.
- Hu, T.-w., J. Kennan, and N. Wallace (2009). Coalition-proof trade and the Friedman rule in the Lagos-Wright model. *Journal of Political Economy* 117(1), 116–137.
- Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review* 95(3), 739–764.

- Kalai, E. (1977). Proportional solutions to bargaining situations: Interpersonal utility comparisons. *Econometrica* 45(7), 1623–1630.
- Kehoe, T. J. and D. K. Levine (1993). Debt-constrained asset markets. *The Review of Economic Studies* 60(4), 865–888.
- Kehoe, T. J. and D. K. Levine (2001). Liquidity constrained markets versus debt constrained markets. *Econometrica* 69(3), 575–598.
- King, R. and M. Watson (1997). Testing long-run neutrality. *Economic Quarterly-Federal Reserve Bank of Richmond* 83, 69–95.
- Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy* 105(2), 211–248.
- Kiyotaki, N. and J. Moore (2005). Liquidity and asset prices. *International Economic Review* 46(2), 317–349.
- Kiyotaki, N. and R. Wright (1989). On money as a medium of exchange. *Journal of Political Economy* 97(4), 927–954.
- Kiyotaki, N. and R. Wright (1993). A search-theoretic approach to monetary economics. *American Economic Review* 83(1), 63–77.
- Kocherlakota, N. R. (1998). Money is memory. *Journal of Economic Theory* 81(2), 232–251.
- Krishnamurthy, A. and A. Vissing-Jorgensen (2012). The aggregate demand for treasury debt. *Journal of Political Economy* 120(2), 233–267.
- Kydland, F. and E. Prescott (1982). Time to build and aggregate fluctuations. *Econometrica: Journal of the Econometric Society*, 1345–1370.
- Lagos, R. (2010). Asset prices and liquidity in an exchange economy. *Journal of Monetary Economics* 57(8), 913–930.
- Lagos, R. (2011). Asset prices, liquidity, and monetary policy in an exchange economy. *Journal of Money, Credit and Banking* 43, 521–552.
- Lagos, R. and G. Rocheteau (2005). Inflation, output and welfare. *International Economic Review* 46(2), 495–522.
- Lagos, R. and R. Wright (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy* 113(3), 463–484.
- Lester, B., A. Postlewaite, and R. Wright (2012). Liquidity, information, asset prices and monetary policy. *Review of Economic Studies* 79, 1209–38.

- Li, Y. and G. Rocheteau (2011, 3). On the threat of counterfeiting. *Macroeconomic Dynamics* 15.
- Li, Y., G. Rocheteau, and P.-O. Weill (2012). Liquidity and the threat of fraudulent assets. *Journal of Political Economy*, forthcoming.
- Lucas, R. E. (1978). Asset prices in an exchange economy. *Econometrica* 46(6), 1429–45.
- Lucas, R. E. (2000). Inflation and welfare. *Econometrica* 68(2), 247–274.
- Lucas, R. E. and E. C. Prescott (1974). Equilibrium search and unemployment. *Journal of Economic Theory* 7(2), 188–209.
- López-Villavicencio, A. and V. Mignon (2011). On the impact of inflation on output growth: Does the level of inflation matter? *Journal of Macroeconomics* 33(3), 455–464.
- Madsen, J. B. (2003). Inflation and investment. *Scottish Journal of Political Economy* 50(4), 375–397.
- McGrattan, E. R., R. Rogerson, and R. Wright (1997). An equilibrium model of the business cycle with household production and fiscal policy. *International Economic Review* 38(2), 267–290.
- Mehra, R. and E. C. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics* 15(2), 145–161.
- Mian, A. and A. Sufi (2011). House prices, home equity-based borrowing, and the us household leverage crisis. *American Economic Review* 101(5), 2132–56.
- Mortensen, D. T. and C. A. Pissarides (1994). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies* 61(3), 397–415.
- Nosal, E. and G. Rocheteau (2011). *Money, Payments, and Liquidity*. MIT Press.
- Reinhart, C. and K. Rogoff (2009). *This time is different: Eight centuries of financial folly*. Princeton University Press.
- Rocheteau, G. (2011). Discussion of "information and liquidity". *Journal of Money, Credit and Banking* 43, 379–384.
- Rocheteau, G., P. Rupert, K. Shell, and R. Wright (2008). General equilibrium with nonconvexities and money. *Journal of Economic Theory* 142(1), 294–317.
- Rocheteau, G., P. Rupert, and R. Wright (2007). Inflation and unemployment in general equilibrium. *Scandinavian Journal of Economics* 109(4), 837–855.
- Rocheteau, G. and R. Wright (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica* 73(1), 175–202.

- Rogerson, R. (1988). Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21(1), 3–16.
- Wallace, N. (2010). The mechanism design approach to monetary economics. In B. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*.
- Williamson, S. and R. Wright (2010). New monetarist economics: Models. In B. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*.
- Williamson, S. D. (2006). Search, limited participation, and monetary policy. *International Economic Review* 47(1), 107–128.
- Wong, R. (2012). A tractable monetary model under general preferences. Mimeo, Bank of Canada.
- Wright, R. (2010). A uniqueness proof for monetary steady state. *Journal of Economic Theory* 145(1), 382–391.

## Appendix: Data Sources

1. Nominal GDP: Table 1.1.5 from NIPA, Bureau of Economic Analysis. Housing services and utilities are deducted.
2. Consumption: Table 1.1.5 from NIPA, Bureau of Economic Analysis. Housing services, utilities and durables are deducted.
3. Nonresidential capital stock: Table 4.1 of the Fixed Asset Account tables.
4. Housing stock: From Davis and Heathcote (2007).
5. Investment (residential and non-residential): Table 1.1.5 from NIPA, Bureau of Economic Analysis.
6. Sweep-adjusted M1: Series M1S in Cynamon et al. (2006).
7. Inflation: Tables 1.1.4 and 2.3.4 from NIPA, Bureau of Economic Analysis.
8. Return on bonds: Nominal returns from the H15 release of the Board of Governors of the Federal Reserve.
9. Treasury bills outstanding: Table L-105 in the Flow of Funds tables of the Board of Governors of the Federal Reserve.
10. Residential mortgages and home equity loans: Table L-218 in the Flow of Funds tables published by the Board of Governors of the Federal Reserve.
11. Labor: Aggregate hours and labor force data published by the Bureau of Labor Statistics.