Designing Pension Plans According to Consumption-Savings Theory*

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Abstract

We derive optimal characteristics of contribution rates into defined-contribution pension plans based on consumption-savings theory. Contribution rates should be age-dependent and adjust to the balance-to-income ratio. Using detailed registry data on household savings behavior in Sweden, we show that individuals adjust savings rates according to these principles. We apply these principles in a quantitative model to design an optimal rule for contribution rates in a mandatory defined-contribution pension plan. Compared to typical rigid designs of contribution rates, our proposed design leads to the same average replacement rate and provides liquidity benefits and insurance against income shocks. The design implies a welfare gain of 1.8 percent in consumption equivalent and reduces the dispersion of replacement rates by more than 40 percent. Our design is robust to time-inconsistent investors.

JEL classification: D91, E21, G11, H55.

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1 Introduction

Developed countries are undertaking reforms that separate pension systems from the fiscal budget. A typical feature of such reforms is to rely more on defined-contribution (DC) pension plans to reduce the fiscal risks associated with the pension system. OECD (2017) reports that 32 out of 34 member countries have mandatory or quasi-mandatory second-pillar pension provision for workers. Fifteen member countries have DC pension plans. Notably, these DC pension plans feature mandatory contributions, which are a constant fraction of income. The shift to DC pension plans raises important questions about contributions that we argue have so far been overlooked, or at least understudied.

In this paper, we study optimal contribution rates to mandatory DC pension plans. We start by demonstrating that constant contribution rates out of income are at odds with basic principles of consumption-savings theory. Using Swedish registry-based data, we show that these principles are empirically relevant: outside the pension system—where individuals can freely choose their savings rate—savings behavior is consistent with these principles. We thus build a rich quantitative life-cycle model, including a detailed pension system, and propose a more flexible design of contribution rates that adheres to these principles. Based on the model, we propose a simple rule for contribution rates that significantly improves consumer welfare compared to an existing pension system with constant contribution rates while maintaining the same average replacement rate. Moreover, the dispersion of replacement rates is substantially reduced compared to the dispersion under constant contribution rates. Finally, we show that the welfare benefits of redesigning contribution rates according to consumption-savings theory are robust to allowing for time-inconsistent investor behavior.

Our first contribution is that we formulate optimal principles for savings relative to income, i.e., savings rates. In a DC plan, participants accumulate an asset balance gradually over the course of their working life. This allows a close comparison and benchmarking of plans to standard consumption-savings theory. We make use of this feature and build a stylized three-period life-cycle model to highlight two guiding principles for optimal savings rates. First, optimal contribution rates depend on expected income growth: the steeper the income profile, the lower the optimal contribution rates for young workers. Plan participants with increasing income over their working life would thus optimally choose contribution rates that increase with age. Second, optimal contribution rates are a function of the asset balance-to-income ratio. The fundamental desire to smooth consumption between working life and retirement implies that contribution rates should be lower if either current income falls or the asset balance increases, for instance due to high returns on previous savings. Constant mandatory contribution rates are at odds with both of these principles.

An aspect that is at the core of the discussion of pension systems is the replacement rate: the ratio of resources during retirement relative to resources available during working life. While the level of replacement rates is important, there is increasing focus on inequality in replacement rates. For instance, Hagen, Laun, and Palme (2022) report that in Sweden there is considerable dispersion in total replacement rates, ranging from 85 percent at the tenth percentile to 120 percent at the 90th percentile. In our three-period model, we show analytically that a constant mandatory savings rate leads to an undesired increase in the dispersion of replacement rates relative to the dispersion resulting from optimal savings rates.

The stylized nature of our three-period model naturally raises the question of whether these principles are empirically relevant. Our second contribution is that we investigate their relevance using Swedish registry-based data. We test whether consumption-savings behavior is consistent with those principles in the sense that individuals adhere to them in their non-mandated savings outside the pension system. Our data sample is representative of the Swedish population and contains comprehensive information about individuals' balance sheets, income and demographics. The panel dimension of the data together with information about each financial security held and its return enables us to measure individuals' savings rates into financial wealth.

We show that individuals' savings behavior is consistent with the predictions from the three-period model and the magnitudes are economically sizable. First, higher expected income growth is associated with lower savings rates and average savings rates increase with age. For example, while the average savings rate of individuals aged 25–35 is 1.3 percent, the corresponding number for individuals aged 56–64 is 3.8 percent. Second, we use both OLS and IV approaches to estimate the response of savings rates to shocks to the asset balance-to-income ratio. We begin by analyzing the reaction to shocks to the asset balance. Individuals reduce their contribution rate by on average 10.8 percentage points after an increase in the asset balance by one year's worth of disposable income. We continue by studying the reaction to income shocks—the second component of the asset balance-to-income ratio. A negative income shock of ten percent reduces the contemporaneous savings rate by 0.26 percentage points. This is consistent with consumption-savings theory as negative income shocks increase the asset balance-to-income ratio and should hence lead to lower savings rates. We thus conclude that individuals strive to adhere to the principles for optimal savings rates, which further motivates that DC pension plans should be based on them.

Our third contribution is that we design a simple policy rule for contribution rates in a realistic economic environment. We build a quantitative, heterogeneous agent, life-cycle model that features risky labor income, a pension system with three pillars of retirement savings, and portfolio choice. We calibrate the model to the Swedish economy—an institutional setting that is often considered a model for other countries. Our proposal is a contribution rate that depends on the individual's age and balance-to-income ratio. Every year, the contribution rate should unconditionally increase by 0.3 percentage points. Furthermore, investors who fall short by 1 percent from the target balance-to-income ratio for their age should increase their contribution rate by 0.15 percentage points.

Through its dependency on age, this rule provides both liquidity and consumption benefits for the first half of working life. For example, the average consumption of 30-year-old investors increases by 3.8 percent. Moreover, the dependency on the asset balance-to-income ratio lets contribution rates counteract shocks to income. Disposable income after pension contributions thereby becomes less volatile so that investors face less period-by-period risk in available cash-flows. Consequently, our proposed rule implies a substantial welfare gain. In terms of consumption equivalent variation, the gain is on average 1.8 percent. Moreover, the standard deviation in replacement rates of the DC account is substantially reduced by more than 40 percent compared to the current rule with constant contribution rates. The insights from the three-period model hence hold in the fully calibrated life-cycle model: Designing contribution rates according to consumption-savings theory can improve welfare and substantially reduce inequality in replacement rates while maintaining the same average level of replacement rates.

Up to this point in our analysis, we have assumed that all investors are rational. However, there is a long history of literature that justifies the existence of mandatory pension or social security systems with the consideration that investors might have time-inconsistent preferences. In a seminal work, Feldstein (1985) analyzed the optimal level of social security if households are myopic, i.e., if they have a shorter planning horizon than their life-span.[1] More recently, empirical analyses link such limited planning horizons to a lack of financial literacy (see, e.g., Lusardi and Mitchell (2011) and van Rooij, Lusardi, and Alessie (2012)). These deviations from rationality can have a sizable impact on financial behavior: Limited planning has been found to lead to lower retirement savings (Lusardi and Mitchell (2007)). In addition, financial literacy or lower cognitive ability more generally has been documented to cause lower stock market participation (Christelis, Jappeli, and Padula

[1]Imrohoroglu, Imrohoroglu, and Joines (2003) conduct a similar analysis in an incomplete markets, heterogeneous agents setup where households have time-inconsistent preferences in the form of quasi-hyperbolic discounting as in Laibson (1997).

(2010), van Rooij, Lusardi, and Alessie (2011)). To check the robustness of our welfare effects to these types of deviations from rationality, we allow investors to have time-inconsistent preferences in the form of limited planning horizons. We recalibrate the model for the degree of myopia such that, within constant contribution rate policies, the existing contribution rate level is optimal. We find that our proposed policy is optimal even under this assumption. Moreover, we show that, for all feasible planning horizons, our proposed rule leads to average welfare benefits equivalent to 1–3 percent of lifetime consumption. Therefore, even though our design was based on a model of rational investors, our proposed rule and its welfare benefits are robust to time inconsistency.

Our analysis relates to three strands of the literature on pension plan design and savings rates of individuals and households. First, there is an ongoing debate about auto-enrollment into pension plans and auto-escalation of contribution rates, in particular for the U.S., where DC pension plan designs vary more (see Beshears, Choi, Laibson, and Madrian (2018) for a discussion of defaults). Our results suggest that designing defaults that involve auto-enrollment and automatic adjustments of the contributions could benefit from conditioning on individual circumstances, as our proposed rule suggests. Second, there is strong concern in the literature that many consumers lack the financial literacy to make informed retirement planning decisions (see Lusardi and Mitchell (2014) for an overview). As a mandatory DC pension plan, the design we propose relieves individuals from the majority of these complex questions. Moreover, in our proposed design, the mandatory contribution rates follow the principles of optimal consumption-savings theory. This allows financially illiterate households to get closer to the optimal retirement savings behavior. Third, our proposed rule features automatic adjustments due to income shocks. It is thus in line with, for instance, Beshears, Choi, Iwry, John, Laibson, and Madrian (2019), who discuss different designs of savings accounts that would enable individuals to build emergency savings and self-insurance against transitory income shocks.

Our analysis also relates to an ongoing policy debate fueled by the COVID-19 pandemic that concerns whether individuals should be able to withdraw balances from retirement accounts, such as 401(k). There are good arguments in favor of either side; on the one hand, if individuals are living hand-to-mouth, the welfare gain from allowing early withdrawals in emergency situations is large. On the other hand, individuals may miss out on high returns as financial markets reverse. Moreover, if investors are time inconsistent, they might withdraw excessively and thus end up in a situation with insufficient savings for retirement. Our proposal does not involve early withdrawals and thus avoids the associated perils. Put differently, the analysis centers attention to the cash-flow relief from automatic adjustments of contributions. We wish to highlight that the cash-flow reliefs associated with our proposed rule for the contribution rate attain more than one-third of the maximum welfare gain associated with a laissez-faire policy under the assumption of rational expectations. This suggests that allowing for pre-withdrawals (possibly for a penalty fee) at most implies an additional average gain of less than two-thirds. In addition, for time-inconsistent investors, we show that too much flexibility can lead to a welfare reduction of up to 9 percent, while our proposed rule with automatic adjustments to the investor's circumstances increases welfare. In other words, our findings imply that a flexible design of contribution rates substantially diminishes the value of outright pre-retirement withdrawals.

Relative to the existing literature, Sandris Larsen and Munk (2022) and Astrup Jensen, Fischer, and Koch (2022) are perhaps most similar to our study. Both study how to design optimal contribution rates given that pension plans are mandatory. Sandris Larsen and Munk (2022) investigate contribution rates that depend on age whereas we derive fundamental principles for consumption-savings theory and hence base our policy proposal on both age and the balance-to-income ratio. In addition, we argue that they are relevant based on empirical evidence on savings. Astrup Jensen, Fischer, and Koch (2022) analyze the interaction of the design of contribution rates with home ownership. Pries (2007) uses a quantitative life-cycle model to study labor supply responses and welfare effects associated with a reform of U.S. Social Security to a system of individual accounts with age-dependent contribution rates.

The paper proceeds as follows. Section 2 sets up a stylized consumption-savings model to illustrate the basic principles of optimal contribution rates for a life-cycle investor. Section 3 shows that, empirically, individuals' consumption-savings behavior is consistent with these principles. Section 4 presents our quantitative life-cycle model, which incorporates a pension system that offers flexibility in contribution rates. Section 5 uses this model to design optimal rules for contribution rates that satisfy the principles of consumption-savings theory, presents their welfare implications, and analyzes robustness to time inconsistency. Finally, Section 6 concludes.

2 Consumption-savings theory and optimal savings rates

In this section, we consider a stylized life-cycle consumption-savings model that highlights the determinants of the optimal savings rate.[2] We derive specific characteristics of optimal savings behavior and discuss their implications for the design of optimal pension plans.

[2] We use the term *savings rate* since the stylized model abstracts from the pension system. In our full model, we use the term *contribution rate*, which is the savings rate that a defined-contribution pension plan stipulates.

2.1 Stylized life-cycle model

The life cycle consists of three distinct periods in life: young working life, middle-aged working life, and retirement. Investors receive exogenous, deterministic income in the two working-life periods but do not receive any income in the retirement period. They optimally choose savings—and hence savings rates out of current income—in the working-life periods to smooth consumption over the life cycle.

An investor i is born in period $t = 1$ with no assets and with an exogenous income profile $Y_{i,1}$ and $Y_{i,2}$ in periods $t = 1$ and $t = 2$, respectively. Savings bear risk-free interest R_1 and R_2 in the two periods. In the retirement period ($t = 3$), investors do not receive any income and thus simply consume the savings that they accumulated from previous periods. Agents maximize discounted lifetime utility where they discount future periods with discount factor β and have logarithmic flow utility. In detail, the agents choose a sequence of consumption $\{C_{i,t}\}_{t=1}^{3}$ and of savings $\{A_{i,t}\}_{t=1}^{2}$ to solve the following optimization problem:

$$\max_{\{C_{i,t}\}_{t=1}^{3},\{A_{i,t}\}_{t=1}^{2}} \quad \log(C_{i,1}) + \beta \log(C_{i,2}) + \beta^2 \log(C_{i,3}) \tag{1}$$

$$\text{s.t.} \quad C_{i,1} = Y_{i,1} - A_{i,1} \tag{2}$$

$$C_{i,2} = A_{i,1}R_1 + Y_{i,2} - A_{i,2} \tag{3}$$

$$C_{i,3} = A_{i,2}R_2. \tag{4}$$

Pension plans are described by their savings rates out of current income. We therefore define savings rates in working life as

$$\lambda_{i,1} = \frac{A_{i,1}}{Y_{i,1}} \tag{5}$$

$$\lambda_{i,2} = \frac{A_{i,2} - A_{i,1}R_1}{Y_{i,2}}. \tag{6}$$

Note that, in $t = 2$, the savings rate measures the additional savings over and above savings brought into the period, as opposed to total savings $A_{i,2}$.

In a first step, we start by analyzing the optimal characteristics of savings rates without imposing any constraints on the investors' savings. This implies that, in period 1, both total savings $A_{i,1}$ and savings rates $\lambda_{i,1}$ can be negative. In period 2, however, total savings $A_{i,2}$ will be positive for all investors to ensure positive consumption in period 3 (see equation (4)). At the same time, in this unconstrained setup, savings rates $\lambda_{i,2}$ can be negative if total savings in period 2 are lower than the asset balance that the investor brought into the period ($A_{i,1}R_1$). In a second step, we introduce restrictions on the savings rates that mimic the constraints implied by mandatory pension systems and analyze their implications.

2.2 Characteristics of optimal savings rates

The following propositions characterize the optimal savings rate for young and middle-aged agents in the setup without constraints on the savings rates. The proofs of all propositions can be found in Appendix A.

Proposition 1 *The optimal savings rate in t=1 decreases in expected income growth.*

Explicitly, the optimal savings rate in $t = 1$ is equal to

$$\lambda_1^* = \frac{(\beta + \beta^2)}{(1 + \beta + \beta^2)} - \frac{1}{(1 + \beta + \beta^2)} \frac{1}{R_1} \frac{Y_{i,2}}{Y_{i,1}}. \tag{7}$$

The intuition for this proposition is that, as income later in working life increases, the agent optimally wants to delay savings for retirement. If income early in working life is sufficiently small compared to later in working life, i.e., income growth is sufficiently large, the agent optimally would like to borrow when she is young.

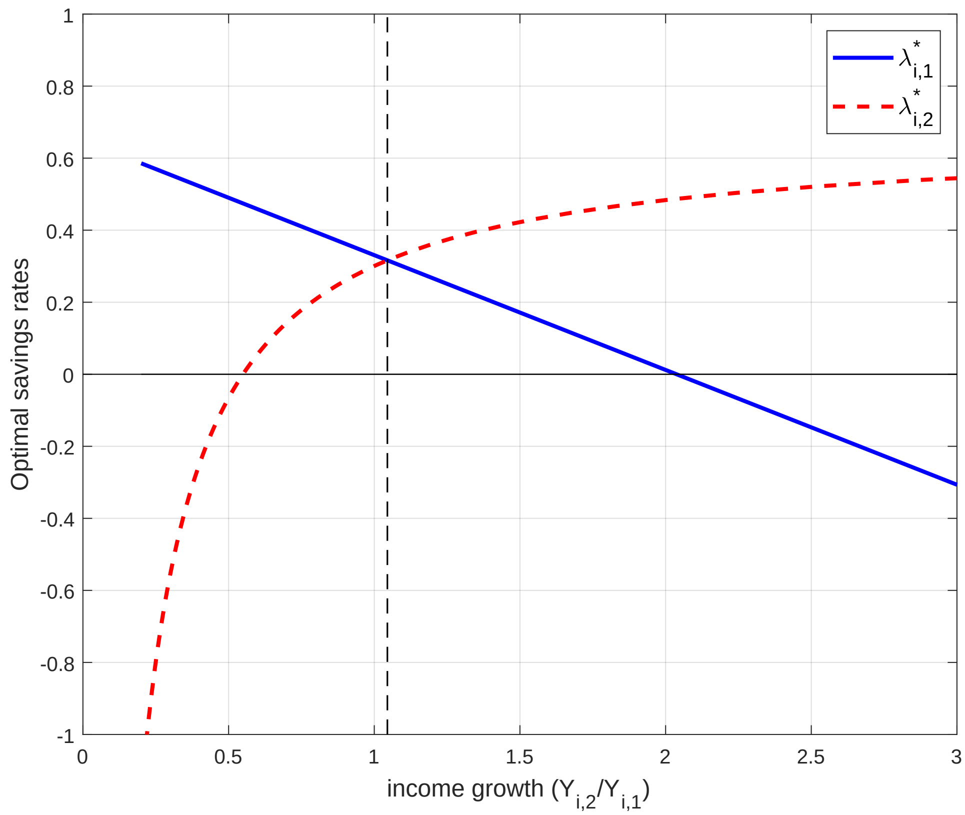
Proposition 2 *The optimal savings rate in t=2 decreases in the balance-to-income ratio.*

Explicitly, the optimal savings rate in $t = 2$ is equal to

$$\lambda_2^* = \frac{\beta}{1 + \beta} - \frac{1}{1 + \beta} \frac{A_{i,1}R_1}{Y_{i,2}}. \tag{8}$$

The intuition for this proposition is that agents want to smooth their consumption between working life and retirement. If they have already accumulated high savings relative to their current income, they optimally do not want to save much in addition to the savings that they have already accumulated. Moreover, if the balance of their savings is sufficiently large relative to their current income, the agent would optimally like to access these savings for current consumption, leading to a negative savings rate out of current income.

Figure 1: Optimal savings rates in the three-period model



Note: The figure illustrates the optimal savings rates in periods $t = 1$ and $t = 2$ as a function of income growth $\frac{Y_{i,2}}{Y_{i,1}}$. The discount factor and return in period 1 are set to $\beta = 0.95$ and $R_1 = 1.1$, respectively. The dashed vertical line indicates the value $\frac{Y_{i,2}}{Y_{i,1}} = \kappa(R_1, \beta)$, where optimal savings rates are exactly constant throughout working life.

Proposition 3 *There exists a specific income profile* $\frac{Y_{i,2}}{Y_{i,1}} = \kappa(R_1, \beta)$ *such that the optimal savings rates in* $t = 1$ *and* $t = 2$ *are exactly identical. For any other income profile, constant savings rates throughout working life are suboptimal.*

Figure 1 illustrates this proposition graphically for a particular combination of discount rates and interest rates. It depicts the optimal savings rates in period $t = 1$ and $t = 2$ as a function of income growth. The knife-edge case where the savings rates are optimally constant throughout working life is indicated with the vertical dashed line. The figure emphasizes that, for an income growth larger than κ, the agent would optimally like to postpone saving for retirement and hence would optimally choose to have a larger savings rate late in working life relative to earlier in working life.

Note that the threshold income growth κ is a function of the discount factor and interest rates. The model assumes for simplicity that all investors face the same interest rates. In reality, returns are likely to be heterogeneous due to both idiosyncratic differences in investment possibilities and common risk factors. In this case, the threshold level of income growth κ would be heterogeneous across investors, and the distribution of thresholds would vary over time with the risk factors. Thus, a constant savings rate for all investors and across all time periods would not be optimal.

So far, we have characterized the optimal savings rates in the absence of a mandatory pension system. In what follows, we introduce mandatory constant savings rates. In this constrained case, investors solve their life-cycle consumption-savings problem according to (1)-(4) subject to two additional constraints on the savings rates:

$$\lambda_{i,1} \geqslant \underline{\lambda} \tag{9}$$

$$\lambda_{i,2} \geqslant \underline{\lambda}, \tag{10}$$

where $\underline{\lambda}$ is the minimum savings rate requirement.

Regulators are often concerned about measuring the replacement rates that a pension system delivers. We therefore define replacement rates as the ratio between consumption in retirement relative to consumption before retirement:

$$RR_i = \frac{C_{i,3}}{C_{i,2}}. \tag{11}$$

Proposition 4 *Mandatory constant savings rates increase the dispersion of replacement rates.*

The intuition for this result is that, in the absence of a mandatory pension system, all investors optimally choose a replacement rate that is independent of their income profile and past returns. Specifically, under homogenous preferences and identical expected returns, investors choose

$$RR_i = \beta R_2 \qquad \forall i. \tag{12}$$

This result reflects the fact that investors choose their replacement rate by optimally smoothing consumption between late working life and retirement in a purely forward-looking manner. Once mandatory savings rates are introduced, however, the constrained-optimal solution is no longer an identical replacement rate for all investors. Instead, there is a region of income growth, $\frac{Y_{i,2}}{Y_{i,1}} < \kappa_2(\underline{\lambda}, \beta, R_1)$, for which investors are constrained. In this region, the constrained-optimal replacement rate has the form

$$RR_i = \beta R_2 \cdot \frac{\beta}{1+\beta^2} \left(\frac{R_1}{1-\underline{\lambda}} \frac{Y_{i,1}}{Y_{i,2}} + \frac{\underline{\lambda}}{(1-\underline{\lambda})} \right). \tag{13}$$

In this case, the replacement rate varies with both past returns to savings R_1 and the steepness of income growth $\frac{Y_{i,2}}{Y_{i,1}}$. This leads to undesired variation in replacement rates due to the pension system. Intuitively, the constrained region is where investors have relatively low income compared to the asset balance that they have already accumulated. They would thus like to smooth their consumption between their remaining working life and retirement by either making very little additional savings or by already consuming part of their accumulated asset balance. Enforcing mandatory constant savings rates, however, does not allow for this behavior and leads to excessive savings. This distorts the behavior of constrained investors away from the optimal (unconstrained) replacement rate.

2.3 Implications for pension plan design

Existing mandatory pension plans typically feature constant savings rates (contribution rates) out of income across all age groups and irrespective of the specific circumstances of an individual worker. We have shown that such a design is at odds with the basic principles of optimal consumption-savings behavior. In particular, we have shown two deficiencies of constant savings rates. First, young individuals, who typically expect their income to grow substantially, would optimally like to postpone saving for retirement. They would thus choose to have a non-constant, increasing age profile of savings rates. Second, we have shown that optimal savings rates later in working life optimally vary with the balance-to-income ratio. The fundamental desire to smooth consumption over working life and retirement implies that workers should optimally reduce their current savings rates if their accumulated asset balance is larger relative to current income. This could either be because they have contributed a lot in the past, they received a particularly good return on their earlier savings, or because their current income is comparatively low. We thus conclude from this section that an optimal design of pension plans should both feature an increasing age profile of savings rates and allow savings rates to adjust as a function of the balance-to-income ratio. We will use these insights from the stylized consumption-savings model when we design an optimal pension plan in the quantitative model.

3 Empirical tests of the principles for consumption and savings

Are the insights from the three-period model relevant for actual empirical behavior of investors? We use Swedish registry-based data to show that individuals' consumption-savings behavior in their liquid savings is consistent with the principles of consumption-savings theory.

3.1 Data

Our dataset is a representative sample of the Swedish population for 2000–2007.[3] It contains comprehensive information about individuals' balance sheets, income and demographics. Through the tax and financial registries, we are able to observe stocks, cash in bank accounts, mutual funds, bonds and various types of financial securities held by individuals. This is possible because a wealth tax was levied on individuals during our sample period, which by law required individuals to disclose their assets, earnings and income to the tax authority. Furthermore, standard sociodemographic variables, such as age and income, are observable. See, e.g., Calvet, Campbell, and Sodini (2007), Dahlquist, Setty, and Vestman (2018), and Di Maggio, Kermani, and Majlesi (2020) for details.

The detailed information in the data combined with the longitudinal dimension enables us to measure both individuals' asset balance in financial wealth and their savings rates in financial wealth. The data challenge in constructing the savings rates lies in the need of longitudinal data and detailed asset returns. To be precise, the balance in financial wealth can be constructed as:

$$A_{it} = \sum_{k=1}^{K} q_{itk} \cdot P_{kt}, \tag{14}$$

where K is the number of financial securities in the portfolio, q_{itk} indicates the number of shares held of security k—a stock, mutual fund or bond—by individual i at the end of year t, and P_{kt} is the price per share. In addition, we measure the balances of bank accounts and capital insurance accounts. Based on this information, we can also compute the gross return:

$$R_{it}^{A} = \sum_{k=1}^{K} w_{ikt-1} \frac{P_{kt} + D_{kt}}{P_{kt-1}}, \tag{15}$$

[3] We start from a population data set of all Swedes for 2000–2007 and then employ a sequence of sample restrictions that improve precision in measurement. See Table A.1 in the Appendix.

where w_{ikt-1} is the portfolio weight for individual i in asset k at time $t-1$ and P_{kt} and D_{kt} are the end-of-year price and financial payouts (e.g., dividends) by asset k at time t, respectively.[4]

We construct a cash-flow measure of savings in liquid financial wealth as follows:

$$\Delta \widetilde{A}_{it} = A_{it} - A_{it-1} \times R_{it}^{A}, \tag{16}$$

and from this flow we construct the savings rate

$$\widetilde{\lambda}_{it} = \frac{\Delta \widetilde{A}_{it}}{Y_{it}^{\text{Disp}}}, \tag{17}$$

where Y_{it}^{Disp} is disposable income (gross labor income minus taxes plus transfers). The variable $\widetilde{\lambda}_{it}$ is the empirical counterpart to the savings rates in Section 2.[5]. This is a cash-flow based measure of the savings rate that resembles the ones constructed by e.g., Fagereng, Blomhoff Holm, Moll, and Natvik (2019) and Bach, Calvet, and Sodini (2018). This is sometimes referred to as the active savings rate since it reflects active decisions to withdraw or contribute during year t from the end-of-year $t-1$ asset balance, A_{it-1}. This makes $\widetilde{\lambda}_{it}$ a measure of the savings rate that is analogue to pension plans' stipulated contribution rates.

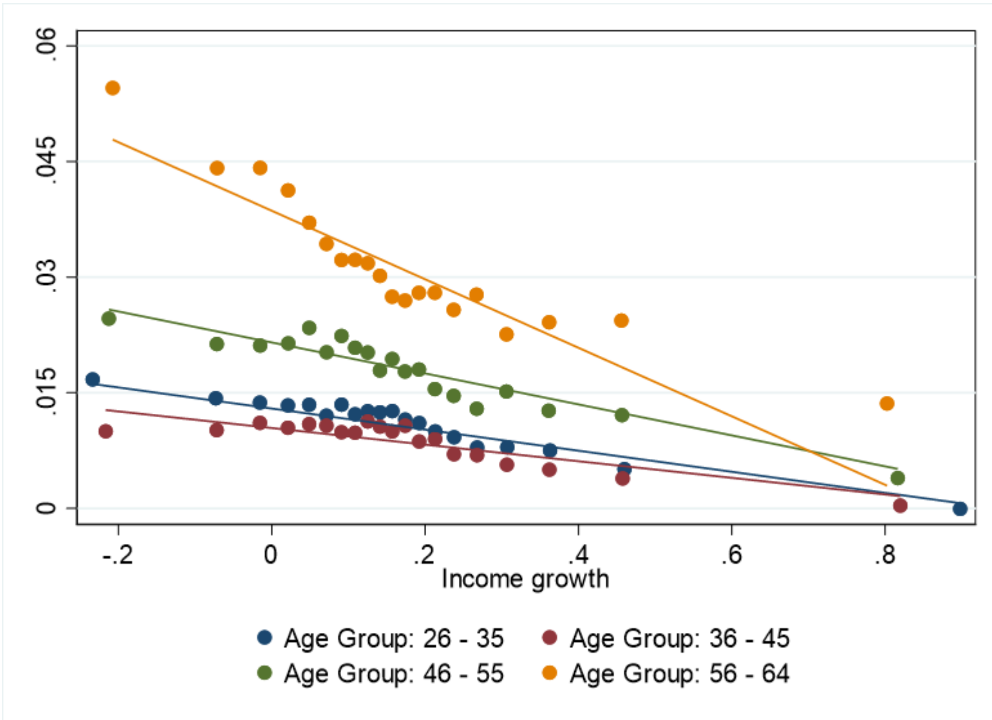
3.2 The role of age and expected income growth

The three-period model predicts that savings rates should decrease in expected income growth (Proposition 1). For most individuals, this implies that savings rates should increase with age. We investigate these relationships based on the following regression:

[4]The gross return for bonds and bank accounts is set to 1, while the gross return for stocks and mutual funds with missing return data is set to the return of the MSCI World Gross Index. The financial securities held in the so-called *capital insurance accounts* are unobservable. Therefore, we set the gross return equal to the return of the all-share Stockholm Stock Exchange, in particular the SIX Return Index.

[5]To be clear, Statistics Sweden also reports asset balances for each asset class. Our way of computing the cash-flow measure of savings, (16), gives almost identical values as an approach that uses Statistics Sweden's precomputed asset balances; that is, $\Delta \widetilde{A}_{it} = A_{it} - A_{it-1} \times R_{it}^{A} \approx \Delta b_{it} + \Delta v_{it} + \Delta \psi_{it} - y_{it}^{v}$, where Δb_{it} is the change in bank account balances, Δv_{it} is the active rebalancing of stocks, mutual funds, and bonds, $\Delta \psi_{it}$ is the net-change in capital insurance accounts, and y_{it}^{v} is after-tax financial income, particularly dividends from stocks, interest payments from bank accounts, and coupons from bonds. The correlation between the left-hand side and the measure after the approximation is 0.98. The difference between the two measures has a mean and median of 356 and 0 SEK, respectively.

Figure 2: Income growth, age and savings rates



Note: The figure is a binned scatter plot of savings rates against income growth rates. The savings rate is the average over two years, $(\widetilde{\lambda}_{i,2002} + \widetilde{\lambda}_{i,2003})/2$. Income growth is defined as $y^{\text{Disp}}_{i,late} - y^{\text{Disp}}_{i,early}$, where $y^{\text{Disp}}_{i,early} = \log((Y^{\text{Disp}}_{i,2002} + Y^{\text{Disp}}_{i,2003})/2)$ and $y^{\text{Disp}}_{i,late} = \log((Y^{\text{Disp}}_{i,2006} + Y^{\text{Disp}}_{i,2007})/2)$.

$$\widetilde{\lambda}_{it} = \beta_0 + \beta_1 \times (y^{Disp}_{i,t+s} - y^{Disp}_{i,t}) + \varepsilon_{it}, \tag{18}$$

where $y^{Disp}_{i,t+s} - y^{Disp}_{i,t}$ is the log difference in disposable income between year $t+s$ and t and ε_{it} is an error term. The coefficient of interest is β_1, which is the elasticity of the savings rate with respect to income growth.

Figure 2 reports a binned scatter plot of this regression, where income growth has been interacted with dummy variables indicating the age group of the individual, and income growth is measured four years ahead of the observed savings rate ($s = 4$). If income is at least partially predictable, then $y^{Disp}_{i,t+4} - y^{Disp}_{i,t}$ is a proxy for expected income growth at t. The interaction with age is intended to isolate the effect of expected income growth since it varies over the life cycle. Table 1 reports these estimates. Columns (1) and (2) show that there is a strong negative

Table 1: Savings rates and the role of expected income growth and age

	(1)	(2)	(3)
$y_{i,t+4}^{Disp} - y_{i,t}^{Disp}$	-0.026***	-0.056***	
	(0.001)	(0.003)	
$\mathbb{1}$[Age $26-35$]		-0.026***	-0.025***
		(0.001)	(0.000)
$\mathbb{1}$[Age $36-45$]		-0.029***	-0.028***
		(0.001)	(0.000)
$\mathbb{1}$[Age $46-55$]		-0.017***	-0.017***
		(0.001)	(0.000)
$(y_{i,t+4}^{Disp} - y_{i,t}^{Disp}) \times \mathbb{1}$[Age $26-35$]		0.036***	
		(0.003)	
$(y_{i,t+4}^{Disp} - y_{i,t}^{Disp}) \times \mathbb{1}$[Age $36-45$]		0.043***	
		(0.003)	
$(y_{i,t+4}^{Disp} - y_{i,t}^{Disp}) \times \mathbb{1}$[Age $46-55$]		0.032***	
		(0.003)	
Constant	0.019***	0.039***	0.038***
	(0.000)	(0.001)	(0.000)
F $\mathbb{1}$[Age $26-35$] = $\mathbb{1}$[Age $36-45$]		128***	147***
F $\mathbb{1}$[Age $36-45$] = $\mathbb{1}$[Age $46-55$]		1,309***	2,380***
F $\mathbb{1}$[Age $46-55$] = $\mathbb{1}$[Age $56-64$]		932***	2,607***
Observations	1,568,521	1,568,521	2,336,042
Adj. R^2	0.002	0.006	0.006

Note: The table reports estimates of regressing savings rates on income growth rates and age dummy variables using OLS. The savings rate is the average over two years, $(\widetilde{\lambda}_{i,2002} + \widetilde{\lambda}_{i,2003})/2$ and income growth is defined as $y_{i,t+4}^{\text{Disp}} - y_{i,t}^{\text{Disp}}$, where $y_{i,t}^{\text{Disp}} = \log((Y_{i,2002}^{\text{Disp}} + Y_{i,2003}^{\text{Disp}})/2)$ and $y_{i,t+4}^{\text{Disp}} = \log((Y_{i,2006}^{\text{Disp}} + Y_{i,2007}^{\text{Disp}})/2)$. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$.

relationship between income growth and savings rates. For individuals aged 26–55, a ten-percent increase in expected income growth is associated with lower savings rates by 0.13 to 0.24 percentage points. For individuals aged 56–64, the same increase in expected income growth is associated with lower savings rates by 0.56 percentage points. The negative relationship between expected income growth and savings rates is consistent with Proposition 1. Additionally, column (3) shows that the savings rates of the individuals aged 56–64 are 1.7 to 2.8 percentage points higher than for younger individuals. This is consistent with Proposition 3. Overall, this shows that individuals take into account their age and expected income growth when choosing their savings rate in liquid financial wealth.

3.3 The role of shocks to the asset balance-to-income ratio

The three-period model predicts that the optimal savings rate decreases in the asset balance-to-income ratio (Proposition 2). We investigate if this is supported empirically by analyzing the responses to shocks to the two components—asset balance and income—separately.

A positive shock to the asset balance ceteris paribus increases the asset balance-to-income ratio. The theory predicts that this should lead to a reduction in the savings rate. We test this by analyzing the response of savings rates to idiosyncratic shocks to returns. To do so, we estimate the following regression using OLS and IV, using the empirical strategy of Di Maggio, Kermani, and Majlesi (2020):

$$\widetilde{\lambda}_{it} = \theta_i + \delta_t + \beta_1 \frac{A_{it-1}}{Y_{it}^{\text{Disp}}} \times R_{it}^A + \beta_2 Y_{it}^{\text{Disp}} + \beta_3 NW_{it-1} + \beta_4 ND_{it,t-1} + \varepsilon_{it}, \tag{19}$$

where θ_i and δ_t are individual and year fixed effects, respectively, and ε_{it} is an error term. Our control variables are disposable income Y_{it}^{Disp}, one-year lagged net worth NW_{it-1}, and a dummy variable $ND_{it,t-1}$ equal to 1 if the individual did not receive a dividend at time t or at time $t-1$. The variable A_{it-1} is the end-of-year $t-1$ (beginning-of-year t) asset balance, and R_{it}^A is the gross return during year t of the portfolio had it been unchanged throughout the year. The coefficient of interest is β_1, which measures the response in the savings rate upon a change in the asset balance-to-income ratio, originating from an idiosyncratic return shock. According to Proposition 2, we expect this response to be negative, i.e. $\beta_1 < 0$.

OLS estimates are reported in columns (1)–(2) of Table 2. The estimates indicate that an increase in the asset balance by one year's worth of disposable income reduces individuals' savings rates by 11 percentage points (holding disposable income constant). This is a strong response since it implies that the response in savings rates to such a change is of the same order of magnitude as typical pension plans' mandated contribution rates.

If individuals simultaneously adjust their portfolios and savings rates during the year, due to e.g., macroeconomic news, then OLS estimates are biased. We therefore instrument $\frac{A_{it-1}}{Y_{it}^{\text{Disp}}} \times R_{it}^A$ with $\frac{A_{it-1}}{Y_{it}^{\text{Disp}}} \times \overline{R_{it}^A}$, where $\overline{R_{it}^A}$ is the individual's return on financial assets given the portfolio held at

Table 2: Savings rates and the role of asset-balance shocks

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS		IV				
$\frac{A_{it-1}}{Y_{it}^{Disp}} \times R_{it}^{A}$	-0.114***	-0.112***	-0.108***	-0.167***	-0.132***	-0.133***	-0.094***
	(0.010)	(0.009)	(0.007)	(0.017)	(0.006)	(0.022)	(0.007)
Age Group	26-64	26-64	26-64	26-35	36-45	46-55	56-64
Controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Individual FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
First-stage F-stat			36,477	9,766	76,424	8,292	36,539
Adj. R^2	0.303	0.305	0.063	0.096	0.075	0.075	0.065
Observations	15,796,927	15,577,583	15,577,583	4,018,451	4,362,187	3,818,800	2,749,725

Note: The control variables are Y_{it}^{Disp}, NW_{it-1} and $ND_{it,t-1}$. Standard errors, clustered at the level of the individual and the individual's largest security holding, are in parentheses. The individual's largest security holding is the particular stock or mutual fund—identified by their International Securities Identification Number (ISIN)—with the largest weight in the individual's financial asset portfolio. When the largest holding is bonds, bank accounts or capital insurance accounts, we classify the largest security holding by its respective asset type. Singleton groups are excluded. Table A.3 reports first-stage IV estimates. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$.

$t-2$.[6]

Our IV estimates are qualitatively similar to the OLS estimates but somewhat smaller. In the full sample, column (3), an increase in the asset balance by one year's worth of disposable income reduces savings rates by 10.8 percentage points. We also consider heterogeneity in savings responses for individuals of different age. There is an essentially monotone relationship between the response in the savings rate and age. The response of individuals who are 26–35 years old is sixty percent stronger than the average (-0.167; column (4)), and the response of individuals who are 56–64 years old is less strong (-0.094; column (7)). This is consistent with young individuals having higher marginal utility of consumption, for instance because of binding borrowing constraints.[7]

The asset balance-to-income ratio can also change due to shocks to its denominator—income. A negative shock to income ceteris paribus increases the asset balance-to-income ratio. Proposition

[6] $\overline{R_{it}^{A}} = \sum_{k=1}^{K} w_{ikt-2} \frac{P_{kt} + D_{kt}}{P_{kt-1}}$

[7] Table A.4 considers an alternative regression specification that measures the response in $\Delta\widetilde{A}_{it}$ to changes in $A_{it-1} \times R_{it}^{A}$. The estimates are qualitatively the same.

Table 3: Savings rates and the role of income shocks

	(1) OLS	(2) IV
$y_{i,t}^{Disp} - y_{i,t-1}^{Disp}$	0.057***	0.026*
	(0.001)	(0.013)
Individual FE	Yes	Yes
Year FE	Yes	Yes
Individual Clusters	Yes	Yes
Employer Clusters	No	Yes
F First-stage		4,212***
Observations	11,035,018	9,340,192
Adj. R^2	0.294	0.001

Note: The table reports estimates of regressing savings rates on changes in income by OLS and IV. The IV estimate in column (2) uses the wage-bill growth of the employer as an instrument for the individual's income growth. Table A.2 reports the first-stage estimate. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$.

2 hence predicts that individuals should decrease their savings rate in response to adverse shocks to income. We test this by estimating the following equation:

$$\widetilde{\lambda}_{it} = \theta_i + \delta_t + \beta_1 \times (y_{i,t}^{Disp} - y_{i,t-1}^{Disp}) + \varepsilon_{it}, \tag{20}$$

where $y_{i,t}^{Disp} - y_{i,t-1}^{Disp}$ is the log difference in disposable income between year t and $t-1$, θ_i and δ_t are individual and time fixed effects, respectively, and ε_{it} is an error term. The coefficient of interest is β_1, which is the elasticity of the savings rate with respect to an income change. Column (1) of Table 3 shows the results. The estimate is consistent with the prediction of the theoretical model: A ten-percent decrease in income from $t-1$ to t reduces the savings rate by 0.57 percentage points.

If individuals simultaneously adjust labor supply and savings rates then the OLS estimate suffers from endogeneity, as discussed in Fagereng, Guiso, and Pistaferri (2018). We therefore adopt their estimation strategy and use IV estimation, where we instrument individual i's income growth with the aggregate wage growth of individual i's employer. Column (2) reports the corresponding estimate. The IV estimate is about half the magnitude but still economically meaningful: A negative income shock of ten percent reduces the savings rate by 0.26 percentage points.

In sum, our empirical analysis shows that individuals' savings into liquid financial wealth indeed follow the principles highlighted by the three-period model. Savings rates increase with age, respond negatively to expected income growth, and respond negatively to increases in the asset balance-to-income ratio. Next, we set up a quantitative consumption-savings life-cycle model with risky labor income and a detailed pension system to analyze the consequences of implementing these principles in a defined-contribution pension plan.

4 A quantitative life-cycle model

We set up a life-cycle model featuring a detailed pension system as currently mandated in Sweden—an institutional setting that is often a model for other countries. The setup is an extension of Dahlquist, Setty, and Vestman (2018), which in turn builds on Viceira (2001), Cocco et al. (2005), and Gomes and Michaelides (2005).[8] It includes risky labor income, a consumption-savings choice, and a portfolio choice. We augment the model with a pension system in which individuals save in two pension accounts, from which they receive their pension as annuities. One of the accounts belongs to the first pillar of the pension system and is pay-as-you-go but with an individual notional balance. The other account is a standard defined-contribution pension account that represents the second pillar of the pension system. The central focus of our analysis is to design optimal contributions into this account.

4.1 Model setup

Next, we describe the model's building blocks.

Demographics

We follow individuals from age 25 until the end of their lives.[9] The end of life occurs at the latest at age 100 but could occur before since individuals face an age-specific survival rate, ϕ_t. The life

[8] Our model relates to Gomes et al. (2009), who consider portfolio choice in the presence of tax-deferred retirement accounts, and to Campanale et al. (2014), who consider a model in which stocks are subject to transaction costs, making them less liquid.

[9] We choose age 25 as the start of the working phase since Swedish workers do not fully qualify for occupational pension plans before that age.

cycle is split into a working, or accumulation, phase and a retirement phase. From the age of 25 to 64, individuals work and receive labor income exogenously. They retire at 65.

Preferences

Individuals have Epstein and Zin (1989) preferences over a single consumption good. At age t, each individual maximizes the following:

$$U_t = \left(C_t^{1-\rho} + \beta \phi_t E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \tag{21}$$

$$U_T = C_T, \tag{22}$$

where β is the discount factor, $1/\rho$ is the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion, and $t = 25, 26, ..., T$ with $T = 100$. For notational convenience, we define the operator $\mathcal{R}_t(U_{t+1}) \equiv E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$.

Labor income

Let Y_{it} denote the labor income of an employed individual i at age t. During the working phase (up to age 64), the individual faces a labor income process with a life-cycle trend and persistent income shocks:

$$y_{it} = g_t + z_{it}, \tag{23}$$

$$z_{it} = z_{it-1} + \eta_{it} + \theta \varepsilon_t, \tag{24}$$

where $y_{it} = \ln(Y_{it})$. The first component, g_t, is a hump-shaped life-cycle trend. The second component, z_{it}, is a permanent labor income component. It has an idiosyncratic shock, η_{it}, which is distributed $\mathrm{N}\big(-\sigma_\eta^2/2, \sigma_\eta^2\big)$, and an aggregate shock, ε_t, which is distributed $\mathrm{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$. The aggregate shock also affects the stock return, and θ determines the contemporaneous correlation between the labor income and the stock return. We allow for heterogeneity in income as early as age 25 by letting the initial persistent shock, z_{i25}, be distributed $\mathrm{N}(-\sigma_z^2/2, \sigma_z^2)$.

During the retirement phase (from age 65 onwards), the individual has no labor income.[10] Pension is often modeled as a deterministic replacement rate relative to the labor income just

[10] Hence, the retirement decision is not endogenous as in French and Jones (2011). More generally, we do not consider endogenous labor supply decisions as in Bodie et al. (1992) and Gomes et al. (2008).

before retirement.[11] However, in our model, the replacement rate is endogenously determined. The individual relies entirely on annuity payments from the savings accounts. Later, we discuss these accounts in detail.

Asset returns

The gross return on the stock market, R_{t+1}, develops according to the following log-normal process:

$$\ln(R_{t+1}) = \ln(R_f) + \mu + \varepsilon_{t+1}, \tag{25}$$

where R_f is the gross return on a risk-free bond and μ is the equity premium. Recall that the shock, ε_t, is distributed $\mathrm{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$, so $E_t(R_{t+1} - R_f) = \mu$. Also recall that ε_t affects labor income in (24) and the correlation between stock returns and labor income is governed by the parameter θ.

Three accounts for retirement savings

An individual has three financial savings accounts: (i) a notional account belonging to the pension system, (ii) a fully-funded DC account in the pension system, and (iii) a liquid account outside the pension system (which we simply refer to as financial wealth).

The first pillar of the pension system is a notional account. It provides the basis for the pension, is income based and evolves at the rate of the risk-free bond. Specifically, during the working phase, its balance evolves as follows:

$$N_{it+1} = (N_{it} + \lambda^{\mathrm{N}} Y_{it}) R_f, \tag{26}$$

where λ^{N} is the contribution rate for the notional account.

The second pillar of the pension system is a defined-contribution pension account. This account is also income based but the investor can choose how to allocate between bonds and stocks. We denote the return on this asset, which depends on the allocation and the aggregate equity return by R^B_{t+1}. It features mandatory contribution rates $\lambda_{it} \geqslant 0$, and these are the contribution rates we wish to design. Going forward, we will label a pension system for which λ_{it} is a positive constant independent of age and individual characteristics (i.e., $\lambda_{it} = \lambda > 0$) as a rigid pension system.

[11] One exception is that of Cocco and Lopes (2011), who model the preferred DB or DC pension plan for different investors.

Before retirement ($t \leqslant 64$), the law of motion for the DC account balance B_{it} is

$$B_{it+1} = (B_{it} + \lambda_{it} Y_{it}) R^B_{t+1}. \tag{27}$$

Upon retirement at age 65, the DC account and the notional account are converted into two actuarially fair life-long annuities. They insure against longevity risk through within-cohort transfers to survivors from individuals who die. The notional account provides a fixed annuity with a guaranteed minimum. If the balance of the account is lower than required to meet the guaranteed level at age 65, we let the individual receive the remainder at age 65 in the form of a one-time transfer from the government, which is annuitized as well. The annuity from the DC account is variable and depends on the choice of the equity exposure as well as realized returns. In expectation, the individual will receive a constant payment each year. The conversion from account balances to annuity payments are functions denoted by $h^B(.)$ and $h^N(.)$. They take the respective balances as arguments. The law of motion for B_{it} after retirement ($t > 64$) is given by

$$B_{it+1} = (B_{it} - h^B(B_{it})) R^B_{t+1} \tag{28}$$

and similarly for N_{it}.

The third pillar of retirement savings is financial wealth, which is an account outside the pension system that is accessible at any time. Each individual chooses freely how much to save and withdraw from it. In contrast, the contributions to the two pension accounts during the working phase are mandated by the pension policy (rather than by the individual) and are accessible only in the form of annuities during the retirement phase. Importantly, in contrast to the two pension accounts, financial wealth does not include insurance against longevity risk.

The individual starts the first year of the working phase with financial wealth, A_{i25}, outside the pension system. The log of initial financial wealth is distributed $\mathrm{N}(\mu_A - \sigma_A^2/2, \sigma_A^2)$. In each subsequent year, the individual can freely access her financial wealth, make deposits, and choose the fraction to be invested in risk-free bonds and in the stock market. However, the individual cannot borrow:

$$A_{it} \geqslant 0, \tag{29}$$

and the equity share is restricted to be between zero and one,

$$\alpha_{it} \in [0, 1]. \tag{30}$$

Taken together, (29) and (30) imply that individuals cannot borrow at the risk-free rate and that they cannot short the stock market nor take leveraged positions in it.

To enter the stock market outside the pension system, the individual must pay a one-time participation cost, κ_i. (The financial wealth and the decision to invest in the stock market are described later.) A one-time entry cost is common in portfolio-choice models (see, e.g., Alan, 2006; Gomes and Michaelides, 2005, 2008).

The state variable, I_{it}, tracks whether stock market entry has occurred between age 25 and age t; its initial value is zero (i.e., $I_{i25} = 0$). The law of motion for I_{it} is given by

$$I_{it} = \begin{cases} 1 \text{ if } I_{it-1} = 1 \text{ or } \alpha_{it} > 0 \\ 0 \text{ otherwise} \end{cases}, \tag{31}$$

where α_{it} is the fraction of financial wealth invested in the stock market. The cost associated with stock market entry then becomes $\kappa_i(I_{it} - I_{it-1})$.

We allow for different costs for different investors. We assume a uniform distribution of the cost:

$$\kappa_i \sim U(\underline{\kappa}, \bar{\kappa}), \tag{32}$$

where $\underline{\kappa}$ and $\bar{\kappa}$ denote the lowest and highest costs among all investors, respectively. We justify the dispersion in cost with reference to the documented heterogeneity in financial literacy and financial sophistication (see Lusardi and Mitchell, 2014, for an overview). By introducing a cost distribution, we can replicate the fairly flat life-cycle participation profile in the data.[12] On average, low-cost investors will enter early in life, whereas high-cost investors will enter later or never at all. With a sufficiently low value of $\underline{\kappa}$, some low-cost investors will enter immediately. At the end of life, more high-cost than low-cost investors will remain non-participants. For simplicity, we assume that the cost is independent of other characteristics.

Budget constraint

The budget constraint at all stages in life is the same:

$$C_{it} + A_{it} + \kappa_i(I_{it} - I_{it-1}) = X_{it}, \tag{33}$$

[12] Fagereng et al. (2015) present an alternative setup to account for the empirical life-cycle profiles on portfolio choice. Their setup involves a per-period cost and a loss probability on equity investments.

where X_{it} denotes (liquid) cash-in-hand. The law of motion for X_{it} is

$$X_{it+1} = \hat{Y}_{it+1} + A_{it}R^a_{t+1} \tag{34}$$

$$\hat{Y}_{it+1} = \begin{cases} Y_{it+1}\text{exp}(\omega_{it+1}) - \lambda_{it+1}Y_{it+1} & \text{if } t < 64 \\ h^B(B_{it+1}) + h^N(N_{it+1}) & \text{otherwise} \end{cases}, \tag{35}$$

where ω_{it+1} is an idiosyncratic expense shock distributed $\text{N}(-\sigma^2_\omega/2, \sigma^2_\omega)$.[13]

4.2 The individual's problem

Next, we describe the individual's problem. To simplify the notation, we suppress the subscript i. Let $V_t(X_t, B_t, z_t, \kappa, I_t)$ be the value of an individual of age t with cash in hand X_t, DC account balance B_t, a persistent income component z_t, cost κ, and stock market participation experience I_t.

The following describes the individual's problem.

The participant's problem

An individual who has already entered the stock market solves the following problem:

$$\begin{aligned} V_t(X_t, B_t, z_t, \kappa, 1) &= \max_{A_t,\alpha_t}\left\{\left((X_t - A_t)^{1-\rho} + \beta\phi_t\mathcal{R}_t\left(V_{t+1}\left(X_{t+1}, B_{t+1}, z_{t+1}, \kappa, 1\right)\right)^{1-\rho}\right)^{\frac{1}{1-\rho}}\right\} \\ &\quad \text{subject to equations (23)–(28).} \end{aligned}$$

The entrant's problem

Let $V^+_t(X_t, B_t, z_t, \kappa, 0)$ be the value for an individual with no previous stock market participation experience who decides to participate at t. This value can be formulated as

$$\begin{aligned} V^+_t(X_t, B_t, z_t, \kappa, 0) &= \max_{A_t,\alpha_t}\left\{\left((X_t - A_t - \kappa)^{1-\rho} + \beta\phi_t\mathcal{R}_t\left(V_{t+1}\left(X_{t+1}, B_{t+1}, z_{t+1}, \kappa, 1\right)\right)^{1-\rho}\right)^{\frac{1}{1-\rho}}\right\} \\ &\quad \text{subject to equations (23)–(28).} \end{aligned}$$

[13]Notice that, in Equation (35), we do not subtract the notional contribution (λ^N) from income $Y_{i,t+1}$ since $Y_{i,t+1}$ is defined as net of the notional account contribution.

The non-participant's problem

Let $V_t^-(X_t, B_t, z_t, \kappa, 0)$ be the value for an individual with no previous stock market participation experience who decides not to participate at t. This value can be formulated as

$$\begin{aligned} V_t^-(X_t, B_t, z_t, \kappa, 0) &= \max_{A_t}\left\{\left((X_t - A_t)^{1-\rho} + \beta\phi_t\mathcal{R}_t\left(V_{t+1}\left(X_{t+1}, B_{t+1}, z_{t+1}, \kappa, 0\right)\right)^{1-\rho}\right)^{\frac{1}{1-\rho}}\right\} \\ &\quad \text{subject to equations (23)–(28).} \end{aligned}$$

Note that as $\alpha_t = 0$, the return on financial wealth is simply R_f.

Optimal stock market entry

Given the entrant's and non-participant's problems, the optimal stock market entry is given by

$$V_t(X_t, B_t, z_t, \kappa, 0) = \max\left\{V_t^+(X_t, B_t, z_t, \kappa, 0), V_t^-(X_t, B_t, z_t, \kappa, 0)\right\}.$$

A novel feature of our model is the design of the DC account's contribution rates. We discuss this component in detail following the calibration, which is based on the current constant contribution rates policy.

4.3 Calibration

In this section, we describe our calibration strategy. Table 4 reports the values of key parameters. Most parameters are set either according to the existing literature or to match Swedish institutional details; these parameters can be said to be set exogenously. Four parameters are set to match the data as well as possible; these parameters can be said to be determined endogenously.

Exogenous parameters

There are six sets of exogenous parameters.

First, we set the elasticity of intertemporal substitution to 0.5, which is a common value in life-cycle models of portfolio choice (see, e.g., Gomes and Michaelides, 2005).

Second, we set the equity premium to 4% and the standard deviation of the stock market return to 18%. These choices are in the range of commonly used parameter values in the literature. We set the simple risk-free rate to zero, which in other calibrations is often set to 1–2%. We argue that this is correct in our model as labor income does not include economic growth. Thus, we deflate

Table 4: Calibration—model parameters

	Notation	Value
Preferences and stock market entry cost		
Discount factor*	β	0.941
Elasticity of intertemporal substitution	$1/\rho$	0.500
Relative risk aversion*	γ	14
Ceiling for stock market entry cost*	$\overline{\kappa}$	29,250
Floor for stock market entry cost*	$\underline{\kappa}$	0
Returns		
Gross risk-free rate	R_f	1.00
Equity premium	μ	0.04
Standard deviation of stock market return	σ_ε	0.18
Labor income, expense shock, and financial wealth		
Standard deviation of idiosyncratic labor income shock	σ_η	0.072
Weight of stock market shock in labor income	θ	0.040
Standard deviation of idiosyncratic expense shock	σ_ω	0.101
Standard deviation of initial labor income	σ_z	0.366
Standard deviation of initial financial wealth	σ_A	1.392
Mean of initial financial wealth	$\exp(\mu_A - \sigma_A^2/2)$	112,500
Floor for notional pension	$\underline{Y}$	85,829
Contribution rates in pension accounts		
DC account	λ	6.54%
Notional account	λ^{N}	14.95%
Life-cycle profiles		
Labor-income profile (scaled by 1.07)	g_t	**
Survival rates	ϕ_t	***

Note: The table presents the parameter values of the model. * The parameter value has been determined endogenously by simulation of the model. ** The labor-income profiles are discussed in detail in the main text. *** The survival rates are computed from unisex statistics provided by Statistics Sweden.

the account returns by the expected growth to obtain coherent replacement rates. As replacement rates in our model are a function of returns rather than a function of final labor income, this choice is more important to the present model than to previous models. Simulations of the labor income process and contributions to the pension accounts validate our strategy. These simulations indicate that replacement rates at age 65 relative to labor income at age 64 are coherent with Swedish

Pensions Agency forecasts.

Third, we set labor income and the parameters of the pension system according to Swedish data. Appendix C contains a detailed description of the Swedish pension system. The level of the income profile (g_t) is first set to match observed labor income. Then the profile is adjusted further to account for the fact that observed labor income in the data is after deductions of DC plan contributions. Typical contribution rates are 7%—the sum of the premium pension account with a contribution rate of 2.5% and the occupational pension account with a typical contribution rate of 4.5%.[14] We therefore scale up the income profile by a factor of 1.07. Following Carroll and Samwick (1997), we estimate the riskiness of labor income. To abstract from other transfers of the welfare state, progressive taxation, etc., we estimate the risk on disposable income. We find that the standard deviation of permanent labor income (σ_η) equals 0.072 and that the standard deviation of transitory risk is 0.101. We use this value for our expense shock (σ_ω). We set the one-year correlation between permanent income growth and stock market returns to 10%. This corresponds to a θ of 0.040. We approximate the distribution of initial labor income and financial wealth using log-normal distributions. The mean financial wealth for 25-year-old default investors is set to SEK 112,500.[15] The cross-sectional standard deviations are set to 0.366 (σ_z) and 1.392 (σ_A) to match the data for 25-year-old individuals.

Fourth, we match the contribution rates to Sweden. As mentioned before, the total contribution rate to DC accounts is 7% of observed labor income. This corresponds to a contribution rate in the model of 6.54% (0.07/1.07). The contribution rate in Sweden for the notional account is 16% of observed labor income. This corresponds to 14.95% in the model (0.16/1.07).

Fifth, we determine the annuity divisor for the notional account in retirement. We use the unisex mortality table of Statistics Sweden to determine ϕ_t. We assume that the notional account continues to be invested in the risk-free bond and allow for inheritances within a cohort from dying to surviving individuals, incorporating these into the returns of the survivors. We then use the standard annuity formula to reach an annuity factor of 5.6% out of the account balance at age 65. We use the same formulas for the DC account, though we adjust the expected return to the endogenous choice of the DC equity share in retirement.

Finally, we determine the DC equity share profile of the calibration. We use glide path 100-

[14]This corresponds to the ITP1 pension plan for birth cohorts 1979 and younger but abstracting from the increase in contributions above the cap of the notional account, which is intended to offset a cap on contribution rates to the DC and notional accounts.

[15]In 2022, the SEK/USD exchange rate was around 10. During our sample period, it has fluctuated between 6 and 10. We henceforth report numbers in SEK.

Table 5: Matched Moments in Data and Model

	Data	Model
Financial wealth-to-labor income ratio	0.92	0.94
Stock market participation	0.51	0.49
Equity share (conditional)	0.44	0.42

Note: The table presents matched moments in the data and model.

minus-age, which is a very common allocation and similar to the default fund in the premium pension plan.

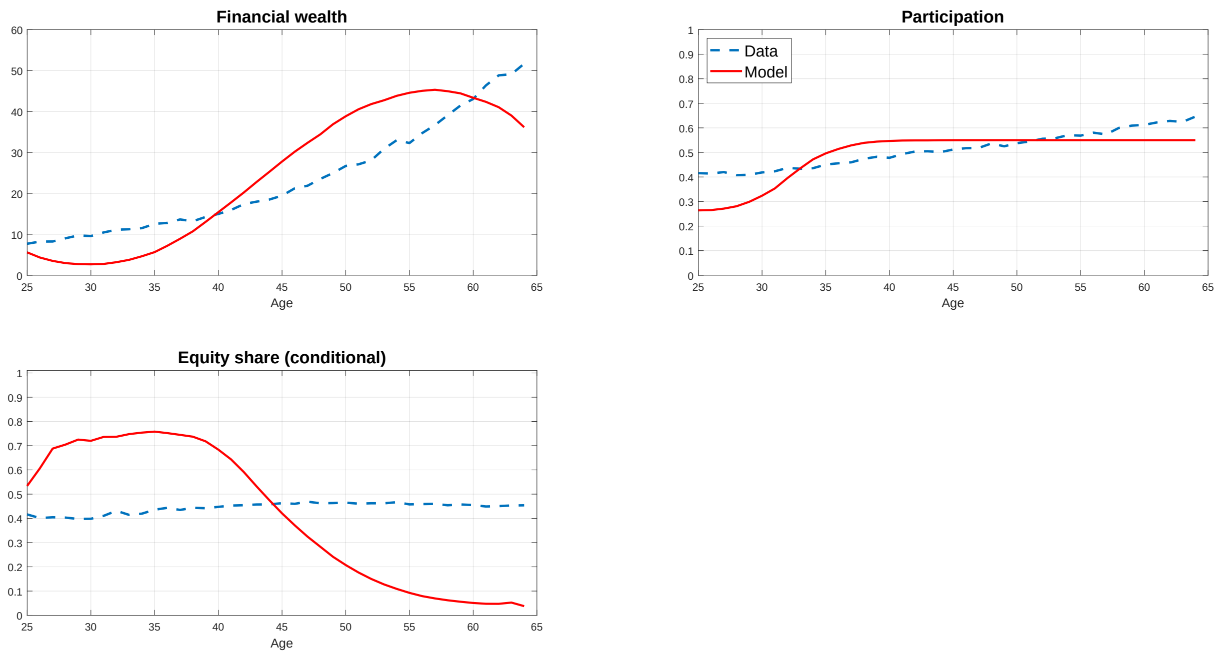
Endogenous parameters and model fit

Four parameters are treated as endogenous in the calibration. We consider data from the working phase.[16] Table 5 shows the close fit between data and model moments. The discount factor (β) is calibrated as 0.941 to match the average ratio of financial wealth to labor income during the working phase (0.94 in the model and 0.92 in data). The top-left panel of Figure 3 shows the full life-cycle profile of financial wealth. The model fits the financial wealth quite well up to age 60 and undershoots after that.

The support of the cross-sectional distribution of participation costs is set so that we match the average stock market participation rate between ages 25 and 64. As can be seen in the top-right panel of Figure 3, participation is almost flat over the life cycle. Intuitively, the parameter $\underline{\kappa}$ affects the participation rate among the young, who are poor in terms of financial wealth and reluctant to enter the stock market if the cost is high. The relatively high participation rate of young individuals therefore leads us to set $\underline{\kappa}$ equal to zero. The parameter $\bar{\kappa}$ is then determined to match the average participation rate from age 25 to 64, which is 0.49 in the model and 0.51 in the data. We obtain this participation rate by setting $\bar{\kappa} = 29,250$. As the distribution is uniform, this corresponds to an average participation cost of SEK 14,625. We find our modeling approach appealing as the uniform distribution of the cost enables the model to replicate the flat participation profile in the

[16]Note that we match the model to data from 2007. This does not allow us to extract cohort or time effects as in, e.g., Ameriks and Zeldes (2004). However, Vestman (2019) finds that cohort and time effects are not strongly present in the data.

Figure 3: Calibration



Note: The figure shows the life-cycle properties of the variables that the calibration targets (targets are their average levels). Financial wealth is expressed in SEK 10,000s.

data.[17]

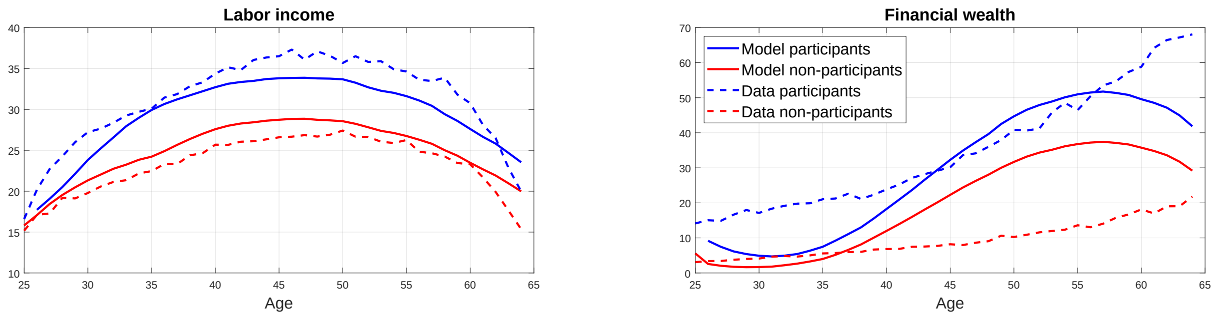
Finally, the relative risk aversion coefficient, γ, determines the conditional equity share. We weigh each age group's equity share equally. A relative risk aversion of 14 provides a good fit. The conditional equity share is 0.42 in the model and 0.44 in the data. The lower-left panel of Figure 3 depicts the life-cycle profile. As is common in life-cycle models such as ours, the model overshoots the data when financial wealth is low and undershoots when liquid financial wealth is high. We are reluctant to increase the relative risk aversion above 14, as this would lead to a worse discrepancy close to retirement age.

Figure 4 shows that the distribution of entry costs produces an endogenous sorting of individuals into stock market participants and non-participants that matches the data well. The left panel shows that the average labor income by participation status is similar in the model and the data. The right panel shows the financial wealth in the model and in the data. The sorting by financial wealth to participants and non-participants is consistent with the data but weaker.[18] Financial

[17] Technically, we approximate the uniform distribution using five equally weighted discrete types (the five costs are equally spaced between zero and SEK 29,250).

[18] It is well known that it is difficult to generate wealth inequality in life-cycle models with incomplete markets. This

Figure 4: Model fit



Note: The figure shows labor income and financial wealth conditional on stock market participation. Financial wealth is expressed in SEK 10,000s.

wealth in the model peaks just before retirement, somewhat earlier than in the data.

Replacement rates in the benchmark pension system

We compute the replacement rate out of the DC account in the baseline and its cross-sectional dispersion, reported in column (1) of Table 6. The mean across individuals is 0.29 with substantial cross-sectional dispersion. The standard deviation is 0.12 and percentiles 95 and 5 are 0.54 and 0.15, respectively. The Swedish Pensions Agency has reported similar dispersion in replacement rates (Pensionsmyndigheten (2021)). This cross-sectional dispersion translates into considerable dispersion in wealth at 65. Panel B of Table 6 considers the thought experiment that the sum of financial wealth and the DC account balance would be annuitized at 65. It would yield a mean replacement rate of 0.94 with a standard deviation of 0.30 and a 95th percentile corresponding to 1.57. These replacement rates can be contrasted to the wealth dispersion if there had been no DC pension plan, which is reported in column (2). In this setting, wealth accumulation is lower, resulting in a mean replacement rate of 0.80. It is, however, noteworthy that the cross-sectional dispersion in replacement rates out of wealth is considerably smaller, with a smaller standard deviation of 0.20 and a 95th percentile of 1.19.

These statistics show that the analytic insights from Proposition 4 carry over to the fully calibrated quantitative life-cycle framework: mandatory constant contribution rates increase the dispersion in replacement rates relative to an environment without a mandatory pension system. This suggests that a constant contribution rate is too rigid compared to principles of optimal

has been addressed by incorporating heterogeneity in discount factors (Krusell and Smith, 1998) or a right-skewed income process (Castaneda et al., 2003).

Table 6: Replacement rates and welfare gains

	(1)	(2)	(3)	(4)	(5)
	Baseline	No DC	Age dependency	B/Y dependency	Both age and B/Y dependency
Panel A: Replacement rate out of the DC account					
Mean	0.29	—	0.29	0.29	0.29
Standard deviation	0.12	—	0.10	0.08	0.07
Percentile 95	0.54	—	0.49	0.45	0.43
Percentile 5	0.15	—	0.17	0.19	0.20
Panel B: Replacement rate out of total wealth					
Mean	0.94	0.80	0.93	0.93	0.92
Standard deviation	0.30	0.20	0.29	0.28	0.27
Percentile 95	1.57	1.19	1.54	1.52	1.50
Percentile 5	0.65	0.60	0.65	0.65	0.65
Panel C: Welfare gain relative to baseline (in percent)					
Mean	—	5.1	1.1	1.2	1.8
Standard deviation	—	0.7	0.2	0.2	0.3
Percentile 95	—	6.0	1.4	1.4	2.0
Percentile 5	—	3.9	0.7	0.7	1.3

Note: Panel A reports moments of replacement rates out of the DC account ($h^B(B_{65}/Y_{64})$. Panel B reports moments of replacement rates out of total wealth ($(h^B(B_{65} + A_{65}) + h^N(N_{65}))/Y_{64}$). Panel C reports moments of ex ante welfare gains associated with a shift from the baseline to each one of the other DC plan designs. The age-dependent policy in column (3) corresponds to $\lambda_{it} = 0.0104 + 0.003t$. The B/Y-dependent policy in column (4) corresponds to $\lambda_{it} = 0.0645 - 0.2\left(\frac{B_{it}}{Y_{it}}/\chi_t - 1\right)$. The policy in column (5) corresponds to $\lambda_{it} = 0.0101 + 0.003t - 0.15\left(\frac{B_{it}}{Y_{it}}/\chi_t - 1\right)$.

consumption-savings theory.

5 Flexible DC contribution rates

We now design a DC pension plan with a flexible contribution rate that follows the principles of consumption-savings theory. We have shown that optimal contribution rates should increase with age and decrease in the balance-to-income ratio. We therefore consider flexible DC contribution rates of the form:

$$\lambda_{it} = \beta_0 + \beta_1 t + \beta_2 \left(\frac{\frac{B_{it}}{Y_{it}}}{\chi_t} - 1 \right), \tag{36}$$

where t indicates the individual's age (minus 24) and $\frac{B_{it}}{Y_{it}}$ indicates the individual's balance-to-income ratio, which is compared to the target balance-to-income ratio χ_t for individuals of age t.[19] Note that we do not consider early withdrawals from the DC account and hence apply a lower bound to contribution rates of 0.

The choice of functional form deserves discussion. First, it nests the current benchmark pension system: if $\beta_1 = \beta_2 = 0$, the contribution rate λ_{it} simplifies to a constant contribution rate β_0 for all investors, irrespective of their age or their balance-to-income ratio. Letting β_1 and/or β_2 be non-zero allows the contribution rate to adjust according to the two principles of optimal consumption-savings behavior. Second, we chose a linear age profile to capture the effect of increasing contribution rates with age. In principle, we could allow for higher order polynomials to improve the fit to the income profile of the population of workers. However, since we already obtain large welfare improvements with the linear rule—and in order to keep the design as simple as possible—we chose to restrict our analysis to this specification. Finally, the adjustment due to the balance-to-income ratio is a linear function of the centered percentage deviations from the target ratio. This functional form has three advantages. First, it gives the parameter β_2 a clear interpretation: It is the semi-elasticity of the contribution rate to the balance-to-income ratio if an investor currently is on target. Second, for $\beta_2 < 0$ (which will be the case of interest), $|\beta_2|$ is the upper bound for how much the contribution rates can theoretically be increased due to the balance-to-income ratio (for investors with $\frac{B_{it}}{Y_{it}} = 0$, i.e., no savings in their DC account). This precludes excessive contribution rates even for extreme shock realizations. Third, the functional form assumption also ensures that the elasticity of disposable income ($Y_{it}^{disp} = (1 - \lambda_{it})Y_{it}$) to gross income is strictly positive for all agents according to the following condition (see appendix D for the proof), which are easily satisfied for all cases of interest:

$$\frac{\partial Y_{it}^{disp}}{\partial Y_{it}} \cdot \frac{Y_{it}}{Y_{it}^{disp}} \geqslant 0 \ \ \forall i, t \qquad \text{iff} \qquad \beta_0 + \beta_1 t - \beta_2 \leqslant 1. \tag{37}$$

Thus, our choice of functional form ensures that the contribution rates are well-behaved across the

[19] The adjustment for an age-specific target replacement rate is necessary in a setup with more than two working-life periods since the average account balance naturally increases with age. In contrast, in the theoretical model of section 2, there was only one period where contribution rates were a function of the balance-to-income ratio. In the theory section, we could thus abstract from this adjustment.

whole state space.

We perform a grid search over the parameters $(\beta_0, \beta_1, \beta_2)$ and compute the maximum welfare gain relative to the baseline calibration.[20] We impose several restrictions in the search. A common restriction in all our searches is that we require the average replacement rate out of the DC account to be maintained at a certain level, for instance 0.29 if we wish to target the DC replacement rate in the baseline setting. Moreover, we only consider parameter combinations that ensure that the average contribution rate does not exceed 15% at any age. The reason is that these DC-account contributions are made in addition to the contributions to the notional account of $\lambda^N = 14.95\%$.

To facilitate comparisons and illustrations of mechanisms, we consider four subsets of contribution rates[21]:

1. Constant contribution rates. We impose $\beta_1 = \beta_2 = 0$ and vary β_0 between the contribution rate of the baseline and the case of the No-DC plan.

2. Age-dependent contribution rates. We impose $\beta_2 = 0$ and then adjust β_0 so that for each value of β_1 we achieve a specific average replacement rate out of the DC account.

3. Balance-to-income (B/Y) dependent contribution rates. We impose $\beta_1 = 0$ and determine the target balance-to-income ratios $\{\chi_t\}_{t=25}^{64}$ so that the average contribution rate is constant over the life cycle.

4. Age and balance-to-income dependent contribution rates. In this case, we impose that the average contribution rate for each age group is equal to $\beta_0 + \beta_1 t$ to facilitate the interpretation of the role of the adjustment due to the balance-to-income ratio.

5.1 Welfare effects and dispersion of replacement rates

Columns (3) to (5) of Table 6 report our findings from grid searches over sets 2, 3, and 4, imposing that the average replacement rate out of the DC account should be equal to the baseline. Panels A and B report the resulting moments for replacement rates out of total wealth and out of the DC account alone, respectively. Panel C shows the associated welfare gains. The welfare gain statistics of a shift to the No-DC setting are reported in column (2). The average ex ante welfare gain of the No-DC setting is substantial at 5.1 percent. Notably, in expectation nobody loses from abolishing

[20]With Epstein-Zin utility, it is straightforward to compute the consumption equivalent. It is proportional to the value function. Our reference to ex ante welfare means that we use the value functions of the 25-year-olds.

[21]See Appendix E for a detailed description of our search algorithm.

the DC plan. This is because our baseline assumes that all individuals are rational and because the insurance value against longevity that the DC plan offers is insufficient to outweigh the rigidity during working life. We will use the gains of moving to a No-DC setting as a yardstick when we evaluate more flexible designs of the contribution rate.

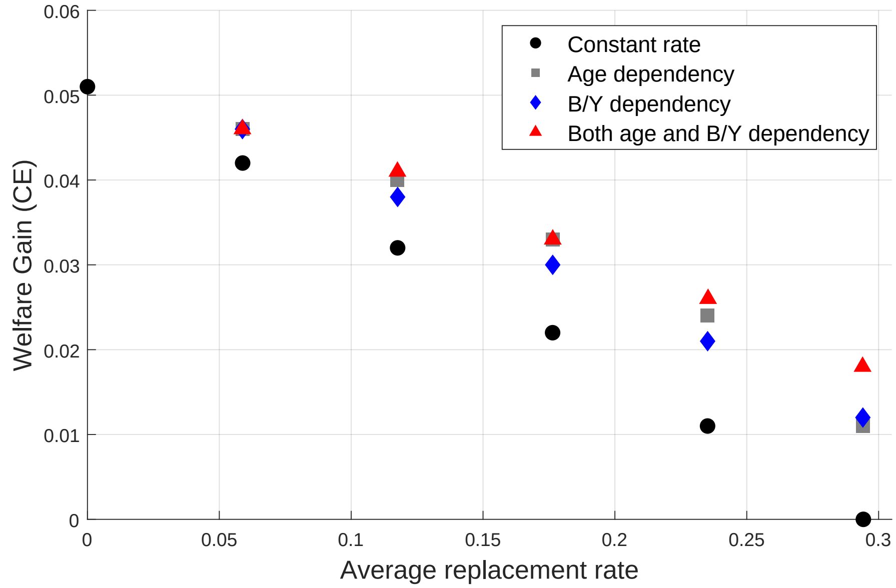
The age-dependent contribution rate that maximizes welfare is $\lambda_{it} = 0.0104 + 0.003\,t$. This formula implies that contribution rates start low (about 1.3% at age 25) and then increase by 0.3 percentage points per year. At 64, the contribution rate peaks at 13% percent. As reported in Panels A and B, this age-dependent contribution rate is able to reduce the cross-sectional dispersion in replacement rates to 0.10 and limits the 95th percentile to 0.49. This results in an average welfare gain of 1.1 percent relative to the baseline. Thus, it bridges 22 percent of the welfare gap between the baseline and the No-DC plan. Intuitively, this is achieved by improving individuals' ability to self-insure early in life when their marginal utility is high as well as reducing the undesired dispersion of the replacement rate out of the DC account.

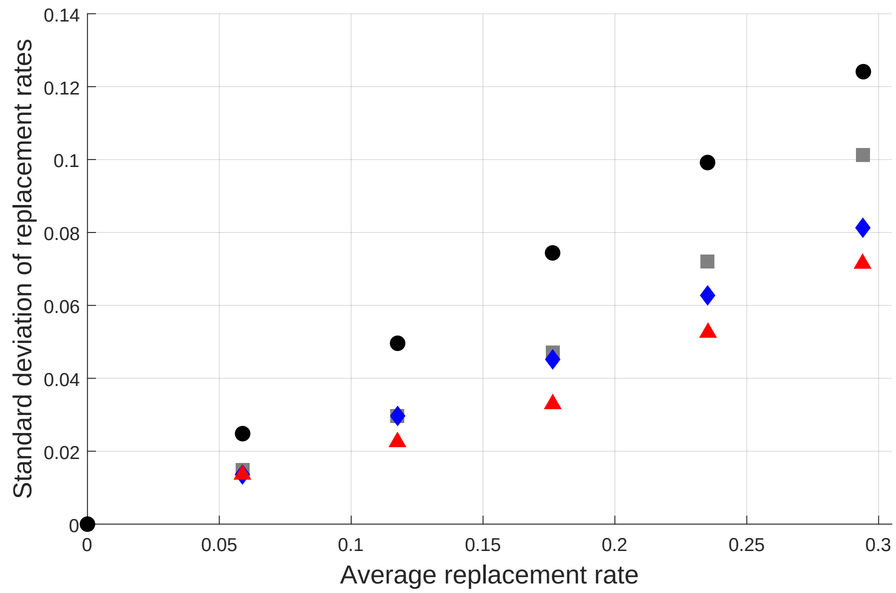
The balance-to-income dependent contribution rate that maximizes welfare is $\lambda_{it} = 0.0645 - 0.2\left(\frac{B_{it}}{Y_{it}}/\chi_t - 1\right)$. This formula implies that an investor who falls short by 1 percent from the balance-to-income target should increase her contribution rate by 0.2 percentage points. A fall in income or an increase in the account balance hence leads to cash-flow benefits. Column (4) of Table 6 reports the associated statistics. According to Panel B, this policy is able to reduce the cross-sectional dispersion in replacement rates even more than the age-dependent policy. The standard deviation is a mere 0.08. Panel C reports welfare gains. Interestingly, at 1.2 percent, this rule for the contribution rate is associated with a slightly higher welfare gain than the best age-dependent contribution rate.

After examining each instrument separately, we now describe our proposal, which combines both instruments. The best combination is $\lambda_{it} = 0.0101 + 0.003\,t - 0.15\left(\frac{B_{it}}{Y_{it}}/\chi_t - 1\right)$. According to that formula, average contribution rates increase by 0.3 percentage points for every year of age and the adjustment based on the balance-to-income ratio implies a semi-elasticity of -0.15, which is slightly weaker than in the pure balance-to-income rule. The results are reported in column (5). Panel B reports a further decline in the cross-sectional distribution, with a standard deviation of only 0.071. Panel C reports the welfare gains. Interestingly, this rule adds an additional welfare benefit, which implies that the two instruments complement each other. The average welfare gain is 1.8 percent. Thus, the best rule for the contribution rate covers 36 percent of the gap between the baseline setting and the No-DC setting.

To gauge the role of flexible contribution rate rules versus target replacement rates, Figure

Figure 5: Replacement rate targets, welfare gains, and dispersion of replacement rates





Note: The figure shows welfare gains (consumption equivalents) and the dispersion of replacement rates out of the DC account against target replacement rates for four types of contribution rate policies: (i) constant contribution rates ($\lambda_{it} = \beta_0$), (ii) only age coefficients ($\lambda_{it} = \beta_0 + \beta_1 t$), (iii) only B/Y coefficients ($\lambda_{it} = \beta_0 + \beta_2(\frac{B_{it}}{Y_{it}}/\chi_t - 1)$), and (iv) age and B/Y coefficients ($\lambda_{it} = \beta_0 + \beta_1 t + \beta_2(\frac{B_{it}}{Y_{it}}/\chi_t - 1)$).

5 reports the outcome of a broader grid search, targeting different DC replacement rates. The frontier of dots represents the designs with successively smaller constant contribution rates. Those correspond to 0.2, 0.4, 0.6 and 0.8 of the actual replacement rate target that we aimed at so far. The top panel depicts welfare gains relative to the baseline economy. As before, the No-DC case serves as a yardstick, corresponding to a replacement rate of zero and a welfare gain of 5.1 percent. The gray squares depict the possible welfare gain from age-only dependencies for the same average replacement rates, and blue diamonds the corresponding the B/Y-only dependencies. The red triangles show the welfare gains achievable with both dependencies for the different replacement rates. One important insight from this figure is how disadvantageous constant contribution rates are relative to our proposals. For all average replacement-rate targets, the three sets of rules (age-only, B/Y-only, and both dependencies) attain a considerable fraction of the gain associated with the No-DC plan. For low average replacement-rate targets, the performance of all three rules is similar. For higher replacement-rate targets, however, the rules that only allow for one of the adjustments are no longer able to achieve the same welfare gains. In contrast, the combined design with both adjustments achieves large welfare benefits even at the high level of replacement rates that are currently implemented in the baseline design.

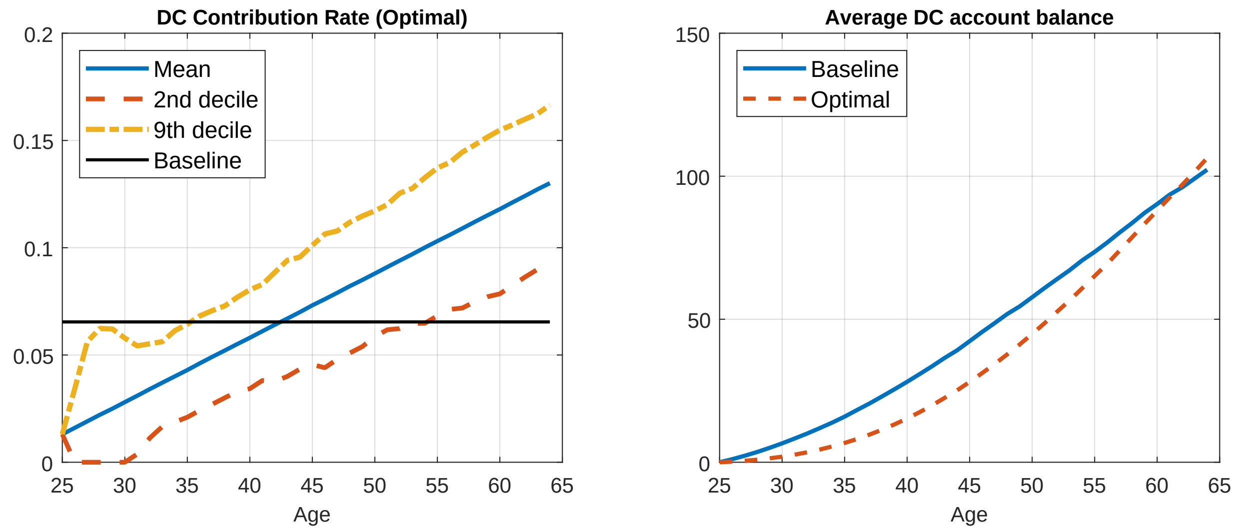
The bottom panel of Figure 5 reports the standard deviation of replacement rates that is associated with the different rules. As the required average replacement rate increases, the associated standard deviation of replacement rates also increases for all types of designs. However, for each target replacement rate, the dispersion can be substantially reduced by redesigning the contribution rates to follow consumption-savings principles. As in the case of the welfare gains, the differential effects across the three rules become particularly pronounced for higher replacement-rate targets: Only the combined design with both dependencies is able to reduce the dispersion of replacement rates consistently by more than 40 percent, even for high replacement-rate targets.

The results from the quantitative model thus confirm that, in a realistically calibrated life-cycle model, the insights from the simple model of section 2 hold: By allowing mandatory contribution rates to follow the principles of optimal consumption-savings theory, it is possible to achieve large welfare gains and at the same time reduce the dispersion of replacement rates without changing the average replacement rate that the pension system provides.

5.2 Impact on investor behavior and mechanisms behind welfare gains

Where do the substantial welfare benefits of the flexible design come from? To shed light on this question, we analyze the effect of our proposed design (the combined rule) on the behavior of

Figure 6: Baseline vs. flexible pension system



Note: The left panel shows the average and the 2nd and 9th deciles of the contribution rate into the DC account (i.e., λ_{it}) for the optimal age- and balance-to-income-dependent design compared to the flat contribution rates of the baseline design. The right panel shows the average DC account balance during working life. Values are expressed in SEK 10,000s.

investors. Figure 6 illustrates the implications of our proposal on the evolution of contribution rates and DC account balances. The left panel details that contribution rates are low early in life. The average contribution is lower than the one in the baseline until age 42 and reaches its highest value at thirteen percent just before retirement. At the same time, heterogeneity in contribution rates is the largest early in working life. This is driven by return shocks: The equity share in the DC account follows a glide path of "100 - age", so for young investors, the DC equity share is the highest. Sequences of extreme return realizations early in working life hence lead to relatively larger swings in the DC account balance and affect the contribution rates relatively more before the balance-to-income adjustment in the contribution rates over time reduces the dispersion again. Despite this larger heterogeneity, even the 9th decile of contribution rates is lower for young investors than the baseline flat contribution rate of 6.54 percent (depicted as the horizontal black line). The right panel shows that this increase in contribution rates over life implies that the DC account balance displays more exponential growth under the proposed rule and on average account balances do not reach the level of the baseline until a few years before retirement.[22]

[22]Figure A.1 in Appendix F shows the plots of the whole distribution of the DC account balance as well as distributions of financial wealth and consumption.

Table 7: Effect of the optimal design on behavior

	Age 30	Age 40	Age 50	Age 60
Panel A: Average Consumption				
Baseline	20.6	26.4	27.7	25.0
Optimal	21.4	26.5	27.1	24.3
Changes (percent)	3.9	0.4	-2.2	-2.8
Panel B: Average Financial Wealth				
Baseline	2.7	15.4	38.8	43.4
Optimal	2.9	16.7	38.2	37.2
Changes (percent)	7.4	8.4	-1.5	-14.3
Panel C: Average Stock Market Participation				
Baseline	0.32	0.55	0.55	0.55
Optimal	0.29	0.39	0.39	0.39
Changes (percent)	-9.4	-29.1	-29.1	-29.1

Note: The table reports average consumption, financial wealth, and stock-market participation at different points in the life cycle. It compares the levels in the baseline calibration and under the optimal design of contribution rates (in SEK 10,000s) and computes the change from baseline to optimal in percent.

The impact of our proposed design on the behavior of investors is detailed in Table 7. It shows for different points in working life the average consumption, financial wealth, and stock market participation in the baseline economy under the proposed flexible design and changes of the flexible design relative to the baseline (in percent). Due to the lower contribution rates when young, investors are able to consume more in the first half of their working life (see panel A). At age 30, the average consumption is higher by 3.8 percent due to the flexible design. Since overall consumption levels are lower at this point in the life cycle, marginal utility is higher than later in life, where consumption levels are typically higher. This shift of consumption towards younger investors with higher marginal utility is one of the main driving forces of the welfare gains of our proposed design.

Financial wealth (panel B) is somewhat higher early in life (by on average SEK 2,000, or 7.4 percent) and lower at age 60 (by on average SEK 62,000, or 14.3 percent). This is the result of two opposing forces.[23] On the one hand, investors have more resources available for saving early

[23]Tables A.5 to A.7 in Appendix F show the corresponding results to Tables 7 and 8 for the contribution-rate designs

Table 8: Changes in disposable income—standard deviation and correlation with shocks

	Standard Deviation	Correlation with Changes in Gross Income	Correlation with Stock Market Returns
Baseline	21.24	1.00	-0.0014
Optimal	19.26	0.95	-0.0074

Note: The table reports statistics of changes in disposable income: average lifetime standard deviation, correlation with changes in gross income, correlation with changes in stock market returns.

in life since their contribution rates are lower. This increases savings. On the other hand, they face lower period-to-period risk since their disposable income is more stable than under the baseline. Table 8 shows that the standard deviation of changes in disposable income is substantially lower under the flexible design than in the system where contribution rates are flat. The reason is that the balance-to-income adjustment in the contribution rate implies that shocks to gross income are partially smoothed by corresponding changes in the contribution rate: All else equal, when income drops the balance-to-income increases, so that contribution rates fall. The correlation of changes in disposable income with changes in gross income is thus only 0.95 while it is 1.00 in the baseline. This reduced variation in disposable income implies that the investor has less need for precautionary savings, and as a result the optimal level of financial wealth decreases. This reduction in disposable-income risk is another driver of the welfare benefits from the flexible design.[24]

In terms of stock market participation, Panel C of Table 7 shows that the average participation rate is lower under the flexible design for all age groups by 3–16 percentage points (or 9–29 percent) depending on age. This again is the outcome of two opposing mechanisms. On the one hand, financial wealth is higher early in life. All else equal, this increases the incentives to enter the stock market. On the other hand, however, Table 8 shows that the correlation of changes in disposable income with stock returns is higher (in absolute terms). In the baseline design with constant contribution rates, disposable income is correlated with stock returns only through the correlation of gross income with aggregate shocks. In contrast, under the flexible design, stock

that are only age-dependent or only B/Y-dependent, respectively. The results for these designs depict the mechanisms described below.

[24] Since income and equity shocks are only partially correlated, disposable income is also affected by equity shocks, which in turn affect the balance-to-income ratio. Quantitatively, however, as we see from Table 8, the total variation in disposable income is lower with the optimal rule.

returns directly affect contribution rates through the balance-to-income adjustment. This increased correlation of disposable income with returns, together with lower financial wealth later in life, implies that investors optimally participate less in the stock market.

5.3 Time-inconsistent preferences

So far, we have assumed that all investors are rational. However, going back to Feldstein (1985), a long history of literature justifies the existence of mandatory pension or social security systems with the consideration that investors might have time-inconsistent preferences. In this section, we show that the welfare benefits of our proposed contribution rates are robust even if investors are time inconsistent.

We model time inconsistency in the form of myopia, i.e., a limited planning horizon of investors. In particular, the objective function of an investor at age t (equations (21)-(22)) is replaced by

$$U_t = \sum_{s=0}^{H-1} \beta^s \phi_{t+s} \frac{C_{t+s}^{1-\gamma}}{1-\gamma}, \tag{38}$$

where H is the planning horizon of the investor. Two characteristics of this specification are noteworthy. First, the preference specification in (38) is time inconsistent since investors at age t only consider their consumption up to age $t+H-1$. However, once they get older, their window of consideration moves to include further periods; investors of age $t+1$, for example, plan including consumption up to age $t+H$, something they did not foresee when they were younger. Second, we simplified the preferences to be von Neumann-Morgenstern expected utility with a constant relative risk aversion (CRRA) felicity function. In the limit, if the horizon $H-1$ is equal to the maximum life-span of the investor T, this model thus represents the results for a rational investor with CRRA preferences. This simplification allows us to abstract from the effects that shorter planning horizons have on the preference for early resolution of uncertainty that is inherent in the Epstein-Zin specification of the main calibration.

We repeat the policy exercise for CRRA as we did for Epstein-Zin preferences and choose the best combined design of age and balance-to-income ratio. As it turns out, the best policy retains the same parameters as before. That the welfare benefits of our proposed design turn out to be robust to the change from Epstein-Zin to CRRA preferences alone is a robustness check in itself. Columns 1 and 2 in Table 9 show the average welfare effects of abolishing the mandatory DC

Table 9: Welfare gains—rational CRRA and myopic agents

	Rational CRRA		Myopic	
	No DC	Both age and B/Y dependency	No DC	Both age and B/Y dependency
Mean	4.6	3.0	-8.9	1.0
Standard deviation	2.1	1.2	7.5	2.2
Percentile 95	6.8	4.7	2.8	3.9
Median	5.2	3.2	-8.6	1.1
Percentile 5	0.4	0.7	-21.7	-2.5

Note: The table reports statistics of the welfare gains (in percent) of having no mandatory DC account “No DC and having our optimal flexible design. Statistics for myopic investors assume a planning horizon of fourteen years. Note that, for both CRRA and myopic preferences, the model has been recalibrated to match data moments (preference parameters β and $\bar{\kappa}$ vary compared to baseline calibration).

account or moving to our proposed flexible design, relative to the baseline constant contribution rate. Compared to Epstein-Zin, the average gain without a DC account is slightly lower (4.6 percent compared to 5.1 percent). Our proposed design achieves almost two thirds of this gain (welfare increase of on average 3 percent) by allowing investors to accumulate pension savings according to consumption-savings theory. The higher gain (both in absolute terms and relative to the no DC design) of CRRA compared to Epstein-Zin preferences is expected since our CRRA specification has a lower elasticity of intertemporal substitution; hence, the smoothing property inherent in our design is even more beneficial.

We use the model with myopic preferences as an alternative setup to the rational-agents model and proceed in the following way. First, for each planning horizon H, we recalibrate the other preference parameters (discount factor β and the ceiling for stock market entry costs $\bar{\kappa}$) to match the target wealth-to-income ratio and the average participation rate in the data.[25] Second, we identify the degree of myopia such that, within constant contribution rate policies, the existing contribution rate level is optimal. For $H = 14$, this is approximately true.[26] Thus, we now have a model that justifies the current contribution rate as a policy instrument to contrive myopia. Third, we repeat the main exercise of finding the optimal policy rule using the same policy state space that was used

[25]We assume that risk aversion γ is equal to 5, independent of the planning horizon.

[26]The optimal flat contribution rate in the economy with $H = 14$ is slightly lower than 6.54%. However, for $H = 13$, the optimal level of constant contribution rates exceeds 6.54%, so we choose to focus on $H = 14$.

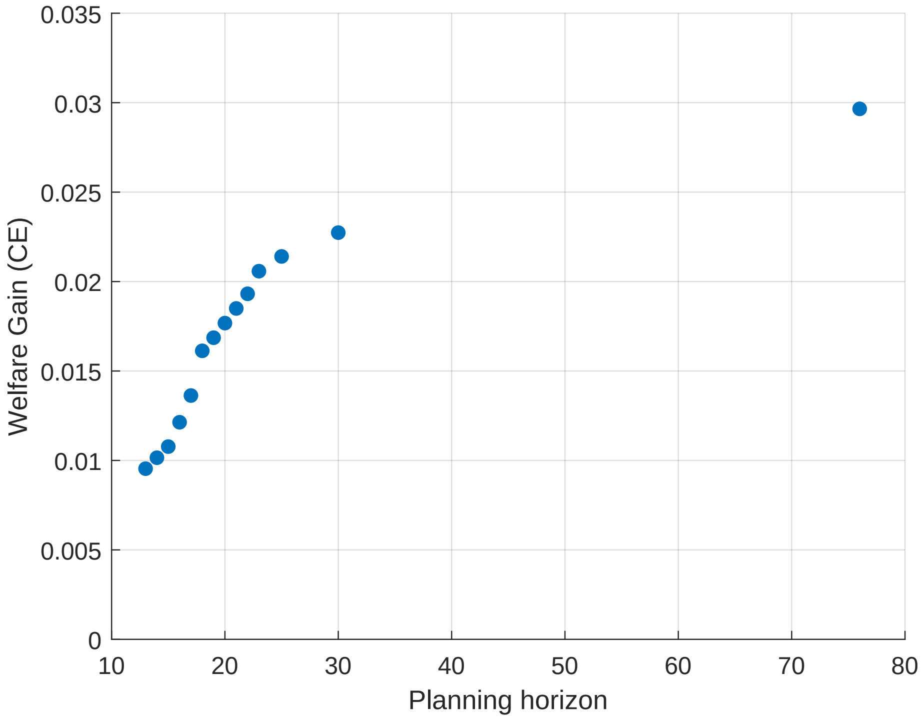
for the rational-agents model. We use ex-post experienced welfare gain as the welfare criterion since ex-ante myopic investors will trivially think that the "No DC setting is unambiguously better since it increases their resources in the horizon that they initially plan for. Finally, we calculate the average welfare gain (as consumption equivalent) of living in an economy with the flexible design as opposed to the baseline pension plan.

Columns 3 and 4 of Table 9 show statistics of the distribution of welfare gains when having no DC account and the DC account with the optimal flexible design, respectively, for the model with myopic investors. As expected, since the baseline constant contribution rate of 6.54% is close to optimal under this degree of time inconsistency, abolishing the mandatory DC account leads to substantial welfare losses (about 9 percent of lifetime consumption on average). The optimal flexible design that we find is exactly the same as in the rational-agents model, with an age coefficient $\beta_2 = 0.003$ and a balance-to-income coefficient $\beta_2 = -0.15$. As the table indicates, changing from a constant contribution rate to the optimal flexible contribution rate (while as usual maintaining the average replacement rate) leads to an average welfare benefit of 1 percent. Moreover, while not every single investor benefits ex post (across all income, expenditure and return shock realizations), the majority of investors have a welfare benefit of at least 1.1 percent.

We thus see that the welfare benefits of our proposed design are robust to the considered time inconsistency despite having been derived under the assumption of rational behavior. Our proposed design ensures the same average replacement rate as the current system, so it does not reduce the mandated amount of pension savings that investors accumulate until retirement. It merely mandates that investors accumulate those savings in line with the principles of consumption-savings theory. While time-inconsistent investors undersave in their (non-mandated) financial assets when they are young, this part of their savings is relatively small compared to the sizable amount of mandated savings. They thus still benefit from the optimal design of mandated savings, which gives them more disposable income when they are young and when they are hit by an adverse income shock.

We further explore the welfare effects to time inconsistency by using other planning horizons as candidates that justify the mandatory retirement policy. We do this since it is not clear that the retirement policy is designed exactly to rectify myopic behavior. In particular, we consider an array of planning horizons that cover a range from strong myopia to rational agents. The shortest horizon we consider is $H = 13$, since for very strong myopia with planning horizons up to 12 years it is not possible to find a discount factor that would allow the model to match the data moments. On the other extreme, we consider $H = 76$, which is the complete horizon for a 25-year-old, hence

Figure 7: Planning horizon and welfare gains



Note: The figure shows the welfare gain (expressed as consumption equivalent) of the optimal design relative to the baseline design for different degrees of myopia. Note that, for each planning horizon, the model has been recalibrated to match data moments (hence, preference parameters β and $\bar{\kappa}$ vary across different horizons).

representing rational preferences. Figure 7 plots the resulting average welfare gain against the planning horizon for the optimal flexible rule for various planning horizons, including that of the rational-agents model.[27] For all planning horizons, our proposed optimal design leads to sizable average welfare gains between 1–3 percent of lifetime consumption. Therefore, even if the current retirement policy is not designed to exactly counterbalance myopia, the welfare gains would be at the same order of magnitude as those we report for $H = 14$. More specifically, there could be reasons other than myopia that also justify the current mandatory policy, such as political economy concerns about bailing out poor retirees. In such cases, myopia would be less severe than in our main myopia exercise (i.e., planning horizons larger than 14) and the welfare gains associated with the optimal policy would be greater.

[27]We solve the model only up to a planning horizon of 30, since the computational burden increases substantially with this horizon (the planning horizon multiplies the existing state space).

6 Concluding remarks

In this paper, we have shown that simple changes to existing DC pension plans—reflecting the principles of optimal consumption-savings theory—could substantially increase welfare while maintaining the same average replacement rate. Specifically, we have demonstrated that optimal contribution rates should be increasing with the age of the investor and decreasing with the balance-to-income ratio. These principles are derived from consumption-savings theory, and we have demonstrated empirically that investor behavior outside of the pension system is consistent with them. Using a quantitative life-cycle model with a detailed pension system, we have shown that introducing contribution rates that incorporate these principles leads to substantial welfare gains without changing the average replacement rate.

Moreover, our proposed rule reduces the dispersion of the replacement rates of retirement income with respect to labor income. We stress that this reduction stems from avoiding both very low and very high replacement rates: Individuals who fall behind the target DC account balance given their income are automatically mandated to save more to avoid insufficient resources during retirement to maintain their consumption level. This avoids very low replacement rates. On the other hand, individuals who have already accumulated more than what the target balance-to-income requires will contribute less. This allows them to smooth their consumption between remaining working life and retirement and avoids excessively high replacement rates.

We have also shown that the optimality of our proposed rule is robust to a model where investors have time-inconsistent preferences. Such preferences are often used as justification for the existence of mandatory pension systems. For this alternative model, we find that the optimal design is the same as the one for the rational-agents model and the mean welfare gain is of the same order of magnitude. Taking all our analyses together, we thus conclude that pension plan designers should take consumption-savings theory into account when setting contribution rates.

Our simple proposed rule relies only on two statistics that are readily available to the pension fund manager, namely age and the account balance. In principle, additional statistics that predict income trends could be used to refine this rule. Such statistics could be, e.g., industry, occupation, or education. Since DC pension plans in practice are often organized separately by occupation or industry, a refinement along those dimensions would be a natural extension. In that case, the welfare benefits and reduction in the dispersion of replacement rates in this paper can be seen as a lower bound for the potential benefits of aligning contribution rates with the principles of consumption-savings theory.

Finally, our analysis focused on the design of contribution rates in a mandatory DC pension plan. Nevertheless, the insights of our analysis are more widely applicable. For example, as part of the Fintech evolution, the importance of software-based, algorithmic financial advice ("robo advising) is steadily increasing. The simple rules for contribution rates that we derived in this paper could guide the advice of such services.

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Appendix

Designing Pension Plans According to Consumption-Savings Theory

Kathrin Schlafmann, Ofer Setty and Roine Vestman

A Proofs of propositions in section 2

To solve the model we reformulate the optimization problem recursively. In period $t = 3$ the agent does not make any decisions and instead simply consumes all available resources:

$$C_{i,3} = A_{i,2}R_2. \tag{39}$$

We can thus define the value function in period $t = 3$, V_3, as

$$V_3(A_{i,2}R_2) = \log(A_{i,2}R_2). \tag{40}$$

Middle-aged agents in period $t = 2$ anticipate this and solve the following optimization problem:

$$V_2(A_{i,1}R_1, Y_{i,2}) = \max_{C_{i,2},A_{i,2}} \log(C_{i,2}) + \beta V_3(A_{i,2}R_2) \tag{41}$$

$$\text{s.t.} \qquad C_{i,2} = A_{i,1} \cdot R_1 + Y_{i,2} - A_{i,2}, \tag{42}$$

where V_2 denotes the value function in period $t = 2$. Finally, young agents anticipate the optimal behavior later in life and solve the following maximization problem:

$$V_1(Y_{i,1}) = \max_{C_{i,1},A_{i,1}} \log(C_{i,1}) + \beta V_2(A_{i,1}R_1, Y_{i,2}) \tag{43}$$

$$\text{s.t.} \qquad C_{i,1} = Y_{i,1} - A_{i,1}. \tag{44}$$

Proof of proposition 2 To solve the optimization problem of the middle-aged agent ($t = 2$) we reformulate the optimization problem in equations (41) and (42) to:

$$V_2(A_{i,1}R_1, Y_{i,2}) = \max_{A_{i,2}} \log(A_{i,1}R_1 + Y_{i,2} - A_{i,2}) + \beta \log(A_{i,2}R_2) \tag{45}$$

This implies the following first-order condition:

$$-\frac{1}{A_{i,1}R_1 + Y_{i,2} - A_{i,2}} + \beta \frac{1}{A_{i,2}} = 0. \tag{46}$$

Solving for the optimal savings $A_{i,2}$ leads to

$$A_{i,2} = \frac{\beta}{1+\beta}(A_{i,1}R_1 + Y_{i,2}). \tag{47}$$

Substituting into the definition of the contribution rate (equation (6)) results in the optimal contribution rate in $t = 2$ and the optimal reactions to shocks to the balance-to-income:

$$\lambda_2^* = \frac{\beta}{1+\beta} - \frac{1}{1+\beta}\frac{A_{i,1}R_1}{Y_{i,2}} \tag{48}$$
$$\frac{\partial \lambda_2^*}{\partial \frac{A_{i,1}R_1}{Y_{i,2}}} = -\frac{1}{1+\beta} < 0.$$

Proof of proposition 1 Substituting equations (42) and (47) and the constraint (44) into the objective function in period $t = 1$ leads to the optimization problem:

$$\max_{A_{i,1}} \quad \log(Y_{i,1} - A_{i,1}) + \beta\left(\log\left(\frac{1}{1+\beta}(A_{i,1}R_1 + Y_{i,2})\right) + \beta^2 \log\left(\frac{\beta}{1+\beta}(A_{i,1}R_1 + Y_{i,2})R_2\right)\right). \tag{49}$$

This implies the following first-order condition:

$$-\frac{1}{Y_{i,1} - A_{i,1}} + \beta\frac{R_1}{A_{i,1}\cdot R_1 + Y_{i,2}} + \beta^2\frac{R_1}{A_{i,1}\cdot R_1 + Y_{i,2}} = 0. \tag{50}$$

Solving for optimal savings in $t = 1$ leads to

$$A_{i,1} = \frac{\beta+\beta^2}{1+\beta+\beta^2}Y_{i,1} - \frac{1}{1+\beta+\beta^2}\frac{Y_{i,2}}{R_1}. \tag{51}$$

Substituting into the definition of the contribution rate (equation (5)) results in the optimal contribution rate and reaction to changes in income growth in $t = 1$:

$$\lambda_1^* = \frac{\beta+\beta^2}{1+\beta+\beta^2} - \frac{1}{1+\beta+\beta^2}\frac{Y_{i,2}}{R_1 Y_{i,1}}$$
$$\frac{\partial \lambda_1^*}{\partial \frac{Y_{i,2}}{Y_{i,1}}} = -\frac{1}{(1+\beta+\beta^2)R_1} < 0.$$

Proof of proposition 3 Substituting the optimal savings in period $t = 1$ (equation (51)) into the optimal contribution rate in period $t = 2$ (equation (8)) leads to the optimal contribution rate in period $t = 2$ as a function of income growth of

$$\lambda_2^* = \frac{1+\beta^2}{1+\beta+\beta^2} - \frac{\beta}{(1+\beta+\beta^2)}\frac{Y_{i,1}R_1}{Y_{i,2}}. \tag{52}$$

Equating the optimal contribution rates in periods $t = 1$ and $t = 2$ we obtain

$$\frac{(\beta+\beta^2)}{(1+\beta+\beta^2)} - \frac{1}{(1+\beta+\beta^2)}\frac{Y_{i,2}}{R_1 Y_{i,1}} = \frac{1+\beta^2}{1+\beta+\beta^2} - \frac{\beta}{(1+\beta+\beta^2)}\frac{Y_{i,1}R_1}{Y_{i,2}}$$
$$\Leftrightarrow \quad 0 = \beta\left(\frac{Y_{i,1}R_1}{Y_{i,2}}\right)^2 - (1-\beta)\frac{Y_{i,1}R_1}{Y_{i,2}} - 1.$$

Solving for $\frac{Y_{i,1}R_1}{Y_{i,2}}$ we obtain

$$\frac{Y_{i,1}R_1}{Y_{i,2}} = \frac{(1-\beta)+\sqrt{(1-\beta)^2+4\beta}}{2\beta}, \tag{53}$$

where the second solution of the quadratic equation was dropped since it is negative. This leads to the following solution for the value of income growth at which the optimal contribution rates are constant throughout working life:

$$\frac{Y_{i,1}}{Y_{i,2}} = \kappa(R_1,\beta) \quad \text{where}$$
$$\kappa(R_1,\beta) = \frac{(1-\beta)+\sqrt{(1-\beta)^2+4\beta}}{2\beta R_1}.$$

Proof of proposition 4 We start by solving for the optimal replacement rate in the absence of a pension system. Inserting the optimal level of savings in period $t = 2$ (equation (47)) into the budget constraints for period $t = 2$ (equation (42)) and period $t = 3$ (equation (39)), respectively, we obtain

$$C_{i,2} = \frac{1}{1+\beta}(A_{i,1}R_1 + Y_{i,2}) \tag{54}$$

and

$$C_{i,3} = \frac{\beta}{1+\beta}(A_{i,1}R_1 + Y_{i,2})R_2. \tag{55}$$

The optimal replacement rate in the absence of a pension system is thus

$$RR_i = \beta R_2.$$

Next, we solve for the thresholds in terms of income growth that separate the regions where either of the constraints (9) and (10) are binding. Since the model is a perfect-foresight model the constraints will not affect the optimal behavior as long as income growth is in between the two thresholds.

The constraint in $t = 2$ (equation (10)) is binding iff $\lambda^*_{i,2} < \underline{\lambda}$. Setting equation (52) smaller than $\underline{\lambda}$ we obtain that agents are constrained iff

$$\frac{Y_{i,2}}{Y_{i,1}} < \kappa_2(\underline{\lambda}, \beta, R_1),$$
$$\text{where} \quad \kappa_2(\underline{\lambda}, \beta, R_1) = \frac{\beta R_1}{1 + \beta^2 - \underline{\lambda}(1 + \beta + \beta^2)}. \tag{56}$$

In this region the agent chooses $\lambda_{i,2} = \underline{\lambda}$, so that $C_{i,2} = (1 - \underline{\lambda})Y_{i,2}$ and $A_{i,2} = A_{i,1}R_1 + \underline{\lambda}Y_{i,2}$. The agent anticipates this in period $t = 1$ and faces the optimization problem

$$\max_{A_{i,1}} \quad \log(Y_{i,1} - A_{i,1}) + \beta \left(\log \left((1 - \lambda_{i,2})Y_{i,2} \right) + \beta^2 \log \left((\lambda_{i,2}Y_{i,2} + A_{i,1}R_1)R_2 \right) \right). \tag{57}$$

We obtain the optimal solution as

$$A_{i,1} = Y_{i,1}\frac{\beta^2}{1 + \beta^2} - Y_{i,2}\frac{\lambda_{i,2}}{R_1(1 + \beta^2)}, \tag{58}$$

which implies:

$$C_{i,3} = \frac{\beta^2}{1 + \beta^2} \left(Y_{i,1}R_1 + \lambda_{i,2}Y_{i,2} \right) R_2. \tag{59}$$

The constrained-optimal replacement rate in the region $\frac{Y_{i,2}}{Y_{i,1}} \leqslant \kappa_2(\underline{\lambda}, \beta, R_1)$ is thus:

$$RR_i = \beta R_2 \cdot \frac{\beta}{1 + \beta^2} \left(\frac{R_1}{1 - \lambda_{i,2}} \frac{Y_{i,1}}{Y_{i,2}} + \frac{\lambda_{i,2}}{(1 - \lambda_{i,2})} \right)$$

The constraint in $t = 1$ (equation (9)) is binding iff $\lambda^*_{i,1} \leqslant \underline{\lambda}$. From (7) we thus obtain that agents are constrained iff

$$\frac{Y_{i,2}}{Y_{i,1}} \geqslant \kappa_1(\underline{\lambda}, \beta, R_1) \tag{60}$$
$$\text{where} \quad \kappa_1(\underline{\lambda}, \beta, R_1) = (\beta + \beta^2)R_1 - \underline{\lambda}R_1(1 + \beta + \beta^2). \tag{61}$$

In that region the agent is constrained in period $t = 1$ and hence chooses $A_{i,1} = \underline{\lambda}Y_{i,1}$. Inserting into (47) we obtain

$$A_{i,2} = \frac{\beta}{1 + \beta} \left(\underline{\lambda}Y_{i,1}R_1 + Y_{i,2} \right), \tag{62}$$
$$C_{i,2} = \frac{1}{1 + \beta} \left(\underline{\lambda}Y_{i,1}R_1 + Y_{i,2} \right), \tag{63}$$
$$C_{i,3} = \frac{\beta}{1 + \beta} R_2 \left(\underline{\lambda}Y_{i,1}R_1 + Y_{i,2} \right). \tag{64}$$

The constrained-optimal replacement rate in the region $\frac{Y_{i,2}}{Y_{i,1}} > \kappa_1(\underline{\lambda}, \beta, R_1)$ is thus

$$RR_i = \beta R_2, \tag{65}$$

the same as in the unconstrained case. We can summarize the solution for the replacement rate as

$$RR_i = \begin{cases} \beta R_2 \cdot \frac{\beta}{1+\beta^2}\left(\frac{R_1}{1-\underline{\lambda}}\frac{Y_{i,1}}{Y_{i,2}} + \frac{\underline{\lambda}}{(1-\underline{\lambda})}\right) & \text{if } \frac{Y_{i,2}}{Y_{i,1}} < \kappa_2(\underline{\lambda}, \beta, R_1) \\ \\ \beta R_2 & \text{if } \kappa_2(\underline{\lambda}, \beta, R_1) \leqslant \frac{Y_{i,2}}{Y_{i,1}} \leqslant \kappa_1(\underline{\lambda}, \beta, R_1) \\ \\ \beta R_2 & \text{if } \frac{Y_{i,2}}{Y_{i,1}} > \kappa_1(\underline{\lambda}, \beta, R_1) \end{cases} \tag{66}$$

Assuming that an economy consists of a continuum of agents who differ in their income profile $(\frac{Y_{i,2}}{Y_{i,1}})$ and in the returns in period $t = 1$ (R_1), this implies that in the unconstrained case all agents choose the same optimal replacement rate $RR_i = \beta R_2$, so the cross-sectional variance of replacement rates is zero. In the constrained-optimal solution, however, all agents whose income growth satisfies $\frac{Y_{i,1}}{Y_{i,2}} < \kappa_2(\underline{\lambda}, \beta, R_1)$ have to choose a different replacement rate. This replacement rate varies with their income growth and returns in period $t = 1$. Constrained-optimal behavior therefore leads to a weakly positive cross-sectional variance of replacement rates. This concludes the proof.

B Details on the empirical analysis

Table A.1: Sampling restrictions

	Type of restriction	Observations	Unique individuals	Age	Disposable income	Net worth	Stock market partic. rate
0.	Full sample	73 202 337	10 152 351	41	114 425	322 563	.58
1.	Excl. year 2000	64 187 119	10 043 652	41	116 966	329 606	.58
2.	Excl. individuals who do not reside in Sweden	63 146 967	9 909 002	40	118 101	331 336	.59
3.	Excl. individuals who are farmers or entrepreneurs	61 541 535	9 813 520	40	117 067	324 751	.58
4.	Excl. individuals who hold financial derivatives in time t or $t-1$	61 345 062	9 810 028	40	116 848	317 472	.58
5.	Excl. individuals who hold stocks or mutuals funds with missing prices or ISIN's in time t or $t-1$	57 633 171	9 651 826	40	112 919	258 860	.56
6.	Excl. individuals who have extreme financial portfolio returns in a given year (top and bottom the 1 percent)	57 179 282	9 646 664	40	112 658	258 696	.56
7.	Excl. individuals who have big changes in net worth (top and bottom 2.5 percent)	55 055 383	9 602 181	39	110 409	222 462	.55
8.	Excl. individuals who own commercial real estate	52 337 385	9 192 787	38	107 979	215 425	.54
9.	Excl. individuals who are aged below 26 or above 64	24 919 674	4 836 354	44	168 884	237 331	.52
10.	Excl. individuals who have gross labor income below the Price Base Amount (plus 5 percent) of time t	19 881 647	4 129 413	44	189 429	260 803	.57
11.	Excl. individuals who have zero or negative disposable income	19 870 094	4 127 867	44	189 610	259 814	.57
12.	Excl. individuals who are not in the sample in t, $t-1$ and $t-2$	17 026 001	3 959 358	44	193 090	272 692	.57
13.	Excl. individuals who have extreme contribution rates relative to their age cohort (top and bottom 1 percent).	16 268 479	3 908 308	44	194 394	258 190	.56

Note: Stock market participation rate in decimals.

Table A.2 reports the first-stage estimate corresponding to Column (5) of Table 1. We instrument income growth with the earnings growth of the individuals' employer. The estimation is based on a subsample of individuals that have the same employer in two consecutive years. The subsample contains 179,929 employers, and the average employee-to-employer ratio per year is: 14.9 (2003), 15 (2004), 14.9 (2005), 15 (2006) and 14.7 (2007).

Table A.2: First-stage IV estimates for income shocks

	(1)
$\Delta w_{it}^{employer}$	0.075***
	(0.001)
Individual FE	Yes
Year FE	Yes
Individual Clusters	Yes
Employer Clusters	Yes
Adj. R^2	0.265
Observations	9,340,192

Note: The table reports the first-stage estimates for Column (5) of Table 1. The variable $\Delta w_{it}^{employer}$ is the aggregate earnings growth of individual i's employer. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$.

Table A.3: First-stage IV estimates of savings rates and asset balance-to-income ratios

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\frac{A_{it-2}}{Y_{it}^{\text{Disp}}} \times R_{it-1}^{A} \times \overline{R_{it}^{A}}$	0.430***	0.385***	0.779***	0.676***	0.661***	0.679***	0.826***
	(0.064)	(0.044)	(0.018)	(0.012)	(0.017)	(0.017)	(0.026)
Controls	No	No	Yes	Yes	Yes	Yes	Yes
Individual FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Age	26-64	26-35	26-64	26-35	36-45	46-55	56-64
F-statistic	45	78	1 115	3 874	985	1 225	335
Observations	15 796 927	4 093 078	10 883 910	2 462 778	3 123 051	2 759 616	2 031 256

Note: The table reports the first-stage estimates corresponding to Table 2. The control variables are Y_{it}^{Disp}, NW_{it-1} and $ND_{it,t-1}$. Standard errors, clustered at the level of the individual and the individual's largest security holding, are in parentheses. The individual's largest security holding is the particular stock or mutual fund—identified by their International Securities Identification Number (ISIN)—with the largest weight in the individual's financial asset portfolio. When the largest holding is bonds, bank accounts or capital insurance accounts, we classify the largest security holding by their respective asset type. Singleton groups are excluded. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$. Dependent variable: $\frac{A_{it-1}}{Y_{it}^{\text{Disp}}} \times R_{it}^{A}$

Table A.4 reports estimates from the following regression on savings amounts:

$$\Delta\widetilde{A}_{it} = \theta_i + \delta_t + \beta_1 A_{it-1} \times R^A_{it} + \beta_2 Y^{\text{Disp}}_{it} + \beta_3 NW_{it-1} + \beta_4 ND_{it,t-1} + \varepsilon_{it}. \tag{67}$$

Table A.4: Response in savings cash-flows to changes in financial wealth

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS		IV: Second-Stage				
$A_{it-1} \times R^A_{it}$	-0.183***	-0.339***	-0.143***	-0.190***	-0.161***	-0.138***	-0.124***
	(0.027)	(0.044)	(0.021)	(0.020)	(0.025)	(0.023)	(0.017)
Controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Individual FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Instr. A_{it-1}	No	No	Yes	Yes	Yes	Yes	Yes
Instr. for R^A_{it}	No	No	Yes	Yes	Yes	Yes	Yes
Age	26-64	26-64	26-64	26-35	36-45	46-55	56-64
Adj. R^2	0.134	0.188	0.102	0.135	0.118	0.105	0.109
Observations	15 796 927	10 883 910	10 883 910	2 462 778	3 123 051	2 759 616	2 031 256

Note: The control variables are Y^{Disp}_{it}, NW_{it-1} and $ND_{it,t-1}$. Standard errors, clustered at the level of the individual and the individual's largest security holding, are in parentheses. The individual's largest security holding is the particular stock or mutual fund—identified by their International Securities Identification Number (ISIN)—with the largest weight in the individual's financial asset portfolio. When the largest holding is bonds, bank accounts or capital insurance accounts, we classify the largest security holding by their respective asset type. Singleton groups are excluded. $* = p < 0.10$, $** = p < 0.05$, $*** = p < 0.01$.

C The Swedish pension system

The Swedish pension system rests on three pillars: public pensions, occupational pensions, and private savings. Below, we describe the public and occupational pensions.

The public pension system was reformed in 2000.[28] It has two major components referred to as the income-based pension and the premium pension. A means-tested benefit provides a minimum guaranteed pension.

The contribution to the income-based pension is 16% of an individual's income, though the income is capped (in 2014, the cap was SEK 426,750, or approximately USD 62,200). The return on the contribution equals the growth rate of aggregate labor income measured by an official "income index". Effectively, the return on the income-based pension is similar to that of a real bond. The income-based pension is notional in that it is not reserved for the individual but is instead used to fund current pension payments as in a traditional pay-as-you-go system. It is worth mentioning that the notional income-based pension is also DC, but to avoid confusion we simply refer to it as the notional pension.

The contribution to the premium pension is 2.5% of an individual's income (capped as above). Unlike the income-based pension, the premium pension is a fully funded DC account used to finance the individual's future pension. Individuals can choose to allocate their contributions to up to five mutual funds from a menu of several hundred. The premium pension makes it possible for individuals to gain equity exposure. Indeed, most of the investments in the system have been in equity funds (see, e.g., Dahlquist et al., 2015). A government agency manages a default fund for individuals who do not make an investment choice. Up to 2010, the default fund invested mainly in stocks but also in bonds and alternatives. In 2010, the default fund became a life-cycle fund. At the time of retirement, the savings in the income-based pension and the premium pension are transformed into actuarially fair life-long annuities.

In addition to public pensions, approximately 90% of the Swedish workforce is entitled to occupational pensions. Agreements between labor unions and employer organizations are broad and inclusive and have gradually been harmonized across educational and occupational groups. For individuals born after 1980, the rules are fairly homogeneous, regardless of education and occupation. The contribution is 4.5% of an individual's income (capped as above) and goes into a designated individual DC account. For the part of the income that exceeds the cap, the contribution rate is greater in order to achieve a high replacement rate even for high-income individuals. While the occupational pension is somewhat more complex and tailored to specific needs, it shares many features with the premium pension. Specifically, it is an individual DC account.

[28] Individuals born between 1938 and 1954 are enrolled in a mix of the old and new pension systems, while individuals born after 1954 are enrolled entirely in the new system.

D Properties of contribution rates

The functional form assumption of the contribution rates in (36) leads to the following semi-elasticities of the contribution rate to changes in income, DC account balance, or balance-to-income ratio:

$$
\begin{aligned}
\frac{\partial \lambda_{it}}{\partial Y_{it}} \cdot Y_{it} &= -b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t}, && (68)\\
\frac{\partial \lambda_{it}}{\partial B_{it}} B_{it} &= b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t}, && (69)\\
\frac{\partial \lambda_{it}}{\partial \frac{B_{it}}{Y_{it}}} \frac{B_{it}}{Y_{it}} &= b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t}. && (70)
\end{aligned}
$$

Thus, for an investor who has a balance-to-income ratio that is right on target, $\frac{B_{it}}{Y_{it}} = \chi_t$, the contribution rate will change by b_2 for a percentage increase in B_{it} or $\frac{B_{it}}{Y_{it}}$ or for a percentage decrease in Y_{it}.

Moreover, the functional form has implications for the elasticity of disposable income ($Y_{it}^{disp} = (1-\lambda_{it})Y_{it}$) to changes in gross income (Y_{it}):

$$
\begin{aligned}
\frac{\partial Y_{it}^{disp}}{\partial Y_{it}} \cdot \frac{Y_{it}}{Y_{it}^{disp}} &= \left(-\frac{\partial \lambda_{it}}{\partial Y_{it}} Y_{it} + (1-\lambda_{it})\right) \cdot \frac{Y_{it}}{Y_{it}^{disp}}\\
&= \left(1 - \lambda_t + b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t}\right) \frac{Y_{it}}{(1-\lambda_{it})Y_{it}}\\
&= 1 + \frac{b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t}}{1-\lambda_{it}}. && (71)
\end{aligned}
$$

Inserting the functional form of the contribution rate (36), the elasticity of disposable income to changes in gross income is positive if and only if

$$
\begin{aligned}
1 + \frac{b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t}}{1-\lambda_{it}} &\geqslant 0\\
-b_2 \frac{\frac{B_{it}}{Y_{it}}}{\chi_t} &\leqslant 1 - b_0 - b_1 t - b_2 \left(\frac{\frac{B_{it}}{Y_{it}}}{\chi_t} - 1\right)\\
b_0 + b_1 t - b_2 &\leqslant 1. && (72)
\end{aligned}
$$

E Algorithm to find optimal policies for the contribution rate

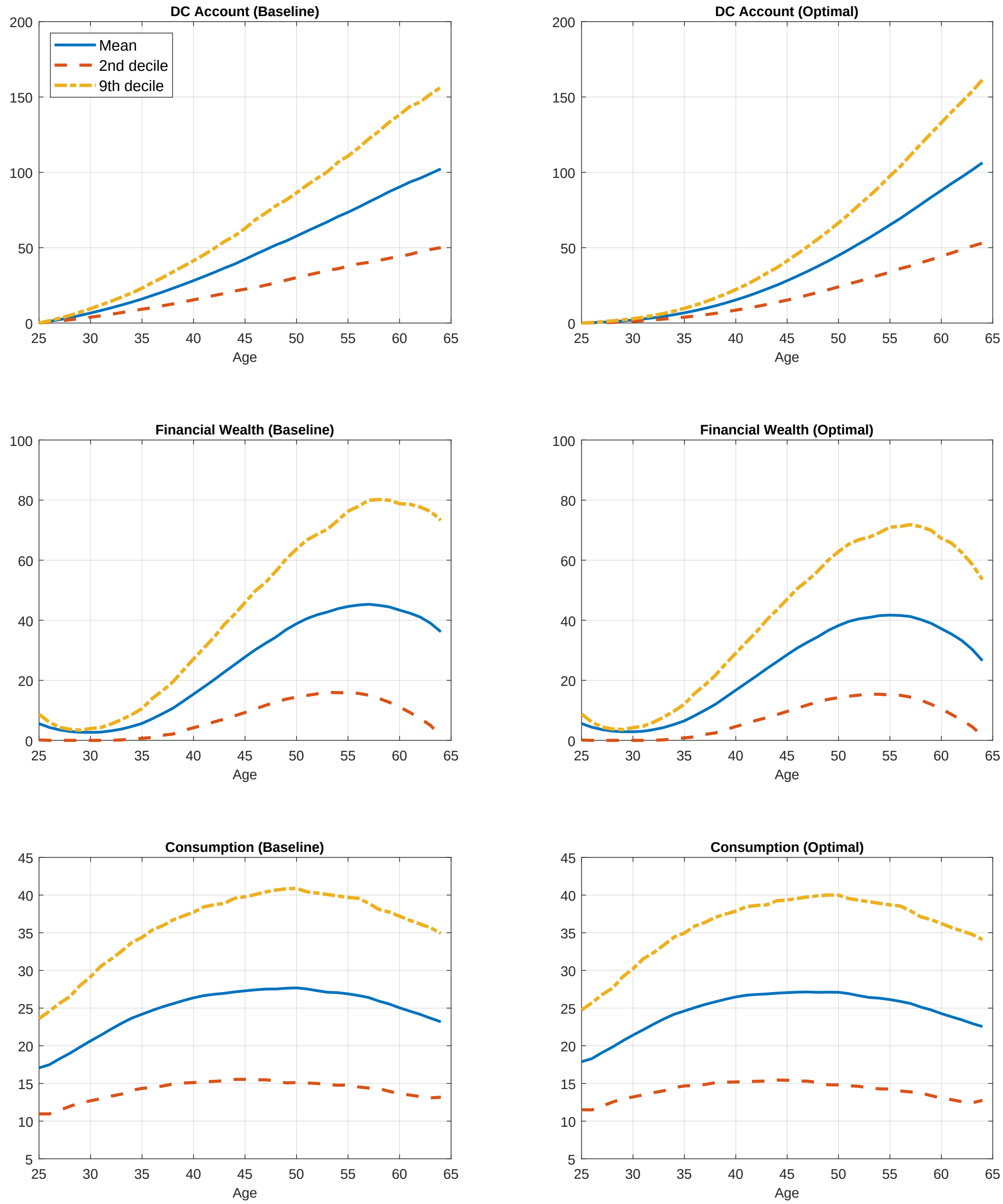
To determine the optimal policy for contribution rates we select the design that delivers the highest welfare gain while achieving the same average replacement rate out of the DC account as the baseline constant contribution rate.

For each replacement-rate target we proceed in four steps:

1. solve for the constant contribution rate that delivers this average replacement rate
2. search for the optimal policy that allows for an age dependency in the policy ($\lambda_{it} = \beta_0 + \beta_1 t$):
   (a) for each candidate coefficient β_1 solve for the required constant β_0 such that the policy achieves exactly the required average replacement rate
   (b) select the design (i.e. candidate coefficient) with the highest welfare gain
3. search for the optimal policy that allows for a dependency on the balance-to-income ratio ($\lambda_{it} = \beta_0 + \beta_2 \left(\frac{\frac{B_{it}}{Y_{it}}}{\chi_t} - 1 \right)$):
   (a) for each candidate for the coefficient β_2, solve for the required constant β_0 and vector of target balance-to-income ratios χ_t such that
      - the design achieves exactly the required average replacement rate and
      - the average contribution rate is constant for all ages
   (b) select the design (i.e. candidate coefficient) with the highest welfare gain
4. search for the optimal policy that allows for dependencies on both age and balance-to-income ratio ($\lambda_{it} = \beta_0 + \beta_1 t + \beta_2 \left(\frac{\frac{B_{it}}{Y_{it}}}{\chi_t} - 1 \right)$):
   (a) for each combination of candidates for the coefficient of β_1 and β_2, solve for the required constant β_0 and vector of target balance-to-income ratios χ_t such that
      - the design achieves exactly the required average replacement rate and
      - the average contribution rate follows exactly the age trend $\beta_0 + \beta_1 t$
   (b) select the design (i.e. combination of coefficients) with the highest welfare gain

F Additional results about flexible contribution rates

Figure A.1: Baseline vs. flexible pension system



Note: The left columns show the distributions for the DC account balance, financial wealth and consumption (the average and the 2nd and 9th deciles) for the baseline calibrations. The right columns show the corresponding distributions for the optimal flexible design. Values are expressed in SEK 10,000s.

Table A.5: Effect of the age-only design on behavior

	Age 30	Age 40	Age 50	Age 60
Panel A: Average Consumption				
Baseline	20.6	26.4	27.7	25.0
Optimal	21.3	26.2	27.4	24.7
Changes (percent)	3.3	-0.6	-1.1	-1.4
Panel B: Average Financial Wealth				
Baseline	2.7	15.4	38.8	43.4
Optimal	3.2	20.9	45.9	43.3
Changes (percent)	19.3	35.9	18.3	-0.0
Panel C: Average Stock Market Participation				
Baseline	0.32	0.55	0.55	0.55
Optimal	0.42	0.72	0.72	0.72
Changes (percent)	30.5	31.4	31.0	31.0

Note: The table reports average consumption, financial wealth, and participation at different points in the life cycle. It compares the levels in the baseline calibration and under the age-dependent design of contribution rates (in SEK 10,000s) and computes the change from baseline to age-dependent in percent.

Table A.6: Effect of the B/Y-only design on behavior

	Age 30	Age 40	Age 50	Age 60
Panel A: Average Consumption				
Baseline	20.6	26.4	27.7	25.0
Optimal	20.7	26.7	27.5	24.7
Changes (percent)	0.3	1.3	-0.7	-1.2
Panel B: Average Financial Wealth				
Baseline	2.7	15.4	38.8	43.4
Optimal	2.3	11.2	31.1	36.3
Changes (percent)	-14.1	-27.5	-19.9	-16.4
Panel C: Average Stock Market Participation				
Baseline	0.32	0.55	0.55	0.55
Optimal	0.23	0.25	0.25	0.25
Changes (percent)	-30.4	-54.5	-54.8	-54.8

Note: The table reports average consumption, financial wealth, and participation at different points in the life cycle. It compares the levels in the baseline calibration and under the B/Y-dependent design of contribution rates (in SEK 10,000s) and computes the change from baseline to B/Y-dependent in percent.

Table A.7: Changes in disposable income—standard deviation and correlation with shocks

	Standard Deviation	Correlation with Changes in Gross Income	Correlation with Stock Market Returns
Baseline	21.24	1.00	-0.0014
Age only	20.98	1.00	-0.0013
BY only	19.06	0.94	-0.0059
Optimal	19.26	0.95	-0.0074

Note: The table reports statistics of changes in disposable income: average lifetime standard deviation, correlation with changes in gross income, and correlation with changes in stock market returns.