The EU's CAM and the Relationship between Output Gaps, Cyclical Indicators, and Inflation

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Abstract:

There has been considerable controversy about the level and sign of the output gap in the period after the financial and Euro crisis. Output gap estimates from the EU's commonly agreed methodology (CAM), as well as from other international institutions, indicated a closing of the output gap until 2018/19. The question was raised whether these estimates can be consistent with the persistently low inflation and the EA's trade balance surplus. We shed light on this question through the lens of (i) statistical dimension reduction methods (PCA) and (ii) a theory-grounded estimated DSGE model. The former shows that an extracted component based on several indicators follows the output gap estimates of the EU's CAM closely. It also supports the view that inflation and external balance indicators do not perform well as indicators of the cycle. A non-linear DSGE model further corroborates this evidence and shows that a decoupling of the output gap and inflation is consistent with estimated persistent adverse demand shocks and wage rigidity.

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The views expressed in this paper are those of the authors and should not be attributed to the European Commission.

Introduction

Based on output gap estimates over the period from 2014 until the outbreak of COVID-19 in 2020, the EU's commonly agreed methodology (CAM), as well as from other international institutions, the euro area's (EA) economy was seen as operating at about its long-term sustainable growth rate (Malin et al., 2018). The EA unemployment rate was also near historic lows, and predictions pointed to a rate well below 8% in 2019. In the past, such conditions have generally been associated with mounting inflationary pressures. Yet, the EA's core inflation[1] was stuck at roughly 1%. This phenomenon that inflation stayed low after 2014 despite an ongoing recovery was labelled the "missing inflation puzzle" (Constânzio, 2015).

However, there was also the alternative view that the lack of inflation was simply showing that output gap estimates must have a problem. For example, Brooks (2019) and others[2] interpreted the historically low (wage and price) inflation and the significant current account surplus as a sign that the EA had not yet reached its potential and that its output gap was still negative. Jarozinsci and Lenza (2016) provided estimates of a negative EA output gap, which was (entirely) based on information about inflation. However, output gap measures based on other business cycle indicators, such as sentiment and capacity utilisation indicators, showed either a positive output gap or a gap that was closing (De Waziers, 2018). In a report, the German Council of Economic Advisors (SVR, 2019) compared output gap estimates of international organisations (based on a production function (PF) methodology) with output gap estimates using an HP-Filter, a Hamilton Filter and a factor model. They found higher positive output gaps (and an earlier closing of the output gap) in 2018.

In this paper, we look at this episode from two angles. In a first exploratory step, we employ a principal component analysis (PCA). This statistical dimension-reduction technique documents co-movements between standard cyclical indicators and inflation as well as CA measures. It brings out the cyclical signal provided by these indicators. As an additional advantage, the PCA identifies other dimensions, such as the existence of medium-term cycles. In particular, the PCA identifies two distinct components, namely one reflecting a business cycle and a second component capturing a medium-term cycle or a trend. All indicators contribute to the first (cyclical) component, while both inflation and the CA also contribute strongly to the second component. The first (cyclical) component identified by the PCA analysis exhibits strikingly similar dynamics to the CAM-based output gap. Given the substantial contribution of inflation and the CA to the second (medium-run) component, this result suggests that cyclical measures, such as the CAM output gap, are only partly correlated with inflation and the CA.

Our second approach employs an estimated DSGE model to assess the structural relationship between the PF output gap and inflation as well as the CA in a theory-grounded manner. For this purpose, we integrate the PF output gap into the Commission's Global Multicountry (GM) model (see Albonico et al. 2019). This extends standard macro models, which do not embed the PF approach for calculating output gaps. A notable exception is Kiley (2013). The model estimation links the (PF) output gap to inflation and the CA in a coherent general equilibrium model. Based on the impulse responses, we study the relationship between the output gap and inflation and we find decoupling between the output gap and inflation as a response to demand shocks, in the sense that the output gap closes before inflation reaches the inflation target. This result holds both under constrained and unconstrained monetary policy. We identify two characteristics in the labour market, and in particular in the wage setting rule, which generates decoupling, namely a hybrid wage Phillips curve and real wage rigidity. The estimated model finally allows us to assess the empirical importance of decoupling as generated by demand shocks, vs

[1] Core inflation is measured using the PCE (personal consumption expenditures) deflator, excluding food and energy.

[2] For example, Jarocinski and Lenza (2016) also stress the importance of inflation as an indicator of the cycle, suggesting that output in the euro area was 6% lower than potential in 2014 and 2015, with this 6% output gap estimate being substantially below comparable institutional estimates.

decoupling generated by other shocks, by looking at shock decompositions for the output gap and inflation.

The rest of the paper proceeds as follows. Section 1 explores the relationship between the output gap and inflation in a small NK model. Section 2 presents results from the PCA, while Section 3 introduces more details on the CAM. Section 4 presents the results of the estimated DSGE model. Section 5 concludes.

1. Demand Shocks, Output Gap and Inflation in a small New Keynesian Model

In order to set the stage for the following discussion, we first discuss the range of possible relationships between the output gap and inflation in a small model. We use the simplest New Keynesian workhorse model (Clarida et al. (1999)) and slightly modify wage setting by allowing for real wage rigidity (see Blanchard Gali (2007)).

Aggregate demand:

$$y_{t+1} - y_t = r_t - \rho - s_t$$

Production:

$$y_t = a + l_t$$

Price setting/labour demand (hybrid Phillips curve $s^f < 1$):

$$\pi_t = s^f \beta \pi_{t,t+1} + \left(1 - s^f\right)\pi_{t-1}\beta + w_t^r$$

Wage setting (real wage rigidity: $\chi > 0$):

$$w_t^r = (1-\chi)(y_t + \kappa)l_t + \chi w_{t-1}^r$$

Taylor rule:

$$i_t = Max(0, \rho + \bar{\pi} + \tau(\pi_t - \bar{\pi})$$

Demand shock :

$$s_t = \rho^s s_{t-1} + e_t > 0 \quad \text{e=0}$$

Since under flexible prices, the real wage is not affected by demand shocks, the employment level in the flex price economy is a constant:

$$w_t^{r,flex} = a = 0$$

This implies constant employment in the flexible economy. Thus, the dynamic response of employment is identical to the output gap.

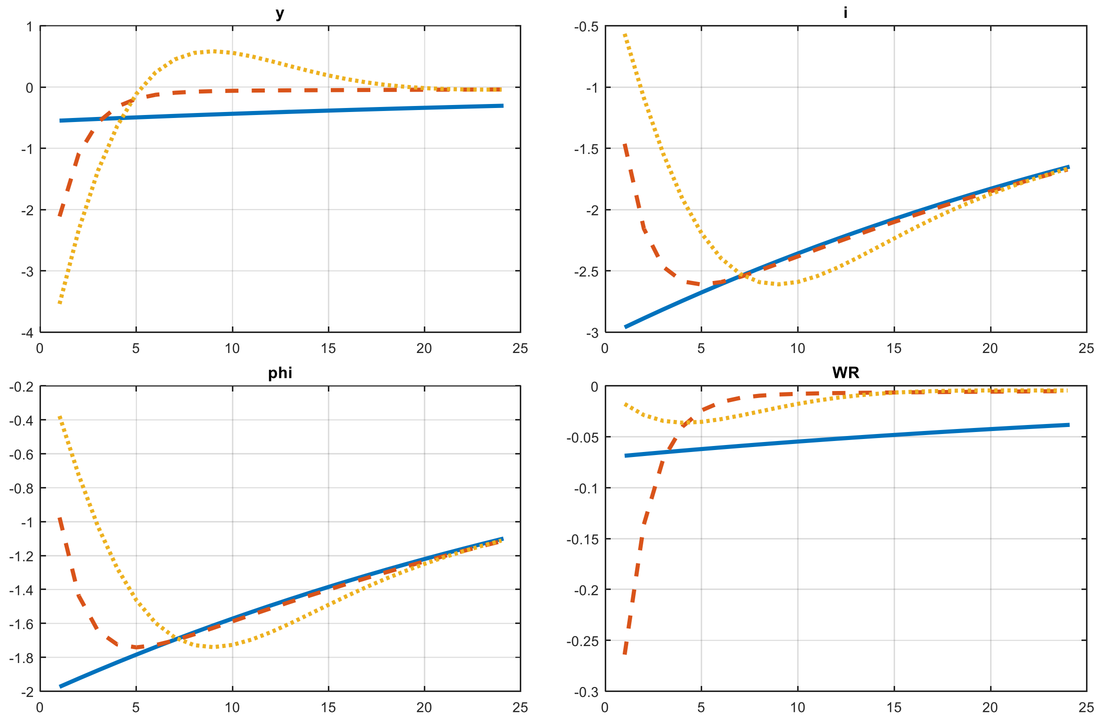
For the simulation experiments we choose the following set of parameters:

$$\rho = 0.005;\ \ \sigma = 1.;\ \ \gamma = 0.125;\ \tau = 1.5;\ s^f = (1;\ \ 0.5);\ \rho^s = 0.975, \chi = (0.,0.96)$$

In all our simulation experiments, we consider an annual frequency, with a rate of time preference of 1%, an interest elasticity of one, a slope of the Phillips curve equal to 0.125. The inflation coefficient in the Taylor rule is set to 1.5. The parameter $s^f \epsilon[0,1]$ provides the degree to which price setting is forward-looking. The upper bound of the real wage rigidity parameter coincides with the estimated parameter in the GM model.

The following impulse responses show the response of this economy to a persistent negative demand shock under three alternative parametrisations, namely first under the standard model, with a forward-looking Phillips curve, second the standard model with a hybrid Phillips curve and third a model with both a hybrid Phillips curve and high degree of real wage rigidity. We show results both for a Taylor rule and a ZLB regime.

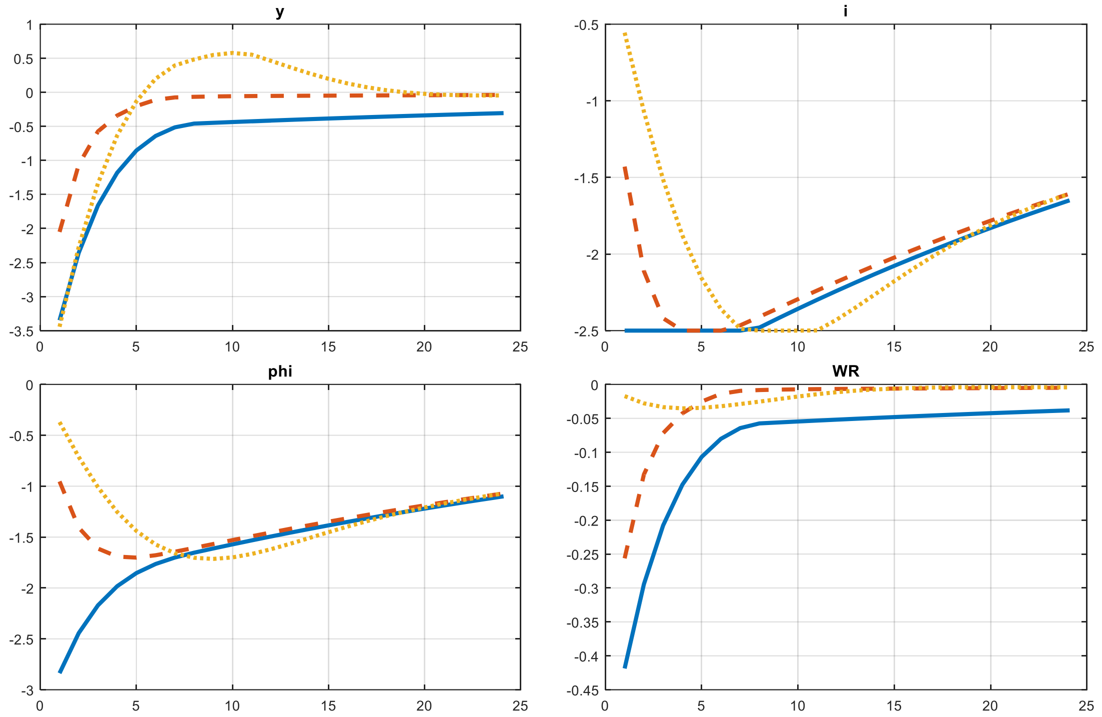
Figure 1a: Persistent demand shock: Taylor rule



Notes: ____ $\mathrm{s^f} = 1, \chi = 0$; ----- $\mathrm{s^f} = 0.5, \chi = 0$; ...... $\mathrm{s^f} = 0.5, \chi = 0.96$. All variables are expressed in deviation from the flexible economy baseline

The standard NK model with a fully forward-looking Phillips curve and no real wage rigidity does not generate decoupling. A persistent demand shock generates a persistent output gap and a long lasting deviation of inflation from target. With a hybrid Philips curve, some decoupling emerges. First, the output gap approaches zero more rapidly, while there is more inflation inertia. In the medium run dynamics of inflation is similar to the forward-looking Phillips curve. The faster closing of the output gap is related to the mark up dynamics induced by the hybrid Phillips curve. While under the forward-looking Phillips curve the markup stays above the average as long as next period expected inflation exceeds current inflation, in the hybrid Phillips curve the mark up is a weighted average of the expected (quasi) inflation differential and the (quasi) difference between actual and past inflation. The former increases the mark up while latter term reduces it. Thus compared to forward looking model the mark up approaches zero more rapidly. Consider for example a fully backward-looking (accelerationist) Phillips curve. In this case, the mark up would be zero at the trough of inflation.

Figure 1b: Persistent demand shock: ZLB



Notes: ____ $s^f = 1, \chi = 0$; ----- $s^f = 0.5, \chi = 0$; …… $s^f = 0.5, \chi = 0.96$. All variables are expressed in deviation from the flexible economy baseline,

With real wage rigidity, the negative employment response/output gap is more negative since wages do not respond strongly. However, one (real) wages have reached a trough, there is the perspective of a slow upward adjustment of wages, which acts like a negative wage mark up shock. This raises employment above the flexible economy benchmark and generates a positive output gap, while there is considerable wage inertia.

The ZLB does not significantly alter the dynamics of output gap and inflation under the three alternative parametrisations.

2. PCA Analysis

This section uses a comprehensive set of cyclical indicators to shed some light on the cyclical signal they provide. We consider the following set of indicators: price and wage inflation, the current account balance, capacity utilisation, sentiment indicator of economic slack[3] , the short term unemployment rate and (lagged) GDP growth. Roeger et al. (2019) provides an additional discussion on the variable selection and technical specification of the PCA.

Principal component analysis (PCA) is a powerful dimension reduction method, which allows one to assess how well individual indicators are correlated with the common cyclical component. It does not

[3] It is important to stress that we do not use all of the available sentiment indicators because of the importance of restricting the analysis to indicators of the output gap cycle (in levels) and not the GDP growth cycle. A restrictive selection process is essential to avoid mixing level and growth signals, and, consequently, we restrict the analysis to just one sentiment indicator, namely the indicator of economic slack.

impose specific restrictions on the data, and it sorts the data series into common components. PCA reduces the dimension of observed data and brings out strong patterns. In the PCA, each common (principal) component (PC_i) is a weighted combination of all the explanatory variables (X); the aforementioned indicators. The number of common components (i) is selected based on the eigenvalue (all components are selected with an eigenvalue above 1).

$$PC_i = w_{1i}X_1 + w_{2i}X_2 + \cdots$$

The dataset includes all 27 European Union countries and the United Kingdom, excluding Ireland and Croatia due to lack of data[4]. The sample period depends on the availability of the data and can differ from one country to another, with the earliest starting in 1996 and the latest in 2008 (Cyprus) and ending in 2018. PCA is sensitive to the scale of the variables. Before performing the analysis, we standardise the variables over the sample for which all variables exist per country.

We select two principal components based on their eigenvalues (as shown in Appendix A below, only the first two components have an eigenvalue above 1). However, the missing cyclical pattern of the second component (which also has a much smaller eigenvalue) suggests to only use the first component in the calculation of a PCA output gap (see Appendix A.1). Table 1 provides the weights (correlation coefficients) of each of the variables.

To get an estimate of an output gap comparable to the official CAM output gap, we rescale the selected components using the standard deviation of the CAM output gap, per country, over the sample for which all variables exist. A mean PCA output gap of zero is assumed. The rescaled PC_1 thus gives the PCA output gap, which we compare to the CAM output gap.

Table 1: Overview of PCA Results

	First component		**Second component**		
	Correlation coefficient	% of variance explained	Correlation coefficient	Additional % of variance explained	Total % of variance explained
Price inflation[5]	0.3	0.3	0.5	0.3	0.6
Wage inflation[6]	0.4	0.4	0.3	0.1	0.6
Current account balance	-0.2	0.1	-0.6	0.5	0.6
Growth in GDP (lagged)	0.4	0.6	-0.2	0.0	0.6
Short term unemployment rate[7]	-0.4	0.6	0.3	0.1	0.7
Economic slack (sentiment indicator)	-0.4	0.7	0.2	0.0	0.7
Capacity utilisation	0.4	0.6	-0.3	0.1	0.7

[4] The variables economic slack (sentiment indicator) and capacity utilisation are based on questions from the Business and Consumer Survey of the European Commission. Price inflation (private consumption expenditure deflator), wage inflation (nominal compensation per employee), current account balance and lagged growth in real GDP come from the AMECO database, Autumn 2018. The short-term unemployment rate is calculated using the harmonised unemployment rate available in AMECO and the long-term unemployment rate from Eurostat.
[5] Price inflation is based on the deflator of private consumption expenditure.
[6] Wage inflation is calculated using nominal compensation per employee.
[7] Data on the long-term unemployment rate in Eurostat is only available for Germany as of 2003, limiting the sample and analysis.

Notes: This table presents loadings of the first two components for the EU28.

Table 1 shows that these two components can explain more than 60% of the variation in each indicator. The first component can explain more than 50% of the variation in most indicators. Several slack indicators, namely capacity utilisation, slack in the economy, and GDP growth, are strongly related to the first component. These are the cyclical variables, which are most closely linked to developments in the EA's product markets. The PCA also picks up several labour market indicators, namely the short-term unemployment rate and wage inflation, although a bit less strongly than the slack indicators.

Table 1 also shows that the CAB and price inflation indicators are less strongly correlated with the first component. Instead, they are more strongly related to the second component. Indeed, Figures 2a confirms that this second component does not have a strong cyclical pattern. This finding is robust across member states (see Appendix A.2) and further corroborated by regression analysis. In particular, the regressions show that a cyclical AR(2)-process approximates the first component well. By contrast, the second component follows more closely a stationary AR(1)-process with a high degree of autocorrelation. The second component implies economic shocks, which have a more persistent effect on inflation and the CA but no persistent effect on the other indicators (zero or low correlation of the second component with the remaining indicators, especially capacity utilisation and sentiment indicators). This result reveals some decoupling between inflation and the CA on the one hand and the other cyclical indicators.

Figure 2a: First and second component of the PCA for the EA (2000-2018)

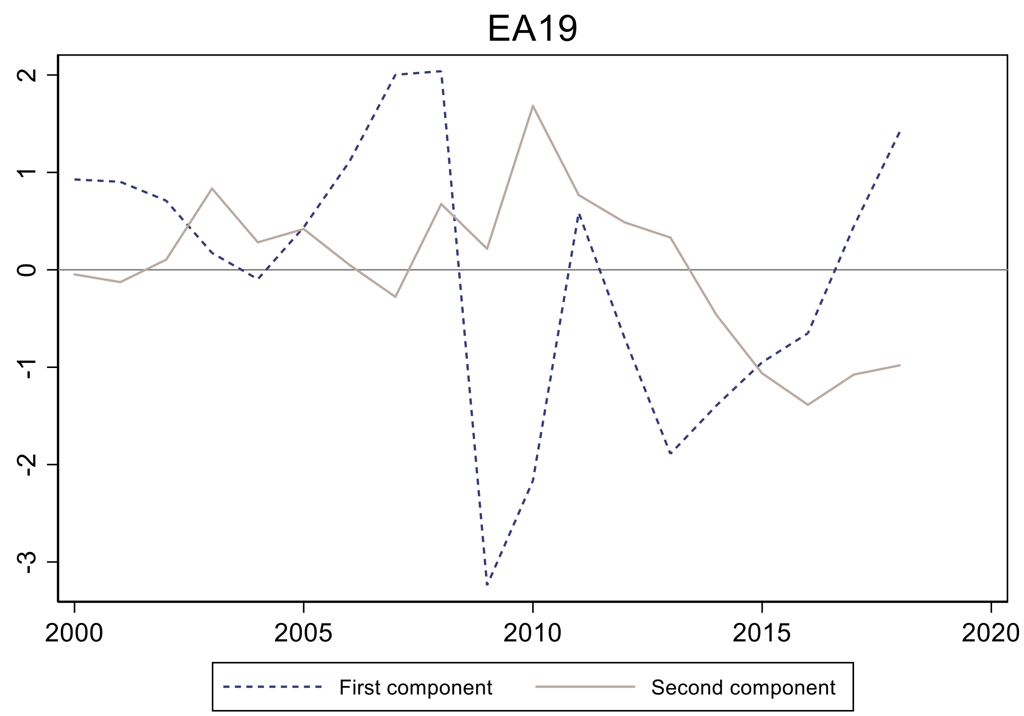
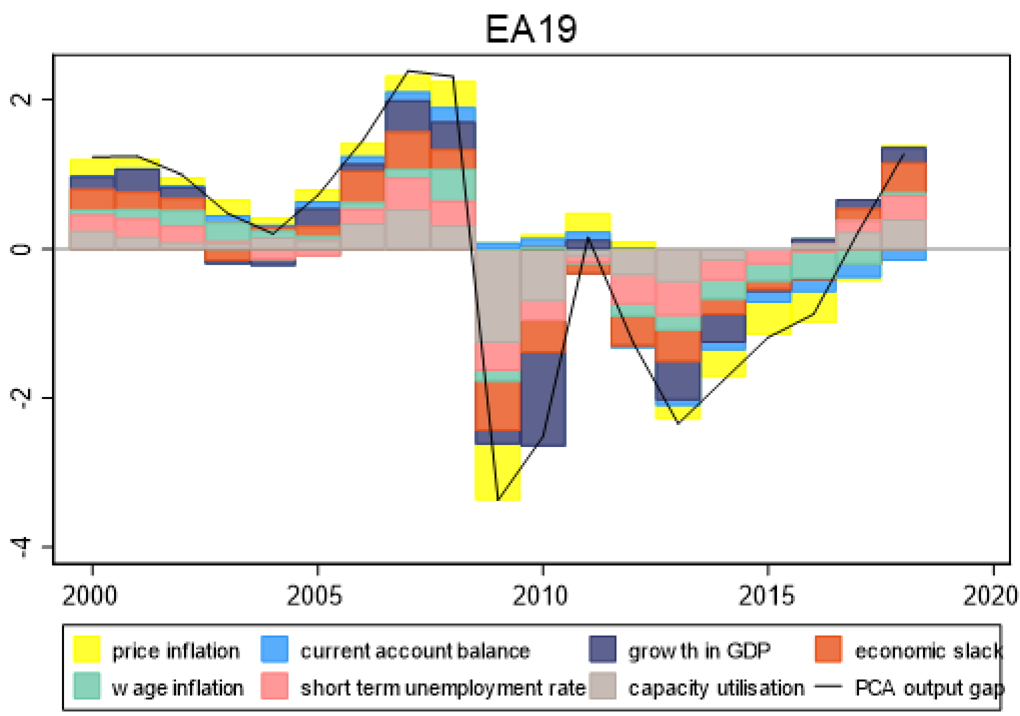


Figure 2b: Indicator contributions to the PCA output gap for the EA (2000-2018)



Notes: The PCA output gap is calculated using only the first component and rescaled to the standard deviation of the CAM output gap.

3. The production function approach for calculating potential output and output gaps

According to the PF concept, potential output is the level of output, which could be obtained, with the current capital stock (K_t) at the target rate of utilisation (normalised to one) and a level of employment equal to the supply of labour at the current wage adjusted for the average wage mark up. We will refer to this employment level as potential employment (N_t^P).[8] Formally, let us denote potential output and the output gap as Y_t^{pot} and Y_t^{gap}, respectively.

$$Y_t^{pot} = N_t^{P^{\alpha}} (K_t)^{1-\alpha} A_t,$$

where α is the labour share and A_t is total factor productivity. Correspondingly, the output gap is a weighted average of the difference between actual employment (L_t) and market clearing employment as well as the difference between the actual level of capacity utilisation and the target level of capacity utilisation (in the absence of adjustment costs), cu_t, with weights corresponding to the output elasticity of labour and capital.

$$Y_t^{gap} = \ln\left(\left(\frac{N_t}{N_t^P}\right)^{\alpha} cu_t^{1-\alpha}\right)$$

This definition of potential differs from the standard concept of the output gap in the DSGE literature, which often defines the output gap as the difference between the actual level of output and the level of output that could be obtained in the absence of nominal rigidities in the past, present and future. In this case, potential output refers to the level of output which a hypothetical economy (without nominal frictions, superscript e) could achieve in period t. Because of the absence of frictions in the past, this

[8] In the absence of utilisation adjustment costs.

hypothetical economy would also operate at a different level of the capital stock and at a different capacity utilisation rate.

$$Y_t^{pot,e} = N_t^{e\,\alpha}(K_t^e cu_t^e)^{1-\alpha}U_t$$

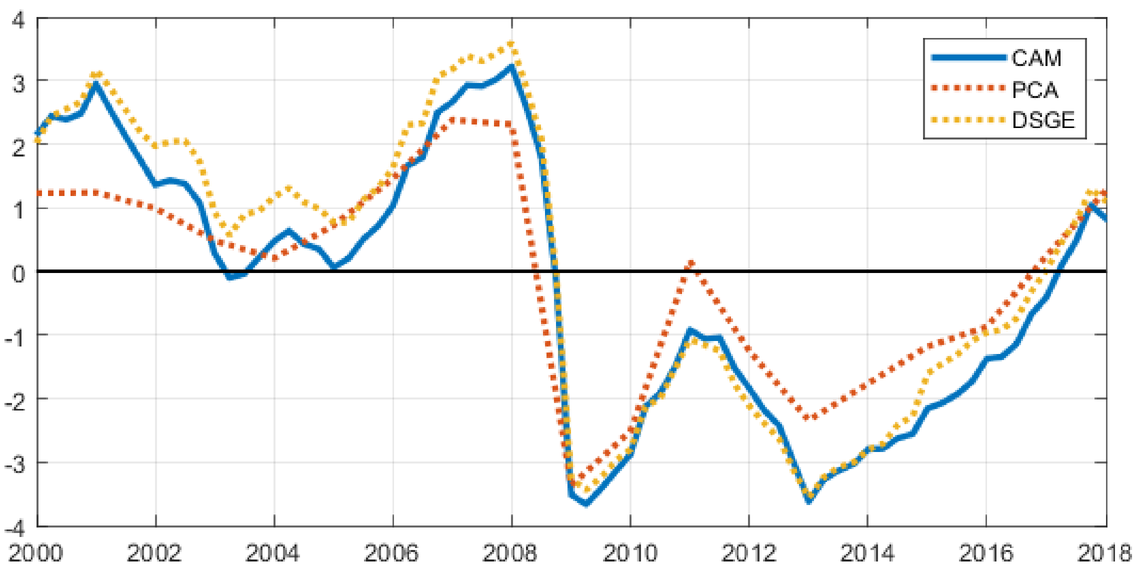
$$Y_t^{gap,e} = \ln\left(\left(\frac{N_t}{N_t^e}\right)^{\alpha}\left(\frac{K_t cu_t}{K_t^e cu_t^e}\right)^{1-\alpha}\right)$$

Since we are interested in the relationship between the PF output gap concept and inflation, we will use the first definition of potential and the output gap to establish a close correspondence between the estimates of a DSGE model and the CAM. Additional methodological criticism of the output gap concept used in the DSGE literature is that the efficient level of output is not necessarily attainable (i.e. it is not equal to what we mean by potential output). For example, Blanchard (2018) finds that the output gap defined as the difference between actual and efficient output "is not a useful definition, because what matters is what output can be today, as opposed to what it could have been in some hypothetical world."

The CAM uses several cyclical indicators. For example, the cyclical component of unemployment (unemployment gap) is extracted using information about real or nominal unit labour cost developments (non-accelerating wage rate of unemployment concept). The TFP gap, which measures the degree of capacity utilisation by firms (tightness in product markets), is informed by a whole set of sentiment indicators (capacity utilisation for manufacturing and business sentiment indicators for services and construction).

Figure 3 shows that the CAM output gap is closely correlated with the first component of the PCA analysis ("the PCA output gap"), as well as with the model-based DSGE output gap.

Figure 3: CAM output gap versus PCA and DSGE estimates for the EA (2000-2018)



Notes: The PCA output gap is calculated using only the first component and rescaled to the standard deviation of the CAM output gap. The DSGE output gap is based on the model-based production function. The vertical axis shows the output gap in percent.

4. Model-based evidence

This section links the results from the CAM and PCA methods to those of a fully-fledged estimated DSGE model. Section 4.1 first outlines the general structure and PF-specific elements of the model. We then shed light on the relationship between the output gap and the current account as well as on the role of adverse demand shocks in explaining the decoupling of inflation and the output gap (Section 4.2). Finally, Section 4.3 provides a model-based decomposition of the key sources of fluctuations in inflation and the output gap.

4.1 An estimated DSGE model

We integrate a PF based output gap into the Global Multicountry (GM) model, the workhorse DSGE model of the European Commission. The following descriptions highlight key model elements given the purpose of this paper. Appendices B and C cover the remaining model and estimation details.

This paper considers a two-region setup consisting of the Euro Area (EA) and the rest of the world (RoW). The EA economy consists of households, a continuum of intermediate goods producers, a final goods firm, import and export sectors, and a government. Wages are sticky and set by trade unions. Prices are sticky, too. Perfectly competitive EA final goods producers use EA intermediate goods as well as imported commodities and manufactured goods as inputs. Trade in goods and a financial asset link the EA with the rest-of-the-world (RoW). Only the RoW supplies commodities.

EA production. We consider a continuum of intermediate goods firms. Each firm $i \in (0,1)$ produces differentiated goods using a Cobb-Douglas technology:

$$Y_{it} = (A_t N_{it})^{\alpha} \left(cu_{it} K_{it-1}^{tot}\right)^{1-\alpha} - A_t FC,$$

where A_t is labour-augmenting productivity common to all firms i. α denotes the output elasticity of labour. N_{it} are hours worked. Total capital K_{it}^{tot} is the sum of private, K_{it}, and public capital K_t^G utilised at rate cu_{it}. By considering both types of capital, K_{it}^{tot} corresponds to the capital measure used in the CAM. FC denotes fixed-costs in production. Each firm i maximises the real firm value V_{it} by setting prices, P_{it}^Y, labour, investment I_{it}, and capacity utilisation subject to adjustment costs. V_{it} is a discounted stream of future dividends

$$V_{it} = d_{it} + E_t\left[\Lambda_{\mathrm{t,t+1}} V_{t+1}\right],$$

where E_t is the expectations operator, and $\Lambda_{\mathrm{t,t+1}}$ the stochastic discount factor. d_{it} denotes period t dividends:[9]

$$d_{it} = \frac{1}{P_t^Y}\left(P_{it}^Y Y_{it} - W_t N_{it} - P_{it}^I I_{it}\right) - adj_{it},$$

where W_t is the wage rate. P_{it}^I is the price for investment goods. Gross investment I_{it} induces a law of motion for capital $K_{it} = I_{it} + (1-\delta)K_{it-1}$ with depreciation rate δ. adj_{it} summarises adjustment costs:

$$adj_{it} = adj_{it}^P\left(P_{it}^Y\right) + adj_{it}^N(N_{it}) + adj_{it}^{cu}(cu_{it}) + adj_{it}^I(I_{it})$$

See Appendix C for additional details on the functional forms of adj_{it} and the corresponding optimality conditions.

Capacity utilisation which enters the output gap is determined by firms as part of their profit maximisation problem, via the first order condition

[9] For the sake of clarity, we abstract in the main text from linear taxes. Appendix C presents additional details.

$$(1-\mu_{it})\frac{\partial Y_{it}}{\partial cu_{it}} = \frac{\partial adj_{it}^{cu}}{\partial cu_{it}}$$

Which equates the marginal revenue product of the firm with marginal capacity adjustment costs. For any shock, the firm can adjust capacity by changing the capital and by adjusting its utilisation rate. The relative magnitude of capital and capacity adjustment will depend on the expected duration of the shock. Given higher investment adjustment costs the firm will tend to adjust capacity more strongly for temporary shocks.

A perfectly competitive final goods sector aggregates intermediate goods with elasticity of substitution σ^Y (inversely related to the price mark-up):

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\sigma^Y-1}{\sigma^Y}} di\right]^{\frac{\sigma^Y}{\sigma^Y-1}}$$

Further upstream, perfectly competitive firms produce output (O_t) by combining domestic value-added (Y_t) and imported commodities (CO_t) in a CES production function

$$O_t = \left[(1-s_t^{CO})^{\frac{1}{\sigma^O}} Y_t^{\frac{\sigma^O-1}{\sigma^O}} + (s_t^{CO})^{\frac{1}{\sigma^O}} CO_t^{\frac{\sigma^O-1}{\sigma^O}}\right]^{\frac{\sigma^O}{\sigma^O-1}},$$

where s_t^{CO} is the stochastic commodity share in total output.[10] σ^O governs the elasticity of substitution between factors.

EA Households

Households consume, provide labour to intermediate good producers, and own the asset portfolio B_{jt} with return R_t^r. B_{jt} consists of risk-free bonds (rf), one international bond (bw), and domestic firms shares (S). Households have this period utility function:[11]

$$U_{jt} = \frac{\left(C_{jt} - hC_{t-1}\right)^{1-\theta}}{1-\theta} - \omega_t^N \frac{\left(N_{jt}\right)^{1+\theta^N}}{1+\theta^N} + \sum_Q B_{jt}^Q \left(\varepsilon_t^Q - \alpha^Q\right),$$

where $\theta, \theta^N > 0$. C_{jt} denotes consumption. h and ω_t^N govern external consumption habits, and stochastic labour supply, respectively. $Q \in (rf, bw, S)$ captures different asset classes. Intercept α^Q captures asset-specific steady-state risk premia. ε_t^Q introduces asset-specific risk-premium shocks. Savers maximise expected lifetime utility $E_0 \sum_{t=0}^{\infty} (\beta_t)^t U_{jt}$ subject to an intertemporal budget constraint:

$$P_t^C C_{jt} + B_{jt+1} = W_t N_{jt} + d_t + R_t^r B_{jt} + T_{jt},$$

where P_t^C denotes the price of consumption goods. T_{jt} summarises taxes and transfers.

[10] Commodities include oil and raw materials.

[11] More specifically, only a share ω^s of households are savers (superscript s) who own firm and financial assets. The remaining households (superscript c) consume each period their labour income in a hand-to-mouth fashion. Their utility function omits the term governing disutility of financial assets, $\sum_Q B_{jt}^Q \left(\varepsilon_t^Q - \alpha^Q\right)$, and their budget constraint is as follows:

$$P_t^C C_{jt}^c = W_t N_{jt}^c + T_{jt}^c.$$

Labour supply and wage setting

The labour market structure follows Ratto et al. (2009) and Kollmann et al. (2016). A trade union that 'differentiates' homogenous EA hours provided by the two domestic household groups into imperfectly substitutable labour services. The union then offers those services to intermediate goods-producing firms. The labour input, N_{it}, in those firms' production functions, is a CES aggregate of these differentiated labour services. The union set wage rates at a mark-up over the marginal rate of substitution between leisure and consumption. The mark-up consists of constant component μ^w (inversely related to the degree of substitution between labour services in production) and a stochastic time-varying mark-up $\tilde{\mu}_t^w$. Wage adjustment costs imply a countercyclical, the mark-up. Real wage rigidity, governed by parameter γ^{wr}, captures the high persistence of employment rate fluctuations in many advanced economies, including the EA. Following Blanchard and Gali (2007), the current period real wage rate set by the union is a weighted average of the desired real wage and the past real wage:

$$\frac{W_t}{P_t^C} = \left(\frac{U_{N,t}}{U_{C,t}} * [1 + \mu^w + \tilde{\mu}_t^w]\right)^{1-\gamma^{wr}} \left(\frac{W_{t-1}}{P_{t-1}^C}\right)^{\gamma^{wr}},$$

where $U_{N,t}/U_{C,t}$ is a weighted average of the two households' marginal rates of substitution between consumption and leisure. P_t^C denotes the net consumption price.

We follow Gali (2011) and define potential employment by the standard labour supply condition adjusted for the average wage mark up.

$$\frac{W_t}{P_t^C} = \left(\frac{U_{N^P,t}}{U_{C,t}} * [1 + \mu^w]\right)^{1-\gamma^{wr}} \left(\frac{W_{t-1}}{P_{t-1}^C}\right)^{\gamma^{wr}}.$$

This equation determines potential employment, which we denote N_t^P.

Model-based output gap estimation

We define the PF-based model-consistent output gap in line with the CAM outlined above. The output gap is a weighted average of the TFP gap (TFP_t^{gap}) and employment gap (N_t^{gap}):

$$Y_t^{gap} = (1 - \alpha)\, TFP_t^{gap} + \alpha\, N_t^{gap},$$

where the TFP gap corresponds to the deviations of capacity utilisation from its normalized steady-state value of unity:

$$TFP^{gap} = cu_{it} - 1$$

The log ratio of hours worked and potential employment (N_t^P) defines the employment gap.

$$N^{gap} = \ln\left(\frac{N_t}{N_t^P}\right).$$

This output gap estimate is consistent with a PF-based approach. Yet, it is not identical to the CAM results. For example, the NAWRU estimation differs. The GM model estimates labour supply and labour participation using a large set of macro variables, including consumption, in a full information Bayesian estimation. By contrast, the CAM uses information about structural indicators in a panel framework. Moreover, the production function specification slightly differs as the GM model includes fixed costs.

Monetary policy outside and at the ZLB

The unconstrained nominal interest rate i_{kt}^{NC} follows a Taylor-type rule where the ECB sets the policy rate in response to the annualised inflation gap, $\pi_t^{c,vat,QA}$, and the annualised output gap:

$$i_t^{NC} - \bar{\iota} = \rho^i(i_{t-1} - \bar{\iota}) + (1 - \rho^i)\left[\eta^{i\pi}/4\left(\pi_t^{c,vat,QA} - \bar{\pi}^{c,vat,QA}\right) + \eta^{i\pi}/4\left(Y_t^{gap,QA}\right)\right]$$

where $\bar{\iota}$ is the steady-state nominal interest rate, and $\eta^{i\pi}$ and η^{iy} are the responses to the inflation gap and output gap, respectively. As long as the actual policy rate is above the lower bound, the nominal interest rate is:

$$i_t = \max(0, i_t^{NC}) + \varepsilon_t^i$$

and the variable i_t^{NC} acts as a 'shadow' interest rate. We set the lower bound for quarterly short-term interest rates, i^{LB}, at 0.0%.

The remainder of the model

The remainder of the model includes fiscal policy reaction functions, linear taxes, rich trade linkages and a RoW block. The model shares many of these elements with other larger estimated DSGE models used in policy institutions and all of the remaining details are relegated to Appendix C.

Non-linear model estimation

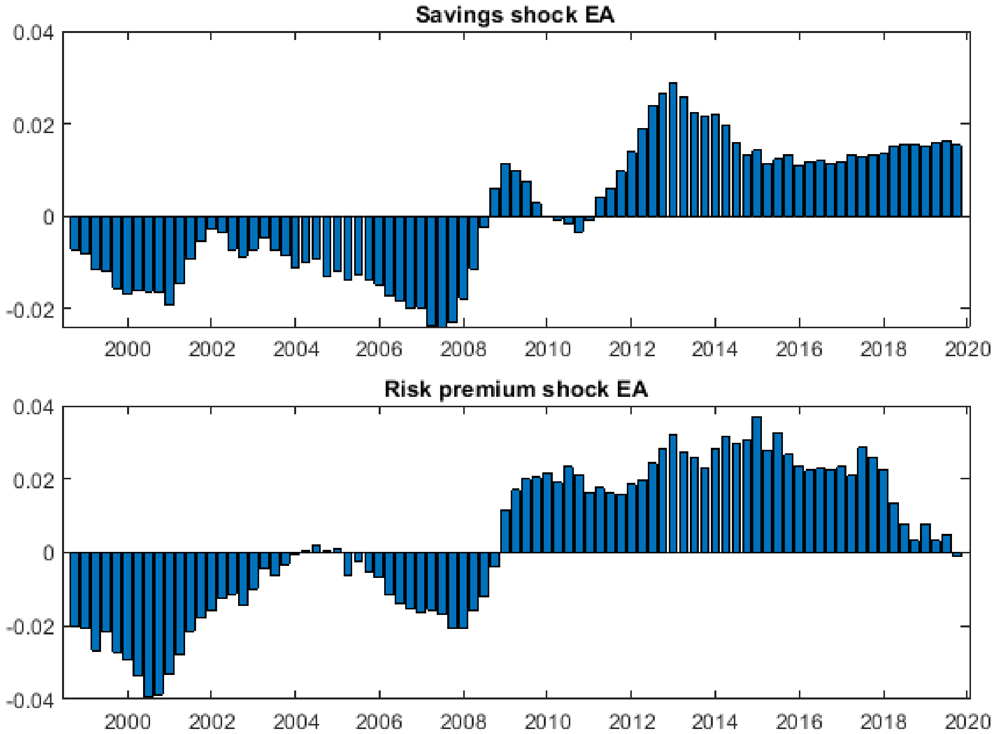
We estimate the model parameters using quarterly data from 1998q4 until 2019q4. Based on the estimated parameter values from the linear model, we run a piecewise linear Kalman filter as in (Giovannini et al., 2021) to identify the structural shocks, accounting for the endogenous ELB periods using the OccBin approach (Guerrieri and Iacoviello, 2015). Unless indicated differently, all shocks follow independent autoregressive processes. Appendix B presents additional details.

4.2 Estimated demand shocks

The model distinguishes two types of demand shocks: Risk premia on investment and savings shocks. We estimate their respective contributions and persistence. This specification gives the model sufficient flexibility to capture strong but temporary financial friction shocks (relevant in 2009 and 2011) and more persistent risk premia and savings shocks. Figure 4a shows that the model identifies two demand components with notable shifts around 2009.

This evidence is consistent with previous work which characterises the period since 2009 as subject to persistent demand shocks (see Kollmann et al., 2016). Also, various estimates of the natural rate of interest (see e. g. Laubach, 2018) show a further drop in the natural rate around 2009. These shocks have been strong enough to move the EA policy rate to the ZLB over an extended period.

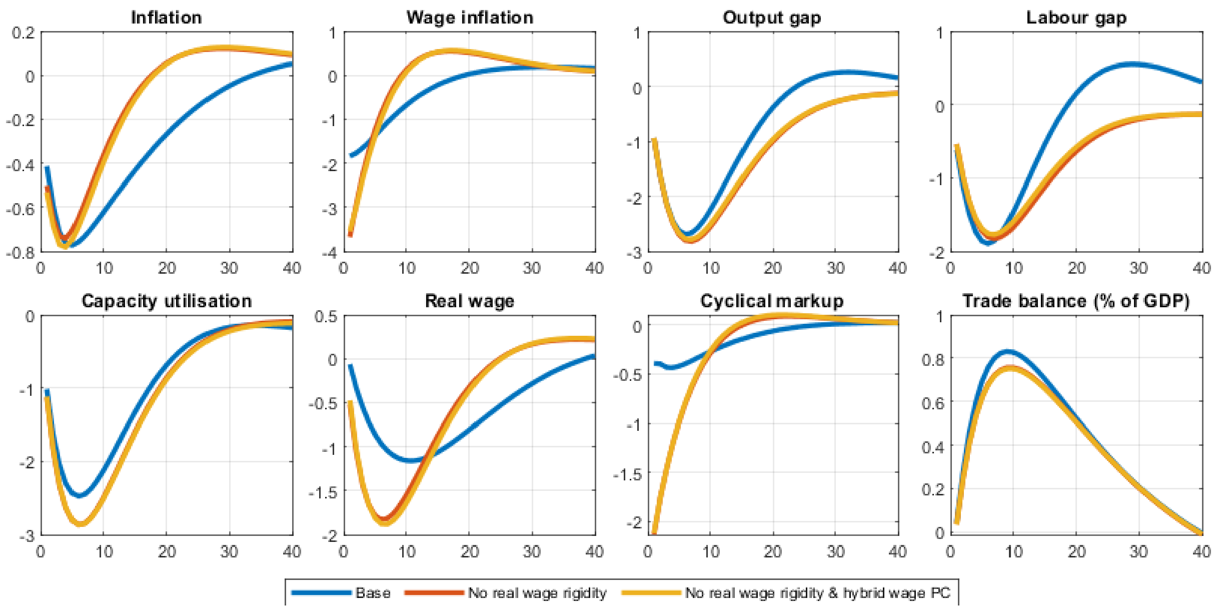
Figure 4a: Estimated demand shock processes



Notes: This figure displays estimated structural shock processes of the domestic demand shocks.

The following figures show the impulse response function to both the savings and the risk premium shock, which constitute the demand shocks in the model

Figure 4b: Dynamic responses to a savings shock

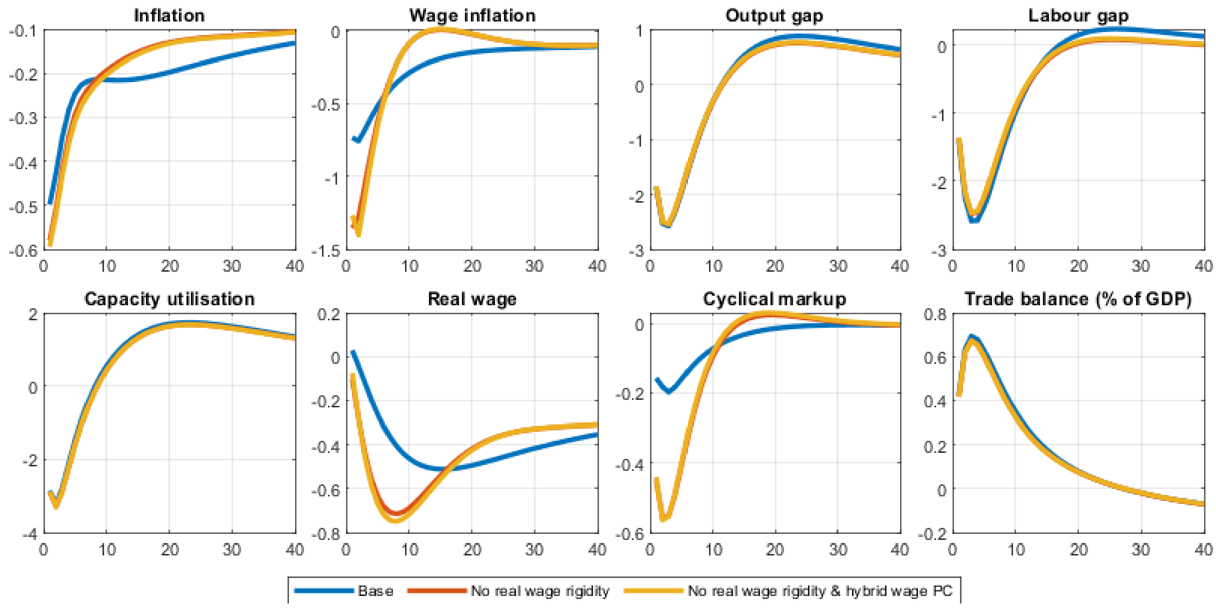


Notes: Inflation rates are annualised. Inflation refers to the growth rate of the consumer price deflator.

The model only exhibits small backward-looking wage response, therefore this feature does not generate substantial decoupling. Most of the decoupling between the output gap and inflation comes from real wage rigidity. The economic intuition for decoupling is the following in this case. Real wage rigiditiy initially reduces the speed in which wages fall but also keeps wages lower more persistently.

It is especially this last feature, which keeps consumer price inflation low. Low persistent wages in turn contribute towards a faster closing of the employment gap, with the gap even turning positive.

Figure 4c: Dynamic responses to a risk premium shock



Notes: Inflation rates are annualised. Inflation refers to the growth rate of the consumer price deflator.

Similar to the savings shock, most of the decoupling is generated by real wage rigidity in case of a persistent risk premium shock. The decoupling mostly results from a slowdown in price inflation due to a more persistent decline in nominal wages (lower wage inflation in the medium term).

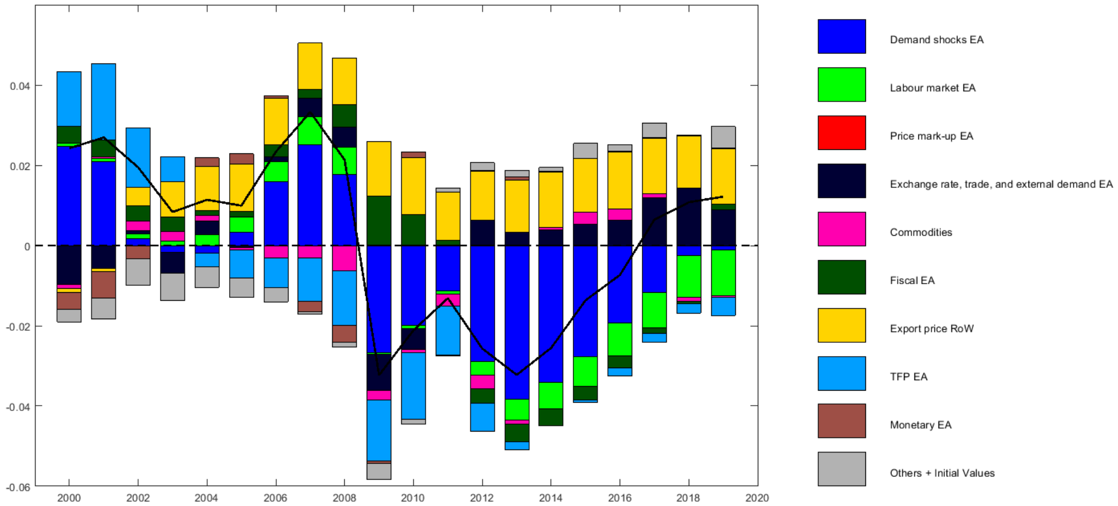
The next section discusses how these three shocks have affected the output gap and inflation (plus trade balance)

4.3 Shock decompositions

This section explores how both the demand shocks and the other shocks account for the dynamics of the output gap, inflation and the trade balance. To quantify the role of different shocks as drivers of endogenous variables in the period 1999-2018, we plot the estimated contribution of the different shocks to historical time series. Figures 4d to 4f show historical decompositions of the following EA variables: the output gap and the year-on-year growth rate of the consumer price deflator and the trade balance/GDP ratio. The continuous lines show the historical series from which we subtracted the sample averages. The stacked vertical bars show the contribution of a different group of shocks (see figure caption for details) to the data. Bars above the horizontal axis represent positive shock contributions, while bars below the horizontal axis show negative contributions. The sum of all shock contributions equals the historical data. We obtain the shock decompositions in the non-linear model with an estimated length of the ZLB regime. Appendix B1 presents additional technical details on the construction and representation of shock contributions in the non-linear model.

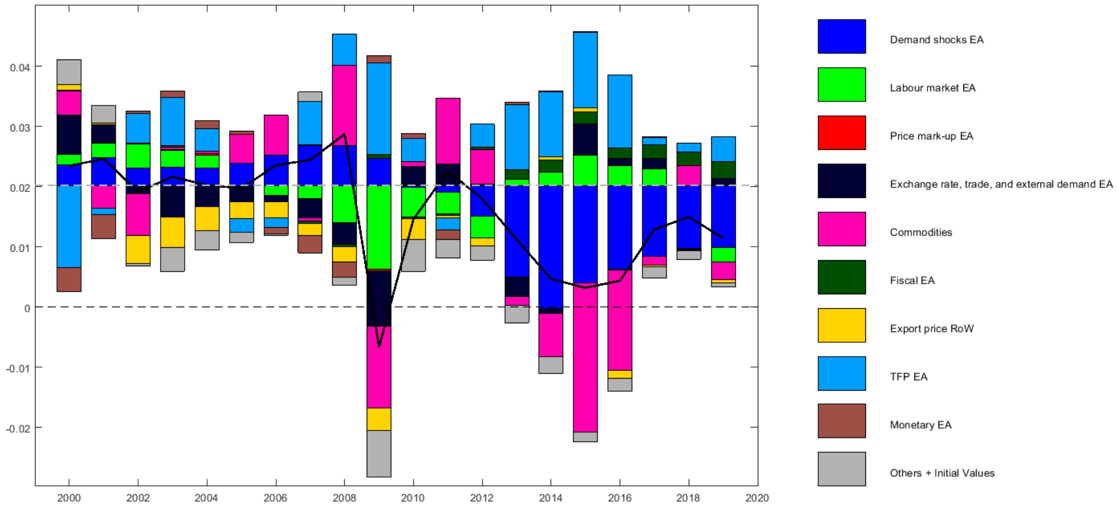
Consider first the historical shock decomposition of the output gap (Figure 4d). Domestic demand shocks account for most of the negative output gap associated with the double-dip recession in the EA. Fiscal measures have offset some of the shock. Domestic technology as well as mark-up shocks contributed little to the output gap.

Figure 4d: Historical decomposition of EA output gap



Notes: This figure shows the estimated shock contributions to the EA output gap. All structural shocks recover the observed time series (black continuous line). We have grouped estimated shocks into 8 broad categories: (1) domestic demand shocks; (2) wage mark-up shocks; (3) price mark-up shocks; (4) exchange rate shocks as well as changes to world demand and international trade, which contains foreign demand and supply shocks and deviations of trade volumes and prices from the estimated export and import demand and pricing equations; (5) commodity price shocks; (6) discretionary fiscal policy shocks which capture deviations from estimated policy rules; (7) export price shocks from the RoW (EA import prices); (8) TFP shocks; (9) deviations of short-term interest rates from the estimated monetary policy rule under the ZLB constraint; (10) any other factors..

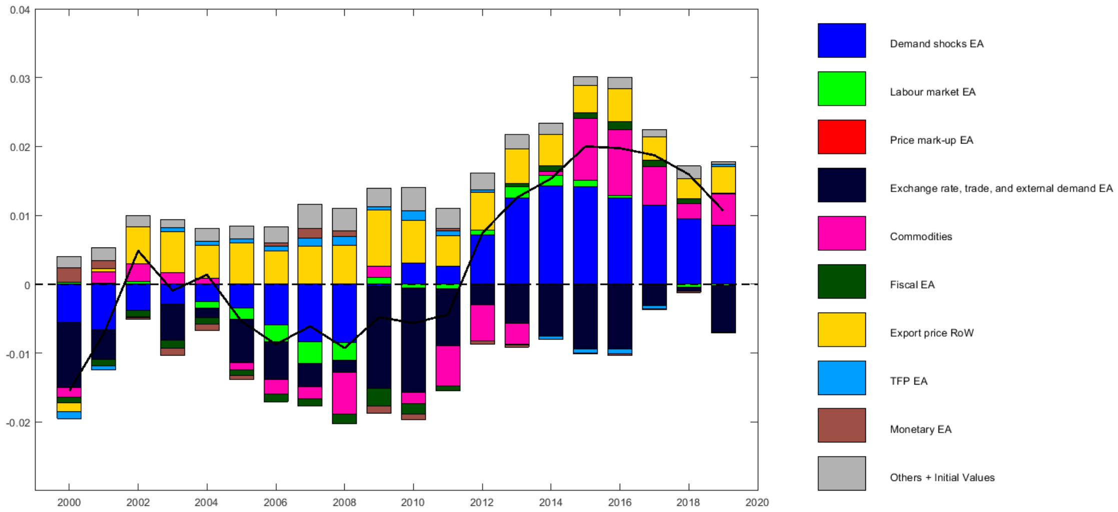
Figure 4e: Historical decomposition of EA consumer price inflation



Notes: This figure shows the estimated shock contributions to the growth rate of the consumer price deflator. All structural shocks recover the observed time series (black continuous line).

Figure 4e presents the historical shock decomposition of consumer price inflation. In the absence of mostly domestic demand shocks, consumer price inflation would have been closer to the 2% target. Falling commodity prices (around 2015), had an additional impact. However, external price shocks cannot explain the persistent decline of inflation. Fiscal policy exerted some stabilisation over the relevant period   Negative TFP shocks and positive wage mark-up shocks prevented a further decline of inflation.

Figure 4f: Historical decomposition of EA trade balance-to-GDP ratio



Notes: This figure shows the estimated shock contributions to the EA trade balance-to-GDP ratio. All structural shocks recover the observed time series (black continuous line).

Figure 4f considers the trade balance-GDP-ratio. The identified demand shocks are tracing well the medium term dynamics of the EA trade balance, in particular the negative contribution of the pre-crisis demand boom and the positive contribution of the post-crisis slack. The fall of commoditiy prices after 2014 had an additional impact on increasing the EA trade balance. RoW export prices exerted persistent upward pressure on the trade balance over the last 20 years. These results also show that trade has been affected by both domestic demand shocks as well as foreign supply shocks.

5. Conclusion

This paper has documented the close correlation between the cyclical component of various indicators of the business cycle and the output gap, as generated by the production function approach used by the EU's commonly agreed methodology (CAM). The paper focusses in particular on the relationship between inflation and the trade balance on the one hand and the output gap on the other; especially highlighting the decoupling which took place between the recovery in real GDP growth and persistently low inflation developments, a phenomenon which has been labelled the "inflation puzzle".

We have looked at this issue from two angles. First, a principal components analysis shows that a common cyclical component from several key cyclical indicators follows the CAM output gap estimates closely. This finding indicates that the output gap of the selected countries (and that of the EA as a whole) was heading towards positive territory despite sluggish inflation developments. The analysis also confirms the ambiguous cyclical signals from indicators such as price inflation and the current account balance.

Our second angle uses a rich DGSE model to provide a more theory-grounded perspective. We show that decoupling between the output gap and inflation can emerge if wage setting is subject to a hybrid Phillips curve and substantial degree of real wage rigidity. The latter seems to be a strong feature in the data. The model estimation identifies that the EA has been subject to persistent positive shocks to consumption and investment before 2009 and persistent negative shocks after the financial crisis. This makes demand shocks a relevant factor for explaining output and inflation dynamics. A further indication that our identification of demand shocks seems reasonable is shown by the ability of these shocks to match the dynamics of the trade balance over the estimation period.

References

Albonico A., L. Cales, R. Cardani, O. Croitorov, F. Ferroni, M. Giovannini, S. Hohberger, B. Pataracchia, F. Pericoli, R. Raciborski, M. Ratto, W. Roeger, L. Vogel (2019). Comparing post-crisis dynamics across Euro Area countries with the Global Multi-country model, *Economic Modelling*, forthcoming.

Blanchard, O. (2018). Should we reject the natural rate hypothesis?, *Journal of Economic Perspectives*, 32(1), 97-120.

Blanchard, O and J. Gali (2007). Real Wage Rigidities and the New Keynesian Model. Journal of Money Credit and Banking, Vol 39, pp. 35-65.

Brooks, R. (2019) "The Campaign Against Nonsense Output Gaps" Twitter.

Constâncio, V. 2015. "Understanding Inflation Dynamics and Monetary Policy." Panel Remarks at the Economic Policy Symposium, sponsored by the Federal Reserve Bank of Kansas City, held in Jackson Hole, Wyoming, August 29.

De Waziers, D. (2018) "What Do Business Surveys Tell us About the Position of the Economy in the Business Cycle?" Ministère de l'Économie et des Finances, Direction générale, Trésor-Economics No. 223, June 2018, pp. 1-8.

European Commission (2018) "European Economic Forecast. Autumn 2018" Institutional Paper 089. Economic and Financial Affairs, November 2018, Brussels, 220pp.

Fleischman, C., and Roberts, J. (2011) "From Many Series, One Cycle: Improved Estimates of the Business Cycle from a Multivariate Unobserved Components Model" Board of Governors, Finance and Economics Discussion Series Working Paper 2011-46.

Gayer, C., and Marc, B. (2018) "A New Modesty? Level Shifts in Survey Data and the Decreasing trend of "Normal" growth" *European Economy Discussion Paper* 083.

Giovannini, M., P. Pfeiffer, and M. Ratto (2021). Efficient and robust inference of models with occasionally binding constraints. Working Papers 2021-03, Joint Research Centre, European Commission (Ispra site).

Graff, M. and Sturm, J.-E. (2010): The Information Content of Capacity Utilisation Rates for Output Gap Estimates, in: Third International Seminar on Early Warning and Business Cycle Indicators, 17 – 19 November 2010, ESA/STAT/AC. 223/S4.4, Moscow

Guerrieri, L., and M. Iacoviello (2015). OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily, Journal of Monetary Economics 70, 22-38.

Guerrieri, Luca, and Matteo Iacoviello (2017), "Collateral Constraints and Macroeconomic Asymmetries," Journal of Monetary Economics, vol. 90, October, pp.28-49.

Havik, K., McMorrow, K., Orlandi, F., Planas, C., Raciborski, R., Roeger, W., Rossi, A., ThumThysen A., and Vandermeulen, V. (2014) "The Production Function Methodology for Calculating Potential Growth Rates and Output Gaps" *European Economy Economic Papers* 535, November 2014.

Jarocinski, M., and Lenza, M. (2016) "How Large is the Output Gap in the Euro Area" *ECB Research Bulletin*, Issue 24/2016.

Kiley, Michael T., 2013. "Output gaps," *Journal of Macroeconomics*, Elsevier, vol. 37(C), pages 1-18.

Kollmann, R., B. Pataracchia, R. Raciborski, M. Ratto, W. Roeger, L. Vogel (2016). The post-crisis slump in the Euro Area and the US: Evidence from an estimated three-region DSGE model, *European Economic Review* 88(C), 21-41.

Malin, A., Szörfi, B., Tóth, M., and Nico Zorell, N. (2018) "Potential Output in the Post-crisis Period" *ECB Economic Bulletin*, Issue 7/2018.

Pfeiffer, P. and Ratto M., (2022) Compact representations for nonlinear shock decompositions, *mimeo*.

Roeger W, K Mc Morrow, A Hristov and V Vandermeulen (2019), "The current debate on output gaps and cyclical indicators", European Economy Discussion Papers, July.

Sachverständigenrat, Jahresgutachten 2019/20. Statistisches Bundesamt.

Tooze, A. (2019) "Output gap nonsense" Social Europe.

Appendix A: PCA

A.1 Eigenvalues of the PCA components

	Eigenvalues
PC_1	3.3
PC_2	1.2
PC_3	0.7
PC_4	0.6
PC_5	0.5
PC_6	0.4
PC_7	0.3

A.2 Cyclical Properties of the Second Component of the PCA

Within the text, it is assumed that the second component of the PCA is not a clear cyclical variable. To decide whether a variable is cyclical, the following criteria were used:

1. Visual inspection: the series should follow a cyclical pattern, and preferably indicate troughs and peaks around the same time of the business cycle troughs and peaks.

2. Consistent signs for the coefficients: all coefficients should have consistent signs, as predicted by the theory of business cycle behaviour.

3. Cyclical process: a cyclical time series is expected to be stationary and to follow a second order autoregressive process.

As reported in the text, the second component does not perform well on the first and second criteria. Looking at Figure A.1, the second component does not have a strong cyclical pattern. The first component does show this pattern, with peaks and troughs at times when expected by general business cycle knowledge. Table A.1 has shown that the indicators of the second component have inconsistent signs, if one wants to interpret it as being cyclical (see the main text for further elaboration).

Regarding the third criterion (i.e. can the time series based on the two principal components be specified using an AR(2) process), there are clear differences between the first and second components. Looking at the panel dataset (Random-effects ML regression) one can see that the first component follows a clear cyclical AR(2) process (first coefficient is positive and smaller than 1, second coefficient is negative, their sum is less than 1). Looking at the second component, the regression shows a positive and significant first coefficient, but also a positive and insignificant second coefficient (p-values are given between brackets). This pattern is much more consistent with a stationary trend process, rather than a cyclical process.

$$PC_1 = 0.77(0.00)\ L.PC1 - 0.29(0.00) L2.PC1$$

$$PC_2 = 0.73(0.00)\ L.PC2 + 0.05(0.31)\ L2.PC2$$

The next two tables confirm this result on a country-by-country basis.

Table A.1: Regression analysis of cyclical properties

First component				
	DE	ES	IT	PT
First lag	0.40	1.17	0.72	0.90
p-value	*(0.16)*	*(0.00)*	*(0.01)*	*(0.00)*
Second lag	-0.42	-0.37	-0.06	-0.13
p-value	*(0.15)*	*(0.10)*	*(0.82)*	*(0.57)*
N	14	21	17	21
R-squared	0.23	0.77	0.51	0.66
Second component				
	DE	ES	IT	PT
First lag	0.38	0.93	0.72	0.55
p-value	*(0.19)*	*(0.00)*	*(0.01)*	*(0.02)*
Second lag	0.18	-0.07	0.13	0.43
p-value	*(0.44)*	*(0.78)*	*(0.65)*	*(0.08)*
N	14	21	17	21
R-squared	0.27	0.71	0.59	0.63

A.3 The PCA for other countries

Figure A.1 Principal components for EA members

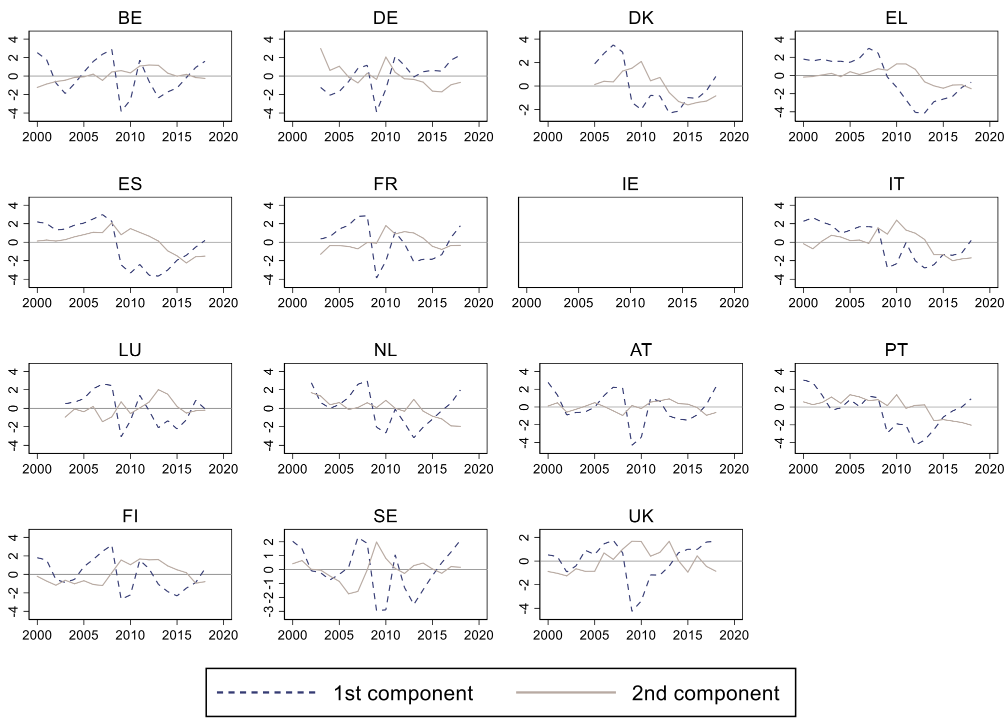
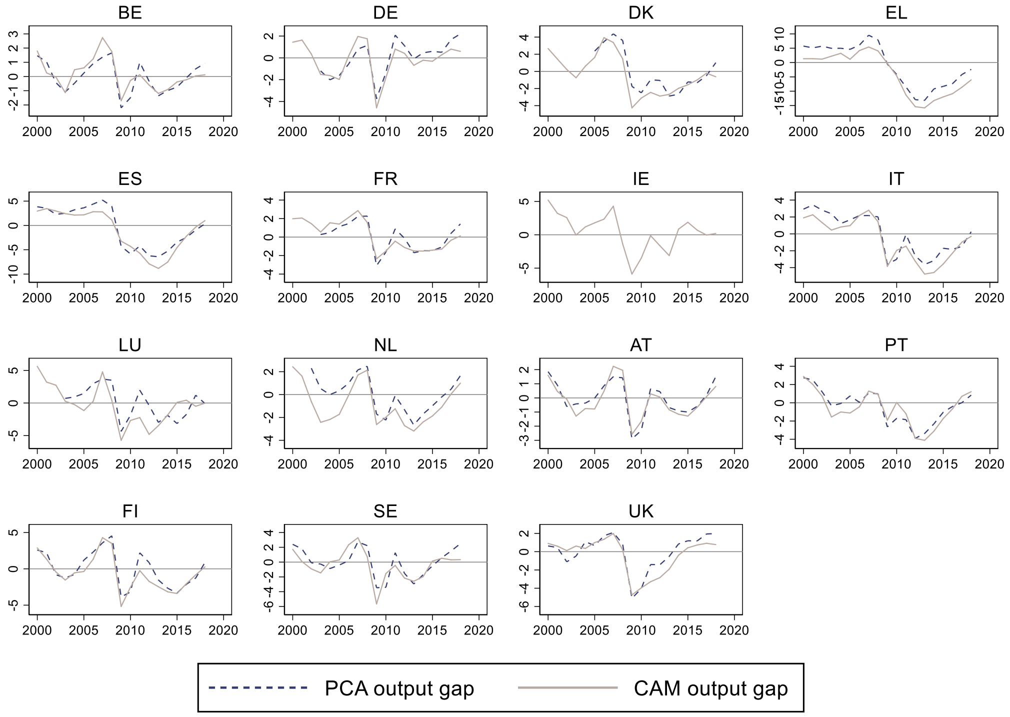


Figure A.2 CAM versus PCA-based output gaps for EA members



Appendix B: Model solution and econometric approach

B.1 Model solution, estimation, and non-linear shock decompositions

We compute an approximate model solution by linearizing the model around its deterministic steady state. Following the recent literature that estimates DSGE models, we calibrate a subset of parameters to match long-run data properties, and we estimate the remaining parameters using Bayesian methods. To obtain parameter estimates and smoothed shocks, we apply a specially adapted Kalman filtering techniques, which take into account the occasionally binding constraint (Ratto 2019).

Following Pfeiffer and Ratto (2022), we can represent the non-linear DSGE model as

$$x_t = T(x_{t-1}, \varepsilon_t)x_{t-1} + C(x_{t-1}, \varepsilon_t) + R(x_{t-1}, \varepsilon_t)\varepsilon_{t-1},$$

where x_t is a vector of endogenous variables. Matrices $T, C,$ and R are state-dependent since they are functions of lagged states in x_{t-1} and current shocks ε_t. Note that in the regime outside the ZLB ("normal times") $C(.) = 0$.

When estimating shock contributions in a non-linear model, additivity of shocks is only a special case. To observe the estimated contributions under the estimated regimes, i.e. without changing the regime sequence, we distribute any non-zero $C(.)$ to the (group of) shocks according to their individual contributions, ε_t. This procedure has the advantage of maintaining the estimated regime sequence. Otherwise, simulating the contribution of each (group of) shocks would alter the implied regime sequence, and the shock contributions in this simulated "artificial" state of the world. Because $C(.)$ captures the non-additive properties of non-linear shock decompositions, this approach allows us to represent the resulting shock decompositions in an additive way – as done in the main text.[12]

B.2 Data

Data for the EA (quarterly national accounts, fiscal aggregates, quarterly interest and exchange rates) are taken from Eurostat. The estimation uses the following time series (1991q1 – 2018q4) for the EA: GDP, GDP deflator, TFP trend, output gap, population, employment (hours worked), active population rate, relative prices with respect to GDP deflator (VAT consumption, government consumption, total investment, government investment, export, import), the policy rate, and nominal shares of GDP (consumption, government consumption, investment, government investment, government transfers, government debt, wage bill, exports, trade balance). We also observe the first observation point (1999q1) of capital stock and international investment positions to provide a reasonable starting point. The list of observables also includes the effective exchange rate of the EA. For the RoW we use data on population, GDP, GDP trend, GDP deflator and the nominal interest rate.

Series for GDP and prices in the RoW starting in 1999 are constructed on the basis of data for the following 59 countries: Albania, Algeria, Argentina, Armenia, Australia, Azerbaijan, Belarus, Brazil, Bulgaria, Canada, Chile, China, Colombia, Croatia, Czech Republic, Denmark, Egypt, Georgia, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Israel, Japan, Jordan, Korea, Lebanon, Libya, FYR Macedonia, Malaysia, Mexico, Moldova, Montenegro, Morocco, New Zealand, Nigeria, Norway, Philippines, Poland, Romania, Russia, Saudi Arabia, Serbia, Singapore, South Africa, Sweden, Switzerland, Syria, Taiwan, Thailand, Tunisia, Turkey, Ukraine, United Arab Emirates, United Kingdom, United States, and Venezuela. The RoW data are annual data from the IMF IFS and WEO databases.

[12] Alternative additive representations require the researcher to specify an ordering of shocks to be "marginalized" (see Guerrieri and Iacoviello, 2017).

B.3 Calibrated parameters

We calibrate the model so that steady-state ratios of main economic aggregates to GDP match average historical ratios for the EA and the RoW.

Table B.1: Selected calibrated parameters

Monetary Policy	
Nominal interest rate in SS	0.004
CPI inflation in SS	0.005
Interest rate persistence	0.92
Response to inflation	2.28
Response to output gap	0.11
Preferences	
Intertemporal discount factor	0.998
Savers share	0.67
Import share in consumption	0.10
Import share in investment	0.15
Import share in exports	0.14
Weight of disutility of labuor	4.23
Production	
Cobb-Douglas labour share	0.65
Depreciation of private capital stock	0.014
Share of oil in total output	0.04
Linear capacity utilization adj. costs	0.03
Value-added demand elasticity	7.32
Fiscal policy	
Consumption tax	0.20
Corporate profit tax	0.30
Labour tax	0.43
Deficit target	0.03
Debt target	3.03

B.4 Posterior parameter estimates

The posterior estimates of key model parameters are reported in Table B.2.

	Prior distribution			Posterior distribution		
	Distr.	Mean	Std.	Mean	10%	90%
Preferences						
Consumption habit persistence	B	0.5	0.1	0.89	0.85	0.95
Risk aversion	G	1.5	0.2	1.63	1.34	2.02
Inverse Frisch elasticity of labour supply	G	2.5	0.5	2.60	2.23	3.61
Import price elasticity	G	2	0.4	1.90	1.33	2.15

Oil price elasticity	G	0.5	0.08	0.40	0.34	0.58
Nominal and real frictions						
Price adjustment cost	G	60	40	32.27	10.04	47.56
Nominal wage adjustment cost	G	15	3	15.86	3.11	18.85
Real wage rigidity	B	0.95	0.02	0.93	0.92	0.99
Employment adjustment cost	G	60	40	1.87	1.02	70.53
Capacity Utilization quadratic adj cost	G	0.03	0.012	0.011	0.007	0.049
Investment adjustment cost	G	60	40	225.54	77.12	335.46
Share of forward looking price setters	B	1	0.05	0.99	0.92	1.00
Share of forward looking wage setters	B	0.5	0.2	0.84	0.01	0.90
Fiscal policy						
Lump sum taxes persistence	B	0.85	0.06	0.91	0.84	0.96
Lump sum taxes response to deficit	B	0.03	0.008	0.03	0.02	0.04
Demand shock processes						
Subjective discount factor - AR(1) coeff.	B	0.5	0.2	0.89	0.78	0.94
Subjective discount factor - std. (%)	G	0.02	0.012	0.005	0.003	0.016
Investment risk prem. - AR(1) coeff.	B	0.5	0.2	0.89	0.84	0.94
Investment risk prem. - std. (%)	G	0.0075	0.004	0.007	0.003	0.011

Table B.2: Priors and posteriors of key model parameters

Appendix C: Model description

C.1 EA production

Each firm $i \in [0;1]$ produces a variety of the domestic good, which is an imperfect substitute for varieties produced by other firms. Given imperfect substitutability, firms are monopolistically competitive in the goods market and face a downward-sloping demand function for goods. Domestic final good producers then combine the different varieties into a homogenous good and sell them to domestic final demand goods producers and exporters.

Differentiated goods are produced using total capital, K_{it-1}^{tot}, and labour, N_{it}, which are combined in a Cobb-Douglas production function:

$$Y_{it} = [A_t^Y\ (N_{it})]^{\alpha}\left(cu_{it}\ K_{it-1}^{tot}\ \right)^{1-\alpha}$$

Where α is the steady-state labour share, A_t^Y is labour-augmenting productivity common to all firms in the differentiated goods sector, and cu_{it} is a level of capital utilization. Total capital, K_{it}^{tot}, is the sum of private installed capital, K_{it}, and public capital, K_{it}^G. Total Factor Productivity, TFP_t, can therefore be defined as:

$TFP_t = (A_t^Y)^{\alpha}$.

Since TFP is not a stationary process, we allow for two types of shocks that are related to a non-stationary process and its autoregressive component:

$$log(\overline{A}_t^Y) - log(\overline{A}_{t-1}^Y) = g_t^{\overline{AY}} + \varepsilon_t^{L\overline{AY}}$$

$$g_t^{\overline{AY}} = \rho^{\overline{AY}} g_{t-1}^{\overline{AY}} + \left(1-\rho^{\overline{AY}}\right) g^{\overline{AY0}} + \varepsilon_t^{g\overline{AY}}$$

where $g^{\overline{AY}}$ and $g^{\overline{AY0}}$ are the time-varying growth and the long-run growth of technology, respectively, and $\varepsilon_t^{L\overline{AY}}$ is a permanent technology shock.

The monopolistically competitive producers maximize the real value of the firm, V_t, equal to a discounted stream of future dividends d_t, $V_t = d_t + E_t[sdf_{t+1}V_{t+1}]$, with the stochastic discount factor:

$$sdf_t = (1 + i^s_{t+1})/(1 + \pi^y_{t+1}) \approx (1 + i^{rf}_t + rprem^s_t)/(1 + \pi^y_{t+1})$$

which depends directly on the investment risk premium, $rprem^s_{kt}$. The dividends are defined as:

$$d_{it} = (1 - \tau^K)\left(\frac{P^Y_{it}}{P^Y_t}Y_{it} - \frac{W_t}{P^Y_t}N_{it}\right) + \tau^K\delta\frac{P^I_t}{P^Y_t}K_{it-1} - \frac{P^I_t}{P^Y_t}I_{it} - adj_{it}$$

where I_{it} is physical investment, P^I_t is the investment price, τ^K is the corporate tax, δ is the capital depreciation rate. Following Rotemberg (1982), firms face quadratic adjustment costs, adj_{it}, Adjustment costs are associated with the output price, P^Y_{it}, and labour input, N_{it}, adjustment or moving capacity utilization, cu_{it}, and investment, I_{it}, away from their optimal level:

$$adj_{it} = adj(P^Y_{it}) + adj(N_{it}) + adj(cu_{it}) + adj(I_{it})$$

where

$$adj^{PY}_{it} = \frac{\gamma^p}{2}Y_t\left(\frac{P^Y_{it}}{P^Y_{it-1}} - 1\right)^2$$

$$adj^N_{it} = \frac{\gamma^n}{2}Y_t\left(\frac{N_{it} - FN_{it}}{N_{it-1} - FN_{it-1}} - 1\right)^2$$

$$adj^{cu}_{it} = \frac{P^I_t}{P^Y_t}K_{it-1}\left(\gamma^{u,1}(cu_{it} - 1) + \frac{\gamma^{u,2}}{2}(cu_{it} - 1)^2\right)$$

$$adj^I_{it} = \frac{P^I_t}{P^Y_t}\left(\frac{\gamma^{I,1}}{2}K_{t-1}\left(\frac{I_{it}}{K_{t-1}} - \delta\right)^2 + \frac{\gamma^{I,2}}{2}\frac{(I_{it} - I_{it-1})^2}{K_{t-1}}\right)$$

where γ-s capture the degree of adjustment costs. The maximization is subject to the production function, standard capital accumulation equation, $K_{it} = (1 - \delta)K_{it-1} + I_{it}$, and the usual demand condition that inversely links demand for variety i goods and the price of the variety:

$$Y_{it} = \left(\frac{P^Y_{it}}{P^Y_t}\right)^{-\sigma^y} Y_t$$

The usual equality between the marginal product of labour and labour cost holds, with a wedge driven by the labour adjustment costs:

$$\mu^y_t\alpha\frac{Y_t}{N_t} - adj^N_{it} = (1 - \tau^k)\frac{W_t}{P^Y_t}$$

with μ^y_t being inversely related to the price mark-up. The capital optimality condition reflects the usual dynamic trade-off faced by the firm:

$$\frac{1 + \pi^y_{t+1}}{1 + i^s_{t+1}}\frac{P^I_{t+1}/P^Y_{t+1}}{P^I_t/P^Y_t}\left(\mu^y_{t+1}(1 - \alpha)\frac{P^Y_{t+1}Y_{it+1}}{P^I_{t+1}K^{tot}_{it}} + \tau^k\delta - \frac{adj^{cu}_t}{K_{it}} + (1 - \delta)Q_{t+1}\right) = Q_t$$

where Q_t has the usual Tobin's interpretation.

FOC w.r.t. investment implies that Tobin's Q varies due to the existence of investment adjustment costs:

$$Q_t = 1 + adj^I_{it}$$

Firms adjust their capacity utilization depending on the conditions on the market via the optimality condition:

$$\frac{\mu_t^y}{P_t^I/P_t^Y}(1-\alpha)\frac{Y_t}{cu_t} = adj_{it}^{cu}$$

Finally, the FOC w.r.t. differentiated output price pins down the price mark-up:

$$\frac{\sigma^y}{(\sigma^y-1)}\mu_t^y = \left(1-\tau^k\right) + \frac{adj_{it}^{PY}}{(\sigma^y-1)} + \varepsilon_t^\mu$$

with ε_t^μ being the mark-up shock. The latter equation, combined with the FOC w.r.t. labour implies the Phillips curve of the familiar form:

$$\gamma_t^y\sigma^Y = \left(1-\tau^k\right)(\sigma^y-1) + \gamma^P\sigma^Y\frac{P_t^Y}{P_{t-1}^Y}[\pi_t^Y-\bar{\pi}] - \gamma^P\sigma^y\left[\frac{1+\pi_{t+1}^Y}{1+i_{t+1}^s}\frac{P_{t+1}^Y}{P_t^Y}\frac{Y_{t+1}}{Y_t}(\pi_{t+1}^Y-\bar{\pi})\right] + \sigma^y\varepsilon_t^\mu$$

where ε_t^μ is the inverse of the price mark-up shock.

C.2 Households

The household sector consists of a continuum of households $j \in [0;1]$. There are two types of households, savers ("Ricardians", superscript *s*) who own firms and hold government and foreign bonds and liquidity-constrained households (subscript *c*) whose only income is labour income and who do not save. The share of savers in the population is ω^s.

Both households enjoy utility from consumption C_{jkt}^r and incur disutility from labour N_{jkt}^r ($r = s, c$). On top of this, Ricardian utility also depends on the financial assets held. Date *t* expected lifetime utility of household *r*, is defined as:

$$U_{jkt}^r = \sum_{s=t}^{\infty} exp(\varepsilon_{kt}^c)\beta^{s-t}\, u_{jkt}^r(\cdot),$$

where β is the (non-stochastic) discount factor (common for both types of households) and ε_{kt}^c is a saving shock.

C.2.1. Ricardian households

The Ricardian households work, consume, own firms and receive nominal transfers T_{jkt}^s from the government. Ricardians are the only households with full access to financial markets. Their financial wealth of household *j* consists of bonds and shares, where P_{kt}^S is the nominal price of shares in *t*, B_{jkt-1}^S the number of shares held by the household and $P_{kt}^{C,vat}$ is the consumption price, including VAT. The period *t* budget constraint of a saver household *j* is:

$$\begin{aligned}\left(1-\tau_k^N\right)W_{kt}N_{jkt}^s + \left(1+i_{kt-1}^g\right)B_{jkt-1}^g + \left(1+i_{lt-1}^{bw}\right)e_{lkt}B_{jlkt-1}^w + \left(1+i_{t-1}^{rf}\right)B_{jkt-1}^{rf} \\ + \left(P_{kt}^S + P_{kt}^Y d_{kt}\right)B_{jkt-1}^S + T_{jkt}^s - tax^s{}_{jkt} = P_{kt}^{C,vat}C_{jkt}^s + A_{jkt} + adj_t^W,\end{aligned}$$

where W_{kt} is the nominal wage rate, N_{jkt}^s is the employment in hours, and τ_k^N the labour tax rate. B_{jkt-1}^g, B_{jkt-1}^{rf} and B_{jlkt-1}^w are domestic government bonds, foreign bonds of region *l*, and risk-free bonds with returns i_{kt-1}^g, i_{lt-1}^{bw} and i_{t-1}^{rf}, respectively. P_{kt}^Y, is the GDP price deflator. e_{lkt} denotes the bilateral exchange rate.[13] T_{jkt}^s are government transfers to savers and T_{jkt}^s are lump-sum taxes paid by savers. Intermediate goods producers pay dividends $P_{kt}^Y d_{kt}$ to savers. adj_t^W denotes wage adjustment costs.

We define the gross nominal return on domestic shares as:

$$1 + i_{kt}^s = \frac{P_{kt}^S + P_{kt}^Y d_{kt}}{P_{kt-1}^S}$$

[13]Note that $e_{kkt} \equiv 1$.

The instantaneous utility functions of savers, $u^s(\cdot)$, is defined as:

$$u^s_{jkt}\left(C^s_{jkt}, N^s_{jkt}, \frac{U^A_{jkt-1}}{P^{c,vat}_{kt}}\right) = \frac{1}{1-\theta_k}\left(C^s_{jkt} - h_k C^s_{kt-1}\right)^{1-\theta_k} - \frac{\omega^N_k \varepsilon^U_{kt}}{1+\theta^N_k}\left(C_{kt}\right)^{1-\theta_k}\left(N^s_{jkt}\right)^{1+\theta^N_k} - \left(C^s_{kt} - h_k C^s_{kt-1}\right)^{-\theta_k}\frac{U^A_{jkt-1}}{P^{c,vat}_{kt}},$$

where $C^s_{kt} = \int_0^1 C^s_{jkt}\,dj$, and $C_{kt} = \omega^s C^s_{kt} + (1-\omega^s)C^c_{kt}$; $h_k \in (0;1)$ measures the strength of external habits in consumption, ω^N_k the weight of the disutility of labour, and ε^U_{kt} captures a labour supply (or wage mark-up) shock.

The disutility of holding risky financial assets, U^A_{jkt-1}, is defined as:

$$U^A_{jkt-1} = \sum_Q B^Q_{jkt-1}\left(\alpha^Q_k + \varepsilon^Q_{kt-1}\right).$$

The asset-specific risk premium shock depends on an asset-specific exogenous shock $\varepsilon^Q, Q \in \{B, S, bw\}$ (bonds, stocks, and foreign assets) and an asset-specific intercept α^Q.[14] Similar to Krishnamurthy and Vissing-Jorgensen (2012) and Fisher (2015), the approach of modelling the disutility of holding risky assets captures the households' preferences for the safe short term bonds, which generates an endogenous wedge between the return on risky assets and safe bonds.[15] As in Benigno (2009) and Ratto et al. (2009), we assume that only the RoW bond is traded internationally. It follows that households in the Euro Area can invest in both national and foreign assets.

The Ricardian household problem leads to the following first-order conditions (FOC):

The FOC w.r.t. savers' consumption produces:

$$\varepsilon^C_{kt}(C^s_{kt} - hC^s_{kt-1})^{-\theta} = \lambda^s_{kt},$$

where λ^s_{kt} is the Lagrange multiplier on the budget constraint.

The stochastic discount factor follows: $\Lambda^s_{t,t+1} = \beta E_t\left[\frac{\lambda^s_{kt+1}}{\lambda^s_{kt}}\right]$

The FOC w.r.t. domestic risk-free bond:

$$\beta E_t\left[\frac{\lambda^s_{kt+1}}{\lambda^s_{kt}}\frac{1+i^{rf}_{kt}}{1+\pi^{C,vat}_{kt+1}}\right] = 1$$

The FOC w.r.t. domestic government bonds:

$$\beta E_t\left[\frac{\lambda^s_{kt+1}}{\lambda^s_{kt}}\frac{1+i^g_{kt} - \varepsilon^B_{kt} - \alpha^{b0}_{kk}}{1+\pi^{C,vat}_{kt+1}}\right] = 1$$

with $\pi^{C,vat}_{kt}$ the consumption deflator inflation rate and ε^B_{kt} the risk-premium on government bonds.

The FOC w.r.t. domestic stocks:

$$\beta E_t\left[\frac{\lambda^s_{kt+1}}{\lambda^s_{kt}}\frac{(1+i^s_{kt+1}) - \varepsilon^S_{kt} - \alpha^{s0}_{kk}}{1+\pi^{C,vat}_{t+1}}\right] = 1$$

[14] Internationally traded bonds are also subject to transaction costs in the form of a function of the average net foreign asset position relative to GDP.

[15] This modification is along the lines of the money-in-utility approach by Sidrauski (1967), in which model agents derive utility from their holdings of money. In our model, it reflects the costs of holding risky assets relative to risk-free assets. A similar framework is used by Vitek (2017).

where ε_{kt}^{S} is the risk premium on stocks. The above optimality conditions are similar to a textbook Euler equation, but incorporate asset-specific risk premia that depend on an exogenous shock ε_{kt}^{A} as well as the size of the asset holdings as a share of GDP, see Vitek (2017) for a similar formulation. Taking into account the Euler equation for the risk-free bond and approximating, the equations simplify to the familiar expressions:

$$i_{kt}^{g} = i_{kt}^{rf} + rprem_{kt}^{g}$$

$$i_{kt}^{s} = i_{kt}^{rf} + rprem_{kt}^{s}$$

In the equations above, $rprem_{kt}^{g}$ is the risk premium on domestic government bonds, and $rprem_{kt}^{s}$ is a risk premium on domestic shares. It is introduced to capture, in a stylized manner, financial frictions that are commonly believed to have contributed to the first phase of the financial crisis and may have contributed to its second phase.[16]

Given the monetary union setting, we assume that an uncovered interest rate parity condition links the interest rate of an EMU country to the EA interest rate (set by the central bank):

$$\left(1 + i_{kt}^{rf}\right) = (1 + i_{EAt}) - \left(\alpha_{k}^{bw1} \frac{e_{RoW,EA,t} B_{kt}^{W}}{P_{kt}^{Y} Y_{kt}} \varepsilon_{kt}^{FQ}\right)$$

where $\alpha_{k}^{bw1} \frac{e_{RoW,EA,t} B_{kt}^{W}}{P_{kt}^{Y} Y_{kt}}$ captures a debt-dependent country risk premium on net foreign asset holdings, as external closure to ensure long-run stability (see, e.g., Schmitt-Grohe and Uribe, 2003; Adolfson et al., 2008). Following Smets and Wouters (2007), we also introduce an additional risk premium shock, ε_{kt}^{FQ} (‘Flight-to-safety’), which creates a wedge between the EA interest rate, i_{EAt}, and the return on domestic risk-free assets, i_{kt}^{rf}. Since a positive shock increases the required return on domestic assets and the cost of capital, it reduces current consumption and investment simultaneously and helps to explain the co-movement of consumption and investment.

The instantaneous utility functions for liquidity-constrained households. $u^{c}(\cdot)$, is defined as:

$$u_{jkt}^{c}\left(C_{jkt}^{c}, N_{jkt}^{c}\right) = \frac{1}{1-\theta_{k}}\left(C_{jkt}^{c} - h_{k} C_{kt-1}^{c}\right)^{1-\theta_{k}} - (C_{kt})^{1-\theta_{k}} \frac{\omega_{k}^{N} \varepsilon_{kt}^{U}}{1+\theta_{k}^{N}} \left(N_{jkt}^{c}\right)^{1+\theta_{k}^{N}}$$

with $C_{kt}^{c} = \int_{0}^{1} C_{jkt}^{c}\, dj$.

while RoW households can only invest in domestic bonds.

C.2.2. Liquidity-constrained household

The liquidity-constrained household consumes her disposable after-tax wage and transfer income in each period ('hand-to-mouth'). The period *t* budget constraint of the liquidity-constrained household is:

$$\left(1 + \tau_{k}^{C}\right) P_{kt}^{C} C_{jkt}^{c} = \left(1 - \tau_{k}^{N}\right) W_{kt} N_{kt}^{c} + T_{kt}^{c} - {tax^{c}}_{jkt}.$$

C.2.3. Wage setting

Households provide differentiated labour services, N_{jkt}^{r}, in a monopolistically competitive market. A labour union bundles labour hours provided by both types of domestic households and resells homogeneous labour services to intermediate goods producing firms.[17] The resulting wage rule equates a weighted average of the marginal utility of leisure to a weighted average of the marginal utility of consumption times the real wage adjusted for a wage mark-up. Adjustment costs allow for nominal

[16] Observationally, this approach is equivalent to exogenous risk premia as well as risk premia derived in the spirit of Bernanke et al. (1996).

[17] Since both households face the same labour demand schedule, each household works the same number of hours as the average of the economy. It follows that the individual union's choice variable is a common nominal wage rate for both types of households.

wage rigidities. The household bears the wage adjustment costs. We also allow for real wage rigidity, as in Blanchard and Galí (2007).

The optimality condition for the equilibrium wage is given by:

$$\left(\mu_k^w \frac{U_{kt}^N}{\lambda_{kt}} \frac{P_{kt}^{C,vat}}{P_{kt}^Y}\right)^{1-\gamma_k^{wr}} \left(\left(1-\tau_k^N\right)\frac{W_{kt-1}}{P_{kt-1}^Y}\right)^{\gamma_k^{wr}}$$
$$= \left(1-\tau_k^N\right)\frac{W_{kt}}{P_{kt}^Y} + \gamma_k^w\left(\frac{W_{kt}}{W_{kt-1}} - 1 - (1-sf_k^w)\left(\pi_{kt-1}^y - \bar{\pi}\right) - \pi^w\right)\frac{W_{kt}}{W_{kt-1}}\frac{W_{kt}}{P_{kt}^Y}$$
$$- \gamma_k^w E_t\left[\tilde{\beta}_{kt}\frac{\lambda_{kt+1}}{\lambda_{kt}}\frac{N_{kt+1}}{N_{kt}}\frac{P_{kt}^{C,vat}}{P_{kt+1}^{C,vat}}\left(\frac{W_{kt+1}}{W_{kt}} - 1 - (1-sf_k^w)\left(\pi_{kt}^y - \bar{\pi}\right)\right.\right.$$
$$\left.\left.- \pi^w\right)\frac{W_{kt+1}}{W_{kt}}\frac{W_{kt}}{P_{kt}^Y}\right] + \varepsilon_{kt}^U\frac{W_{kt}}{P_{kt}^Y}$$

where μ_k^w is the wage mark-up, γ_k^{wr} is the degree of real wage rigidity, γ_k^w is the degree of nominal wage rigidity and sf_k^w is the degree of forward-looking behaviour in the labour supply equation. U_{kt}^N is the marginal disutility of labour and defined as:

$$U_{kt}^N = \omega_k^N (C_{kt})^{1-\theta_k}(N_{kt})^{\theta_k^N}.$$

C.2.4. Trade: Import sector

The EA MS final aggregate demand component goods, C_t (private consumption good), I_t, (private investment good), G_t (government consumption good), and I_t^G (government investment good), as well as X_t (export good) are produced by perfectly competitive firms by combining domestic output, Y_t^Z, with imported goods, M_t^Z, where $Z = \{C, I, G, I^G, X\}$, using the following CES technology:

$$Z_t = A_t^{p^z}\left[(1-\varepsilon_t^M s^{M,Z})^{\frac{1}{\sigma^z}}(Y_t^Z)^{\frac{\sigma^z-1}{\sigma^z}} + (\varepsilon_t^M s^{M,Z})^{\frac{1}{\sigma^z}}(M_t^Z)^{\frac{\sigma^z-1}{\sigma^z}}\right]^{\frac{\sigma^z}{\sigma^z-1}}$$

where $A_t^{p^z}$ is a shock to productivity in the sector producing goods, ε_t^M is a shock to the share, $s^{M,Z}$, of good-specific import demand components, and σ^z is the import elasticity of substitution between goods varieties. It follows that the demand for Y_t^Z and imported goods M_t^Z are given by:

$$Y_t^Z = \left(A_t^{p^z}\right)^{\sigma^z-1}(1-\varepsilon_t^M s^{M,Z})\left(\frac{P_t^Y}{P_t^Z}\right)^{-\sigma^z} Z_t$$

$$M_t^Z = \left(A_t^{p^z}\right)^{\sigma^z-1}\varepsilon_t^M s^{M,Z}\left(\frac{P_t^M}{P_t^Z}\right)^{-\sigma^z} Z_t$$

where P_t^Y and P_t^M are the price deflators associated with Y_t^Z and M_t^Z, respectively, and the total final good deflator P_t^Z is such that:

$$P_t^Z = \left(A_t^{p^z}\right)^{-1}\left[(1-\varepsilon_t^M s^{M,Z})(P_t^Y)^{1-\sigma^z} + \varepsilon_t^M s^{M,Z}(P_t^M)^{1-\sigma^z}\right]^{\frac{1}{1-\sigma^z}}$$

Economy-specific final imports demand

Final imported goods, M_t, are produced by perfectly competitive firms combining economy-specific homogenous goods, M_t^*, using a CES production function:

$$M_t = \left[(s^M)^{\frac{1}{\sigma^{FM}}}(M_t^*)^{\frac{\sigma^{FM}-1}{\sigma^{FM}}}\right]^{\frac{\sigma^{FM}}{\sigma^{FM}-1}}$$

where σ^{FM} is the price elasticity of demand for RoW goods. The demand for goods from RoW is then:

$$M_t^* = s^M \left(\frac{P_{RoWt}^M}{P_t^M}\right)^{-\sigma^{FM}} M_t$$

while the import price:

$$P_t^M = \left[s^M (P_{RoWt}^M)^{1-\sigma^{FM}}\right]^{\frac{1}{1-\sigma^{FM}}}$$

with P_{RoWt}^M being the country-specific (RoW) import goods prices. Since all products from RoW are initially purchased at export price P_{RoWt}^X, the economy-specific import goods price can be expressed as:

$$P_{RoWt}^M = e_t P_{RoWt}^X$$

C.2.5. Export sector

The exporting firms are competitive and export a good that is a combination of domestic output and import content. The corresponding export price is given by:

$$P_t^X = exp(\varepsilon_t^X)\left[(1 - s^{M,Z})(P_t^Y)^{1-\sigma^z} + s^{M,Z}(P_t^M)^{1-\sigma^z}\right]^{\frac{1}{1-\sigma^z}}$$

where ε_t^X captures an export-specific price shock.

C.2.6 Monetary policy

Monetary policy is modelled by a Taylor rule where the ECB sets the policy rate i_t in response to the annualized inflation gap, $\pi_t^{c,vat,QA}$, and the annualized output gap. The policy rate adjusts sluggishly to deviations of inflation from their respective target level and to the output gap; it is also subject to random shocks:

$$\begin{aligned} i_t - \bar{\imath} = {} & \rho^i (i_{t-1} - \bar{\imath}) \\ & + (1 - \rho^i)\left[\eta^{i\pi} 0.25(\pi_t^{c,vat,QA} - \bar{\pi}^{c,vat,QA}) \right. \\ & \left. + \eta^{iy}\left(log\left(0.25 \sum_{r=1}^{4} Y_{t-r}\right) - log\left(0.25 \sum_{r=1}^{4} Y_{t-r}^{pot}\right)\right)\right] + \varepsilon_t^i \end{aligned}$$

where $\bar{\imath} = r + \bar{\pi}^{Yobs}$ is the steady-state nominal interest rate, equal to the sum of the steady-state real interest rate and GDP inflation. The policy parameters (ρ^i, $\eta^{i\pi}$, η^{iy}) capture interest rate inertia and the response to the annualized inflation gap and output gap, respectively.

C.2.7 Fiscal policy

The government collects taxes on labour, τ^N, capital, τ^K, and consumption, τ^C, as well as lump-sum taxes, and issues short and long-term bonds to finance government consumption, G_t, investment, I_t^G, transfers, T_t , and the servicing of the outstanding debt. The stock of government debt, B_t^G, follows

$$B_t^G = B_{t-1}^G + P_t^G G_t + P_t^{IG} I_t^G + P_t T_t - R_t^G$$

where nominal government revenues, R_t^G, are defined as:

$$R_t^G = \tau^K (P_t Y_t - W_t N_t - P_t^I \delta K_{t-1}) + \tau^N W_t N_t + \tau^C P_t^C C_t + tax_t$$

Government consumption, investment and transfers follow autoregressive processes. Government expenditure and receipts can deviate temporarily from their long-run levels in systematic response to budgetary or business cycle conditions and in response to idiosyncratic shocks.

The government uses lump-sum taxes for budget closure and increases (decreases) taxes when the level of government debt and government deficit is above (below) the debt, $\bar{B}^G$, and deficit target, DEF^T, respectively:

$$\frac{tax_t}{\overline{P_t^Y Y_t}} = \rho^{tax}\left(\frac{tax_{t-1}}{\overline{P_t^Y Y_t}} - \overline{tax}\right) + \eta^{DEF}\left(\frac{\Delta B_{t-1}^G}{P_{t-1}^Y Y_{t-1}} - DEF^T\right) + \eta^B\left(\frac{B_{t-1}^G}{P_{t-1}^Y Y_{t-1}} - \bar{B}^G\right) + \varepsilon_t^{tax}$$

C.2.8 RoW

The model of the RoW block is simplified in structure. Specifically, it consists of a budget constraint for the representative Ricardian household, demand functions for domestic and imported goods (derived from CES consumption good aggregators), a production technology for manufacturing output that uses labour as the only factor of production, a New Keynesian Phillips curve, and a Taylor rule. The RoW block abstracts from capital accumulation. There are shocks to labour productivity, price mark-ups for manufacturing output, the subjective discount rate, the relative preference for domestic vs. imported goods, as well as monetary policy shocks in the RoW.

The budget constraint for the representative household is:

$$P_{RoWt}^Y Y_{RoWt} = P_{RoWt}^C C_{RoWt} + TB_{RoWt}$$

where P_{RoWt}^Y and Y_{RoWt} are the price and volume of RoW final goods output, and TB_{RoWt} is the trade balance.

The intertemporal equation for aggregate demand follows from the FOC for consumption:

$$\beta_t \frac{\lambda_{RoWt+1}}{\lambda_{RoWt}} \frac{1 + i_{RoWt}}{1 + \pi_{RoWt+1}^C} = 1$$

with $\beta_t = e^{\varepsilon_{RoWt}^C}\beta$, $(C_{RoWt} - hC_{RoWt-1})^{-\theta} = \lambda_{RoWt}$ and ε_{RoWt}^C as the RoW demand shock.

Final aggregate demand C_{RoWt} (in the absence of investment and government spending in RoW) is a combination of domestic output, Y_{RoWt}, and imported goods, M_{RoWt}, using the following CES function:

$$C_{RoWt} = A_{RoWt}^p \left[(1 - \varepsilon_{RoWt}^M s^M)^{\frac{1}{\sigma}} \left(Y_{RoWt}^C\right)^{\frac{\sigma-1}{\sigma}} + (\varepsilon_{RoWt}^M s^M)^{\frac{1}{\sigma}} \left(M_{RoWt}^C\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

where ε_{RoWt}^M is a shock to input components and s^M the import share. From profit maximization we obtain the demand for domestic and foreign goods:

$$Y_{RoWt} = \left(A_{RoWt}^p\right)^{\sigma-1} (1 - \varepsilon_{RoWt}^M s^M) \left(\frac{P_{RoWt}^Y}{P_{RoWt}^C}\right)^{-\sigma} C_{RoWt}$$

$$M_{RoWt} = \left(A_{RoWt}^p\right)^{\sigma-1} \varepsilon_{RoWt}^M s^M \left(\frac{P_{RoWt}^M}{P_{RoWt}^C}\right)^{-\sigma} C_{RoWt}$$

where the consumer price deflator, P_{RoWt}^C , is given by:

$$P_{RoWt}^C = \left(A_{RoWt}^p\right)^{-1} [(1 - \varepsilon_{RoWt}^M s^M)(P_{RoWt}^Y)^{1-\sigma} + \varepsilon_{RoWt}^M s^M (P_{RoWt}^M)^{1-\sigma}]^{\frac{1}{1-\sigma}}$$

The intermediate good producers use labour to manufacture domestic goods given a linear production function:

$$Y_{RoWt} = A_{RoWt}^Y N_{RoWt}$$

where A^Y_{RoWt} captures a trend in productivity.

Price setting follows a New Keynesian Phillips curve:

$$\pi^Y_{RoWt} - \bar{\pi}^Y_{RoW} = \beta \frac{\lambda_{RoWt+1}}{\lambda_{RoWt}} (E_t \pi^Y_{RoWt+1} - \bar{\pi}^Y_{RoW}) + \varphi^Y_{RoW} \ln(Y_{RoWt} - \bar{Y}_{RoW}) + \varepsilon^Y_{RoWt}$$

where $\lambda_{RoWt} = (C_{RoWt} - h_{RoW} C_{RoWt-1})^{-\theta}$ is the marginal utility of consumption, and ε^Y_{RoWt} is a cost push shock.

Monetary policy in RoW follows a Taylor-type rule:

$$\begin{aligned} i_{RoWt} - \bar{\iota} &= \rho^i_{RoW}(i_{RoWt-1} - \bar{\iota}) \\ &\quad + (1 - \rho^i_{RoW}) \Bigg[\eta^{i\pi}_{RoW} 0.25 (\pi^{c,vat,QA}_{RoWt} - \bar{\pi}^{c,vat,QA}_{RoW}) \\ &\quad + \eta^{iy}_{RoW} \left(log\left(0.25 \sum_{r=1}^{4} Y_{RoWt-r}\right) - log\left(0.25 \sum_{r=1}^{4} Y^{pot}_{RoWt-r}\right)\right)\Bigg] + \varepsilon^i_{RoWt} \end{aligned}$$

Total nominal exports of final goods are defined as:

$$P^X_{RoWt} X_{RoWt} = P^M_{EAt} M_{EAt}$$

with the export price being defined as the domestic price subject to a price shock:

$$P^X_{RoWt} = \exp(\varepsilon^{PX}_{RoWt})\, P^Y_{RoWt}$$

C.2.9 Closing the model

Market clearing requires that:

$$Y_t P^Y_t = P^C_t C_t + P^I_t I_t + P^{IG}_t IG_t + TB_t$$

where the trade balance, TB_t, is defined as the difference between exports and imports with domestic importers buying the imported good at the price P^X_{RoWt}:

$$TB_t = P^X_t X_t - e_t P^X_{RoWt} M_t$$

Exports of EA are given by:

$$X_t = M_{RoWt}$$

where M_{RoWt} stands for imports of RoW from the EA. Imports are defined as:

$P^M_t M_t = P^M_t (M^C_t + M^I_t + M^G_t + M^{IG}_t)$.

Net foreign assets, B^*_t, evolve according to:

$$e_t B^*_t = (1 + i^*_{t-1}) e_t B^*_{t-1} + TB_t + ITR\, \overline{P^Y_t Y_t}$$

Since we allow for a non-zero trade balance in the steady-state, we include an international transfer, ITR, calibrated to satisfy zero NFA in equilibrium.

Finally, the net foreign assets of each country globally sum to zero:

$$NFA_t + NFA_{RoWt} = 0$$